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THE EFFECT OF INFLATION OF THE DISTRIBUTION OF ECONOMIC WELFARE

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I. INTRODUCTION

In recent years, restraint of inflation has been one of the most important considerations of macro-economic policy. On several occasions drastic measures have been taken to halt inflation. Most often these policies have led to significant unemployment and loss of output. But after a quarter century's experience with inflation, most economists and politicians are looking for new and less draconian tools to combat inflationary tendencies.

While most analysts agree that price stability is desirable, there is wide disagreement about the costs of inflation. In general, three reasons for price stability are mentioned. First, price stability encourages a favorable external balance. In an open economy with fixed exchange rates and difficulties of adjusting these exchange rates, there can be little doubt that inflation produces serious social costs. Second, it is sometimes alleged that inflation leads to inefficient resource allocation. There is, however, no evidence that the allocational effects of the mild inflations observed in advanced countries are significant. Third, it is often

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argued that inflation introduces a significant, arbitrary, and regressive redistribution of income.

The present study is concerned mainly with the third of these costs of inflation. More precisely, we attempt to determine the effects of alternative inflationary policies on the distribution of lifetime income.[1] It is useful to lay out briefly the procedures used.

1. The study is basically a simulation study of the effects of inflation; in this respect it follows all the micro-economic studies.[2] Starting with an observed distribution of income and wealth for 1962, movements in variables are predicted and the results analyzed.

2. Most studies examine the effect of inflation on the distribution of income. The concept of income used in the present study is per capita lifetime consumption annuity (or annuity for short).[3] Given wealth and current labor income it is possible to predict future labor income and thence total lifetime wealth. From this data and from specific assumptions about individual utility functions, the path of potential consumption is then derived.

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[1] This study follows many others concerned with the same general question. See in particular Bach and Ando [1957], Metcalf [1969], Schultz [1969], Brownlee and Conrad [1963], Hollister and Palmer [1969], Mirer [1971], Budd and Seiders [1971], and Thurow [1970].

[2] Without a detailed longitudinal study of households it is not possible to infer the effects of inflation. The aggregate, time series studies (such as Schultz [1969] and Thurow [1970]) do not allow us to disentangle movements of the aggregate distribution of income from movements within the distribution.

[3] A more complete formal description of this procedure is given in Section II.

The income annuity of a household is the maximum constant per capita consumption the household could consume over the household's expected lifetime, subject of course to the household's lifetime wealth. The utility-equivalent annuity is that constant per capita consumption stream which would give the same utility as the optimal consumption plan.

The advantage of using the annuity concept rather than either income or wealth is obvious: First, it allows comparison of different age groups and groups with different wealth-income ratios. Since there are very marked differences in the wealth-income ratio with age (especially money-fixed valued wealth assets) it is possible that the distributional effects of inflation may be misstated by looking at other concepts. Second, it may be argued that the concern of government policy should be with a consumption variable rather than income or wealth variables.

3. Since we are concerned with the long-run effects of inflation (the effects on lifetime consumption rather than the effects of inflation over a few years), we must consider a long-run macro-economic model. We thus leave aside many important and relatively well-understood details of short-run models and examine the more speculative features of long-run systems.

4. The analysts of inflation do not often agree on the economic mechanism which generates and transmits inflation. For this reason it is crucial to distinguish the effects of inflation in different economic systems. In this study we analyze three different kinds of economies, which we call classical, neoclassical, and Keynesian.[1]

[1] These titles are merely suggestive and, as will be seen, do not correspond exactly to any author or school of thought.

a. The classical system considered is one which has a unique equilibrium for all real variables independent of the level of prices or the rate of inflation. This system is closely related to competitive, general equilibrium systems which are in continuous long-run equilibrium. In this system inflation alters this distribution of income and wealth according to the relative holdings of money-fixed assets.

b. The neo-classical system examined is a generalization from the classical system in that it explicitly introduces production and savings functions and a mechanism for income distribution. The level of utilization of resources is constant. In this regime, inflation affects capital accumulation and, via the neoclassical production and distribution system, inflation changes real wages and real interest rates. These lead to change in income, wealth, and consumption.

c. The Keynesian system imposes on the neoclassical system a very simple cyclical mechanism. Over the cycle, inflation varies inversely with unemployment as in the well-known Phillips curve. The distributional effects of an inflationary package can be examined, where the employment and inflationary effects are combined.

5. We assume that any inflation initially occurring is fully anticipated. The system is then shocked by an inflationary episode, and the anticipated inflation gradually adjusts to the new inflation rate. In the classical and neoclassical models, the systems are completely neutral to

fully anticipated inflation, while in the Keynesian system the level of inflation is associated with a given level of unemployment.[1]

Since the system is determined by a set of continuous functions, the effects of deflation are exactly the opposite of the effects of inflation for small changes.

In Section II of the paper we analyze more carefully the concepts and measurement of the personal distribution of the annuity concept. Section III then analyzes the effects of inflation in the three economic systems under consideration.

II. THE DISTRIBUTION OF ECONOMIC WELFARE

It is customary to discuss the effect of policies on "the distribution of income." By almost any standard, income is a rudimentary concept to employ in serious studies of policy effects. The objections to income are well-known. Which definition of income should be used? How can one measure and how should one correct for the transitory component in income? How should wealth be included? Should income be lifetime or annual, total or per capita?

It may appear that too much importance has been attributed to income in distributional studies. This is particularly surprising in light of the general philosophical position of economists that consumption, not production or receiving income, is the goal of economic activity. A man's economic welfare should be measured by his consumption activities, or potential consumption not by his income.

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[1]Neutral means that the real variables of the system are independent of the rate of fully anticipated inflation.

The income-based measure of economic welfare has a great many problems which evaporate if a utilitarian, consumption-based, concept is used. Other problems arise, however. The most important problem involves the form of the utility function.

A. The Theoretical Propositions

1. The Simplest Case

We consider the simplest case of a single consumer with a lifetime of T years and an iso-elastic utility function. His total lifetime utility is

$$U = \int_0^T u[c(t)]e^{-\delta t}dt \tag{1}$$

where δ is the subjective discount rate and $u = AC^{\beta}/\beta$ is the instantaneous utility function, where $-\infty < \beta < 1$.[1] In this simplest formulation, bequests, taxes, and transfers are ruled out.

The household has flows of labor income (y_L) and property income (y_P) , so conventionally measured income (y) is given by

$$y = y_L + y_P \quad . \tag{2}$$

In addition the household has initial wealth, K_0 . Assuming a constant real rate of return on wealth of ρ , we can calculate discounted income,

[1] In the case of $\beta = 0$, the formulae below must be changed so that $u[c(t)] = A \log [c(t)]$. When $\beta = 1$, we get a corner solution.

or lifetime wealth, W^*, as follows:

$$W^* = K_0 + \int_0^T y_L(t)e^{-\rho t}dt \quad (3)$$

Our household is assumed to maximize lifetime utility, that is the household maximizes (1) subject to (3). Integrating (1) for constant growth rates of consumption gives

$$U = \frac{A}{\beta}\,\frac{c_0^{\beta}[1 - e^{(g\beta-\delta)T}]}{\delta - g\beta} . \quad (4)$$

The wealth constraint is independent of the consumption plan. We thus have the budget constraint:

$$W^* = \int_0^T c_0 e^{(g-\rho)t}dt = \frac{c_0[e^{(g-\rho)T} - 1]}{g - \rho} \quad (5)$$

or

$$c_0 = \frac{(\rho-g)W^*}{1 - \exp[(g-\rho)T]} .$$

Substituting (5) into (4):

$$U = \frac{A}{\beta}\left\{\frac{(\rho-g)}{1 - \exp[(g-\rho)T]}\right\}^{\beta}\left\{\frac{1 - \exp[(g\beta-\delta)T]}{\delta - g\beta}\right\}(W^*)^{\beta} . \quad (6)$$

Maximizing (6) with respect to the rate of growth of consumption, g, yields:

$$U'(g) = \frac{\beta U}{\delta - g\beta} - \frac{\beta U}{\rho - g} + \frac{\beta T U \exp[(g-\rho)T]}{\{1 - \exp[(g-\rho)T]\}} - \frac{\beta T U \exp[(g\beta-\delta)T]}{\{1 - \exp[(g\beta-\delta)T]\}} .$$

Thus $0 = \beta U \left\{ \frac{1}{\delta - g\beta} - \frac{1}{\rho - g} \right\} + \beta T U \left[\frac{\exp[(g-\rho)T]}{\{1 - \exp[(g-\rho)T]\}} - \frac{\exp[(g\beta-\delta)T]}{\{1 - \exp[(g\beta-\delta)T]\}} \right] .$

It can easily be verified that $(\rho - g) = \delta - g\beta$ is the only solution such that $U'(g) = 0$ for all lifetimes T. Thus:

$$\rho - g = \delta - g\beta$$

or

(7) $$g = \frac{\rho - \delta}{1 - \beta} .$$

The consumption profile is given by

(8) $$c(t) = \frac{(\rho - g)W^* \exp(gt)}{1 - \exp[(g-\rho)T]} .$$

Actual wealth at a point of time $(K(t))$ is then given by

(9) $$K(t) = K(0)e^{\rho t} + \int_0^t [y_L(v) - c(v)]e^{\rho(t-v)}dv .$$

The case of varying rates of return is easily handled. Let $\rho(t)$ be the instantaneous rate of return and let $R(t) = \int_0^t \rho(v)dv$ be the total rate of return to time t. Then lifetime wealth is

(3') $$W^* = K_0 + \int_0^T y_L(t) \exp[-R(t)]dt .$$

The optimal consumption trajectory then implies

$$(7') \qquad c(t) = c(0)\, \exp\left[\frac{R(t) - \delta t}{1 - \beta}\right] = c(0)\, \exp[G(t)]$$

where $G(t) = \int_0^T [(\rho(v) - \delta)/(1-\beta)]dv$. Thus

$$(5') \qquad c(0) = W^* \left[\int_0^T \exp[G(t) - R(t)]dt\right]^{-1} .$$

2. The Concepts of Annuity Income (AI) and Utility-Equivalent Annuity Income (UAI)

We now turn to the problem of finding a concept of economic welfare which will allow comparisons of groups with differing life expectancies, wealth-income ratios, and future labor incomes. Since our investigation is oriented toward welfare, it is natural that we choose an index of welfare such that if two individuals have equal utilities they have equal index values. We thus propose the following measures:

> As index of economic welfare, we define the annuity income (AI) as that constant annual per capita consumption level which exhausts lifetime income; the utility-equivalent annuity income (UAI) is the value of that constant consumption stream which gives a total utility equal to the optimal consumption profile.

Thus two households have equal AI if the maximum (per capita) annuities they can buy with their income are equal. Speaking loosely, they have equal UAI if the average annual utility levels are equal. For households with equal lifetimes, to have equal AI means that the value of lifetime wealth of the households are equal, while equal UAI means that the utility of households are the same.

The concepts can be illustrated in Figure 1 for a two-period problem. W^* on the horizontal axis represents the value of lifetime wealth, and has slope $-(1+\rho)$. At the market interest rate ρ, the household chooses the point A, on indifference curve U_2, with associated consumption $\{\hat{c}_1, \hat{c}_2\}$ as his consumption plan. Alternatively he could have bought an annuity at point D, with consumption (AI, AI) which would have left him slightly worse off. This is the annuity-income (AI). The utility-equivalent annuity, along indifference curve U_2, is B with consumption $\{UAI, UAI\}$. This is our definition of utility-equivalent annuity (UAI).

