

# The Effects of Pick Density on Order Picking Areas With Narrow Aisles

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## **Abstract**

The cost and service performance of an order fulfillment center are determined partly by how workers are organized into an order picking system. One common approach is batch picking, in which workers circumnavigate a picking area with other workers, gathering items on a pick list. In some systems with high space utilization, narrow aisles prohibit workers from passing one another when in the same aisle, and this leads to congestion. We build analytical and simulation models of these systems to investigate their behavior under different levels of activity. Among other things, our results suggest that when the system is busier and pick density is high — that is, when workers stop often to make picks — congestion is *less* of a problem and workers are more productive.

# 1 Order picking

A distribution center typically consists of a reserve area (or warehouse) and an order fulfillment center (an order picking forward area and possibly downstream sortation). Among the decisions related to the order fulfillment center, one of the most significant from a cost and service perspective is the design of the order picking system. Frazelle (2002) reports that order picking typically comprises about 50% of the total operating costs of a distribution center; and, to some extent, order picking determines customer service because it is one of several time-consuming tasks that occur after a customer places an order. It also affects service quality because certain designs are more susceptible to mispicks and clerical errors than others.

Designers of order picking systems face a complex set of decisions, which interact mainly to satisfy three design goals:

- Throughput — How many picks per hour will be required?
- Space utilization — How many items must be stored in how much space? and
- Cost — What will be the fixed and variable costs?

For a greenfield design, the objective is usually to minimize cost, subject to constraints on throughput and space utilization. But this may change over the life of the distribution center: as the business grows, throughput requirements often increase, and managers attempt to maximize throughput; or storage space could become a problem, and managers would focus on space utilization. Regardless of the objective, these design goals involve tradeoffs: a higher throughput system typically requires more workers or automation and therefore has a higher cost. Higher space utilization may require one-way aisles, leading to worker congestion and lower throughput, or to automation and its increased cost.

To achieve these goals, designers must specify two primary characteristics of the system: the physical configuration (racks, aisles, picking vehicles, etc.) and the order picking *policy*; that is, how workers are organized to pick and consolidate items for orders. The physical

configuration determines space utilization by specifying the type of storage racks (single-deep, double-deep, flow-rack, etc.) and the width of aisles. Narrow aisles result in higher space utilization, but can lead to increased travel and congestion associated with one-way travel.

The order picking policy specifies how workers are organized to retrieve orders. Tompkins et al. (2003) identify three major order picking policies: (1) *discrete order picking*, in which a worker picks all the items for a single order on each tour, (2) *batch picking*, in which a worker collects items for several orders on each tour, and (3) *zone picking*, in which each worker is assigned to a specific area of the warehouse and portions of orders are assigned to appropriate zones. (Here, we consider discrete order picking to be a form of batch picking, and therefore use the term *batch picking* to refer to either method.) Naturally, each method has advantages and disadvantages: discrete order picking is simple to implement and not prone to mispicks, but is labor-intensive; batch picking has higher productivity due to less worker travel per item picked, but can result in congestion and mispicks; zone picking has high productivity (if there is sufficient picking activity to keep workers busy), but requires downstream sortation, which can be costly. In an attempt to garner the advantages from multiple methods, some warehouses use hybrid policies. We also note that multiple workers are allowed in a zone in some zone picking systems, but this is rare in our experience.

Notice that the physical configuration and the order picking policy might interact. If, in an attempt to increase space utilization, the designer specifies aisles too narrow to allow passing (which is not uncommon in practice), then having multiple workers in the same aisle — as in batch picking — could lead to congestion. This is the situation we consider in our work. Specifically, we are interested in narrow aisle picking systems (hereafter we mean this to imply no passing within the aisles) when batch picking is the order picking policy; that is, when there can be congestion among workers. We have visited narrow aisle systems with batch picking that were designed this way and one, which we describe below, that was forced into batch picking by a need for more throughput.

But we believe our work is relevant for another reason. Our experience suggests that batch

picking is not very common in narrow-aisle systems because managers believe that if required to produce high throughput, congestion would be a major problem. Therefore, managers often choose zone picking when narrow aisles prevent workers from passing, without (in our opinion) carefully weighing the disadvantages of zone picking, such as the cost of downstream sortation, the difficulty of balancing zones, which leads to the possibility of idle workers due to zone imbalances, and so on.

The models we present suggest that the effect of congestion in narrow aisle systems — particularly busy ones — is misunderstood. The idea for our work came after discussions with a distribution center manager who told us he used zone picking rather than batch picking in a narrow aisle picking area because the pick density (the probability that a worker will pick from any given location during his tour) was so *low* (corresponding to low throughput). Several months later, when we visited again, he had converted the area to batch picking, citing a sharp increase in requests from this area and a need to increase the throughput; that is, pick density was *high*. This was not in keeping with our observation of industry practice, which seems to use zone picking to avoid congestion in busy picking areas with narrow aisles.

His statement led us to pose the following questions:

- Does lower pick density lead to *more* congestion in narrow-aisle systems? Why? What is the relationship between pick density and congestion, and how should this influence the design of an order picking system?
- Could we develop insights that would help designers know if congestion would be a problem in narrow-aisle, batch picking areas?

Here, we present analytical and simulation models of narrow-aisle batch picking systems and show that our DC manager was mostly correct: as throughput increases to high levels in a narrow-aisle system, congestion among workers does (according to our models) tend to *decrease*.

