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# The Effects of Temperature, Humidity and Barometric Pressure on Short Sprint Race Times 

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#### Abstract

A numerical model of 100 m and 200 m world class sprinting performances is modified using standard hydrodynamic principles to include effects of air temperature, pressure, and humidity levels on aerodynamic drag. The magnitude of the effects are found to be dependent on wind speed. This implies that differing atmospheric conditions can yield slightly different corrections for the same wind gauge reading. In the absence of wind, temperature is found to induce the largest variation in times $\left(0.01 \mathrm{~s}\right.$ per $10^{\circ} \mathrm{C}$ increment in the 100 m ), while relative humidity contributes the least (under 0.01 s for all realistic conditions for 100 m ). Barometric pressure variations at a particular venue can also introduce fluctuations in performance times on the order of a 0.01 s for this race. The combination of all three variables is essentially additive, and is more important for headwind conditions that for tail-winds. As expected, calculated corrections in the 200 m are


magnified due to the longer duration of the race. The overall effects of these factors on sprint times can be considered a "second order" adjustment to previous methods which rely strictly on a venue's physical elevation, but can become important in extreme conditions.

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## 1 Introduction

Adjusting athletic sprinting races for atmospheric drag effects has been the focus of many past studies [1-18]. Factors influencing the outcome of the races are both physiological and physical. The latter has included the effects of wind speed and variations in air density, primarily introduced through altitude variations. This is due in part to the International Association of Athletics Federation's (IAAF) classification of "altitude assistance" for competitions at or above 1000 m elevation.

For the 100 m , with the exception of a few references it is generally accepted that a world class 100 m sprint time will be 0.10 s faster when run with a $2 \mathrm{~ms}^{-1}$ tail-wind, where as a rise of 1000 m in altitude will decrease a race time by roughly 0.03 s . The figures reported in [6] are slight underestimates, although one of the authors has since re-assessed the calculations and reported consistent values [13] to those mentioned above. A 0.135 s correction for this windspeed is reported in [11], which overestimates the accepted value by almost $40 \%$.

Wind effects in the 200 m are slightly more difficult to determine, based on lack of data (the race is around a curve, but the wind is measured in only one direction). As a result, the corrections are less certain, but several References [16, 18] do agree that a $2 \mathrm{~ms}^{-1}$ wind will provide an advantage between $0.11-0.12 \mathrm{~s}$ at sea level. Altitude adjustments are much more spread out in the literature, ranging between $0.03-0.10 \mathrm{~s}$ per 1000 m increase in elevation. The corrections discussed in [2] and [11] are mostly overestimates, citing (respectively) 0.75 s and 0.22 s for a $2 \mathrm{~ms}^{-1}$ tail-wind, as well as 0.20 s and 0.08 s for 1000 m elevation change (although this latter altitude
correction is fairly consistent with those reported here and in [16]. Conversely, an altitude change of 1500 m is found to assist a sprinter by 0.11 s in the study presented in [18].

Several other factors not generally considered in these analyses, which are nonetheless crucial to drag effects, are air temperature, atmospheric pressure, and humidity level (or alternatively dew point). There is a brief discussion of temperature effects in [6], but the authors conclude that they are largely inconsequential to the 100 m race. However, the wind and altitude corrections considered therein have been shown to be underestimates of the now commonly accepted values, so a more detailed re-evaluation of temperature variations is in order.

This paper will investigate the individual and combined effects of these three factors, and hence will discuss the impact of density altitude variations on world class men's 100 m and 200 m race times. The results suggest that it is not enough to rely solely on physical altitude measurements for the appropriate corrections, particularly in the 200 m event.