In the general case, the annuity income (y^*) is simply the solution to

$$W^* = \int_0^T y^* e^{-\rho t} dt$$

or

$$y^* = \frac{\rho W^*}{1 - \exp(\rho T)} . \tag{10}$$

The utility-equivalent annuity (c^*) is the solution to

$$\int_0^T (c^*)^\beta e^{-\delta t} dt = \int_0^T [\hat{c}(t)]^\beta e^{-\delta t} dt$$

where $\hat{c}(t)$ is the optimal consumption profile. Substituting from (8), we have after some manipulation:

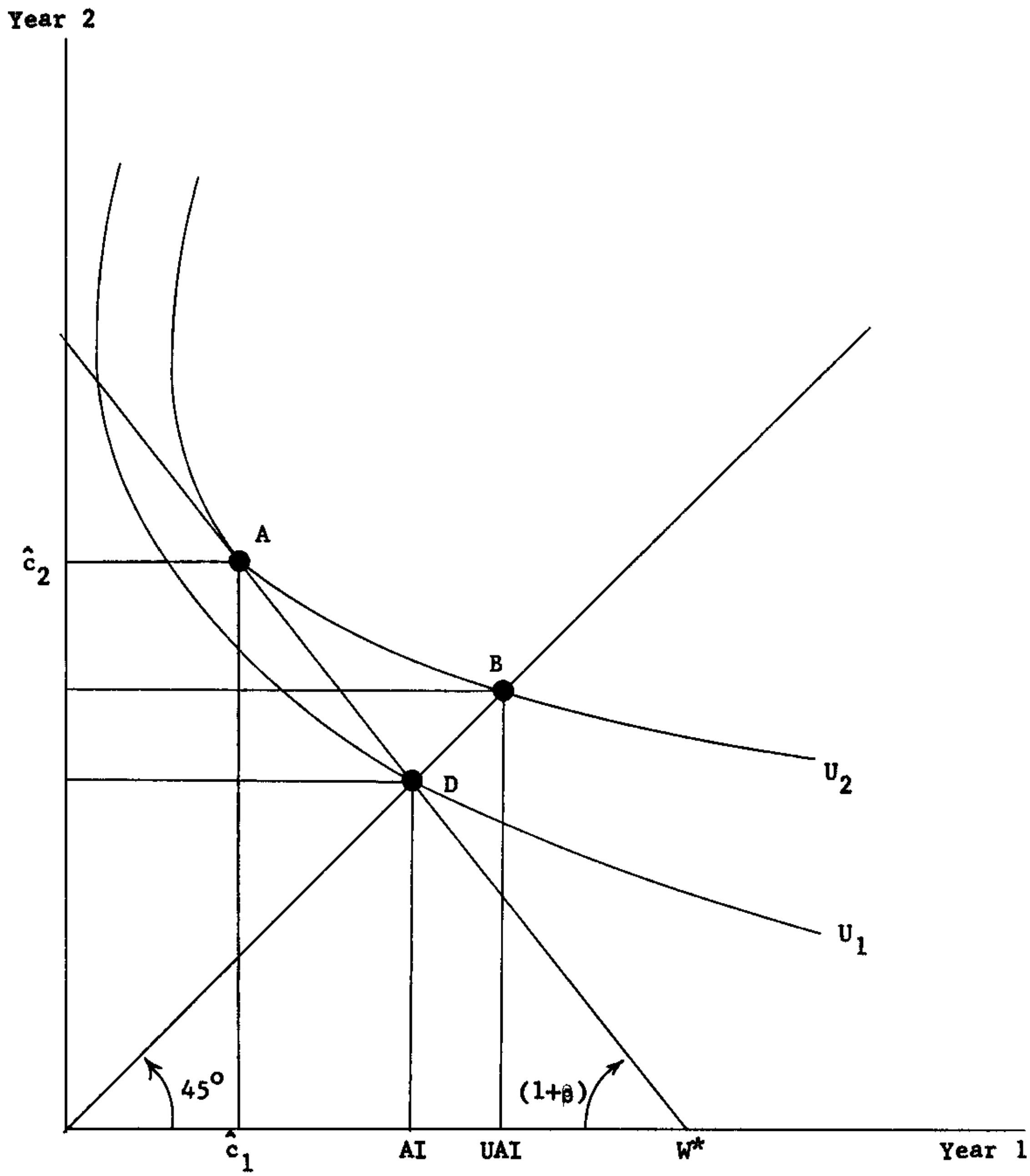

FIGURE 1. Consumption, Wealth, AI, and UAI

$$c^{*\beta} = (W^*)^\beta \left(\frac{\delta}{1 - e^{-\delta T}}\right) \frac{(\rho - g)^\beta}{\delta - g\beta} \frac{1 - e^{(g\beta - \delta)T}}{[1 - e^{(g-\rho)T}]^\beta} .$$

Note that if $\delta = 0$, we substitute T^{-1} for $\delta/(1 - \exp(-\delta T))$ in all expressions. Thus UAI is[1]

$$\text{(11)} \qquad c^* = \frac{W^*(\rho - g)}{1 - e^{(g-\rho)T}} \left(\frac{\delta[1 - e^{(g\beta - \delta)T}]}{(1 - e^{-\delta T})(\delta - g\beta)}\right)^{1/\beta}$$

where $g = (\rho - \delta)/(1 - \beta)$. This is more easily understood if the first-order Taylor expansion is taken:

$$c^* \cong \left\{ \left(\frac{\rho - g}{g\beta - \delta}\right)^\beta \left(\frac{\delta}{1 - [1 - \delta T]}\right) \left(\frac{1 + (g\beta - \rho)T - 1}{[1 + (g - \rho)T - 1]^\beta}\right) \right\}^{1/\beta} W^*$$

or

$$\text{(12)} \qquad c^* \cong \frac{W^*}{T} .$$

Thus the level of economic welfare is approximately the lifetime wealth

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[1] With variable rate of return, the formulae for AI and UAI are:

$$\text{(10')} \qquad y^* = W^* \left[\int_0^T \exp[-R(t)]dt \right]^{-1}$$

and

$$\text{(11')} \quad c^* = W^* \left[\int_0^T \exp[G(t) - R(t)] \right]^{-1} \left(\frac{\delta}{1 - \exp(-\delta T)}\right)^{\frac{1}{\beta}} \left(\int_0^T \exp[G(t)\beta - \delta t]dt \right)^{\frac{1}{\beta}} .$$

over years of life.[1]

3. Extensions

Varying family size. The cases just examined pertain to a single individual. For a household with varying size, we assume utility is linear in the number of persons times the utility of per capita consumption. If the household has size P_t , C_t total consumption c_t per capita consumption, the utility function is

$$U = \int_0^T P_t U\left(\frac{C_t}{P_t}\right) e^{-\delta t} dt \quad (13)$$

Subject to the same budget constraint in (5). Substituting we get

$$U = \frac{A}{\beta} \int_0^T P_t^{1-\beta} C_t^{\beta} e^{-\delta t} dt .$$

If g = the rate of growth of per capita consumption, then

$$U = \frac{A}{\beta} \int_0^T P_t c_0^{\beta} e^{(g\beta-\delta)t} dt \quad (14)$$

where

$$W^* = \int c_t P_t e^{-\rho t} dt = \int c_0 e^{(g-\rho)t} P_t dt$$

[1]The approximation is not always accurate. If $\delta = .01$, $\rho = .06$, $\beta = .5$, $T = 40$, the approximation yields $c^*/W^* = .025$, while the correct figure is .09084. The difference between the two values gives some peculiar results in the estimates below.

or

$$c_0 = W^* \left[\int_0^T e^{(g-\rho)t} P_t dt \right]^{-1} . \tag{15}$$

Maximizing (14) with respect to g subject to (15)

$$U'(g) = \frac{U \int_0^T t\beta e^{(g\beta-\delta)t} P_t dt}{\int_0^T e^{(g\beta-\delta)t} P_t dt} - \frac{U \int_0^T \beta t e^{(g-\rho)t} P_t dt}{\int_0^T e^{(g-\rho)t} P_t dt} = 0 . \tag{16}$$

It is easily seen that $\delta - g\beta = \rho - g$ is the only solution to equation (16).

We can thus use the results in Section II.A.1 above as long as we interpret all variables as per capita magnitudes and replace the original budget constraint by that in (15).

Uncertainty. The treatment of uncertainty is a serious problem. For the most part, we assume that households act on the basis of certainty equivalence rather than expected utility maximization. The latter poses such large computational problems that it is infeasible for us (not to mention the problems it poses for a household without sophisticated computational techniques).

The one partial exception to this rule is the household's investment policy. In the quantitative work to follow, we assume that there are two investment assets, fixed yield and variable yield. The fixed yield (called "bonds") give rate of return i with subjective certainty, while the variable yield (called "equities") have rate of return r which is normally distributed with mean α and standard deviation σ .

Under these assumptions, and the utility function used above, it can be shown that the consumption profile and portfolio composition is completely determined by the parameters of utility function and security returns. More precisely, the household divides its portfolio in fixed proportions between bonds and equities, with the proportion of equities, e^*, being

$$e^* = \frac{\alpha - i}{\sigma^2(1-\beta)} . \tag{17}$$

The consumption profile is the same as above except that ρ is no longer the interest rate, but becomes a risk-corrected discount rate.[1]

Future labor income is probably the single most important uncertainty faced by households. Formally, we can think of the problem as uncertainty with an asset that cannot be bought or sold. A reasonable treatment, and the one which is used here, is to have individuals make the best possible point estimate of future labor income.

Demography. Households face two general kinds of demographic uncertainty: First, they do not know the life expectancies of existing household members. Second, they do not know the exact number of new additions which will be made to the household. Here again we have our households behaving according to certainty-equivalent behavior. For calculations

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[1]More precisely ρ should now be interpreted as that riskless rate of return which gives the same utility as the optimal portfolio. See Samuelson and Merton [1970] and Merton [1969]. More precisely $(1+\rho)^\beta =$ $\int[(1-e^*)i + e^*r]^\beta f(r)dr$. To a first order approximation $\rho = (1-e^*)i + e^*\alpha$.

of the life expectancy, we take the average life expectancy of a member of the population with age of the head. Second, we assume no further births, marriage, or divorce in the population. We do, however, account for the process of separation of children from the household, which is assumed to occur at 21 years. The major shortcoming of this procedure is that it may over-estimate the per capita lifetime wealth of a young household by under-estimating the number of children. The fact that we underestimate future gifts and bequests of young households may offset this underestimate of household size.

Taxes. We have not explicitly calculated the taxes for each household. To do this correctly would require not only an allocation of each of the many kinds of Federal, state, and local taxes, but explicit assumptions about the macro-economic incidence of each.[1] This is beyond the scope of the present paper.

To obtain the distribution of after-tax income, we could assume that the present value of lifetime taxes is proportional to per capita lifetime wealth. Given the great uncertainty about the incidence of taxes, and in particular as to whether the total system of taxation (including state and local taxation) progressive or regressive, this is at least a useful first approximation.[2]

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[1] It was necessary to make some assumptions about the incidence of existing taxes in order to use national income data. We have used the following conventional assumptions about existing taxes: corporate income taxes and labor taxes are absorbed by capital and labor, while all indirect business taxes are completely shifted forward.

[2] It should be recalled that personal income taxes constitute only 38% of total government revenues. A flat-yield sales tax meets the above assumption.

Similarly we assume that the benefit of government expenditures on goods and services is proportional to lifetime wealth.

B. Empirical Estimation of the Distribution of Economic Welfare

1. Sources

We now present enough estimates of the levels and distribution of economic welfare. The basic data are the Federal Reserve Survey of Financial Characteristics of Consumers.[1]

The survey gives reasonably complete and detailed estimates of current wealth and income of 2,557 families. It is necessary to complement this with estimates of future labor income. Variable definitions are given in Appendix A. The parameter estimates used in deriving AI and UAI are given in Appendix B. The details of the estimation of future labor income are given in Appendix C.

2. Results

Tables 1 and 2 and Figure 1 below give the results of the estimates of the distribution of AI and UAI described above. These are also compared with the distribution of income and net worth as conventionally measured.

Table 1 shows the numerical distribution in 22 groups for annuity income and by age and other groups. Figure 1 shows the Lorentz curves for different measures. Other summary measures are given in Table 2.

[1] See Survey [1966].

