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The Effects of a Varied Method of Instruction on Student Achievement, Transfer, Situational Interest, and Course Retention Rates in Community College Developmental Mathematics

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Dissertation Abstract

The Effects of a Varied Method of Instruction on Student Achievement, Transfer, Situational Interest, and Course Retention Rates in Community College Developmental Mathematics

The purpose of this quasi-experimental study was to compare the effects of a varied method of instruction on student achievement, knowledge transfer, situational interest, and course retention rates, relative to a non-varied method of instruction, in community college developmental mathematics. The varied method of instruction consisted of active learning teaching practices with foundations in social constructivism, whereas the non-varied method of instruction was founded in Cognitive Load Theory and consisted primarily of explicit instruction and individual practice.

An initial sample of 139 students who enrolled in six sections of Beginning Algebra at an urban community college in Northern California participated in the study. Given the quasi-experimental nature of the study, considerable effort was taken to control for school, teacher, student, and curriculum implementation variables. As such, the six sections were divided equally among three instructors, with each instructor teaching one varied class and one non-varied class. Additionally, students were assessed on the following entry characteristics: preferences for working in groups, personal interest in mathematics, reasoning ability, verbal ability, and prior mathematics knowledge.

The dependent variables were conceptual understanding, procedural application near transfer, far transfer, situational interest, and course retention rates. Conceptual understanding and procedural application were assessed three times throughout the study, whereas the remaining variables were measured after eight weeks of instruction.

No statistically significant differences in conceptual understanding, procedural application, near transfer, far transfer, or course retention rates were obtained between the varied and non-varied classes while controlling for individual differences. There was a statistically significant difference of medium effect in situational interest; the students in the varied classes enjoyed their classes to a lesser extent than students in the non-varied classes.

Overall, both methods of instruction were equally ineffective in teaching basic algebraic concepts and procedures. Therefore, it appears that manipulating methods of instruction is not an adequate solution to the high failure rates in developmental mathematics. Instead, developmental mathematics education may better benefit from other reforms, such as learning communities, contextualized curricula, and mandatory support services. Future studies may be conducted to investigate the effects of these reforms, both in isolation and in combination.

This dissertation, written under the direction of the candidate's dissertation committee and approved by the members of the committee, has been presented to and accepted by the Faculty of the School of Education in partial fulfillment of the requirements for the degree Doctor of Education. The content and research methodologies presented in this work represent the work of the candidate alone.

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CHAPTER I

STATEMENT OF THE PROBLEM

The College Board of Mathematical Sciences estimated that over 2 million students were enrolled in a mathematics course at a community college in 2010, approximately 57% of which enrolled in pre-collegiate courses, such as Prealgebra, Geometry, and Algebra (Blair, Kirkman, & Maxwell, 2013). Pre-collegiate mathematics courses such as these are often labeled as developmental because their purpose is to provide a review of basic mathematics concepts that are usually included in a K-12 curriculum. Despite the relatively low complexity of material, however, success rates within developmental courses are very low; failure rates have been reported to be as high as 70% (Attewell, Lavin, Domina, & Levey, 2006; Bahr, 2008).

The high failure rates in developmental mathematics courses is particularly troubling given that these courses contain a disproportionate amount of underrepresented students; using a nationally representative sample of 8th graders collected by the U.S. Department of Education, Bailey, Jenkins, and Leinbach (2005) estimated that of all first-year community college African American and Latino students, 76% and 78% enrolled in developmental courses, respectively, compared to 55% of Caucasian students. Additionally, they also observed that approximately 33% of entering community college students were first-generation students. These data imply that community colleges are serving as an outlet for underrepresented and first-generation students who enroll in developmental courses at relatively high rates.

In another study using the same data set, Adelman (2005) attempted to identify enrollment patterns in post-secondary education with respect to age, socioeconomic status, and gender. Adelman discovered that approximately 40% of traditionally-aged students (ages 16-20) began their post-secondary education at community college, compared to 65% of students over the age of 24. Additionally, within the group of non-traditionally aged students, approximately 80% of them did not transfer to four-year institutions. From these data it may be inferred that community colleges are also serving as an outlet for older students whose goals might be vocational as opposed to transferring to four-year institutions. With respect to socioeconomic status and gender, no statistically significant differences in enrollment between collegiate and pre-collegiate courses were observed.

Findings similar to those reported by Bailey et al. (2005) and Adelman (2005) were later published in 2006 by the American Mathematical Association of Two Year Colleges (AMATYC) in which an estimated 33% of the total enrollment was comprised of students of color. Further, AMATYC reported that the average age of a community college student was approximately 29 years, and about 61% of all students were enrolled part-time; 80% reported having part-time jobs, and 41% reported having full-time jobs. Together, the enrollment and demographic data presented by Bailey et al., Adelman, and AMATYC indicated that student characteristics within developmental mathematics classes vary widely with respect to ethnicity, age, goals, and socioeconomic status. Further, students of color are enrolling in developmental courses at disproportionate rates, which motivates an investigation into how developmental education may be improved to increase access to college-level courses for all students.

The problem of low success rates in developmental mathematics has been investigated from a variety of angles. For example, several studies have attempted to predict student success by investigating affective factors, such as self-regulation, self-efficacy, and attitudes toward mathematics at community colleges (Cortes-Suarez, 2008; Otts, 2011; Subocz, 2008). Additionally, given that research has long suggested that success cannot be attributed entirely to student factors (Gates & Creamer, 1984), research has also been conducted regarding the effects of centralized developmental programs and levels of administrative support on student success in developmental mathematics (Boylan, 2009; Center for Student Success, 2005). Overall, developmental education is complex and may be investigated from many perspectives.

Researchers have put forth several explanations for the poor success rates in developmental mathematics, including a lack of teacher training in remedial education (Boylan, 2002; Epper & Baker, 2009), as well as negative teacher attitudes toward the abilities of developmental students (Roueche & Wheeler, 1973). Additionally, researchers have suggested that a misalignment between high school and community college mathematics curricula is partly to blame (McCabe, 2003). Lastly, from a pedagogical perspective, researchers have also proposed that student failure may be attributed to ineffective instructional practices (Grubb & Gabriner, 2013). All together, these studies illustrate that there are many possible causes for the low success rates within developmental mathematics courses.

Relative to methods of instruction, Kaestle, Campbell, Finn, Johnson, and Mikulecky (2001), as well as McCabe (2003), have determined that secondary mathematics instruction has been largely unsuccessful; seemingly, high school students are falsely learning that mathematics is a subject that may be mastered simply by memorizing discrete facts without any reinforcement in qualitative literacy or problem-solving skills (Stigler, Givvin, & Thompson, 2010). Moreover, in conjunction with the findings by Grubb and Gabriner (2013), it appears that a similar instructional method is being replicated in many developmental mathematics classrooms, which could be viewed as a reason for the high student failure rates.

In terms of possible solutions, researchers have observed that successful developmental mathematics programs used a variety of instructional practices, as opposed to the traditional lecture-seatwork method observed by Grubb and Gabriner (2013). For example, Boylan (2002) and his team studied 36 community colleges nationwide that were successful in delivering developmental mathematics education and discovered that courses were organized into learning communities and used a variety of instructional practices, such as small-group work, peer reviews, and whole-class discussions.

Along the same lines, Epper and Baker (2009) observed over a dozen community colleges deemed to be successful in delivering remedial mathematics in order to discover trends in effective developmental mathematics programs. Their findings were consistent with the conclusions made by Boylan (2002). A review of the successful programs yielded several common characteristics, including the implementation of learning communities, contextualization of mathematics curricula, implementation of project-based learning, and the integration of a variety of instructional practices. Similarly, in a quasi-experimental design, Fowler and Boylan (2010) observed positive effects on student outcomes from the implementation of an innovative developmental mathematics program that utilized a variety of instructional practices in conjunction with several other reforms, including mandatory orientation classes and academic advising.

Considered jointly, these findings imply that a varied method of instruction, which is a method that implements a variety of instructional practices in addition to traditional lecturing and individual practice, is a key component of successful developmental mathematics programs. However, two of the aforementioned studies were not experimental studies, and the other was poorly controlled, in which selection effects, teacher effects, and curricular effects could have been operating simultaneously, resulting in the inability to identify a causal link between instructional methods and student outcomes. For example, in the study by Fowler and Boylan (2010), students received a varied method of instruction in conjunction with orientation courses, peer tutoring services, and mandatory advising. Therefore, the extent to which each individual component contributed to the positive outcomes remains unknown.

Purpose of the Study

Researchers investigating student outcomes in community college developmental mathematics have identified several key successful reforms, such as the implementation of learning communities, contextualized curricula, support services, and a varied method of instruction (Boylan, 2002; Epper & Baker, 2009; Fowler & Boylan, 2010). However, in each study, several of these reforms were operating simultaneously resulting in the inability to ascertain the effects of instructional method on student outcomes. Therefore, the purpose of this study was to investigate the effects of instructional method on student outcomes in developmental mathematics courses while controlling for extraneous variables, such as style of the instructor, learning communities, contextualized curricula, and student support services.

To accomplish this purpose, a two-group comparative quasi-experiment was conducted using six developmental mathematics classes at a local community college: three courses received a varied method of instruction and three courses received a non-varied method of instruction. Given that students were not randomly assigned to these courses, any preexisting differences in student characteristics, such as prior knowledge, reasoning ability, and verbal ability, were investigated. To control for extraneous variables, varied and non-varied classes were taught by the same instructor, and no other reforms were integrated into the courses. Consequently, any differences in student outcomes may be attributed to the method of instruction as opposed to other factors.

Significance of the Study

This study is important for several reasons. First, developmental mathematics courses fill a niche in higher education in that they serve as a channel into post-secondary education for minority, first-generation, and non-traditionally aged students (Adelman, 2005; AMATYC, 2006; Bailey, Jenkins, & Leinbach, 2005). Additionally, researchers hypothesized that at-risk students, such as those typically enrolled in developmental mathematics, would benefit from experiencing a varied method of instruction because it is likely that their previous experience with mathematics in the K-12 system was similar to that described by Grubb and Gabriner (2013), as direct instruction, from which they did not benefit (Goldrick-Rab, 2007; O’Neil, 1990). Therefore, if community college missions of equity and equal opportunity are to be satisfied, then success rates must increase so that students may meet their goals (Perin, 2006). Especially in a changing job market in which it has been speculated that 80% of all new jobs will require some level of post-secondary education (McCabe, 2003).

Second, disentangling pedagogical factors that lead to student success will begin to fill a gap in the existing literature pertaining to developmental mathematics instruction. Specifically, given the common recommendation in the literature to implement a varied method of instruction (AMATYC, 2006; Boylan, 2002; Fowler & Boylan, 2010; Higbee & Thomas, 1999; Levin & Calcagno, 2007), the results from this study may provide empirical evidence in support of, or against, this recommendation. Moreover, such causal links between instruction and student achievement are an understudied area in developmental mathematics (Mesa, 2008); it is believed that of the few studies pertaining to instruction, many are of poor quality and lack operational definitions of instructional treatments (Hiebert & Grouws, 2006; Mesa, 2008). Therefore, this study contributed to the literature on developmental mathematics instruction by including detailed descriptions of teaching practices and investigating the effects of a varied method of instruction on student outcomes while controlling for extraneous variables.

Third, this study is important because the findings may explicate the relative effects of instruction on student outcomes compared to the other reforms observed by Boylan (2002), Epper and Baker (2009), and Fowler and Boylan (2010). For example, if no statistically significant differences in student achievement and course retention rates are observed between treatment groups, then it may be inferred that non-instructional reforms have a stronger impact on achievement than does the method of instruction. This finding would be of practical importance to community college educators and administrators in terms of professional development and resource allocation.

Theoretical Framework

A social constructivist framework was adopted as a theoretical foundation for this study because the recommendations from the literature were to include a variety of instructional practices encouraging student interaction and active involvement in the learning process. Social constructivist perspectives of learning and instruction focus on the interdependence of social and individual processes in the co-construction of knowledge (Palincsar, 1998). Social constructivism is in contrast to behaviorist theories of learning, such as direct instruction, in which the instructor assumes full control of the teaching and learning process (Baumann, 1988).

One main tenet of social constructivism is that social interactions lead to higher levels of reasoning and learning, a claim that has received much empirical support (Bell, Grossen, & Perret-Clermont, 1985; Bereiter & Scardamalia, 1989; Lua, Singh, & Hwa, 2009). From this perspective, as learners participate in a range of interactive activities, they acquire new strategies and gain knowledge from one another (Palincsar, 1998). Social construvtivism is consistent with the literature on developmental mathematics education in which implementing a variety of practices and encouraging student interactions are frequently recommended (AMATYC, 2006; Boylan, 2002).

One set of practices consistent with social constructivism is called active learning. Meyers and Jones (1993) defined active learning to be in contrast to a traditional method of instruction in which instructors actively present information while students passively receive it. Specifically, active learning occurs when students are given opportunities to talk, listen, read, and write about concepts in order to organize and clarify their thinking and create new mental structures (Meyers & Jones, 1993; Millis & Cottell, 1998). To facilitate active learning, researchers have suggested utilizing interactive lecturing and questioning, collaborative and cooperative activities, and writing assignments to encourage the active processing of information among students (AMATYC, 2006; Borich, 2007, Meyers & Jones, 1993).

An additional learning theory pertinent to the current study is Cognitive Load Theory (CLT), which posits that due to the limited capacity of our short-term memory (Miller, 1956), students need to focus their available cognitive resources on activities that are beneficial to learning (Sweller, Van Merrienboer, & Paas, 1998). When presented with material, it is believed that students encounter three types of cognitive load: (1) intrinsic load, which is determined by the complexity of the material; (2) extrinsic load, which is a characteristic of the presentation of the material; and (3) germane load, which is cognitive activity that is beneficial for learning. Evidence suggests that receiving explicit instruction reduces extraneous load, thereby increasing cognitive resources available for germane load (Owen & Sweller, 1985).

Alternatively, unguided problem solving imposes a high cognitive load on students because they need to attend to the current state of the problem, as well as to the desired goal, while simultaneously making decisions about which procedure to apply (Ward & Sweller, 1990). But, when provided with explicit instruction, students may focus on the current problem state and how to apply the correct procedure, rather than searching for the next step in the procedure. Therefore, it may be the case that social constructivist strategies increase the cognitive load experienced by students, which may result in underdeveloped mental structures relative to students receiving the non-varied method of instruction.

Social constructivism and Cognitive Load Theory are both pertinent to the study because they support the teaching practices included in the varied and non-varied methods of instruction. More specifically, the varied method of instruction included teaching practices consistent with both theories, whereas the non-varied method of instruction only consisted of direct instruction, which is entirely consistent with Cognitive Load Theory.

In sum, it is believed that a varied method of instruction is one of many key factors in increasing student success in developmental mathematics (AMATYC, 2006; Boylan, 2002; Epper & Baker, 2009). From a social constructivist perspective, a varied method of instruction may be characterized by opportunities for students to discuss concepts with each other through interactive lecturing, group activities, and sharing of writing assignments (AMATYC, 2006; Meyers & Jones, 1993). Additionally, a varied method of instruction also consists of periods of explicit instruction, which may be used to decrease the cognitive processing demands on students during the learning process. Therefore, this study investigated the effects of two overlapping methods of instruction: one consisting of a combination of social and explicit practices (varied), and another consisting entirely of explicit instruction (non-varied).

Background and Need

Approximately 99% of the 1,150 community colleges in the United States offer developmental courses in reading, writing, and mathematics, with mathematics receiving the highest enrollment (AMATYC, 2006). Depending on the college, anywhere between 15% and 60% of community college students require at least one developmental mathematics course (Bettinger & Long, 2005; Massachusetts Community College

Executive Office [MACCEO], 2006). Additionally, Bahr (2008) discovered that of all community-college students who did not reach their goals of degree attainment or transfer to four-year institutions, 84% failed their remedial mathematics courses.

Also, in light of the research by Bailey et al. (2005) that reported about a third of entering community college students are first-generation and students of color are enrolling in developmental mathematics at a much greater rate than Caucasians, developmental mathematics courses are preventing underrepresented students from reaching college-level courses. Therefore, student success in developmental mathematics is inextricably linked to the goals of a large and diverse group of students.

In addition, there is evidence confirming that students who completed their developmental mathematics courses obtained comparable academic outcomes to similar students who did not need remediation (Bahr, 2008). Further, McCabe (2000) discovered that approximately 90% of successfully remediated students became employed in skilled labor positions. Thus, it appears that remediation is beneficial to students who pass developmental courses, which warrants an investigation into reforms that may increase the number of successful students.

In sum, developmental mathematics instruction needs to be explored because of the large number of students who enroll in developmental mathematics courses coupled with the historically low number of students who are successful, and especially those who are first-generation and historically underrepresented. Hence, this study will address an important aspect of community college education. As such, what follows is a review of several important studies pertaining to developmental mathematics instruction.

In a landmark study, Boylan (2002) attempted to identify effective teaching practices for community college developmental mathematics instruction. Beginning in the Fall of 1999 and ending in the Summer of 2000, Boylan and his colleagues identified 60 community colleges nationwide that were effective in delivering remedial mathematics education. Of these institutions, 36 community colleges, deemed best-practice colleges, agreed to participate in the study by responding to several surveys and participating in site visits. Results yielded several effective instructional practices that were common across the colleges.

Primarily, Boylan (2002) and his team pointed out that successful developmental mathematics programs used a varied method of instruction, which incorporated a variety of instructional practices, such as lecturing, individual problem-solving, small-group work, peer reviews, and whole-class discussions. Additionally, the instructors at these colleges made the content relevant to students by linking the developmental outcomes with college-level outcomes in an attempt to teach mathematics in context as opposed to teaching isolated sets of mathematical skills. In other words, the observations by Boylan implied that successful developmental mathematics programs utilized a varied method of instruction aimed at delivering concepts in the context of practical problem-solving situations.

The findings by Boylan (2002) were later supported in a publication by AMATYC (2006), titled *Beyond Crossroads*, which is a standards document created to facilitate the continued improvement of developmental mathematics education at community colleges. The standards were divided into three categories: standards for content, standards for intellectual development, and standards for pedagogy. Within the pedagogy standard, it was suggested that developmental mathematics instructors should implement a varied method of instruction to appeal to the various learning styles typically present in a developmental classroom, a recommendation consistent with the conclusions of Boylan.

Similarly, Epper and Baker (2009) presented their findings from a review of the literature regarding instructional practices in community college developmental mathematics. Similar to Boylan (2002), Epper and Baker also observed several community colleges deemed to be successful in delivering remedial mathematics in order to discover trends in their developmental mathematics programs. In the end, their findings were consistent with the conclusions of Boylan (2002) and AMATYC (2006). A review of the successful programs yielded several common characteristics, contextualizing the mathematics curriculum and the incorporation of a variety of instructional practices to actively engage students in the learning process.

A quasi-experiment conducted by Fowler and Boylan (2011) provided empirical support for the observations made by Boylan (2002) and Epper and Baker (2009), as well as for the recommendations put forth by AMATYC (2006). Student outcomes were compared between developmental mathematics students enrolled in Pathways to Success (PWAY), which was a reformed developmental mathematics course taught using a varied method of instruction, and similar students enrolled in a traditional developmental mathematics course. As part of PWAY, students received in-class counseling, advising, and tutoring services in addition to a varied method of instruction. Student achievement outcomes and retention rates were compared between students in the two groups, yielding statistically significant differences; students in the PWAY program obtained higher grades and were less likely to fail the course or dropout of college.

Overall, these studies were consistent in identifying a varied method of instruction as a key component of developmental mathematics education. Regarding plausible explanations for the observed success using such an instructional method, sources have claimed that community college developmental mathematics students are diverse with respect to learning styles, and therefore respond positively to instruction that is equally diverse (AMATYC, 2006; Thomson & Mascazine, 1997). Although much of the literature has been inconsistent in supporting the claim that using a variety of instructional techniques to accommodate students' learning styles results in increased achievement (AMATYC, 2006; Brown, 2003; Davis, 1993; Meyers & Jones, 1993), much of the literature is consistent in recognizing the existence of learning styles and recommends implementing a variety of instructional techniques to address them (AMATYC, 2006; Guild & Garger, 1985; Midkiff & Thomasson, 1993). In this way, developmental mathematics instructors can provide students with an equal opportunity to learn in their preferred ways, which may or may not increase student achievement (Banks, 1988; Guild & Garger, 1985, Midkiff & Thomasson, 1993).

As another plausible explanation, Herbert and Grouws (2006) conducted a literature review of mathematics instruction and concluded that different teaching practices provided different opportunities to learn, which may have yielded different kinds of learning. In other words, certain instructional practices may be more effective for certain learning outcomes than for others. This conclusion is consistent with that made by Mesa (2008) after a review of the literature in developmental mathematics instruction. Both reviews stated that a varied method of instruction is appropriate for mathematics courses with varied learning outcomes.

With respect to the different types of learning outcomes expected in developmental mathematics courses, Mesa (2010) observed seven successful developmental mathematics instructors and discovered that instruction focused on three basic cognitive processes. Using the terminology set forth by Anderson and Krathwohl (2001) in their revision of Bloom's taxonomy (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956), the three basic cognitive processes were remembering, understanding, and applying. This finding was also supported in the literature review by Herbert and Grouws (2006) who identified skill efficiency, defined as the application of mathematical procedures, and conceptual understanding, defined as the ability to make connections between rules, ideas, and procedures, as the two most valued learning outcomes in lower-level mathematics courses.

Further, given the recent recommendations from the Common Core State Standards for Mathematics to solve algebraic equations while explaining each step in the process (California Department of Education, 2013), it may be inferred that developmental mathematics instruction needs to attend to a variety of learning outcomes and cognitive processes, especially conceptual understanding and procedural application. Therefore, the extent to which a varied method of instruction affects these two learning outcomes was investigated in the current study.

An additional learning outcome thought to be related to instruction is the ability to apply knowledge outside of the context in which it was learned, or in other words, the ability to transfer knowledge (Klahr & Nigam, 2004). One hypothesis offered by Klahr and Nigam is called the path-independence hypothesis, which predicts that knowledge transfer is a function of what was learned and not a function of how the concepts were taught. This hypothesis is in contrast to the belief that constructivist approaches may yield better transfer skills compared to explicit instruction (Hmelo-Silver, Duncan, & Chinn, 2007; Matlen & Klahr, 2013). Consequently, given that the varied method of instruction contained a mix of social constructivist and explicit instructional practices, and the non-varied method of instruction contained only explicit instruction, the extent to which transfer was facilitated under these conditions was studied.

Relative to non-achievement outcomes, researchers suggested that instruction may influence students' interest (Hulleman, Godes, Hendricks, & Harackiewicz, 2010). Two types of interest are believed to exist: situational interest, defined as an affective reaction that is triggered by conditions in the learning environment, and personal interest, defined as an individual's predisposition in a particular context (Hidi & Renninger, 2006; Mitchell, 1993). With respect to methods of instruction, it is believed that various teaching practices may influence situational interest (Mitchell, 1993; Rotgans & Schmidt, 2011). As such, it may be possible that a varied method of instruction affects situational interest to a different extent than does a non-varied method of instruction. This may result in more interested and possibly more motivated students, which has been hypothesized to correlate with student achievement (Koller, Baumert, & Schnabel, 2011; Middleton, 2013). Therefore, the extent to which situational interest is triggered by varied and non-varied methods of instruction was investigated.

An additional non-achievement outcome that was investigated in the current study was course retention. Course retention is defined as the percentage of students who were enrolled in the course that did not withdraw, which may be calculated by dividing the number of students without 'W' grades by the total number of students enrolled in the course (The Research and Planning Group for California Community Colleges, 1996). Researchers believe that course retention rates may be related to college-level structures, such as learning communities and academic support services (Visher, Butcher, & Cerna, 2010; Waldron & Yungbluth, 2007), and were thus investigated in the study.

Research Questions

This study investigated the following research questions with respect to developmental mathematics education at community colleges:

(1) To what extent does a varied method of instruction facilitate conceptual understanding and procedural application more effectively than a non-varied method of instruction?

(2) To what extent does a varied method of instruction facilitate students' knowledge transfer more effectively than a non-varied method of instruction?

(3) To what extent does a varied method of instruction affect students' situational interest compared to a non-varied method of instruction?

(4) To what extent does a varied method of instruction affect course retention rates compared to a non-varied method of instruction?

Definition of Terms

Active-Learning: A method of instruction that provides students with opportunities to talk, listen, read, and write about concepts in order to organize and clarify their thinking and create new mental structures (Meyers & Jones, 1993).

Cooperative Activity: Students working in pairs or small groups to complete a highly structured learning activity and achieve specific learning goals (Barkley Cross, & Major, 2004; Ellis, 2005).

Course Retention: The percentage of students who were enrolled in a course that did not withdraw (The Research and Planning Group for California Community Colleges, 1996).

Developmental Mathematics: Pre-collegiate mathematics courses offered at community colleges, such as Pre-Algebra, Algebra, and Geometry (AMATYC, 2006).

Instruction: In-class teaching practices, such as lectures, discussions, and assignments, that facilitate students' achievement of learning outcomes.

Instructional Method: An adopted set of teaching practices.

Learning Style: The preferences, tendencies, and strategies exhibited by students while learning (Thomson & Mascazine, 1997).

Lecture-Seatwork Method of Instruction: A method of instruction consisting primarily of lecturing and individual assignments, both of which are characterized by few opportunities for student-teacher and student-student interactions.

Personal Interest: An individual's predisposition in a particular context (Hidi & Renninger, 2006).

Social Constructivism: A theory of learning and instruction that focuses on the interdependence of social and individual processes in the co-construction of knowledge (Palincsar, 1998).

Situational Interest: An affective reaction that is triggered by conditions in the learning environment (Hidi & Renninger, 2006).

Transfer: Applying knowledge outside of the context in which it was learned (Klahr & Nigam, 2004).

Varied Method of Instruction: A method of instruction consisting of a variety of reaching practices, such as lectures, discussions, individual assignments, group-work, peer reviews, and writing assignments, characterized by high levels of student-teacher and student-student interactions.

CHAPTER II

REVIEW OF THE LITERATURE

The concept of remediation in post-secondary education is not a new one; in an historical review by the Massachusetts Community College Executive Office (MACCEO, 2006), Harvard College was reported to offer special tutoring in Greek and Latin for selected underprepared students during the 17th Century. Additionally, the University of Wisconsin is credited with creating the first developmental program for reading in 1849 (MACCEO, 2006). By 1889 it was estimated that 80% of the nation's colleges and universities were offering pre-collegiate programs (Canfield, 1889). Currently, it is estimated that 99% of the 1,150 community colleges in the United States offer developmental courses in reading, writing, and mathematics, with mathematics receiving the highest enrollment rates; depending on the college, anywhere between 15 and 60 percent of community-college students require at least one developmental mathematics course (Bettinger & Long, 2005; MACCEO, 2006).

Further, the American Mathematical Association of Community Colleges (AMATYC) estimated that across the United States, approximately 1.3 million students were enrolled in mathematics courses, more than half of which were not considered college level (AMATYC, 2006). Within these non-transferable courses, the success rates are typically very low; researchers have found that failure rates in developmental mathematics courses can reach as high as 70% (Attewell, Lavin, Domina, & Levey, 2006; Bahr, 2008). Moreover, an estimated 1 billion dollars in tax-payer funds are spent on developmental education nationwide each year (Bettinger & Long, 2005). This combination of low success rates and high funding is troubling given that developmental courses are necessary to meet many transfer and graduation requirements. Therefore, given the high demand and low success rates, the purpose of this literature review is to investigate methods of instruction that may be used to positively affect student outcomes in developmental mathematics courses at community colleges.

First, an overview of developmental mathematics education is provided. Then, research pertaining to a varied method of instruction is discussed. Last, student outcomes that may be affected by instructional practices will be described.

Overview of Developmental Education

Developmental education affects over one million students that are diverse with respect to several variables (Blair, Kirkman, & Maxwell, 2013). Additionally, the research investigating the effectiveness of developmental mathematics programs has focused on an equally diverse set of factors, such as administrative support, academic support services, and curriculum reforms (Epper & Baker, 2009). Therefore, what follows is a review of several studies investigating student diversity and a brief summary of perspectives from which to investigate developmental education at community colleges.

Diversity in Learning Styles

The term learning styles refers to the preferred strategies and behaviors exhibited by students when gathering, interpreting, and organizing information (AMATYC, 2006; Davis, 1993; Gabriel, 2008; Thomson & Mascazine, 1997). These behaviors and strategies can be organized into a variety of categories. For example, Claxton and

Murrell (1987) described four categories of learning styles: personality characteristics, such as being an introvert or extrovert; information-processing characteristics, such as learning concepts holistically or in a step-by-step approach; social interaction characteristics, such as working individually or cooperatively; and instructional preference characteristics, such as listening to a lecture or reading from a book. From other perspectives, O'Neil (1990) suggested the three categories of cognitive, affective, and physiological behaviors as factors of a student's learning style, whereas Thomson and Mascazine (1997) described five categories: environmental, emotional, sociological, physical, and psychological factors.

Overall, it is evident that an array of student learning preferences and behaviors is likely to be present within any given classroom. Relative to a developmental mathematics classroom, however, an even greater number of leaning styles is expected to be present given that there is evidence to suggest that learning styles are a function of ethnicity, socioeconomic status, and age (Banks, 1988; Brown, 2003). Therefore, not only do developmental mathematics students vary widely in terms of goals, age, ethnicity, and socioeconomic status, they most likely vary widely in the ways they prefer to learn.

Of these various learning styles, the most common styles explored in the research on instruction are students' instructional modality preferences, categorized as auditory, tactile, and visual (AMATYC, 2006; Davis, 1993; O'Neil, 1990). Said differently, students may have a preference to learn by listening, doing, or watching. In terms of mathematics instruction, however, it may be difficult to isolate and measure the extent to which students prefer learning mathematics in these ways given that mathematics instruction typically contains elements of all three modalities simultaneously. Alternatively, sociological preferences of learning styles proposed by Dunn and Dunn (1972) may be more appropriate for the current study given the social constructivist underpinnings of active learning practices. Dunn and Dunn defined sociological preferences as the preference for learning individually or through interactions with a partner or small group.

In terms of the relationship between learning styles and achievement, researchers have been inconsistent in supporting the claim that matching teaching styles with learning styles results in increased achievement (AMATYC, 2006; Brown, 2003; Davis, 1993; Meyers & Jones, 1993). However, much of the literature is consistent in recognizing the existence of learning styles and has suggested implementing a variety of instructional practices that may accommodate them (AMATYC, 2006; Guild & Garger, 1985; Midkiff & Thomasson, 1993). As a result, developmental mathematics instructors may better meet the equity mission of the community college system by providing all students with an equal opportunity to learn in their preferred ways (Banks, 1988; Guild & Garger, 1985, Midkiff & Thomasson, 1993). Therefore, the current study included a measure of students' sociological preferences as part of the measures pertaining to their entry characteristics.

Diversity in Personal Interest

In a dissertation by Subocz (2008), Prealgebra students from 13 different community college classrooms were administered a modified version of the Mathematics Attitudes Survey (Fennema & Sherman, 1976). Items were based on a 5-point Likert scale rated from 1 (*strongly agree*) to 5 (*strongly disagree*). Although the descriptive statistics were not provided, Subocz claimed that students varied in terms of the extent to which they enjoyed learning mathematics, as well as in their comfort levels and perceived utility value of learning mathematical content.

Additionally, in a study on interest in secondary mathematics education, Mitchell (1993) used factor analyses to identify two distinct types of math interest: personal and situational. Personal interest may be defined as an individual's predisposition in a particular context, and situational interest may be defined as an affective reaction that is triggered by conditions in the learning environment (Hidi & Renninger, 2006; Mitchell, 1993). In relation to the previously discussed study by Subocz (2008), given that the survey was administered to students prior to instruction, it may be inferred that Subocz gathered data on students' personal interest in developmental mathematics.

Therefore, there is evidence to support that students' personal interest toward developmental mathematics at community colleges varies, which is consistent with the definition of personal interest offered by Hidi and Renninger (2006); given that developmental mathematics students have presumably already received several years of mathematics instruction, they may arrive to community college with an emerging or well-developed interest in mathematics, or lack thereof. More importantly, researchers have discovered that students' disposition and interest towards mathematics is correlated with mathematics achievement, although the strength of this relationship is often weak and the mechanisms by which interest and achievement affect each other remains largely unknown (Koller, Baumert, & Schnabel, 2011; Middleton, 2013). Consequently, due to the potential variation in interest levels and the plausible interaction between interest and mathematics achievement, a measure of personal interest was administered to participants prior to the start of instruction.

Diversity in Prior Knowledge and Ability

Between 2009 and 2010, Grubb and Gabriner (2013) selected over a dozen California community colleges to participate in a qualitative study on developmental education in mathematics, reading, and writing. The 13 colleges were selected by various criteria that yielded a mixed sample from rural, urban, and suburban areas in Northern and Southern California. At each college, approximately 16 instructors were observed and interviewed, resulting in a total of 144 classroom observations. As a result of the observations, the researchers determined that developmental classrooms contained a highly heterogeneous student population consisting of at least five different kinds of students with respect to ability and prior knowledge: refresher students, incorrectly placed students, underprepared students, students with learning disabilities, and students with mental health problems. Each type of student will be described in turn.

First, refresher students were defined by Grubb and Gabriner (2013) as students who have mastered basic skills in the past, but have forgotten them. These students may have not taken a math class in several years and are in need of a brief review. Second, misplaced students are students who may actually know all the concepts and procedures but did not take the placement test, or who took the placement test but did not recognize its importance. Third, and the most common type of student identified by Grubb and Gabriner, were underprepared students. These students genuinely needed to receive instruction in basic skills because they learned very little in their prior schooling.

Fourth, although the researchers were not trained to identify learning disabilities, the researchers hypothesized and confirmed through interviews with students, teachers, and counselors, that students in basic skills courses often suffer from learning disabilities. Given that the observed instructors were not trained in special education, and in some cases the colleges had no institutionalized mechanism for which to diagnose and accommodate these students, the actual percentage of learning disabled students could not be established. Finally, Grubb and Gabriner identified several students as suffering from mental health disorders, such as compulsions, anxiety, depression, and post-traumatic stress disorder. Similar to students with learning disabilities, however, this conclusion was reached after interviews and was not based on clinical evaluations.

In any event, it may be inferred from the observations of Grubb and Gabriner (2013) that developmental mathematics students vary with respect to abilities and prior knowledge. Therefore, given the purpose of the current study to investigate the effects of a varied method of instruction, each type of student identified by Grubb and Gabriner may exhibit different learning outcomes as a result of varying instructional practices. Accordingly, measures of ability and prior knowledge were administered to the participants.

Various Research Perspectives

The problem regarding low success rates and retention in developmental mathematics has been investigated from a variety of angles. For example, several studies have attempted to predict student success by investigating affective factors such as self-regulation, self-efficacy, and attitudes toward mathematics at community colleges (Cortes-Suarez, 2008; Otts, 2011; Subocz, 2007). Additionally, given that research has long suggested that retention and success cannot be attributed entirely to student factors (Gates & Creamer, 1984), researchers have also investigated the effects of centralized developmental programs and levels of administrative support on student success in developmental mathematics (Boylan, 2009). From another perspective, research has also been conducted in the area of student support services for developmental mathematics students such as providing in-class counselors and offering supplemental instruction (Boylan, 2002). Considered together, developmental education emerges as a complicated matter that may be investigated from a wide array of perspectives.

With respect to poor success rates in developmental mathematics, several explanations have been posited by the extant literature including a lack of teacher training in remedial education (Boylan, 2002; Epper & Baker, 2009; Grubb & Gabriner, 2013), as well as negative teacher attitudes towards the abilities of developmental students (Roueche & Wheeler, 1973). Additionally, it has been suggested that high schools are partly to blame because their curricula are not aligned with college-level mathematics, which leads to an inadequate level of preparation (McCabe, 2003).

From a similar curricular standpoint, researchers have also observed that the content in a typical developmental mathematics class is not practical for students whose majors are not in the Science and Engineering fields, therefore offering alternative pathways to college-level courses has been recommended (Bryk & Triesman, 2010). Lastly, from another standpoint, researchers have hypothesized that antiquated instructional techniques are to blame for student failure (Grubb & Associates, 1999; Grubb and Gabriner, 2013). Altogether, from these studies it may be inferred that there are a multitude of possible causes for the low success rates in developmental mathematics, which is commensurate with the various perspectives from which this problem may be approached.

Summary

In conclusion, developmental mathematics students are diverse with respect to goals, age, socioeconomic status, ethnicity, learning styles, personal interest, ability, and prior knowledge. Further, factors that may affect these students' success are equally diverse, with each factor deserving an equal amount of attention and future research. However, pertaining to the purpose of this study, focus was placed on instructional practices that may be used to increase student outcomes under such diverse conditions. Accordingly, an analysis of instructional practices in developmental mathematics is provided next.

Developmental Mathematics Instruction

In order to understand the landscape of developmental mathematics instruction, current practices will first be described and evaluated, followed by recommendations for improved practices and future research, and concluded with a description of a varied method of instruction.

Current Methods

Grubb and Associates (1999) conducted a study in which 42 developmental mathematics instructors were observed while teaching in community colleges across the United States. The observations revealed that instructors were delivering remedial instruction to community college students in the same ways that it was delivered to the students in elementary and high school: through a traditional method of instruction beginning with the delivery of a rule, followed by an example, and then concluded by assigning an excess of similar problems (Grubb & Associates, 1999). According to the researchers, each part of this instructional method was implemented without any context or relevance to real-world situations. That is, Grubb and Associates discovered that developmental mathematics students are being exposed to isolated mathematical forms and skills devoid of meaning and practicality, a strategy that is typically used in high-school education. These findings need to be interpreted with caution, however, because no descriptive statistics were provided regarding the number of instructors who were delivering this type of instruction. Therefore, overall conclusions appear to have been drawn from this study without much evidence regarding the extent to which instructors were utilizing such a non-varied approach.

The findings by Grubb and Associates (1999) were replicated in a similar study by Grubb and Gabriner (2013) in which 144 developmental mathematics classrooms were observed at 13 different colleges across the state of California. Observations were consistent with those in the 1999 study in that developmental mathematics instruction consisted largely of the drill and practice of small skills without any real-world applications; a method Grubb and Gabriner called “remedial pedagogy” (p 52). Additionally, it was discovered that a large part of instruction in the observed courses was dedicated to learning tricks for getting a right answer instead of attending to the underlying concepts and procedures. Overall, Grubb and Gabriner concluded that remedial pedagogy lacks opportunities for students to play an active role in their learning, which is partly responsible for students’ maladaptive college behaviors, such as arriving late, participating in off-topic conversations, and using mobile devices during class, in addition to their low conceptual understanding and inability to transfer mathematical knowledge to real-world problems.

Grubb and Gabriner (2013) offered several reasons for the prevalence of non-varied instructional practices in developmental mathematics classrooms, with the most likely being that community colleges instructors have no formal training in methods of instruction. In other words, when provided with a textbook and a syllabus, instructors are likely to teach as they have been taught in high school and college, instead of experimenting with different approaches. As such, Grubb and Gabriner classified a lecture-seatwork method of instruction as the "default position" (p. 72) for the majority of developmental mathematics instructors.

