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The effects of the atmosphere and oceans on the Earth's wobble – I. Theory

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Summary. The theory of wobble excitation for a non-rigid earth is extended to include the effects of the earth's fluid core and of the rotationally induced pole tide in the ocean. The response of the solid earth and oceans to atmospheric loading is also considered. The oceans are shown to be affected by changes in the gravitational potential which accompany atmospheric pressure disturbances and by the load-induced deformation of the solid earth. These various improvements affect the excitation equations by about 10 per cent. Atmospheric and oceanic excitation can be computed using either an angular momentum or a torque approach. We use the dynamical equations for a thin fluid to relate these two methods and to develop a more general, combined approach. Finally, geostrophic winds and currents are shown to be potentially important sources of wobble excitation, in contrast to what is generally believed.

1 Introduction

The Earth's instantaneous pole of rotation follows a roughly circular path in its motion about the Earth's mean figure axis. Practically all of the power in this polar motion is concentrated at periods of 12 month (the 'annual wobble' with an amplitude of 0.1 arcsec) and 435 sidereal day (the 'Chandler wobble', also with a typical amplitude of 0.1 arcsec). The annual wobble is apparently due to seasonal effects in the atmosphere and oceans. Most important is the large seasonal pressure variation over Asia associated with the monsoon (see, e.g. Kikuchi 1971; Siderenkov 1973; Wilson & Haubrich 1976a; Jochmann 1976; Daillet 1981). Also demonstrably important are the oceanic response to this pressure variation, the seasonal variation in ground water storage (Van Hylckama 1970), and, to a lesser extent, the seasonal change in sea-level height accompanying the wind-driven circulation of the ocean (O'Connor 1980). The effects of winds and ocean currents are unknown, but are generally assumed to be small, since it is erroneously believed (see below) that geostrophic winds and currents cannot excite wobble.

The Chandler wobble is an apparently randomly exicited free mode of the Earth. Although the motion was discovered nearly a century ago by S. C. Chandler, no single geophysical mechanism, meteorological or otherwise, has been shown capable of maintaining the wobble at its observed amplitude (see, e.g. Lambeck 1980). From an analysis of global

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atmospheric pressure data (and by including a model for the oceanic response to this pressure) Munk & Hassan (1961) concluded that perturbations in the global pressure distribution, which are so important in exciting the annual wobble, seem to play only a minor role in driving the Chandler wobble. In contrast, Wilson & Haubrich (1976a) recently repeated the analysis using a longer data set and including the additional effects of mountain torques, and found that atmospheric excitation may, in fact, be important. However, the effectiveness of Wilson & Haubrich's excitation depended to a large extent on an *ad hoc* adjustment of the data meant to correct for an apparently inadequate analysis over Asia, and the strength of their conclusions suffers as a result. No other sources of meteorological or oceanographic excitation of the Chandler wobble have been considered, to my knowledge.

This paper is the first step in a re-evaluation of the meteorological and oceanographic excitation of the Earth's wobble. The initial goal was to strengthen (or reject) Wilson & Haubrich's conclusions by essentially repeating their analysis using a more reliable interpolation of the meteorological data over Asia. However, a preliminary theoretical study showed that the theory of wobble excitation for a non-rigid earth (see, e.g. Munk & MacDonald 1960; Lambeck 1980) needed to be extended in a number of ways. In this paper, which is a description of these preliminary results, we extend the theory to account more completely for the Earth's fluid core and the rotationally induced pole tide in the ocean (Section 2), for the load-induced deformation of the solid earth (Section 3), and for the response of the ocean to the gravitational perturbations which accompany atmospheric pressure fluctuations (Section 4). These extensions affect the equations for wobble excitation by about 10 per cent.

The effects of winds and currents on wobble can be computed using either an angular momentum approach or a torque approach. In Section 5 we use the first-order Eulerian equations of motion for a fluid to relate these two methods to one another. In Section 6 we develop a hybrid technique which uses both methods. Finally, by considering the effects of a variable thickness atmosphere and ocean, we conclude (Section 7), that geostrophic winds and ocean currents are potentially important sources of wobble excitation, in contrast to what is generally believed. I am at present applying the theoretical results described in this paper in an analysis of atmospheric and synthetic oceanographic data. Results will be reported in a future paper.

2 The excitation function

The equations for wobble excitation for a non-rigid earth separate into terms proportional to the wobble amplitude and terms dependent on relative particle displacements within the Earth. The displacement terms are usefully combined into a single 'excitation function'. In this section we derive the excitation equations and the excitation function which describe the wobble of the earth (or, what is observationally more pertinent, of the mantle) at periods of a few hundred days or longer. Our derivation will extend earlier results by including more completely the effects of the pole tide in the ocean and motion in the fluid core. Although we will refer specifically to excitation by the atmosphere and oceans, our results are sufficiently general to accommodate any long-period geophysical excitation process.

The effects of the atmosphere and oceans on the rotation of the mantle can be easily understood qualitatively. Suppose there are no astronomical torques on the oceans, the atmosphere or the solid earth ('solid earth' is used here and below to denote the mantle and core). Then, any change in the angular momentum of the atmosphere and oceans is accompanied by an opposite change of equal magnitude in the angular momentum of the

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solid earth. Physically, this angular momentum exchange is the result of atmospheric and oceanic torques on the solid earth. If the solid earth were rigid, this change in its angular momentum would appear entirely as a perturbation in the angular velocity of rotation, and the wobble could be easily determined. For a non-rigid earth some of the angular momentum is also absorbed into a perturbation of the Earth's inertia tensor and into an incremental rotation of the fluid core relative to the mantle. Clearly, to determine the wobble in this case we must first model these deformational contributions to the angular momentum. The deformation is induced both by the incremental centrifugal force which accompanies the change in the Earth's rotation, and by the surface stresses and gravitational effects caused by displacements in the atmosphere and oceans. The response of the Earth to centrifugal effects is considered in this section. The deformation caused by the direct forcing from the atmosphere and oceans is considered in Section 3.

We begin by defining an equilibrium reference state for the solid earth + atmosphere + oceans where, for simplicity, the system is assumed to be rigidly rotating with angular velocity $\mathbf{\Omega} = \Omega\hat{\mathbf{z}}$ and to have an inertia tensor

$$\mathbf{I}_0 = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{pmatrix}. \tag{2.1}$$

The angular momentum of our equilibrium earth ('earth' refers to the solid earth + atmosphere + oceans) is

$$\mathbf{H}_0 = \Omega C\hat{\mathbf{z}}. \tag{2.2}$$

Note that our equilibrium state ignores all mean flow. This omission is not likely to be important here since mean particle velocities in the atmosphere and oceans are much smaller than the velocity due to the earth's rotation.