As would be expected, the comprehensive measures of economic welfare are more unequally distributed than income, but less unequally distributed than wealth.[1] The reason is that annuity incomes are (complicated) functions of both labor income and wealth. There is no significant difference between the two annuity versions (AI and UAI).[2]

C. Measures of the Distributional Incidence of Policies

The conventional measures of inequality are inadequate for a complete assessment of the distributional impact of policies. The usual method of analyzing distributional effects is to examine some aggregate measure (such as Gini coefficient) or to examine the impact by economic or demographic class. The problems which these measures are (1) that the aggregate measures do not correspond to a reasonable social preference ordering (see Atkinson [1970]); and (2) that the dispersion and statistical significance of the incidence, and general reshuffling of households, is ignored.

For the purpose of the present study, we distinguish two classes of measures of the incidence of inflation.

Aggregate measures. For aggregate measures of incidence, we have two sets of calculations. (1) We calculate the first three sets of normalized moments of the distribution-mean, coefficient of variation, and coefficient

[1]The distribution of wealth is much more concentrated with Survey estimates than using the conventional Estate Tax method. The estimate of the share of the top 1% of households is 31%, while for the same year the estate method gives an estimate of 15%. The major differences are the share of the top group and the fact that the Estate Tax method uses extraneous total wealth estimates while we use those estimated by Survey.

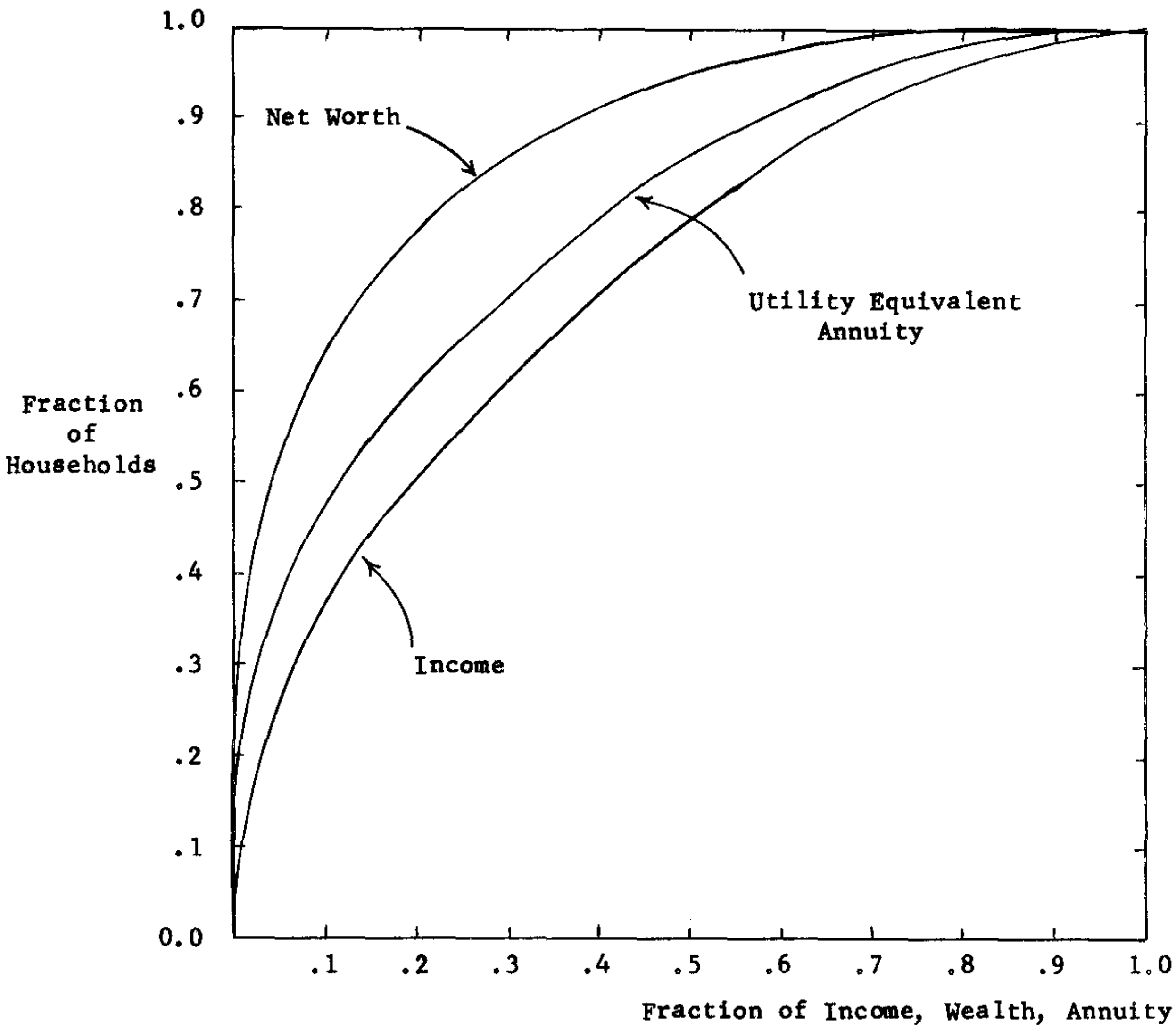
[2]The distribution of annuity by age group shows a feature of distribution which has not been noted before, the dip in age--annuity profile by age in the 60-69 age bracket. This dip may be explained by the effect of the Great Depression.

TABLE II.1. Distribution of Per Capita Annuity, Different Classes, 1962

Income Class	Mean Annuity (dollars)	Sample Size	Lower Limit of Class (dollars)	Upper Limit of Class (dollars)
1	6.25155	64	20.1	33.1
2	46.17748	10	33.1	54.6
3	69.55664	9	54.6	90.0
4	113.82186	19	90.0	148.4
5	199.59049	33	148.4	244.7
6	326.16945	63	244.7	403.4
7	536.73351	105	403.4	665.1
8	873.10310	156	665.1	1096.6
9	1431.84589	247	1096.6	1808.0
10	2392.56158	332	1808.0	2981.0
11	3813.80325	462	2981.0	4914.8
12	6226.09216	396	4914.8	8103.1
13	10156.72229	244	8103.1	13359.7
14	15852.50500	149	13359.7	22026.5
15	26901.65161	94	22026.5	36315.5
16	43052.78174	66	36315.5	59874.1
17	75150.67285	43	59874.1	98715.8
18	130546.25684	30	98715.8	162754.8
19	199624.45508	10	162754.8	268337.3
20	306410.15234	11	268337.3	442413.4
21	572418.89844	3	442413.4	729416.4
22	771768.21875	2	729416.4	1202604.3
Age Class				
10-19	2403.44363	5		
20-29	5365.72595	246		
30-39	4405.81024	435		
40-49	3228.53036	590		
50-59	2580.67984	606		
60-69	1951.78925	424		
70-79	2467.34225	199		
80-89	2916.73422	40		
90+	1481.12770	3		
Other				
Retired	2192.80975	231		
Widows	1287.33987	283		
Nonwhite	1502.22806	156		
All Households	3306.	2557		

Note: The estimates in this table and in Table II.2 correspond to the distribution using the individually estimated ρ with a mean $\rho = 5.6$. The fact that equities are progressively distributed leads to the higher estimates of annuity income than Table II.3.

TABLE II.2. Lorentz Curves for Income, Net Worth and Utility Equivalent Annuity



Source: Table 1.

TABLE II.3. Other Measures of Inequality, 1962

	Income	Net Worth	Per Capita Lifetime Wealth	Annuity Income	Utility-Equivalent Annuity Income
Mean (μ)	6377	20,955	28,804	2096	2167
Standard Deviation (σ)	7297	124,595	81,022	6453	6620
Coefficient of Variation (σ/μ)	1.144	5.946	2.813	3.077	3.054
Coefficient of skewness (λ/μ)	2.766	23.20	12.35	11.66	11.68

Note: If x_i is the relevant variable, w_i is the population weight estimated in Survey [1966], then:

$$\mu = (\Sigma x_i w_i)/(\Sigma w_i)$$

$$\sigma^2 = [\Sigma(x_i^2 - \mu)w_i]/(\Sigma w_i)$$

$$\lambda = [\Sigma(x_i - \mu)^3 w_i]/(\Sigma w_i)$$

For this table all concepts were estimated with a constant 5 percent real rate of return on wealth (i.e., $\rho = .05$).

of skewness.[1] These allow general evaluations of what has happened to the distributions. (2) We have further chosen to value the outcome by an explicit "social welfare function" as an aggregate objective. This allows the policymaker to make an explicit judgment about the desirability of redistributive policies. We have used the following four social welfare functions:

$$SW_1 = A + B(\Sigma\, w_i y_i)$$

$$SW_2 = A + B(\Sigma\, w_i \log(y_i))$$

$$SW_3 = A + B(\Sigma - 1.8 w_i y_i^{-1.8})$$

$$SW_4 = A + B(\Sigma - 5 w_i y_i^{-5})$$

where SW_i is the value of the social welfare function, w_i are the estimated sampling weights of each household given in Survey [1966], and y_i is the measure of economic welfare (e.g., per capita annuity income) for household i . The four cases correspond to four different assumptions about the marginal social utility of income. With SW_1 the function is linear; SW_2 is the Bernoulli or logarithmic form; SW_3 uses the estimate of the elasticity derived from household behavior; and SW_4 uses the very low value which might come from an extreme egalitarian.[2]

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[1] See notes to Table II.2 for definitions of these terms.

[2] It should be noted that there is no presumption that the social welfare function uses the same parameter as the individual; the latter embodies no value judgments, but is more usefully interpreted as a representation of a particular ordinal structure of intertemporal choice.

To see the extent to which the SW_4 is egalitarian, note that the social marginal utility of income to a man with $10,000 is .0312 times the marginal social utility of a man with income of $5,000.

Individual measures. In addition to the aggregate measures of incidence described above, there are three calculations used to measure the impact on individual households. (1) The first set are calculations of the impact of inflation by annuity class and by a few demographic characteristics. In addition to calculating the mean impact by class, we also indicate the variability of that experience. (2) A second set of measures are logarithmic regressions of change in annuity against annuity before inflation. The regression coefficient indicates whether the inflation is significantly progressive or regressive in its impact. We will call the estimated coefficient the coefficient of regressiveness. If the sign is positive, this indicates that higher incomes gain a larger fraction from the inflation than lower incomes, and this is regressive; the converse holds for negative signs. (3) A final measure is the average change in the fortunes of individual households, designated by the coefficient of variation of individual income and called the coefficient of mobility. This indicates the extent to which aggregate changes in distribution are accompanied by a general reshuffling of the economic welfare of households.[1]

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[1]Thus if the relation between relative welfare before (y_1) and after (y_0) is $y_{1i} = y_{0i} + \varepsilon_i$, the variance after a policy (or aggregate inequality) is $\sigma_1^2 = \sigma_0^2 + \sigma_\varepsilon^2 + 2\ \mathrm{cov}[y_0, \varepsilon]$. The variance of individual welfare from original (σ_2^2) if $\bar{y}_1 = \bar{y}_0$ is $\sigma_2^2 = \Sigma(y_{1i} - \bar{y}_1 - y_{0i} + \bar{y}_0)^2 = \sigma_1^2 + \sigma_0^2 - 2\ \mathrm{cov}[y_0, y_1]$. While most social planners wish to reduce aggregate inequality (σ_1^2) it is not clear whether individual mobility (σ_2^2) is desirable or not. The present author would think it undesirable.

Since $\sigma_2^2 = (\sigma_1^2 - \sigma_0^2) + 2\sigma_0(\sigma_0 - \rho\sigma_1)$, there is a constraint on the amount of redistribution that can occur for a given amount of individual wealth.

III. THE EFFECT OF INFLATION OF THE DISTRIBUTION OF LIFETIME INCOME

There are widely disparate views on the impact of inflation on individual economic welfare. As in most economic controversies, the differences of opinion stem largely from differing views of the economic process or different time periods under consideration. Virtually all the empirical work on the distributional effects of inflation considers the effects on short-run income, with little consideration of effects on assets; virtually all theoretical work considers long-run effects in equilibrium systems.