Rationale for Change

The logic in using similar instructional practices in both high school and community college classrooms needs to be evaluated. In other words, community college students are arriving from high school in need of remediation in mathematics, suggesting that their high school instruction was ineffective. For example, in a study by Kaestle, Campbell, Finn, Johnson, and Mikulecky (2001), a random sample of 13,600 Americans over the age of 16 were interviewed to gather information regarding literacy skills in adults. About 400 trained interviewers went into households across the United States to assess three scales of literacy: prose, document, and quantitative. Relevant to the current study was the quantitative literacy scale that was comprised of 5 skill levels of increasing difficulty beginning with single arithmetic operations, Level I, and ending with multiple sequential operations embedded within the context of a real-world situation, Level V.

Results indicated that the average quantitative literacy skill level for participants with a high-school degree or equivalent was representative of Level II: performing single operations with numbers that are either stated in a task or easily located in the question. Further, approximately 47% of all high school graduates placed into the first two levels, suggesting that a high-school mathematics education might not be providing sufficient training that is necessary to succeed in college. These findings were supported by McCabe (2003) in which an estimated 42% of high-school graduates were considered not ready for college-level work.

The findings by Kaestle et al. (2001) and McCabe (2003) imply that the typical high school instructional model has been traditionally unsuccessful; it seems that high-school students are falsely learning that mathematics is a subject that can be mastered simply by memorizing discrete facts without any reinforcement in qualitative literacy or problem-solving skills (Stigler, Givvin, & Thompson, 2010). Moreover, in conjunction with the findings by Grubb and Associates (1999) and Grubb and Gabriner (2013), this is also the strategy that is replicated in many community college classrooms. These findings beg the following question: if a non-varied method of instruction did not work for students in high school, then why are educators doing the same thing for students who arrive at community colleges? It seems that a new set of instructional practices needs to be adopted in order to increase the success of students who are arriving underprepared and in need of remediation in mathematics.

Recommended Instructional Practices

Despite the largely negative review of instructional practices put forth by Grubb and Gabriner (2013), there are several innovative departments that have been experimenting with alternate methods of instruction across the United States. Consequently, beginning in the Fall of 1999 and ending in the Summer of 2000, Boylan (2002) set forth to identify methods of instruction that were common to successful developmental mathematics programs. Although the exact criteria for inclusion in this study were not provided, Boylan identified 60 community colleges nationwide that were deemed effective in delivering remedial mathematics education. Of these institutions, 36 community colleges, deemed best-practice colleges, agreed to participate in the study by responding to several surveys and participating in site visits. Quantitative and qualitative data were gathered from the surveys and classroom observations and the results were analyzed by Boylan and his colleagues. The results of the study included the identification of several effective instructional practices that were common across the colleges.

Primarily, the observations by Boylan (2002) revealed that successful developmental mathematics programs used a variety of instructional practices, such as lecturing, individual problem-solving, small-group work, peer reviews, and whole-class discussions. Additionally, the instructors at these colleges made the content relevant to students by linking the developmental outcomes with college-level outcomes in an attempt to teach mathematics in context as opposed to teaching isolated sets of mathematical skills. Therefore, Boylan concluded that successful developmental mathematics instruction should utilize a variety of instructional practices aimed at delivering concepts in the context of practical problem-solving situations.

The findings by Boylan (2002) were later supported in a publication by AMATYC (2006) titled *Beyond Crossroads,* a standards document created to facilitate the continued improvement of developmental mathematics education at community colleges. The recommendations included teaching with an emphasis on quantitative reasoning and problem-solving, instead of delivering a string of unrelated mathematical topics, as well as incorporating many different teaching practices.

Along the same lines, Epper and Baker (2009) presented their findings from a recent review of the literature regarding instructional practices in community college developmental mathematics. Epper and Baker investigated 14 community colleges deemed to be successful in delivering remedial mathematics in order to discover trends in effective developmental mathematics programs. Similar to Boylan (2002), the exact criteria for inclusion in their report were not provided, however, their findings were consistent with the conclusions of Boylan (2002) and AMATYC (2006); a review of the successful programs yielded several reforms, including contextualized curricula and teaching using a variety of instructional practices that engaged in the learning process.

Further support in favor of implementing a varied method of instruction was provided in a quasi-experimental study by Fowler and Boylan (2011). GPA, course success rates, and freshman-sophomore retention rates were compared between 434 students who enrolled in an innovative mathematics program during the 2008-2009 academic year, titled Pathways to Success (PWAY), and 453 equivalent students who were enrolled in regular developmental mathematics classes prior to the inception of PWAY, during the 2003-2004 academic year. There were at least two advantages to this sampling technique. First, the program was already in existence for five years, which helped control for extraneous variables related to potential problems that may have occurred while the program was in its nascent state. Second, the study was able to capture all developmental mathematics students during each academic year, which allowed for larger sample sizes and eliminated potential self-selection bias.

PWAY consisted of four central components believed to affect student achievement: clear student guidelines; mandatory orientation and first-year experience; prescriptive, developmental, and intrusive advising; and developmental mathematics coursework. More specifically, PWAY was structured so that students were linked together in a cohort and provided with clear expectations, academic and personal advising and counseling, and a variety of instructional practices with frequent assessment and feedback. Additionally, the students' daily schedules were fixed to include a training course in metacognitive skills as well as to include blocks of time dedicated to mandatory tutoring.

The results of the study yielded statistically significant differences between the PWAY and non-PWAY students in GPA (PWAY = 2.15, non-PWAY = 1.50), Prealgebra success rates (PWAY = 51%, non-PWAY = 30%), and freshman-sophomore retention rates (PWAY = 52%, non-PWAY = 29%). These results must be interpreted with caution, however, because all of the students belonged to the same community college and there was no way to determine which features of the program caused the observed differences. Additionally, despite the high level of support provided to students, the improved success rate was just over 50%, which is still low.

Considered in combination, the results of these just-reviewed studies were all consistent in that a varied method of instruction was observed to be one part of an effective developmental mathematics program. However, the extent to which instruction caused success is yet to be determined. In turn, a discussion of future directions for research in developmental mathematics education is provided.

Future Directions

Despite the prevalence of a varied method of instruction in what Boylan (2002) and Epper and Baker (2009) identified as effective developmental mathematics programs, there is still a lack of research supporting a causal link between developmental mathematics instruction and student learning outcomes; in a literature review by Mesa (2008), a search of databases (e.g., ERIC, PsychInfo, etc…), research journals (e.g., *The Journal of Higher Education*, *The Journal of Community College Research and Practice*, etc…), and disciplinary websites (e.g., AMATYC, National Council of Teachers of Mathematics, etc…) yielded just 47 studies pertaining to mathematics instruction, only 12 of which used community-college student data. Further, Mesa described the quality of these studies to be poor in terms of inadequate descriptions of the samples, treatments, and instruments.

With respect to the current study, Mesa (2008) observed that instructional studies often left out descriptions of the specific teaching practices that were implemented by the instructors for both the treatment and control groups, which generated nearly useless data when the intent was to study the effects of instruction on student outcomes. For example, the quasi-experimental study previously described by Fowler and Boylan (2011) did not actually include a description of the instructors' day-to-day activities. Further, there was no mention as to the method of instruction that was applied in the developmental mathematics classes before PWAY began. Thus, although using lectures and discussions was mentioned as part of the PWAY program, not enough details were provided to replicate the methods of instruction that were being compared.

Consequently, the purpose of the current study was to investigate conditions under which student outcomes are facilitated by a varied method of instruction while controlling for the other reforms of successful developmental programs identified by Boylan (2002) and Epper and Baker (2009), such as required academic and personal support services. Additionally, the current study attempted to clearly describe the instructional practices that were used in both treatment and control groups. As such, what follows is a description of a set of instructional practices, named active learning, which served as a foundation for the varied method of instruction that was used in the current study.

Active Learning

Meyers and Jones (1993) defined active learning to be in contrast with the traditional instructional model in which instructors actively present information while students passively receive it. Specifically, active learning occurs when students are given opportunities to talk, listen, read, and write about concepts in order to organize and clarify their thinking and create new mental structures (Meyers & Jones, 1993; Millis & Cottell, 1998). Instructors who subscribe to an active-learning philosophy typically believe that learning is an active, collaborative, and dynamic process in which instruction should allow opportunities for students to reason with each other and apply problem-solving skills (AMATYC, 2006; Doyle, 2008; Stein, Grover, & Henningsen, 1996; Stevenson, 1921).

Active learning is consistent with the recommendations for implementing a variety of teaching practices because it provides opportunities for all students to interact with concepts at a deeper level (Meyers & Jones, 1993). It is important to recall that this method was not recommended as a replacement for lecturing; advocates of Cognitive Load Theory claim that students with low-prior knowledge benefit from receiving explicit instruction and watching instructors demonstrate how to solve problems (Sweller & Cooper, 1985). Rather, active learning pratices are viewed as a supplement to explicit instruction in order to provide students of all learning styles and backgrounds an opportunity to engage in the learning process (Meyers & Jones, 1993).

To facilitate active learning, multiple researchers have recommended utilizing interactive lecturing and questioning, collaborative and cooperative activities, and writing assignments to encourage the active processing of information among students (AMATYC, 2006; Borich, 2007, Meyers & Jones, 1993). Thus, what follows is a description of these practices and how they may be applied to developmental mathematics instruction.

Interactive Lecturing and Questioning

Interactive lecturing and questioning may be characterized by various student-teacher and student-student interactions in which the teacher encourages students to ask questions, effectively manages wait-time between questions, and also encourages students to answer their own questions (AMATYC, 2006; Davis, 1993). To facilitate these interactions, prior to lecturing an instructor may organize seats into a circular or semi-circular shape to encourage students to ask questions. Additionally, when students ask questions, the instructor may wait a few seconds before responding to allow students to reflect on the question and attempt to answer it themselves. Consequently, students are actively participating in the lecture.

Mesa (2010) investigated the extent to which successful developmental mathematics instructors, as determined by student and administrative evaluations, facilitated interactive lecturing in their courses and discovered that over a third of the questions asked by instructors were yes or no questions, and over two-thirds of the questions were never answered by students because the questions where rhetorical or the instructor provided the answer immediately after asking. From these observations it may be reasoned that interactive-lecturing and questioning is not a commonly applied instructional practice. In terms of generalizability, however, the results of this study do not extend very far because only five instructors were observed and they all worked at the same community college. But, on the other hand, these findings are consistent with the observations by Grubb and Gabriner (2013) previously discussed.

Cooperative Learning

Many researchers agree that collaboration and cooperation may be viewed as opposite ends of a spectrum relative to the degree of structure and control provided by the instructor; collaborative activities typically have little structure in which students are free to choose how to complete a task, or in some cases even have a choice about the task itself, whereas cooperative activities have a high degree of structure in which groups work toward a specific goal (AMATYC, 2006; Barkley, Cross, & Major, 2005; Millis & Cottell, 1998; Springer, Stanne, & Donovan, 1999). Relevant to the current study on developmental mathematics education, coupled with the work by Sweller and Cooper (1985) declaring that explicit instruction is more effective for students of low prior knowledge, cooperative activities are more appropriate than unstructured problem-solving activities for developmental education in which students are typically underprepared. Therefore, cooperative activities were included as part of the varied method of instruction whereas collaborative activities will be excluded.

Cooperative learning is defined as students working in pairs or small groups to complete a structured learning activity and achieve learning goals (Barkley et al., 2005, Ellis, 2005). Many studies and meta-analyses on cooperative learning activities have suggested that cooperative activities foster deep learning and increase affective and cognitive outcomes, such as self-efficacy and increased critical-thinking skills (AMATYC, 2006; Barkley et al., 2005; Borich, 2007; Millis & Cottell, 1998; Qin, Johnson, & Johnson, 1995; Slavin, 1996; Wright et al., 1998). However, in order for the benefits of cooperative learning to be realized, activities must be purposefully chosen and carefully structured (Millis & Cottell, 1998).

A key characteristic of an effective cooperative-learning activity is group interdependence, which occurs when the individual outcomes of group members are affected by the actions of the other members in the group (Johnson & Johnson, 2009; O'Donnell, 1996; Millis & Cottell, 1998). That is, cooperative activities will be more effective if the group members depend on each other to complete a common task. Therefore, simply placing students into groups and having them talk about problems or work individually is not considered an effective cooperative-learning activity (Johnson & Johnson, 2009; Millis & Cottell, 1998).

In addition to group interdependence, there are also several other factors that have been hypothesized to increase the effectiveness of cooperative learning. The importance of group processing, which is allowing group members to self-evaluate their progress as a team toward the desired learning objectives, was identified as a fundamental component to a cooperative learning environment (Ellis, 2005; Johnson & Johnson, 2009). Additionally, clearly describing the goal and structure of each activity, and assigning roles, such as facilitator, recorder, timekeeper, and reporter, to the members of each group are considered effective characteristics of cooperative activities (Barkley et al., 2005; Borich, 2007; Johnson & Johnson, 2009; Millis & Cottell, 1998).

In regard to the current study, the varied method of instruction contained several cooperative activities, but there were also opportunities for individual and non-cooperative group work. Therefore, the varied instructional method did not cultivate a purely cooperative environment, which made group processing somewhat inappropriate for the current study. However, to encourage group interdependence, roles were assigned to students to ensure that every member of the group had a role to play in the groups' successful completion of the in-class assignments.

For example, when practicing mathematical procedures by working on problems in groups, students were assigned the following roles recommended by Millis and Cottell (1998): facilitator, reporter, and timekeeper. The facilitator was responsible for keeping the group on task and monitoring discussions, the reporter was responsible for serving as the spokesperson when asked to share responses, and the timekeeper was responsible for keeping members aware of time constraints and monitoring the groups' progress toward the objective.

Writing

A final key element of active learning is the incorporation of brief writing assignments into instruction (AMATYC, 2006; Meyers & Jones, 1993), the purpose of which is to focus learners' attention onto their level of understanding and to promote deep learning through reflection, synthesis, and evaluation of concepts (AMATYC, 2006; Borich, 2007). For example, recommendations from AMATYC included beginning class by asking students to write about the main concepts from the previous night's homework, or to finish a class by asking students to write to an absent student explaining the key concepts of the day. As a result, students are encouraged to re-organize their mental structures as they actively process the information from homework or in-class lectures and activities.

Summary

Grubb and Gabriner (2013) declared that developmental mathematics instruction focuses primarily on the delivery of isolated procedures devoid of real-world applications, an instructional method common across secondary and higher education. Further, researchers have claimed that students are entering community colleges unprepared for college-level work in mathematics (Kaestle et al., 2001; McCabe, 2003). In response, several community colleges have begun to implement innovative programs to increase students' success that include using a variety of instructional practices (Boylan, 2002; Epper & Baker, 2009; Fowler & Boylan, 2011). However, what is missing from the literature is a causal link between using a varied method of instruction and increased student outcomes, as well as detailed descriptions of instructional treatments (Mesa, 2008). Therefore, the current research drew upon active learning practices (Meyers & Jones, 1993) to define a varied method of instruction and investigated the extent to which it affects student outcomes while controlling for extraneous variables.

Student Outcomes

Another weakness in the research on developmental mathematics instruction identified by Mesa (2008) was an inadequate description of dependent variables. For example, Mesa noted that many studies used course grade as a dependent variable without explicitly stating how the grades were computed. It may be possible that several scores, such as those associated with participation, homework, quizzes, or exams, could have been included in the final grade. As such, interpreting the effects that instruction may have on student learning becomes problematic without more information. Therefore, what follows is a description of the dependent variables investigated in the current study.

Conceptual and Procedural Knowledge

Anderson and Krathwohl (2001) put forth a revision of Bloom's Taxonomy of Educational Objectives (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956) that outlined four types of knowledge: factual, conceptual, procedural, and metacognitive. The current study focused on factual, conceptual, and procedural knowledge, each of which will be described in turn. Factual knowledge was defined as the knowledge of basic elements needed in order to solve problems within a discipline, such as terminology and other specific details. Conceptual knowledge was defined as the knowledge of how the basic elements (factual knowledge) are interrelated within larger structures that enable the basic elements to function together. Lastly, procedural knowledge was defined as the knowledge associated with how to do something, such as using skills, algorithms, and techniques.

For example, in developmental mathematics, knowing the definition of an ordered pair may be considered factual knowledge, knowing how a collection of ordered pairs form solution sets and graphs may be considered conceptual knowledge, and knowing how to solve or graph an equation would be considered procedural knowledge. These three knowledge types are considered to exist on a continuum of less complex (factual) to more complex (procedural), and it is also assumed that knowledge on the lower end of the continuum is required to advance to more complex knowledge structures. For instance, it is presumed that knowing the definition of an ordered pair is required to know how the ordered pairs come together to form solution sets, which also precedes being able to solve an equation.

Anderson and Krathwohl (2001) also defined six types of cognitive processing for each of the previously discussed knowledge types: remember, understand, apply, analyze, evaluate, and create. Similar to the knowledge dimension, these cognitive processes are also considered to exist on a continuum of less complex (remember) to more complex (create). Remembering is defined as the retrieval of knowledge from long-term memory, understanding is defined as the ability to construct meaning from what is known, application is defined as the ability to carry out or use a procedure, analyzing is defined as the ability to break material into parts and to see how the parts relate to each other and the whole, evaluating is defined as the ability to make judgments based on certain criteria, and creating is defined as the ability to put elements together to form a new structure.

For example, prompting a student to write the definition of a solution set requires remembering factual knowledge, prompting a student to explain why one equation cannot have two graphs requires understanding of conceptual knowledge, and asking a student to solve an equation requires application of procedural knowledge. However, it may be the case that an instructor has already dedicated instructional time to explaining why one equation cannot have two graphs, in which case this problem would now only require remembering instead of understanding. Therefore, Anderson and Krathwohl (2001) emphasized that care must be taken when determining the knowledge and cognitive processes required for a particular learning outcome; if a problem has already been shown to the students, then prompting for the same response would automatically fall under the remember category. In other words, the knowledge and cognitive dimensions are a function of student preparedness and prior knowledge.

With respect to previous studies in developmental mathematics instruction, particular attention has been given to understanding conceptual knowledge and applying procedural knowledge (Hiebert & Grouws, 2006; Mesa, 2010). For example, in an observational study of seven developmental mathematics instructors, Mesa (2010) discovered that 98% of classroom activities elicited remembering (23%), understanding (39%), and applying (36%). Further, 52% of the activities were identified as developing procedural knowledge.

Interestingly, the seven instructors were included in the study because of their success in delivering remedial education; each instructor had received well above average student and administrative evaluations prior to the study. Therefore, it appears that successful developmental mathematics instructors tend to focus on lower-order cognitive processing skills. These results may not extend beyond the study, however, because of the small sample size of instructors from a single college. However, these finding were consistent with the observations by Grubb and Gabriner (2013) who used a much larger sample size to conduct their observations across the state of California; they also concluded that a majority of developmental mathematics instruction is dedicated to executing procedures.

The emphasis placed on understanding concepts and applying procedures may be credited in part to the fact that developmental mathematics courses are prerequisites for college-level mathematics courses in which students are expected to be able to apply routine procedures, such as solving equations, in the context of real-world applications commonly arising in courses such as Statistics and Calculus. Put differently, instructors tend to emphasize these two specific parts of the taxonomy because it is believed that students need these particular skills in order to be successful in subsequent courses (Mesa, 2010). Another plausible explanation for the prevalence of teaching conceptual understanding and procedural application comes in the form of publications by the American Mathematical Association of Two-year Colleges (AMATYC, 2006), the Common Core State Standards (California Department of Education, 2013), and the National Council of Teachers of Mathematics (NCTM, 2000), which all include recommendations for teaching understanding in addition to routine procedures.

In sum, conceptual understanding, defined as the ability to explain and exemplify interrelationships among basic elements, and procedural application, defined as the ability to use a procedure or problem-solving technique (Anderson & Krathwohl, 2001), are two learning outcomes that deserve special attention in developmental mathematics given that the courses are prerequisites to more complex courses in addition to the recommendations by existing organizations promoting standards in mathematics instruction (AMATYC, 2006; California Department of Education, 2013; NCTM, 2000). Further, it appears that these two specific outcomes already receive the majority of instructional time (Grubb & Gabriner, 2013; Mesa, 2010). Therefore, the current study investigated the extent to which conceptual understanding and procedural application are affected by a varied method of instruction.

Transfer

An additional learning outcome thought to be related to instruction is the ability to apply knowledge outside of the context in which it was learned, or in other words, the ability to transfer knowledge (Klahr & Nigam, 2004). One hypothesis offered by Klahr and Nigam is called and the path-independence hypothesis, which predicts that transfer is a function of the knowledge that was gained and not a functions of how the material was presented. This hypothesis is in contrast to the belief that constructivist approaches may yield better transfer skills compared to explicit instruction (Hmelo-Silver, Duncan, & Chinn, 2007; Matlen & Klahr, 2013).

For example, Matlen and Klahr (2013) investigated the transfer abilities of 57 third-grade students trained to design unconfounded experiments using a Control of Variables Strategy, or CVS. The independent variable was the level of guidance (high vs low) provided during instruction on how to design an experiment that investigated factors affecting the distance rolled by a ball released on a ramp, such as steepness and surface type. The high guidance group received direct instruction and inquiry questions regarding experimental design, whereas the low guidance group only received the inquiry questions.

The dependent variables consisted of scores on a series of near transfer assessments and far transfer assessments; near transfer was defined as the ability to apply CVS in a situation similar to one used during instruction, and far transfer was defined as the ability to apply CVS in a new context. For instance, one of the near transfer tasks targeted students' abilities to design unconfounded experiments to investigate factors affecting the length that a spring could stretch, such as spring length and wire size. Alternatively, one of the far transfer assessments measured students' abilities to evaluate written descriptions of experiments from a range of contexts, such as plant growth and cookie baking, using CVS criteria.

Matlen and Klahr (2013) discovered that the high guidance students outperformed the low guidance students on both types of transfer tests and concluded that direct instruction may be more efficient than unguided discovery approaches for teaching CVS, both in terms of near and far transfer of knowledge. Although this study used third-grade students, the results are relevant to the current study because the varied method of instruction will contain opportunities for students to discover mathematical concepts under low levels of guidance from the instructor. Concurrently, the non-varied method of instruction will always provide high levels of guidance and explicit instruction. Therefore, it could be possible that students' near and far transfer abilities might differ based on instructional method. Based on these findings, the current study investigated the extent to which near and far transfer were affected by a varied method of instruction.

Situational Interest

Relative to non-achievement outcomes, researchers have suggested that instruction may influence students' interest (Hulleman, Godes, Hendricks, & Harackiewicz, 2010; Rotgans & Schmidt, 2011). Two types of interest are believed to exist: situational interest, defined as an affective reaction that is triggered by conditions in the learning environment, and personal interest, defined as an individual's predisposition in a particular context (Hidi & Renninger, 2006; Mitchell, 1993). Situational interest was further defined as consisting of two phases: a triggered phase, defined as a psychological state of interest resulting from short-term changes in environmental features, and a maintained phase, defined as a state of interest following the triggered phase that involves focused attention and persistence (Hidi & Renninger, 2006).

In terms of mathematics instruction, Mitchell (1993) posited that using group work triggers situational interest, which may be maintained if the to-be-learned content is meaningful to students and they are provided with opportunities to be involved in the learning process. As such, it may be possible that a varied instruction affects situational interest to a different extent than does non-varied instruction because the students who received a varied instruction engaged in cooperative actives whereas students who received a non-varied method of instruction worked in isolation. Thus, the current study included a measure of situational interest that was administered to both groups at the end of the study.

Course Retention

Course retention may be defined as the percentage of students who were enrolled in a course that did not withdraw, which may be calculated by dividing the number of students without ‘W’ grades by the total number of students enrolled in the course (The Research and Planning Group for California Community Colleges, 1996). Researchers believe that retention may be related to course-level and college-level structures, such as learning communities and academic support services (Visher, Butcher, & Cerna, 2010; Waldron & Yungbluth, 2007).

For example, Bloom and Sommo (2005) posited that students who enrolled in learning communities were more likely to pass their developmental courses and enroll in a subsequent college-level course compared to an equivalent control group. These findings advanced what was known about the effectiveness of community college learning communities because the researchers were able to randomly assign first-year students to either the leaning community or the general population of students, whereas most other research on learning communities has had to deal with self-selected samples (Bloom & Sommo, 2005). Although the students in the study were not entirely developmental mathematics students, the results still indicated that course retention may be affected by classroom-related factors.

As another example, Tinto (1997) proposed that participating in learning communities lead to increased retention and campus involvement among developmental students at community colleges. Tinto believed that placing students into cohorts and teaching them contextualized concepts were some of the factors that lead to the positive outcomes exhibited among these students. Additionally, he credited the social interactions that occurred within the classrooms as part of the success of learning communities. Therefore, given that the current study included varying degrees of social interaction between students in the varied and non-varied classrooms, it may be the case that course retention rates were affected.

Summary

The current study investigated the effects of a varied method of instruction on the following dependent variables: conceptual understanding, procedural application, near and far transfer, situational interest, and course retention rates. First, conceptual understanding was included because of the emphasis placed by mathematics education publications on understanding mathematics (AMATYC, 2006; California Department of Education, 2013; NCTM, 2000). Second, procedural application was assessed because of the need for procedural fluency in subsequent college-level courses (Hiebert & Grouws, 2006; Mesa, 2008). Third, near and far transfer were investigated because researchers have argued that varying levels of explicit instruction yield different degrees of deep learning as defined by the ability to transfer knowledge in familiar and new contexts (Hmelo-Silver, Duncan, & Chinn, 2007; Matlen & Klahr, 2013). Fourth, situational interest was compared between students in the varied and non-varied classes because situational interest is hypothesized to be triggered and maintained by varied teaching practices, such as group work and opportunities for involvement (Hidi & Renninger, 2006; Mitchell, 1993). Last, course retention rates were compared between the two groups because it has been proposed that social interaction is partly responsible for the increased course retention rates observed within learning communities (Bloom & Sommo, 2005; Tinto, 1997).

Summary

Community college students enrolled in developmental mathematics are a highly heterogeneous group; researchers have acknowledged that the students are diverse with respect to goals, age, socioeconomic status, ethnicity, learning styles, personal interest, ability, and prior knowledge (Adelman, 2005; AMATYC, 2006; Bailey et al., 2005; Grubb & Gabriner, 2013; Subocz , 2008). Additionally, observations of successful developmental mathematics programs revealed that varying instructional practices was an effective method of instruction for these students (Boylan, 2002; Epper & Baker, 2009; Fowler & Boylan, 2011). However, the effectiveness of these programs cannot be attributed entirely to the method of instruction; inadequate descriptions of instructional treatments coupled with the simultaneous inclusion of various support services has made the causal effects of instruction difficult to ascertain (Fowler & Boylan, 2011; Mesa, 2008). Therefore, the purpose of this study was to investigate the conditions under which conceptual understanding, procedural knowledge, near and far transfer, situational interest, and course retention rates were affected by a varied instructional method in developmental mathematics education while controlling for additional reforms, such as learning communities, contextualized curricula, and mandatory counseling and academic support services.

CHAPTER III

METHODOLOGY

Researchers investigating student outcomes in community college developmental mathematics have identified several successful reforms, such as implementation of learning communities, contextualized curricula, academic support services, and a varied method of instruction (Boylan, 2002; Epper & Baker, 2009; Fowler & Boylan, 2011). However, in each study, several of these components were operating simultaneously resulting in the inability to ascertain the effects of instructional method on student outcomes. Therefore, the purpose of this study was to investigate the effects of a varied method of instruction on conceptual understanding, procedural application, transfer, situational interest, and course retention rates while controlling for other reforms identified by Boylan (2002), Epper and Baker (2009), and Fowler and Boylan (2011).

What follows is a description of the methodology that was used in the current study. First, the research design and sample will be described, followed by an explanation of the instruments and instructional treatment. Then, the procedures and pilot procedures will be outlined. Finally, the chapter concludes with descriptive statistics and preliminary analyses of the scores obtained from the instruments.

Research Design

This study addressed the following research questions with respect to developmental mathematics education at community colleges:

(1) To what extent does a varied method of instruction facilitate conceptual understanding and procedural application more effectively than a non-varied method of instruction?

(2) To what extent does a varied method of instruction facilitate students' near and far knowledge transfer more effectively than a non-varied method of instruction?

(3) To what extent does a varied method of instruction affect students' situational interest compared to a non-varied method of instruction?

(4) To what extent does a varied method of instruction affect course retention rates compared to a non-varied method of instruction?

To answer the research questions, a two-group comparative quasi-experiment was conducted using six Beginning Algebra classes at a Northern California community college in the Bay Area. Beginning Algebra was chosen because it was in the middle of the developmental mathematics sequence at the participating institution and consists of material commonly offered in 8th or 9th grade. The independent variable was method of instruction; a varied method of instruction was implemented in three classes, and a non-varied method of instruction was implemented in another three classes.

Three volunteer instructors participated, and each instructor taught both methods of instruction. The dependent variables were conceptual understanding, procedural application, near and far transfer, situational interest, and course retention rate. Additionally, to determine whether the treatment was delivered as intended, weekly instructional checklists and time logs were collected from the participating instructors. The study began on the first day of the 16-week Spring 2015 semester and concluded after covering three units of content, which required nine weeks of instruction and testing. Table 1 summarizes the research design.

To control for additional extraneous variables besides instructor, such as class start time and day of the week, the varied and non-varied classes were scheduled as displayed in Figure 1. Additionally, neither class included the other reforms identified by Boylan (2002) and Hunter and Boylan (2009), such as learning communities, contextualized curricula, or mandatory advising. Consequently, any differences in the dependent variables may be better attributed to the method of instruction.

Table 1
Summary of the Research Design

Background Variables	Covariates	Classes	Teachers	Dependent Variables
Demographics	Fluid Intelligence	Varied 1	Instructor 1	Conceptual Understanding
Units and Employment Status	Crystallized Intelligence	Non-Varied 1	Instructor 1	Procedural Application
Learning Disabilities	Prior Math Knowledge	Varied 2	Instructor 2	Near and Far Transfer
Prior Math Experience		Non-Varied 2	Instructor 2	Situational Interest
Learning Styles		Varied 3	Instructor 3	Course Retention
Personal Interest		Non-Varied 3	Instructor 3	

To address the research questions, differences in the dependent variables between the varied and non-varied classes were investigated for each teacher. Next, the data from the varied classes (Varied 1, Varied 2, and Varied 3) were combined and analyzed as a single data set. Likewise, scores on the dependent variables from the non-varied classes (Non-Varied 1, Non-Varied 2, and Non-Varied 3) were combined into a single data set.

Then, differences between the varied and non-varied groups on the dependent variables were investigated.

Time	Monday	Tuesday	Wednesday	Thursday
9:15 am - 11:40 am	Varied 1 Instructor 1	Non-Varied 1 Instructor 1	Varied 1 Instructor 1	Non-Varied 1 Instructor 1
10:45 am - 1:10 pm	Non-Varied 2 Instructor 2	Varied 2 Instructor 2	Non-Varied 2 Instructor 2	Varied 2 Instructor 2
12:15 pm - 2:40 pm	Varied 3 Instructor 3	Non-Varied 3 Instructor 3	Varied 3 Instructor 3	Non-Varied 3 Instructor 3

Figure 1. An outline of the schedule used in the current study for the six two-day per week classes taught by three different instructors.

Sample

What follows is a description of the participating institution, instructors, and students in the current study. First, the participating institution will be described relative to the characteristics of successful developmental mathematics programs identified by Boylan (2002). Then, the backgrounds of the participating instructors will be summarized. Finally, demographics of the participating students will be provided.

The study was conducted at an urban community college in Northern California that serves approximately 13,500 students each semester and has an approximate success rate of 50% for students enrolled in Beginning Algebra (California Community Colleges Chancellor's Office, 2015). With respect to the successful reforms identified by Boylan (2002) and Epper and Baker (2009), the community college in the current study did not have learning communities, contextualized curricula, mandatory academic support services, nor professional development opportunities for implementing varied instructional practices. Further, instructors were allowed to teach developmental mathematics courses using their preferred methods of instruction, as opposed to agreeing to implement and develop a common set of teaching practices.

Three instructors volunteered to participate in the study: Instructor 1, Instructor 2, and Instructor 3. No incentives were provided to the instructors to participate in the study, so each instructor volunteered by their own accord. When asked to explain their reasons for participating in the study, all three instructors claimed that they are frequently looking for new ways to reflect on their teaching practices and improve their students' success, and viewed the study as an opportunity to do so. Additionally, Instructor 1 mentioned a proclivity for active-learning strategies, which added to her interest in the study. Overall, the participating instructors joined the study because they wanted an opportunity to investigate the effects of instruction on student learning in community college developmental mathematics.

Table 2 summarizes instructor demographics, experience, and comfort levels with implementing in-class activities. Comfort level was measured using a Likert-type item on a 5-point scale ranging from *very uncomfortable* to *very comfortable*. It is important to notice that the three instructors varied widely with respect to their experience in implementing structured cooperative learning activities; Instructor 1 had experience implementing such activities on a daily basis whereas Instructor 2 rarely used such activities. However, when the definition of an in-class activity was broadened to include group work, worksheets, and/or discussions, the range in responses decreased, which implies that all three instructors had experience implementing in-class activities, although perhaps not as structured (e.g., assigning group roles) as the ones that were used in the study.

Another important distinction between the instructors is that Instructor 2 had much less experience teaching Beginning Algebra than the other two instructors. In fact, Instructor 2 had only taught Beginning Algebra one time before the start of the study. A final significant observation is that all three instructors felt very comfortable using in-class discussions, which is a key component of the varied method of instruction.

Table 2
Participating Instructors' Demographics, Teaching Experience, and Comfort Levels

Variable	Instructor 1	Instructor 2	Instructor 3
Gender	Female	Male	Female
Age Range	40 - 49 Years Old	25 - 29 Years Old	60 - 69 Years Old
Ethnicity	Caucasian	Multiracial	Asian
Math Teaching Experience	9 Years	6 Years	11 Years
Community College Math Experience	8 Years	3 Years	10 Years
Developmental Math Experience	8 Years	3 Years	9 Years
Beginning Algebra Experience	8 Years	< 1 Year	7 Years
Structured Activity Implementation	Once per Class	Once per Month	Every 2-3 Weeks
Structured Activity Comfort Level	Very Comfortable	Somewhat Comfortable	Very Comfortable
Other Activity Implementation	Once per Class	Every Other Class	Every Other Class
Discussion Comfort Level	Very Comfortable	Very Comfortable	Very Comfortable

Table 3 summarizes student demographics for participants in the current study compared to college-wide student demographics based on data from the California

Community Colleges Chancellor's Office (2013) for the 2012-2013 academic year. Participants differed from the general student population on several variables. First, the females in the study outnumbered the males by a ratio of approximately 2 to 1, even though less than half of the students at the college are female. This observation is surprising in light of the findings by Adelman (2005) and Bailey, Jenkins, and Leinbach (2005) who observed that no gender differences in enrollment existed in a nationally representative sample of developmental mathematics students.

Table 3

Percentages Regarding Gender, Age, and Ethnicity of Students at the Participating Institution and Students Participating in the Study

Variable	College-Wide ($n = 13{,}750$)	Varied ($n = 61$)	Non-Varied ($n = 78$)	Total Sample ($n = 139$)
Gender				
Female	44	67	65	66
Male	56	33	35	34
Age				
< 20 years old	18	29	38	35
20 - 24 years old	34	35	35	35
25 - 39 years old	33	31	21	25
≥ 40 years old	15	5	6	5
Ethnicity				
African American	8	9	11	10
American Indian	1	1	3	2
Asian	22	9	6	7
Filipino	4	2	0	1
Hispanic	39	60	56	58
Pacific Islander	1	2	4	3
White	17	16	16	16
Unknown	8	1	4	3

Second, as expected, the participating students appeared to be younger, on average, than other students at the institution. Finally, although approximately 40% of the students on campus self-identify as Hispanic, they comprised almost 60% of the students in the sample, which supports the claim that developmental mathematics education disproportionately enrolls minority students (Adelman, 2005; AMATYC, 2006; Bailey et al., 2005).

Table 4 contains additional background information of the participating students. Overall, the majority of students were enrolled in 9-15 units, were employed part-time, and had taken a math class within the last year. Surprisingly, almost half of the participants were taking the course for at least the second time. Additionally, about half of the participants had completed the prerequisite course, and about 20% of them needed multiple attempts to succeed. Therefore, it appears that many of the participants have recently been exposed to basic algebraic concepts, but have struggled to pass their courses.

Protection of Human Subjects

To ensure the protection of the participants' rights, approval was obtained by the Institutional Review Board for the Protection of Human Subjects (IRBPHS) at the University of San Francisco (see Appendix A) and the guidelines explicated by the IRBPHS were followed (University of San Francisco, 2008). Additionally, consent to conduct the study was granted by the review board at the participating community college and their guidelines were also followed.

Instrumentation

Table 5 summarizes the twelve instruments administered to students in the study. Given that students were not randomly assigned to the treatment groups, the Background Survey, Social Preference Questionnaire, Personal Interest Questionnaire, ability assessments, and Prior Knowledge Test were administered during the first two weeks of instruction to provide evidence of student equivalence on these measures. The remaining

Table 4

Percentages Regarding Number of Units, Employment Status, Learning Disabilities, and Previous Math Experience

	Instructor 1		Instructor 2		Instructor 3		Total	
Variable	Varied ($n = 21$)	Non-Varied ($n = 31$)	Varied ($n = 25$)	Non-Varied ($n = 28$)	Varied ($n = 15$)	Non-Varied ($n = 19$)	Varied ($n = 61$)	Non-Varied ($n = 78$)
Units								
≤ 5	15	9	8	4	7	11	10	8
6 – 8	0	9	20	4	13	16	12	9
9 - 12	40	38	32	43	27	21	33	35
12 – 15	45	31	40	39	53	53	45	40
> 15	0	13	0	11	0	0	0	9
Employment								
Unemployed	40	28	44	36	33	42	40	34
Part-time	50	60	44	36	40	32	45	44
Full-time	10	12	12	28	27	26	15	22
Learning Disability	15	3	8	7	0	16	7	8
Retaking Class	60	53	36	61	40	37	45	52
Completed Prerequisite	40	66	52	57	40	53	45	60
Multiple Attempts	30	14	8	25	33	20	21	19
Placed in Class	45	34	36	32	40	32	38	35
Last Math Class								
< 1 year	60	59	48	64	60	63	57	63
1 – 2 years	15	25	24	11	20	16	21	17
3 – 4 years	15	13	12	7	0	16	10	11
> 4 years	10	3	16	18	20	5	16	9

Note. No statistically significant differences were found among treatment groups, teachers, nor classes at the .05 significance level.