## 2 The quasi-physical model with hydrodynamic modifications

The quasi-physical model introduced in [14] involves a set of differential equations with five degrees of freedom, of the form

$$
\begin{align*}
\dot{d}(t) & =v(t) \\
\dot{v}(t) & =f_{s}+f_{m}-f_{v}-f_{d} \tag{1}
\end{align*}
$$

where the terms $\left\{f_{i}\right\}$ are functions of both $t$ and $v(t)$. The first two terms are propulsive (the drive $f_{s}$ and the maintenance $f_{m}$ ), while the third and fourth terms are inhibitory (a speed term $f_{v}$ and drag term $f_{d}$ ). Explicitly,

$$
\begin{align*}
f_{s} & =f_{0} \exp \left(-\sigma t^{2}\right) \\
f_{m} & =f_{1} \exp (-c t) \\
f_{v} & =\alpha v(t) \\
f_{d} & =\frac{1}{2}\left(1-1 / 4 \exp \left\{-\sigma t^{2}\right\}\right) \rho A_{d}(v(t)-w)^{2} . \tag{2}
\end{align*}
$$

The drag term $f_{d}$ is that which is affected by atmospheric conditions. The component of the wind speed in the direction of motion $w$ is explicitly represented, while additional variables implicitly modify the term by changing the air density $\rho$.

The motivation for using time as the control variable in the system of equations (2) stems from the idea that efficient running of the short sprints is an time-optimization problem. This has been discussed in the context of world records in Reference [19].

### 2.1 The definitions of altitude

In general, altitude is not a commonly-measured quantity when considering practical aerodynamics, simply because the altitude relevant to such problems is not the physical elevation. It is much easier to measure atmospheric pressure, which is implicitly determined by the altitude $z$ via the differential relationship $d P(z)=-\rho g d z$, known as the hydrostatic equation [20]. Thus, the pressure $P(z)$ at some altitude $z$ may be obtained by integration assuming the functional form of the density profile $\rho$ is known. The air den-
sity can depend non-trivially on factors such as vapor pressure (i.e. relative humidity) and temperature.

It is useful to embark on a slight digression from the analysis to consider several possible definitions of altitude. Including density altitude, there are at least four possible types of altitudes used in aerodynamic studies: geometric, geopotential, density, and pressure. Focus herein will be primarily on the first three.

Both geometric and geopotential altitudes are determined independently of any atmospheric considerations. Geometric altitude is literally the physical elevation measured from mean sea level to a point above the surface. Geopotential altitude could just as easily be defined as equipotential altitude. It is defined as the radius of a specific (gravitational) equipotential surface which surrounds the earth.

Density and pressure altitude, on the other hand, are defined as the altitudes which yields the standard altitude for a given set of parameters. The variation of pressure as a function of height $z$ is

$$
\begin{equation*}
\frac{d p}{d z}=-\rho(z) g(z)=-\frac{p g(z)}{R T(z)} \tag{3}
\end{equation*}
$$

which can be rearranged to give the integrals

$$
\begin{equation*}
\int_{p_{0}}^{p(H)} \frac{d p}{p}=-\frac{1}{R} \int_{0}^{H} \frac{d z g(z)}{T(z)} . \tag{4}
\end{equation*}
$$

For the small altitude changes considered herein, it is more than appropriate to adopt the approximations $g(z) \approx g \equiv$ constant and the linear temperature gradient $T(z)=T_{0}-\Lambda z$, where $\Lambda$ is the temperature lapse rate. In this case, Equation 4 may be solved to give

$$
\begin{align*}
\ln \left(\frac{p(H)}{p_{0}}\right) & =\frac{g}{R \Lambda} \ln \left(\frac{T_{0}}{T(H)}\right) \\
& =\frac{g}{R \Lambda} \ln \left(\frac{T_{0}}{T_{0}-\Lambda H}\right) \tag{5}
\end{align*}
$$

After some mild algebra, the pressure altitude $H_{p}$ is determined to be

$$
\begin{equation*}
H_{p} \equiv \frac{T_{0}}{\Lambda}\left\{1-\left(\frac{p(H)}{p_{0}}\right)^{\frac{R \Lambda}{g}}\right\} \tag{6}
\end{equation*}
$$