The thrust of the present work is long-run considering lifetime income and wealth. Three different mechanisms (classical, neoclassical, and Keynesian) have been used in order to span most current views about the macro-economic mechanism. Moreover, two different kinds of inflationary episodes are considered: a once and for all burst of inflation of 10 percent in the first year then returning to the usual rate; we call this "one-shot inflation." The second kind of inflation consists of an increase in the rate of inflation of 1 percent for all future periods above the usual rate; this we call "continual inflation."[1]

We present no explicit discussion of the behavior of the macro-economic systems in the paper.

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[1] These figures were chosen as the approximate magnitude of the differential inflation over the period 1962 to 1971 relative to prior experience.

A. Classical Inflation

1. Theory

The first and simplest economic mechanism we consider is the classical mechanism, closely related to current monetarist thought. To oversimplify slightly, inflation makes no difference to the real variables of the system in the long run. The only effects are the transient ones due to incorrectly anticipated inflation.

More precisely, we assume that the system is one where all markets clear instantaneously, and where relative prices of reproducible goods and labor are unaffected by the level of the real interest rate. There is no outside money or noninterest bearing debt.

We examine a burst of inflation caused by an increase in demand due to a war or shift in expectations. This demand is assumed to be completely and instantaneously choked off by a rise in prices of 10 percent for one-shot inflation, or by an increase in the rate of inflation of 1 percent in the continuous case. There are thus no changes in any real magnitudes except for the distribution of current wealth.

2. Estimates

Given a burst of one-shot inflation of 10 percent we can easily calculate the effect of inflation on the distribution of economic welfare. All "bonds" (i.e., the net value of fixed yield assets) are revalued downwards by the fraction $1/(1 + .10)$ while the real value of equities and future labor income remains unchanged.[1]

In the case of continual inflation, the real interest rate adjusts slowly to regain its original value.[2] Thus bonds have a lower value in consumption than they would have in the absence of inflation.

The results of inflation on annuity are shown in Tables III.1 to III.3. Table III.1 indicates that there is a very small decrease in average annuity (AI). This decrease is 0.03% for both kinds of inflation. UAI, on the other hand, increases substantially.

The result on both measures of inequality is a slight decrease in both the coefficient of variation and the coefficient of skewness. These measures of inequality are reduced from one to two percent in the case of one-shot inflation and 0.3% for continual inflation.

Finally, the coefficient of mobility indicates that there is a significant amount of reshuffling of annuities, especially for one-shot inflation.

The annuity regressions point to the same general conclusion. According to these, both one-shot and continual inflation are progressive measures as indicated by the fact that the regression coefficients are less than zero. Except for regression 4 these coefficients are significantly less than zero.

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[1]It is recognized that a certain inconsistency results from this method. Since the net value of fixed-yield assets does not net to zero, but rather to an average value of $423 per household (two percent of net worth), there is in addition a small decrease in the aggregate real value of wealth (i.e. the Pigou effect).

[2]The adjustment of the real interest rate is described below, p. 32 and in Appendix D.

TABLE III.1. Effects of Inflation in Classical Regime

	No Inflation	One-Shot Inflation	Continual Inflation
Mean:			
AI	3306.	3305.	3305.
UAI	3921.	3974.	3944.
Coefficient of Variation:			
AI	2.069	2.045	2.062
UAI	1.927	1.910	1.920
Coefficient of Skewness:			
AI	7.332	7.209	7.295
UAI	6.411	6.221	6.341
Coefficient of Mobility (percent):			
AI	-----	4.07	1.27
UAI	-----	4.06	1.40

Notes: The definitions of the mean, coefficients of variation, and coefficient of skewness are given in Table II.3 above.

The coefficient of mobility is defined as the coefficient of variation of individual income.

TABLE III.2. Annuity Regressions for Classical Regime, Percent

[all coefficients in percentages]

One-Shot Inflation

1. $\ln AI(1) - \ln AI(0) = \underset{(.340)}{1.58} - \underset{(.041)}{.204} \ln AI(0)$ $\qquad R^2 = .9998$

2. $\ln UAI(1) - \ln UAI(0) = \underset{(.341)}{.868} - \underset{(.0407)}{.0817} \ln UAI(0)$ $\qquad R^2 = .9999$

Continual Inflation

3. $\ln AI(1) - \ln AI(0) = \underset{(.106)}{.456} - \underset{(.0128)}{.0591} \ln AI(0)$ $\qquad R^2 = .9998$

4. $\ln UAI(1) - \ln UAI(0) = \underset{(.117)}{.266} - \underset{(.0140)}{.0156} \ln UAI(0)$ $\qquad R^2 = .9999$

Notes to regressions (Tables III.2, III.5, III.8). We have expressed coefficients in percentages to facilitate reading. The numbers in parentheses are standard errors.

TABLE III.3. Effect of Inflation on Annuity by Annuity, Age, and Other Classes
Classical Inflation: Ratio of Annuity After to Annuity Before

Income Class		Instantaneous Inflation Mean	Instantaneous Inflation Stan.Dev.	Continuous Inflation Mean	Continuous Inflation Stan.Dev.	Sample Size
1		1.03904	0.04549	1.01321	0.00508	64
2		0.96279	0.00118	0.99108	0.00006	10
3		0.99078	0.00047	0.99613	0.00006	9
4		0.98805	0.00393	0.99387	0.00039	19
5		0.99576	0.00309	0.99500	0.00035	33
6		1.01697	0.02091	1.00148	0.00058	63
7		1.00049	0.00772	0.99855	0.00036	105
8		0.99784	0.00278	0.99843	0.00012	156
9		1.00586	0.00335	1.00146	0.00023	247
10		0.99955	0.00079	1.00006	0.00008	332
11		1.00041	0.00041	1.00026	0.00004	462
12		1.00225	0.00031	1.00059	0.00003	396
13		0.99845	0.00012	0.99955	0.00001	244
14		0.99966	0.00019	0.99998	0.00002	149
15		0.99206	0.00053	0.99804	0.00004	94
16		0.99006	0.00026	0.99780	0.00002	66
17		0.98757	0.00029	0.99693	0.00002	43
18		0.98389	0.00014	0.99632	0.00001	30
19		0.99107	0.00018	0.99767	0.00001	10
20		1.00025	0.00046	0.99959	0.00002	11
21		0.97623	0.00005	0.99242	0.00001	3
22		0.98664	0.00013	0.99759	0.00000	2
Age Class						
1		0.99849	0.00000	0.99865	0.00000	5
2		1.01709	0.00539	1.00323	0.00029	246
3		1.02098	0.01791	1.00538	0.00149	435
4		1.00264	0.00015	1.00051	0.00004	590
5		0.99995	0.00046	0.99946	0.00007	606
6		0.99126	0.00142	0.99711	0.00013	424
7		0.98075	0.00115	0.99574	0.00006	199
8		0.97118	0.00094	0.99541	0.00002	40
9		0.99879	0.00000	0.99981	0.00000	3
Other						
Retired	41	0.98152	0.00120	0.99612	0.00006	231
Widows	42	1.00208	0.01451	1.00104	0.00155	283
Nonwhites	43	1.02356	0.02304	1.00716	0.00264	156

B. Neoclassical Inflation

The classical model used above abstracts from some important features of the economic system. The effects have long been considered crucial in discussing distributional effects of inflation. First, steady inflation erodes the real value of money-fixed assets. To the extent that these are in the aggregate assets of the household sector (as is generally the case), an inflationary episode reduces to the real value of wealth. Second, since inflation affects the value of real interest and profit rates and the wealth-income ratio, the system will generally be out of long-run equilibrium. This leads to saving in tangible assets which changes factor prices. It is usually thought that this revaluation of wealth and labor income flows will redistribute economic welfare toward labor and lower income groups.

The neoclassical regime considers an economy at a fixed level of utilization growing according to the principles of a standard neoclassical model.[1]

1. The Theory

To analyze the effects of neoclassical inflation we combine our sample of households with a very simple neoclassical macro-economic model. To make the model operational, we have estimated the equations using annual U.S. data.

The following assumptions are made about the economy.

[1] For a recent exposition, see Solow [1970].

1) Inflation is assumed to be generated by conditions of aggregate excess demand or supply. In the neoclassical case (like the classical but unlike the Keynesian case) the reaction of prices is assumed to be so swift that deflation or inflation wipes out excess supply or demand immediately and no change in relative prices takes place. In terms of current macro-economic thought, the short-run Phillips curve is vertical. We can in this case treat inflation as an exogenous variable since there is no feedback of the system's equilibrium on the rate of inflation. Whether inflation is a policy variable or a true exogenous variable is, from our point of view, unimportant. What becomes important is the anticipated rate of inflation in the system (π^e) ; this is in turn a geometric distributed lag over past experienced inflation rates:

$$\pi_t^e = \sum_{i=1}^{\infty} \pi_{t-i}\theta_\pi^i(1 - \theta_\pi)/\theta_\pi \tag{1}$$

2) Aggregate potential output (Q) is determined by a Cobb-Douglas production function:[1]

$$Q = AK^{\alpha}[L(1-\bar{u})]^{1-\alpha}e^{\gamma t} \tag{2}$$

where K is replacement cost of capital L the labor force,

[1] Note that we define potential output to be the level of output at a "normal" rate rather than at four percent, which is the customary definition. This definition makes the model compatible with a vertical Phillips curve at an arbitrary rate. It also resolves complications which arise in the Keynesian regime where the economy functions at alternative unemployment rates.

$\bar{u}$ the normal unemployment rate, and $\gamma/1-\alpha$ the rate of Harrod-neutral technological change. The rate of profit on capital (r) and the real wage rate (w) are determined by marginal productivity conditions:

$$(3) \qquad r = \alpha Q/K$$

$$(4) \qquad w = (1-\alpha)Q/(L(1-\bar{u})) .$$

3) Employment is a fixed fraction--96 percent--of the labor force.

4) The money value of government debt (GD) is given and the money value of debt increases at 2 percent. Taxation is neutral.[1] Wealth is composed of the real value of government debt plus the replacement cost of capital:[2]

$$(5) \qquad W = \frac{GD}{p} + K = D + K .$$

where $D = GD/p$.

5. Discrepancies between potential output and the amount of consumption or investment predicted by the model are assumed to be completely made up by government expenditures neutrally financed.

6. The relation between the rate of profit on capital and the real interest rate is given by the portfolio equation (17):

---

[1] A "neutral" tax is defined as one which does not alter the distribution of economic welfare, here the distribution of c^* or of consumption. A comprehensive flat-rate sales or consumption tax is neutral, whereas a flat rate income tax is not.

[2] We have overlooked the problem of the effect of inflation on financing the interest payments on government debt. Formally, we can assume that interest payments are financed by a flat-rate sales tax (which is not altogether unreasonable for state issues). For a complete study, the incidence through reduction of the real value of government debt depends on the incidence of the marginal tax payment. This is a speculative issue.

(6) $$i_t = \pi_t^e + r_t + \lambda\left(\frac{K}{K + D}\right) .$$

7. The desired wealth-income ratio is determined by the life-cycle consumption model discussed in Section II. The long-run relation is assumed to have constant elasticity with respect to the effective rate of return on assets, ρ .[1] The short-run relation adjusts with a distributed lag:

(7) $$\left(\frac{W}{Q}\right)^* = B_0\rho^{\beta_1}$$

(8) $$\Delta\left(\frac{W}{Q}\right)_t = \beta_2\left[\left(\frac{W}{Q}\right)^*_{t-1} - \left(\frac{W}{Q}\right)_{t-1}\right] .$$

Estimates of equations (1) through (8) are shown in Table III.7, and methods are discussed in Appendix D .

2. The Results

Tables III.4 through III.6 show the estimates of the results of neoclassical inflation. The general conclusion is that one-shot inflation is more equalizing and progressive than continual inflation.

In Table III.4 the effects of neoclassical inflation on our summary statistics is shown. As in the case of classical inflation, there is a decline in the average annuity and utility-equivalent annuity of households in three of four cases. This reflects the fact that the loss of income due

[1] See Section II for a discussion of this concept.