Table 5

Summary of Constructs, Instruments, Variables, Number of Items, Item Type, Administration Time, and Score Range

Construct	Instrument	Variable(s)	# of Items	Item Type	Time	Score Range
Background Information	Background Survey	Demographics	3	MC	3 min	--
		Units and Employment Status	2	MC		
		Learning Disability	1	MC		
		Prior Math Classes	4	MC		
Learning Styles	Social Preference Questionnaire	Social Preference	4	Likert	3 min	4-28
Personal Interest	Personal Interest Questionnaire	Personal Interest	4	Likert	3 min	4-28
Fluid Intelligence (*Gf*)	*Gf* Assessment	Letter Series	10	MC	10 min	0-10
		Letter Sets	15	MC	15 min	0-15
		Figure Analogies	12	MC	12 min	0-12
Crystallized Intelligence (*Gc*)	*Gc* Assessment	Synonyms	12	MC	8 min	0-12
		Sentence Completion	10	MC	8 min	0-10
Prior Knowledge	Prior Knowledge Test	Prior Math Knowledge	50	MC	60 min	0-50
Unit 1 Achievement	Unit 1 Achievement Test	CU	7	Written	60 min	0-10
		LCPA	10	MC		0-10
		HCPA	5	Written		0-20
Unit 2 Achievement	Unit 2 Achievement Test	CU	6	Written	60 min	0-10
		LCPA	10	MC		0-10
		HCPA	5	Written		0-20
Unit 3 Achievement	Unit 3 Achievement Test	CU	6	Written	60 min	0-10
		LCPA	8	MC		0-8
		HCPA	5	Written		0-22
Transfer	Transfer Test	Near Transfer	1	Written	8 min	0-3
		Far Transfer	1	Written	8 min	0-3
Situational Interest	Situational Interest Questionnaire	Situational Interest	3	Likert	2 min	3 - 21

Note. All times are approximate. MC = Multiple Choice; CU = Conceptual Understanding; LCPA = Low Complexity Procedural Application; HCPA = High Complexity Procedural Application.

instruments measured the dependent variables. Additionally, Treatment Implementation Logs and Researcher Notes were utilized to assess the fidelity of treatment implementation and to collect feedback from the instructors regarding their experiences. Each instrument is described in more detail below.

Background Survey

The Background Survey was used to collect data pertaining to student demographics, number of units, employment status, previous math coursework, and current math placement (see Appendix B). The survey was administered electronically on the first day of the semester and required no more than four minutes to complete.

Social Preference Questionnaire

To determine the extent to which students preferred to work in groups as opposed to working in isolation, the Social Preference Questionnaire was administered (see Appendix C). Each of the four items used a 7-point Likert-type scale ranging from 1 (*strongly disagree*) to 7 (*strongly agree*). For each item, participants were expected to select the extent to which they agree or disagree with statements pertaining to working with others, such as “During math class, I enjoy working in small groups” and “I like working by myself during math class” (negative). The items were administered electronically and were completed in less than three minutes. The sum of the scores was calculated to obtain a total score ranging from 4 to 28 that reflected participants’ preferences for working in groups; the items were coded so that high scores corresponded to a preference for working in groups, and lower scores represented a preference for working alone. Cronbach’s coefficient alpha was then calculated to obtain an estimate of internal consistency reliability, which was .80.

Personal Interest Questionnaire

The Personal Interest Questionnaire consisted of four items: two items used in a study by Mitchell (1993), and two items used in a study by Hulleman et al. (2010), both of who were investigating personal interest in mathematics classrooms (see Appendix D). Each of the four items used a 7-point Likert-type scale ranging from 1 (*strongly disagree*) to 7 (*strongly agree*). Similar to the Social Preference Questionnaire, participants were expected to select the extent to which they agreed or disagreed with statements pertaining to interest in mathematics, such as “Compared to other subjects, I feel relaxed studying mathematics” and “I do not enjoy working on mathematics problems” (negative). The items were administered electronically and required no more than three minutes to complete.

The sum of the scores was calculated to obtain a total score ranging from 4 to 28 that reflected participants’ personal interest in mathematics; the items were coded so that high scores corresponded to high levels of personal interest. Cronbach’s coefficient alpha for the personal interest scores was .87.

Ability Assessments

Carroll (1993) performed factor analyses using data from studies on human abilities and found evidence to suggest the following three-stratum model of intelligence: at the highest level is a general factor of intelligence, *g*, followed by eight broad abilities at the second level, and approximately 70 narrow abilities at the lowest level. Of interest in the current study is *g* and two of the eight broad intelligences that correlated with *g*: fluid intelligence, *Gf*, and crystallized intelligence, *Gc*. Fluid intelligence may be defined as the ability to reason and solve problems, whereas crystallized intelligence may be defined as the ability to define words and comprehend text (Mackintosh, 2011). A description of the instruments that will be administered to measure both of these broad abilities is provided next.

Gf Assessment

The *Gf* assessment consisted of three different tests of fluid intelligence: a letter series test, a letter sets test, and a figure analogy test. The letter series test consisted of 10 items from the publicly available and out-of-print test of Primary Mental Abilities (Thurnstone, 1962). The items consisted of a sequence of letters, followed by five choices; the task for the participants was to choose from the five choices the next letter that logically continues the sequence. For example, a letter series may look like the following: a b a c a d a e a __. Then, the participant must select from five choices the next logical letter, which in this case is f. The letter series test was administered electronically and required no more than 10 minutes to complete. Each item was scored as correct or incorrect yielding total scores that ranged from 0 to 10, with higher scores reflecting higher reasoning ability. Cronbach's coefficient alpha for these scores was .77.

The second test of fluid intelligence was a letter sets test from the Ekstrom Kit of Factor-Referenced Cognitive Tests (Ekstrom, French, Harman, & Darman, 1976). This test consisted of 15 items reproduced with permission from the publisher (see Appendix E). Each item consisted of five sets of four letters each, and the task was to select the set of letters that differs from the other four. Similarly to the letter series test, the letter sets test was administered electronically and required no more than 15 minutes to complete. Each item was scored as correct or incorrect yielding total scores that ranged from 0 to

15, with higher scores reflecting higher reasoning ability. Cronbach's coefficient alpha for these scores was .80.

Lastly, with permission from the publisher (see Appendix E), the figure analogy test consisted of 12 items from the Cognitive Abilities Test (CogAT) Form 4 (Thorndike & Hagen, 1986). Each item contained two figures from which a relationship is to be inferred, and an additional third figure; the task was to select from among five options the figure that shares the same relationship with the third figure that existed between the first two figures. The items were administered electronically accompanied with a test booklet, and required no more than 12 minutes to complete. Each of the 12 items was scored as correct or incorrect yielding a range of scores from 0 to 12, with higher scores reflecting higher reasoning ability. Cronbach's coefficient alpha for these scores was only .56.

Gc Assessment

The *Gc* assessment consisted of two tests of crystallized intelligence: a synonym test and a sentence completion test. The synonym test consisted of 12 items from the Test of Primary Mental Abilities (Thurnstone, 1962). The items consisted of a single word, followed by five choices; the task was for the participants to choose from the five choices the word that means the same as the given word. As another test of verbal ability, the sentence completion test consisted of 10 items from the CogAT Form 4 (Thorndike & Hagen, 1986). Each item consisted of a sentence in which one word is missing; the task was to choose the word that makes a complete and sensible sentence from among five choices.

The synonym and sentence completion tests were administered electronically and required no more than 8 minutes each to complete. Further, each item was scored as correct or incorrect, yielding total scores that ranged from 0 to 12 for the synonyms test and from 0 to 10 for the sentence completion test, with higher scores reflecting higher verbal ability. Cronbach's coefficient alphas for the synonym and sentence completion scores were .65 and .67, respectively.

Prior Knowledge Test

A 50-item multiple-choice was administered to students to measure their prior knowledge in mathematics (see Appendix F). The Prior Knowledge Test was administered in paper and pencil format using a scantron to record responses and required no more than an hour of class time. Each item was scored as correct or incorrect, so the total score ranged from 0 to 50, with higher scores representing higher levels of knowledge regarding rules, concepts, and procedures typically taught in Prealgebra, which is the prerequisite for the courses used in the study.

The test items were selected from a bank of items provided that accompanied the textbook, which provides some evidence of content validity. Further, the Prior Knowledge Test was reviewed by the participating instructors prior to the start of the study to ensure that the items were consistent with knowledge that should be known prior to a course in Beginning Algebra. Cronbach's coefficient alpha for the Prior Knowledge scores was .85.

Achievement Tests

As previously discussed, two of the dependent variables for the current study were conceptual understanding and procedural application. Further, procedural application was analyzed as consisting of two subscales: low complexity and high complexity. Low complexity problems may be defined as problems requiring relatively few steps, and high complexity problems may be defined as problems requiring relatively many steps. As such, each of the achievement tests generated scores for three dependent variables: conceptual understanding (CU), low complexity procedural application (LCPA), and high complexity procedural application (HCPA). The CU and HCPA items required written responses, whereas the LCPA items were multiple choice, and together each achievement test required no more than an hour to complete during class in paper-and-pencil format (see Appendix G).

Responses to CU items were scored on a discrete scale in which one point was awarded for each correctly explained concept. For example, one CU item was “Provide an example of a linear equation with a solution of 3.” This item may be graded as correct or incorrect. As another example, consider the following: “Is it possible for an equation to have two different graphs? Explain why or why not.” This item may be graded on a scale of 0 to 2 points; one point for a correct answer to the question and another point for a correct explanation. Partial credit was awarded at the instructors' discretion in half-point increments.

Notice that it is highly unlikely that either of the previously discussed CU examples may be answered by recall from long-term memory and therefore require cognitive processing consistent with the definition of understanding provided by Anderson and Krathwohl (2001). Further, notice that although the number of CU items on each of the three achievement tests varies slightly (see Table 5), the total points generated by the items on all three tests ranged from 0 to 10, with higher totals reflecting higher conceptual understanding of the corresponding content.

Similarly to the CU items, the number of LCPA items varied slightly between the three tests. But, given that these items did not require many steps, they were all scored as correct or incorrect for a maximum total score equivalent to the number of LCPA items. An example of a LCPA item is “Solve: $x + 8 = -12$.” This problem was categorized as LCPA because it may be solved by applying a procedure that consists of only one step (subtract 8 from both sides of the equation). Given that LCPA items were relatively simple, it may have been the case that students solved LCPA items using conceptual understanding in lieu of procedural application, which is why these items were separated from more complex problems requiring the application of many steps. Ultimately, the total scores for the LCPA items ranged from 0 to 10, with higher scores reflecting higher achievement relative to procedural application. Cronbach's coefficient alphas for the three LCPA subtests were .31, .63, and .49, respectively, which were low.

Lastly, each achievement test also contained five HCPA items graded on a discrete scale in which one point was awarded for each correctly written step of the required procedure. For example, “Solve: $4(3x - 2) - 32 = 8x - 4$” may be graded on a four-point scale because the procedure required to solve the equation consisted of at least four steps (simplifying, using the addition property of equality, using the multiplication property of equality, and obtaining a final solution). Additionally, similar to the CU items, partial credit was awarded at the instructors' discretion in half-point increments. Total scores on the HCPA items ranged from 0 to 22, with higher scores reflecting higher achievement relative to procedural application.

With respect to reliability of the CU and HCPA subtest scores, during training the instructors and researcher reached 100% inter-rater reliability on student responses to the CU and HCPA items collected on the pilot tests using rubric scoring and a criterion of at most one point difference in scores. Therefore, the instructors were trained to score these tests individually. However, during the study, the instructors requested to score the achievement tests collaboratively with the researcher so that any questionable responses could be graded together. During these sessions the instructors remained on task, constantly referred to the rubrics issued to them, and raised questions to the group whenever an item was difficult to grade. As such, the CU and HCPA subtests were reliably scored.

With respect to the validity of each test, the majority of the test items were selected from a bank of items that accompanied the textbook for the course, which provides some evidence of content validity; the only exception being a few of the CU items that were written by the researcher and verified by the participating instructors. Further, each achievement test was also reviewed by the participating instructors prior to administration to ensure that the items were consistent with the intended content domain.

Transfer Test

The Transfer Test included two items: one item measuring near transfer and one item measuring far transfer, each graded on a discrete scale of 0 to 3, with higher scores reflecting higher transfer abilities (see Appendix H). The near transfer item required students to solve a system of three linear equations in two variables, which is a new but relatively similar problem to those found in Unit 3 that required students to solve systems of two linear equations in two variables. One point was awarded for obtaining correct graphs, one point was awarded for a correct conclusion, and one point was awarded for a valid explanation. Additionally, partial credit was awarded at the instructors' discretion in half-point increments.

The far transfer item required students to draw inferences from a line graph depicting the projected minimum wage in four cities. In order to make meaningful conclusions, students needed to apply their cumulative knowledge from Unit 1 to Unit 3 to identify key characteristics of the line graph. One point was awarded for making a conclusion based on the slope or trajectory of any of the lines (e.g., the minimum wage will continue to increase over time), another point was awarded for a conclusion based on the intersection of two or more lines (e.g., two cities have the same minimum wage in a given year), and a final point was awarded for a conclusion based on the relative heights of the lines (e.g., one city has the highest minimum wage during a span of several years). To earn all three points, at least one conclusion must have been provided from each category. Therefore, even if a student made more than one conclusion regarding the trajectory of a line, only one point was awarded. Also, as with the near transfer item, partial credit may have be awarded at the instructor's discretion in half-point increments.

Situational Interest Questionnaire

The Situational Interest Questionnaire consisted of three items modified from a subset of items used by Mitchell (1993) to investigate situational interest in secondary mathematics (see Appendix I). Each item used a 7-point Likert-type scale ranging from 1 (*strongly disagree*) to 7 (*strongly agree*). Similar to the Personal Interest Questionnaire, participants were expected to select the extent to which they agree or disagree with statements pertaining to their current math class, such as "Our math class is fun." The items were administered electronically and required no more than two minutes to complete.

The sum of the scores was calculated to obtain a total score ranging from 3 to 21 that reflected participants' current situational interest in mathematics; the items were coded so that high scores corresponded to high levels of situational interest. Cronbach's coefficient alpha for these scores was .71.

Treatment Implementation Log

In addition to the instruments that were administered to the participating students, Treatment Implementation Logs were distributed to the participating instructors to ensure that the varied and non-varied methods of instruction were implemented as intended. Given that the instructors agreed to follow the exact same lesson plans, the Treatment Implementation Logs consisted of the detailed lesson plans for each class session in both varied and non-varied conditions, and also included space to indicate the time that was allocated to each teaching practice (see Appendix J). Additionally, a comment section was provided on the back of each page of the lesson plans for instructors to jot down their comments and concerns relative to their experiences in implementing the prescribed methods. The instructors completed the logs daily and reviewed them at the end of each unit during face-to-face meetings with the researcher.

Researcher Notes

During face-to-face meetings with the instructors, the researcher jotted down notes. Many of the meetings were conducted informally, often in between classes, so the notes consisted primarily of paraphrased comments as opposed to direct quotes. Most of the meetings were one-on-one, but some of the meetings occurred with more than one instructor present. At the conclusion of the study, the notes were collated and organized by date and the comments were labeled by instructor.

Treatment

This section has two subsections: student enrollment to the treatment and treatment procedures. First, a brief explanation of how students enrolled in each class and the resulting background characteristics of students in each class is provided. Second, procedures for implementing the varied and non-varied methods of instruction are outlined.

Student Enrollment to Treatment

Unfortunately, it was not possible to randomly assign students to the varied and non-varied courses at the start of the study. Therefore, students enrolled themselves in classes taught by the three volunteer instructors. In order to register for their classes, students had to complete the following registration procedure: (1) enroll in the college by completing an online application, (2) complete a mathematics assessment test to determine which level of math is appropriate based on their prior knowledge, (3) meet with a counselor to create an education plan and outline the subsequent courses needed to reach the students' goals, and (4) register for courses electronically via the campus web portal or in-person at the admissions and records office. Steps (2) and (3) were not required, so it may be the case that students enrolled in a math class without being placed or advised.

Treatment Procedures

Two methods of instruction were implemented during the current study: a varied method of instruction and a non-varied method of instruction. Due to constraints placed by the Accrediting Commission for Community and Junior Colleges (ACCJC), the two methods of instruction were expected to achieve the same learning outcomes in approximately the same amount of time (ACCJC, 2014; see Appendix K for the student learning outcomes of the courses in the current study, as well as the accompanying calendar and syllabus). However, the means by which instructors choose to achieve the learning outcomes is free to vary. Therefore, the two methods of instruction only differed in the instructional practices implemented in the classroom to achieve the same learning outcomes.

To facilitate the conceptualization of the instructional treatments, each class session was viewed as consisting of three sequential stages: development, practice, and closing. The development stage consisted of time allotted for students to learn new rules, concepts, and procedures. Subsequent to the development stage was the practice stage, in which opportunities were provided for students to practice the just-learned material. Finally, each session usually consisted of a closing stage during which the targeted learning outcomes were reviewed.

Given that each class session was scheduled to last 145 minutes, it was often be the case that the three stages were cycled through at least twice with a brief break after about 70 minutes. Additionally, the time allocated to each stage fluctuated from day to day depending on the difficulty of the content and the implemented teaching practices. Overall, the majority of class time was allocated to development and practice, with a small percentage of time devoted to closing.

Each of the aforementioned stages may be conducted in a variety of ways. For example, developing a concept may be done by using an explicit lecture in which the instructor explains a concept and then provides a series of examples demonstrating how the concept is applied. Alternatively, an instructor may begin by showing a series of examples and then allow students an opportunity to infer a concept from the examples. Additionally, it may be possible to implement a cooperative activity that facilitates the learning of a concept. Overall, there are a variety of teaching practices that may be applied during each stage of the class session to meet the desired learning outcome. As such, what follows is a description of the teaching practices that were implemented as part of the varied and non-varied methods of instruction.

Varied Method of Instruction

Recall that a varied method of instruction was characterized by the following components of active learning: interactive lecturing, cooperative learning activities, and writing assignments (AMATYC, 2006; Meyers & Jones, 1993). As such, the varied method of instruction used in the current study included a variety of these activities in addition to traditional lecturing and individual seatwork.

For example, consider a class session devoted to learning how to solve linear equations in a single variable. The development stage may consist of the instructor facilitating an interactive lecture that includes examples of solving linear equations. Then, for the practice stage, the instructor may transition into individual seatwork during which students work on assigned problems from the textbook while receiving feedback from the instructor. Last, for the closing stage, the instructor may facilitate a writing activity to summarize the procedure for solving a linear equation in one variable. See Figure 2 for a list of teaching practices that may be applied during each stage of class.

Development	
• Lecture	The instructor describes a rule, concept, or procedure, then shows examples of it. Students sit in rows and the instructor answers questions directly as they arise.
• Interactive Lecture	Similar to a regular lecture, however, students may sit in a U-shape arrangement and the teacher redirects questions to other students and effectively utilizes wait time as questions arise. Alternatively, the instructor may begin with a series of examples, and then facilitate a whole-class discussion aimed at discovering a rule, concept, or procedure.
• Cooperative Activity	In contrast to lectures, the instructor does not describe concepts or demonstrate examples. Rather, the instructor facilitates a structured group activity focusing on the development of a rule, concept, or procedure.
Practice	
• Problems Individually	Students work alone to complete instructor-assigned problems from a textbook or other resource.
• Problems in Pairs	In contrast to working alone, students are encouraged to discuss the assigned problems, check answers with each other, and compare problem-solving strategies.
• Cooperative Activity	A structured activity that focuses on practicing already-learned rules, concepts, or procedures.
Closing	
• Lecture	The instructor summarizes a rule, concept, or procedure. Students sit in rows and the instructor answers questions directly as they arise.
• Interactive Lecture	The instructor facilitates a whole-class discussion aimed at summarizing a rule, concept, or procedure. Students sit in rows or a U-shape arrangement and the teacher redirects questions to other students and effectively utilizes wait time as questions arise.
• Cooperative Activity	A structured activity that focuses on summarizing the just-practiced rule, concept, or procedure.
• Writing Activity	The instructor provides writing prompts to the class and requires that all students write complete responses on their own paper. Then, students may be asked to report their written summaries to the whole class, within small groups, or with a partner.

Figure 2. An outline of various teaching practices that may be applied in each stage of a class session to form varied and non-varied methods of instruction.

Each participating instructor was provided with daily lesson plans that outlined various teaching practices to be applied at each stage of any given class session (see Appendix J). If necessary, the instructors were allowed to modify their individual lesson plans as they saw appropriate, however, none of the instructors chose to do so. See Figure 3 for an example schedule of teaching practices for three sections of content using a varied method of instruction.

Unit 1	Section 1	Section 2	Section 3
Development			
Lecture	X		
Interactive Lecture		X	X
Cooperative Activity	X		
Practice			
Problems Individually		X	
Problems in Pairs	X		
Cooperative Activity			X
Closing			
Lecture		X	
Interactive Lecture			X
Cooperative Activity			
Writing Activity	X		X

Figure 3. An outline of the proposed teaching practices that may be applied for the first three sections of Unit 1using a varied method of instruction.

It is important to notice in Figure 3 that it was acceptable to include multiple practices during a single stage of class. For example, Section 1 was scheduled to include elements of a traditional lecture along with a cooperative activity. Additionally, it was also acceptable to use the same teaching practice on consecutive days, as long as it does not happen so frequently that the other teaching practices are ignored. In the event that a cooperative activity was scheduled, the participating instructors were provided with the required learning materials and directions for implementing the activity (see Appendix L for general descriptions of cooperative activities and Appendix M for the accompanying handouts). For all other teaching practices, each instructor was allowed to use their own materials during the development stage, such as lecture notes and questions for discussion, but were required to assign the same set of in-class exercises during the practice stage and the same set of questions during the closing stage.

Overall, the varied method of instruction included a variety of active-learning teaching practices during the development, practice, and closing stages of each class session. The practices included any combination of interactive lectures, cooperative activities, and writing assignments, in addition to traditional lecturing and individual seatwork, in order to achieve the required student learning outcomes.

Non-Varied Method of Instruction

In contrast to the varied method of instruction, the non-varied method of instruction consisted only of lecturing and individual practice. See Figure 4 for an example schedule of teaching practices for three sections of content using a non-varied method of instruction. Although a non-varied method of instruction could technically consist of classes that implement cooperative activities on a daily basis, the current study defined a non-varied method of instruction to be consistent with the instructional practices identified by Grubb and Gabriner (2013); that is, each day began with a lecture, followed by individual practice, and concluded with a summary lecture.

Each stage of the non-varied class session was characterized by high amounts of teacher-student interactions, but minimal student-student interactions. During the development stage of class the teacher was expected to clearly explain and demonstrate all the to-be-learned material before assigning individual practice problems. During the practice stage, students were not encouraged to ask each other for help or discuss the problems, and instead were expected to ask the instructor for feedback and guidance.

Lastly, the closing stage of class was facilitated by the instructor and consisted of a summary of the concepts and procedures that were just practiced.

Unit 1	Section 1	Section 2	Section 3
Development			
Lecture	X	X	X
Interactive Lecture			
Cooperative Activity			
Practice			
Problems Individually	X	X	X
Problems in Pairs			
Cooperative Activity			
Closing			
Lecture	X	X	X
Interactive Lecture			
Cooperative Activity			
Writing Activity			

Figure 4. An outline of the proposed teaching practices that may be applied for the first three sections of Unit 1using a non-varied method of instruction.

As in the varied condition, the participating instructors were expected to have their own lecture notes and were therefore not provided with lecture materials. Additionally, and also consistent with the varied condition, the instructors were required to assign the same set of in-class exercises during the practice stage. Overall, the main facet of the non-varied method of instruction was for instructors to consistently explain all of the rules, concepts, and procedures to students as well as to provide opportunities for students to practice and receive individual feedback.

Procedures

First, approval was obtained by faculty and administrators within the researcher's mathematics department to conduct a study comparing methods of instruction using students from the researcher's institution. Then, full-time and part-time mathematics instructors were lobbied to volunteer as instructors in the study under the condition that they were willing to implement specific methods of instruction and teach two sections of

Beginning Algebra at the same time on alternating days. After the instructors were identified, training sessions were scheduled to occur during the semester prior to the start of the study.

The participating instructors attended five training sessions addressing the following topics: overview of the study, methods of instruction, achievement tests and scoring, course policies, and summary of teaching expectations. During the two-hour overview session, instructors were provided with a prospectus of the study. At the conclusion of this session, the instructors were familiar with the purpose of the study, the supporting literature, and the proposed methodology.

The methods of instruction session was conducted about one week later. During this two-hour session, instructors were provided with lesson plans, treatment logs, cooperative activity structures, and accompanying handouts (see Appendices J, L, and M). After reviewing the materials, each instructor was given an opportunity to practice implementing a cooperative activity using the other instructors as mock-students. At the conclusion of this session, the instructors claimed to be comfortable with the required teaching practices for both conditions.

After about another week, the third session was conducted. This two-hour session was dedicated to reviewing the achievement measures and rubrics (see Appendices F, G, and N). After reaching consensus on the validity of each test, the instructors were given opportunities to practice-score student responses collected during pilot testing. After three rounds of scoring and discussion the instructors reached 100% agreement, defined as scoring an item within one point of one another on items that ranged from one to six points. At the conclusion of the session, the instructors claimed to be familiar with the test administration and scoring procedures required for the study.

Less than one week later the fourth session was conducted in which the instructors agreed on a common syllabus and calendar to be used in all of the classes (see Appendix K). This session lasted about an hour and a half. At the end of the session, all of the instructors were in agreement on common pacing and class policies, such as grade weights and attendance policy.

The final session occurred about a month later and lasted approximately two hours. The purpose of this session was to once again review the procedures for the study after allowing sufficient time for the instructors to familiarize themselves on a deeper level with all of the materials provided to them during the previous four sessions. At the conclusion of this session, the instructors claimed to feel comfortable with what was expected from them in terms of methods of instruction and test administration.

Beginning around the same time as the training sessions, students were self-enrolling into various sections of Beginning Algebra at the participating institution. As such, on the first day of the semester in which the study occurred, the researcher attended each participating class and explained the purpose of the study before distributing and collecting student consent forms (see Appendix O). After consent was granted, the participants were taken to a computer lab to begin the administration of background instruments.

The Background Survey, Social Preference Questionnaire, Personal Interest Questionnaire, and ability assessments were administered during the first class session, which required about an hour to complete for the average student. Ninety minutes were allotted for testing and students were not allowed to leave until the official end-of-class time, so there was no incentive to finish early. The Prior Knowledge Test was administered during the fourth class session and was listed in the syllabus as 5% of the students' overall course grades; as such, students were encouraged to do their best on the Prior Knowledge Test.

With the exception of the Prior Knowledge Test, all of the data from the background instruments were collected using an online survey and downloaded into SPSS for analysis. The participating instructors were not granted access to these data while the study was being conducted. The Prior Knowledge Test, on the other hand, was administered to students by their instructor using a paper-and-pencil format with scantron scoring. These scores accounted for a percentage of the students' overall class grades and were thus known by the instructors.

After the first three class sessions, the instructors began Unit 1 and correspondingly applied the varied and non-varied instructional methods within their respective classes. Throughout the following eight weeks, the Treatment Implementation Logs were collected by the researcher at the end of each unit during face-to-face meetings. At the conclusion of a unit, the instructors administered the corresponding achievement test to their students. Grading was conducted collaboratively with the researcher present so that any uncertainty in the rubric scoring procedure could be discussed as a group and settled immediately. Afterwards, the students' scores were entered into SPSS.

At the conclusion of Unit 3, the instructors administered the Transfer Test in their classes as an extra-credit opportunity for students. As with the achievement tests, the

Transfer Test was also graded by both the researcher and the participating instructors using rubric scoring (see Appendix N for the rubric used on the Transfer Test), and the scores were entered into SPSS. Then, after the Transfer Test, the classes were taken to a computer lab to complete the very brief Situational Interest Questionnaire that was administered electronically.

Also at the conclusion of Unit 3, the instructors were asked to meet for a final debriefing meeting during which the overall experiences of each instructor were discussed as well as recommendations for the remainder of the semester. Two weeks later, the instructors were required to submit a copy of their current class roster so that course retention rates could be calculated.

Pilot Procedures

Most of the procedures and instruments were pilot tested with students enrolled in the researcher's Beginning Algebra course during the semester prior to the study. The students were self-enrolled so it is likely that the students were similar to those who participated in the study, and as such the findings from the pilot procedures may be generalizable to other students enrolled in Beginning Algebra at the same institution. However, the sample size was small; only 25 students were administered all of the instruments, so results needed to be interpreted with caution.

For each pilot-tested instrument, descriptive data and reliability estimates were obtained and are summarized in Table 6. To improve instrument items, a combination of judgmental and empirical techniques were implemented (Popham, 2000). Student judgments were collected through informal discussion with students after the test administration, and difficulty indices, which are obtained by dividing the number of correct responses by the total number of responses, were obtained for each item as an empirical technique. More information on the administration of each instrument is provided next.

Table 6

Descriptive Statistics and Cronbach's Coefficient Alphas for Each Pilot-Tested Instrument

Instrument	Min	Q1	Med	Q3	Max	M	SD	α
Social Preference Questionnaire	12.0	22.0	27.0	30.0	33.0	25.5	5.6	.82
Personal Interest Questionnaire	4.0	11.0	16.0	22.0	28.0	16.7	6.2	.92
Letter Series	1.0	7.0	8.0	9.0	10.0	7.7	2.1	.74
Letter Sets	3.0	5.0	9.0	13.0	15.0	9.3	4.0	.84
Figure Analogies	1.0	8.0	10.0	12.0	13.0	9.1	3.5	.79
Synonyms	2.0	6.0	7.0	8.0	11.0	6.7	2.0	.45
Sentence Completion	3.0	4.8	6.0	9.0	10.0	6.2	2.3	.67
Prior Knowledge Test	26.0	36.0	39.0	42.0	45.0	38.5	6.2	--
Unit 1 Achievement Test	5.0	17.5	23.0	27.0	33.0	21.3	6.7	.76
Unit 2 Achievement Test	3.0	19.9	25.0	28.1	35.0	23.3	7.9	.81
Unit 3 Achievement Test	3.0	12.5	19.3	23.6	33.0	18.5	7.7	.78
Near Transfer	0.0	0.0	0.5	2.5	3.0	1.0	1.1	N/A
Far Transfer	0.0	1.0	1.5	2.0	3.0	1.4	1.0	N/A
Situational Interest Questionnaire	9.0	13.5	15.5	18.0	21.0	15.3	3.2	.85

First, the students were notified on the first day of class that the Background Survey, Social Preference Questionnaire, Personal Interest Questionnaire, ability measures, and Situational Interest Questionnaire would be administered throughout the semester for extra credit participation points. The Prior Knowledge Test, achievement tests, and Transfer Test were not mentioned because these assessments were written into the course syllabus as mandatory tests that contributed to the students' overall grades.

During the third week of class, the Background Survey and Social Preference Questionnaire were administered to students using an online survey program via a link sent to students' emails. The Background Survey exhibited no technical problems and all the data were appropriate for the desired variables. The Social Preference Questionnaire initially contained 15 items attempting to measure three constructs based on Dunn and Dunn's (1972) sociological element of learning styles: preferences for working alone, preferences for working in pairs, and preferences for working in groups. Additionally, 12 items were included to measure preferences for three learning modalities consistent with Dunn and Dunn's perceptual elements of learning styles: auditory, perceptual, and tactile. A principal components analysis on the 15 sociological items yielded one component for all the items, which was interpreted as a preference for working with others. Therefore, several items were eliminated for redundancy resulting in a final set of four items. A principal components analysis on the 12 learning modality items yielded inconclusive and anomalous results, and were therefore dropped from the study.

Similarly, the Personal Interest Questionnaire was administered electronically to students during the fourth week of the semester. The original survey consisted of eight items used by Mitchell (1993) and Hulleman et al. (2010) in studies on mathematics interest. A principal components analysis yielded a single construct. However, upon further analysis, several items were dropped for redundancy until a final set of four items was obtained.

All three tests that were included in the *Gf* Assessment were pilot tested. The letter series test contained 10 items from the original 20 in the test of Primary Mental Abilities (Thurnstone, 1962), but four of them were too easy with difficulty indices greater than .90. Therefore, two of the items were replaced with items that were likely to have more appropriate difficulty indices. The letter sets test originally contained 30 items, but 15 were dropped for redundancy and inappropriate difficulty indices, resulting in a final test of 15 items.

The figure analogies test contained 17 items, but five were dropped for having high difficulty indices, resulting in a final set of 12 items. Scores on the letter series and figure analogies tests correlated highly ($r = .85$), which suggested that both assessments measured fluid intelligence. With respect to time, the last students to finish each test required an average of 1 minute per item, therefore it was determined that the letter series, letter sets, and figure analogies tests would require approximately 10 minutes, 15 minutes, and 12 minutes, respectively.

Similarly, both tests comprising the *Gc* Assessment, the synonyms test and sentence completion test, were pilot-tested and several items of inappropriate difficulty were identified and dropped resulting in a set of 12 synonym items and 10 sentence completion items. Scores on these two assessments correlated highly ($r = .70$) which indicated that both tests measured verbal ability. With respect to time, the last students to finish each test required much less time per average than on the fluid ability tests; just over half a minute per item. Therefore, the 12-item tests will require at most eight minutes to administer.

Overall, the five ability tests loaded onto one component in a principal component analysis, which provided evidence to support the construct of a general intelligence factor, *g*. Additionally, the *Gf* and *Gc* Assessments loaded onto two distinct constructs as expected. Therefore, the current study consisted of all five ability tests.

On the third day of class, after two days of brief review, the students were administered the 50-item Prior Knowledge Test in paper-and-pencil format with a scantron. Then, the distribution of scores and difficulty indices were analyzed and eight items with indices less than .25 (just above the probability of guessing a correct answer from 5 choices) or more than .90 were removed and replaced by items that are likely to have more appropriate difficulty indices to increase the variability in test scores. Given that the items were scored using a traditional scantron machine, the distribution of scores for each individual item were unattainable, therefore Cronbach's coefficient alpha could not be obtained. With respect to time, approximately 50% of the students completed the test within 30 minutes, and everyone was done within 60 minutes.

The three achievement tests were also pilot tested and analyzed. In each of the three tests, one or two items from the three subscales were replaced due to inappropriate difficulty indices. Additionally, based on student judgments, several of the conceptual understanding (CU) items were reworded to be more explicit. For example, two-part CU items including the directive "identify and interpret", were rewritten to emphasize that the answer requires two distinct responses: an identification and an explanation of meaning. In regard to time, all students were finished with each achievement test in under an hour.

Overall, the three tests loaded onto one component in a principal components analysis, implying the existence of a single construct that may represent overall mathematics achievement. Additionally, the three total scores from the achievement tests correlated significantly with each other, ranging from .72 to .81.

In terms of the subtests on each achievement test, a mean correlation of .54 was observed between low-complexity procedural application (LCPA) and high-complexity procedural application (HCPA) items, a mean correlation of .52 was observed between CU and LCPA items, and a mean correlation of .58 was observed between CU and HCPA problems. Additionally, over the duration of the semester, average correlations of .62, .31, and .71 were observed for CU, LCPA, and HCPA items, respectively. The low average correlation between LCPA items was somewhat surprising, but this could be due in part to the small sample sizes and items with inappropriate difficulty indices. Overall, each subtest appeared to be measuring overall mathematics achievement in addition to the targeted constructs of conceptual understanding and procedural application.

The Situational Interest Questionnaire was administered to students on the same day as the Unit 3 Achievement Test. The original survey consisted of seven items used by Mitchell (1993) and Hulleman et al. (2010) in studies on situational interest in math classrooms. A principal components analysis yielded two constructs and a reliability analysis identified multiple items that were inconsistent with the overall scale. Therefore, several items were removed until the items loaded onto a single factor and a sufficient reliability estimate was obtained. The correlation between the final items on the Situational and Personal Interest Questionnaires was .44, suggesting that the two constructs are somewhat related, but overall distinct; this finding was confirmed by a principal components analysis on the individual items that resulted in two components.

Lastly, the Transfer Test was administered to students after the Unit 3 Achievement Test. The scores from the near transfer item did not have sufficient variability in scores; there was a floor effect in which have the students received no more than half a point out of a possible three. Therefore, the equations contained in the item were replaced with easier ones. Alternatively, the far transfer test yielded scores with sufficient variation and will not be changed.

Data Analyses

This section has two subsections: preliminary analyses and analyses for research questions. First, missing data procedures will be described. Then, descriptive statistics, reliabilities, and correlations for instrument scores will be provided. Next, component analyses will be presented resulting in the reduction of the final set of variables under investigation. Finally, an outline of the analyses that were used to address each research question is provided.

Preliminary Analyses

Missing Data

Given that the Background Survey, Social Preference Questionnaire, Personal Interest Questionnaire, and ability assessments were administered electronically on the first day of class, there were no missing data for these instruments. Regarding the remaining instruments, Table 7 contains a summary of the number of missing scores, which never exceeded 10% of the enrollment. Prior to computing the descriptive statistics, missing scores on each assessment were replaced with overall sample means.

Table 7

Number of Drops Before Each Achievement Test, Current Enrollment at the Time of Each Achievement Test, and the Number of Missing Test Scores out of the Current Enrollment on each Achievement Test

	Instructor 1		Instructor 2		Instructor 3		Total	
	Varied	Non-Varied	Varied	Non-Varied	Varied	Non-Varied	Varied	Non-Varied
Initial Enrollment	21	31	25	28	15	19	61	78
Prior Knowledge Missing	1	0	2	1	0	1	3	2
Unit 1								
Drops	1	3	2	3	2	6	5	12
Enrollment	20	28	23	25	13	13	56	66
Missing	0	0	3	2	0	3	3	5
Unit 2								
Drops	0	0	0	1	0	1	0	2
Enrollment	20	28	23	24	13	12	56	64
Missing	0	0	2	2	0	0	2	2
Unit 3								
Drops	3	0	1	1	2	0	6	1
Enrollment	17	28	22	23	11	12	50	63
Missing	0	2	1	2	1	1	2	5
Final Enrollment[a]	16	26	16	22	10	10	42	58
Total Dropped	5	5	9	6	5	9	19	20

Note. [a]Represents the number of students enrolled two weeks after the completion of Unit 3.

Descriptive Statistics

Table 8 summarizes the total scores obtained by participants on the Social Preferences Questionnaire, Personal Interest Questionnaire, ability assessments, and Prior Knowledge Test for each class and combined treatment group (Table P1 in Appendix P contains additional descriptive statistics for each instrument using the overall sample). Total scores of 16 on the Social Preference Questionnaire and Personal Interest Questionnaire may be interpreted as neutral scores, whereas scores greater than 16 reflect higher levels of each construct, and scores less than 16 reflect lower levels of each construct. Overall, it appears that students have a slight preference for working in groups and have a slightly negative view towards mathematics.

Scores on the three ability tests may be interpreted in terms of percentage correct by dividing the Letter Series, Letter Sets, and Figure Analogies total scores by 10, 15, and 12, respectively. Therefore, total scores of 5, 7.5, and 6 represent 50% correct on the Letter Series, Letter Sets, and Figure Analogies tests, respectively. Given that the scores were not compared against a national norm, these scores may not be interpreted beyond relative standing among classes, teachers, and treatment groups.