Since it is of greater interest to find an expression for altitude as a function of air density, substituting $p(z)=\rho(z) R T(z)$ in Equation 5 yields

$$
\begin{equation*}
H_{\rho}=\frac{T_{0}}{\Lambda}\left\{1-\left(\frac{R T_{0} \rho(H)}{\mu P_{0}}\right)^{\left[\frac{\Lambda R}{g \mu-\Lambda R}\right]}\right\} \tag{7}
\end{equation*}
$$

which is the density altitude.
Here, $P_{0}$ and $T_{0}$ are the standard sea level pressure and temperature, $R$ the gas constant, $\mu$ the molar mass of dry air, $\Lambda$ the temperature lapse rate, and $g$ the sea level gravitational constant. The explicit values of these parameters are given in [21] as

$$
\begin{aligned}
T_{0} & =288.15 \mathrm{~K} \\
P_{0} & =101.325 \mathrm{kPa} \\
g & =9.80665 \mathrm{~ms}^{-2} \\
\mu & =2.89644 \times 10^{-2} \mathrm{~kg} / \mathrm{mol} \\
\Lambda & =6.5 \times 10^{-3} \mathrm{~K} \cdot \mathrm{~m}^{-1} \\
R & =8.31432 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}
\end{aligned}
$$

In the presence of humidity, the density $\rho$ is a combination of both dry air density $\rho_{a}$ and water vapor density $\rho_{v}$. This can be written in terms of the associated pressures as [22]

$$
\begin{equation*}
\rho=\rho_{a}+\rho_{v}=\frac{P_{a}}{R_{a} T}+\frac{P_{v}}{R_{v} T}, \tag{8}
\end{equation*}
$$

where each term is derived from the ideal gas law, with $P_{a}$ and $P_{v}$ the pressure of dry air and water vapor, and $R_{a}=287.05$ and $R_{v}=461.50$ the corresponding gas constants. Since the total pressure is simply the sum of both dry air pressure and vapor pressure, $P=P_{a}+P_{v}$, the previous equation may be simplified to give

$$
\begin{equation*}
\rho=\frac{P-P_{v}}{R_{a} T}+\frac{P_{v}}{R_{v} T} . \tag{9}
\end{equation*}
$$

The presence of humidity lowers the density of dry air, and hence lowers aerodynamic drag. However, the magnitude of this reduction is critically dependent on temperature. A useful and generally accurate approximation to calculate vapor pressure is known as the Magnus Teten equation, given by [22]

$$
\begin{equation*}
P_{v} \approx 10^{7.5 T /(237.7+T)} \cdot \frac{H_{r}}{100} \tag{10}
\end{equation*}
$$

where $H_{r}$ is the relative humidity measure and $T$ is the temperature in ${ }^{\circ} \mathrm{C}$.
Figures 1 and 2 show the calculated density altitudes for the temperature range in question, including curves of constant relative humidity, for normal pressure ( 101.3 kPa ) as well as high-altitude pressure ( 75 kPa ). The latter is representative of venues such as those in Mexico City, where world records were set in every sprint race and jumping event at the 1968 Olympic Games.

It is somewhat striking to note that a $20^{\circ} \mathrm{C}$ temperature range at a fixed pressure can yield an effective altitude change of over 600 m , and even greater
values with the addition of humidity and lowered station pressure. This suggests that a combination of the effects considered in this paper can have significant influence on sprint performances (especially in the 200 m which has been shown to be strongly influenced by altitude [16]).

### 2.2 A note on meteorological pressure

A review of reported meteorological pressure variations seems to suggest that annual barometric averages for all cities range between about $100-102 \mathrm{kPa}$. This might seem contradictory to common sense, since one naturally expects atmospheric pressure to drop at higher altitudes. Much in the same spirit as this work, it is common practice to report sea-level corrected (SL) pressures instead of the measured (station) atmospheric pressures, in order to compare the relative pressure differences experienced between weather stations, as well as to predict the potential for weather system evolution.

Given a reported SL pressure $P_{\text {SL }}$ at a (geophysical) altitude $z$, the station pressure $P_{\text {stn }}$ can be computed as [23]

$$
\begin{equation*}
P_{\mathrm{stn}}=P_{\mathrm{SL}}\left(\frac{288-0.0065 z}{288}\right)^{5.2561} \tag{11}
\end{equation*}
$$

The present study uses only station atmospheric pressures, but a conversion formula which can utilize reported barometric pressure will be offered in a subsequent section.