TABLE III.4. Effects of Inflation in Neoclassical Regime

	No Inflation	One-Shot Inflation	Continual Inflation
Mean:			
AI	3350.	3350.	3348.
UAI	3887.	3876.	3907.
Coefficient of Variation:			
AI	2.083	2.065	2.069
UAI	1.952	1.942	1.938
Coefficient of Skewness:			
AI	7.412	7.327	7.338
UAI	6.632	6.570	6.517
Coefficient of Mobility (percent):			
AI	-----	7.96	6.26
UAI	-----	7.81	6.23

TABLE III.5. Annuity Regression for Neoclassical Regime

[coefficients in percentages]

One-Shot Inflation

5. $\ell n\ AI(1) - \ell n\ AI(0) = \underset{(.669)}{-1.89} + \underset{(.0804)}{.203}\ \ell n\ AI(0)$ $\quad R^2 = .9992$

6. $\ell n\ UAI(1) - \ell n\ UAI(0) = \underset{(.658)}{-2.22} + \underset{(.0784)}{.212}\ \ell n\ UAI(0)$ $\quad R^2 = .9999$

Continual Inflation

7. $\ell n\ AI(1) - \ell n\ AI(0) = \underset{(.526)}{-1.74} + \underset{(.0632)}{.187}\ \ell n\ AI(0)$ $\quad R^2 = .9995$

8. $\ell n\ UAI(1) - \ell n\ UAI(0) = \underset{(.525)}{-1.99} + \underset{(.0625)}{.230}\ \ell n\ UAI(0)$ $\quad R^2 = .9999$

TABLE III.6. Effects of Inflation on Annuity by Annuity, Age, and Other Classes
Neoclassical Inflation: Ratio of Annuity After to Annuity Before

Income Class	Instantaneous Inflation Mean	Instantaneous Inflation Stan.Dev.	Continuous Inflation Mean	Continuous Inflation Stan.Dev.	Sample Size
1	1.00551	0.00097	1.00614	0.00109	63
2	0.98236	0.00047	0.98988	0.00008	10
3	1.00554	0.00110	0.99975	0.00031	8
4	0.99253	0.00052	0.99328	0.00021	18
5	1.01172	0.00249	1.00320	0.00150	33
6	0.99746	0.00031	0.99426	0.00029	64
7	1.00550	0.00684	1.00070	0.00135	97
8	1.00234	0.00156	0.99863	0.00080	159
9	1.00005	0.00077	0.99923	0.00042	247
10	0.99940	0.00022	0.99912	0.00017	319
11	1.00036	0.00026	1.00025	0.00015	469
12	1.00159	0.00005	1.00058	0.00006	400
13	1.00036	0.00005	0.99962	0.00002	246
14	0.99867	0.00005	0.99930	0.00003	155
15	0.99323	0.00009	0.99608	0.00004	89
16	0.99090	0.00003	0.99573	0.00002	69
17	0.99063	0.00004	0.99509	0.00002	43
18	0.98789	0.00003	0.99382	0.00002	32
19	0.99108	0.00002	0.99480	0.00001	10
20	0.99816	0.00016	0.99836	0.00002	10
21	0.98608	0.00001	0.98511	0.00003	4
22	0.98897	0.00003	0.99641	0.00000	2
Age Class					
10-19	1.00215	0.00000	0.99718	0.00001	5
20-29	1.01289	0.00171	1.00502	0.00112	246
30-39	1.00912	0.00264	1.00505	0.00081	434
40-49	1.00142	0.00007	0.99944	0.00011	590
50-59	0.99888	0.00012	0.99766	0.00011	606
60-69	0.99391	0.00020	0.99520	0.00018	424
70-79	0.98824	0.00028	0.99489	0.00006	199
80-89	0.98049	0.00037	0.99510	0.00002	40
90+	0.99857	0.00001	0.99963	0.00000	3
Other					
Retired	0.98792	0.00030	0.99518	0.00007	231
Widows	0.99606	0.00067	0.99849	0.00048	282
Nonwhites	1.00452	0.00046	1.00229	0.00049	155

to the lowering of the real interest rate in the early period is not sufficiently offset by the increase in capital and real wages in the later periods as a result of the decline in real government debt. For a decline of 0.1% in AI and no change in UAI. The figures for continual inflation are -0.3% and +0.5% respectively.

The results on the distribution of annuity are very similar to that in the classical case. In particular, there is a reduction of between 0.5% and 1.0% in the coefficient of variation.

It is noticable that there is also a fair amount of mobility as a result of neo-classical inflation.

In the regression analysis, the conclusion about the distribution incidence is reversed. In the unweighted regressions, inflation is regressive (as evidenced by the positive sign) and for both AI and UAI is significantly so at the 99 percent level.

C. Keynesian Inflation

1. General Considerations

Although perhaps a step toward reality, the neoclassical simulation overlooks one entire feature of the inflationary process--the role of short-run disequilibrium in the generation and damping of inflation. (Indeed, some economists feel that this is the entire story about inflation.)

In modern macro-economic thought, inflation is part--and only one part--of the adjustment process that occurs when a system is shocked by exogenous changes or by structural shifts. For example, if an economy is in macro-economic equilibrium (with whatever inflation is occurring perfectly anticipated) and there is a sudden outbreak of war or an exogenous burst of consumption or investment, part of the adjustment--perhaps most of the adjustment in the very short run--comes through a higher level of output and employment. Only after those markets which are price-adjusters have responded and the response filters through the system does a larger fraction of the response come from prices.

In terms of the modern theory of the Phillips curve, a higher level of output and employment leads to higher settlements in money and real wages (or wage inflation). As producers mark up costs in their prices, the whole structure of prices moves up. If the inflation is gradually choked off, the system settles down to a new system of absolute prices, perhaps with some small adjustments in relative prices.

The importance of considering the Keynesian case is that for the most part inflation does not occur as an isolated incident. If we are to believe the vast outpouring of statistical work on the wage-price problem, then it is clear that inflation (here meaning deviations of the rate of inflation from the historical trend) occurs as a result of, and after, periods of high levels of utilization of the economy. Deflation, conversely, occurs during periods of relative slack. Because of the complicated series of lags relating the wage-price system, it is not easy to disentangle the exact causes of lags in the inflationary process, nor do we know the exact functional form of the trade-off between resource utilization and inflation. There can be little doubt that a trade-off, however unstable, does exist.

In discussing the effect of inflation in the Keynesian system, we must consider the effect of inflationary packages on the distribution of economic welfare. An inflationary package means that inflation is part of the package that comes when resource utilization is very high. The important effects of an inflationary package are the following:

(1) Rates of inflation and price levels are raised.

(2) Employment, hours, and output are raised.

(3) The composition of output shifts, generally toward manufacturing and durable goods.

(4) Depending on the cause of the expansionary program on aggregate demand, there may be effects on interest rates, profit rates, liquidity, and tax rates.

The present study focused on effects (1), (2) and to a certain extent (4). The effects on composition of output is beyond the scope of the present study.

2. The Macro-Economic Model

The Keynesian model used here constitutes only the most rudimentary modification of the neoclassical model discussed above.

1. The most important modification is the introduction of a trade-off between inflation and unemployment. For this we use the simplest form of Phillips curve, with the rate of inflation inversely related to unemployment and positively related to past inflation:

(9) $$\pi = a_0 + a_1 \frac{1}{u} + \sigma\pi^e \qquad 0 \leqq \sigma \leqq 1 .$$

Equation (9) is the Phillips curve with inflation (π) a function of unemployment (u) and "expected" inflation (π^e). Expected inflation follows the distributed lag over past inflation discussed above.[1]

2. The aggregate employment rate (E) is equal to employment divided by the noninstitutional population over 16. This is a function of unemployment and time:

(10) $$E = j_0 + j_1 u + j_2 t .$$

Equation (10) tells us the average fraction of available time worked by the adult population as a whole.

3. Finally, we recognize that factor rewards are subject to cyclical influences. We continue to assume that full-employment wages, profits, and interest rates are determined as in the neoclassical system above.[2] Moreover, real wages are set at the full-employment marginal product, rather than fluctuating over the business

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[1]As noted in Appendix D, this simple Phillips curve proved unable to distinguish between $\sigma = 0$ and $\sigma = 1$. We therefore chose $\sigma = 0$. Simulations of the macroeconomic model with $\sigma = 1$ indicated, not surprisingly, that with rapid reaction speeds (or low θ_π in equation (1) above) the Keynesian model behaves very much like the neoclassical model. Intermediate values of σ or of reaction speeds would lie between the neoclassical and Keynesian results. There does not seem to be much point in trying different values of σ.

[2]It would be easy to introduce "exploitation" of one factor by the other, if for example labor were paid two-thirds its marginal product.

cycle.[1] Since it is the residual income, the rate of profit on income will vary positively over the cycle. We thus represent this as:

$$(11) \qquad r = \frac{\alpha Q}{K} [\gamma_1(u-\bar{u}) + \gamma_0]$$

where Q is potential output and $(u-\bar{u})$ is the difference between unemployment and normal unemployment.

The rate of interest follows the same relation as in the neoclassical model above.

Finally, the normal unemployment rate is a distributed lag over past rates:

$$(12) \qquad \bar{u} = \sum_{i=0}^{\infty} u_{t-i}\theta_u^i .$$

3. Individual Behavior

To translate our simple macro-economic system into households, the only requirement is that the aggregate employment rate be distributed among the households. There is at present no satisfactory way of calculating the distribution of unemployment and changes in participation rates by the fine classifications we require. In any case, we are unable to translate the reported employment rate of our households (number of months worked

[1]Ignoring the short lag of prices behind wages, the assumption about fluctuations in the real-wage are consistent with the "normal-pricing" hypothesis often used in inflation studies.

in 1962) into a "permanent" or expected unemployment rate; the problem is very similar to that of calculating future labor income.

The technique used here makes the following assumption: the employment response of an individual household is proportional to amount of unemployed time. Thus if E_i^0 is the observed employment rate of household i in 1962, $E_i(t)$ the predicted employment rate in year t, $\overline{E^0}$ and $\overline{E(t)}$ the averages of E_i^0 and $E_i(t)$ for the entire population, then

$$(13) \qquad E_i(t) = E_i^0 + \left(\frac{1 - E_i^0}{1 - \overline{E^0}}\right)[\overline{E(t)} - \overline{E^0}] \; .$$

This assumption is easily seen in Figure 2. This shows on the vertical axis how a given household (say E_1) responds as a function of the aggregate employment rate on the horizontal axis.[1]

[1]The assumption about employment rates is probably a good one for those who are continually in the labor force or are out of the labor force for economic reasons. For those who are out of the labor force for non-economic reasons (say, sickness, retirement, or school), this is undoubtedly a poor assumption. A recent study (Bowen and Finegan [1971]) indicates the kind of assumption used above is fairly accurate for labor force participation. For the effect of group unemployment on aggregate unemployment, the hypothesis does quite well (see Thurow [1965]).