## 2 Related literature

The literature on order picking is extensive, with several papers written on the operational problem of how to execute a batch ordering system (van den Berg and Gademann, 1999). Hwang and Lee (1988) use cluster analysis in an attempt to improve upon existing approaches to order batching. Gibson and Sharp (1992) use spacefilling curves to form batches of orders based on the expected retrieval tours. Pan and Liu (1995) investigate twelve algorithms for order batching and find that the capacity of the picking vehicle (a man-on-board automated storage and retrieval system) has an impact on which algorithm performs best. de Koster et al. (1998) investigate heuristics for forming batches in a warehouse, noting that since these problems are solved frequently, robustness and solution time are important criteria when selecting an algorithm. de Koster and van der Poort (1998) provide a computational comparison of heuristic order picking tours with optimal tours, noting that a heuristic tour is frequently more than 10% longer than the optimal tour.

Among distribution center design papers, Gray et al. (1992) propose a hierarchical design methodology that uses a sequence of mathematical programming models to address problems such as warehouse layout, equipment selection, item location, picker assignment, and order batching. Simpson and Erenguc (2001) model the order picking function at the middle of a supply chain to illustrate how cost structures influence how much inventory is held and the impact of policies on overall supply chain inventory costs. Ashayeri and Gelders (1985) propose that the most practical approach to distribution center design lies in using simulation and analytical tools together.

Our model of worker congestion in a batch picking system addresses the interaction of workers in a set of picking aisles when there is no passing, which we will model as a circle. Some related work has been published for production systems on a line. Muth (1984) models a production line of  $k$  stations, where the service time at each station is given by a random variable. Weber and Weiss (1994) describe an interesting problem called the “cafeteria problem.” Customers move through a series of stations at which they receive service lasting one time unit from a single, randomly chosen server. As in our model, there is no passing,

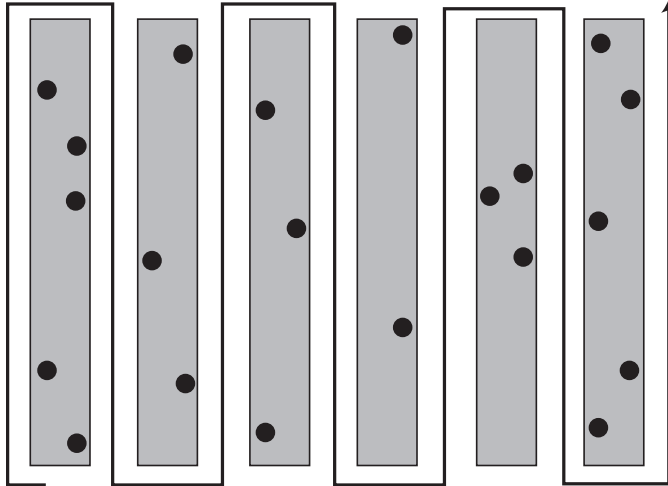


Figure 1: A graphical representation of a batch picking system with narrow aisles. Workers travel one-way within aisles gathering items (indicated by black dots) as required.

and customers can be blocked by a customer receiving service ahead. Unlike our model, customers receive service at only a single station before exiting the system. Eisenstein (2001) examines the design of picking lines in a zone picking system. Il-Choe et al. (1992) discuss the tradeoffs between zone picking and batch picking, but do not examine in-the-aisle blocking. Rouwenhorst et al. (2000) raise, but do not address, the question of batch versus zone picking in discussing a preliminary model of how to design a warehouse.

### 3 Analytical models

In the system we model (see Figure 1) workers circumnavigate unidirectional aisles making picks where required, and there is no passing. If a worker’s next pick is beyond the location of a downstream worker, the upstream worker is blocked until the downstream worker moves beyond the upstream worker’s next pick. We assume that the workers in the batch picking system travel in an aisle-by-aisle fashion due to the one-way aisle structure (i.e., to skip an aisle, the worker would have to look ahead and see that he does not have any picks in the next *two* aisles — an event that rarely occurs). We address other limitations of the model

in the next section.

In this section we develop a simple throughput model for this batch picking system. The model depends on the fraction of time a worker is blocked, and so we develop analytical models for that fraction.

### 3.1 Throughput model

One measure of the effectiveness of an order picking strategy is its throughput capacity, where throughput is the number of picks made per time. We assume that a worker is either picking, walking, or idle due to blocking.

Our model is built on the following assumptions:

- The picking area has  $n$  picking points. In a typical application, a picking point might represent a column of pallet rack, a bay of flow rack, or a section of bin shelving. Each picking point, in reality, might contain several picking locations (e.g., a column of pallet rack might be 3 or 4 pallets high), but for our purposes the place at which a worker stops to make picks is the picking point. Our definition of picking point could include both sides of an aisle because the worker would stop at the same point to make a pick from either side.
- Workers stop at each picking point independently with probability  $p$ . This is a surrogate for pick density, or alternatively, for the average number of picks per tour ( $np$ ). (The probability of not having to make a pick at a picking point is  $q = 1 - p$ .)
- The time to pick is a constant  $t_p$ , meaning we ignore the variation in the actual time to pick at a picking point and instead use just an average time.
- The time to walk past a location is  $t_w$ . We ignore the effects of acceleration and deceleration in our model, although this time could be added to the picking time  $t_p$ .