## 3 Results

The model parameters used for the 100 meter analysis in this study are identical to those in (14],

$$
\begin{array}{lll}
f_{0}=6.10 \mathrm{~ms}^{-2} & f_{1}=5.15 \mathrm{~ms}^{-2} & \sigma=2.2 \mathrm{~s}^{-2} \\
c=0.0385 \mathrm{~s}^{-1} & \alpha=0.323 \mathrm{~s}^{-1} & A_{d}=0.00288 \mathrm{~m}^{2} \mathrm{~kg}^{-1} \tag{12}
\end{array}
$$

as well as the ISA parameters described previously. By definition, these give a density altitude of 0 meters at $15^{\circ} \mathrm{C}, P_{0}=101.325 \mathrm{kPa}$ and $0 \%$ relative humidity.

Since few track meets are held at such "low" temperatures, the standard performance to which all others will be compared will be for the conditions $P=101.3 \mathrm{kPa}, T=25^{\circ} \mathrm{C}$, and a relative humidity level of $50 \%$, which produces a raw time (i.e. excluding reaction) of 9.698 s . Note that the corresponding density altitude for this performance under the given conditions would be $H_{\rho}=418$ meters. Thus, if the performance were physically at sea level, the conditions would replicate an atmosphere of elevation $H_{\rho}$. Also, the relative humidity level is never measured at $0 \%$. The range of possible relative humidity readings considered herein are thus constrained between $25 \%$ and $100 \%$.

The associated performance adjustments in the 100 meter sprint for various conditions are displayed in Figures 3 through 8, while Figures 9 through 13 show possible adjustments to the 200 meter sprint (see Section for further discussion). General features of all graphs include a narrowing parameter space for increasing wind speeds. This implies that regardless of the conditions, stronger tail winds will assist performances by a smaller de-
gree with respect to the "base" performance than will head winds of similar strength. This is completely consistent with previous results in the literature, and furthermore is to be expected based on the mathematical form of the drag component.

Ranked in terms of magnitude of effect, humidity variations show the least impact on race times. Small changes in atmospheric pressure that one might expect at a given venue show slightly more influence on times. Temperature changes over the range considered show the greatest individual impact. However, it is the combination of the three factors that creates the greatest impact, as is predictable based on the calculated density altitudes.

### 3.1 Temperature and relative humidity variations

At fixed pressure and temperature, the range of realistic humidity variations shows little influence on 100 meter race times (Figure 3), yielding corrections of under 0.01 s for the range considered. Since race times are measured to this precision, the effects would be no doubt negligible. The corrections for low pressure regions are also less than the 0.01 s . Due to the extremely slim nature of the parameter space, the effects of wind will essentially be the same regardless of the relative humidity reading.

Similarly, if temperature is allowed to vary at a fixed humidity level, the corrections grow in magnitude but are still relatively small (Figure 4). Although one would realistically expect variations in barometer reading depending on the venue, the chosen values are to reflect a sampling of the possible range of pressures recorded at the events. On its own, temperature does not have a profound impact on the simulation times over the $\Delta T=20^{\circ} \mathrm{C}$
range considered herein. With no wind or relative humidity at fixed pressure ( 101.3 kPa ), a 100 m performance will vary only 0.023 s in the given temperature range. Overall, this corresponds to approximately a 750 m change in density altitude (the $15^{\circ} \mathrm{C}$ condition corresponds roughly to the standard atmosphere except for the non-zero humidity, and thus a density altitude not significantly different from 0 m ). Compared to the standard performance, temperatures below $25^{\circ} \mathrm{C}$ are equivalent to running at a lower altitude (e.g. below sea level).