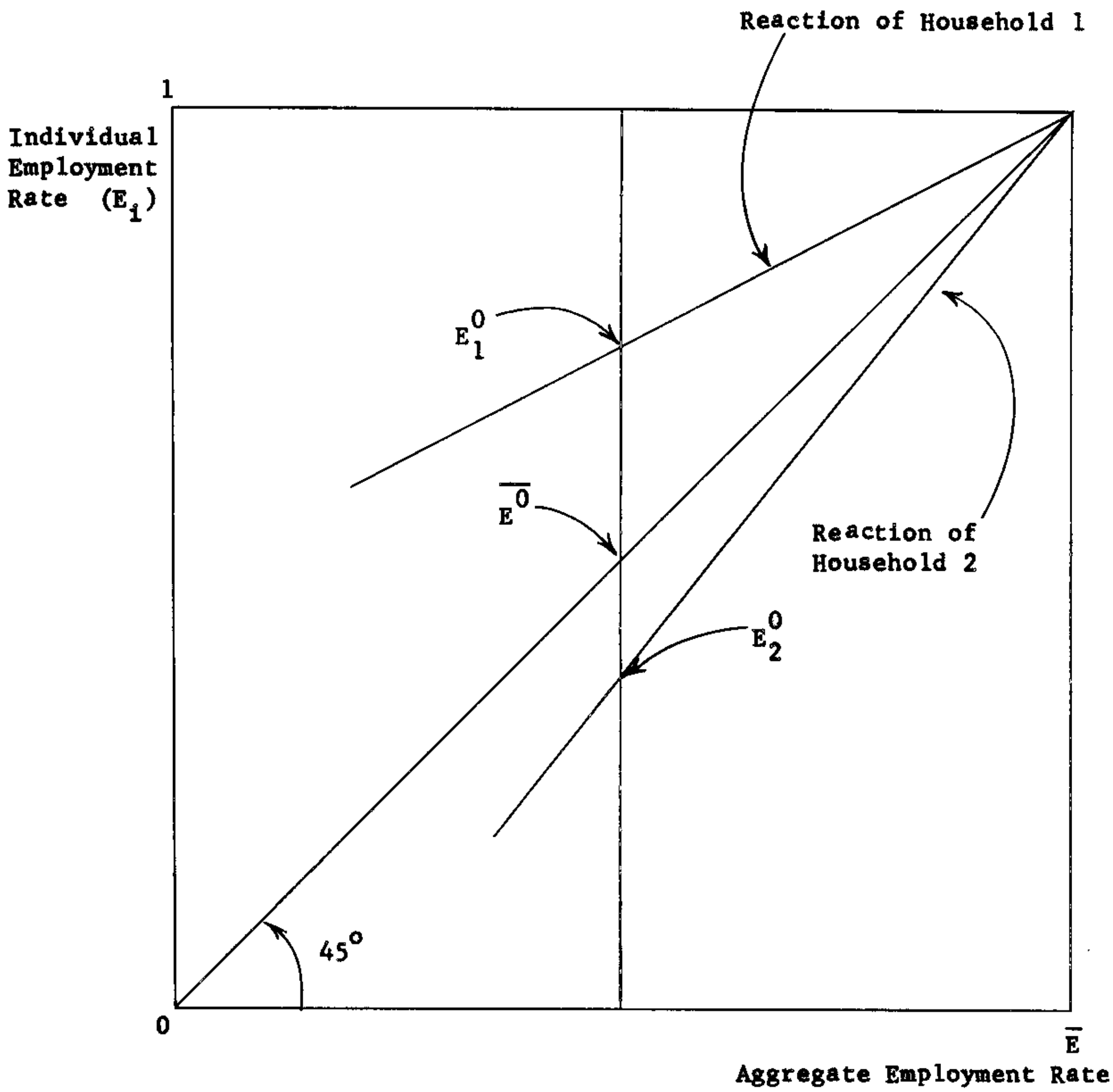

FIGURE 2. Employment Response for Two Households

Finally, we assume (as above) that individuals have perfect foresight about incomes and rates of return. This allows us to calculate easily the effect of inflation in the Keynesian model.

To calculate income, we take "normal" labor income (as calculated in Appendix C) and augment this by the ratio of employment rate in a given year over 1962 employment rate.[1] That is,

$$y_{Li}(t) = y_{Li}^{N}(t)\left(\frac{E_i(t)}{E_i(1962)}\right) \tag{14}$$

where $y_{Li}^{N}(t)$ is predicted normal labor income of household i in year t from Appendix C, $y_{Li}(t)$ is predicted labor income at employment rate $E_i(t)$.

4. Results

The results of inflation in the Keynesian regime are shown in Tables III.8 through III.10. The general impression is quite different from that of the classical or neoclassical cases. Keynesian inflation is considerably more equalizing than the other kinds, mainly because of the powerful employment effect associated with inflation.

Whereas classical inflation and neoclassical inflation decreased the

[1] The employment rate is calculated in "head-of-household equivalents," where adults are weighted by full-time earnings. Thus for a married couple in which the husband worked 9 months and the wife 2 months, the head-of-household equivalent employment rate is $(9x1 + 2x.6)/(12x1 + 12x.6) = 0.54$. The 0.6 figure is the ratio of full-time female to male earnings.

TABLE III.7. **Estimated Equations**

Neoclassical

(1) $\pi_t^e = 4.\{\sum_{i=1}^{t-1947} \pi_{t-i}(0.2)^i + (.04)(0.2)^{t-1946}\}$

(2) $Q_t = 4.13\ K^{.23}EN^{.77}e^{.0153(t-1961)}$, $EN = LF(1-\bar{u})$

(3) $r = rF = .23\ Q/K$

(4) $w = .77\ Q/EN$

(5) $i = \pi^e + r - .035\ \frac{K}{K+D}$

(6) $\ell n(W/Q)^* = 1.688 + 0.304\ \ell n\ \rho$

(7) $\Delta(W/Q) = 0.1[(W/Q)^* - (W/Q)_{-1}]$

Keynesian

(8) $\pi = -.007299 + .001486\ 1/u$

(9) $E = .600 - 0.747u + .00031(t-1930)$

(10) $r/rF = .5255 - .0157(u-\bar{u})$

(11) $\bar{u} = 10\{\sum_{i=1}^{t-1947} u_{-i}(.1)^i + .04(.1)^{t-1946}\}$

Source: Appendix D.

Variables:

π = Rate of Inflation
π^e = Expected Rate of Inflation
Q = Potential Output
K = Capital
EN = Potential Employment
LF = Labor Force
u = Unemployment
$\bar{u}$ = Normal Unemployment Rate
r = Rate of Profit
rF = Normal Rate of Profit
i = Nominal Interest Rate
D = Real Government Debt
W = Net Worth
ρ = see text
w = wage per man-year

real value of annuity and utility equivalent annuity, the Keynesian case had little effect on the one-shot case and a strong increase on annuity in the continual inflation case. The continual inflation case corresponds, in the simulations, to raising the rate of inflation from 2% to 3%, which lowers the unemployment rate from 5.4% to 3.9%. This in turn leads to an increase in AI of 0.7% and in UAI of 1.2%.

The effect on the measures of inequality are considerably more powerful in this continuous case than the most powerful effects in the classical and neoclassical cases. The coefficient of variation is reduced by 1.2% and the coefficient of skewness is reduced by two percent.

The coefficient of mobility is lower in the case of Keynesian inflation than neoclassical inflation, reflecting the fact that the incidence of unemployment is less uneven within annuity classes.

The regressions confirm our a priori notions about the incidence of unemployment. These indicate that for both cases and measures, Keynesian inflation is progressive and significantly so.

D. Inflation and Money[1]

It has been customary to associate the social costs of inflation with the notion that inflation is a tax on real balances. Inflation is a tax because the real rate of return on cash balances is equal to the negative of the rate of inflation ($-\pi$ in the notation used above).

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[1]This section in light of discussion at the conference.

TABLE III.8. Effects of Inflation in Keynesian Regime

	No Inflation	One-Shot Inflation	Continual Inflation
Mean:			
AI	3397.	3404.	3419.
UAI	3942.	3940.	3991.
Coefficient of Variation:			
AI	2.059	2.038	2.034
UAI	1.931	1.918	1.908
Coefficient of Skewness:			
AI	7.308	7.208	7.186
UAI	6.537	6.461	6.380
Coefficient of Mobility (percent):			
AI	-----	3.39	3.89
UAI	-----	3.15	3.92

TABLE III.9. Annuity Regressions for Keynesian Regime

[coefficients in percentages]

One-Shot Inflation

9. $\ell n\ AI(1) - \ell n\ AI(0) = \underset{(.285)}{2.72} - \underset{(.0341)}{.310}\ \ell n\ AI(0)$ $\qquad R^2 = .9998$

10. $\ell n\ UAI(1) - \ell n\ UAI(0) = \underset{(.265)}{2.41} - \underset{(.0316)}{.296}\ \ell n\ UAI(0)$ $\qquad R^2 = .9999$

Continual Inflation

11. $\ell n\ AI(1) - \ell n\ AI(0) = \underset{(.327)}{3.64} - \underset{(.0392)}{.371}\ \ell n\ AI(0)$ $\qquad R^2 = .9999$

12. $\ell n\ UAI(1) - \ell n\ UAI(0) = \underset{(.330)}{3.38} - \underset{(.0392)}{.320}\ \ell n\ UAI(0)$ $\qquad R^2 = .9999$

TABLE III.10. Effects of Inflation on Annuity by Annuity Class, Age and Other Classes Keynesian Inflation: Ratio of Annuity After to Annuity Before

Income Class	Instantaneous Inflation		Continuous Inflation		Sample Size
	Mean	Stan.Dev.	Mean	Stan.Dev.	
1	1.01240	0.00104	1.02295	0.00173	63
2	0.98921	0.00091	0.99867	0.00033	9
3	1.01800	0.00126	1.02153	0.00109	8
4	1.00080	0.00094	1.00547	0.00144	19
5	1.01261	0.00263	1.00587	0.00128	31
6	1.00083	0.00044	1.00823	0.00208	60
7	1.01068	0.00638	1.01251	0.00200	102
8	1.00427	0.00149	1.00473	0.00118	153
9	1.00369	0.00075	1.00801	0.00078	243
10	1.00255	0.00035	1.00669	0.00033	317
11	1.00271	0.00028	1.00791	0.00028	467
12	1.00383	0.00005	1.00806	0.00010	402
13	1.00229	0.00006	1.00593	0.00008	251
14	1.00093	0.00010	1.00539	0.00044	160
15	0.99504	0.00010	0.99973	0.00008	92
16	0.99139	0.00003	0.99639	0.00002	69
17	0.99087	0.00004	0.99546	0.00002	43
18	0.98804	0.00003	0.99409	0.00002	32
19	0.99130	0.00002	0.99523	0.00001	10
20	0.99834	0.00016	0.99856	0.00002	10
21	0.98612	0.00002	0.98521	0.00003	4
22	0.98915	0.00003	0.99664	0.00000	2
Age Class					
10-19	1.00635	0.00001	1.02968	0.00099	5
20-29	1.01458	0.00161	1.01656	0.00160	246
30-39	1.01178	0.00257	1.01680	0.00128	434
40-49	1.00465	0.00011	1.00936	0.00034	590
50-59	1.00375	0.00024	1.00637	0.00042	606
60-69	0.99728	0.00033	0.99919	0.00031	424
70-79	0.99074	0.00035	0.99802	0.00013	199
80-89	0.98143	0.00041	0.99592	0.00004	40
90+	1.00473	0.00009	1.00339	0.00002	3
Other					
Retired	0.98988	0.00037	0.99762	0.00012	231
Widows	0.99844	0.00072	1.00496	0.00121	282
Nonwhites	1.00957	0.00058	1.01344	0.00090	155

The present study has completely, but consciously, ignored non-interest-bearing debt. The reason for this omission was that after a little thought it appeared that the empirical importance of the "tax on real balances" was negligible relative to the other costs of inflation. The following considerations led the author to this conclusion.

1. Money is a concern because the yield on non-interest-bearing debt cannot adjust to inflation; any change in the rate of inflation will reduce the real yield on this debt by the same amount. Since the alternative assets have yields, the inflation can be considered a tax on holdings of non-interest-bearing debt, with the difference between zero and alternative yields on assets accruing to the government in the form of expenditure foregone.

2. This problem has both a distributive and an allocative aspect. The long-run distributional impact of inflation will be uneven insofar as the interest-elasticities of the demand for real balances differ across income or annuity classes. (The short-run distributional impact is, in principle, estimated above.)

The long-run allocational effect relies on a reallocation of resources from socially costless real balances to socially scarce substitutes (trips to the bank, etc.). This effect will be significant only if there is a significant aggregative interest-elasticity of the demand for real balances.

3. Although the data are not conclusive, it appears that the empirical significance of these effects discussed in paragraph 2 above is negligible.