For a batch picking system with  $k$  workers, we say that each worker is blocked a fraction

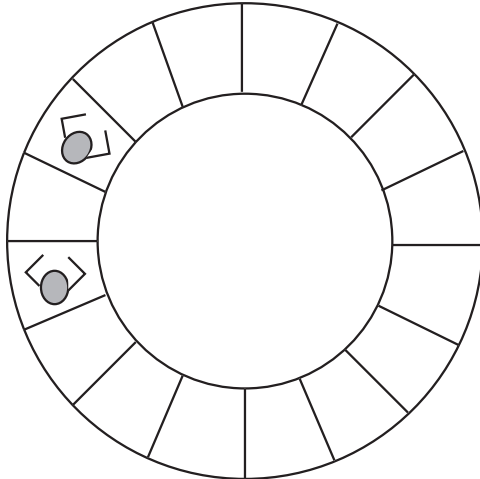


Figure 2: A simple representation of a batch picking system with no passing. Workers move from location to location making picks where required.

of the time,  $b(k)$ , where  $0 \leq b(k) \leq 1$ , and so throughput is

$$\lambda(k) = k \left[ \frac{p}{pt_p + t_w} \right] (1 - b(k)). \quad (1)$$

## 3.2 Congestion models

If there is no passing within or between aisles in a batch picking system (see Figure 1) and if we ignore possible interaction among workers at the pickup-deposit point, then we can model the picking area with a circle, as in Figure 2. We model two cases. In the first case, we model the circular picking area with 2 workers as a Markov process, where the time to walk past a location is equal to the time to pick at a location; i.e., the pick time to walk time ratio is 1:1. In the second case, we model workers with “infinite speed,” where that ratio is  $\infty:1$ .

### 3.2.1 Case: Walk time equals pick time

Consider two workers working in a circular picking area with  $n$  picking points. At each time instant, or *epoch*, a worker makes a pick with probability  $p$  at his current point and spends the epoch at that point (i.e., he does not move). If he does not make a pick, he moves to



the next picking point in this epoch, unless that point is occupied by another worker making a pick. In that case he is blocked for that epoch. Since the picking area is circular, either worker can block the other. We assume that once a pick is made at a point, the worker will move past the current point in the next epoch with probability 1. Assume that workers have the same picking time, walking speed, and probability of stopping at any picking point.

Consider the distance  $D_t$  between Worker 1 and Worker 2 at epoch  $t$ , where worker numbers are assigned arbitrarily, *a priori*, and never change. The distance is always equal to  $(n + (\text{Worker 1 position}) - (\text{Worker 2 position})) \bmod n$ . Since the workers are not allowed to be at the same picking point,  $1 \leq D_t \leq n - 1$ . For example, in a picking area with  $n = 6$  picking points, the workers could be as close as points 2 and 1 ( $D_t = 1$ ) or as far away as points 6 and 1 ( $D_t = 5$ ).

The sequence  $\{D_t\}$  is not a Markov chain because if a worker picked during the last epoch he must move during this epoch; therefore,  $D_{t+1}$  depends on  $D_t$  and on the action of the workers in the previous step. To establish the Markov property we must include the history of the previous step in the state. We label states as  $r_{xy}$ , where  $x$  and  $y$  represent the previous action of Workers 1 and 2, respectively. For example, state  $3_{pw}$  says that workers are 3 picking points apart, Worker 1 just picked, and Worker 2 just walked. The state  $1_{wp}$  is not achievable because it indicates that workers are adjacent immediately after Worker 1 walked and Worker 2 picked, which could only happen if the workers occupied the same picking point in the previous epoch. We called this the *blocked* state instead. All other states are achievable except state  $(n - 1)_{pw}$ , which we also label as *blocked*.

We order the states  $\{1_{pp}, 1_{pw}, \text{blocked}, 1_{ww}, 2_{pp}, 2_{pw}, 2_{wp}, 2_{ww}, \dots, (n - 1)_{pp}, \text{blocked}, (n - 1)_{wp}, (n - 1)_{ww}\}$ . Figure 3 illustrates the transitions. The resulting transition matrix is

$$A = \begin{bmatrix} D_1 & U & 0 & \cdots & 0 \\ L & D & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & D & U \\ 0 & \cdots & 0 & L & D_{n-1} \end{bmatrix},$$

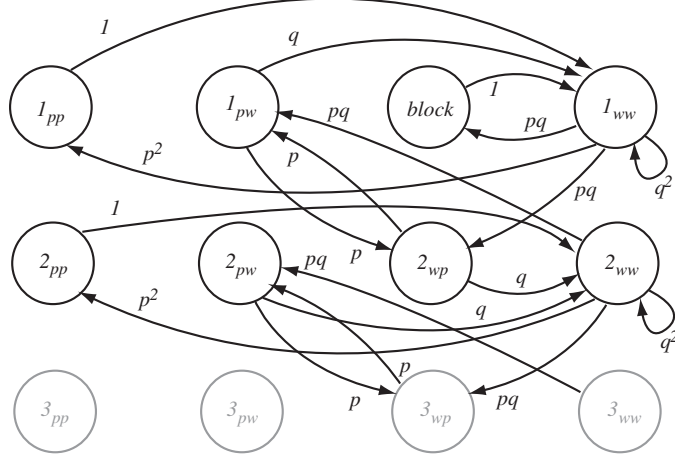


Figure 3: State space and transitions for the Markov chain model when picking time equals travel time.