Scores on the Synonym, Sentence Completion, and Prior Knowledge tests may be interpreted similarly, with maximum scores of 12, 10, and 50, respectively. Therefore, scores of 6, 5, and 25 represent 50% correct on the three tests. One additional interpretation relative to the Prior Knowledge test was that a score of 35 represented 70% correct, which may be considered low given that the test consisted of prerequisite material from a Pre-Algebra course.

Table 8

Means and Standard Deviations of Social Preference, Personal Interest, Ability, and Prior Knowledge Measures

	Instructor 1				Instructor 2				Instructor 3				Total			
	Varied (n = 21)		Non-Varied (n = 31)		Varied (n = 25)		Non-Varied (n = 28)		Varied (n = 15)		Non-Varied (n = 19)		Varied (n = 61)		Non-Varied (n = 78)	
Variable	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
Social Preference	16.9	4.9	18.2	4.5	19.1	4.5	18.2	5.6	17.8	4.8	19.6	5.7	18.0	4.7	18.5	5.2
Personal Interest	13.6	5.6	15.1	5.7	15.8	6.1	14.3	5.2	14.5	6.5	15.8	5.2	14.8	6.0	15.0	5.4
Gf																
Letter Series	4.4	2.5	5.0	2.8	6.0	2.4	4.8	2.9	6.5	2.2	5.1	2.4	5.6	2.5	5.0	2.7
Letter Sets	7.6	3.3	8.2	3.2	9.8	3.3	8.8	3.9	9.9	4.1	7.8	3.3	9.1	3.6	8.3	3.5
Figure Analogies	6.1	2.4	5.8	2.4	6.4	2.1	5.6	2.4	6.3	2.5	4.9	2.3	6.3	2.3	5.5	2.4
Gc																
Synonyms	5.9	2.6	6.2	2.1	6.4	2.7	7.3	2.6	7.3	2.1	6.6	2.6	6.5	2.5	6.7	2.4
Sentence Completions	4.8	2.4	5.3	1.9	6.2	2.1	5.6	2.2	6.7	2.1	5.8	2.7	5.8	2.3	5.5	2.2
Prior Knowledge	34.1	6.9	33.8	6.6	37.6	5.7	34.8	8.0	32.7	7.9	33.5	5.5	35.2	6.9	34.1	6.8

Note. No statistically significant differences were obtained among classes, teacher, nor treatment groups at the .05 significance level.

It is important to point out that no statistically significant differences in average total scores among classes or teachers were found at the .05 significant level. Additionally, there were also no statistically significant differences in mean scores between the treatment groups; the largest difference was in Figure Analogy scores, which was not statistically significant ($F(1, 137) = 3.68,\ p = .057$).

In other words, there was evidence to suggest that the varied and non-varied groups were equivalent with respect to preferences for learning socially, personal interest in mathematics, reasoning ability, verbal ability, and prior knowledge. This is an important finding because students were not randomly assigned to the varied and non-varied groups, so it was possible that the two groups differed on one or more of these variables.

Table 9 includes means and standard deviations of the total scores obtained on the three achievement tests, Transfer Test, and Situational Interest Questionnaire (Table P1 in Appendix P includes additional descriptive statistics of each instrument using the overall sample). Each Conceptual Understanding (CU) subtest was out of 10 points, so the scores in Table 9 reflected very low levels of conceptual understanding across all classes. The first two Low Complexity Procedural Application (LCPA) subtests were also out of 10 points, and the third LCPA subtest was out of 8 points. The percentage correct on the LCPA subtests ranged from 68% to 76%, which may be considered low given that LCPA items were, by definition, of low complexity in terms of the thought processes required to solve them.

Table 9

Means and Standard Deviations of Achievement, Transfer, and Situational Interest Measures

	Instructor 1				Instructor 2				Instructor 3				Total			
	Varied		Non-Varied		Varied		Non-Varied		Varied		Non-Varied		Varied		Non-Varied	
Variable	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
Unit 1																
CU	4.4	2.4	4.7	2.3	3.9	1.9	3.5	2.0	4.7	2.2	3.7	2.0	4.2	2.1	4.0	2.2
LCPA	7.2	1.6	7.1	1.8	7.6	1.8	7.1	2.0	7.5	1.8	7.3	1.4	7.4	1.7	7.1	1.8
HCPA	13.9	5.5	14.9	4.4	15.5	4.1	13.4	5.3	12.3	5.9	13.1	4.3	14.2	5.2	14.0	4.8
Unit 2																
CU	5.1	2.0	5.9	2.9	3.6	1.9	4.5	1.9	4.5	2.1	3.6	2.1	4.4	2.1	5.0	2.5
LCPA	7.5	2.2	7.6	1.9	6.8	1.8	7.1	2.5	6.8	2.0	7.4	2.2	7.0	2.0	7.4	2.2
HCPA	11.7	5.2	12.1	4.2	9.7	5.0	10.7	5.1	7.8	4.4	9.6	6.0	9.9	5.1	11.1	4.9
Unit 3																
CU	3.3	2.4	3.7	2.3	3.5	2.5	4.2	2.2	3.4	2.0	2.7	2.0	3.4	2.3	3.7	2.3
LCPA	5.4	1.6	5.5	1.6	5.6	1.5	5.6	1.5	5.8	1.7	5.4	1.7	5.6	1.6	5.7	1.6
HCPA	11.0	4.7	10.8	6.6	8.6	6.5	11.2	6.8	7.3	4.0	7.1	4.6	9.1	5.5	10.2	6.5
Transfer																
Near	0.9	0.7	0.7	0.6	0.7	0.8	0.9	0.9	1.7	1.1	1.3	0.7	1.0	0.9	0.9	0.8
Far	2.1	0.8	1.8	0.9	2.0	0.9	2.0	0.5	2.2	0.8	2.6	0.5	2.1	0.9	2.0	0.8
Situational Interest	12.2	3.7	15.4	3.8	14.7	3.0	15.4	3.0	11.7	4.5	13.8	2.9	13.1	3.8	15.1	3.4

Note. Statistics are based on the number of students currently enrolled at the time of the test (see Table 1). CU = Conceptual Understanding; LCPA = Low Complexity Procedural Application; HCPA = High Complexity Procedural Application.

Similarly, the first two High Complexity Procedural Application (HCPA) subtests were out of 20 possible points, and the third HCPA subtest was out of 22 points. The percentage correct on the HCPA subtests ranged from 32% to 78%. The high variability in percentages may be due in part to the increased difficulty of each unit, with the Unit 3 Achievement Test consisting of the most difficult problems relative to the other two achievement tests. Overall, the mean totals reflected achievement scores consistent with that of the participating institution in which an estimated 55% of first-time Beginning Algebra students passed the course in the Spring semester of the previous year (California Community Colleges Chancellor's Office, 2015).

Near Transfer and Far Transfer were each assessed using one item graded on a three-point rubric. The percentage correct on the near transfer item ranged from 23% to 57%, whereas the percentage correct on the far transfer item ranged from 60% to 73%. Therefore, students performed better on the far transfer item than on the near transfer item across all classes, with the latter appearing to be very challenging for students.

Scores on the Situational Interest Questionnaire may be interpreted similarly to the Personal Interest Questionnaire, however, a total score of 12 reflected a neutral stance because there were only 3 items. Therefore, for the most part, students appeared to enjoy their mathematics classes; only one class showed an average response just below 12 whereas the others obtained averages up to 15, which reflects an average response in agreement with the situational interest items.

Table 10 contains correlations among the variables in the study, with reliability estimates of the scores along the main diagonal. The variables were organized into three groups: entry characteristics, achievement throughout the study, and additional measures

Table 10

Pearson Product-Moment Correlations Among All Instruments for the Total Sample with Cronbach's Coefficient Alphas Along the Main Diagonal

Variable	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1. Social Preference	*.80*																				
2. Personal Interest	-.05	*.87*																			
3. Letter Series	-.04	-.02	*.77*																		
4. Letter Sets	-.02	.10	.60	*.80*																	
5. Figure Analogies	.02	.01	.50	.52	*.56*																
6. Synonyms	-.20	-.18	.22	.19	.18	*.65*															
7. Sentence Comp	-.15	-.26	.39	.25	.22	.57	*.67*														
8. Prior Knowledge	-.16	.05	.36	.32	.27	.21	.26	*.85*													
9. CU1	-.10	.03	.33	.12	.31	.20	.23	.32	--												
10. LCPA1	-.18	.14	.34	.16	.32	.29	.24	.56	.48	*.31*											
11. HCPA1	-.10	.12	.29	.18	.32	.19	.22	.56	.64	.66	--										
12. CU2	-.20	.01	.21	.09	.08	.14	.23	.31	.56	.51	.55	--									
13. LCPA2	-.06	.01	.33	.20	.29	.36	.23	.50	.51	.49	.53	.49	*.63*								
14. HCPA2	-.05	.01	.23	.19	.23	.22	.20	.45	.60	.52	.68	.64	.64	--							
15. CU3	-.04	-.02	.41	.35	.37	.22	.37	.46	.49	.41	.47	.45	.56	.52	--						
16. LCPA3	-.10	-.09	.33	.24	.20	.40	.32	.36	.35	.39	.38	.27	.41	.30	.43	*.49*					
17. HCPA3	-.12	.06	.17	.15	.19	.15	.15	.42	.53	.41	.59	.58	.56	.71	.62	.41	--				
18. Near Transfer	.00	.03	.30	.19	.21	.30	.30	.36	.35	.33	.34	.20	.34	.30	.44	.29	.36	--			
19. Far Transfer	-.06	.01	.09	.05	.09	.13	.25	.07	.07	.22	.12	.07	.12	.12	.09	.12	.05	.26	--		
20. Situational Interest	.02	.16	.16	.02	.00	.04	.01	.10	.14	.17	.23	.15	.22	.24	.21	.26	.37	.04	.08	*.71*	
21. Retention[a]	.04	.07	-.05	-.03	.02	.07	-.21	.18	.24	.23	.33	.23	.22	.38	.17	.10	.19	.18	-.08	.15	--

Note. Correlations among entry characteristics (1 through 8) were based on the original sample of 139 students, whereas correlations involving variables 9 through 21 were based on sample sizes equal to the total enrollment at the time of the assessment (see Table 1). CU = Conceptual Understanding; LCPA = Low Complexity Procedural Application; HCPA = High Complexity Procedural Application.

[a]Point-Biserial correlations are provided for Retention

*Correlations greater than or equal to .25 were statistically significant at the .01 significance level.

given at the end of the study. A correlation coefficient of .00 represents no linear relationship, whereas a correlation of 1.00 represents a perfect linear relationship between the two variables. Correlations of .20, .40, .60, and .80 represent weak, moderate, strong, and very strong linear relationships between two variables, respectively (Salkind, 2008).

Notice that the three affective variables correlated weakly with all variables, including themselves. The three *Gf* measures correlated moderately to strongly among themselves, as did the two *Gc* measures. Also of importance are the moderate to strong correlations among the achievement variables and Prior Knowledge. Additionally, there were weak to moderate correlations between Near Transfer and the achievement variables, but mostly very weak correlations among Far Transfer and the achievement variables. Finally, notice that Retention correlated very weakly with all variables.

Component Analyses

Data were collected on 21 variables for each participant: two pre-instruction affective variables (Social Preferences and Personal Interest), five cognitive ability variables (Letter Series, Letter Sets, Figure Analogies, Synonyms, and Sentence Completions), one prior knowledge variable, nine achievement variables (CU, LCPA, and HCPA for each of three achievement tests), one near transfer variable, one far transfer variable, one post-instruction affective measure (Situational Interest), and one class retention variable. In the case where more than one variable was used to collect data on a particular construct, namely affect, ability, and achievement, a principal component analysis was conducted to investigate whether each set of variables could be reduced to a smaller set of components, where each component represented a linear combination of the original variable scores. An additional component analysis on the scores from the Situational Interest Questionnaire was conducted to obtain evidence of the validity of including a situational interest construct in addition to personal interest.

Components with eigenvalues greater than one were extracted from the analyses. The scores on the resulting components were standardized with means of 0 and standard deviations of 1, whereas the original variable scores represented total scores obtained from each instrument. Additionally, each analysis utilized a Varimax rotation so that multiple components extracted from the same set of variables were uncorrelated. The results of each analysis are described in turn.

Pre-instruction affective variables. A principal components analysis with Varimax rotation on the total set of 8 items comprising the Social Preference Questionnaire and Personal Interest Questionnaire resulted in the extraction of 2 components. Table 11 contains the loadings of each item onto the two components. Based on these results it may be inferred that the set of 8 items indeed measured two distinct constructs: preferences for working with groups, called Social Preference, and levels of interest in mathematics, called Personal Interest.

Scores on Social Preference and Personal Interest represented a linear combination of the scores on each of their respective items using weights equal to the component loadings shown in Table 11, which then become standardized with a mean of 0 and standard deviation of 1. For example, assume that the scores for a particular student on the Personal Interest Questionnaire were 4, 6, 3, and 4, yielding a total score of 17. This student's Personal Interest score was approximately $4(.838) + 6(.809) + 3(.840) + 4(.903) = 14.34$, which was then standardized using the mean and standard deviation of the set of Personal Interest scores resulting in a final Personal Interest score of about 0.49. In this case, the score of 0.49 signifies that this particular student scored about half a standard deviation above the average Personal Interest score.

Table 11
Component Loadings from the Component Analysis on Items from the Social Preference and Personal Interest Questionnaires

Item	Loading on Component 1	Loading on Component 2
SP 1	.717	
SP 2	.862	
SP 3	.730	
SP 4	.840	
PI 1		.838
PI 2		.809
PI 3		.840
PI 4		.903

Note. Loadings less than .3 are omitted. SP = Social Preference ; PI = Personal Interest.

Ability variables. A similar analysis was conducted on the scores from the five ability measures. However, because there were 61 total items, the analysis was conducted using the total scores generated by the five instruments as opposed to using the individual item scores. Two components were extracted, and the loadings on each component are presented in Table 12. As expected, the three *Gf* measures loaded onto one component, and the two *Gc* measures loaded onto another. Therefore, the two components will henceforth be called Reasoning Ability and Verbal Ability.

Table 12
Component Loadings from the Component Analysis on Total Scores on the Ability Assessments

Instrument	Loadings on Component 1	Loadings on Component 2
Letter Series	.805	
Letter Sets	.846	
Figure Analogy	.808	
Synonyms		.888
Sentence Completion		.859

Note. Loadings less than .3 are omitted.

Achievement variables. A principal components analysis was conducted on the total scores obtained on the Conceptual Understanding (CU), Low Complexity Procedural Application (LCPA), and High Complexity Procedural Application (LCPA) subtests of the Unit 1 Achievement Test, resulting in the extraction of a single component. Table 13 contains the loadings on the extracted component that will hereafter be called Unit 1 Achievement, which represents a linear combination of scores obtained on both the conceptual understanding and procedural application items.

The fact that the three variables loaded onto the same component indicated that students scored similarly on items requiring conceptual understanding and items requiring procedural application. Put differently, students who scored well on one subtests tended to score well on the others, which is evidenced by the moderate correlations ranging from .43 to .66 among the three subtests in Table 10. As such, it may be assumed for the remainder of the study that differences in Unit 1 Achievement reflect differences in conceptual understanding as well as procedural application.

Table 13

Component Loadings from the Component Analysis on Total Scores of the Unit 1 Achievement Test Variables

Variable	Loading on Component 1
CU1	.820
LCPA1	.835
HCPA1	.905

Note. CU = Conceptual Understanding; LCPA = Low Complexity Procedural Application; HCPA = High Complexity Procedural Application.

Similar component analyses were conducted on the Unit 2 and Unit 3 Achievement Test variables, which also resulted in the extraction of a single component. Table 14 and Table 15 contain the component loadings for the Unit 2 and Unit 3 analyses, respectively. The resulting components are interpreted similarly to Unit 1 Achievement, and will hereafter be called Unit 2 Achievement and Unit 3 Achievement.

Table 14

Component Loadings from the Component Analysis on Total Scores of the Unit 2 Achievement Test Variables

Variable	Loading on Component 1
CU2	.830
LCPA2	.829
HCPA2	.900

Note. CU = Conceptual Understanding; LCPA = Low Complexity Procedural Application; HCPA = High Complexity Procedural Application.

Table 15

Component Loadings from the Component Analysis on Total Scores of the Unit 3 Achievement Test Variables

Variable	Loading on Component 1
CU3	.856
LCPA3	.726
HCPA3	.847

Note. CU = Conceptual Understanding; LCPA = Low Complexity Procedural Application; HCPA = High Complexity Procedural Application.

Situational interest. A component analysis was conducted on the three items of the Situational Interest Questionnaire and the four items on the Personal Interest Questionnaire to obtain evidence for the inclusion of these two types of interest in the study. Component loadings from the analysis are shown in Table 16. Notice that Item 3 on the Situational Interest Questionnaire was inconsistent with the other two items, resulting in a lower loading.

Overall, the loadings in Table 16 indicated that the two questionnaires successfully measured two distinct constructs: personal interest in mathematics and situational interest in math class. The scores on component 1 represented a linear combination of scores on the situational interest questionnaire and will continue to be called Situational Interest.

Table 16
Component Loadings from a Component Analysis on Items from the Situational Interest and Personal Interest Questionnaires

Item	Loading on Component 1	Loading on Component 2
SI 1	.921	
SI 2	.895	
SI 3	.546	
PI 1		.882
PI 2		.789
PI 3		.858
PI 4		.898

Note. Loadings less than .3 are omitted. SI = Situational Interest ; PI = Personal Interest.

Summary

Table 17 contains the correlations among the components resulting from the just-described component analyses and the other variables. Additionally, Cronbach's coefficient alphas were calculated for each component (Kaiser, 1991) and provided along the main diagonal. As before, the rows are organized into three categories: entry characteristics, achievement throughout the study, and data collected at the end of the study.

Notice that as a result of the Varimax rotation, correlations of 0 were obtained between components extracted from the same component analysis. Also note that no coefficient alphas existed for the Near Transfer, Far Transfer, and Retention variables because they each consisted of single items and therefore lacked reliability estimates. Most of the reliabilities were consistent with those presented in Table 10, with the exception of Verbal Ability and Situational Interest; the reliabilities of these components were both very low despite adequate reliability of the original scores.

Table 17

Pearson Product-Moment Correlations Among All Component Scores for the Total Sample with Cronbach's Coefficient Alphas Along the Main Diagonal

Variable	1	2	3	4	5	6	7	8	9	10	11	12
1. Social Preference	*.69*											
2. Personal Interest	.00	*.76*										
3. Reasoning Ability	.01	.07	*.75*									
4. Verbal Ability	-.20	-.26	.00	*.20*								
5. Prior Knowledge	-.16	.05	.35	.21	*.85*							
6. Unit 1 Achievement	-.14	.12	.32	.26	.57	*.81*						
7. Unit 2 Achievement	-.11	.02	.24	.28	.49	.76	*.81*					
8. Unit 3 Achievement	-.10	-.01	.34	.32	.51	.65	.70	*.74*				
9. Near Transfer	.00	.03	.23	.31	.36	.40	.33	.45	--			
10. Far Transfer	-.07	.00	.06	.21	.07	.16	.12	.10	.26	--		
11. Situational Interest	.06	.14	.05	-.03	.06	.19	.22	.32	.02	-.10	*.55*	
12. Retention[a]	.04	.08	.00	-.16	.18	.31	.33	.17	.17	-.07	.14	--

Note. Correlations among entry characteristics (1 through 5) were based on the original sample of 139 students, whereas correlations involving variables 6 through 12 were based on sample sizes equal to the total enrollment at the time of the assessment (see Table 1).

[a]Point-Biserial correlations are reported for Retention

*Correlations greater than or equal to .25 were statistically significant at the .01 significance level.

Analyses For Research Questions

The first three research questions will be answered by comparing mean scores on Unit 1 Achievement, Unit 2 Achievement, Unit 3 Achievement, Near Transfer, Far Transfer, and Situational Interest between the varied and non-varied groups. In each case, an Analysis of Covariance (ANCOVA) will be conducted at the .05 significance level using the method of instruction as the grouping variable. Any entry variables that correlated significantly with the dependent variables will be used as covariates in the corresponding analysis, assuming there were no differences between the two groups on the covariates. If no covariates are identified, then an Analysis of Variance (ANOVA) will be conducted.

Prior to conducting the statistical tests, the required assumptions of homogeneity of variance and a normally distributed sampling distribution will be explored (Field, 2009). The homogeneity of variance assumption will be analyzed using Levene's test, and both ANCOVA and ANOVA provide alternative test statistics if this assumption is violated. The assumption of normality will be addressed using the Central Limit Theorem, which states that distributions of means calculated from samples of size greater than 30 are normally distributed. However, if the samples are not sufficiently large, then a Mann-Whitney test will be conducted, which is the non-parametric equivalent to ANOVA and does not assume normally distributed data.

Additionally, in the case of ANCOVA, another assumption is that the regression slopes obtained by regressing the dependent variable onto the covariate are equivalent for each group, called homogeneity of regression slopes. Violations in this assumption indicate that the method of instruction did not affect students within the same group equally due to an interaction with the covariate. For example, a violation in the homogeneity of regression slopes may occur if the varied method of instruction was more effective for students with high levels of prior knowledge than for students with low levels of prior knowledge. As a result, the students receiving each method of instruction may react differently depending on their individual characteristics, which renders meaningless any main effects due to instruction (Field, 2009).

The assumption of homogeneity in regression slopes will be tested using a custom ANCOVA that includes an interaction term between the covariate and method of instruction in addition to the main effects from an ordinary ANCOVA. Statistically significant p-values for the interaction term at the .05 level indicate a violation in the assumption. Additionally, violations in the assumption may be signaled by large differences in correlation coefficients among the covariates and dependent variables between the varied and non-varied groups. For example, if Prior Knowledge correlates .20 with Unit 1 Achievement for students in the varied group, but correlates .50 with Unit 1 Achievement in the non-varied group, then the assumption of homogeneity of regression slopes might not be tenable.

After the main analysis, statistically significant differences between the varied and non-varied groups at the .05 significance level will be analyzed at the class level to determine the extent to which the classes differed for each instructor. Additionally, Cohen's d effect size estimates will be calculated in which values of 0.20, 0.50, and 0.80 represent differences of small, medium, and large effect, respectively (Cohen, 1992).

The fourth research question will be answered using a 2 (varied or non-varied) x 2 (dropped or still enrolled) Chi-square test of independence at the .05 significance level.

The null hypothesis is that dropping the course was independent of the method of instruction for the course, the alternative being that the method of instruction affected course retention. The only assumption needing to be verified is that the number of students expected in each group is more than five. If this assumption is violated, then Fisher's exact test may be used in lieu of Pearson's Chi-square test.

CHAPTER IV

RESULTS

This chapter has seven sections: fidelity of treatment implementation, descriptive statistics, an analysis for each of the four research questions, and a summary. First, data are presented to demonstrate that the instructional treatments were administered as prescribed. Then, means, standard deviations, and correlation coefficients will be presented for all variables separated by treatment group. Last, analyses addressing each research question are outlined and a summary is provided.

Fidelity of Treatment Implementation

An analysis of the Treatment Implementation Logs and Researcher Notes revealed that the instructors implemented the lesson plans as planned. In the varied classes, the instructors reported developing, practicing, and closing for an average of 29.5 min ($SD = 13.0$), 32.8 min ($SD = 17.5$), and 9.6 min ($SD = 6.5$), respectively. In the non-varied classes, the instructors reported developing, practicing, and closing for an average of 28.0 min ($SD = 11.0$), 32.6 min ($SD = 17.8$), and 8.4 min ($SD = 6.2$), respectively. These data are very rough estimates, however, because the instructors provided only approximate times on their logs and included a margin of error of approximately five minutes.

In addition to adhering to the time recommendations for each stage of class, there was also evidence in the Treatment Implementation Logs to indicate that the instructors implemented a majority of the assigned teaching practices. However, all three instructors noted that they did not have enough time in the varied classes to implement some of the writing assignments during the closing stage. The instructors cited the extra time required for the development and practice activities as the main reason for not having enough time for the closing activities, and would sometimes replace a closing activity with a brief closing lecture. Overall, the instructors' logs indicated that skipping a writing activity occurred no more than half of the time one was recommended in the varied class, which represented a small percentage of overall class time. But, nonetheless, the inconsistent implementation of the writing assignments may have resulted in fewer opportunities for students to construct knowledge than were originally planned.

Despite the similar lengths of time spent in the varied and non-varied classes on each stage of class, the instructors reported that the non-varied classes were able to complete more problems and often progressed at a faster rate than students in their varied classes. For instance, Instructor 1 reported that several of her higher achieving students left her non-varied class early after they completed their individual assignments (personal communication, March 10, 2015). Additionally, Instructor 2 reported that his non-varied class was given an extra full-day review prior to the Unit 2 test because the class had finished the Unit 2 material a day ahead of the varied class (personal communication, March 27, 2015).

A common theme appearing in the Researcher Notes was that students in the varied classes experienced difficulties with the cooperative activities, such as the Jigsaw and Group Discovery (see Appendix L). For example, Instructor 3 observed that students did not feel comfortable learning concepts from their classmates in the Jigsaw activity and preferred to complete each part of the Jigsaw on their own (personal communication, February 26, 2015). Additionally, all three instructors observed that many students did not read the directions on discovery handouts, or had poor reading comprehension (personal communication, March 27, 2015). Although it is not clear exactly how many students exhibited these difficulties, all three instructors did indicate that implementing activities involving reading were challenging for students.

In sum, the Treatment Implementation Logs and Researcher Notes provided evidence that the instructors successfully implemented the varied and non-varied methods of instruction in their respective classes. Although there were a few instances of schedule changes and unexpected challenges, both methods of instruction were overall implemented as planned.

Descriptive Statistics

To assist in the forthcoming analyses, descriptive statistics for each variable are provided below. Table 18 contains the means and standard deviations of each standardized variable for both groups. The means in Table 18 represent the average distance of students' scores from the overall mean in standard deviation units. For example, consider the Social Preference means in Table 18; the score of .04 within the non-varied group signifies that the average Social Preference score of students in the non-varied group was .04 standard deviations above the mean score for all students in the study, whereas students in the varied group scored .05 standard deviations below the overall mean, on average. In other words, positive means in Table 9 reflect above-average scores, and negative means reflect below-average scores, with respect to the total sample.

Table 18
Means and Standard Deviations of all Standardized Variables for the Varied and Non-Varied Groups

	Varied		Non-Varied	
Variable	M	SD	M	SD
1. Social Preference	-0.05	0.93	0.04	1.05
2. Personal Interest	-0.03	1.05	0.03	0.96
3. Reasoning Ability	0.20	0.96	-0.15	1.01
4. Verbal Ability	-0.04	1.01	0.03	1.00
5. Prior Knowledge	0.08	1.00	-0.07	1.00
6. Unit 1 Achievement	0.06	1.00	-0.05	1.00
7. Unit 2 Achievement	-0.14	0.98	0.12	1.01
8. Unit 3 Achievement	-0.09	0.95	0.07	1.04
9. Near Transfer	0.09	1.08	-0.08	0.93
10. Far Transfer	0.03	1.07	-0.02	0.94
11. Situational Interest	-0.28	1.02	0.23	0.93

Table 19 contains the correlation coefficients among the variables for the participants in the varied group, and Table 20 contains the same for the non-varied group. As before, correlations near 1.0 reflect very strong linear relationships whereas correlations near 0.0 reflect very weak linear relationships. By comparing the correlations in the two tables, any differential effects of instruction on these variables may become apparent, which will assist in validating the forthcoming homogeneity of regression slopes assumption of ANCOVA.

Table 19

Pearson Product-Moment Correlations Among All Component Scores for the Varied Group

Variable	1	2	3	4	5	6	7	8	9	10	11	12
1. Social Preference	--											
2. Personal Interest	-.18	--										
3. Reasoning Ability	-.01	.05	--									
4. Verbal Ability	.03	-.20	.00	--								
5. Prior Knowledge	-.17	-.04	.43	.17	--							
6. Unit 1 Achievement	-.22	.20	.23	.27	.54	--						
7. Unit 2 Achievement	-.26	.07	.24	.25	.50	.78	--					
8. Unit 3 Achievement	-.05	.00	.33	.31	.52	.60	.70	--				
9. Near Transfer	.02	.04	.23	.21	.38	.37	.29	.49	--			
10. Far Transfer	-.13	.05	.19	.13	.12	.30	.21	.23	.32	--		
11. Situational Interest	.10	.27	.00	-.08	.08	.04	.06	.12	-.12	-.13	--	
12. Retention[a]	.05	.05	-.07	-.12	.10	.28	.29	.09	.30	-.19	-.07	--

Note. Correlations among entry characteristics (1 through 5) were based on the original sample of 139 students, whereas correlations involving variables 6 through 12 were based on sample sizes equal to the total enrollment at the time of the assessment (see Table 1).

[a]Point-Biserial correlations are reported for Retention

*Correlations greater than or equal to .32 were statistically significant at the .01 significance level.

Table 20

Pearson Product-Moment Correlations Among all Component Scores for the Non-Varied Group

Variable	1	2	3	4	5	6	7	8	9	10	11	12
1. Social Preference	--											
2. Personal Interest	.13	--										
3. Reasoning Ability	.03	.10	--									
4. Verbal Ability	-.35	-.32	.01	--								
5. Prior Knowledge	-.15	.12	.28	.25	--							
6. Unit 1 Achievement	-.07	.04	.39	.25	.59	--						
7. Unit 2 Achievement	-.01	-.04	.29	.30	.51	.77	--					
8. Unit 3 Achievement	-.14	-.02	.38	.33	.52	.71	.70	--				
9. Near Transfer	-.01	.02	.21	.42	.34	.42	.39	.44	--			
10. Far Transfer	-.02	-.04	-.06	.28	.03	.02	.04	.00	.21	--		
11. Situational Interest	.02	.03	.18	.01	.09	.39	.32	.47	.22	-.06	--	
12. Retention[a]	.02	.09	.07	-.19	.25	.39	.36	.24	.03	.08	.31	--

Note. Correlations among entry characteristics (1 through 5) were based on the original sample of 139 students, whereas correlations involving variables 6 through 12 were based on sample sizes equal to the total enrollment at the time of the assessment (see Table 1).

[a]Point-Biserial correlations are reported for Retention

*Correlations greater than or equal to .32 were statistically significant at the .01 significance level.

Research Question 1

To what extent will a varied method of instruction facilitate conceptual understanding and procedural application more effectively than a non-varied method of instruction?

Unit 1 Achievement

At the time of the Unit 1 Achievement Test, 56 students were enrolled in the varied group and 66 students were enrolled in the non-varied group. Given that each sample size was greater than 30, the Central Limit Theorem states that the sampling distribution of the means were normally distributed. Additionally, the results of Levene's test implied that the variance in Unit 1 Achievement for each group were not significantly different ($F(1, 120) = 0.13,\ p > .05$).

No statistically significant correlations were obtained between Unit 1 Achievement and Social Preference or Personal Interest. But, there were statistically significant linear relationships between Unit 1 Achievement and Reasoning Ability ($r = .32,\ p < .01$), Verbal Ability ($r = .26,\ p < .01$), and Prior Knowledge ($r = .57,\ p < .01$). There was no statistically significant difference in Verbal Ability ($F(1, 137) = 0.16,\ p > .05$) or Prior Knowledge ($F(1, 137) = 0.78,\ p > .05$) between the varied and non-varied groups, but there was a statistically significant difference in Reasoning Ability ($F(1,137) = 4.35,\ p < .05, d = 0.36$), with students in the varied group scoring slightly higher than the students in the non-varied group, on average, as seen in Table 18. However, the effect size of the difference was small, and given the lack of statistically significant differences in the three *Gf* measures between the varied and non-varied groups reported in Table 9, Reasoning Ability was still used as a covariate in the analysis along with Verbal Ability and Prior Knowledge.

The assumption of homogeneity of regression slopes was explored and the results implied that regression slopes under both methods of instruction were equivalent for Reasoning Ability ($F(1, 120) = 0.64,\ p > .05$), Verbal Ability ($F(1, 120) = 0.00,$ $p > .05$), and Prior Knowledge ($F(1, 120) = 0.18,\ p > .05$). This finding was consistent with the correlations among these variables in Table 19 and Table 20; Reasoning Ability, Verbal Ability, and Prior Knowledge correlated similarly with Unit 1 Achievement in both groups. Therefore, there was no interaction effect on Unit 1 Achievement between the method of instruction and students' levels of Reasoning Ability, Verbal Ability, and Prior Knowledge.

Relative to the main analysis, no statistically significant differences in Unit 1 Achievement were found between the varied and non-varied groups while controlling for individual differences in Reasoning Ability, Verbal Ability, and Prior Knowledge ($F(1, 120) = 0.03,\ p > .05$), which is consistent with the similar means shown in Table 18.

Unit 2 Achievement

Fifty six students were enrolled in the varied group and 64 students were enrolled in the non-varied group at the time of the Unit 2 Achievement test. Similar to the analysis of Unit 1 Achievement, the assumption of a normally distributed sampling distribution was satisfied by the Central Limit Theorem and the results of Levene's test implied that the variance in Unit 2 Achievement within each method of instruction were equal ($F(1, 118) = 0.03,\ p > .05$).

Unit 2 Achievement correlated significantly only with Verbal Ability ($r = .28$, $p < .01$) and Prior Knowledge ($r = .49,\ p < .01$). Additionally, as previously stated, there were no differences in Verbal Ability or Prior Knowledge between the varied and non-varied groups, so both variables were appropriate covariates in the analysis of Unit 2 Achievement.

The assumption of homogeneity of regression slopes was explored and the results indicated that regression slopes under both methods of instruction were equivalent for Verbal Ability ($F(1, 118) = 0.19,\ p > .05$) and Prior Knowledge ($F(1, 118) = 0.07$, $p > .05$), which was consistent with the similar correlations among these variables between Table 19 and Table 20. Therefore, there was no interaction effect on Unit 2 Achievement between the method of instruction and Verbal Ability or Prior Knowledge.

Relative to the main analysis, no statistically significant differences in Unit 2 Achievement between the varied and non-varied groups were found while controlling for individual differences in Verbal Ability and Prior Knowledge ($F(1, 118) = 3.48,\ p > .05$). This result is somewhat surprising based on the means in Table 18, and even more so given that the means after being adjusted for differences in Verbal Ability and Prior Knowledge were -0.16 for the varied group and 0.14 for the non-varied group. But, it appears to be the case that the difference in adjusted mean scores on Unit 2 Achievement was not large enough to reach statistical significance at the .05 level.

Unit 3 Achievement

By the time of the Unit 3 Achievement test, there were 52 students enrolled in the varied classes and 64 students enrolled in the non-varied classes. The Central Limit Theorem was invoked once again to satisfy the assumption of normality, and the results of Levene's Test implied that the variances in Unit 3 Achievement for both groups were equivalent ($F(1, 114) = 0.62,\ p > .05$).

Unit 3 Achievement correlated significantly with Reasoning Ability ($r = .34$, $p < .01$), Verbal Ability ($r = .32,\ p < .01$), and Prior Knowledge ($r = .51,\ p < .01$). No statistically significant differences were found in Verbal Ability ($F(1, 137) = 0.16$, $p > .05$) or Prior Knowledge ($F(1, 137) = 0.78,\ p > .05$) between the varied and non-varied groups, but, as discussed previously, there was a small difference in Reasoning Ability ($F(1,137) = 4.35,\ p < .05, d = 0.36$). Similarly to the analysis of Unit 1 Achievement, Reasoning Ability was still used as a covariate in addition to Verbal Ability and Prior Knowledge for the analysis of Unit 3 Achievement.

As with the analyses of Unit 1 Achievement and Unit 2 Achievement, no interaction effects on Unit 3 Achievement were found between the method of instruction and Reasoning Ability ($F(1, 114) = 0.14,\ p > .05$), Verbal Ability ($F(1, 114) = 0.14$, $p > .05$), nor Prior Knowledge ($F(1, 114) = 0.11,\ p > .05$). Therefore, the assumption of homogeneity of regression slopes was tenable, which is consistent with the correlations among these variables shown in Table 19 and Table 20.

No statistically significant differences in Unit 3 Achievement between the varied and non-varied groups were obtained while controlling for individual differences in Reasoning Ability, Verbal Ability, and Prior Knowledge ($F(1, 114) = 3.48,\ p > .05$). Although the means in Table 18 appear to be somewhat different, they only differ by about .16 standard deviations, which was not enough to reach statistical significance.

Research Question 2

To what extent will a varied method of instruction facilitate students' knowledge transfer more effectively than a non-varied method of instruction?

Near Transfer

Exactly 116 students completed the near transfer item: 52 students in the varied classes and 64 students in the non-varied classes. Given that the sample sizes were greater than 30, the Central Limit Theorem applied and satisfied the assumption of a normally distributed sampling distribution of the means. Results of Levene's test implied that the variances in Near Transfer within both groups were equal ($F(1, 114) = 1.75$, $p > .05$).

Verbal Ability ($r = .31,\ p < .01$) and Prior Knowledge ($r = .36,\ p < .01$) correlated significantly with Near Transfer, both of which did not differ significantly between the varied and non-varied groups. An analysis of the homogeneity of regression slopes resulted in no statistically significant interaction effects of Verbal Ability ($F(1{,}114) = 1.19,\ p > .05$) or Prior Knowledge ($F(1{,}114) = 0.22,\ p > .05$) on Near Transfer. Therefore, Verbal Ability and Prior Knowledge were used as covariates in the analysis of Near Transfer.

No statistically significant difference in Near Transfer was obtained between the varied and non-varied groups while controlling for individual differences in Verbal Ability and Prior Knowledge ($F(1, 114) = 0.56,\ p > .05$). Thus, although Table 18 indicates an observable difference in Near Transfer means in favor of the varied group, the difference was not large enough to reach statistical significance.

Far Transfer

The same number of students completed the far transfer item as completed the near transfer item, so the Central Limit Theorem also applied to the analysis of Far Transfer, thus satisfying the normality assumption. Results of Levene's test implied that the variances in Far Transfer within the varied and non-varied groups were equal ($F(1, 114) = 1.95,\ p > .05$).

None of the entry characteristic variables correlated significantly with Far Transfer, so there was no need to include a covariate in the analysis. No statistically significant difference in Far Transfer was obtained between the varied and non-varied groups ($F(1, 114) = 0.08,\ p > .05$), which was consistent with the means in Table 18.

Research Question 3

To what extent will a varied method of instruction affect students' situational interest compared to a non-varied method of instruction?

The assumption of normally distributed data was satisfied by the Central Limit Theorem because 52 and 64 students were enrolled in the varied and non-varied classes, respectively, at the time of the Situational Interest Questionnaire. Additionally, the results of Levene's test verified that the homogeneity of variances assumption was tenable ($F(1, 114) = 1.15,\ p > .05$).

None of the entry characteristic variables correlated significantly with Situational Interest, so there was no need to include a covariate in the analysis. An analysis of differences in Situational Interest between the varied and non-varied groups resulted in a main effect due to instruction ($F(1, 114) = 7.75,\ p < .05,\ d = 0.52$). This result is consistent with the means in Table 18 that show students in the varied classes reported lower levels of Situational Interest than students in the non-varied classes, on average.