The survey used for our data estimated demand deposits to be $409 per household. If we apply the observed aggregate currency-demand deposit ratio of 27 percent for 1962, we get an estimate of $110 of currency per household, for a total money holding of $520. This is about 0.5 percent of household lifetime wealth (see Table II.3 above).[1]

It appears very unlikely, however, that the entire holding of money is the appropriate tax base. In the long run, it seems quite likely that a sizable fraction of the rise in yields will show up in the implicit yield on demand deposits. The relevant tax base is thus currency. Using the numbers given above, per capita currency is about $30.

A rise in the rate of inflation of the order comtemplated in this paper is one percent per annum. Viewed as a tax, this amount of inflation would involve a deadweight loss of <u>at most</u> $0.30 per year per capita. This is about 0.001 percent of average per capita annuity income.

By comparison with the estimates given above on the cost of inflation, we see: <u>The "tax on real balances" coming through inflation is on the order of one-tenth to one-hundredth as important as other effects considered.</u>

4. All of the above assume the demand for real balances is quite elastic. It also ignores the question of whether the "inflation tax" is an efficient tax in an economy where the government must raise considerable revenue. If the demand for real balances is inelastic, the estimates of efficiency loss are overstated. Moreover, if the demand is sufficiently inelastic, inflation may be a very efficient way of collecting revenue.

The considerations outlined above imply that the importance of the effect of inflation on real balances has been greatly overemphasized.

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[1] <u>Survey</u> [1966], Appendix.

IV. CONCLUSION

We have described a method for estimating the long-run effects of inflation on the distribution of economic welfare. It should be noted that the technique used is quite complicated, is sensitive to the assumptions, and makes severe demands on the survey data employed. The author feels that without this or a similar method of estimation, calculating the effects of inflation may be quite misleading.

We can summarize the results in Tables IV.1 and IV.2. Table IV.1 shows the effects of inflation for our six cases on both the average annuity and the coefficients of variation and mobility and the rate of regressivity.[1]

Table IV.2 shows the results of calculating the value of the social welfare function $W = \sum \alpha y^{\alpha}$ discussed above in Section II.

Subject to the limitations of the methods used here, the overall conclusions are the following:

1. In all three systems examined and for both kinds of inflation, inflation is an equalizing factor. Both coefficients of variation and skewness are reduced.
2. Associated with the reduction of aggregate inequality we find a high degree of individual mobility. For every one percent reduction in the aggregate inequality (as measured by the coefficient of variation) we find from three to ten percent mobility

[1]For definitions of these, see Section III. The rate of regressiveness is the coefficient of ℓn AI or ℓn UAI in the regression equations above. A positive coefficient indicates that the measure has a larger percentage impact on higher incomes, and conversely for a negative coefficient.

TABLE IV.1. Summary Effects of Inflation

	Classical		Neoclassical		Keynesian	
	0	C	0	C	0	C
Percentage Change:						
Mean Annuity Income (AI)	-.03	-.03	.00	-.06	.21	.65
Mean Utility Equivalent Annuity Income (UAI)	1.35	.58	-.28	.51	-.05	1.24
Coefficient of Variation: AI	-1.16	-.34	-.86	-.67	-1.02	-1.21
UAI	-.88	-.36	-.51	-.72	-.67	-1.19
Other Statistics:						
Coefficient of Mobility (percent): AI	4.07	1.27	7.96	6.26	3.39	3.89
UAI	4.06	1.40	7.81	6.23	3.15	3.92
Rate of Regressiveness (percent): AI	-.204	-.059	.203	.187	-.310	-.317
UAI	-.082	-.016	.212	.230	-2.96	-.320

0 = once and for all inflation of 10%

C = continual inflation of 1%

Sources: Tables III.1, III.2, III.4, III.5, III.8, III.9.

TABLE IV.2. Values of Social Welfare Function

	Classical		Neoclassical		Keynesian	
Value of α	0	C	0	C	0	C
α = 10000(log of SW^a/SW^b)						
AI	-2	-1	-1	-5	21	63
UAI	133	57	-29	53	-6	122
$\alpha = 0$ $10000(SW^a - SW^b)$						
AI	107	-38	202	-203	1379	3003
UAI	1351	735	-958	447	221	3671
$\alpha = -1.8$ $10000(SW^a - SW^b)$						
AI	33	46	368	433	145	247
UAI	24	41	387	441	128	216
$\alpha = -5$ $10000(SW^a - SW^b)$						
AI	45	28	89	116	65	149
UAI	40	27	91	115	60	138

Notes: 0 = once and for all inflation of 10%.

C = continual inflation of 1%.

We have used four trial values. $\alpha = 1$ implies the function is linear, or that the marginal social value of income is constant. $\alpha = 0$ implies a Bernoulli or logarithmic utility function. $\alpha = -1.8$ is the estimated elasticity of individual utility functions. $\alpha = -5$ is the value which might be used by an extreme egalitarian.

of individuals. There is therefore a considerable amount of unsystematic reshuffling of individual fortunes.

We can calculate the following coefficients of efficiency of redistribution for the different regimes. The efficiency of a redistributive policy varies from unity for a progressive tax on annuities to zero for a "neutral" or flat-rate consumption tax or random poll taxes. Although we have no realistic standards of comparison, it would appear that (from a purely redistributive point of view) inflation is an inefficient means of redistributing economic welfare.

TABLE IV.3. Efficiency of Redistribution of AI

Inflation	Classical	Neoclassical	Keynesian
Once and for all	.29	.11	.30
Continual	.27	.11	.31

Note: The efficiency of a redistributive policy is calculated as the ratio of the change in coefficient of variation of the aggregate $(\sigma_1 - \sigma_0)$ to the coefficient of variation of individuals (σ_2). The relationship is shown in the footnote on p. 22 above.

3. The judgment as to the desirability of the aggregate distribution of annuities resulting from inflation depends on the curvature of the objective function. Keynesian inflation (of the magnitude discussed here) is desirable for all social welfare functions except one.

Moreover, the two more egalitarian distributions approve of <u>all</u> <u>six</u> inflationary cases, for the egalitarian zeal of these outweighs any income loss. In the two less egalitarian functions, there is a mixed evaluation, with classical inflation generally approved and neoclassical inflation generally disapproved.

Whatever other conclusions may come from this study, it demonstrates that the policy conclusions depend crucially on both the economic mechanism one has in mind and the methods of evaluation.

It seems unlikely that we can completely support or completely condemn inflationary policies, except those in depression, without some consideration of the type of inflation, the macroeconomic response, and the criterion function.

APPENDIX A. Variable Definitions for Households

The definitions of variables used in constructing income and wealth of households. The list in the column headed "definition" corresponds to the name of the variable in Survey [1966].

Variable	Definition
Wage and Salary Income	Wage and Salary Income
Business Income	Earnings from profession, partnership, farm and closely held corporations
Total Labor Income	Wage and Salary Income plus Business Income minus (.05 x Business Assets)
Total Income	Wage and Salary Income, Business Income, Earnings on Fixed and Variable Priced Assets, Income from Trusts and Estates
Business Assets	Market value or book value, profession, partnership, farm, or closely held corporation
Net Money Fixed Assets ("Bonds")	(Demand deposits, balances in savings accounts, credit balances in brokerage accounts, preferred stock, bonds, mortgage assets, loans) less (Debt)
Net Variable Assets ("Equities")	Common stock, real estate, shares in mutual funds, estates, consumer durables, market value of residences, business assets, and miscellaneous nonliquid assets
Net Worth	Net Money fixed assets plus Net Variable Assets

APPENDIX B. Value of Parameters for Households

The lifetime wealth of a household depends on current wealth, future labor income, and the rate of return on wealth (ρ). Thus annuity income AI depends only on the assumption about ρ. To calculate UAI, we need in addition the parameters β, δ, and g. The parameters are constrained by equation (7) so that $g = (\rho-\delta)/(1-\beta)$. The following calculations give the parameters:

Rate of return (ρ). Recall that ρ is interpreted as the risk-corrected rate of return on wealth. (See p. 15 above.) For quantitative work, we have assumed that ρ is a weighted average rate of return on wealth, with weights equal to the aggregate relative shares of "bonds" and "equities." For 1962 the real rates of return on bonds (as measured by the AAA rate less the rate of change of the GNP deflator) and equities (as measured by the ratio of after-tax capital earnings to replacement cost of private capital) were 2.1 percent and 6.3 percent respectively.[1] We thus calculate $\rho = 2.1 \times .17 + 6.3 \times .83 = 5.6$ percent.

Rate of growth of consumption (g). A second parameter which can be directly observed is the rate of growth of per capita consumption. Using several surveys of income and savings[2] we can calculate consumption by cohort over the last 20 years. Time series indices of consumption by cohort were

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[1]Data are from The Economic Report of the President [1971] and IISR [1971 Appendix A].

[2]For useful summaries, see Survey [1966], and Friedman [1957].

constricted and--using estimates of family size by age--per capita consumption paths were calculated. The average rate of growth of per capita consumption was calculated to be 2.0 percent per annum.

Subjective parameters (β and δ). Given estimates of ρ and g, equation (7) constrains β and δ. For $\rho = 5.75$ and $g = .02$ the following sets of parameters meet the constraints:[1]

β	δ (percent)
1.0	5.6
0.0	3.6
-1.8	0.0
-2.5	-1.4

In a different context $\beta = -2.5$ has been found by Fellner [1967]. Tobin [1967] and Nordhaus and Tobin [1971] use $\beta = 0.0$.

In the present work, we use $\delta = 0.0$, $\beta = -1.8$. This particular assumption is important for calculating utility-equivalent annuity, but does not affect any of the other calculations.

[1]There is, perhaps, one further constraint--the elasticity of the wealth income ratio (or of the savings rate) with respect to the rate of return (η). In studies cited above η is much higher than empirical estimates. Since η is positively associated with β a lower β is probably preferable to a higher one. $\beta = 0$ and $\beta = 1$ are, strictly speaking, limiting cases of the formulae since the formulae do not hold for these values.

APPENDIX C. Calculations of Future Labor Income

1. Lifetime wealth estimates are very sensitive to the assumptions made about future labor earnings. The basic problem is to decide whether observed labor income can be used as permanent income or whether it has a large transitory component. The method of projection used here rejects the idea that labor income will remain constant in the future, but it recognizes that the transient component has a high degree of autocorrelation.

The problem can be seen more easily if we assume that a household's income (y_i) is determined by three components: observed constant exogenous variables, z_i (such as sex, age,[1] education, and race); unobserved constant exogenous variables, x_i (such as intelligence, personality, or quality of education); observed endogenous variables (such as months worked, region or city size, or occupation) and other variables (such as luck or experience), u_i. Scaling variables so that each contributes unit income, we have:

(C.1) $$y_i = z_i + x_i + u_i \ .$$

We assume all variables are independent.

The variables z_i and x_i are fixed and can be considered "permanent income," while u_i is "transitory income." For a given year θ years ahead, expected income is:

[1]Strictly speaking, of course, age is exogenous but not constant. The treatment of age is to predict labor income in the future as a function of age, where age moves forward over time.

(C.2) $$E[y_i(t+\theta)] = z_i + x_i + E[u_i(t+\theta)] \ .$$

The $u_i(\theta)$ is assumed to be a stationary moving average process with constant variance.