where

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & pq & 0 & 0 \end{bmatrix}, \quad U = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & pq & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & q \\ 0 & 0 & 0 & q \\ p^2 & 0 & 0 & q^2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & q \\ 0 & 0 & 0 & 1 \\ p^2 & 0 & pq & q^2 \end{bmatrix}, \quad \text{and } D_{n-1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & q \\ p^2 & pq & 0 & q^2 \end{bmatrix}.$$

We can verify that

$$\mathbf{v} = [\overbrace{p^2, p, pq, 1}^{r=1}, \overbrace{p^2, p, p, 1}^{r=2}, \dots, \overbrace{p^2, p, p, 1}^{r=n-2}, \overbrace{p^2, pq, p, 1}^{r=n-1}]$$

satisfies  $\mathbf{v}\mathbf{A} = \mathbf{v}$ . The stationary density for the Markov process is determined by scaling by  $\|\mathbf{v}\|_1$ . Since the blocked state for Worker 2 is represented by the third row or column in the transition matrix  $A$ , we have

$$b_1(2) = \frac{v_3}{\|\mathbf{v}\|_1} = \frac{pq}{(n-3)(p^2 + 2p + 1) + 2(p^2 + pq + p + 1)} = \frac{pq}{(n-1)(p+1)^2 - 2p^2}, \quad (2)$$

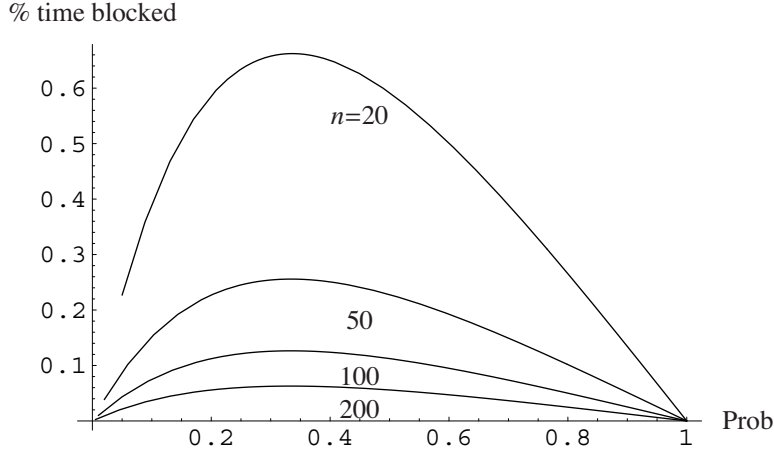


Figure 4: Percentage of time workers are blocked in a 2-worker system in which the time to pick from a picking point equals the time to walk past it, as the number of picking points  $n$  varies.

where  $b_1(2)$  is the average amount of time that Worker 2 is blocked. And since the blocked state for Worker 1 (the  $n-3$  row or column in  $A$ ) is occupied with the same frequency,  $b_1(2)$  also represents the average amount of time that Worker 1 is blocked. Therefore,  $b_1(2)$  is the average amount of time that each of the two workers is blocked. Figure 4 shows the result for several values of  $n$ .

From the derivative of (2), the value of  $p$  that maximizes blocking in a 1:1 system with  $n$  picking points is

$$p_1^{max}(n) = \frac{1 - n + \sqrt{6 - 10n + 4n^2}}{-5 + 3n}, \quad (3)$$

which leads to

**Theorem 1** *Congestion is worst in a 1:1 batch picking system when  $1/3 \leq p \leq \frac{\sqrt{3}-1}{2}$ , or approximately  $0.33 \leq p \leq 0.37$ .*

**Proof** Taking the derivative of (3) with respect to  $n$ , we see that  $p_1^{max}(n)$  decreases as  $n$  increases. Evaluating (3) with  $n = 3$  establishes the upper bound at  $\frac{\sqrt{3}-1}{2}$ . To establish the lower bound, we note that  $\lim_{n \rightarrow \infty} p_1^{max}(n) = 1/3$ .  $\square$

### 3.2.2 Case: Workers travel with infinite speed

At the other extreme of worker speed, the ratio of pick time to walk time is  $\infty:1$  (the *infinite speed* case). Again, we represent the distance between the workers as  $1 \leq D_t \leq n - 1$ , where  $D_t$  is defined as before. We model this system as a Markov chain, with State  $S_t = 0$  representing Worker 1 blocking Worker 2 and State  $S_t = n$  representing Worker 2 blocking Worker 1. States  $\{1, 2, \dots, n - 1\}$  are given by  $S_t = D_t$ .