Table 21 summarizes the means, standard deviations, and results from a Mann-Whitney test of differences in mean Situational Insterest for each instructor. Only the differences between classes taught by Instructor 1 were statistically significant at the .05 significance level. However, it is important to notice that the observed differences in average Situational Interest for all three instructors favored the non-varied classes, which likely contributed to the overall main effect of method of instruction on Situational Interest.

Table 21
Sample Sizes, Means, Standard Deviations, and Results of Mann-Whitney Tests for Situational Interest

	Varied			Non-Varied					
Instructor	n	M	SD	n	M	SD	z	p	d
Instructor 1	18	-0.59	1.02	28	0.31	1.06	-2.98	.003	0.89
Instructor 2	22	0.18	0.76	24	0.29	0.82	-0.56	.575	--
Instructor 3	12	-0.65	1.17	12	-0.11	0.82	-1.54	.123	--

Research Question 4

To what extent will a varied method of instruction affect course retention rates compared to a non-varied method of instruction?

Table 22 summarizes the results of a Chi-square test of independence between the method of instruction and the frequency of students who dropped. Notice that the expected values in each cell were greater than five, which satisfied the only assumption for the test. Overall, results indicated that there was no statistically significant deviation from the number of drops under each method of instruction that were expected by chance ($\chi^2(1) = 0.51,\ p > .05$).

Table 22
Observed and Expected Frequencies of Students Who Dropped From Each Method of Instruction

Status	Varied	Non-Varied	Total
Dropped			
Observed	19	20	39
Expected	17.1	21.9	39.0
Still Enrolled			
Observed	42	58	113
Expected	43.9	56.1	113.0
Total			
Observed	61	78	139
Expected	61.0	78.0	139.0

Summary

The assumptions of a normally distributed sampling distribution and homogeneity of variance were satisfied by the Central Limit Theorem and Levene's Test, respectively, for all significant tests. Additionally, no interaction effects were found between Reasoning Ability, Verbal Ability, or Prior Knowledge on any of the dependent variables, so the homogeneity of regression slopes assumption was validated for ANCOVA. Main analyses resulted in no statistically significant differences in Unit 1 Achievement, Unit 2 Achievement, Unit 3 Achievement, Near Transfer, or Far Transfer between the varied and non-varied groups. There was, however, a statistically significant difference in Situational Interest; students in the non-varied classes tended to enjoy their classes to a greater extent than students in the varied classes, on average. Lastly, a Chi-square test of independence between method of instruction and course retention yielded no statistically significant results.

CHAPTER V

SUMMARY, LIMITATIONS, DISCUSSION, AND IMPLICATIONS

This chapter begins with a summary of the study leading up to the research questions. Then, a summary of the findings is given, followed by a discussion of the limitations of the study. Subsequently, a discussion of the findings in light of the limitations is provided, which will lead to the conclusions of the study. This chapter then finishes with implications for research and practice.

Summary of Study

Developmental mathematics courses at community colleges may be defined as mathematics courses that contain content traditionally included in a K-12 curriculum, such as Pre-Algebra, Algebra, and Geometry. Success rates within these courses are typically very low; researchers have observed success rates as low as 30% (Attewell, Lavin, Domina, & Levey, 2006). The low success rates in developmental courses are a concern for students and educators because these courses are prerequisites for college-level courses needed to meet degree and transfer requirements. Moreover, developmental courses tend to enroll students of color at disproportionate rates; it has been estimated that over 75% of first-year community college African American and Latino students enrolled in developmental courses, compared to just over 50% of Caucasian students (Bailey, Jenkins, & Leinbach, 2005). Therefore, factors that affect student outcomes in developmental mathematics need to be investigated in order to increase student success rates and to provide equitable access to subsequent college-level courses.

There are many perspectives from which to investigate student outcomes in developmental mathematics, such as student and teacher motivation, support services on campus, and levels of administrative support to developmental programs. However, this study focused on methods of instruction, defined as an adopted set of in-class practices that facilitate student learning, such as lectures, discussions, and activities.

Based on observations of almost 150 developmental classrooms, Grubb and Gabriner (2013) concluded that the most prevalent method of instruction in developmental mathematics consisted of lecturing followed by individual seatwork. However, researchers investigating developmental mathematics programs identified a varied method of instruction, defined as instruction containing opportunities for student involvement in the learning process, as a common component among successful programs (Boylan, 2002; Epper & Baker, 2009). Therefore, it appears that a varied method of instruction may be related to increased learning, however, most developmental mathematics instructors are implementing a non-varied approach, consisting only of lectures and isolated practice.

Recommendations for implementing a varied method of instruction are prevalent in the literature. For example, Goldrick-Rab (2007) outlined the ineffectiveness of traditional developmental mathematics programs and proposed that active methods of instruction may better serve students, especially those who arrive at community college underprepared. Additionally, AMATYC (2006) recommended implementing varied instructional practices as a response to students' diverse learning styles. The underlying assumption was a varied method of instruction may provide all students with equal opportunities to learn in their preferred ways, although the resulting effects on student achievement remain debatable (Pashler, McDaniel, Rohrer, & Bjork, 2009).

Unfortunately, few experimental studies have attempted to isolate the effects of methods of instruction on student outcomes in developmental mathematics at the community college level (Mesa, 2008). Although Boylan (2002) and Epper and Baker (2009) provided support for using a varied method of instruction, it is also important to know that a varied method of instruction was just one of several reforms identified by the researchers. Other reforms included learning communities, contextualized curricula, and mandatory support services (Boylan, 2002; Epper & Baker, 2009; Fowler & Boylan, 2010). Hence, in these studies it was impossible to determine the effects of any single reform on the observed student outcomes. In other words, it remains unclear the relative impact, if any, the varied method of instruction had on student learning relative to the effects of learning communities, contextualized curricula, and support services. Therefore, the purpose of this study was to investigate the effects of a varied method of instruction on various student outcomes compared to a non-varied method of instruction, while controlling for the other reforms identified by Boylan (2002) and Epper and Baker (2009).

This study is important because students of color are enrolling in developmental mathematics classes at disproportionate rates. As such, the teaching practices applied within the classroom are being received by a diverse group of students that is not reflective of the general community college student population. Additionally, disentangling pedagogical factors that increase student success in developmental mathematics will contribute to the existing literature that is currently lacking in this area (Mesa, 2008).

A social constructivist framework was adopted as a theoretical foundation for the study because the varied method of instruction consisted primarily of various activities facilitating student learning through social interactions. Social constructivist perspectives of learning and instruction focus on the interdependence of social and individual processes in the co-construction of knowledge (Palincsar, 1998). One main tenet of social constructivism is that social interaction leads to deeper learning (Bereiter & Scardamalia, 1989; Lua, Singh, & Hwa, 2009). From this perspective, as learners participate in a range of interactive activities, they acquire new strategies and gain knowledge from one another (Palincsar, 1998).

One set of instructional practices that follows from social constructivist beliefs is called active learning. Meyers and Jones (1993) defined active learning as a method of instruction consisting of opportunities for students to talk, listen, read, and write about concepts in order to construct well-developed mental structures of the material. A similar method of instruction was also recommended by the American Mathematical Association of Two Year Colleges (AMATYC) in a standards publication for developmental mathematics instruction (AMATYC, 2006). Therefore, the varied method of instruction implemented in the current study utilized active learning strategies, which are grounded in social constructivism and recommended by a national organization aimed at improving mathematics instruction.

Due to the inclusion of explicit instruction in both the varied and non-varied methods of instruction, an additional theoretical framework for the current study was

Cognitive Load Theory (CLT). CLT posits that due to the limited capacity of our short-term memory (Miller, 1956) students need to focus their available cognitive resources on activities that are beneficial to learning (Sweller, Van Merrienboer, & Paas, 1998). When presented with material, it is believed that students encounter three types of cognitive load: (1) intrinsic load, which is determined by the complexity of the material; (2) extrinsic load, which is a characteristic of the presentation of the material; and (3) germane load, which is cognitive activity that is beneficial for learning. Researchers have posited that receiving explicit instruction reduces extraneous load, thereby increasing cognitive resources available for germane load (Owen & Sweller, 1985).

Using the aforementioned theoretical perspectives as a foundation, three volunteer teachers were trained to implement both varied and non-varied methods; each teacher was assigned to teach one class using each method. Consequently, differences in student outcomes that may be attributed to the style of the instructor were controlled. As a requirement of their participation, the instructors had to attend a series of training sessions addressing methods of instruction, course policies, and grading procedures.

All three instructors worked at the same community college, which was located in an urban setting in Northern California. The college was ideal for the study because it did not offer learning communities, contextualized curricula, or mandatory support services. Therefore, the effects of a varied method of instruction could be examined in the natural absence of other successful reforms identified by Boylan (2002) and Epper and Baker (2009).

Prior to the start of the Spring 2015 semester, students began enrolling themselves into one of 14 sections of Beginning Algebra offered at the institution using standard enrollment procedures, six sections of which were designated to participate in the study without the students' knowledge at the time of their registration. Because there was no random assignment, students were required to complete a series of questionnaires and assessments to provide evidence of equivalence on their entry characteristics, such as demographics, previous math experience, preferences for working in groups, personal interest in math, reasoning ability, verbal ability, and prior knowledge.

Throughout the ensuing nine weeks of instruction and testing, data were collected on several cognitive and affective student outcomes. First, conceptual understanding and procedural application were assessed because these two knowledge domains are considered fundamental to learning basic mathematics concepts (Hiebert & Grouws, 2006; Mesa, 2010). Second, knowledge transfer, defined as the ability to apply knowledge in novel contexts, was assessed because researchers have argued that varying levels of explicit instruction may affect the extent to which students can transfer knowledge (Hmelo-Silver, Duncan, & Chinn, 2007; Matlen & Klahr, 2013).

Third, with respect to the affective domain, situational interest was compared between students in the varied and non-varied classes because situational interest is thought to be triggered and maintained by varied teaching practices, such as group work and opportunities for involvement (Hidi & Renninger, 2006; Mitchell, 1993). Last, course retention rates were investigated because it has been proposed that social interactions may increase students' desires to remain enrolled in college programs (Tinto, 1997).

Using these data, this study was able to address the following research questions with respect to developmental mathematics education at community colleges:

(1) To what extent does a varied method of instruction facilitate conceptual understanding and procedural application more effectively than a non-varied method of instruction?

(2) To what extent does a varied method of instruction facilitate students' knowledge transfer more effectively than a non-varied method of instruction?

(3) To what extent does a varied method of instruction affect students' situational interest compared to a non-varied method of instruction?

(4) To what extent does a varied method of instruction affect course retention rates compared to a non-varied method of instruction?

Summary of Findings

This study had four findings. First, a varied method of instruction was determined to be just as effective as a non-varied method of instruction in facilitating conceptual knowledge and procedural application. In fact, both methods of instruction were equally ineffective as evidenced by the low achievement scores across all classes, which is consistent with the overall success rates of Beginning Algebra students at the participating institution (California Community Colleges Chancellor's Office, 2013). Second, the two methods of instruction were equally effective in facilitating transfer of knowledge as evidenced by equivalent scores on Near Transfer and Far Transfer test items, with the former being much more difficult for all students.

Third, students in the varied classes tended to enjoy their classes to a lesser extent than students in the non-varied classes as evidenced by a statistically significant difference in mean scores on the Situational Interest Questionnaire; students in the varied classes tended to remain neutral whereas students in the non-varied classes tended to agree with positive statements about their classes. However, the difference was only statistically significant for Instructor 1, whereas differences in the situational interest for the other two instructors favored the non-varied classes but did not reach statistical significance. Finally, the varied method of instruction had an equivalent effect on student retention rates as the non-varied method of instruction signified by similar frequencies of student dropouts.

Limitations

Internal integrity was defined by Krathwohl (2009) as the power of a study to support a causal link between variables. In other words, internal integrity reflects levels of conceptual and empirical support used to reach a credible finding. Krathwohl also defined external generality as the power of a study to support the generalization of a causal relationship. Limitations affecting both the internal integrity and external generality of the study are described below.

Internal Integrity

The current study attempted to increase internal integrity by eliminating as many alternative explanations as possible. However, in this regard, there were still at least seven limitations to the current study. First, there was no random assignment of students to treatment groups, which means that differences in student outcomes may be attributed to initial student differences at the time of enrollment. This was not a major limitation in the study, however, because data were collected on several variables believed to be correlated with student achievement, such as ability and prior knowledge, and only a very small difference in Reasoning Ability was observed between the treatment groups. In fact, no differences between the two groups existed on the three individual *Gf* measures, but a difference of small effect was observed on the overall composite score. Therefore, the findings are not limited by pre-existing group differences.

Second, even though the participating instructors were required to teach one varied class and one non-varied class to control for the styles of the instructors, the number of students who enrolled with Instructor 1 and Instructor 2 were much larger than the enrollments for Instructor 3. Therefore, it could be the case that the outcomes due to the styles of Instructor 1 and Instructor 2 had a disproportionate impact on the final analyses. As such, any statistically significant differences in means at the treatment level were followed up with analyses at the instructor level to determine the extent to which the findings were limited by disproportionate sample sizes.

A third limitation was the exclusion of student motivation variables, such as self-regulated learning (Zimmerman, 2002) and degree of in-class engagement, as well as student study skills and behaviors outside of class. As a result, there were several variables left unaccounted for that could help interpret the findings. However, attempting to control for all such variables is unfeasible. Further, the study did not completely ignore the affective domain; social preferences, personal interest, and situational interest were measured in the study. Therefore, although the study did not account for all possible metacognitive and affective variables, data were collected on three such variables to aid in interpreting the effects of a varied method of instruction.

The fourth limitation of the study was related to the fidelity of treatment implementation. As a consequence of implementing active learning strategies, the instructors reported not having enough time to implement a few of the writing activities during the closing stage of class. Therefore, one main facet of active learning was not implemented as originally planned. However, the Treatment Implementation Logs indicated that the writing activities were skipped less than half the time one was scheduled. Thus, this was not a major limitation to the findings.

The fifth limitation of the study was that the methods of instruction were only implemented for eight weeks (the first week consisted of introductions, background testing, and review), which might not have been sufficient for the effects of the varied instruction to manifest. It may be possible that students may have needed additional time to adjust to the non-traditional teaching practices. Additionally, eight weeks may not have been enough time for the instructors to become comfortable implementing the active learning practices. However, the instructors did agree that they were comfortable as a result of the training they received, and their logs did not reflect any notions of inadequate preparation, so there was evidence to suggest that the instructors felt comfortable with cooperative activities throughout the eight weeks despite having little prior experience with their implementation. Therefore, the findings do not seem to be limited by the length of the study in terms of the instructors' comfort levels, but it is possible that a longer study may yield different findings.

Relative to the teaching materials used in the current study (see Appendix L), the sixth limitation was that the active learning materials were created by the researcher and may not have elicited the desired cognitive processes from students. But, although the handouts were not official, they were pilot tested and improved based on student feedback prior to the study. Additionally, the participating instructors were provided the worksheets during training for validation. Therefore, because each activity was tested and validated prior to the start of the study, the findings were probably not limited by the quality of the activities.

The seventh and final limitation discussed in this section is that the Near Transfer and Far Transfer measures both consisted of single items, and the observed scores were heavily skewed. Therefore, the measurements used to assess students' transfer abilities were potentially unreliable or invalid. To increase reliability, scoring was conducted collaboratively among instructors and the researcher using a three-point rubric. However, approximately 75% of students obtained one point or less on the near transfer item, and 75% of students obtained two points or more on the far transfer item, suggesting inappropriate item difficulties. The contrasting score distributions may also be credited to the fact that the near transfer item required primarily procedural knowledge, whereas the far transfer item required primarily conceptual knowledge. As such, the items may have been measuring different types of knowledge in addition to different types of transfer abilities, resulting in anomalous results. Thus, the data collected by the transfer items should be interpreted with caution.

External Generality

Given that the study was conducted in live classroom environments and not in a highly controlled laboratory setting, the findings may generalize to similar developmental mathematics courses. However, given that the students were not randomly selected to participate, the results may not be generalizable to the general population of developmental mathematics students. But, the study was conducted using 50% of the available morning and afternoon sections of Beginning Algebra, so the sample was likely representative of all non-evening Beginning Algebra students at the participating institution.

The extent to which the results may generalize to other developmental mathematics courses, such as Arithmetic, Prealgebra, and Intermediate Algebra, remains unknown. Students who enroll in other levels of basic skills courses likely differ from Beginning Algebra students in terms of ability and prior knowledge. Further, the degree of difficulty in these courses varies considerably, so effective teaching practices for one course may not be effective in the others, even within the same institution.

In terms of generalizing the findings beyond the participating institution, the demographics presented in Table 3 were consistent with national demographics in that students of color comprised a large share of the enrollment. However, the participating institution did not enroll a significant percentage of African American students, and the majority of participants were female, which differs from the nationally representative samples described by AMATYC (2006) and Attewell, Lavin, Domina, and Levey (2006). Therefore, the results do not appear to generalize to the national level, but may still generalize to Beginning Algebra classes at local institutions with similar demographics.

Discussion of Findings

This study set forth to investigate the effects of a varied method of instruction in the absence of other successful reforms identified by Boylan (2002) and Epper and Baker (2009), such as the implementation of learning communities, contextualized curricula, and mandatory support services. As such, this study compared the effectiveness of two methods of instruction based on two competing learning theories: social constructivism and Cognitive Load Theory (CLT).

The varied method of instruction included instructional practices consistent with both theories, whereas the non-varied method of instruction included practices consistent with CLT. More specifically, the varied classes included opportunities for knowledge construction through social interactions in conjunction with occasional explicit instruction, and the non-varied classes consisted entirely of explicit instruction followed by individual practice. Instructors were trained to implement both methods of instruction, and procedures were put in place to ensure that the instructors received the resources and support necessary to successfully implement both methods.

Additionally, for a two-group comparative study, Cohen (1992) stated that sample sizes of at least 26, 64, and 393 are required in each group to have an 80% chance of detecting differences of large, medium, and small effects, respectively. Given that the sample sizes in the varied and non-varied groups ranged between 52 and 78, this study had a high probability of detecting medium to large differences in student outcomes as a result of the method of instruction should one exist. Therefore, appropriate steps were taken to ensure the internal integrity of the study and the statistical tests had sufficient power to detect statistically significant differences.

The findings are that both methods of instruction are equally effective in facilitating conceptual understanding, procedural application, and knowledge transfer for students in Beginning Algebra at the participating institution. Additionally, there is no difference in course retention rates between the two groups, but there is a statistically significant difference of medium effect with respect to situational interest. Each finding is discussed in more detail below.

Math Achievement

The lack of differences in conceptual understanding is somewhat surprising because it is believed that active learning practices result in better conceptual knowledge (AMATYC, 2006; NCTM, 2000). However, no statistically significant differences in conceptual understanding scores occurred. Therefore, it appears that encouraging Beginning Algebra students to discuss basic mathematics concepts is no better at facilitating conceptual understanding than directly explaining the concepts to them. But it could also be the case that the study did not last long enough for students to become accustomed to the social practices to an extent that would produce results.

The lack of differences in procedural application was equally surprising because it is believed that explicit instruction results in better procedural application (Sweller & Cooper, 1985). One plausible explanation for the equivalent test averages is that the occasional explicit instruction delivered as part of a varied method of instruction had an equivalent effect on students' procedural knowledge as did the daily explicit instruction delivered in the non-varied classes. Alternatively, given that student behaviors and meta-cognitive strategies were not measured in the current study, it could be the case that students in the study were spending sufficient time outside of class to ensure that they could implement the basic algebraic procedures regardless of the instruction they received.

Obtaining no differences in overall math achievement is inconsistent with two previously-discussed assumptions supporting the implementation of varied instructional practices. First, a varied method of instruction was recommended for underprepared students given that a traditional method of instruction was likely ineffective for them in secondary school, leading to placement in developmental mathematics (Goldrick-Rab, 2007). Second, it was hypothesized that a varied method of instruction is beneficial in classes consisting of students with a variety of learning styles and preferences (AMATYC, 2006; Thomson & Mascazine, 1997). But, because students in the varied classes performed just as well as students in the non-varied classes, neither assumption is supported by the findings of this study. In fact, the extent to which students preferred working in groups did not correlate with any achievement variables under either method of instruction.

Overall, however, the average test scores for students receiving both methods of instruction were below proficient. As such, discussing plausible explanations for the equivalently low levels of mathematics achievement may be moot. The low scores observed in this study are consistent with national success rate data that estimates only a third of students who place into developmental mathematics eventually complete the sequence (Bailey, Jeong, & Cho, 2010). The situation is the same at the state level of the participating institution; only 54% of the approximately 90,500 students enrolled in developmental mathematics courses during the Spring semester of 2013 were successful (California Community Colleges Chancellor's Office, 2013). Therefore, the findings regarding the effects of math achievement indicate that neither method of instruction was capable of instilling knowledge of basic mathematics concepts or procedures.

This finding is consistent with the conclusions made by Grubb and Gabriner (2013) who stated that developmental education will continue to be a roadblock for students unless substantial reforms are made to move away from "remedial pedagogy" (p. 52). Even though the varied method of instruction was considered to be non-traditional, the learning objectives and the actual material being conveyed were still traditional and lacked real-world relevance, which may ultimately be the cause for the observed failure of both methods of instruction.

Transfer

The finding of no statistically significant differences in near or far transfer is consistent with Klahr and Nigam's (2004) path-independence hypothesis that states knowledge transfer is not a function of the method of instruction, but rather dependent on the knowledge that was obtained. In other words, the equivalent scores on the transfer assessments may be attributed to the equivalent outcomes in overall mathematics achievement. Thus, it seems that the ability to apply basic mathematical knowledge in new contexts does not depend on whether the instruction was social or direct in nature. However, this finding must be interpreted with caution because each of the measures consisted of individual items with heavily skewed scores that tapped different types of knowledge.

Situational Interest

Despite the similar scores in conceptual understanding, procedural application, near transfer, and far transfer resulting from the two methods of instruction, the non-varied method of instruction produced higher levels of situational interest. This finding should be interpreted with caution because there was only a statistically significant difference between classes for Instructor 1, whereas the differences in classes taught by the other two instructors failed to reach statistical significance.

The difference in attitude for Instructor 1's students was of a large effect, which is interesting because Instructor 1 had the most experience implementing cooperative activities prior to the start of the study. As such, the differences in attitude cannot be attributed to teacher inexperience with active learning practices. Additionally, in combination with the observed differences in favor of the non-varied classes for all three instructors, an overall difference of medium effect in situational interest appears credible.

The difference in situational interest favoring the non-varied method of instruction is to some extent inconsistent with the research on situational interest, which claims that in-class activities may trigger situational interest, and active involvement in the learning process may help maintain it (Mitchell, 1993). However, Mitchell also posited that situational interest is maintained through meaningful content. Further, Mitchell studied adolescents, who appeared to enjoy working together and being social. Therefore, given that the students in the current study were adults and the content was not made relevant, perhaps inconsistent findings to those found by Mitchell should not be a surprise.

With respect to a plausible explanation for the observed differences in attitude, the instructors claimed that the students preferred being told how to solve a problem as opposed to following directions as part of an activity leading to the discovery of a concept (personal communication, March 27, 2015). This finding is consistent with Cognitive Load Theory that states explicit instruction reduces cognitive load experienced by students (Sweller & Cooper, 1985), which may result in a more positive attitude toward the learning process. Additionally, it could be possible that students are not used to a varied method of instruction as a result of their primary and secondary mathematics education and therefore felt uncomfortable with various activities.

In sum, students who received a varied method of instruction enjoyed their classes to a lesser extent than students who received explicit instruction. This may be due in part to students feeling uncomfortable with active learning practices. Additionally, the learning materials that accompanied the activities may have induced extraneous challenges for students, resulting in decreased enjoyment.

Course Retention Rates

No statistically significant differences in class retention rates were obtained, which is somewhat inconsistent with what was expected based on the work of Tinto (1997) who asserted that social interactions positively influence class retention. However, Tinto's work was conducted at the college-level measuring levels of social integration into the greater campus community, so perhaps Tinto's results do not apply to classroom environments and instructional methods.

Additionally, given that students in the varied classes reported lower levels of situational interest, it could be that any positive effects of social interaction on course retention were negated by the decreased levels of enjoyment. Alternatively, the equivalent retention rates may be a result of the equivalent achievement levels between the varied and non-varied groups. In other words, given that each method of instruction facilitated the same levels of achievement, students were equally likely to stop attending class as a result of poor grades.

As another plausible explanation, previous studies finding increased retention rates have typically contained additional reforms such as learning communities and mandatory counseling (Bloom & Sommo, 2005; Visher, Butcher, & Cerna, 2010).

Therefore, it is possible that the independence of retention and method of instruction is due to the lack of other reforms encouraging higher levels of interaction among students.

Summary

The main finding is that both methods of instruction were equally ineffective in facilitating student learning of basic algebraic concepts and procedures, as evidenced by average percentages of total scores ranging from 65% on the Unit 1 Achievement Test to 45% on the Unit 3 Achievement Test. In other words, regardless of the method of instruction that was implemented, average test scores failed to reach a proficiency level of 75% that is required to pass Beginning Algebra at the participating institution. Therefore, even though Boylan (2002) and Epper and Baker (2009) identified a varied method of instruction as part of successful developmental programs, the effect of a varied method of instruction on student achievement is not significant when offered within traditional curricula and without other academic support services.

The apparent failure of both methods of instruction can be interpreted in light of instructor comments paraphrased in the Researcher Notes. All three instructors reported that many students exuded an air of overconfidence in their math abilities (personal communication, March 27, 2015). Additionally, the instructors mentioned that several of their students should have been enrolled in Pre-Algebra (personal communication, March 27, 2015). These comments are consistent with the observations reported by Grubb and Gabriner (2013); recall that two common types of students in developmental mathematics observed by Grubb and Gabriner were misplaced and underprepared. Therefore, the low scores could be due in part to a significant number of unprepared students and students who overestimated their knowledge prior to assessment.

Another plausible explanation for the low scores emerging from the Researcher Notes is the prevalence of disruptive and misbehaved students in each of the classes (personal communication, March 27, 2015). Instructor 1 cited a particularly troubling student in her varied class who constantly challenged the rationale of almost every concept to the extent that the class was slowed down (personal communication, March 19, 2015). These behavioral issues may be a result of the lack of meaningfulness inherent in traditional basic skills curricula because students may be losing interest during class and not see the value in what is being taught, regardless of how the material is delivered. However, because no qualitative data were collected from students, this hypothesis cannot be tested by the current study. The explanation is plausible, however, because it is consistent with the previous findings by Boylan (2002) and Epper and Baker (2009) who identified that a varied method of instruction was successful in conjunction with contextualized curricula.

In addition to being consistent with the observations of Boylan (2002) and Epper and Baker (2009), the findings of this study are also consistent with the recent recommendations of the Student Success Task Force formed by the California Community Colleges Board of Governors; Recommendation 5 explicates the need to restructure developmental mathematics programs by implementing learning communities, contextualized curricula, and additional academic support services, such as supplemental instruction and team teaching (California Community Colleges Student Success Task Force, 2012). Overall, then, the findings of this study are consistent with current publications regarding the problem of high failure rates in developmental mathematics.

The consensus appears to be that developmental education is in need of an overhaul that extends beyond the mere manipulation of instructional methods.

Conclusions

This study has two conclusions. First, implementing active learning practices resulted in lower situational interest among Beginning Algebra students at the participating institution. It was not the case that the varied students did not enjoy their classes, rather, the students tended to be impartial, which was in contrast to students in the non-varied classes who tended to enjoy their classes.

Second, and more importantly, a varied method of instruction is not capable of increasing student success without the aid of other reforms, such as learning communities, contextualized curricula, and mandatory academic support services. This conclusion is a result of the observed ineffectiveness of both methods of instruction in facilitating mathematics achievement and knowledge transfer among students in Beginning Algebra at the participating institution.

Implications for Research

This study generated at least three potential avenues for future research. First, exploring the effects of a varied method of instruction over a longer period of time would address a limitation of the present study. For example, providing instructors with longer training sessions and tracking the effects of active learning practices on student outcomes throughout an entire developmental mathematics sequence, which may last anywhere from six months to two years, may provide different findings. It may be the case that students need additional time to become accustomed to working together and discussing math concepts.

Second, reasons for why students enjoyed the varied classes to a lesser extent than the non-varied classes could be explored in greater depth, perhaps through the collection of qualitative data. Plausible explanations for the observed differences in attitude were offered in the previous section, but a discussion with students regarding their preferences for the subfacets of situational interest identified by Mitchell (1993), such as group work, meaningfulness, and involvement, may be more insightful.

Third, given the conclusion that a varied method of instruction will not increase student success all alone, future studies may begin to address the effectiveness of a varied method of instruction combined with other reformations in developmental mathematics, such as learning communities, contextualized curricula, and support services. For instance, a future study may compare the effects of a varied method of instruction on student outcomes in developmental mathematics using contextualized curricula to courses without contextualized curricula. Or, a traditional method of instruction may be compared to a varied method of instruction during which students in both classes are required to attend supplemental instruction workshops. Overall, future research is needed to continue investigating the effects of the developmental mathematics reforms identified by AMATYC (2006), Boylan (2002), the California Community Colleges Student Success Task Force (2013), and Epper and Baker (2009) on student achievement, knowledge transfer, situational interest, and course retention rates.

Implications for Practice

Two main implications for practice resulted from the conclusions of this study. First, Beginning Algebra courses consisting of traditional curricula should probably be taught using traditional methods, given that students who received a non-varied method of instruction enjoyed their classes to a greater extent. By using traditional methods of instruction, any potential extraneous difficulties resulting from active learning practices may be avoided without a loss in achievement.

Second, because both methods of instruction were equally inadequate in generating student knowledge, it would be fitting for community college mathematics instructors to seek administrative support to begin implementing some of the other successful developmental mathematics reforms identified in the literature. For example, committees may be formed to investigate alternative curricula or feasible academic support services that may be integrated in developmental mathematics classes.

One such alternative program for developmental mathematics students created by the Carnegie Foundation is called Statway. The goal of Statway is to provide students with the basic algebraic concepts and procedures that are required for a course in statistics, and recent data have indicated that 50% of entering Beginning Algebra students successfully completed college-level statistics within one year (Sowers & Yamada, 2015). Statway is just one example of a reformed developmental mathematics program that may increase student success. Overall, a recommended course of action is for mathematics faculty and administrators to begin restructuring developmental mathematics programs in order to increase access to higher education for all students.

Summary

Students in developmental mathematics at community colleges are failing at alarming rates, which has a significant impact on students' personal and transfer goals (Attewell, Lavin, Domina, & Levey, 2006; Bahr, 2008). Additionally, students of color are enrolling in developmental mathematics courses at disproportionate rates (Bailey, Jenkins, & Leinbach, 2005). Therefore, in order to increase access to higher education for a diverse group of students, reforms in developmental mathematics need to be investigated.

Researchers have identified a combination of several reforms that appear to be successful in increasing student success in developmental mathematics, such as varied instruction, learning communities, contextualized curricula, and mandatory support services (Boylan, 2002; Epper & Baker, 2009). In an effort to begin investigating the relative impact of each reform on student outcomes, this study compared the effects of a varied method of instruction, supported by social constructivism, to that of a non-varied method of instruction, supported by Cognitive Load Theory, on student outcomes while controlling for other successful reforms identified by researchers. The data indicated that neither method of instruction has the power to increase student learning by itself, a finding that is consistent with the work of Boylan (2002), Epper and Baker (2009), and the California Community Colleges Student Success Task Force (2013).

These findings were limited, however, by the length of the treatment, so an avenue of future research may include the effects of a varied method of instruction that has been accepted and assimilated by students over a longer period of time. Additionally, further investigations into the effects of developmental mathematics reforms on student outcomes may be conducted to determine which combination of reforms, if any, is most beneficial for all students.

REFERENCES

Accrediting Commission for Community and Junior Colleges. (2014). *Accreditation standards.* Retrieved from http://www.accjc.org/wpcontent/uploads/2014/07/ Accreditation_ Standards_Adopted_June_2014.pdf

Adelman, C. (2005). *Moving into town and moving on : The community college in the lives of traditional-age students*. Washington, DC : Office of Vocational and Adult Education, U.S. Dept. of Education.

American Mathematical Association of Two Year Colleges. (2006). *Beyond crossroads: Implementing mathematics standards in the first two years of college.* Retrieved from http://beyondcrossroads.matyc.org/doc/PDFs/BCAll.pdf

Anderson, L.W., & Krathwohl, D. R. (2001). *A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives.* New York, NY: Addison Wesley Longman, Inc.

Attewell, P. A., Lavin, D. E., Domina, T., & Levey, T. (2006). New evidence on college remediation. *Journal of Higher Education*, *77*(5), 886–924.

Bahr, P. (2008). Does mathematics remediation work? A comparative analysis of academic attainment among community college students. *Research in Higher Education*, *49*(5), 420–450. doi:10.1007/s11162-008-9089-4

Bailey, T., Jenkins, D., & Leinbach, T. (2005). *Community college low-income and minority student completion study: Descriptive statistics from the 1992 high school cohort*. Retrieved from http://ccrc.tc.columbia.edu/media/k2/attachments /low-income-minority-completion-study.pdf

Bailey, T., Jeong, D.W., & Cho, S. W. (2010). Referral, enrollment and completion in developmental education sequences in community colleges. *Economics of Education Review, 29*(2), 255-270.

Banks, J. A. (1988). Ethnicity, class, cognitive, and motivational styles: Research and teaching implications. *The Journal of Negro Education*, 4, 452. doi:10.2307/ 2295689

Barkley, E. F., Cross, K. P., & Major, C. H. (2004). *Collaborative learning techniques: A handbook for college faculty*. San Francisco, CA : Jossey-Bass.

Baumann, J. F. (1988). Direct instruction reconsidered. *Journal of Reading*, *31*, 712–718.

Bell, N., Grossen, M., & Perret-Clermont, A.-N. (1985). Sociocognitive conflict and intellectual growth. *New Directions for Child & Adolescent Development*, (29), 41-54.

Bereiter, C., & Scardamalia, M. (1989). Intentional learning as a goal of instruction. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser.* (pp. 361–392). Hillsdale, NJ England: Lawrence Erlbaum Associates, Inc.

Bettinger, E. P., & Long, B. T. (2005). Remediation at the community college: Student participation and outcomes. *New Directions for Community Colleges*, (129), 17–26.

Blair, R., Kirkman, E. E., & Maxwell, J.W. (2013). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States. Fall 2010 CBMS Survey.* Retrieved from http://www.ams.org/profession/data/cbms-survey/cbms2010-Report.pdf

Bloom, B., Englehart, M., Furst, E., Hill, W., & Krathwohl, D. (1956). *Taxonomy of educational objectives: The classification of educational goals. Handbook I: Cognitive domain*. New York, Toronto: Longmans, Green.

Bloom, D., & Sommo, C. (2005). *Building learning communities: Early results from the opening doors demonstration at Kingsborough Community College*. Retrieved from http://www.mdrc.org/sites/default/files/full_36.pdf

Borich, G. D. (2007). *Effective teaching methods : Research-based practice*. Upper Saddle River, NJ: Pearson Merrill Prentice Hall.

Boylan, H. R. (2002). *What works: Research-based best practices in developmental education.* Boone, NC: Continuous Quality Improvement Network/National Center for Developmental Education.

Boylan, H. R. (2009). Targeted Intervention for Developmental Education Students (T.I.D.E.S). *Journal of Developmental Education, 32*(3), 14–23.

Brown, B. L. (2003). Teaching Style vs. Learning Style. *ERIC Clearinghouse on Adult, Career, and Vocational Education, 26*. Retrieved from http://www.calpro-online.org/eric/docs /mr41.pdf

Bryk, A. S., & Treisman, U. (2010). Make math a gateway, not a gatekeeper. *Chronicle of Higher Education*, *56*(32), B19–B20.

California Community Colleges Chancellor's Office. (2013). *San Jose City College student success scorecard*. Retrieved from http://scorecard.cccco.edu/reports /OneYear/472_One Year.pdf

California Community Colleges Chancellor's Office. (2015). *Basic skills progress tracker*. Retrieved from http://datamart.cccco.edu/Outcomes/BasicSkills_ Cohort_ Tracker.aspx

California Community Colleges Student Success Task Force. (2013). *Advancing student success in the California Community Colleges: Recommendations of the California Community Colleges Student Success Task Force.* Retrieved from http://www.californiacommunitycolleges.cccco.edu/Portals/0/Executive/StudentSuccessTaskForce/SSTF_Final_Report_1-17-12_Print.pdf

California Department of Education. (2013). *California common core state standards: Mathematics.* Retrieved from http://www.cde.ca.gov/be/st/ss/documents/ccssmathstandardaug2013.pdf

Canfield, J. H. (1889). *The opportunities of the rural population for higher education*. Nashville, TN: National Council on Education.

Carroll, J. B. (1993). *Human cognitive abilities: A survey of factor-analytic studies.* New York, NY: Cambridge University Press.

Center for Student Success. (2005). *Environmental scan: A summary of key issues facing California community colleges pertinent to the strategic planning process.* Retrieved from http://files.eric.ed.gov/fulltext/ED496115.pdf

Claxton, C. S., & Murrell, P. H. (1987). *Learning styles: Implications for improving educational practices.* Washington, DC: Association for the Study of Higher Education.

Cohen, J. (1992). A power primer. *Psychological Bulletin, 112*(1), 155-159.

Cortes-Suarez, G., & Sandiford, J. R. (2008). Causal attributions for success or failure of students in College Algebra. *Community College Journal of Research and Practice*, *32*(4), 325–346.

Davis, B. G. (1993). *Tools for teaching.* San Francisco, CA : Jossey-Bass.

Doyle, T. (2008). *Helping students learn in a learner-centered environment : A guide to facilitating learning in higher education*. Sterling, VA: Stylus Publishing, LLC.

Dunn, R., & Dunn K. (1972). *Practical approaches to individualizing instruction.* Englewood Cliffs, NJ: Parker Division of Prentice-Hall.

Ekstrom, R. B., French, J. W., Harman, H. H., & Darman, D. (1976). *Kit of factor-referenced cognitive tests*. Princeton, NJ: Educational Testing Service.

Ellis, A. K. (2005). *Research on educational innovations.* Larchmont, NY: Eye On Education.

Epper, R. M., & Baker, E. D. (2009). *Technology solutions for developmental math: An overview of current and emerging practices.* Retrieved fromhttp://www.rpgroup.org/sites/default/ files/environmental%20scan.pdf

Fennema, E., & Sherman, J. (1976). Instruments designed to measure attitudes towards the learning of mathematics by males and females. *JSAS Catalog of Selected Documents in Psychology*, *6*, 31.

Field, A. (2009). *Discovering statistics using SPSS*. Los Angeles, CA: Sage Publications.

Fowler, P. R., & Boylan, H. R. (2010). Increasing student success and retention: A multidimensional approach. *Journal of Developmental Education*, *34*(2), 2–10.

Gabriel, K. (2008). *Teaching underprepared students: Strategies for promoting success and retention in higher education.* Virginia: Stylus Publishing, LLC.