A $+2 \mathrm{~ms}^{-1}$ tailwind and standard conditions will assist a world class 100 m performance by 0.104 s using the input parameters defined in Equation 12. This is essentially identical to the prediction in [14], since the standard 100 m performance has been defined to be under these conditions. Increasing the temperature to $35^{\circ} \mathrm{C}$ yields a time differential of 0.111 seconds over the standard race, whereas decreasing the temperature to $15^{\circ} \mathrm{C}$ shows a difference of 0.097 seconds. Hence, the difference in performance which should be observed over the $20^{\circ} \mathrm{C}$ range is roughly 0.02 s .

In combination, these factors expand the parameter space from that of temperature variation alone. Figure 5 shows the region bounded by the lowest density altitude conditions (low temperature and low humidity) and the highest (higher temperature and higher humidity). In this case, with no wind the performances can be up to 0.026 s different (effectively the additive result of the individual temperature and humidity contributions). Although such adjustments to the performances seem small, it should be kept in mind that a difference of 0.03 s is a large margin in the 100 meter sprint, and can cause an athlete to miss a qualifying time or even a record.

### 3.2 Pressure variations

As one would expect, the greatest variations in performances next to wind effects is introduced by changes in atmospheric pressure. Barometric pressure changes will be addressed in two ways. First, variations in pressure for a fixed venue will be considered. Diurnal and seasonal fluctuations in barometer reading are generally small for a given location [24], at most 1 kPa from average.

Model simulations for performances run at unusually high pressure ( 102.5 kPa ) and unusually low pressure ( 100.5 kPa ) with constant temperature and relative humidity actually show little variation (under 0.01 s ). However, if the temperature and humidity are such as to allow extreme under-dense and over-dense atmospheres, then the effects are amplified. Figure 6 demonstrates how such ambient weather could potentially affect performances run at the same venue. With no wind, the performances can be different by almost 0.03 seconds for the conditions considered, similar to the predicted adjustments for the combined temperature and humidity conditions given in the previous subsection.

The performance range for races run at high-altitude venues under similar temperature and humidity extremes is plotted in Figure 7. The base performance line is given as reference for the influence of larger pressure changes. At the lower pressure, the width of the parameter space for zero wind conditions is 0.021 seconds, but the lower density altitude point is already 0.071 s faster than the base performance. Thus, depending on the atmospheric conditions at the altitude venue races could be upward of 0.1 seconds faster than those run under typical conditions at sea level.

### 3.3 Combined effects of temperature, pressure, and humidity

The largest differences in performances arise when one considers combined effects of pressure, temperature, and relative humidity. Figure 8 shows the allowed parameter space regions for performances run in extremal conditions: $75 \mathrm{kPa}, 100 \%$ humidity, and $35^{\circ} \mathrm{C}$ (yielding the least dense atmosphere) and $101.3 \mathrm{kPa}, 25 \%$ humidity, $15^{\circ} \mathrm{C}$. The total allowed performance space is shown in Figure 8

Note that the difference in extremes is exceptionally amplified for headwinds. Based on the total horizontal width in the Figure, a performance run with a strong tail-wind (top of left curve) at high altitude can be almost half a second faster than the same performance run with an equal-magnitude headwind in extremely low density altitude conditions.

## 4 Back-of-the-envelope conversion formula

Reference [14] gives a simple formula which can be used to correct 100 meter sprint times according to both wind and altitude conditions,

$$
\begin{equation*}
t_{0,0} \simeq t_{w, H}\left[1.03-0.03 \exp (-0.000125 \cdot H)\left(1-w \cdot t_{w, H} / 100\right)^{2}\right] . \tag{13}
\end{equation*}
$$

The time $t_{w, H}(\mathrm{~s})$ is the recorded race time run with wind $w\left(\mathrm{~ms}^{-1}\right)$ and at altitude $H(\mathrm{~m})$, while the time $t_{0,0}$ is the adjusted time as if it were run at sea level in calm conditions.

It is a simple task to modify Equation 13 to account instead for density altitude. The exponential term represents the change in altitude, thus it can
be replaced with a term of the form $\rho / \rho_{0}$, where $\rho$ is the adjusted density, and $\rho_{0}$ is the reference density. Alternatively, the original approximation in Equation 13 may be used with the altitude $H$ being the density altitude.