We now slightly perturb the system in some arbitrary, time-dependent manner. We assume only that the period of the perturbation is much longer than one day. To describe the resulting motion we attach a coordinate system to the earth in an as yet unspecified way (we will uniquely define this system later on). Suppose that this coordinate system rotates with respect to inertial space with angular velocity

$$\boldsymbol{\omega} = \Omega(m_1, m_2, 1 + m_3) \tag{2.3}$$

where each $m_i \ll 1$ (i.e. the incremental rotation is small). The new inertia tensor in this system is

$$\mathbf{I} = \mathbf{I}_0 + \mathbf{c} \tag{2.4}$$

where each $c_{ij} \ll C$. The perturbation of the angular momentum of the Earth relative to the equilibrium state is, to first order in small quantities

$$\delta\mathbf{H} = \mathbf{I}_0 \cdot (\boldsymbol{\omega} - \mathbf{\Omega}) + \mathbf{c} \cdot \mathbf{\Omega} + \mathbf{h}. \tag{2.5}$$

Here,

$$\mathbf{h} = \int \rho \mathbf{r} \times \mathbf{v} \tag{2.6}$$

is the angular momentum due to motion relative to our perturbed coordinate system and ρ is the material density.

We now explicitly define our coordinate system so that there is no contribution to **h** from

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relative motion in the mantle. This is the 'Tisserand mean mantle' coordinate system (see, e.g. Munk & MacDonald 1960) and is equivalent to orienting $\boldsymbol{\omega}$ along the mean rotation axis of the mantle. In this way $\mathbf{h}$ is easier to compute and, since astronomical observatories rotate with the mantle, the resulting $\boldsymbol{\omega}$ can be usefully compared with observations. Specifically, m_1 and m_2 represent wobble of the mantle (i.e. a reorientation of the mantle's rotation axis relative to fixed points in the mantle) and m_3 represents a change in the rate of rotation of the mantle. Liouville's equation for the conservation of angular momentum in our rotating coordinate system is (Munk & MacDonald 1960; Lambeck 1980)

$$\partial_t \delta\mathbf{H} + (\boldsymbol{\omega} - \boldsymbol{\Omega}) \times \mathbf{H}_0 + \boldsymbol{\Omega} \times \delta\mathbf{H} = \mathbf{0} \tag{2.7}$$

where we have assumed there to be no external torque on the earth. Using (2.3) for $\boldsymbol{\omega}$ and (2.5) for $\delta\mathbf{H}$ and keeping only those terms first order in small quantities, (2.7) reduces to

$$\begin{aligned} &\frac{A}{\Omega[C-A]}\,\partial_t m_1 + m_2 = \psi_1 \\ &\frac{A}{\Omega[C-A]}\,\partial_t m_2 - m_1 = \psi_2 \\ &\frac{1}{\Omega}\,\partial_t m_3 = \psi_3 \end{aligned} \tag{2.8}$$

where

$$\begin{aligned} \psi_1 &= [-\partial_t(h_1 + \Omega c_{13}) + \Omega(h_2 + \Omega c_{23})]/\Omega^2[C-A] \\ \psi_2 &= [-\partial_t(h_2 + \Omega c_{23}) - \Omega(h_1 + \Omega c_{13})]/\Omega^2[C-A] \\ \psi_3 &= -\partial_t[h_3 + \Omega c_{33}]/C\Omega^2. \end{aligned} \tag{2.9}$$

The apparent separation in (2.8) between geophysically measurable quantities (the ψ_i) and astronomically measurable quantities (the m_i) is deceptive. The perturbed rotation of the mantle produces an incremental centrifugal force which causes displacements in the solid earth and ocean (there is probably very little response in the atmosphere because of its low density). Consequently, the ψ_i will include terms which depend on the m_i. Assuming the solid earth and oceans respond linearly to this centrifugal force, then to first order in the m_i

$$\psi_i(t) = P_{ij} m_j(t) + \psi_i'(t) \tag{2.10}$$

where the P_{ij} are linear differential or integral operators in the time domain which depend on the dynamics of the solid earth and oceans, and the ψ_i' represent all contributions to ψ_i not induced by centrifugal forces. For an elliptical earth without oceans $P_{13} = P_{23} = P_{31} = P_{32} = 0$ and there is no spin-wobble coupling (that is, a change in m_3 does not induce a change in m_1 or m_2 or vice versa). For an earth with an equilibrium ocean these spin-wobble coefficients are non-zero but each is smaller than P_{11}, P_{22} or P_{33} by a factor of at least 2000 (Dahlen 1976). The theoretical results of Carton (1982) and Carton & Wahr (1982) indicate that at periods of a few hundred days or longer the contributions to the P_{ij} from the deep ocean differ from the equilibrium contributions by at most a few per cent. Consequently, using (2.10) we find that in this long-period limit

$$\begin{pmatrix} \dfrac{A}{\Omega[C-A]}\,\partial_t - P_{11} & 1 - P_{12} \\ -1 - P_{21} & \dfrac{A}{\Omega[C-A]}\,\partial_t - P_{22} \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} \psi_1' \\ \psi_2' \end{pmatrix} \tag{2.11}$$

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$$\frac{1}{\Omega}\partial_t m_3 - P_{33} m_3 = \psi'_3. \tag{2.12}$$

P_{33} is almost entirely due to the induced angular momentum of the fluid core. For an elliptical core–mantle boundary and no frictional coupling, the core does not participate in the incremental rotation (see Merriam 1980; Yoder, Williams & Parke 1981; Wahr, Sasao & Smith 1981) and

$$P_{33} = \frac{C_c}{C}\frac{1}{\Omega}\partial_t \tag{2.13}$$

where C_c is a principal moment of inertia of the core. Consequently, (2.12) becomes

$$\frac{C - C_c}{C}\frac{1}{\Omega}\partial_t m_3 = \psi'_3. \tag{2.14}$$

Equation (2.14) describes the geophysical effects on the length of day and will not be considered further here.

Excitation of wobble is described by (2.11). Using equations (4.9), (4.11), (4.16) and (5.1) of Smith & Dahlen (1981) we find that at long periods and for an earth with an elastic mantle, a homogeneous incompressible fluid core and an equilibrium ocean, the P_{ij} for $i, j = 1, 2$ reduce approximately to

$$\begin{aligned} P_{11} &= P_{22} = \frac{A_c}{C - A}\frac{1}{\Omega}\partial_t \\ P_{12} &= (a^5\Omega^2/3G[C - A])0.34051 \\ P_{21} &= -(a^5\Omega^2/3G[C - A])0.35092 \end{aligned} \tag{2.15}$$

where A_c is a principal moment of inertia of the core, a is the Earth's radius, and G is the gravitational constant. The relative errors in the approximation (2.15) are of order 1/300 and $1/T$ where T is the period of the perturbation in days. So, for periods of a few hundred days or longer, (2.15) is accurate to better than 1 per cent.