Gates, A. G., & Creamer, D. G. (1984). Two year college attrition: Do student or institutional characteristics contribute most? *Community/Junior College Quarterly of Research and Practice*, *8*(1), 39–51.

Goldrick-Rab, S. (2007). *Promoting academic momentum at community colleges: Challenges and opportunities*. New York: Columbia University, Teachers College, Community College Research Center.

Grubb, W. N., & Associates (1999). *Honored but invisible : An inside look at teaching in community colleges.* New York, NY: Routledge.

Grubb, W. N., & Gabriner, R. (2013). *Basic skills education in community colleges: Inside and outside the classrooms.* New York, NY: Routledge Taylor and Francis Group.

Guild, P. B., & Garger, S. (1998). *Marching to different drummers.* Alexandria, VA: Association for Supervision and Curriculum Development.

Hidi, S., & Renninger, K. A. (2006). The four-phase model of interest development. *Educational Psychologist*, *41*(2), 111-127.

Hiebert, J., & Grouws, D. (2006). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook for research in mathematics teaching and learning* (pp. 371-404). Reston, VA: National Council of Teachers of Mathematics

Higbee, J. L., & Thomas, P. V. (1999). Affective and cognitive factors related to mathematics achievement. *Journal of Developmental Education*, *23*(1), 8.

Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scafffolding and achievement in problem-based inquiry learning: A response to Kirschner, Sweller, and Clark (2006). *Educational Psychologist, 42*(2), 99-107.

Hulleman, C. S., Godes, O., Hendricks, B. L., & Harackiewicz, J. M. (2010). Enhancing interest and performance with a utility value intervention. *Journal of Educational Psychology*, *102*(4), 880-895.

Johnson, D. W., & Johnson, R. T. (2009). An educational psychology success story: Social interdependence theory and cooperative learning. *Educational Researcher*, *38*(5), 365–379.

Kaestle, C. F., Campbell, A., Finn, J. D., Johnson, S. T., & Mikulecky, L. J. (2001). Adult literacy and education in America. *Education Statistics Quarterly*, *3*(4), 67–72.

Kaiser, H. F. (1991). Coefficient alpha for a principal component and the Kaiser-Guttman rule. *Psychological Reports, 68*, 855-858.

Klahr, D., & Nigam, M. (2004). The equivalence of learning paths in early science instruction. *Psychological Science, 15*(10), 661-667.

Koller, O., Baumert, J., & Schnabel, K. (2011). Does interest matter? The relationship between academic interest and achievement in mathematics. *Journal for Research in Mathematics Education*, *32*(5), 448-470.

Krathwohl, D. R. (2009). *Methods of educational and social science research: The logic of methods.* Long Grove, IL: Waveland Press, Inc.

Lau, P. N., Singh, P., & Hwa, T. Y. (2009). Constructing mathematics in an interactive classroom context. *Educational Studies in Mathematics*, *72*(3), 307–324.

Levin, H. M., & Calcagno, J. C. (2007). *Remediation in the community college: An evaluator's perspective.* NY: Community College Research Center, Columbia University.

Mackintosh, N. J. (2011). *IQ and human intelligence.* New York, NY: Oxford University Press.

Massachusetts Community College Executive Office. (2006). *100% math initiative: Building a foundation for student success in developmental math.* Retrieved from http://www.masscc.org/sites/massc.drupalgardens.com/files/mathinitiativefinal.pdf

Matlen, B. J., & Klahr, D. (2013). Sequential effects of high and low instructional guidance on children's acquisition of experimentation skills: Is it all in the timing? *Instructional Science*, *41*, 621-634.

McCabe, R. H. (2000). *No one to waste: A report to public decision makers and community college leaders*. Washington, DC: Community College Press.

McCabe, R. H. (2003). *Yes we can!: A community college guide for developing America's underprepared*. Phoenix, AZ : League for Innovation in the Community College.

Mesa, V. (2008). *Mathematics instruction in community colleges: A literature review.* Ann Arbor, MI: University of Michigan.

Mesa, V. (2010). Student participation in mathematics lessons taught by seven successful community college instructors. *Adults Learning Mathematics, 5*(1), 64-88.

Meyers, C., & Jones, T. B. (1993). *Promoting active learning: strategies for the college classroom*. San Francisco, CA: Jossey-Bass.

Middleton, J. A. (2013). More than motivation: The combined effects of critical motivational variables on middle school mathematics achievement. *Middle Grades Research Journal*, *8*(1), 77-95.

Midkiff, R. B., & Thomasson, R. D. (1993). *A practical approach to using learning styles in math instruction*. Springfield, IL: Thomas Books.

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review*, *63*(2), 81-97.

Millis, B. J., & Cottell, P. G. (1998). *Cooperative learning for higher education faculty.* Phoenix, AZ: Oryx Press.

Mitchell, M. (1993). Situational interest: Its multifaceted structure in the secondary school mathematics classroom. *Journal of Educational Psychology, 85*(3), 424-436.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM

O'Donnell, A. M. (1996). Effects of explicit incentives on scripted and unscripted cooperation. *Journal of Educational Psychology*, *88*(1), 74-86.

O'Neil, J. (1990). Making sense of style. *Educational Leadership*, (2), 4-9.

Otts, C. D. (2010). *Self-Regulation and math attitudes: Effects on academic performance in developmental math courses at a community college* (Doctoral dissertation). Available from ProQuest Dissertations and Theses Database. (UMI No. 3434592)

Owen, E., & Sweller, J. (1985). What do students learn while solving mathematics problems? *Journal of Educational Psychology*, *77*(3), 272-84.

Palincsar, A. S. (1998). Social constructivist perspectives on teaching and learning. *Annual Review of Psychology, 49*, 345-375.

Perin, D. (2006). Can community colleges protect both access and standards? The problem of remediation. *Teachers College Record*, *108*(3), 339–373.

Popham, W. J. (2000). *Modern educational measurement: Practical guidelines for educational leaders.* Needham, MA: Allyn & Bacon, a Pearson Education Company.

Qin, Z., Johnson, D. W., & Johnson, R. T. (1995). Cooperative versus competitive efforts and problem solving. *Review of Educational Research*, *65*(2), 129-143.

Rotgans, J. I., & Schmidt, H. G. (2011). Situational interest and academic achievement in the active-learning classroom. *Learning and* Instruction, *21*, 58-67.

Roueche, J. E., & Wheeler, C. L. (1973). Instructional procedures for the disadvantaged. *Improving College & University Teaching*, *21*, 222–225.

Salkind, N. J. (2008). *Statistics for people who think they hate statistics.* Thousand Oaks, CA: Sage Publications, Inc.

Slavin, R. (1996). Research on cooperative learning and achievement: What we know, what we need to know. *Contemporary Educational Psychology*, *21*(1), 43–69.

Sowers, N., & Yamada, J. (2015). *Pathways impact report.* Retrieved from http://cdn. carnegiefoundation.org/wpcontent/uploads/2015/01/pathways_impact_report_2015.pdf

Springer, L., Stanne, M. E., & Donovan, S. S. (1999). Effects of small-group learning on undergraduates in science, mathematics, engineering, and technology: A meta-analysis. *Review of Educational Research*, *69*(1), 21–51.

Stein, M. K., Henningsen, M., & Grover, B. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, *33*(2), 455–488.

Stevenson, J. A. (1921). *The project method of teaching*. New York, NY: Macmillan.

Stigler, J. W., Givvin, K. B., & Thompson, B. J. (2010). What community college developmental mathematics students understand about mathematics. *MathAMATYC Educator*, *1*(3), 4–16.

Subocz, S. L. (2008). *Attitudes and performance of community college students receiving metacognitive strategy instruction in mathematics courses* (Doctoral dissertation). Available from ProQuest Dissertations and Theses Database. (UMI No. 3284066)

Sweller, J., & Cooper, G. A. (1985). The use of worked examples as a substitute for problem solving in learning algebra. *Cognition and Instruction*, *2*(1), 59-89.

Sweller, J., Van Merrienboer, J. J. G., & Paas, F. (1998). Cognitive architecture and instructional design. *Educational Psychology Review*, *10*(3), 251-296.

The Research and Planning Group for California Community Colleges (1996). *Operational definitions.* Retrieved fromhttp://cms.cerritos.edu/uploads/Research andPlanning/ Operational DefsRPGroup_%28021111%29.pdf

Thomson, B. S., & Mascazine, J. R. (1997). *Attending to learning styles in mathematics and science classrooms.* Columbus, OH: ERIC Digest. Retrieved from http:// www.ericdigests.org/2000-1/attending.html

Thorndike, R. L., & Hagen, E. (1986). *Cognitive abilities test, form 4.* Chicago, IL: Riverside Publishing Company

Thurnstone, T. G. (1962). *Test of primary mental abilities: For grades 9-12.* Chicago, IL: Science Research Associates, Inc.

Tinto, V. (1997). Classrooms as communities. *Journal of Higher Education*, *68*(6), 599-623.

University of San Francisco. (2008). *Institutional review board for the protection of human subjects (IRBPHS): Manual 2008*. Retrieved from http://www.usfca.edu /uploadedFiles/Destinations/School_of_Education/documents/IRBPHS/irbManual .pdf

Visher, M. G., Butcher, K. F., & Cerna, O. S. (2010). *Guiding developmental math students to campus services: An impact evaluation of the Beacon program at South Texas College.* New York, NY: MDRC

Waldron, V. R., & Yungbluth, S. C. (2007). Assessing student outcomes in communication-intensive learning communities: A two-year longitudinal study of academic performance and retention. *Southern Communication Journal*, *72*(3), 285-302.

Ward, M., & Sweller, J. (1990). Structuring effective worked examples. *Cognition and Instruction*, *7*(1), 1-39.

Wright, J. C., Millar, S. B., Kosciuk, S. A., Penberthy, D. L., Williams, P. H., & Wampold, B. E. (1998). A novel strategy for assessing the effects of curriculum reform on student competence. *Journal of Chemical Education*, *75*(8), 986-992.

Zimmerman, B. J. (2002). Becoming a self-regulated learner: An overview. *Theory Into Practice, 41*(2), 64-70.

APPENDICES

Appendix A

IRBPHS Approval Letters

UNIVERSITY OF SAN FRANCISCO — *Protocol Exemption Notification*

To: Kevin McCandless
From: Terence Patterson, IRB Chair
Subject: Protocol #364
Date: 12/08/2014

The Institutional Review Board for the Protection of Human Subjects (IRBPHS) at the University of San Francisco (USF) has reviewed your request for human subjects approval regarding your study.

Your project (IRB Protocol **#364**) with the title **Dissertation IRB** has been approved by the University of San Francisco IRBPHS as **Exempt** according to 45CFR46.101(b). Your application for exemption has been verified because your project involves minimal risk to subjects as reviewed by the IRB on 12/08/2014.

Please note that changes to your protocol may affect its exempt status. Please submit a modification application within ten working days, indicating any changes to your research. Please include the Protocol number assigned to your application in your correspondence.

On behalf of the IRBPHS committee, I wish you much success in your endeavors.

Sincerely,

Terence Patterson,
Chair, Institutional Review Board for the Protection of Human Subjects
IRBPHS - Univeristy of San Francisco
IRBPHS@usfca.edu

December 3, 2014

Mr. Kevin McCandless
Principal Investigator

Dear Kevin

I am pleased to inform you that your request to conduct research at a college within the Community College District has been approved.

You have been approved to conduct the project entitled, *The Effects of a Varied Method of Instruction on Student Achievement, Transfer, Situational Interest, and Course Retention Rates in Community College Developmental Mathematics* at . This approval expires on **June 30, 2016**. After this date, you will have to reapply for IRB permission to continue to do research on our campus.

PLEASE NOTE THAT APPROVAL DOES NOT COMPEL STUDENTS, STAFF OR FACULTY TO PARTICIPATE IN YOUR STUDY AND THAT ULTIMATE APPROVAL IS DEPENDENT UPON AGREEMENT FROM THE ADMINISTRATOR OF THE PROGRAM OF INTEREST TO YOUR STUDY.

Thank you for your interest in the t. We look forward seeing the results of your study.

Sincerely,

Interim Vice Chancellor
Institutional Effectiveness and Student Success

Appendix B

Background Survey

Student Information

***1. Last Name:**

***2. Student ID:**

***3. Age:**

- ◯ 17 years or less
- ◯ 18 - 20 years
- ◯ 21 - 24 years
- ◯ 25 - 29 years
- ◯ 30 - 34 years
- ◯ 35 - 39 years
- ◯ 40 - 49 years
- ◯ 50 - 59 years
- ◯ 60 or over

***4. Gender identity:**

- ◯ Female
- ◯ Male
- ◯ Other (please specify)

***5. Ethnicity:**

- ◯ American Indian or Alaska Native
- ◯ Asian
- ◯ Black or African American
- ◯ Hispanic or Latino
- ◯ Native Hawaiian or Other Pacific Islander
- ◯ White
- ◯ Unknown
- ◯ Other (please specify)

***6. How many units are you enrolled in this semester?**

◯ 5 or less

◯ 6 - 8

◯ 9 - 12

◯ 12 - 15

◯ More than 15

***7. What is your employment status?**

◯ Unemployed

◯ Employed part-time

◯ Employed full-time

◯ Other (please specify)

***8. Class:**

◯ Randall

◯ Soman

◯ DeSousa

***9. Are you retaking this course (Math 111 - Beginning Algebra)?**

◯ Yes

◯ No

***10. Did you complete the optional prerequisite course, Prealgebra, either here at SJCC (Math 311) or at another community college?**

◯ Yes

◯ No

Student Information

***11. Did you need to retake Prealgebra before completing it?**

- ◯ Yes
- ◯ No

***12. How long has it been since you completed Prealgebra?**

- ◯ 1 semester
- ◯ 2 semesters
- ◯ 3 - 4 semesters
- ◯ Over 4 semesters

Student Information

***13. Approximately how long has it been since you last completed a math class?**

- ◯ Less than 1 year
- ◯ 1 - 2 years
- ◯ 3 - 4 years
- ◯ Over 4 years

***14. Why did you enroll in Math 111 (Beginning Algebra) instead of the optional prerequisite course (Math 311-Prealgebra)?**

- ◯ My score on the placement test qualified me for Math 111
- ◯ My score on the placement test did NOT qualify me for Math 111, but a counselor still referred me to Math 111
- ◯ My score on the placement test did NOT qualify me for Math 111, but I decided to enroll in Math 111 anyway
- ◯ I did not take the placement test and was referred to Math 111 by a counselor
- ◯ I did not take the placement test and chose Math 111 on my own
- ◯ Other (please specify)

Appendix C

Social Preferences Questionnaire

<u>Directions:</u> Draw upon your experience with mathematics and previous mathematics classes and then select the option that most accurately reflects the extent to which you agree or disagree with each statement.

1. I like working by myself during math class.

Strongly Disagree   Disagree   Disagree Somewhat   Neutral   Agree Somewhat   Agree   Strongly Agree

2. Solving math problems in a group is more motivating than working alone.

Strongly Disagree   Disagree   Disagree Somewhat   Neutral   Agree Somewhat   Agree   Strongly Agree

3. Working in math groups often wastes my time.

Strongly Disagree   Disagree   Disagree Somewhat   Neutral   Agree Somewhat   Agree   Strongly Agree

4. During math class, I enjoy working in small groups.

Strongly Disagree   Disagree   Disagree Somewhat   Neutral   Agree Somewhat   Agree   Strongly Agree

Appendix D

Personal Interest Questionnaire

<u>Directions:</u> Draw upon your experience with mathematics and then select the option that most accurately reflects the extent to which you agree or disagree with each statement.

1. Compared to other subjects, I feel relaxed studying mathematics.

Strongly Disagree   Disagree   Disagree Somewhat   Neutral   Agree Somewhat   Agree   Strongly Agree

2. Compared to other subjects, mathematics is exciting to me.

Strongly Disagree   Disagree   Disagree Somewhat   Neutral   Agree Somewhat   Agree   Strongly Agree

3. I like learning new mathematics concepts.

Strongly Disagree   Disagree   Disagree Somewhat   Neutral   Agree Somewhat   Agree   Strongly Agree

4. I do not enjoy working on mathematics problems.

Strongly Disagree   Disagree   Disagree Somewhat   Neutral   Agree Somewhat   Agree   Strongly Agree

Appendix E

Permissions to use Cognitive Tests

Houghton Mifflin Harcourt Riverside

October 10, 2014

Reference No.: G14304169

University of San Francisco
School of Education, Department of
Learning and Instruction
2130 Fulton Street
San Francisco, California 94117

Dear Kevin McCandless:

Thank you for your interest in the *Cognitive Abilities Test™ (CogAT®), Form 4*. This letter is in response to your recent request for use of these materials in your research project entitled *The effects of a varied method of instruction on student achievement, transfer, and interest in community college developmental mathematics.*

Riverside Publishing is happy to offer you permission to use this material in your research, based on the following details you provided to us:

- You will test items from theCogAT4 Level G Sentence Completion subtest using SurveyMonkey.
- You will use the CogAT4 Level G Figure Analysis subtest in a printed form in combination with other nonverbal tests.
- You will not print or electronically distribute any other materials from the test form.

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Sincerely,

April Wills
Contracts and Permissions

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Kevin McCandless
2785 S. Bascom Avenue #9
Campbell, CA 95008

(Hereinafter called "Licensee"),

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Name: Kevin McCandless	Name: Stella DeVries
Title: Doctoral Student	Title: Copyright Administrator Sr.
Date: 11.19.14	Date:

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Appendix F

Prior Knowledge Test

Prior Knowledge Test
Math 111
Spring 2015

Name: ____________________

Directions: Choose the one alternative that best completes the statement or answers the question and fill in the corresponding selection on your scantron using a pencil. When finished, turn in your scantron and the test. Good luck!

Evaluate the algebraic expression for the given value(s).

1) $5(x + 2)$, given 3

A) 30 B) 25 C) 17 D) 5

Write the English phrase as an algebraic expression. Let the variable x represent the number.

2) five less than the product of 17 and a number

A) $5 - 17x$ B) $17x - 5$ C) $17x + 5$ D) $5x - 17$

Determine if the number 2 is a solution to the equation.

3) $6(p - 1) = 3p$

A) solution B) not a solution

Write the sentence as an equation. Let the variable x represent the number.

4) The quotient of 12 and a number is $\frac{1}{2}$.

A) $12x = \frac{1}{2}$ B) $x + 12 = \frac{1}{2}$ C) $\frac{12}{x} = \frac{1}{2}$ D) $\frac{x}{12} = \frac{1}{2}$

Identify the natural number as prime or composite. If the number is composite, find its prime factorization.

5) 90

A) $2 \cdot 3 \cdot 5$ B) $2 \cdot 2 \cdot 3 \cdot 5$ C) $2 \cdot 3 \cdot 3 \cdot 5$ D) prime

Write the fraction in lowest terms.

6) $\frac{28}{32}$

A) $\frac{14}{16}$ B) $\frac{7}{4}$ C) $\frac{7}{8}$ D) $\frac{4}{8}$

Perform the indicated operation. Where possible, reduce the answer to its lowest terms.

7) $12 \cdot \frac{3}{4}$

A) $\frac{36}{48}$ B) $\frac{3}{4}$ C) 9 D) $\frac{36}{4}$

8) $\frac{3}{17} \div \frac{7}{11}$

A) $\frac{31}{119}$ B) $\frac{33}{119}$ C) $\frac{32}{119}$ D) $\frac{11}{39}$

9) $\frac{1}{8} + \frac{5}{8}$

A) $\frac{2}{4}$ B) $\frac{6}{8}$ C) $\frac{3}{8}$ D) $\frac{3}{4}$

10) $\frac{5}{6} + \frac{3}{2}$

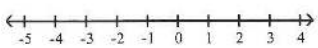
A) $\frac{2}{3}$ B) $\frac{7}{3}$ C) 1 D) $\frac{5}{4}$

11) $\frac{5}{7} - \frac{1}{2}$

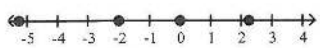
A) $\frac{1}{7}$ B) $\frac{4}{9}$ C) $\frac{4}{5}$ D) $\frac{3}{14}$

Graph the numbers on a number line.

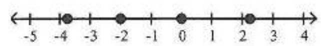
12) −4.75, −2, 0, 2.25



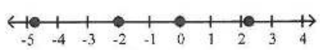
A)



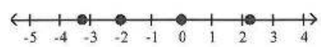
B)



C)



D)



Express the rational number as a decimal.

13) $\frac{9}{2}$

A) 4.5 B) 0.45 C) 45 D) 0.222

14) $\frac{13}{15}$

A) $0.\overline{86}$ B) $0.\overline{866}$ C) 0.86 D) $0.8\overline{6}$

List all the elements of B that are elements of the given set.

15) B = {5, $\sqrt{8}$, −8, 0, 2.3} Natural numbers

A) 5, 0, 2.3 B) 5, 0 C) 5, 2.3 D) 5

16) B = {9, $\sqrt{6}$, −5, 0, $\frac{0}{2}$, $\sqrt{4}$, $\frac{-5}{0}$, 0.42} Rational numbers

A) $\sqrt{6}$, $\sqrt{4}$

B) $\sqrt{6}$, $\frac{0}{2}$, 0.42

C) 9, 0, $\sqrt{4}$

D) 9, −5, 0, $\frac{0}{2}$, $\sqrt{4}$, 0.42

Determine whether the inequality is true or false.

17) $11 \geq 11$

A) True
B) False

Insert either < or > in the area between the pair of numbers to make a true statement.

18) $-\frac{8}{9}$ ___ $-\frac{9}{10}$

A) >
B) <

Find the absolute value.

19) $|\sqrt{13}|$

A) 0
B) $-\sqrt{13}$
C) $\sqrt{13}$
D) $\sqrt{-13}$

An algebraic expression is given. What is the numerical coefficient of the first term ?

20) $5x + 7$

A) 1
B) 5x
C) 5
D) 7

Use the commutative property of multiplication to write an equivalent algebraic expression.

21) 9x

A) $\frac{9}{x}$
B) 9x
C) x + 9
D) x9

Use an associative property to rewrite the algebraic expression. Once grouping has been changed, simplify the resulting algebraic expression.

22) $5 + (8 + x)$

A) $(5 + 8) + x$; 13
B) $5 + (8 + x)$; $5 + 8x$
C) $(5 + 8) + x$; 13x
D) $(5 + 8) + x$; $13 + x$

Use the distributive property to rewrite the algebraic expression without parentheses. Simplify.

23) $-3(x - 7)$

A) $-3x + 7$
B) $-3x - 7$
C) $-3x + 21$
D) $-3x - 21$

Simplify the algebraic expression.

24) $4x + 2x$

A) 8x
B) 6x
C) $6x^2$
D) 6

Write the English phrase as an algebraic expression. Let x represent the number.

25) nine times the sum of 3 and a number.

A) $9(3) + x$
B) $9(3 + x)$
C) $9 + 3x$
D) $9(3x)$

Find the sum.

26) $27 + (-11)$

A) 297
B) -297
C) 16
D) -16

Simplify the algebraic expression.

27) $-6x + 6x$

A) 12x
B) 0
C) x
D) $-12x$

28) $-6 + 4x + 5 + (-9x)$

A) $-5x + 1$
B) $-5x - 1$
C) $-13x - 1$
D) $5x - 1$

Solve.

29) A deep-sea diver dives from the surface to 237 meters below the surface and then swims up 9 meters, down 19 meters, down another 29 meters, and then up 22 meters. Find the diver's depth after these movements.

A) 272 meters below the surface
B) 254 meters below the surface
C) 158 meters below the surface
D) 196 meters below the surface

Perform the indicated subtraction.

30) 8 - (-2)

A) -10 B) 10 C) 6 D) 16

31) -0.15 - (0.16)

A) -0.31 B) 0.31 C) -0.01 D) 0.024

Simplify the algebraic expression.

32) 7 - (-3x) + 8x - (-4)

A) 11 + 11x B) 11 + 5x C) 3 + 11x D) 11 - 5x

Identify the terms in the algebraic expression.

33) -4x - 8xy + y

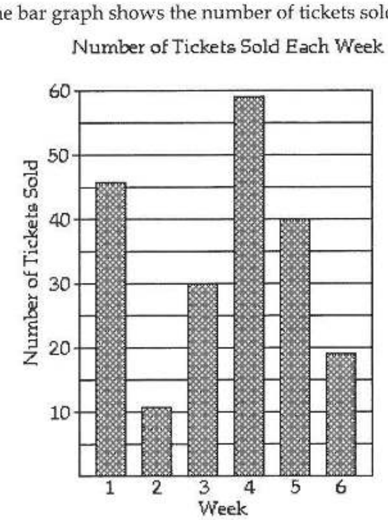
A) -4x, -8xy B) 4x, 8xy, y C) -4x, -8xy, y D) -4x, -8xy, -y

Solve.

34) City A has an elevation of 14,745 feet above sea level while city B has an elevation of 142 feet below sea level. Find the difference in elevation between those two cities.

A) 14,987 ft B) -14,503 ft C) -14,603 ft D) 14,887 ft

35) The bar graph shows the number of tickets sold each week by the garden club for their annual flower show.



What is the difference in tickets sold from week 1 and week 3?

A) 76 tickets B) 16 tickets C) 21 tickets D) 26 tickets

Perform the indicated multiplication.

36) 3.9(-6.22)

A) -2.32 B) 10.22 C) 10.12 D) -24.258

Perform the indicated multiplication. Where possible, reduce the answer to its lowest terms.

37) $\frac{1}{4}(-8)$

A) $-\frac{8}{32}$ B) -2 C) $-\frac{1}{32}$ D) $-\frac{8}{4}$

38) $-\frac{49}{45} \cdot \left(\frac{5}{7}\right)$

A) $-\frac{7}{9}$ B) $-\frac{49}{45}$ C) $\frac{9}{7}$ D) $\frac{7}{9}$

Find the multiplicative inverse.

39) 23

A) $-\frac{1}{23}$ B) 1 C) -23 D) $\frac{1}{23}$

Rewrite the division as multiplication involving a multiplicative inverse.

40) $\frac{-42}{-7}$

A) $-42 \cdot \left(\frac{1}{-7}\right)$ B) $42 \cdot \left(-\frac{1}{7}\right)$ C) $-\frac{1}{42} \cdot (-7)$ D) $-42 \cdot \left(\frac{1}{7}\right)$

Perform the indicated division or state that the expression is undefined.

41) $-85 \div 0$

A) 85 B) 0 C) 1 D) undefined

Simplify the algebraic expression.

42) 2(10y + 3) - 4(6y + 6)

A) -4y + 6 B) -48y C) -4y - 18 D) 2y + 2

43) -9x + x

A) -8x B) -10x C) 8x D) 10x

Determine whether the number -6 is a solution of the equation.

44) 6(4 - z) + 10z = 0

A) not a solution B) solution

Solve.

45) A company's cost per radio when producing x thousand radios in a month is given by the algebraic expression $\frac{5x + 39}{x}$. Find the cost per radio when 3 thousand radios are produced in a month.

A) $18 B) $44 C) $36 D) $28

Simplify the algebraic expression by removing parentheses and brackets.

46) $4 - 2[3 - (7x + 2)]$

A) $14x - 6$ B) $14x + 2$ C) $2 - 14x$ D) $10 - 14x$

Simplify the algebraic expression, or state that the expression cannot be simplified.

47) $5x^6 + 8x^3$

A) $13x^9$ B) $13x^{18}$

C) cannot be simplified D) $13x^6$

Evaluate the exponential expression.

48) -9^2

A) -81 B) 18 C) 81 D) -18

Use the order of operations to simplify the expression.

49) $24 \div 6 \cdot (-3)$

A) $-\frac{4}{3}$ B) -12 C) 8 D) 12

50) $7^2 - 5 \cdot 9$

A) 4 B) 126 C) 396 D) 36

Appendix G

Achievement Tests

Math 111
Chapter 2 Exam
Spring 2015

Name: ______________________

Score: ________ / 40

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the equation.

1) $-x = -15$

A) {15} B) {0} C) {-15} D) {-1}

2) $x + 8 = -12$

A) {-4} B) {20} C) {-20} D) {4}

Solve the equation for y.

3) $x = 5y + 9$

A) $y = \frac{x-9}{5}$ B) $y = 5x - 9$ C) $y = x - \frac{9}{5}$ D) $y = \frac{1}{5}x - 9$

Solve the problem.

4) The amount of water in a leaky bucket is given by the formula $f = 126 - 10t$, where f is in ounces and t is in minutes. Find the amount of water in the bucket after 3 minutes.

A) 156 oz B) 116 oz C) 96 oz D) 30 oz

5) The time it takes to travel a given distance at constant speed is given by the formula $t = \frac{d}{r}$, where t is the time, d is the distance, and r is the rate of travel. At 0.9 mile per minute, what distance can be traveled in 20 minutes?

A) 9 mi B) 18 mi C) 36 mi D) 3.6 mi

6) Forensic scientists use the lengths of certain bones to calculate the height of a person. When the femur (the bone from the knee to the hip socket) is used, the following formula applies for men: $h = 69.09 + 2.24f$, where h is the height and f is the length of the femur (both in centimeters). Find the height of a man with a femur measuring 57 centimeters.

A) 196.77 cm B) 126.09 cm C) 5.40 cm D) 4065.81 cm

Solve.

7) What percent of 20 is 0.6?

A) 0.03% B) 12% C) 33.33% D) 3%

Solve the problem.

8) Jeans are on sale at the local department store for 15% off. If the jeans originally cost $65, find the sale price. (Round to the nearest cent, if necessary.)

A) $74.75 B) $9.75 C) $55.25 D) $64.03

Solve the formula for the specified variable.

9) $A = \frac{1}{2}bh$ for b

A) $b = \frac{h}{2A}$ B) $b = \frac{A}{2h}$ C) $b = \frac{Ah}{2}$ D) $b = \frac{2A}{h}$

Solve the inequality.

10) $4x - 8 \geq 16$

A) $(-\infty, 2]$ B) $(-\infty, 6]$ C) $[6, \infty)$ D) $[2, \infty)$

WRITTEN. Provide responses in the space below each item. Use complete sentences when appropriate.

11) (1 pt) Give an example of a linear equation in one variable whose solution is 2.

12) (1 pt) Model the following information with a linear equation in one variable: If 5 times a number is added to 6, the result is 11 times the number.

13) (2 pts) Determine whether the following equation is a linear equation in one variable. Explain why or why not.

$3(x+1) = x - 2$

14) (2 pts) Using your own words, explain the addition property of equality.

15) (1 pt) Give an example of a linear equation in one variable with infinitely many solutions.

16) (2 pts) Explain why interval notation and graphs on a number line are used to represent solutions to linear inequalities and why they are not typically used with solutions of linear equations.

17) (1 pt) Give an example of a linear inequality in one variable with no solution.

SHORT ANSWER. Show all of your work in the space provided below each item.

Solve the equation by first writing an equivalent equation without fractions. (3 pts)

18) $\frac{3}{2}x + \frac{6}{5} = \frac{7}{5}x$

Solve the equation. (4 pts)

19) $4(3x - 2) - 32 = 8x - 4$

Solve the equation. (3 pts)

20) $5x - 8 + 5x - 7 = 6x + 4x - 18$

Solve the inequality. Write the solution set in interval notation and graph it on the number line. (5 pts)

21) $5x - 7 \geq 7x - 11$

Solve the problem using algebra techniques; that is, identify the variable, use it to model the situation with an equation, solve the equation, and state your conclusion. (5 pts)

22) A promotional deal for long distance phone service charges a $15 basic fee plus $0.05 per minute for all calls. If Joe's phone bill was $68 with this promotional deal, how many minutes of phone calls did he make? Round to the nearest integer, if necessary.

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MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question and fill in the corresponding selection on your scantron (815-E).

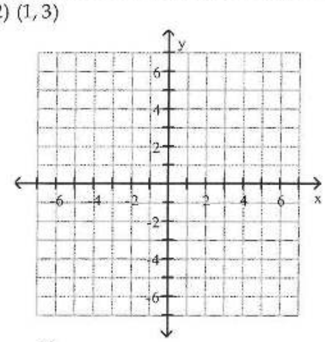
Indicate in which quadrant the point lies.

1) $\left(-\frac{1}{2}, \frac{3}{5}\right)$

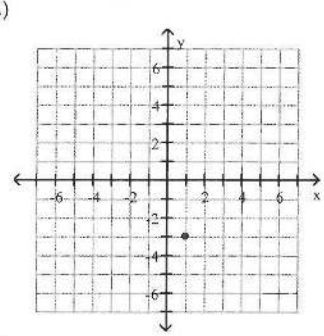
A) III B) IV C) II D) I

Plot the given point in a rectangular coordinate system.

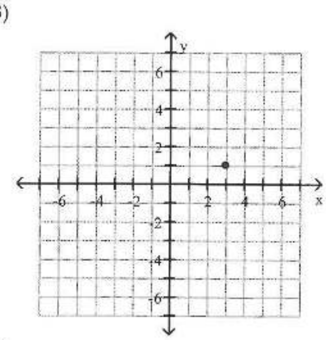
2) (1, 3)



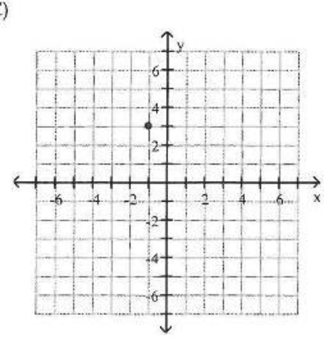
A)



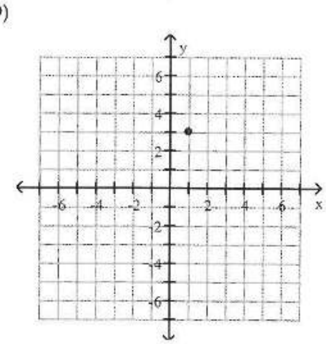
B)



C)



D)



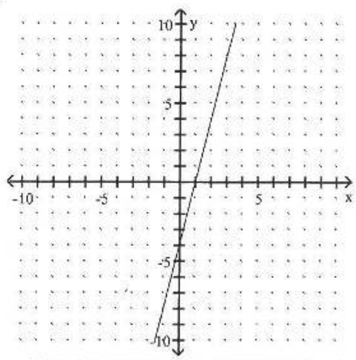
Find the x-intercept and the y-intercept of the graph of the equation. Do not graph the equation.

3) $-4x + y = 8$

A) x-intercept = -1; y-intercept = 4
B) x-intercept = -2; y-intercept = 0
C) x-intercept = -2; y-intercept = 8
D) x-intercept = 8; y-intercept = -2

Use the graph to identify the x- and y- intercepts or state that there is no x- or y-intercept.

4)



A) x-intercept = -1; y-intercept = -4
B) x-intercept = 1; y-intercept = -4
C) x-intercept = 1; y-intercept = 4
D) x-intercept = -4; y-intercept = 4

Find the slope of the line passing through the pair of points or state that the slope is undefined.

5) (-1, -7) and (6, -3)

A) $\frac{7}{4}$
B) -2
C) $\frac{4}{7}$
D) $-\frac{4}{7}$

Find the slope of the line.

6) $-5x + y = 12$

A) -5
B) $-\frac{1}{5}$
C) 12
D) 5

Find the y-intercept of the line.

7) $2y = 6 + 8x$

A) 3
B) 6
C) 4
D) 8

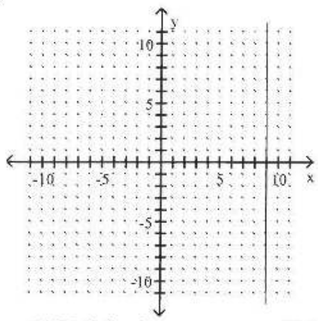
Find the point-slope form of the equation of the line satisfying the given conditions and use this to write the slope-intercept form of the equation.

8) Slope = 3, passing through (5, 2)

A) $y = -3x + 13$
B) $y = 3x - 13$
C) $y = 3x + 13$
D) $y = \frac{1}{3}x - \frac{13}{3}$

Find the slope of the line, or state that the slope is undefined.

9)

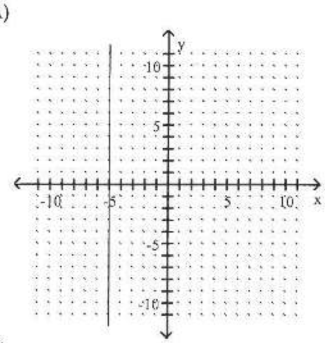


A) Undefined B) 9 C) 0 D) 1

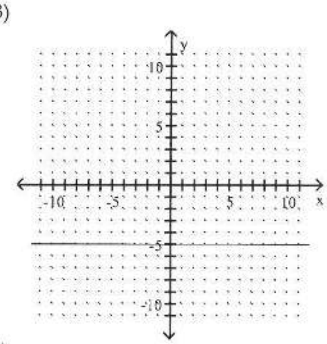
Graph the equation.

10) $y + 5 = 0$

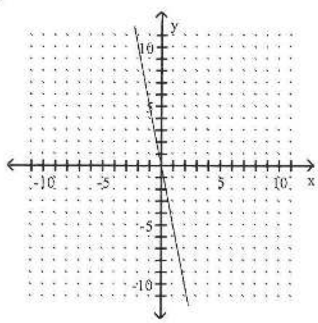
A)



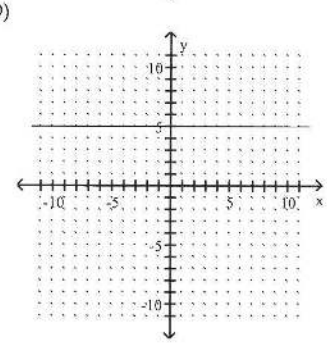
B)



C)



D)



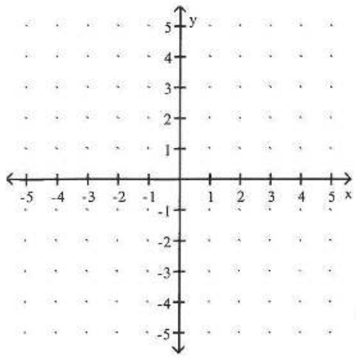
WRITTEN. Provide responses in the space below each item. Use complete sentences when appropriate.

11) (2 pts) Determine whether the equation $x^2 + y = 5$ is a linear equation in two variables. Explain why or why not.

12) (1 pt) Explain the difference between a solution to a linear equation in *one variable* and a solution to a linear equation in *two variables.*

13) (2 pts) Is it possible for one equation to have two different graphs? Explain why or why not.

14) (1 pt) Draw a line with a slope of -2.



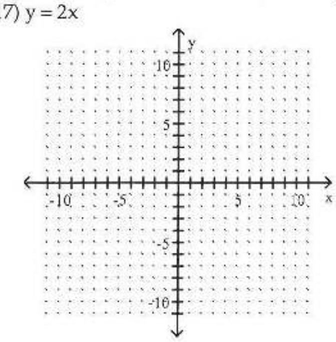
15) (2 pts) A customer at a store bought 5 apples and 2 candy bars for a total cost of \$5.25. Write a linear equation in two variables that reflects the given conditions, and be sure to clearly label what your variables represent.