Let random variables  $X_t^1$  and  $X_t^2$  represent the “desired” number of locations moved in epoch  $t$  by Workers 1 and 2, respectively. In a circular picking area the sample space of these random variables is infinite, and has the geometric distribution

$$f(x) = q^{x-1}p \quad \text{for} \quad x = 1, 2, \dots$$

(Note that  $x$  cannot equal 0 since we do not permit re-picks at the same location.) Let  $Y_t = X_t^1 - X_t^2$  be the change in separation between the two workers. Since these variables are independent and identically distributed (*iid*), we need calculate only one distribution,

$$\begin{aligned} g(y) &= \text{P}(Y_t = y) \\ &= \sum_{x=1}^{\infty} \text{P}(X_t^1 = x + y) \text{P}(X_t^2 = x) \\ &= \sum_{x=1}^{\infty} f(x + y) f(x). \end{aligned}$$

Because the distribution must be symmetric, we consider only non-negative values for  $y$ , so

$$\begin{aligned} g(y) &= \sum_{x=1}^{\infty} q^{x+y-1} p q^{x-1} p \\ &= p^2 q^y \sum_{x=1}^{\infty} (q^2)^{x-1} \\ &= p^2 q^y \frac{1}{1 - q^2} \\ &= p^2 q^y \frac{1}{(1 - q)(1 + q)} \\ &= q^{|y|} \frac{p}{1 + q} \quad \text{for } y \text{ integer.} \end{aligned}$$

Note that  $Y_t$  (with probability density function  $g(y)$ ) has an infinite sample space and describes the desired change in distance. However, the actual change in distance is bounded

by the physical blocking phenomenon. Suppose  $D_{t-1} = r$ . Our goal is to determine the probabilities associated with transition from one state to another.

The easy cases are when there is an event with no blocking:

$$P(Y_t = y) = g(y) = q^{|y|} \frac{p}{1+q} \quad \text{for} \quad 1-r \leq y \leq n-1-r.$$

(To understand these boundaries, think of the specific example  $n = 8$  and  $r = 3$ . If the distance is reduced by 2, blocking does not occur, so  $Y_t$  can be as small as  $-2 = 1 - r$ . The distance can be at most 7, so for no blocking,  $Y_t$  can be as large as  $4 = n - 1 - r$ .)

The next step is calculating the probability of events with blocking. We note that there will be blocking at State 0 if  $Y_t \leq -r$ . We take advantage of the fact that  $g(y)$  is symmetric and calculate

$$P(Y_t \geq r) = \sum_{y=r}^{\infty} q^y \frac{p}{1+q} = \frac{p}{1+q} q^r \frac{1}{1-q} = \frac{q^r}{1+q}.$$

Similarly, we calculate the probability of blocking at the upper boundary as

$$P(Y_t \geq n-r) = \frac{q^{n-r}}{1+q}.$$

The only transitions left to address are how we exit the events with blocking. We take advantage of the fact that the selection or non-selection of a part is governed by Bernoulli trials, and therefore is a memoryless process. At the lower blocking position, the physical distance is  $D_t = 1$ . At epoch  $t + 1$ , the two workers are both moving, conducting Bernoulli trials, and determining when to pick up a part. It does not matter to Worker 2 that he was previously blocked; all he knows is that once he starts moving, he checks to see if he needs that part. The situation is identical to the one in which the two workers were at a distance of 1, but both had picked up a part. Therefore, the transitions from State 0 have the same probabilities as the transitions from State 1. Similarly, the transitions from State  $n$  are identical to those from State  $n - 1$ .

The transition matrix for this Markov process is:

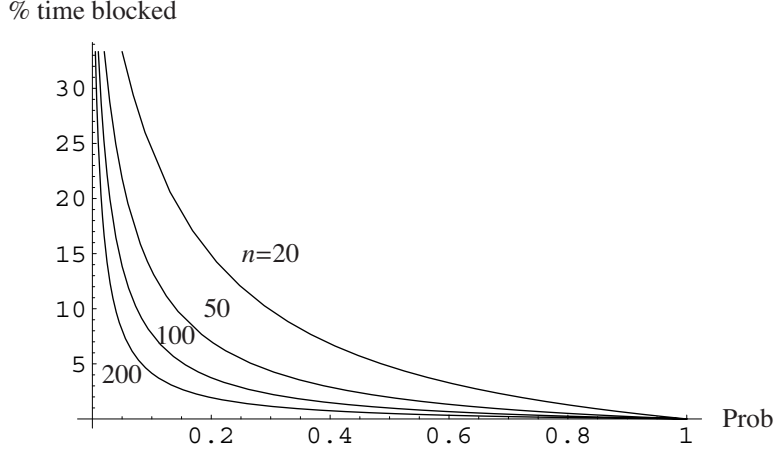


Figure 5: Percentage of time workers are blocked in a 2-worker, infinite speed system as the number of locations  $n$  varies.

$$A = \begin{bmatrix} \frac{q}{1+q} & \frac{p}{1+q} & \frac{pq}{1+q} & \dots & \frac{pq^{n-3}}{1+q} & \frac{pq^{n-2}}{1+q} & \frac{pq^{n-1}}{1+q} \\ \frac{q}{1+q} & \frac{p}{1+q} & \frac{pq}{1+q} & \dots & \frac{pq^{n-3}}{1+q} & \frac{pq^{n-2}}{1+q} & \frac{pq^{n-1}}{1+q} \\ \frac{q^2}{1+q} & \frac{pq}{1+q} & \ddots & \ddots & \ddots & \frac{pq^{n-3}}{1+q} & \frac{q^{n-2}}{1+q} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ \frac{q^{n-2}}{1+q} & \frac{pq^{n-3}}{1+q} & \ddots & \ddots & \ddots & \frac{pq}{1+q} & \frac{q^2}{1+q} \\ \frac{q^{n-1}}{1+q} & \frac{pq^{n-2}}{1+q} & \frac{pq^{n-3}}{1+q} & \dots & \frac{pq}{1+q} & \frac{p}{1+q} & \frac{q}{1+q} \\ \frac{q^{n-1}}{1+q} & \frac{pq^{n-2}}{1+q} & \frac{pq^{n-3}}{1+q} & \dots & \frac{pq}{1+q} & \frac{p}{1+q} & \frac{q}{1+q} \end{bmatrix}. \quad (4)$$