It is useful to define complex variables

$$\begin{aligned} m &= m_1 + i m_2 \\ \psi' &= \psi'_1 + i\psi'_2. \end{aligned} \tag{2.16}$$

Then, using (2.15) in (2.11) gives

$$\left[\frac{i\,\partial_t}{\sigma_{cw}} + 1\right] m = i\,\frac{C - A}{A_m}\,\frac{\Omega}{\sigma_{cw}}\,\psi' \tag{2.17}$$

where

$$\sigma_{cw} = \Omega\,\frac{C - A - [a^5\Omega^2/3G]\,0.3464}{A_m} \tag{2.18}$$

is the Chandler wobble frequency for our model earth and A_m is a principal moment of inertia for the mantle. To derive (2.17) we ignored terms of order

$$\frac{P_{12} + P_{21}}{2\sigma_{cw}}\,\Omega\,\frac{C - A}{A_m} \approx 0.0085 \tag{2.19}$$

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which is equivalent to ignoring the ocean-induced ellipticity of the Chandler wobble. These terms slightly couple m and ψ' to $\psi_1'-i\psi_2'$ and m_1-im_2, respectively.

The close agreement between (2.18) and the observed Chandler wobble frequency (see Smith & Dahlen 1981) indicates that at frequencies near σ_{cw} the results (2.15) for the P_{ij} are probably close to the correct values. In this case, (2.17) can be tentatively extended to the real Earth by replacing σ_{cw} with the observed (complex) frequency

$$\sigma_0 = \frac{2\pi}{T}[1 + i/2Q] \tag{2.20}$$

(Wilson & Haubrich 1976a) where T is the observed period of the Chandler wobble and Q is the observed quality factor. (The replacement of σ_{cw} with σ_0 in (2.17) can be justified if the difference between σ_{cw} and σ_0 is due to mantle anelasticity or to a non-equilibrium ocean response, and if neither the anelastic properties nor the oceanic response are notably different at the frequencies σ and σ_0.) Then, using $(C-A)/A = 1/304.4$, $A/A_m = 1.129$, $\Omega/\mathrm{Re}(\sigma_0) = 435$ (see, e.g. Smith & Dahlen 1981) we define the excitation function

$$\phi = i\,\frac{C-A}{A_m}\,\frac{\Omega}{\mathrm{Re}(\sigma_0)}\,\psi' = \frac{1.61}{\Omega^2[C-A]}\,(-i\,\partial_t + \Omega)(h' + \Omega c') \tag{2.21}$$

where

$$h' = h_1' + ih_2'$$
$$c' = c_{13}' + ic_{23}' \tag{2.22}$$

and the primes imply that the motion induced by the incremental centrifugal force is not included. Then (2.17) becomes

$$\left[i\,\frac{\partial_t}{\sigma_0} + 1\right] m = \phi \tag{2.23}$$

(replacing σ_0 with $\mathrm{Re}(\sigma_0)$ in the denominator of ϕ in (2.21) introduces errors in ϕ and so in m of less than 1 per cent).

This result (2.23) is the equation for wobble excitation which we are seeking. Given ϕ as a function of time, we may invert (2.23) to find the wobble parameter, m. Note that m is resonant at the Chandler wobble frequency, σ_0. In fact, (2.23) demonstrates that it is easiest to excite wobble at periods close to $2\pi/\mathrm{Re}(\sigma_0) \approx 435$ day. To examine geophysical excitation mechanisms it is more useful to work with ϕ than with m. In this case we use (2.23) together with observations of m to find an astronomically 'observed' time series for the excitation function, ϕ (see, e.g. Wilson & Haubrich 1976b). The results may then be compared with known geophysical contributions to ϕ computed using (2.21).

This derivation of the equations for wobble excitation (2.21) and (2.23) extends earlier results by including the effects of the core and equilibrium ocean on the effective excitation function, ϕ. The most important new result is the factor of A_m instead of A in the denominator of (2.21), which increases ϕ by about 10 per cent. The reason for this factor is that the core does not participate in the wobble, and so there is less inertia to resist the excitation than there would be if the Earth were everywhere solid. We have also tenatively corrected for unmodelled effects in the Earth's dynamical behaviour (i.e. mantle anelasticity and a non-equilibrium pole tide) by increasing ϕ by a factor of $[\sigma_{cw}/\mathrm{Re}(\sigma_o)]-1$ or by about 2 per cent.

There are contributions to h' and c' in (2.21) from the atmosphere and oceans and from

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the load-induced deformation of the solid earth. To emphasize the distinction between these contributions, we separate h' and c' into

$$h' = h^{\mathrm{L}} + h^{\mathrm{E}}$$
$$c' = c^{\mathrm{L}} + c^{\mathrm{E}} \tag{2.24}$$

where the superscripts L and E refer to the contributions from the atmosphere and oceans and from the solid earth, respectively (we will argue in Section 3 that $h^{\mathrm{E}} \simeq 0$). Then, (2.21) is equivalent to

$$\phi = \frac{1.61}{\Omega^2[C-A]}(-i\,\partial_t + \Omega)(h^{\mathrm{L}} + \Omega c^{\mathrm{L}} + h^{\mathrm{E}} + \Omega c^{\mathrm{E}}). \tag{2.25}$$

The (non-rotationally induced) perturbation in the angular momentum of the atmosphere and oceans is

$$\mathbf{H}^{\mathrm{L}} = \mathbf{h}^{\mathrm{L}} + \mathbf{c}^{\mathrm{L}} \cdot \mathbf{\Omega}. \tag{2.26}$$

Conservation of angular momentum for the atmosphere and oceans gives

$$\partial_t \mathbf{H}^{\mathrm{L}} + \mathbf{\Omega} \times \mathbf{H}^{\mathrm{L}} = -\mathbf{L} \tag{2.27}$$

where $\mathbf{L}$ is the torque on the Earth from the atmosphere and oceans. It is easy to show from (2.26) and (2.27) that

$$L \equiv L_1 + iL_2 = -(\partial_t + i\Omega)(h^{\mathrm{L}} + \Omega c^{\mathrm{L}}). \tag{2.28}$$

Consequently, (2.25) is equivalent to

$$\phi = \frac{1.61}{\Omega^2[C-A]}[iL + (-i\,\partial_t + \Omega)(h^{\mathrm{E}} + \Omega c^{\mathrm{E}})]. \tag{2.29}$$

(The torque result (2.29) could have been derived, as is more usual, by assuming the atmosphere and oceans were external to 'the earth' and by including L on the right-hand side of (2.7). However, in that case the derivation would have been complicated by the fact that the equilibrium inertia tensor, represented in our derivation by (2.1), would not include contributions from the ocean. There would also have been an offsetting atmospheric and oceanic torque on 'the earth' due to wobble-induced pressure forces at the surface.)