16) (2 pts) When a tow truck is called, the cost of the service is given by the linear function $y = 3x + 45$, where y is in dollars and x is the number of miles the car is towed. First, find the slope and y-intercept of the linear equation. Then, in practical terms, interpret the meaning of the slope and y-intercept. In other words, describe the meaning of the slope and y-intercept in this particular context.

SHORT ANSWER. Show all of your work in the space provided below each item. Pay careful attention to the instructions and be sure to apply the requested procedure.

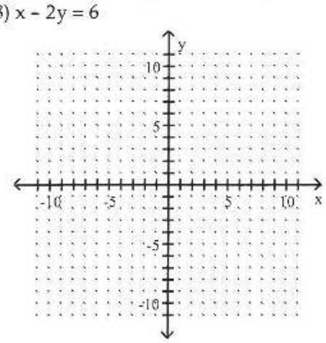
Graph the linear equation in two variables ***by making a table of at least 3 solutions.*** **(4 pts)**

17) $y = 2x$



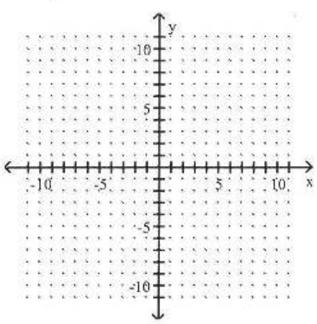
Find the y- and x-intercepts for the equation. Then graph the equation ***using the intercepts and a check-point.*** **(4 pts)**

18) $x - 2y = 6$



Identify the slope and y-intercept, then ***use the slope and y-intercept to graph the equation.*** **(3 pts)**

19) $y = -\frac{1}{2}x + 3$



Determine whether the lines through each pair of points are parallel, perpendicular, or neither. Explain. (4 pts)

20) (-5, 5) and (15, 9); (3, 10) and (-7, 12)

Solve the problem using algebra techniques; that is, identify the variables, use them to model the situation with an equation, solve the equation, and state your conclusion. (5 pts)

21) The graph below shows the average retail price of the least-expensive DVD player available at Mega Mart over the past few years. Use the two points whose coordinates are given to find the slope-intercept form of an equation that models the data. Then, use the model to estimate the average retail price of the least-expensive DVD player in 2007.

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MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question and fill in the corresponding selection on your scantron (815-E).

Determine whether the ordered pair is a solution of the system.

1) (6, -2)
$$\begin{cases} 4x + y = 26 \\ 3x + 4y = 26 \end{cases}$$
A) Yes
B) No

Solve the system by the substitution method. If there is no solution or an infinite number of solutions, so state. Use set notation to express the solution set.

2) $\begin{cases} x - 6 = y \\ y + 6 = x \end{cases}$
A) no solution; ∅
B) {(3, 6)}
C) infinitely many solutions; {(x, y)| x - 6 = y} or {(x, y)| y + 6 = x}
D) {(6, 3)}

3) $\begin{cases} x + y = -3 \\ y = -2x \end{cases}$
A) {(-3, -6)}
B) {(3, 6)}
C) {(-3, 6)}
D) {(3, -6)}

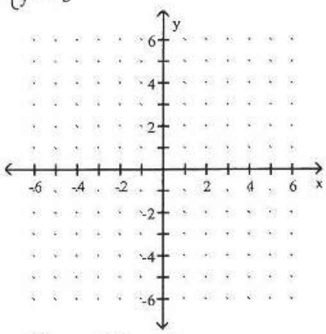
Solve the system by the addition method. If there is no solution or an infinite number of solutions, so state. Use set notation to express the solution set.

4) $\begin{cases} x + y = -8 \\ x + y = 3 \end{cases}$
A) {(-8, 3)}
B) {(0, 0)}
C) {(0, -5)}
D) no solution; ∅

5) $\begin{cases} x + y = -5 \\ x - y = 9 \end{cases}$
A) no solution; ∅
B) {(2, -7)}
C) {(-2, -6)}
D) {(1, -6)}

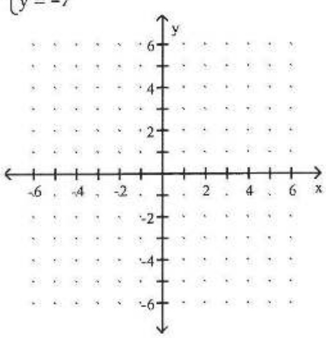
Solve the system by graphing. If there is no solution or an infinite number of solutions, so state. Use set notation to express the solution set.

6) $\begin{cases} x = 5 \\ y = -3 \end{cases}$



A) no solution; ∅
B) infinitely many solutions; $\{(x, y) | x = 5\}$ or $\{(x, y) | y = -3\}$
C) $\{(5, -3)\}$
D) $\{(-3, 5)\}$

7) $\begin{cases} y = 0 \\ y = -7 \end{cases}$



A) infinitely many solutions; $\{(x, y) | y = -7\}$ or $\{(x, y) | y = 0\}$
B) $\{(0, -7)\}$
C) no solution; ∅
D) $\{(-7, 0)\}$

Solve the problem.

8) Devon purchased tickets to an air show for 7 adults and 2 children. The total cost was $136. The cost of a child's ticket was $4 less than the cost of an adult's ticket. Find the price of an adult's ticket.

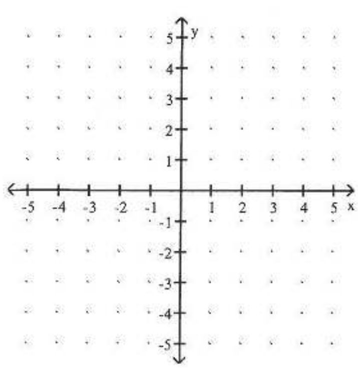
A) $16 B) $18 C) $17 D) $15

WRITTEN. Provide responses in the space below each item. Use complete sentences when appropriate.

9) (2 pts) Determine whether $\begin{matrix} 2x + y = 5 \\ 4y - x = 1 \end{matrix}$ is a system of linear equations in two variables. Explain why or why not.

10) (1 pt) Give an example of a system of linear equations in two variables whose solution is (0,3).

11) (1 pt) Draw a system of linear equations in two variables with exactly one solution.



12) (1 pt) Give an example of a system of linear equations in two variables with no solution.

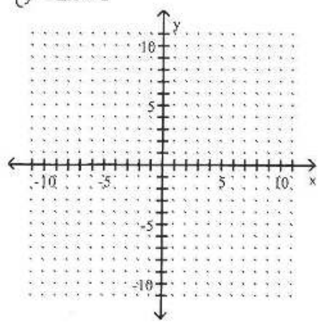
13) (2 pts) The sum of two numbers is –6. Five times the first number equals 3 times the second number. Define two variables (clearly write out what each of your two variables represents) and use them to write a system of linear equations in two variables that reflects the given conditions.

14) (3 pts) A company's cost for producing x magazines is given by the equation y = x + 1500. The revenue for selling x magazines is given by the equation y = 3x. The solution to this system is (750, 2250). Interpret these coordinates in practical terms. In other words, write one or two complete sentences that explain the meaning of the given solution using the context of the problem. Note: you do not need to solve the system or check that the solution is correct

SHORT ANSWER. Show all of your work in the space provided below each item. Pay careful attention to the instructions and be sure to apply the requested procedure.

Solve the system by graphing. If there is no solution or an infinite number of solutions, so state. Use set notation to express the solution set. (3 pts)

15) $\begin{cases} y = x + 1 \\ y = 2x + 4 \end{cases}$



Solve the system by the substitution method. If there is no solution or an infinite number of solutions, so state. Use set notation to express the solution set. (3 pts)

16) $\begin{cases} 2x + y = 10 \\ 8x + 4y = 40 \end{cases}$

Solve the system by the addition method. If there is no solution or an infinite number of solutions, so state. Use set notation to express the solution set. (4 pts)

17) $\begin{cases} -4x + 9y = -59 \\ -2x + 4y = -28 \end{cases}$

Solve the problem using algebra techniques; that is, identify the variables, use them to model the situation with a system of equations, solve the system, and state your conclusion. (6 pts)

18) A rectangular lot whose perimeter is 1600 feet is fenced along all four sides. An expensive fencing along the lot's length costs $15 per foot, whereas an inexpensive fencing along the lot's width costs only $5 per foot. The total cost of the fencing comes to $12,000. What are the lot's dimensions?

Solve the problem using algebra techniques; that is, identify the variables, use them to model the situation with a system of equations, solve the system, and state your conclusion. (6 pts)

19) Chris invested part of his $10,000 bonus in a certificate of deposit that paid 6% annual interest, and the remainder in a mutual fund that paid 11% annual interest. If his total interest for that year was $900, how much did Chris invest in each account?

Appendix H

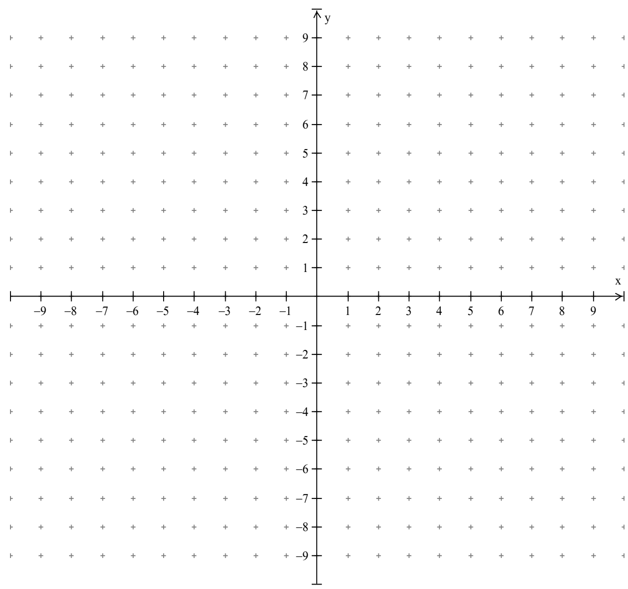
Transfer Test

Math 111 Name:______________________________

Directions: Complete the following two problems as best you can.

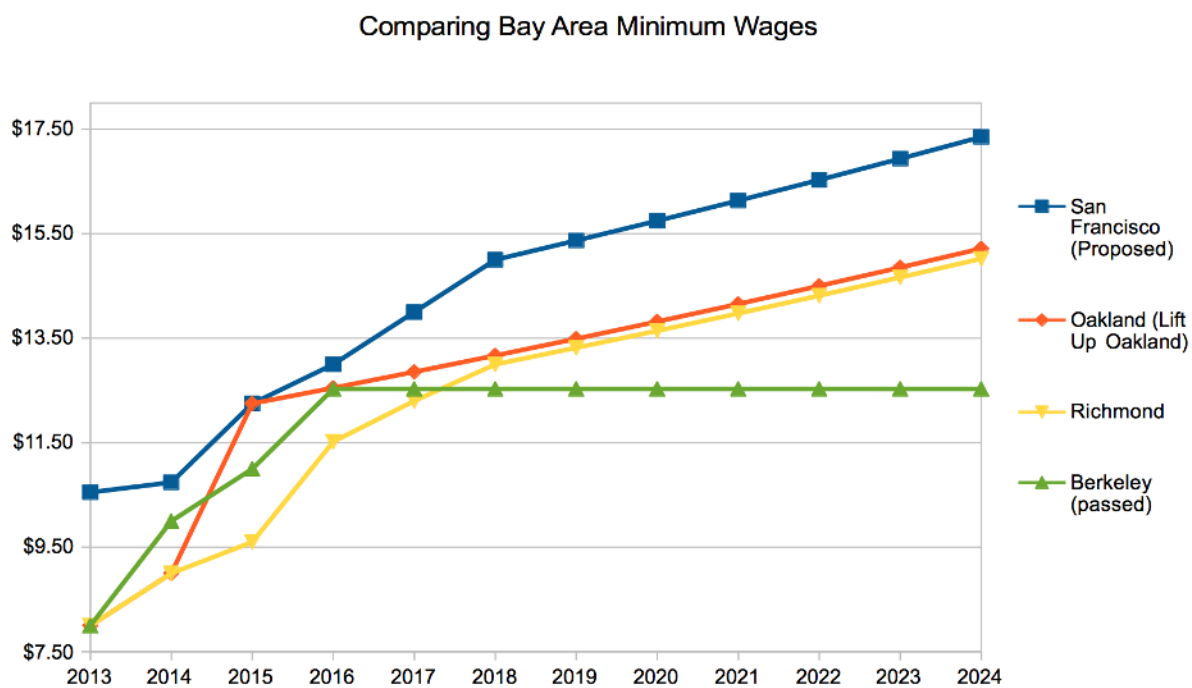
1. Solve the system of equations by graphing and justify your answer in one or two sentences.

$$\begin{cases} y = 5 - 2x \\ y = 3x - 1 \\ y = \dfrac{1}{2}x - 3 \end{cases}$$



Solution and explanation:

2. Apply your knowledge of systems of equations to analyze the graph below. Then, write as many logical conclusions as possible that may be inferred from the graph. In other words, write down specific facts that you can see regarding the minimum wages in San Francisco, Oakland, Richmond, and Berkeley.



Conclusions:

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Appendix I

Situational Interest Questionnaire

Directions: Draw upon your experience with your current math class and then select the option that most accurately reflects the extent to which you agree or disagree with each statement.

1. Our math class is fun.

Strongly Disagree   Disagree   Disagree Somewhat   Neutral   Agree Somewhat   Agree   Strongly Agree

2. I actually look forward to attending math class this semester.

Strongly Disagree   Disagree   Disagree Somewhat   Neutral   Agree Somewhat   Agree   Strongly Agree

3. I have not enjoyed going to math class this semester.

Strongly Disagree   Disagree   Disagree Somewhat   Neutral   Agree Somewhat   Agree   Strongly Agree

Appendix J

Lesson Plans and Implementation Logs

Chapter 2	2.1-Addition Property of Equality	2.2-Multiplication Property of Equality	2.3 - Equations with Fractions and Decimals, Contradictions and Identities	2.4 - Formulas and Percents
Development	min	min	min	min
Lecture	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way
Interactive Lecture				
Cooperative Activity				
Practice	min	min	min	min
Problems Individually	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*
Problems in Pairs				
Cooperative Activity				
Closing / Summary	min	min	min	min
Lecture	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...
Interactive Lecture				
Cooperative Activity				
Writing				

Activity			
Chapter 2	2.5 - Intro to Problem Solving	2.7 - Inequalities	Review
Development	min	min	min
Lecture	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Go over the study guide that describes the type of questions that will be asked on the test and how they are graded, maybe do one or two sample problems
Interactive Lecture			
Cooperative Activity			
Practice	min	min	min
Problems Individually	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign problems from Ch. 2 practice test in book, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*
Problems in Pairs			
Cooperative Activity			
Closing / Summary	min	min	min
Lecture	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...
Interactive Lecture			
Cooperative Activity			
Writing Activity			

Chapter 3	3.1-Intro to Graphing	3.2-Intercepts, Horizontal and Vertical Lines	3.3–Slope, Parallel and Perpendicular Lines
Development	min	min	min
Lecture	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way
Interactive Lecture			
Cooperative Activity			
Practice	min	min	min
Problems Individually	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*
Problems in Pairs			
Cooperative Activity			
Closing / Summary	min	min	min
Lecture	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...
Interactive Lecture			
Cooperative Activity			
Writing Activity			

Chapter 3	3.4–Slope-Intercept Form	3.5–Pt-Slope	Review
Development	min	min	min
Lecture	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Go over the study guide that describes the type of questions that will be asked on the test and how they are graded, maybe do one or two sample problems
Interactive Lecture			
Cooperative Activity			
Practice	min	min	min
Problems Individually	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign problems from Ch. 3 practice test in book, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*
Problems in Pairs			
Cooperative Activity			
Closing / Summary	min	min	min
Lecture	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...
Interactive Lecture			
Cooperative Activity			
Writing Activity			

Chapter 4	4.1 – Graphing Method	4.2 - Substitution	4.3 – Addition
Development	min	min	min
Lecture	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way
Interactive Lecture			
Cooperative Activity			
Practice	min	min	min
Problems Individually	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*
Problems in Pairs			
Cooperative Activity			
Closing / Summary	min	min	min
Lecture	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...
Interactive Lecture			
Cooperative Activity			
Writing Activity			

Chapter 4	4.4 – Applications	Review
Development	min	min
Lecture	X – Present all relevant vocabulary, formulas, procedures, and/or concepts. Lead students through several examples pointing out concepts, connections to previous material, and common mistakes along the way	X – Go over the study guide that describes the type of questions that will be asked on the test and how they are graded, maybe do one or two sample problems
Interactive Lecture		
Cooperative Activity		
Practice	min	min
Problems Individually	X - assign self-checks, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*	X - assign problems from Ch.4 practice test in book, walk around and check, allow students to practice even problems from exercise sets if done early, but *no homework in class*
Problems in Pairs		
Cooperative Activity		
Closing / Summary	min	min
Lecture	X - Summarize all key points, important concepts, common mistakes, important connections, etc...	X - Summarize all key points, important concepts, common mistakes, important connections, etc...
Interactive Lecture		
Cooperative Activity		
Writing Activity		

Chapter 2	2.1-Addition Property of Equality	2.2-Multiplication Property of Equality	2.3 - Equations with Fractions and Decimals, Contradictions and Identities	2.4 - Formulas and Percents
Development	min	min	min	min
Lecture	X – Present vocabulary: Equation, solve, solution, etc...		X - Briefly explain the LCD and decimal methods, and also explain when an equation has infinite solutions or no solution	
Interactive Lecture		X - Begin with an example requiring the multiplication property. Lead discussion about similarities to the addition property (doing operations to both sides to isolate a variable, opposite operations are key)		
Cooperative Activity	X – **Team Discovery** 2.1 Handout with scales and examples, guide students to infer the addition property of equality			X - **Jigsaw** 2.4 Handouts using 4 groups: Manipulate formulas, and solving for a variable in A=PB
Practice	min	min	min	min
Problems Individually		X - Self-checks individually, walk around and check		
Problems in Pairs				
Cooperative Activity	X – **Structured Problem Solving:** Keep students in same groups from the handout activity and assign self-checks using roles		X - **Pass the Problem:** Use the envelopes and problems for 2.3	X – **Structured Problem Solving:** Keep students in same groups from the jigsaw activity and assign self-checks using roles
Closing / Summary	min	min	min	min
Lecture		X – Summarize how to solve a linear equation using both procedures		X - Provide a summary of what should have been learned as a result of the activity and practice.
Interactive Lecture			X - Review students' solutions using projector, lead discussion on common mistakes	
Cooperative Activity				
Writing Activity	X – Individually, ask students to write in their own words the meaning of solve and solution. If time, ask for students to share.		X - Ask students to individually describe the three possible outcomes of solving linear equations. If	

			time, ask for students to share.	

Chapter 2	2.5 - Intro to Problem Solving	2.7 - Inequalities	Review
Development	min	min	min
Lecture	X - Walk students through an example or two using the general problem solving strategy		X – Go over the study guide that describes the type of questions that will be asked on the test and how they are graded, maybe do one or two sample problems
Interactive Lecture		X – Lead discussion on solving a linear inequality, ask students for the steps. Emphasize the same properties, ask students to identify differences. Include one example of switching the inequality	
Cooperative Activity		X – **Team Discovery** (partners) After above discussion, use the 2.7 handout with intervals and graphs, infer interval notation	
Practice	min	min	min
Problems Individually			
Problems in Pairs	X - assign self-checks to pairs, walk around and check work		
Cooperative Activity		X - **Pass the Problem** Keep students in same pairs from interval handout, use 2.7 problems and envelopes	X - **Structured Problem Solving:** Use problems from the Chapter 2 practice test in the book
Closing / Summary	min	min	min
Lecture			
Interactive Lecture	X - Lead discussion around the difficulties with solving word problems, summarize the process	X - Review students' solutions using projector, lead discussion on common mistakes, connect to equations	
Cooperative Activity			X – **Summary Report** Keep in the same groups, give handout of summary questions-- provide more time than with a usual summary to complete the handout.
Writing Activity	X - Ask students to write down the process in their own words, if they can, otherwise just copy from you		

Chapter 3	3.1-Intro to Graphing	3.2-Intercepts, Horizontal and Vertical Lines	3.3–Slope, Parallel and Perpendicular Lines
Development	min	min	min
Lecture	Part I: Introduce all terminology relevant to the coordinate plane, then do Battleship. Part II: After, introduce the definition of a solution to a linear equation in two variables, then do Team Discovery		
Interactive Lecture			X – Review slope terminology, then lead discussion on perpendicular and parallel lines (students tend to know what parallel and perpendicular mean, try to pull it out of them, then introduce the negative reciprocal idea)
Cooperative Activity	X-**Team Discovery** Use 3.1 handout for graphing a line	X-**Jigsaw** Use 3.2 Jigsaw handouts, four groups (intercepts x2, horizontal and vertical lines)	
Practice	min	min	min
Problems Individually			X - Assign self-checks, walk around and check work
Problems in Pairs	X – If time after Battleship and Discovery, assign self-checks to pairs, walk around and check work		
Cooperative Activity	X - **Battleship**	X – **Structured Problem Solving:** Keep students in same groups from the jigsaw activity and assign self-checks using roles	
Closing / Summary	min	min	min
Lecture		X - Provide a summary of what should have been learned as a result of the activity and practice.	
Interactive Lecture			X - lead a Q&A from the individual practice
Cooperative Activity			
Writing Activity	X – Ask students to write down what a graph of an equation represents (key--they are solution sets, visuals of the solutions to an equation, liner equations produce lines)		X – Ask students to describe what slope represents, and the different values it may take on

Chapter 3	3.4–Slope-Intercept Form	3.5–Pt-Slope	Review
Development	min		min
Lecture		X – Introduce point-slope form and demonstrate an application problem	X – Go over the study guide that describes the type of questions that will be asked on the test and how they are graded, maybe do one or two sample problems
Interactive Lecture			
Cooperative Activity	X-**Team Discovery** Use 3.4 handout for slope-intercept, check answers before practicing		
Practice	min	min	min
Problems Individually			
Problems in Pairs	X - Split groups from activity into pairs, assign self-checks, walk around and check work		
Cooperative Activity		X – **Structured Problem Solving** assign self-checks using roles	X –**Card Sort** use the linear cards as a review activity, distribute handout with questions
Closing / Summary	min	min	min
Lecture			
Interactive Lecture		X – Lead summary discussion on the three different forms of an equation of a line (standard, slope-int, point-slope) including the purpose of each	
Cooperative Activity	X – **Summary Report** Use 3.4 handout with prompts		X – **Summary Report** Use the questions from the card sort handout
Writing Activity		X – After the discussion, ask students to write down a summary of the discussion around the three forms.	

Chapter 4	4.1 – Graphing Method	4.2 - Substitution	4.3 – Addition
Development	min	min	min
Lecture		X – Walk students through one example, explain all of the steps.	
Interactive Lecture	X – Lead students through one example of each type, discussing connections to Ch.3 (emphasize that each line represents the solution set to an equation) Try to have students identify when one of your examples has no solution		
Cooperative Activity		X-**Jigsaw** After one example, but students in three groups for jigsaw activity	X-**Team Discovery** Use 4.3 handout, brief activity on importance of adjusting
Practice	min	min	min
Problems Individually	X - Assign self-checks, walk around and check		
Problems in Pairs			X - Split groups into pairs, assign self-checks
Cooperative Activity		X – **Structured Problem Solving:** Keep students in same groups from the jigsaw activity and assign self-checks using roles	
Closing / Summary	min	min	min
Lecture		X - Provide a summary of what should have been learned, mainly connections to Chapter 2 and 3.1-	
Interactive Lecture			X – point out several options for adjusting coefficients
Cooperative Activity	X – **Summary Report** Put students in groups, give 4.1 handout		
Writing Activity		X – Ask students to describe the three possible outcomes of solving a system of linear equations	X - ask students to write about their favorite of the three methods (graph, sub, or add) of solving systems with reasons

Chapter 4	4.4 – Applications	Review
Development	min	min
Lecture	X - Lead students through several problems using the problem solving strategy from Chapter 2, making connections to earlier problems.	X – Go over the study guide that describes the type of questions that will be asked on the test and how they are graded, maybe do one or two sample problems
Interactive Lecture		
Cooperative Activity		
Practice	min	
Problems Individually		
Problems in Pairs	X - Assign self-checks, walk around and check	
Cooperative Activity		X – **Structured Problem Solving** Assign problems from Chapter 4 practice test in book
Closing / Summary	min	
Lecture		
Interactive Lecture	X -Lead students through summary of the process and ask for difficulties students have	
Cooperative Activity		X – **Summary Report** Use Chapter 4 Summary handout
Writing Activity	X – Ask students to write about the similarities/differences between Chapter 2 and Chapter 4 word problems/strategies	

Appendix K

Course Information for Beginning Algebra

Beginning Algebra Student Learning Outcomes:

Upon completion of this course, students should be able to:

1. Perform operations on real numbers using properties of real numbers and appropriate symbols.
2. Simplify and evaluate algebraic expressions, including exponential, polynomial, and rational expressions.
3. Find the equation of a line, graph it, and determine whether two lines are parallel or perpendicular.
4. Solve linear, quadratic, and rational equations and inequalities in one variable, and represent the solution set of the linear inequalities on the number line and using interval notation.
5. Solve systems of linear equations in two variables by graphing, substitution, and addition methods.
6. Solve application problems using systems of linear equations, quadratic, and rational equations.

MATH & SCIENCE DEPARTMENT
SPRING 2015

INSTRUCTOR	**ELEMENTARY ALGEBRA**
OFFICE:	**MATH 111-103**
HOURS:	**REG ID#:**
PHONE:	**ROOM:**
EMAIL:	**TIME:**
COURSE WEBSITE:	

LEARNING MATERIALS:

Required:

Textbook: *Introductory Algebra for College Students*, 6th Edition, by Robert Blitzer
ISBN-10: 0321758951, ISBN-13: 978-0321758958
Textbooks can be found online (Amazon.com, Chegg.com, ABE.com) for about $75 (used) to $125 (new).

Scientific Calculator: Mandatory for quick and accurate computations (in my opinion). My recommendation: TI-30 (~$10)

Scantrons: Form 882-E (large green forms with 50 lines per side) available in the bookstore. (~$1.50)

Pencils: To help organize written work and needed for scantrons.

Graph Paper: Graphing should be done on graph paper to create accurate graphical representations of solutions. Print free at http://www.printfreegraphpaper.com/ .

Three-Ring Binder with Dividers and Binder Paper: To organize notes and class activities. This will be collected for part of your grade (more information to come).

Optional:

Student Solution Manual: Available from various online retailers (Amazon:$20 new, $10 used), just be sure to verify the edition (6th edition).

PREREQUISITE:
Three units of MATH310 (Basic Math) or MATH311 (Pre-Algebra), or placement based on assessment.

COURSE DESCRIPTION:
This is a five-unit course in elementary algebra. Students will cover topics including operations on real numbers and algebraic expressions, solving linear equations and inequalities, algebraic methods for solving application problems, graphing linear equations and inequalities, solving systems of linear equations, laws of exponents and operations on polynomials, factoring polynomials and solving quadratic equations by factoring, and operations on rational expressions and solving rational equations.

STUDENT LEARNING OUTCOMES: Upon completion of this course, students should be able to:
1. Perform operations on real numbers using properties of real numbers and appropriate symbols.
2. Simplify and evaluate algebraic expressions, including exponential, polynomial, and rational expressions.
3. Find the equation of a line, graph it, and determine whether two lines are parallel or perpendicular.

4. Solve linear, quadratic, and rational equations and inequalities in one variable, and represent the solution set of the linear inequalities on the number line and using interval notation.
5. Solve systems of linear equations in two variables by graphing, substitution, and addition methods.
6. Solve application problems using systems of linear equations, quadratic, and rational equations.

CLASSROOM EXPECTATIONS

- No talking during presentations, lectures, or reviews of student work
- Bring required materials every day (book, calculator, pencils, graph paper, and binder)
- All electronics are expected to be turned off and put away in bags (no texting, checking emails, etc….) If you are expecting an important phone call or message, please notify me before class.
- Focus on in-class assignments and stay on task
- Dry snacks and sealed drinks are ok
- If arriving late or leaving early, notify instructor before class and sit close to the door

CALCULATOR POLICY: At a minimum, a scientific calculator is required and should be brought to class every day. *No cell phones* nor other mobile devices may be used in lieu of calculators.

OFFICE HOURS: Office hours (top left of page 1) are an opportunity to receive free tutoring from your instructor. This is your chance to ask any questions you may have from studying or doing your homework or to discuss your grade. Additional office hours are available by appointment so please let me know if you are unable to attend these hours.

SPECIAL ACCOMODATIONS: If you have a learning or physical need that requires special accommodations in this class, please make an appointment with the Disabilities Support Program and Services (DSP&S) in room and notify me as soon as possible. The DSP&S staff and I would like to work cooperatively to ensure your equal access to learning materials, supportive services, and appropriate accommodations as early in the semester as possible. To make a counseling appointment or request services, visit the DSP&S office in the Student Services Building, room

AMERICAN DISABILITIES ACT (ADA): is committed to maintaining an environment free of sexual harassment or discrimination based on race, religious creed, color, national origin, ancestry, disability, medical condition, marital status, political beliefs, organizational affiliation, sexual orientation, gender or age. Information on this can be found in Chapter 5 page 36 of the College Catalog available at

PARTICIPATION AND ATTENDANCE POLICY

Class participation ensures that you will be getting the most out of your educational investment. You have chosen to take this class, and along with that choice comes a responsibility to participate. Therefore, the following attendance policies will be in effect:

- a*ny student who attains more than 2 consecutive unexcused absences may be dropped*
- a*ny student who attains more than 5 total absences may be dropped*
- If these limits are reached after the drop date, grades will be lowered 5% for each 1/2 absence thereafter
- arriving more than 15 minutes late, or leaving early for any amount of time, will each be counted as half an absence
- students are responsible for obtaining the notes from their classmates and completing the corresponding in-class assignments for any missed days

MAKEUP POLICY: No makeup exams are available. Instead, your worst exam score, or a zero from a missed exam, will be dropped.

ACADEMIC HONESTY : Cheating on tests or copying someone else's work is not allowed as it adversely affects and impedes student learning. During a quiz or test, students are not to look at other students' work, share calculators or scratch paper, nor talk to other classmates. If students are caught copying the work of others on any assignments or assessments, their work and the work of the "lender" will both *receive zero points*. If subsequent cheating occurs, the students' names will be forwarded to the Dean of Math and Science and the students will be dropped from the course.

DROP/WITHDRAWAL POLICY: Students are responsible for adding and dropping their classes. A student may drop the class using MyWeb, or in-person at the Office of Admissions and Records. This should be done prior to the drop deadline to avoid a withdrawal notation, "W", on the student's permanent record. If a student drops the course after this date, the student will receive a withdrawal notation, "W", on their permanent record. Students will no longer be able to drop the class after the final drop deadline and must receive a grade in the course. The drop deadline, as well as other important dates, can be found on the website:

IMPORTANT DATES: For a more exhaustive list, use the link above.

**Friday, February 6th*	Last day to drop with eligibility for refund
**Sunday, February 8th*	Last day to add
**Monday, February 9th*	Last day to drop without a W
February 13th – 16th	No School (President's Day Holiday)
March 30th – April 3rd	No School (Spring Break and Cesar Chavez Day)
**Thursday, April 23rd*	*Last day to drop with 'W'*
Friday, May 22nd	Spring Semester ends
Monday, June 8th	Grades Available on MyWeb

*Note: These dates are the responsibility of the student. Late additions will **NOT** be processed and never assume that you have been dropped from the class.

CAMPUS SAFETY: Offenses, accidents, and all emergencies that occur on campus should be reported to Campus Police at . Red emergency call-boxes are available at various locations to contact Campus Police. Emergency call-boxes are marked by blue signs and blue lights for easy visibility. For further Campus Safety information, please visit
safety

SUPPORT SERVICES ON CAMPUS: Please utilize the free tutoring and support service has to offer. These support services will not only help you maintain consistent study habits but will also be crucial in determining your success in the class. It is important that you do not fall behind and seek help if you do not understand a particular math concept or need additional help.

Campus Tutoring Center,

Monday-Thursday 8:30am-7:00pm, Friday 8:30am-2:00pm

The Tutoring Center offers free tutoring in multiple subject areas. students are welcome to drop in, check the subject schedules, and receive help. No appointments are necessary. Subjects tutored include Accounting, English, ESL, History, Music, Economics, Psychology, Spanish, and Vietnamese. Small group workshops in reading, listening & speaking are also offered every semester. Call the Campus Tutoring Center at

GRADING POLICY

Grades will be based on 5 criteria: Chapter 1 Test, Class Participation, Binders, Exams, and the Final Exam.

Chapter 1 Test: A Chapter 1 Test will be administered during the second week of class covering material typically presented in a Pre-Algebra course. This material will be briefly reviewed during class. This test will consist of 50 multiple-choice items. The purpose of the Chapter 1 test is to measure students' prior knowledge of mathematics, specifically pre-algebra knowledge, which is essential to be successful in Math 111. Therefore, the test is mandatory and the score may not be dropped.

Class Participation: Students will be required to participate and engage in problem-solving related assignments and/or discussions. Actively participating in the assignments will count toward your overall grade. See the participation rubric posted on Moodle (https://sjeccd.remote-learner.net/login/) for more details.

Binders: Students will be expected to keep an organized binder containing all in-class assignments and homework. A checklist of items to be included in the binder, which also serves as the rubric, will be provided in-class (also available on Moodle if misplaced). Binders will be submitted on exam days for credit and students are responsible for obtaining the notes from their classmates and completing the corresponding in-class assignments for any missed days. Assignments that are made-up from absences should be labeled with "missed class" at the top of each page in their binders.

Chapter Exams: There will be six individual in-class chapter exams, of which only 5 will count toward your overall grade (lowest score is dropped). See the calendar posted on Moodle for a tentative schedule. The exams will consist of about 10 multiple-choice problems and about 15 short-answer problems. Each exam is closed notes and closed book. The purpose of the exams is for students to demonstrate their knowledge of mathematics and to provide the instructor with a means of measuring students' understanding.

Final Exam: The final exam will be a *cumulative* exam given on **our last day of class** during our regularly scheduled class time. There will be no opportunity to take the final exam at any other time (not early, not late), and t*he final exam must be taken to receive a passing grade in the course.* The final exam will be closed notes and closed book. The purpose of the final exam is for students to pull together earlier material and connect it to new material, resulting in a full and in-depth understanding of the subject.

Summary:

Ch. 1 Test	5%
Class Participation	10%
Binders	15%
Exams	50%
Final Exam	20%
	100%

P	*75% - 100%
NP	0% - 74%

*The math department has decided that 75% is required to pass because history suggests that students who have not mastered Algebra I topics are less likely to be successful in Algebra II.

Math 111 MW *Tentative* Planning Calendar
pring 2015

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
26 Intro and Consent Surveys	27	28 Chapter 1 Review	29	30
February 2 Chapter 1 Review 2.1	3	4 **Ch. 1 Test** 2.2	5	6
9 2.3, 2.4	10	11 2.4, 2.5	12	13 **No School President's Day**
16 **No School President's Day**	17	18 2.7	19	20
23 Test Review 3.1	24	25 **Ch. 2 Test** 3.1	26	27
March 2 3.2, 3.3	3	4 3.3, 3.4	5	6
9 3.5 Test Review	10	11 **Ch. 3 Test** 4.1	12	13 **No School PDD**
16 4.2, 4.3	17	18 4.3, 4.4	19	20
23 Test Review 5.1	24	25 **Ch. 4 Test** 5.2	26	27
30 No School Spring Break	**31 No School Spring Break**	**April 1 No School Spring Break**	**2 No School Spring Break**	**3 No School Spring Break**
6 5.3, 5.4	7	8 5.5, 5.6	9	10
13 5.7 Test Review	14	15 **Ch. 5 Test** 6.1	16	17
20 6.1, 6.2	21	22 6.3, 6.4	23	24
27 6.5, 6.6	28	29 Test Review 7.1	30	May 1
4 **Ch. 6 Test** 7.2	5	6 7.3, 7.4	7	8
11 7.5, 7.6	12	13 7.6, 7.7	14	15
18 Review	19	20 **Ch. 7 Test & Final Ch 1-7**	21	22

Math 111 TR *Tentative* Planning Calendar

ɘ Spring 2015

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
26	27 Intro	28	29 Chapter 1 Review	30
February 2	3 Chapter 1 Review 2.1	4	5 **Ch. 1 Test** 2.2	6
9	10 2.3, 2.4	11	12 2.4, 2.5	13 **No School** **President's Day**
16 **No School** **President's Day**	17 2.7	18	19 Test Review 3.1	20
23	24 **Ch. 2 Test** 3.1	25	26 3.2, 3.3	27
March 2	3 3.3, 3.4	4	5 3.5 Test Review	6
9	10 **Ch. 3 Test** 4.1	11	12 4.2, 4.3	13 **No School** **PDD**
16	17 4.3, 4.4	18	19 Test Review 5.1	20
23	24 **Ch. 4 Test** 5.2	25	26 5.3, 5.4	27
30 **No School** **Spring Break**	**31** **No School** **Spring Break**	**April 1** **No School** **Spring Break**	**2** **No School** **Spring Break**	**3** **No School** **Spring Break**
6	7 5.5, 5.6	8	9 5.7 Test Review	10
13	14 **Ch. 5 Test** 6.1	15	16 6.1, 6.2	17
20	21 6.3, 6.4	22	23 6.5, 6.6	24
27	28 Test Review 7.1	29	30 **Ch. 6 Test** 7.2	May 1
4	5 7.3, 7.4	6	7 7.4, 7.5	8
11	12 7.6	13	14 7.7	15
18	19 Review	20	21 **Ch. 7 Test & Final** **Ch 1-7**	22

Appendix L

Cooperative Activity Structures

Member Roles

Facilitator: The Facilitator is responsible for monitoring discussions and keeping group members on task. The Facilitator also ensures that students are working at the same pace so that no member falls behind.

Time Keeper: The Time Keeper is responsible for keeping the group aware of time constraints. The Time Keeper is expected to work with the Facilitator to ensure that everyone completes the assignment in the allotted time.

Evaluator: The Evaluator is responsible for checking the work of group members and confirming that everyone has the same correct answers. When applicable, the Evaluator is also responsible for identifying mistakes on the board.

Writer: The Writer is responsible for writing the groups' responses on the board or on handouts. The Writer must be sure that the group is in agreement before writing responses and should check with the Evaluator.

Reporter: The Reporter is responsible for speaking for the group when called upon. As with the Writer, the Reporter must be sure that the group is in agreement before speaking.

Spy: The Spy is responsible for covertly looking at the work of other groups, if necessary. The Spy reports back to the group any information that was obtained for evaluation.