The correlation of incomes of identical households θ years apart is:

$$\rho(\theta) = \frac{\mathrm{cov}[y_i(t),\ y_i(t+\theta)]}{\sigma[y_i(t)]\sigma[y_i(t+\theta)]}$$

$$= \sum_i \frac{[z_i + x_i + u_i(t)][z_i + x_i + u_i(t+\theta)]}{\sigma_y^2}$$

(C.3) $$\rho(\theta) = \alpha_z + \alpha_x + \rho_u(\theta)\alpha_u$$

where $\alpha_{\cdot} = \sigma_{\cdot}^2/\sigma^2 y$ and $\rho_u(\theta)$ is the autocorrelation of the u component θ years apart. Since $E[u(t+\theta)] = \rho_u(\theta)u(t)$, expected income in θ years is:

(C.4) $$E[y_i(t+\theta)] = z_i + x_i + \rho_u(\theta)u_i(t) \ .$$

To make this definition operational, we assume that we have an unbiased estimate of z_i from the labor income regression below. We therefore have an unbiased estimate of the residual:

(C.5) $$v_i = x_i + u_i \ .$$

Moreover, we assume that for all households v_i is divided between

x_i and u_i in the same proportion:

(C.6)
$$\begin{cases} x_i = kv_i , & \text{all } i . \\ u_i = (1-k)v_i \end{cases}$$

To calculate expected future labor income in (C.4) we need k and $\rho_u(\theta)$. To get k we introduce one final assumption: the autocorrelation of the transient component is zero after 6 years.[1] Define $\tilde{\rho} = \rho(\theta)$ for $\theta \geq 6$. If this is so, from (C.3), $\tilde{\rho} = \alpha_z + \alpha_x$, or

(C.7)
$$\alpha_x = \tilde{\rho} - \alpha_z .$$

This states that the autocorrelation in household income after 6 years is due entirely to the permanent component: for period less than 6 years, the autocorrelation will be higher than $\tilde{\rho}$ because of the autocorrelation of the transient component. Since: $\alpha_u = \alpha_x = 1 - \alpha_z$, we have:

(C.8)
$$k = \alpha_x/(\alpha_u + \alpha_x) = (\tilde{\rho} - \alpha_z)/(1 - \alpha_z) .$$

Finally, we calculate expected future labor income as:

(C.9)
$$\begin{aligned} E[y_i(t+\theta)] &= z_i + x_i + \rho_u(\theta)u_i \\ &= \hat{z}_i + k\hat{v}_i + \rho_u(\theta)(1-k)\hat{v}_i = \hat{z}_i + \hat{v}_i[k + \rho_u(\theta)(1-k)] . \end{aligned}$$

[1]The reason for this assumption is given in the explanation to Table C.1 below.

2. The empirical estimates of future labor income depend crucially on the regression chosen.

The first question--and a perennial problem for distributional studies --is the question of allocation of business income between labor and capital. The method of allocation was to assume that business capital earns the same rate of return as other equities, about five percent.[1] This imputed capital income was then subtracted from business income to derive estimated labor income in business. This was added to wage and salary income to obtain total labor income.

The second question involves choice of determining variables (the z_i in the discussion above). The procedure outlined above indicates that only exogenous variables should be included. Endogenous variables (such as months worked, region, or occupation) are clearly one of the important determinants of (and reason for autocorrelation in) transitory income.

The following regression was chosen as the permanent labor income equation:[2]

[1]This procedure does not flow naturally from the data. A cross-section regression of business income (y_B) on business assets (A_B) yields:

$$y_B = \underset{}{3601} + \underset{(.0011)}{.0094}\, A_B\,, \quad R^2 = .03\,, \quad \text{SEE} = 16442.\,, \quad \overline{y_B} = 4117\,.$$

Whatever the rate of return on business wealth is, this shows that the rate of withdrawals rises quite slowly.

[2]The regression was run for the entire sample with unweighted observations. The unweighted form is preferable if the purpose is to predict individual labor incomes with minimum error.

$$\text{(C.10)}\quad y = \underset{[3.08]}{7.15} + \underset{[0.73]}{-.0822}\, x_1 + \underset{[1.57]}{.00349}\, x_2 - \underset{[2.97]}{.0000416}\, x_3 - \underset{[3.20]}{.987}\, x_4$$

$$- \underset{[5.50]}{1.60}\, x_5 + \underset{[5.15]}{1.31}\, x_6 + \underset{[2.56]}{.240}\, x_7 - \underset{[0.75]}{.232}\, x_8 + \underset{[1.08]}{.260}\, x_9$$

$$- \underset{[2.23]}{.625}\, x_{10} + \underset{[0.66]}{.288}\, x_{11} - \underset{[0.66]}{.870}\, x_{12} - \underset{[0.34]}{.0239}\, x_{13} + \underset{[1.06]}{.0212}\, x_{14}$$

$$R^2 = .392\ ,\quad SEE = 3.19\ ,\quad \bar{y} = 7.55$$

where

$y = \ell n_e$ (labor income)

x_1 = age of head of household

$x_2 = x_1^2$

$x_3 = x_1^3$

x_4 = dummy variable ($x_4 = 0$ if age < 65 , $x_4 = 1$ if age ≥ 65)

x_5 = sex of head of household ($x_5 = 1$ if male , $x_5 = 2$ if female)

x_6 = marital status ($x_6 = 1$ if married, spouse present; $x_6 = 0$ otherwise)

x_7 = years of education, head

x_8 = dummy variable ($x_8 = 0$ if $x_7 < 8$, $x_8 = 1$ if $x_7 \geq 8$)

x_9 = dummy variable ($x_9 = 0$ if $x_7 < 12$, $x_9 = 1$ if $x_7 \geq 12$)

x_{10} = dummy variable ($x_{10} = 0$ if $x_7 < 16$, $x_{10} = 1$ if $x_7 \geq 16$)

x_{11} = dummy variable ($x_{11} = 0$ if $x_7 < 20$, $x_{11} = 1$ if $x_7 \geq 20$)

x_{12} = race ($x_{12} = 1$ if white, $x_{12} = 2$ if nonwhite)

x_{13} = race-education interaction ($x_{13} = x_{12} \cdot x_7$)

x_{14} = race-age interaction ($x_{14} = x_{12} \cdot x_1$)

The equation explains income variations reasonably well, especially given that it omits any endogenous variables. When variables for months worked, occupation, region, and city size are added, the R^2 improves to 0.65.

3. The calculation of expected labor income uses the predictions of the labor income regression in (C.10). The autocorrelations and the fraction of the residual added back is shown in column (3) of Table C.1.

The important figures for our purpose are the last column of Table C.1. These show what fraction of the residual in the labor income equation should be added back to calculate expected labor income. This calculation yields a sizable figure for the transitory component, perhaps larger than one might first expect.

There are a large number of reservations about the procedure followed. The assumptions above may be incorrect, especially independence of the components and equal discrepancy between the x and u components. The data problems are also serious. Moveover, in the period covered (1929-1935), there were larger than usual macroeconomic disturbances. The correlations reported are for incomes rather than logarithms of incomes. Finally, the incomes included property incomes whereas we consider only labor incomes.

TABLE C.1. <u>Fraction Regression to Permanent Income</u>

Years between Observation (θ)	(1) Correlation between Incomes of Identical Units $\rho(\theta)$	(2) $\rho_u(\theta)$	(3) Fraction of Regression to Permanent Income $[k + \rho_u(\theta)(1-k)]$
1	0.83	.45	.71
2	0.78	.29	.68
3	0.76	.23	.60
4	0.71	.07	0.52
5	0.70	.03	0.50
6	0.69	.00	0.49
More than 6	0.69	.00	0.49

Source: Column (1) is the correlation of the arithmetic levels of incomes of Wisconsin families over the period 1929-1935 calculated in Hanna [1948].

Column (2) = $(1 - \rho(\theta))/(1-\tilde{\rho})$, where $\tilde{\rho} = 0.69$. The value for more than 6 is assumed constant in light of the constancy of $\rho(\theta)$ after 4 years.

Column (3) is derived from equation (C.9) and equals $k + \rho_u(\theta)(1-k)$ where $k = (\tilde{\rho} - \alpha_z)/(1 - \alpha_z) = (.69 - .392)/(1 - .392) = 0.49$.

APPENDIX D. Parameter Estimates for Macroeconomic Models

In this appendix we discuss very briefly the sources and methods of estimating our macroeconomic models.

All aggregate income data are from official national income statistics. Replacement cost of the capital stock and government debt for 1953 to 1968 are from IISR [1971], and the Federal Reserve Bulletin. Labor force employment and population are from ERP [1971]. The price index is the 1958 based GNP deflator. The rate of interest is the AAA bond rate.

The rate of profit is after-tax property income divided by the replacement cost of capital. Property income equals rental income, corporate profits after tax, and 40 percent of proprietors' income.

1. Neoclassical Equations

Equations (2), (3), (4). The production function for potential output was estimated by the method of relative share (for a discussion of this technique, see Nerlove [1965]). The coefficient of capital was estimated as the full-employment pre-tax share of profits in national income. The other coefficients (A and γ) were estimated by taking 1953 and 1968 and fitting a line through these.

Equation (6). The parameter λ is such that the equation fits exactly over the period 1960-1970.

Equations (7) and (8). Although most studies indicate little interest-elasticity (β_1) of the desired wealth-income ratio, Wright [1967] found time series elasticity of consumption with respect to the interest rate of

−.025, while Tobin [1967] and Nordhaus and Tobin [1971] used a priori estimates of the elasticity of the desired wealth-income ratio with respect to the interest rate of about 3 in a model similar to the one used here.

Using our own data, we obtain the equation in Table III.1. This estimate is compatible with Wright's. The response rate is assumed a priori.

Equation (1). The expected inflation equation determines the distributed lag reaction of interest rates to inflation. There are two indirect methods of inferring this. The first is the speed and magnitude of the response of money interest rates to inflation. The second (which becomes important in the Keynesian case) is the response of money wage rates to inflation. Several studies have been performed of the former and some are reported in Table D.1.

There is clearly little consensus on either the average lag or the factional response.

We have chosen an intermediate length lag--mean lag 5 years--and assume complete response-- $\mu = 1$.

2. Keynesian Equations

Equation (9). For this equation we tested simple regressions of rate of change of the GNP deflator on unemployment and expected inflation as defined by equation (1). The expected inflation term was insignificantly different from either zero or one. We used both these regressions, with the following result:

TABLE D.1. Estimates of Effects of Inflation on Long-Term Interest Rates

Author	Mean Lag (years) $(1/\theta)$	Fraction of Reaction (μ)	Sample Period
1. Fisher [1930]			
U.S.A.	7.3	n.a.	1900-27
Great Britain[c]	8.0	n.a.	1820-1924
2. Sargent [1969]	30.[a]	0.116[a]	1902-1940
3. Yohe and Kurnosky [1969][b]			
A	0.78	0.803	1961-69
B	1.9	0.834	1961-69
4. Gordon [1970]	1.1[a]	0.193[a]	1952-69

[a]These coefficients are not directly noted but are reasonably clear from presentation of author.

[b]Version A excludes any variables but inflation, while B includes real income, change in real income, and deflated money supply.

[c]Figure for Great Britain is average of 3 subperiods.

(9-A) $$\pi = -.00730 + .149\ 1/u \qquad R^2 = .20$$
$$(.0151) \quad (.065)$$

(9-B) $$\pi - \pi^e = -.0300 + .128\ 1/u \qquad R^2 = .13$$
$$(.072)$$

(9-A) is the equation used. (9-B) was tested for purposes of comparison.[1]

Equation (10). E is employment over population, u the civilian unemployment rate. The equation used is:

$$E = .600 - .747\ u + .0311\ (T-1929) \qquad R^2 = .80\ .$$
$$(.086) \quad (.014)$$

Equation (11). This equation was estimated as

$$\left(\frac{r}{r^*}\right) = .9023 - .0157\ (u-u^e) - .0120\ (T-1930) \qquad R^2 = .79$$
$$(.0078) \quad (.0013)$$

where $r^* = \alpha Q/K$ = estimated full employment rate of profit.[2]

Equation (11). The movement in the expected rate of unemployment is an attempt to show how producers and factor markets respond to changing levels of utilization of the economy. The quantity $LF(1-\bar{u})$ in equation (2) is the planned employment of plant, and full employment marginal products are calculated at this level of employment. Since the average age of plant and equipment is approximately ten years, a value of 0.1 for θ_u seems appropriate.

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[1]See footnote p. 39 above.

[2]The constant term for 1962 is 0.60 instead of 1.0 because the rate of return is calculated <u>net</u> of capital and profit taxes, but gross of income taxation.

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