More generally, suppose we separate the atmosphere and oceans into two arbitrary disjoint volumes, V and W, which are fixed with respect to the solid earth. We then generalize (2.25) and (2.29) to

$$\phi = \frac{1.61}{\Omega^2[C-A]}[iL_W + (-i\,\partial_t + \Omega)(h_V^{\mathrm{L}} + \Omega c_V^{\mathrm{L}} + h^{\mathrm{E}} + \Omega c^{\mathrm{E}})] \tag{2.30}$$

where $h_V^{\mathrm{L}} + \Omega c_V^{\mathrm{L}}$ is the angular momentum of the fluid in V, and L_W represents the torque on the solid earth and on V from the fluid in W *plus* the rate of flow of angular momentum carried out of W by winds and currents. If $W = 0$ or $V = 0$, then this hybrid result (2.30) reduces to (2.25) or (2.29) respectively.

The decision of whether to use (2.25), (2.29) or (2.30) to compute ϕ must depend on the data available. In principle, the angular momentum result (2.25) is most useful, since it uses only directly measurable properties (h^{L} and c^{L}) of the atmosphere and oceans. The torque, L, in (2.29) (and L_W in (2.30)) requires not only observations but also knowledge of how

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the atmosphere and oceans are coupled to the solid earth. In practice, however, the torque approach (2.29) and the hybrid approach (2.30) are potentially useful, since they may be less sensitive to inadequate atmospheric data coverage over the oceans than the angular momentum approach (see below). We will discuss the angular momentum approach in the next section, the torque approach in Section 5, and the hybrid approach in Section 6.

3 Angular momentum approach

In this section we discuss the angular momentum representation (2.25) of the excitation function, ϕ. We first model the load-induced deformation of the solid earth (c^E and h^E) and then express ϕ as a function of pressure at the surface of the solid earth and of particle velocities within the atmosphere and oceans.

The solid earth response is induced by pressure and frictional stresses at the earth's surface and by the incremental gravitational forces which accompany surface mass loading. Since c^E and h^E cannot be directly measured, they must be modelled. It is usually tacitly assumed that the induced h^E for the fluid core is negligible compared with the induced Ωc^E for the solid earth. (h^E for the mantle vanishes identically due to the definition of our coordinate system.) Although this assumption is likely to be valid, and we will adopt it here, the dynamical behaviour of the core is not well enough understood at present to allow us to be certain. The assumption is supported by results from the extended nutation and body tide model of Sasao, Okubo & Saito (1980) (see, particularly, equations (3.2) and (3.16) of Sasao & Wahr 1981, setting $\tilde{m} = \tilde{\phi} = 0$.) In this model, the induced rotation of the core is assumed to be rigid, which is equivalent to assuming that the only normal mode in the core which includes appreciable rotation and which is likely to be notably excited by the applied force is the 'free core nutation'. This assumption might be violated in certain frequency bands for an inhomogeneous or compressible core, although even in this case the only excited core modes of interest to us would be those modes where the 'average' rotation (i.e. h^E) did not vanish. Note that the discussion in Section 2 of the rotationally induced motion in the solid earth assumed the core to be homogeneous and incompressible and so ignored the possibility of exciting other rotational modes. The good agreement with the observed Chandler wobble frequency is comforting in this case but of course not conclusive.

To model the induced c^E we need to examine the surface tractions from the atmosphere and oceans. These tractions are caused by pressure forces acting along the local surface normal and by frictional shearing stresses acting tangentially to the surface. Since frictional stresses in the atmosphere and oceans are much smaller than pressure stresses, and since the solid earth's response to shear is of the same order of magnitude (in fact somewhat less) than its response to radial traction (Molodensky 1977; Saito 1978), we conclude that deformation induced by frictional stresses can be ignored. Tangential forces can also be produced by pressure acting along a local surface normal which is not exactly in the radial direction. The non-radial components of the surface normal are due to the Earth's ellipticity and to topography. Both of these effects cause relative perturbations in the normal of less than 1 per cent when averaged over a few hundred kilometres. Since low-frequency pressure variations are unlikely to have spatial wavelengths of less than a few hundred kilometres, and since the month-to-month variation of the $l = 2$, $m = 1$ spherical harmonic component of surface pressure (this is the component which induces c^E) is invariably relatively large, we conclude finally that tangential tractions can be ignored when computing the induced deformation of the solid earth. As we will see (Section 5) these tangential tractions cannot be ignored when computing the torque on the solid earth, since the predominant radial pressure traction gives no net torque.

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We assume then that the only forces acting on the solid earth are radial pressure and the incremental gravitational force accompanying the load. To find the induced c^{E} we assume the solid earth responds to these forces as though it were spherical and non-rotating (here, again, we must assume that the effects of the unknown core modes are negligible). Then, an application of McCaullagh's theorem (see, e.g. Munk & MacDonald 1960, section 5.2) shows that

$$c^{\mathrm{E}} = k_2' c^{\mathrm{L}} \tag{3.1}$$

where k_2' is the second degree harmonic potential load Love number for the solid earth.

With this result, and using $k_2' = -0.3088$ from Dahlen (1976), we organize (2.25) into a sum of four terms:

$$\phi = \frac{1.12}{\Omega^2[C-A]}\,[\Omega^2 c^{\mathrm{L}} + 0.44\, i\Omega\, \partial_t c^{\mathrm{L}} + 1.44\,\Omega\,(h^{\mathrm{L}} - i\,\partial_t c^{\mathrm{L}}) - i\,1.44\,\partial_t h^{\mathrm{L}}]. \tag{3.2}$$

This result (3.2) extends earlier results by including more completely the effects of the fluid core and the rotationally induced pole tide in the ocean, resulting in the overall multiplicative factor of 1.12 instead of 1.00, and by more completely modelling the effects of load-induced deformation of the solid earth, resulting in the term proportional to $0.44 i\Omega\, \partial_t c^{\mathrm{L}}$. The first correction can be easily made to any previous numerical results for wobble excitation by simply increasing the computed excitation function by 12 per cent. The second correction is less important since for long periods $\partial_t \ll \Omega$ and so $\partial_t c^{\mathrm{L}}$ is negligible compared with Ωc^{L}.