The first and last columns of  $A$  represent the states with blocking events, States 0 and  $n$ . To determine  $b_\infty(2)$ , we normalize by  $\|\mathbf{v}\|$ . The first component of the result then yields

$$b_\infty(2) = \frac{q}{2q + (n-1)p} = \frac{1-p}{2(1-p) + (n-1)p}. \quad (5)$$

The fraction  $b_\infty(2)$  is strictly decreasing in  $p$  for  $p > 0$ ; that is, blocking decreases as  $p$  increases. Figure 5 illustrates the results for an infinite speed system with 2 workers. Our results lead to the following theorem:

**Theorem 2** *For a system with two workers, blocking in the infinite speed case is strictly greater than blocking in the 1:1 case, for all values of  $n > 2$  and  $0 < p < 1$ .*

**Proof** Directly computing  $b_\infty(2) - b_1(2)$  and performing algebraic simplification, we find

$$b_\infty(2) - b_1(2) = \frac{(1-p)(p(2n-4) + n - 1)}{(p(-3+n) + 2)(p^2(-3+n) + p(-2+2n) + n - 1)}.$$

For  $n \geq 2$  and  $0 < p < 1$ , we see that each factor is positive.  $\square$

The theorem suggests that the faster workers travel (i.e., the larger the pick time to walk time ratio), the more they will be blocked. (Our simulation results, which will be presented later, support this observation.)

In many picking operations the number of workers will be greater than two. Although our model for the 1:1 case is not easily extended to the general case of  $k$  workers, it is possible to extend the infinite speed case. Skufca (2005) gives the main result for blocking for  $k < n$  workers traveling at infinite speed:

$$b_\infty(k) = \frac{\sum_{a=0}^{k-2} \binom{k-1}{a} \binom{n-a-2}{k-a-2} \left(\frac{1-p}{p}\right)^{a+1}}{\sum_{a=0}^{k-1} \binom{k}{a} \binom{n-a-1}{k-a-1} \left(\frac{1-p}{p}\right)^a}. \quad (6)$$

Figure 6 plots  $b_\infty(k)$  for several values of  $k$  and  $p$  in a system with  $n = 50$  locations. As expected, there is more congestion when there are more workers. The plot also makes clear that congestion in the infinite speed case is much more of a problem when pick density is low.

With an expression for the fraction of time that workers are blocked in a batch picking system,  $b(k)$ , we can use (1) to estimate throughput as picking activity increases. To do so here, we use the  $k$ -worker result (6) in (1). Figure 7 shows the expected throughput for different values of  $k$  and  $p$ . Notice that system throughput can decrease, but apparently only when  $p < 0.5$  and  $k > n/2$  — an unrealistic number in practice, especially considering that workers would approach this level only in cases of very high demand, which are precisely the cases that would tend to create  $p > 0.5$ .

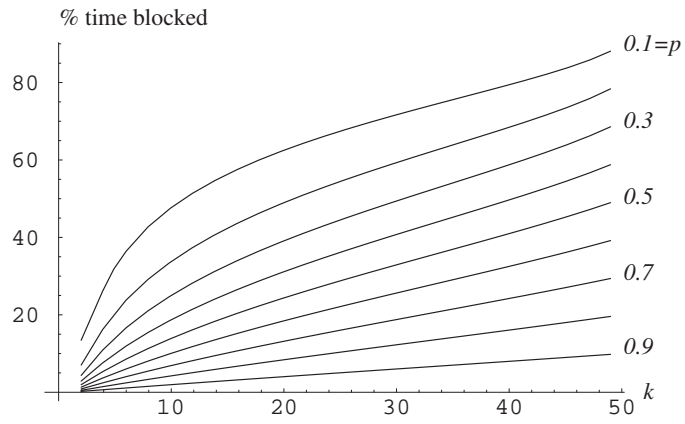


Figure 6: The fraction of time workers are blocked in an  $n = 50$  location system with  $k$  workers.

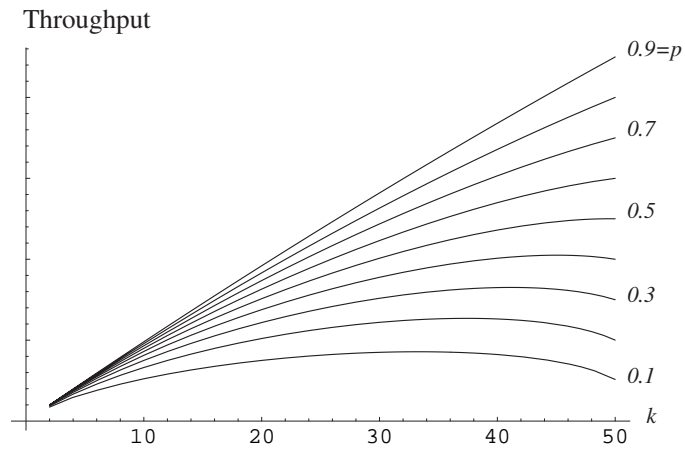


Figure 7: Throughput in a  $k$ -worker, infinite speed system with different values of  $k$  and  $p$ .