Table 1 shows how the top 100 meter performances are re-ordered according to the given approximation. These are compared with the original back-of-the-envelope approximations given in Reference [14, based exclusively on altitude. The weather conditions have been taken from the NC DC database [24]. The relative humidity has been calculated from the mean dew-point. The cited temperature is the maximum temperature recorded on that particular date, which is assumed to be reflective of the conditions near the surface of the track (since the surface reflection and re-emission of heat from the rubberized material usually increases the temperature from the recorded mean). Typical trackside temperatures reported in [25] support this argument. For example, the surface temperature during the 100 m final in Atlanta ( $9.84 \mathrm{~s},+0.7 \mathrm{~ms}^{-1}$ performance in Table (1) was reported as $27.8^{\circ} \mathrm{C}$.

The effects of density altitude are for the most part overshadowed by wind effects and are generally within 0.01 s of each other after rounding to two decimal places. However, larger variations are observable in certain cases. The most notable differences are in the 9.80 s performance (Maurice Greene, USA) at the 1999 World Championships in Seville, ESP, due to the unusually high temperature, as well as the 9.85 s performance in Ostrava, CZE (Asafa Powell, Jamaica). The temperature measurement during the latter was reportedly a chilly $10^{\circ} \mathrm{C}$ and humidity levels near $100 \%$.

At the time of writing of this manuscript, the world record in the men's 100 meter sprint is 9.77 seconds by Asafa Powell, set on 14 June 2005 in

Athens, Greece (note that although there is a faster performance listed in the Table, only races with winds under $+2 \mathrm{~ms}^{-1}$ are eligible for record status). The wind reading for this performance was $+1.6 \mathrm{~ms}^{-1}$. According to Table Powell's world record run actually adjusts to about 9.85 s , very close to his earlier time from Ostrava.

In fact, prior to this the world record was 9.78 s by Tim Montgomery of the USA. The wind in this race was just at the legal limit for performance ratification $\left(+2 \mathrm{~ms}^{-1}\right)$. This time adjusts to an even slower performance of 9.87 s with density altitude considerations ( 9.88 s using the older method). Both of these times are eclipsed by the former world record of Maurice Greene, who posted a time of $9.79 \mathrm{~s}\left(+0.1 \mathrm{~ms}^{-1}\right)$ in Athens roughly six years to the day prior to Powell's race. This time adjusts to between $9.80-9.81 \mathrm{~s}$ depending on the conversion method.

## 5 Corrections to 200 meter race times

Aerodynamic drag effects in the 200 m sprint have been found to be compounded due to the longer duration and distance of the race. In Reference [16], it was suggested that "extreme" conditions such as high wind and altitude can considerably affect performances for better or worse. That is, a 1000 m altitude alone can improve a world class sea-level performance by up to 0.1 s , the equivalent assistance provided by a $+2 \mathrm{~ms}^{-1}$ tail-wind in the 100 m sprint. Higher altitudes can yield even greater boosts. Thus, the variability of density altitude would seem to be all the more relevant to the 200 m sprint.

This section does not seek to provide a comprehensive analysis for the 200 m (e.g. the influence of cross-winds or lane dependence), so only the results for a race run in lane 4 of a standard IAAF outdoor track will be reported. The model equations (11) are adapted for the curve according to Reference [16],

$$
\begin{equation*}
\dot{v}(t)=\beta\left(f_{s}+f_{m}\right)-f_{v}-f_{d} \tag{14}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta\left(v(t) ; R_{l}\right)=\left(1-\xi v(t)^{2} / R_{\uparrow}\right) \tag{15}
\end{equation*}
$$

and $R_{l}$ is the radius of curvature (in meters) for the track in lane $\uparrow$,

$$
\begin{equation*}
R_{l}=36.80+1.22(\uparrow-1), \tag{16}
\end{equation*}
$$

compliant with the standard IAAF track. For the 200 m , slightly different parameters used are to reflect the more "energy-conservative" strategy for the race [16]: $f_{0}=6.0 \mathrm{~ms}^{-2} ; f_{1}=4.83 \mathrm{~ms}^{-2} ; c=0.024 \mathrm{~s}^{-1}$. The other parameters remain unchanged. The curve factor is $\xi=0.015$. As with the 100 meter sprint, the standard performance adopted for comparison is 19.727 s , run in lane 4 at $25^{\circ} \mathrm{C}$ and $50 \%$ relative humidity. Under these conditions, a $2 \mathrm{~ms}^{-1}$ wind assists the sprinter by approximately 0.113 s .