Since c^{L} is determined by perturbations, ρ_1, in the atmospheric and oceanic density field, then the hydrostatic approximation ($\partial_r P_1 = -\rho_1 g$) can be used to express the first term on the right-hand side of (3.2) in terms of the incremental pressure, P_1, at the surface of the solid earth (g is the gravitational acceleration at the surface of the solid earth). The third term on the right-hand side of (3.2) can be expressed entirely in terms of horizontal winds and currents. The reason is that $\partial_t c^{\mathrm{L}}$ is a global integral with integrand proportional to $\partial_t \rho_1$. Using the first-order equation of mass conservation for a fluid ($\partial_t \rho_1 + \nabla \cdot (\rho_0 \mathbf{v}) = 0$ where $\mathbf{v}$ and ρ_0 are the particle velocity and equilibrium density) and ignoring vertical velocities, we replace $\partial_t \rho_1$ with terms dependent on the horizontal components of the fluid velocity (i.e. the winds and currents). If we ignore the small terms in (3.2) dependent only on $\partial_t c^{\mathrm{L}}$ and $\partial_t h^{\mathrm{L}}$, we get finally

$$\phi \simeq \phi_{\mathbf{matter}} + \phi_{\mathbf{motion}} \tag{3.3}$$

where

$$\phi_{\mathbf{matter}} = \frac{1.12}{C-A}\, c^{\mathrm{L}} = \frac{-1.12 a^4}{g[C-A]} \int_{S_{\mathrm{E}}} P_1 \sin^2\theta \cos\theta \exp(i\lambda)\, d\theta\, d\lambda \tag{3.4}$$

$$\phi_{\mathbf{motion}} = \frac{1.61}{\Omega[C-A]}\,[h^{\mathrm{L}} - i\,\partial_t c^{\mathrm{L}}] = -2\,\frac{1.61 a^3}{\Omega[C-A]} \int_V \rho\,[iv\cos\theta + u]\exp(i\lambda) \times \cos\theta \sin\theta\, d\theta\, d\lambda\, dr. \tag{3.5}$$

Here, u and v are, respectively, the eastward and northward components of the winds and currents, S_{E} is the surface of the solid earth, V is the volume of the atmosphere and oceans, and θ and λ are colatitude and east longitude. To derive (3.5) we used the fact that the velocity normal to the boundary of V (i.e. to S_{E}) is zero. This result for $\phi_{\mathbf{motion}}$ will be

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modified when we consider the hybrid approach in Section 6, where we assume V is no longer the entire atmosphere or ocean.

Time-dependent results for ϕ_{matter} could be found, in principle, using (3.4) together with global observations of atmospheric and oceanic pressure at the surface of the solid earth. Alternatively, for a barotropic (i.e. constant density) ocean, P_1 at the ocean floor could be found from observations of atmospheric pressure at the air–sea interface plus observations of non-steric changes in sea-level. Results for ϕ_{motion} could be computed from (3.5) using global observations of winds and ocean currents.

Unfortunately, none of the necessary oceanic data exist. Furthermore, wind data are virtually non-existent over the oceans and so may not permit a reliable global calculation of ϕ_{motion}. Only the atmospheric pressure data are sufficiently global (ϕ_{matter} turns out to be essentially independent of the pressure over the ocean – see Section 4) and exist for a long enough time period to be reliable. This general lack of data makes the angular momentum approach less useful and motivates the derivation of the torque approach in Section 5.

4 The response of the ocean to variations in atmospheric pressure

Because of the lack of necessary data, the excitation function cannot be directly computed for the oceans. However, practically all non-tidal and non-rotationally induced displacements in the ocean are caused by atmospheric effects. Consequently, given the appropriate atmospheric data it might be possible to model the behaviour of the ocean and to include the results in the integrals (3.4) and (3.5). The required atmospheric data would of course come from that portion of the atmosphere directly over the ocean where observations are extremely sparse. Although this is a difficulty for modelling wind-induced oceanic motion, it is less of a problem for computing the effects of atmospheric pressure. The reason is that at long periods the ocean probably responds to variations in atmospheric pressure as though it were an inverted barometer (or nearly so, as we shall see) and with very little induced currents. Since in this limit there is no net variation in the pressure at the ocean floor, the contribution to ϕ_{matter} from the oceans will cancel the contribution from that portion of the atmosphere above the oceans, and the total ϕ_{matter} will depend only on the atmospheric pressure variations over land.

For an equilibrium ocean, however, the inverted barometer assumption is not strictly valid. One previously recognized problem is that a constant must be added to the inverted barometer solution in order to conserve mass. Another problem is that perturbations in atmospheric density, which cause variations in atmospheric pressure, also produce changes in the atmospheric gravitational potential. This potential acts to deform the oceans and its effects must be added to the direct forcing from the surface pressure. In addition, the solid earth responds to the pressure variations and to the perturbed gravitational potential, as well as to the resulting ocean load. This solid earth response also affects the oceans. Finally, a consistent treatment must include the gravitational self-attraction of the oceans.

Consider an applied atmospheric pressure variation at the Earth's surface

$$P_a(\theta, \lambda, t) = \sum_{l,m} a_l^m(t)\, Y_l^m(\theta, \lambda) \tag{4.1}$$

where the Y_l^m are spherical harmonics (their normalization is unimportant) and the a_l^m are independent of position. Then (see, e.g. Farrell 1972) the accompanying perturbation in the atmospheric gravitational potential at the earth's surface is

$$V_a(\theta, \lambda, t) = \sum_{l,m} \frac{3}{\rho_E(2l+1)}\, a_l^m(t)\, Y_l^m(\theta, \lambda) \tag{4.2}$$

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where $\rho_E = 5.517\,\mathrm{g\,cm^{-3}}$ is the mean density of the solid earth. To derive (4.2) we used $4\pi Ga = 3g/\rho_E$. The total induced change in sea-level height can be expanded as

$$\eta(\theta, \lambda, t) = \sum_{l,m} \eta_l^m(t)\, Y_l^m(\theta, \lambda) \tag{4.3}$$

and so the gravitational potential from the ocean will be

$$V_0 = \sum_{l,m} \eta_l^m \frac{3g}{2l+1} \frac{\rho_0}{\rho_E} Y_l^m \tag{4.4}$$

where $\rho_0 = 1.030\,\mathrm{g\,cm^{-3}}$ is the mean density of the ocean. Both the atmospheric and the oceanic response will load, and so deform, the solid earth. The total gravitational potential, V_E, and radial surface displacement, U_E, due to the solid earth deformation are

$$V_E = \sum_{l,m} \frac{3}{\rho_E(2l+1)} k_l' Y_l^m [a_l^m + g\rho_0\eta_l^m] \tag{4.5}$$

$$U_E = \sum_{l,m} \frac{3}{g\rho_E(2l+1)} h_l' Y_l^m [a_l^m + g\rho_0\eta_l^m]$$

where the k_l' and h_l' are load Love numbers (see, e.g. Longman 1962; Farrell 1972).