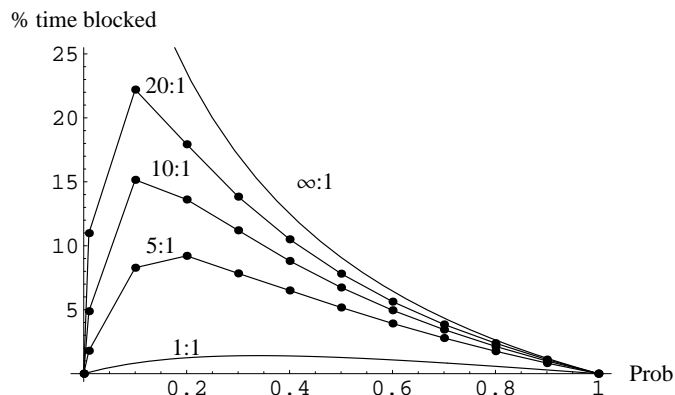


Figure 8: Results of a simulated circular warehouse with two workers and ten picking points, for different pick time to walk time ratios.

## 4 Simulation models

In practice, workers are neither infinitely fast nor as slow as a 1:1 ratio suggests. To address more realistic worker speeds, we built a simulation model with which we could vary worker speeds and other conditions. Discussions with practitioners and our own experience suggest that pick time to walk time ratios are typically 20:1 or less. Since we already have analytical results for the  $\infty$ :1 and 1:1 cases in a circular warehouse, we simulated the ratios 20:1, 10:1, and 5:1.

Figure 8 shows the results of the simulations for a system with 2 workers, along with our analytical results. The faster the workers travel (i.e., the higher the ratio), the greater the congestion. We believe this is because, given that one worker is making a pick, the probability that the trailing worker will be blocked is greater if workers travel faster. For very low densities, however, it appears that there is not enough picking to cause significant congestion no matter the speed of workers. Notice also that the picking density that maximizes congestion seems to get smaller as worker speeds increase.

Our analytical models also assume that workers could never pass one another. In practice, even when aisles are too narrow to allow passing, workers might pass one another between aisles or at the end of their tours.

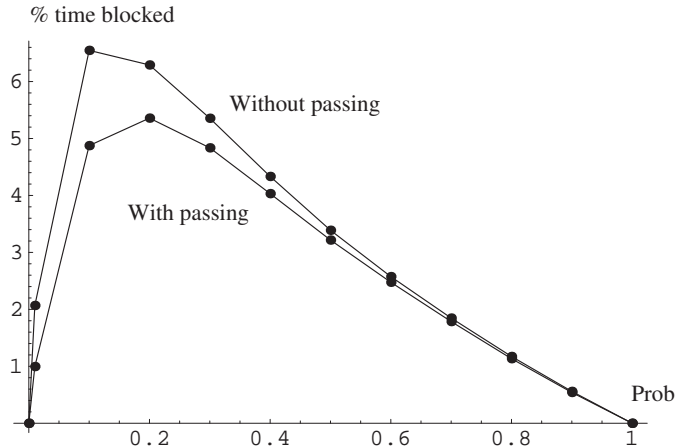


Figure 9: Simulation results for 5-worker systems, one with passing at the ends of aisles and one without.

To see how passing affects our results, we simulated a warehouse in which workers circumnavigate unidirectional aisles, as in our analytical models, but they have picking vehicles that can carry a limited number of items. When the vehicle is full, the simulated worker must empty it at the end of an aisle (presumably at a conveyor, which runs perpendicular to the aisle). After unloading his vehicle, the worker continues on his tour. For one set of simulations, while one worker is unloading a trailing worker may pass him. For the other set of simulations, the trailing worker is blocked. In both cases, if the trailing worker also must empty his vehicle at the end of the aisle, he queues up behind the lead worker to unload his items. We force a simulated worker to empty his vehicle at the end of an aisle if he has more than 10 items on his vehicle. It takes 30 seconds to unload the entire vehicle, 20 seconds to make a pick, and 2 seconds to walk past a picking point (i.e., a 10:1 ratio).

Figure 9 shows the results of a sample of our simulation results, in this case for systems with 5 workers and 50 picking points — one system with passing and one without. The curves have the same general shape, and congestion is slightly lower when passing is allowed.