Jigsaw	
Purpose	Development of several related learning objectives
Materials	For each objective, a handout, textbook passage, and/or other learning resources are required
Grouping Arrangement	Groups of 3-4
Member Roles	N/A: Each group member will be assigned to a unique learning objective
Implementation	1. Arrange students into "home" groups, where each person in the group is assigned one of the objectives. 2. Instruct students to form new groups consisting of all students assigned to the same objective. 3. Provide each group with the materials necessary to master their objective. 4. Allow students enough time to learn the objective well enough to explain and demonstrate it to the members of their original group. This may be facilitated by requiring students to answer questions on a different handout prior to returning to the home group. 5. Instruct students to return to their home groups. 6. Allow sufficient time for each member of the group to report back what they learned about their objective. 7. Additionally, provide students with additional prompts that may be answered by combining their knowledge.
Group Interdependence	The entire group's participation points will be based on adherence to member roles as well as the completion and correctness of one randomly chosen group member's notes. Additionally, one member may be randomly chosen to summarize one of the objectives.
Individual Accountability	All students need a complete set of notes in the in-class notes section of their binders.

Team Discovery	
Purpose	Development of a rule, concept, or procedure
Materials	Examples and explanation prompts requiring written responses
Grouping Arrangement	Groups of 2-4
Member Roles	Facilitator, Time Keeper, Reporter, and Spy
Implementation	1. Arrange students into groups of two to four 2. Assign roles to group members. 3. Instruct groups to work together to study the examples and answer the prompts. 4. Facilitate groups’ reports to the class

Group Interdependence	The entire group's participation points will be based on adherence to member roles as well as the completion and correctness of one randomly chosen group member's notes with written responses.
Individual Accountability	All students need a complete set of notes in the notes section of their binders.

Pass the Problem	
Purpose	Practice procedural knowledge
Materials	Envelopes with selected exercises/problems written on the front. Digital projector to display students' work to the class.
Grouping Arrangement	Groups of 2-3
Member Roles	Facilitator, Writer, and Reporter
Implementation	1. Arrange students into groups of two or three. 2. Assign roles to group members: 3. Distribute an envelope to each group. 4. Instruct groups to first solve the problem individually on their binder paper, then compare solution steps. This is moderated by the Facilitator. 5. The Writer in the group then writes a final answer on a slip of paper to place in the envelope. 6. Instruct groups to not look at other solutions in the envelop, and to exchange envelopes with another group when finished with a problem. 7. After several exchanges, instruct groups to empty the solutions from their last envelope and evaluate them for correctness and efficiency. 8. The Reporter shares with the class their group's evaluations while displaying the problems using a digital projector.
Group Interdependence	The entire group's participation points will be based on adherence to member roles as well as the completion and correctness of one randomly chosen group member's work and notes.
Individual Accountability	All students need a record of the completed problems in the class-work section of their binders.

Structured Problem Solving	
Purpose	Practice conceptual and/or procedural knowledge
Materials	Exercise sets (textbook, handout, etc...)
Grouping Arrangement	Groups of 3-4
Member Roles	Facilitator, Time Keeper, Evaluator, and Writer
Implementation	1. Arrange students into groups of three or four 2. Assign roles to group members.

	3. Instruct groups to work together to solve all the problems in the exercise set. 4. Assign each group one problem from the exercise set to write on the board. Instruct group to write problems simultaneously at end of activity. 5. Instruct groups to compare the work on the board with the work of their group. This may be done in a "gallery walk" style where groups rotate from problem to problem.
Group Interdependence	The entire group's participation points will be based on adherence to member roles as well as the completion and correctness of one randomly chosen group member's work and notes.
Individual Accountability	All students need a record of the completed problems in the class-work section of their binders.
Battleship	
Purpose	Practice terminology associated with the coordinate plane
Materials	Graph paper for students with axes labeled [-15,15] x [-15,15] , this is called the search grid
Grouping Arrangement	Groups of 2-3
Member Roles	N/A: New roles of Commander and Searcher
Implementation	1. Arrange students into groups of two to three (multiple searchers ok) 2. Assign roles to group members. • Commander – Choses an ordered pair and writes it down where the Searcher cannot see. The ordered pair is thought of as the location of a ship. As the Searcher guesses, the Commander eliminates the corresponding regions on the search grid for the Searcher. • Searcher – Guesses the location of the ship by asking the commander yes/no questions using correct terminology. The Searcher has 10 guesses to locate the ship. Using incorrect terminology is an automatic miss and loss of a question. Also responsible for checking the work of the Commander to ensure the correct regions are eliminated. 3. Demonstrate one round of the game with you as the Commander and the whole class as Searchers using the projector. Encourage appropriate questions, such as "is the x-coordinate positive?", or "is the ship in the first quadrant?" Answer yes or no, then eliminate the resulting section from the search grid. 4. Allow sufficient time for everyone to have a turn as Commander.
Group Interdependence	The entire group's participation points will be based on adherence to member roles as well as the completion and correctness of one randomly chosen group member's written

	responses.
Individual Accountability	Each student needs a search grid in their notes.

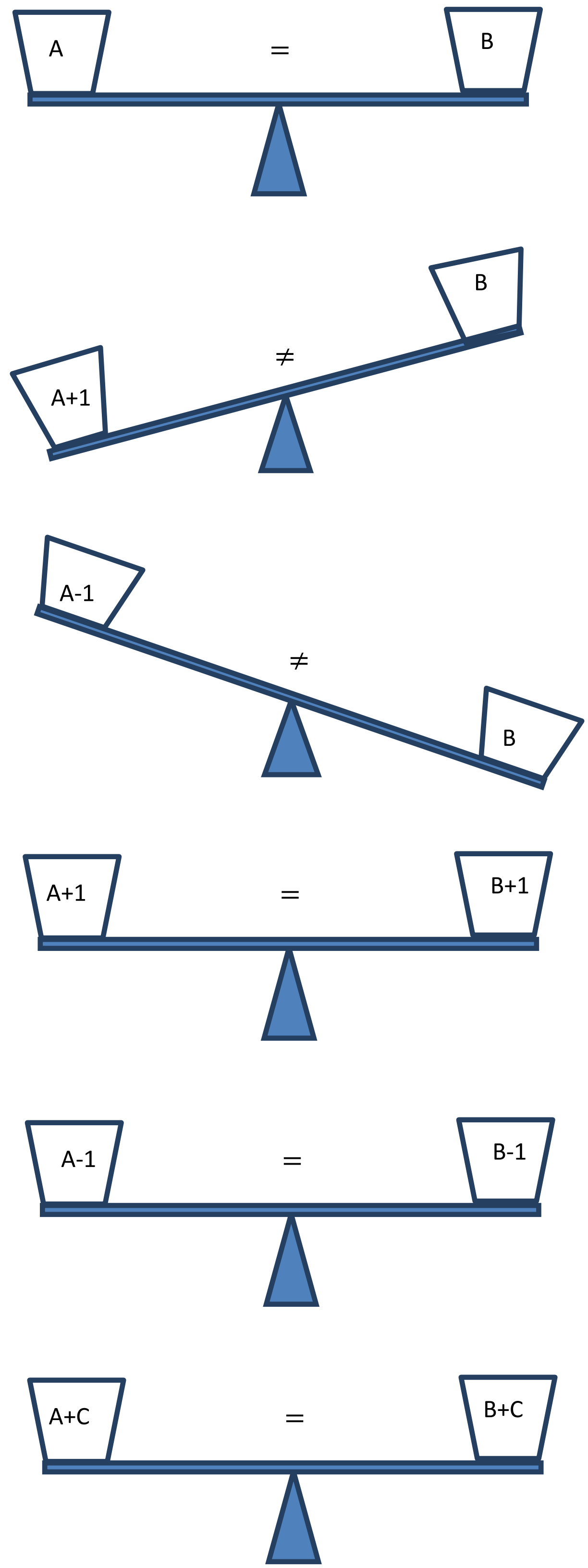
Card Sort	
Purpose	Practice graphing linear equations in 2 variables
Materials	Cards with linear equations and corresponding tables, slopes, intercepts, and graphs, and a handout with summary questions for students to answer.
Grouping Arrangement	Groups of 2-3
Member Roles	Facilitator, Time Keeper, and Spy
Implementation	1. Arrange students into groups of two to three 2. Assign roles to group members. 3. Distribute a stack of cards to each group. 4. Instruct students to organize the cards in a matrix so that each row corresponds to a single linear equation, and each column corresponds to a different property or form of the equation (equation, graph, table, slope, and intercepts). 5. When finished, students are expected to answer the summary questions using the organized cards for support.
Group Interdependence	The entire group's participation points will be based on adherence to member roles as well as the completion and correctness of one randomly chosen group member's written responses.
Individual Accountability	All students need a complete set of notes in the notes section of their binders.

Summary Report	
Purpose	Summarize a rule, concept, or procedure
Materials	Prompts requiring a synthesis of the just-practiced material
Grouping Arrangement	Groups of 2-4
Member Roles	Facilitator, Time Keeper, Reporter, and Spy
Implementation	1. Arrange students into groups of two to four 2. Assign roles to group members. 3. Instruct groups to work together to answer the prompts 4. Facilitate groups' reports to the class
Group Interdependence	The entire group's participation points will be based on adherence to member roles as well as the completion and correctness of one randomly chosen group member's written responses.
Individual Accountability	All students need a complete set of notes in the notes section of their binders.

Appendix M

Learning Materials

2.1 – Addition Property of Equality



Questions:

If both sides of an equation are equal, what is the outcome if one side is increased but not the other?

If both sides of an equation are equal, what is the outcome if one side is decreased but not the other?

When both sides of an equation are equal, what is the outcome if both sides are increased or decreased by the same amount?

Example: Solve $5x - 4 = 4x - 3$

$5x - 4 + 4 = 4x - 3 + 4$ Add 4 to both sides (addition property of equality) and simplify

$5x = 4x + 1$ ← This equation is *equivalent* to the original equation, meaning it will have the same solution as the original equation because we haven't "changed the balance" by increasing both sides of the equal sign by the same amount.

$5x - 4x = 4x - 4x + 1$ Subtract $4x$ from both sides and simplify

$x = 1$ This equation is also *equivalent* to the original equation, meaning that our original equation is true when the variable *x* is equal to 1.

Example: Solve $3x - 2 = 2x + 5$

2.4 - Formulas and Percents

Group 1: Manipulating Formulas

Directions: Read pgs. 145-146 and Example 1, then try Check Point 1. After, answer the two questions below.

Check Point 1work goes here:

1. What does it mean to "solve a formula for one of its variables"?

2. How is the procedure for solving a formula similar to/different from the procedure for solving a linear equation (sections 2.1-2.3)?

2.4 - Formulas and Percents

Group 2: Finding A in $A = PB$

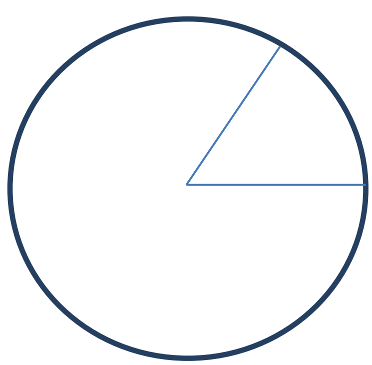
<u>Directions</u>: Read pgs. 148-149 and Example 5, then try Check Point 5. After, answer the three questions below.

Check Point 5 work goes here:

1. What do A, P, and B represent?

2. If you know the value of a whole (B) and are given a certain percentage of that whole (P), how do you find the corresponding amount (A)?

3. What does A represent in the figure? Label it.



2.4 - Formulas and Percents

Group 3: Finding B in $A = PB$

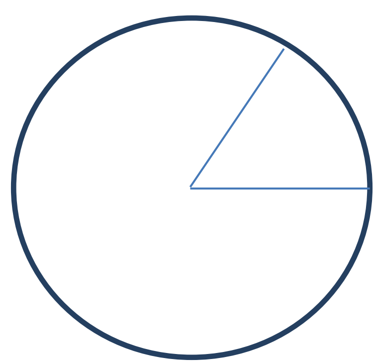
<u>Directions</u>: Read pgs. 148-149 and Example 6, then try Check Point 6. After, answer the three questions below.

Check Point 6 work goes here:

1. What do A, P, and B represent?

2. If you know a part of a whole (A) and the corresponding percentage of the part to the whole (P), how do you find the value of the whole (B)?

3. What does B represent in the figure? Label it.



2.4 - Formulas and Percents

Group 4: Finding P in $A = PB$

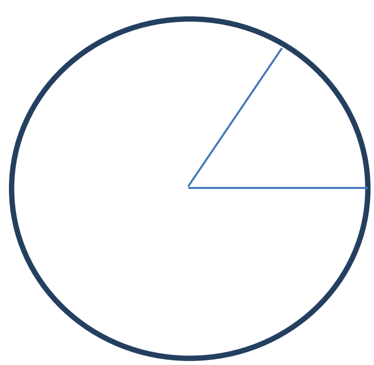
Directions: Read pgs. 148, 150, and Example 7, then try Check Point 7. After, answer the three questions below.

Check Point 7 work goes here:

1. What do A, P, and B represent?

2. If you know the value of a whole and a corresponding piece of the whole, how do you find the corresponding percentage?

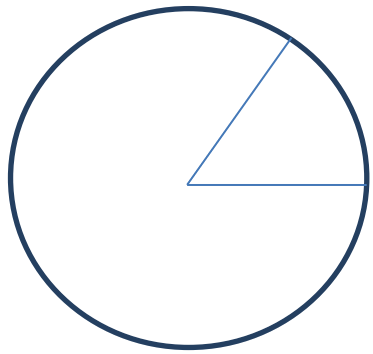
3. What does P represent in the figure? Label it.



2.4 - Formulas and Percents

Directions: As a group, complete the following problems.

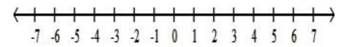
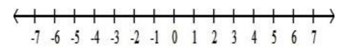
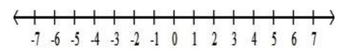
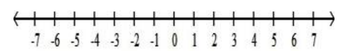
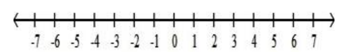
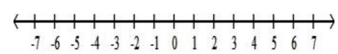
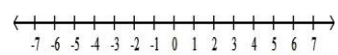
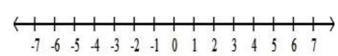
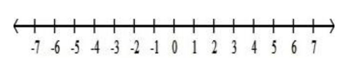
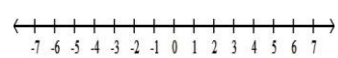
1. Label the figure with A, P, and B:



2. How are finding P and B similar? How are they both different than finding A?

3. What is the connection between group 1 (solving for a variable) and groups 2, 3, and 4?

2.7 – Solving Liner Inequalities

Inequality	Interval Notation	Graph
$x > 3$	$(3, \infty)$	
$x \geq 3$	$[3, \infty)$	
$x < 3$	$(-\infty, 3)$	
$x \leq 3$	$(-\infty, 3]$	
$-3 < x < 3$	$(-3, 3)$	
$-3 \leq x < 3$	$[-3, 3)$	
$-3 \leq x \leq 3$	$[-3, 3]$	
$3 < x$	$(3, \infty)$	
$-3 > x$	$(-\infty, -3)$	
$-\infty < x < \infty$	$(-\infty, \infty)$	

Questions:

When are brackets used in an interval?

When are parentheses used in an interval?

When are brackets used on a graph?

When are parentheses used on a graph?

When is ∞ used in an interval?

When is $-\infty$ used in an interval?

When are $-\infty$ and ∞ NOT used in an interval?

On which side of an interval does ∞ always appear?

On which side of an interval does $-\infty$ always appear?

Which symbol always goes next to $-\infty$ and ∞ in an interval?

When does a graph go to the right?

When does a graph go to the left?

What is the relationship between the inequality symbols and the corresponding graphs?

Chapter 2 Summary Report

1. How is the procedure for solving linear equations similar to the procedure for solving linear inequalities? How are they different?

2. What are the three possible outcomes for solution sets when solving linear equations?
   How do these possibilities relate to the possibilities for linear inequalities?

3. Why do we use interval notation for solution sets of linear inequalities and not typically for solution sets of linear equations?

4. What are the steps in the general problem-solving strategy?

5. What does it mean to "solve for a variable"?

3.1 – Graphing Linear Equations

Example: We are going to *graph* $x + y = 2$, which means we are going to find all of the *solutions* to $x + y = 2$.

Begin by finding *some* solutions. Complete the tables below to yield 15 solutions.

x	y
1	
-8	
-5	
5	
6	

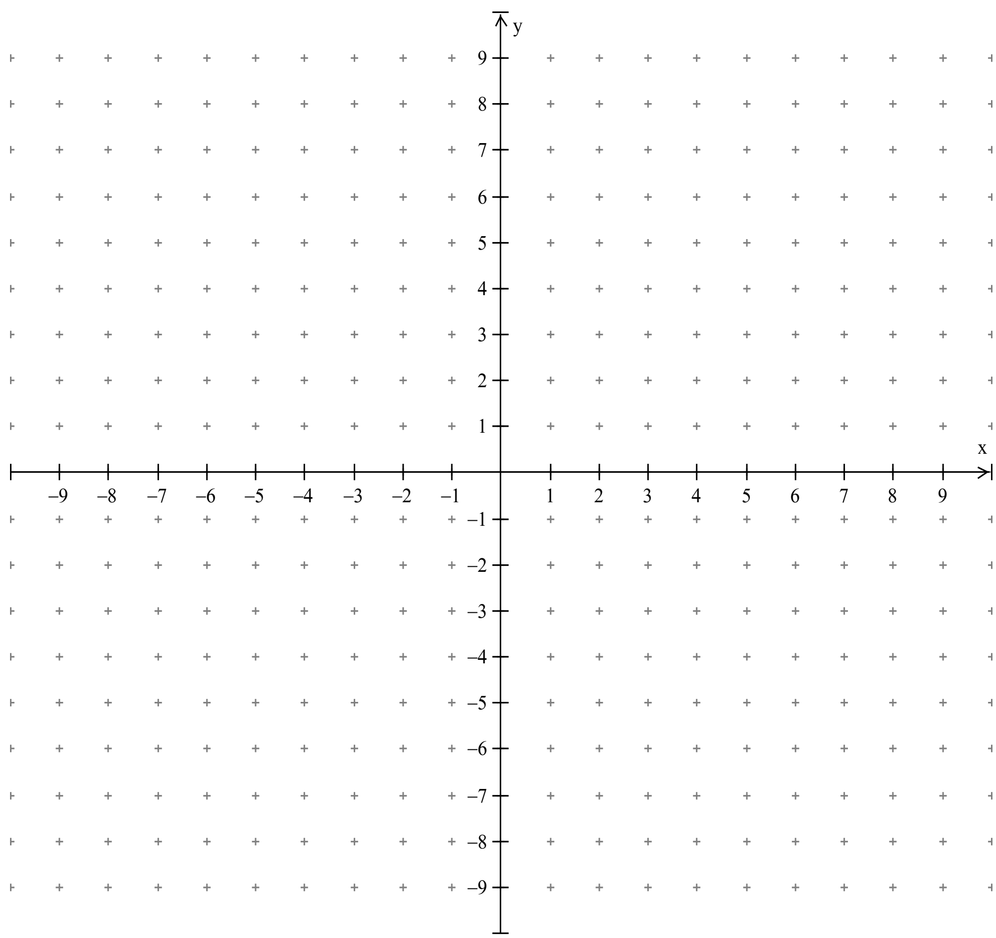
x	y
	-1
	5
	0
	2
	-2

x	y
8	
	8
-1	
	6
7	

Now find 5 more solutions not included in the tables above.

x					
y					

Plot all the solutions here (20 solutions = 20 dots), WITHOUT connecting the dots:



Questions:

In general, how many possible solutions are there to $x + y = 2$?

What pattern is created by the plot of the solutions?

Are there any points that do not fit the pattern? If so, check these points for mistakes. Once all points are checked, connect the dots with the pattern.

Can you guess what pattern will arise when graphing *any* linear equation in two variables?

Was it necessary to find as many solutions as we did to obtain the same graph?

What do you think is the minimum number of solutions needed to obtain the correct graph?

What does a line represent in the context of a linear equation in two variables?

3.2 – Graphing Using Intercepts

Group 1: x-intercepts

Directions: Read the definition on pg. 224 and Examples 1 & 2. Then try Check Point 2. After, answer the two questions below.

Check Point 2 work goes here:

3. In your own words, what is an x-intercept?

4. How do you find an x-intercept for a linear equation?

3.2 – Graphing Using Intercepts

Group 2: y-intercepts

Directions: Read the definition on pg. 224 and Examples 1 & 3. Then try Check Point 3. After, answer the two questions below.

Check Point 3 work goes here:

1. In your own words, what is a y-intercept?

2. How do you find a y-intercept for a linear equation?

3.2 – Graphing Using Intercepts

Group 3: Horizontal Lines

Directions: Read the definition on pg. 230 and Example 7. Then try Check Point 7. After, answer the two questions below.

Check Point 7 work goes here:

1. In your own words, what does the GRAPH of a horizontal line look like?

2. In your own words, what does the EQUATION of a horizontal line look like?

3.2 – Graphing Using Intercepts

Group 4: Vertical Lines

Directions: Read the definition on pg. 230 and Example 8. Then try Check Point 8. After, answer the two questions below.

Check Point 8 work goes here:

1. In your own words, what does the GRAPH of a vertical line look like?

2. In your own words, what does the EQUATION of a vertical line look like?

3.2 – Graphing Using Intercepts

Directions: As a group, complete the following problems.

- Does every line have an x-intercept? Explain.

- Does every line have a y-intercept? Explain.

- What is the maximum number of x-intercepts a line can have? Explain.

- What is the maximum number of y-intercepts a line can have? Explain.

- How can we notice that the graph of a linear equation will be horizontal or vertical just by looking at the EQUATION?

Chapter 3 Card Sort Review Questions

Directions: Use the organized cards to help answer the following questions.

What is a linear equation in two variables?

What is a graph of a linear equation in two variables?

What is the slope of a horizontal line? Vertical line?

What is the equation of a horizontal line? Vertical line?

What is the slope of a line that increases? Decreases?

What types of graphs have equations that are missing a variable?

Which equations are in slope-intercept form?

What is the relationship between the intercepts and the equations in slope-intercept form?

Which equations are in standard form?

What is the relationship between the tables and the slopes?

What is the relationship between the tables and the graphs?

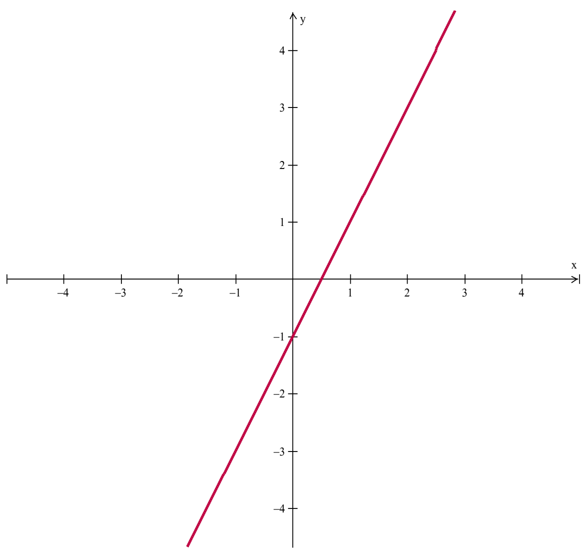
3.4 – Slope-Intercept Form of the Equation of a Line

Directions: Study the examples before answering the questions. In particular, look for patterns and relationships between each graph and the corresponding linear equation.

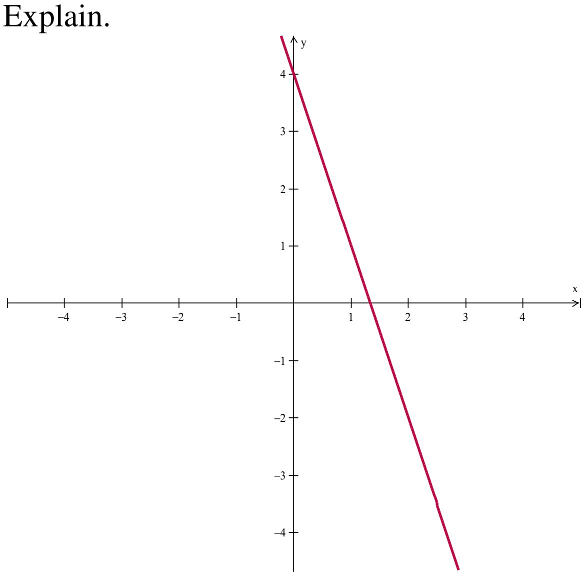
If a linear equation is of the form $y = mx + b$, what will be the slope?

If a linear equation is of the form $y = mx + b$, what will be the y-intercept?

Below is the graph of $2y = 4x - 2$. Does this graph fit the pattern you found above? In other words, is the slope of the graph equal to 4 and is the y-intercept $(0, -2)$? Explain.



Below is the graph of $y = 4 - 3x$. Does this graph fit the pattern you found above? Explain.



4.1 – Summary Report for Graphing

Directions: Compare your answers to the practice problems and then answer the following questions.

- What are the three possible outcomes when solving a system of equations?

- If you made any mistakes, what were they?

(Use #51 to answer the following questions)

- What does it mean with respect to cost when the red line is above the blue line?

- How are the lines related before the intersection? How are the lines related after the intersection?

- What does the intersection point represent?

4.2 – Substitution (1 of 3)

Procedure

- Step 1: Select one of the equations and solve for one of the variables. There are 4 possible ways to complete this step, chose wisely!
- Step 2: Substitute the expression found in Step 1 for the appropriate variable in the equation that was NOT used in Step 1.
- Step 3: Solve the resulting linear equation in 1 variable.
- Step 4: Substitute the solution obtained in Step 3 into the equation in Step 1 and solve for the other variable.

Example With 1 Solution

Solve $\begin{cases} 2x + y = 1 < ----First \\ 3x + 2y = 4 < ----Second \end{cases}$

- Step 1: The easiest variable to isolate is the y variable in the *first* equation. This may be done by subtracting $2x$ from both sides.

- Step 2. Now we replace the y variable in the *second* equation with $1 - 2x$ because the equation above indicates that y is equal to $1 - 2x$.

- Step 3: Solve this new equation that should only contain one variable—either x or y.

- Step 4: Notice that the equation in Step 3 has *one solution* (recall Ch. 2, the equation in Step 3 is only *true when x equals* -2). Now, the corresponding y value is obtained by substituting -2 into the equation in Step 1.

Therefore, the **Final Answer** is**:**

4.2 – Substitution (2 of 3)

Procedure

- Step 1: Select one of the equations and solve for one of the variables. There are 4 possible ways to complete this step, chose wisely!
- Step 2: Substitute the expression found in Step 1 for the appropriate variable in the equation that was NOT used in Step 1.
- Step 3: Solve the resulting linear equation in 1 variable.
- Step 4: Substitute the solution obtained in Step 3 into the equation in Step 1 and solve for the other variable.

Example With 0 Solutions

Solve $\begin{cases} 2x + y = 1 < - - - - First \\ 4x + 2y = 3 < - - - - Second \end{cases}$

- Step 1: The easiest variable to isolate is the y variable in the *first* equation. This may be done by subtracting $2x$ from both sides.

- Step 2. Now we replace the y variable in the *second* equation with $1 - 2x$ because the equation above indicates that y is equal to $1 - 2x$.

- Step 3: Solve this new equation that should only contain one variable—either x or y.

- Step 4: Notice that this step cannot occur because the equation in Step 3 has *no solution* (recall Ch. 2, the equation in Step 3 is *never* true).

Therefore, the **Final Answer** is:

4.2 – Substitution (3 of 3)

Procedure

- Step 1: Select one of the equations and solve for one of the variables. There are 4 possible ways to complete this step, chose wisely!
- Step 2: Substitute the expression found in Step 1 for the appropriate variable in the equation that was NOT used in Step 1.
- Step 3: Solve the resulting linear equation in 1 variable.
- Step 4: Substitute the solution obtained in Step 3 into the equation in Step 1 and solve for the other variable.

Example With Infinitely Many Solutions

Solve $\begin{cases} 2x + y = 1 < - - - - \mathit{First} \\ 4x + 2y = 2 < - - - - \mathit{Second} \end{cases}$

- Step 1: The easiest variable to isolate is the y variable in the *first* equation. This may be done by subtracting $2x$ from both sides.

- Step 2. Now we replace the y variable in the *second* equation with $1 - 2x$ because the equation above indicates that y is equal to $1 - 2x$.

- Step 3: Solve this new equation that should only contain one variable—either x or y.

- Step 4: Notice that this step cannot occur because the equation in Step 3 has *infinitely many solutions* (recall Ch. 2, the equation in Step 3 is *always* true).

Therefore, the **Final Answer** is**:**

4.3 – Addition Method

Example 1: Solve the system of equations using the addition method.

$$\begin{aligned} 2x + y &= 1 \\ x - y &= 5 \end{aligned}$$

Goal: Eliminate a variable by adding the two equations together.

$$\begin{array}{rr} & 2x + y = 1 \\ + & x - y = 5 \\ \hline & 3x + 0 = 6 \end{array}$$

$$3x = 6$$

Notice that the y-variable was *eliminated*, resulting in a new equation in the variable *x*. This equation may be easily solved using methods from Chapter 2.

$$\frac{3x}{3} = \frac{6}{3}$$

$$x = 2$$

Now, we know from section 4.1 that solutions to systems of equations are ordered pairs, so now we need to find the corresponding value of *y* when $x = 2$.

$$x - y = 5$$

$$2 - y = 5 \qquad \text{Substitute 2 for } x\text{, solve for } y$$

$$y = -3$$

Therefore the solution to the system of equations is $(2, -3)$.

Question:
What are the key characteristics of the original system of equations that lead to the elimination of the *y* variable? In other words, what was the relationship between the terms with the y variable that lead to their cancellation when added together?

Example 2: Solve.

$$2x + y = 1$$
$$x + 2y = 5$$

Would any variables be eliminated if the following two equations were added together? Explain.

Notice that the system needs to be changed in order for a variable to be eliminated. There are four possible ways to do this. One way is to change the *x* in the second equation to a $-2x$.

This may be done by using the multiplication property of equality: multiply both sides of the second equation by -2, and do nothing to the first equation.

$$\begin{matrix} 2x + y = 1 \\ x + 2y = 5 \end{matrix} \rightarrow \begin{matrix} 2x + y = 1 \\ -2(x + 2y) = -2(5) \end{matrix} \rightarrow \begin{matrix} 2x + y = 1 \\ -2x - 4y = -10 \end{matrix}$$

This step is called **adjusting the coefficients**.

Now, the problem may be finished as in the first example.

$$\begin{array}{r} 2x + y = 1 \\ + \quad -2x - 4y = -10 \\ \hline 0 - 3y = -9 \end{array}$$

$$y = 3$$

$x + 2(3) = 5$ Substitute 3 for *y*, solve for *x*

$$x = -1$$

Therefore, the solution is $(-1,3)$.

Questions:
What was the major difference between Example 1 and Example 2?

Summarize the procedure for solving systems of equations using addition. Hint: Try to identify 4 steps.

Appendix N

Rubrics for Achievement and Transfer Tests

Chapter 2: High-Complexity Procedural Application Rubric

Responses earn 1 point for each correctly written and executed step below. *Note:* Not all steps apply to every problem.

- Define the variable
- Model the problem with an equation
- Simplify expressions on both sides of the equation
- Use the addition property of equality
- Use the multiplication property of equality
- State the final answer
- Write answer in interval notation
- Express answer graphically

Chapter 3: High-Complexity Procedural Application Rubric

Responses earn 1 point for each correctly written and executed step below. *Note:* Not all steps apply to every problem.

- Define the variable(s)
- Calculate the slope
- Model the problem with an equation
- Manipulate the model
- Provide the requested ordered pair, or other characteristic of a line, and plot it
- Make a graph using the requested ordered pairs or characteristics of the graph
    - One point deduction for plotting points but not graphing the line
- State your conclusion
- Explain your reason(s) when prompted to explain

Chapter 4: High-Complexity Procedural Application Rubric

Responses earn 1 point, unless otherwise noted, for each correctly written and executed step below. *Note:* Not all steps apply to every problem.

- Define the variables
- Model the problem with a system of equations (2 pts)
- Solve the model (2 pts)
- Graph a linear equation
- Isolate a variable
- Substitute and solve for a variable
- Adjust the coefficients
- Add equations and solve for a variable
- State your conclusion

Math 111 Name:______________________________

Directions: Complete the following two problems as best you can.

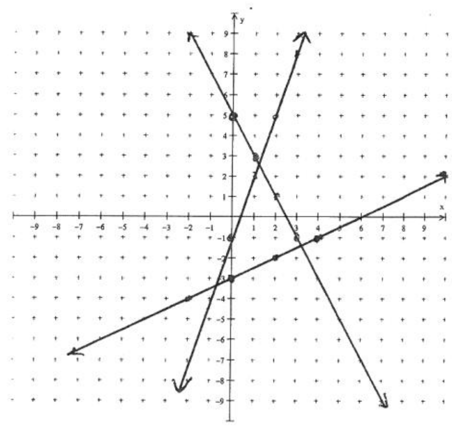
1. Solve the system of equations by graphing and justify your answer in one or two sentences.

$$\begin{cases} y = 5 - 2x \\ y = 3x - 1 \\ y = \frac{1}{2}x - 3 \end{cases}$$

+0 for no graph, or not 3 lines

+.5 for graphing 3 lines, but not correctly.

+1 for graphing the 3 lines correctly.

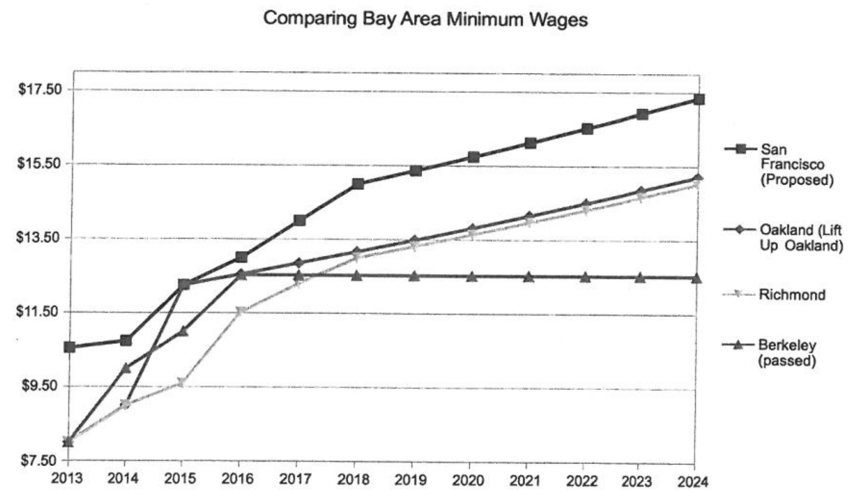


Solution and explanation: +1

The system has no solution because there is no ordered pair that is a solution to all three equations. In other words, the three lines above do not intersect at a single point.

+1 for a valid explanation

2. Apply your knowledge of systems of equations to analyze the graph below. Then, write as many logical conclusions as possible that may be inferred from the graph. In other words, write down specific facts that you can see regarding the minimum wages in San Francisco, Oakland, Richmond, and Berkeley.



Conclusions:

Give one point for any reference to an **intersection point**. *E.g.,* Oakland and Berkeley will have the same minimum wage in 2016, or Oakland and Richmond had the same minimum wage in 2014.

Give one point for any reference to a **slope**, or rate of change, in the minimum wage. *E.g.,* Berkeley's minimum wage will be constant after 2016, or the minimum range will steadily increase in Oakland after 2015.

Give one point for any reference to the $\boldsymbol{y}$ **− values** in the figure. *E.g.,* San Francisco has a higher minimum wage than Berkeley, or the minimum wage in Richmond is just over $9.50 this year.

Appendix O

Student Consent Form

Consent Form

The Effects of a Varied Method of Instruction on Student Achievement, Transfer, Situational Interest, and Course Retention in Community College Developmental Mathematics

Introduction and Purpose:

This study will explore student outcomes as a result of implementing two different methods of instruction. Both methods of instruction are grounded in educational research and are believed to be effective. However, the extent to which these methods are effective for community college developmental mathematics students is yet to be determined. Therefore, this study will compare student outcomes between students taught using the two methods.

Procedures:

As study participants, students are not expected to behave any differently than would be normally expected for any community-college course. That is, students are expected to attend class and abide by the policies described in the course syllabus. However, during the first week of class, students will be required to complete a brief series of surveys and ability tests that will be used to assess the background characteristics of students.

Confidentiality:

All of the data collected from the surveys and ability tests will be kept confidential and known only to the principal investigator--not even your instructor will see these results. The data will be collected electronically so that here are no hard-copies, and all personal information will be recoded using a numeric system to ensure the anonymity of participants.

Voluntary Participation:

Participation in this research is voluntary. Declining to participate will in no way impact your relationship or academic status (*e.g.*, grades) with the instructor, principal investigator, the mathematics department, or college district. If you decide to be in the study, you have the right to drop out at any time by notifying the instructor and/or lead investigator.

Consent Statement:

I understand the procedures described above. My questions have been answered to my satisfaction, I have been given the option to receive a copy of this consent, and I agree to participate in this study.

Print Name: ____ Date: ____________

Participant's Signature: ____

Appendix P

Descriptive Statistics for Instruments

Table P1

Total-Sample Descriptive Statistics for All Instruments

Variable	M	SD	Min	Q_1	Med	Q_3	Max
Social Preference	18.3	5.0	5.0	16.0	19.0	22.0	28.0
Personal Interest	14.9	5.6	4.0	11.0	15.0	19.0	28.0
Gf							
Letter Series	5.2	2.7	0.0	3.0	5.0	7.0	10.0
Letter Sets	8.6	3.5	1.0	6.0	8.0	12.0	15.0
Figure Analogies	5.8	2.3	0.0	4.0	6.0	8.0	11.0
Gc							
Synonyms	6.6	2.5	1.0	5.0	6.0	8.0	12.0
Sentence Completions	5.7	2.2	0.0	4.0	6.0	7.0	10.0
Prior Knowledge	34.6	6.9	14.0	31.0	35.0	40.0	48.0
Unit 1							
CU	4.1	2.1	0.0	2.5	4.0	6.0	9.0
LCPA	7.3	1.7	2.0	7.0	7.3	8.0	10.0
HCPA	14.1	4.6	0.0	12.0	15.0	18.0	20.0
Unit 2							
CU	4.7	2.4	0.0	2.6	5.0	6.0	10.0
LCPA	7.2	2.1	1.0	6.0	7.0	9.0	10.0
HCPA	10.6	5.1	0.0	6.3	11.0	14.8	20.0
Unit 3							
CU	3.6	2.3	0.0	2.0	3.6	5.0	9.0
LCPA	5.6	1.6	1.0	5.0	6.0	7.0	8.0
HCPA	9.7	6.0	0.0	4.3	9.7	14.0	22.0
Transfer							
Near	1.0	0.9	0.0	0.1	1.0	1.0	3.0
Far	2.1	0.8	0.0	2.0	2.0	3.0	3.0
Situational Interest	14.2	3.7	3.0	12.0	14.2	17.0	21.0

Note. Statistics are based on the number of students currently enrolled at the time of the test (see Table 1). CU = Conceptual Understanding; LCPA = Low Complexity Procedural Application; HCPA = High Complexity Procedural Application.