We now assume that the ocean responds to the applied pressure and gravitational forces as though it were in a continual state of equilibrium adjustment. This is the traditional approximation and is probably valid at long periods (see, e.g. Munk & MacDonald 1960; Wunsch 1972). Then there are no induced currents and the gravitational potential energy at the ocean surface will balance the surface atmospheric pressure to within a spatial constant. Since the total perturbation of the sea surface is $\eta + U_E$, and since the change in potential at the surface due to moving the surface through the unperturbed gravitational potential of the earth is $-g(\eta + U_E)$, we find

$$\eta = \left[\frac{1}{g}[V_a + V_o + V_E] - U_E - \frac{P_a}{g\rho_0} + d\right]\mathscr{C} \tag{4.6}$$

where

$$\mathscr{C}(\theta, \lambda) = \begin{cases} 0 & \text{over land} \\ 1 & \text{over the ocean} \end{cases} \tag{4.7}$$

is the ocean function and d is the arbitrary constant necessary to conserve mass in the ocean (i.e. d is determined from $\int \eta = 0$). So:

$$\eta = \mathscr{C}\left[-\frac{P_a}{g\rho_0} + \frac{\rho_0}{\rho_E} \sum_{l,m} \frac{3}{2l+1} \gamma_l' \left(\frac{a_l^m}{g\rho_0} + \eta_l^m\right) Y_l^m + d\right] \tag{4.8}$$

where $\gamma_l' = 1 + k_l' - h_l'$.

Note that for localized pressure disturbances, where l is large, the gravitational and deformational effects are small, and the inverted barometer + constant approximation

$$\eta = \mathscr{C}\left[-\frac{P_a}{g\rho_0} + d\right] \tag{4.9}$$

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is valid. However, for global disturbances, where l is small, the effects of gravity and deformation become surprisingly important. For example, for $l=2$ we find $\gamma_2'=1.7$ and the ratio of gravitational and deformational effects to pressure effects is $3\rho_0\,1.7/5\rho_E \approx 1/5$. For an equilibrium ocean, however, the relative importance of these gravitational and deformational effects is reduced due to oceanic self-attraction. It turns out, in fact, that for an atmospheric pressure disturbance solely over the ocean, and for no net change in atmospheric mass, the ocean will respond exactly as an inverted barometer. To see this, suppose $P_a = P_a\,\mathscr{C}$, so that there is no pressure variation over land. Then (4.8) is equivalent to

$$\eta + \frac{P_a}{g\rho_0} = \mathscr{C}\left[\sum_{l,m} \frac{3\rho_0}{(2l+1)\rho_E}\,\gamma_l'\left(\frac{a_l^m}{g\rho_0} + \eta_l^m\right) Y_l^m + d\right]. \tag{4.10}$$

If there is no net change in atmospheric mass then

$$\int\left[\eta + \frac{P_a}{g\rho_0}\right] = 0 \tag{4.11}$$

where the integral in (4.11) is over the entire globe, and so (4.10) is

$$\eta + \frac{P_a}{g\rho_0} = \mathscr{C}\sum_{l,m} \frac{3\rho_0}{(2l+1)\rho_E}\,\gamma_l'\left[\frac{a_l^m}{g\rho_0} + \eta_l^m\right]\left[Y_l^m - \frac{\int \mathscr{C} Y_l^m}{\int \mathscr{C}}\right]. \tag{4.12}$$

Substitution shows that the solution to (4.12) is

$$\eta + \frac{P_a}{g\rho_0} = 0 \tag{4.13}$$

which is simply the usual inverted barometer response. In this case there is no change in pressure at the ocean floor and so no net contribution to ϕ_{matter}. This is the reason ϕ_{matter} is nearly (see below) insensitive to variations in atmospheric pressure over the ocean. The reason the response is an inverted barometer in this case is because in the inverted barometer state every increase in atmospheric mass is balanced by an identical decrease in the underlying oceanic mass. So, the gravitational attraction from the extra atmospheric mass is exactly balanced by the negative self-attraction due to the absence of oceanic mass. (Note that for a non-equilibrium ocean there is no guarantee that the gravitational and deformational effects can be ignored.)

There are, then, two situations where the gravitational and deformational effects are important. First, the ocean responds to pressure variations over land, since these variations are associated with perturbations in the gravitational potential. Second, if there is a spatially uniform change in atmospheric pressure everywhere over the ocean there will be an induced non-uniform response in sea-level. This is in contrast to the inverted barometer + constant solution (4.9) where there is no effect on sea-level. The point here is that because of the irregular ocean-continent distribution, a uniform incremental pressure over the oceans is associated with a non-uniform perturbation in the gravitational potential which in turn deforms the ocean.

We conclude from this discussion that the net contribution to ϕ_{matter} from the atmosphere and oceans is completely determined by the atmospheric pressure over the continents and by the net change in atmospheric mass over the oceans. This latter quantity can be

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found by subtracting the net change in atmospheric mass over the continents from an estimate of the total change in atmospheric mass.

To get an idea of the possible effects of the gravitational and deformational terms in (4.8), we numerically solve (4.8) and (4.9) for the case where

$$P_a = 5 \sin 2\theta \cos\lambda. \tag{4.14}$$

In this case P_a is proportional to $(Y_2^1 + Y_2^{-1})$ and so the gravitational and deformational effects are relatively large. On the other hand, only $l=2$, $m=\pm 1$ terms in the surface pressure contribute to ϕ_{matter} in (3.4), and so (4.14) is a relevant choice for the problem of wobble excitation. Fig. 1(a) shows P_a as a function of latitude and longitude. If there were no oceanic response to P_a, then using $P_1 = P_a$ in (3.4) would give

$$\phi_{\text{matter}} = -\left[\frac{1.12a^4}{g(C-A)}\,\frac{8\pi}{3}\right]. \tag{4.15}$$

We now ignore all gravitational and deformational effects on the ocean and numerically compute $P_1 = P_a + \rho_0 g\eta$, using the inverted barometer + constant response (4.9). The ocean function, $\mathscr{C}$, is found by partitioning the Earth's surface into rectangular elements of dimensions 5° in longitude × 2.5° in latitude, setting $\mathscr{C}=1$ for any element with an average elevation above sea-level of less than 1 m, and setting $\mathscr{C}=0$ otherwise. Numerical results for $P_1 = P_a + \rho_0 g\eta$ are shown in Fig. 1(b). Note that P_1 is small and constant at the ocean floor, as we might expect. Using these results for P_1 in (3.4) and numerically integrating gives

$$\phi_{\text{matter}} = -\left[\frac{1.12a^4}{g(C-A)}\,\frac{8\pi}{3}\right][0.237 + i\,0.017]. \tag{4.16}$$

We see from (4.15) and (4.16) that the effect of the ocean is to reduce ϕ_{matter} by about 75 per cent. This is consistent with the fact that about 70 per cent of the Earth's surface is ocean.