Figure 10 shows the results for greater numbers of workers and larger picking areas, with worker passing. In general, congestion increases as the number of workers increases. This is expected because, in the case of  $k \geq 3$  workers, a downstream worker may block

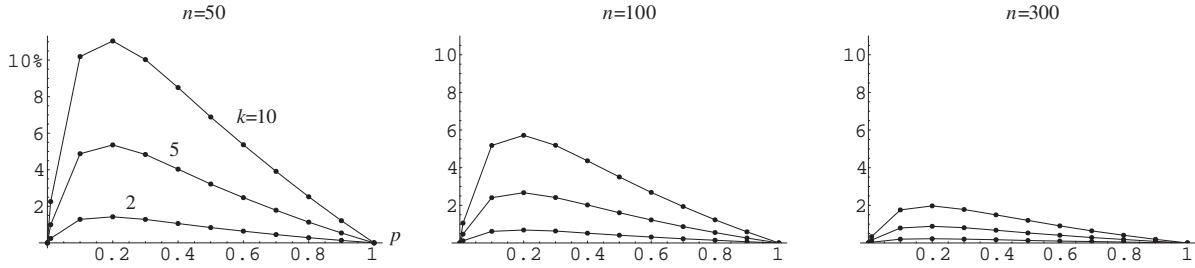


Figure 10: Simulation results for  $k$ -worker systems with vehicle capacities and passing at the end of aisles. Plots show the percentage of time workers were blocked (averaged over all  $k$  workers) for each scenario.

an upstream worker when he is picking *or* when he himself is blocked by a worker further downstream. The figure also shows that, all other things being equal, larger picking areas are less susceptible to congestion than smaller ones, as expected.

We ran a final set of simulations to test the sensitivity of our results to the assumption that picking times are deterministic. We modeled picking time at a picking point with a random variable having a triangular distribution (minimum 10 seconds; most likely 20 sec.; maximum 30 sec.) and found that expected blocking in every case was only slightly higher than the expected blocking when pick times were deterministic.

## 5 Case study

Orchard Supply Hardware (OSH) is a hardware retail chain owned by Sears, which operates a single distribution center (DC) in Lathrop, CA, just west of San Francisco. One of the picking areas in the Lathrop DC is configured as a set of 8 aisles from which workers pick while riding man-aboard picking vehicles. The aisles are wide enough for only a single vehicle, so there is no passing.

When we first toured the DC, this picking area operated a zone picking system, with one worker assigned to each of the 8 aisles. Consolidation was done in a “lay down area” near the shipping doors (in a manual sortation system). As mentioned previously, the DC

manager stated that zone picking was used because of the low pick density. During our second visit, about 2 years later, the same picking area had shifted to a form of batch picking. Managers said the shift was due to a significant increase in business and the need to increase throughput. The requirement for increased throughput translated into a higher pick density. The shift to batch picking meant having to have multiple workers per aisle, and this did lead to some congestion. At the time of our second visit, there were 13 workers in the 8 aisles, which contained a total of 144 picking points.

Workers dealt with congestion in a different way than we have modeled, loosely following this protocol: when one worker is blocked by another, they quickly confer to decide if a pass is appropriate based on the number of picks each has remaining in the aisle. If appropriate, the downstream worker drives through the aisle and circles back behind the blocked worker, thus allowing him to pass. The interesting part of this protocol is that there is still a productivity penalty for blocking, but some of it is shifted to the blocking worker.

While the Lathrop DC case does not validate our models, it does provide anecdotal evidence that increasing pick density in a narrow-aisle system should lead managers to consider batch picking.

## 6 Conclusions

Our work is an effort to gain insights into the effects of pick density on operator blocking in systems with high space utilization. These systems typically disallow passing due to the limited width of aisles, and so are susceptible to worker congestion when firms use batch picking.

In general, we find that congestion among workers can be a significant issue in batch picking areas with high space utilization. Blocking was between 2–11% for systems with passing allowed (at the end of the aisles) and reasonable numbers of workers, picking points, and pick density (see Figure 10). As expected, we also find that increasing the number of workers in a batch picking area tends to reduce productivity for all workers in that area. As

the number of workers in a batch picking area increases, throughput indeed increases, but at the cost of additional congestion for all (see Figure 7). If it is easy to hand-off orders and workers are arranged on a picking line, bucket brigades may be appropriate because there is no congestion under that protocol. Also as expected, we find that congestion is less of a problem for larger batch picking areas. Our results for picking areas with more than 100 picking points had blocking percentages less than 3% for systems with 5 or fewer workers (see Figure 10), which seems quite tolerable. Because the number of picking points is so large compared to the number of workers, workers are more likely to “spread out,” thus avoiding congestion.

Our major finding is that

*As a narrow-aisle batch picking area gets busier, congestion among the workers becomes **less** of a concern.*

Irrespective of the number of workers, size of the picking area, and the ability of workers to pass at the end of the aisles, blocking as a function of picking density tends to follow the same pattern (see Figures 8–9). For extremely low values of pick density, there is very little blocking because workers make almost no picks. For higher levels of pick density, congestion increases as workers make sufficient stops to cause interaction. But as pick density continues to increase, workers tend not to block one another because they spend more time picking and less time traveling. It is only while traveling that a worker can be blocked, so blocking is less of a concern for these high levels of worker activity.

Our results suggest a possible progression of order picking approaches for existing systems with high space utilization: when pick density is very low, batch picking is preferred because there is likely not enough activity to keep workers sufficiently busy in zones (or the zones would be unreasonably large). For higher levels of pick density, zone picking may be preferred due to increased congestion of equivalent batch picking systems, depending on the number of workers, actual pick density, number of picking points, and so on. But as pick density increases yet again, say, due to an increase in business of the firm, batch picking might be preferred again for two reasons. First, there is little loss due to congestion, as our results

suggest, and second, the zone picking system may be unable to deliver sufficient throughput because zones typically contain only a single worker. For example, at OSH Hardware, the picking area employed more workers than it contained aisles, and some worker interaction was unavoidable.

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