The effects of humidity alone are again effectively inconsequential, in this case showing less than a 0.01 s differential with no wind (Figure 9, For wind speeds of $2 \mathrm{~ms}^{-1}$ at the same pressure value combined with a $50 \%$ increase in humidity, the advantage over the base conditions grows to 0.118 s . Thus, for all wind speeds considered, the effects of humidity are negligible.

Temperature plays a much stronger role in the longer sprint. Figure 10 shows this effect for fixed pressure and relative humidity. In both $15^{\circ} \mathrm{C}$ and
$35^{\circ} \mathrm{C}$ weather, a world class 200 meter sprint can be approximately 0.03 s slower or faster, giving a total differential of 0.065 s over the entire $20^{\circ} \mathrm{C}$ range.

Figure 11 demonstrates the effects of pressure variation at a specific venue (at constant temperature and relative humidity). At higher pressures ( 102.5 kPa ), the simulation shows the race time slowed by 0.011 s , while at unusually low pressures ( 100.5 kPa ) the race by only 0.007 s .

In extremal conditions, including temperature and humidity variations as well causes these differentials to dramatically change (Figure 12 For unusually high pressure, low temperature and humidity conditions, the race time is 0.045 s slower as compared to the base conditions. On the other hand, the lower pressure, higher temperature and humidity conditions yield a 0.048 s decrease in the race time. Thus, the difference between two 200 m races run at the same venue but under these vastly differing conditions could up up to 0.1 seconds different even if there is no wind present.

The most striking difference is observed when considering differences between venues or large station pressure differences (Figure 13). With no wind, the difference between the base conditions and those at high altitude (at the same temperature and relative humidity) can be 0.23 s , which is consistent with the figures reported in Reference [16] for a 2500 m altitude difference. Recall that the difference in density altitude between these two conditions is roughly 3000 m .

A $2 \mathrm{~ms}^{-1}$ wind improves the low pressure performance by only 0.083 s over still conditions at the same pressure, but when compared to the base conditions this figure becomes 0.313 s . In fact, the complete horizontal span of the
parameter space depicted in Figure 13 is almost 0.64 seconds, demonstrating the exceeding variability that one could expect in 200 m race times. Again, this assumes that the wind is blowing exclusively in one direction (down the 100 meter straight portion of the track), and is unchanged throughout the duration of the race.

### 5.1 Can density altitude explain the men's 200 meter world record?

Michael Johnson (USA) upset the standard in the men's 200 m event in 1996 when he set two world records over the course of the summer. At the USATF Championships in Atlanta, his time of 19.66 s (wind $+1.6 \mathrm{~ms}^{-1}$ ) erased the 25 year old record of 19.72 s (set in Mexico City, and thus altitude assisted). However, it was his performance of 19.32 s (wind $+0.4 \mathrm{~ms}^{-1}$ ) at the Olympic Games which truly redefined the race. Reference [16] offers a thorough analysis of the race which addresses wind and altitude effects. It was suggested that overall, the race received less than a 0.1 s boost from the combined conditions (Atlanta sits at roughly 350 m above sea level).

In light of the current analysis, however, the question can be posed: "By how much could Johnson's race have been assisted?". Extensive trackside meteorological data was recorded at the Games, and is reported in Reference [25]. According to this data, the mean surface temperature during the race (21:00 EDT, 01 August 1996) was $34.7^{\circ} \mathrm{C}$, with a relative humidity of $67 \%$. From the NCDC database, the adjusted sea-level pressure was recorded as 101.7 kPa . Combined, these values give a density altitude of 1175 meters (Atlanta's physical elevation is roughly 315 meters).