Next, we include gravitational and deformational effects and compute the induced sea-level using an iterative technique to solve (4.8). At the jth iteration, η is approximated by n_j, where n_j is the solution to

$$\eta_j = \mathscr{C}\left[-\frac{P_a}{g\rho_0} + \frac{\rho_0}{\rho_E}\sum_{l,m}\frac{3}{2l+1}\gamma_l'\left(\frac{a_l^m}{g\rho_0} + (\eta_{j-1})_l^m\right)Y_l^m + d\right] \tag{4.17}$$

and η_0 is the inverted barometer + constant solution (4.9). Convergence is fast and we find that only three iterations are needed for better than 1 per cent accuracy. To speed up the calculations we truncate the sum over l in (4.17) to $l \leqslant 7$. This does not seem to notably affect our results, probably because P_a has power only at angular degree $l=2$. In Fig. 1(c) we show the difference between our results for $P_1 = P_a + \rho_0 g\eta$ computed here and the inverted barometer + constant solution (Fig. 1b). This difference, of course, vanishes over land. By comparing Figs 1(a), 1(b) and 1(c) we conclude that the gravitational and deformational effects on sea-level are roughly 5–10 per cent of the pressure-induced effects. Using the results shown in Fig. 1(c) in (3.4) gives

$$\phi_{\text{matter}} = -\left[\frac{1.12a^4}{g(C-A)}\,\frac{8\pi}{3}\right][0.261 + i\,0.012]. \tag{4.18}$$

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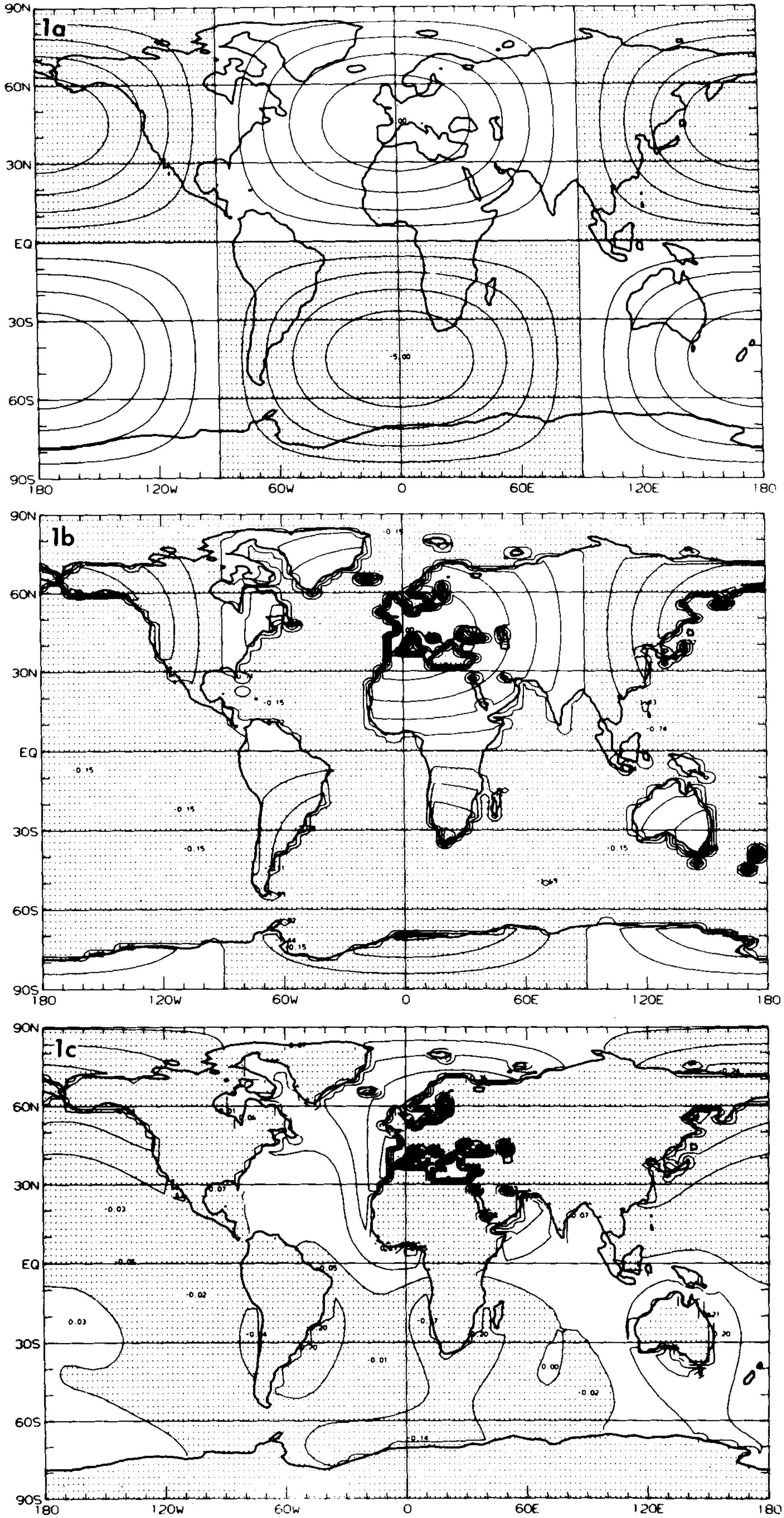

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Comparing (4.18) with (4.16) we see that the wobble excitation function is modified by about 10 per cent due to the gravitational and deformational effects on the ocean. Using real atmospheric data for P_a, I have found that typically the effect on ϕ_{matter} is nearer 5 per cent, due probably to the fact that for the pure $l = 2$ P_a considered in this example, the induced gravitational potential from the atmosphere is relatively large.

5 Torque approach

The angular momentum result for ϕ_{motion} (3.5) for the atmosphere is potentially sensitive to inadequate wind data over the oceans. Consequently, it is useful to discuss the torque result (2.29) for the excitation function, ϕ. To evaluate the torque L, on the solid earth in terms of directly measurable quantities, we derive an angular momentum balance equation for the atmosphere and oceans. Although we specifically refer to the atmosphere at first, the results are extended later to the ocean.