As compared to the base conditions (a density altitude of about 400 m ), this represents an 800 m change in effective altitude. According to the results of Reference [16, such an altitude increase results in an advantage of approximately 0.05 seconds. When the minimal wind is taken into account $\left(0.3 \mathrm{~ms}^{-1}\right)$, the difference rises to 0.1 s . Previously, the "corrected" value of the World Record was reported as 19.38 s [16] using only wind-speed and physical altitude, so the inclusion of density altitude enhances the correction by 0.04 s and thus adjusts the time to a base 19.42 s .

While this does not explain the enormous improvement over the previous record, is does highlight that density altitude considerations become increasingly important in the 200 meter sprint, and no doubt even moreso for the 400 meter event.

## 6 Conclusions

This report has considered the effect of pressure, temperature, and relative humidity variations on 100 and 200 meter sprint performances. It has been determined that the influence of each condition can be ranked (in order of increasing assistance) by humidity, pressure, and temperature. The combined effects of each are essentially additive. When wind conditions are taken into account, the impact is amplified for head-winds but dampened for tail-winds.

All in all, the use of density altitude over geophysical (or geopotential) altitude seems somewhat irrelevant, since the associated corrections are different by a few hundredths of a second at best. Nevertheless, it is the difference of these few hundredths which can secure a performance in the record
books, or lead to lucrative endorsement deals for world class athletes.

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| $t_{w}(w)$ | Venue $(H)$ | Date | $T, P_{\mathrm{SL}}, \mathrm{RH}$ | $t_{\mathrm{DA}}$ | $t_{\mathrm{PA}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $9.69(+5.7)$ | El Paso, TX (1300) | $13 / 04 / 96$ | $100.5,25.6,13$ | 9.910 | 9.915 |
| $9.77(+1.6)$ | Athens, GRE (100) | $14 / 06 / 05$ | $101.5,28.9,36$ | 9.853 | 9.851 |
| $9.78(+2.0)$ | Paris, FRA (50) | $14 / 09 / 02$ | $102.2,21.7,39$ | 9.873 | 9.877 |
| $9.79(+0.1)$ | Athens, GRE (100) | $16 / 06 / 99$ | $101.7,32.8,38$ | 9.804 | 9.799 |
| $9.80(+0.2)$ | Seville, ESP (0) | $22 / 08 / 99$ | $101.3,41.7,23$ | 9.824 | 9.811 |
| $9.82(-0.2)$ | Edmonton, AB (700) | $05 / 08 / 01$ | $101.2,25,40$ | 9.831 | 9.832 |
| $9.84(+0.7)$ | Atlanta, GA (310) | $27 / 07 / 96$ | $102.2,27.2,70$ | 9.885 | 9.885 |
| $9.85(+0.6)$ | Ostrava, CZE (250) | $09 / 06 / 05$ | $101.6,10.0,100$ | 9.874 | 9.889 |
| $10.03(-2.1)$ | Abbotsford, BC (50) | $19 / 07 / 97$ | $101.9,27.1,44$ | 9.901 | 9.901 |

Table 1: Adjusted top performances using back-of-the-envelope conversion algorithm for density altitude $\left(t_{\mathrm{DA}}\right)$, as compared to physical altitude correction $\left(t_{\mathrm{PA}}\right)$. All times are expressed in seconds ( s ), elevation $H$ in meters (m), temperature $T$ in ${ }^{\circ} \mathrm{C}$, SL-pressure $P_{\text {SL }}$ in kPa , and relative humidity ( RH ) in \%.




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Figure 3: Corrections to world class 100 meter sprint times for humidity variations between $25 \%$ and $100 \% \mathrm{RH}$ at fixed pressure ( 101.3 kPa ) and temperature $\left(25^{\circ} \mathrm{C}\right)$. The "base" time is 9.698 s in still conditions $(w=$ $0 \mathrm{~ms}^{-1}$ ) at sea level, $P=101.3 \mathrm{kPa}, T=25^{\circ} \mathrm{C}$, and $50 \%$ relative humidity