5.1 THE ATMOSPHERE

Consider some atmospheric volume, W, which is fixed to the solid earth and does not follow the fluid. The lower surface, S_a, is the outer surface of either the solid earth or the ocean, and $S_\perp$ represents the surface dividing W from the rest of the atmosphere. $S_\perp$ vanishes when W is the entire atmosphere. The Eulerian equations of motion for the fluid in W are (see, e.g. Pedlosky 1979)

$$\rho\left[\frac{d}{dt}\mathbf{v} + 2\boldsymbol{\Omega}\times\mathbf{v} + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})\right] = \rho\mathbf{g} - \nabla P + \nabla\cdot\mathbf{T} \tag{5.1}$$

where ρ is the total density of the fluid, $\mathbf{v}$ is the instantaneous particle velocity, P is the total fluid pressure, $\mathbf{g}$ is the gravitational acceleration due to the underlying earth ($\mathbf{g}$ is directed into the Earth), and $\mathbf{T}$ represents all frictional stresses. We take $\mathbf{g}$ to be constant and ignore all other body forces. We assume that our fluid is perturbed by surface tractions on either S_a or $S_\perp$.

For simplicity we linearize these equations in terms of small departures from an equilibrium state. Let

$$\begin{aligned} \rho &= \rho_0 + \rho_1 \\ P &= P_0 + P_1 \end{aligned} \tag{5.2}$$

where the subscripts 0 and 1 denote the equilibrium value and the non-equilibrium deviation, respectively. We assume that in the equilibrium state $\mathbf{v} = \mathbf{T} = 0$. (The effects of a mean flow, $\mathbf{v}_0$, on our equation for the angular momentum balance will be to modify the angular momentum flux term in (5.6) below. This term has no effect on the Earth's wobble, as we shall see in Section 6.) To first order in these deviations we get

$$\rho_0[\partial_t\mathbf{v} + 2\boldsymbol{\Omega}\times\mathbf{v}] + \rho_1\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}) = \rho_1\mathbf{g} - \nabla P_1 + \nabla\cdot\mathbf{T}. \tag{5.3}$$

Figure 1. (a) Shows $P_a = 5\sin 2\theta\cos\lambda$ as a function of latitude and longitude. The contour interval is 1.0. We apply this pressure field, P_a, to the ocean and compute the oceanic response. (b) Shows the total pressure at the surface of the solid earth, computed by ignoring all gravitational and deformational effects on the ocean (i.e. by using 4.9). The contour interval is 1.0. (c) Shows the difference in the total seafloor pressure between the equilibrium solution (4.8) and the inverted barometer + constant solution (4.9) These results reflect the effects of the gravitational and deformational forcing which accompanies the applied pressure, P_a. The contour interval is 0.1.

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The first-order perturbation in the angular momentum of W is

$$\mathbf{H}_W = \int_W [\rho_0 \mathbf{r} \times \mathbf{v} + \rho_1 \mathbf{r} \times (\boldsymbol{\Omega} \times \mathbf{r})]\, dV. \tag{5.4}$$

Taking $\int_W \mathbf{r} \times (5.3)\, dV$ and using (5.4) together with

$$\partial_t \rho_1 = -\nabla \cdot (\rho_0 \mathbf{v}) \tag{5.5}$$

and the divergence theorem, we get

$$\begin{aligned}\partial_t \mathbf{H}_W + \boldsymbol{\Omega} \times \mathbf{H}_W = &\int_W \rho_1 \mathbf{r} \times \mathbf{g}\, dV + \int_{S_a} \mathbf{r} \times \hat{\mathbf{n}} P_1\, dS + \int_{S_\perp} \mathbf{r} \times \hat{\mathbf{n}} P_1\, dS \\ &- \int_{S_a} \mathbf{r} \times (\hat{\mathbf{n}} \cdot \mathbf{T})\, dS - \int_{S_\perp} \mathbf{r} \times (\hat{\mathbf{n}} \cdot \mathbf{T})\, dS + \int_{S_\perp} \hat{\mathbf{n}} \cdot \mathbf{v} \rho_0 [\boldsymbol{\Omega} r^2 - \mathbf{r}\mathbf{r} \cdot \boldsymbol{\Omega}]\, dS\end{aligned} \tag{5.6}$$

where $\hat{\mathbf{n}}$ represents the normal to either S_a or $S_\perp$ and is chosen to point into W. The terms on the right-hand side of (5.6) represent, in order, a gravitational torque, pressure torques at the Earth's surface and from the surrounding atmosphere, frictional torques at the Earth's surface and from the surrounding atmosphere, and a flux of angular momentum out of W and into the surrounding atmosphere. We can separate the normal to the solid earth and oceans into

$$\hat{\mathbf{n}} = \hat{\mathbf{n}}_0 + \boldsymbol{\delta}\mathbf{n} \tag{5.7}$$

where $\hat{\mathbf{n}}_0$ would be the surface normal if the solid earth were everywhere in hydrostatic equilibrium:

$$\hat{\mathbf{n}}_0 = \frac{-\mathbf{g} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})}{|-\mathbf{g} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})|} \tag{5.8}$$

and $\boldsymbol{\delta}\mathbf{n}$ is the correction to $\hat{\mathbf{n}}$ due to topography. Assuming the atmosphere to be thin and using a hydrostatic approximation for P_1 gives

$$\begin{aligned}\partial_t \mathbf{H}_W + \boldsymbol{\Omega} \times \mathbf{H}_W = &\int_{S_a} \frac{\mathbf{r} \times \boldsymbol{\Omega}\, \boldsymbol{\Omega} \cdot \mathbf{r}}{g} P_1\, dS + \int_{S_a} \mathbf{r} \times \boldsymbol{\delta}\mathbf{n} P_1\, dS + \int_{S_\perp} \mathbf{r} \times \hat{\mathbf{n}} P_1\, dS \\ &- \int_{S_a} \mathbf{r} \times (\hat{\mathbf{n}} \cdot \mathbf{T})\, dS - \int_{S_\perp} \mathbf{r} \times (\hat{\mathbf{n}} \cdot \mathbf{T})\, dS \\ &+ \int_{S_\perp} \hat{\mathbf{n}} \cdot \mathbf{v} \rho_0 [\boldsymbol{\Omega} r^2 - \mathbf{r}\mathbf{r} \cdot \boldsymbol{\Omega}]\, dS.\end{aligned} \tag{5.9}$$

If we let W be the entire atmosphere, then $S_\perp \to 0$, and $\partial_t \mathbf{H}_W + \boldsymbol{\Omega} \times \mathbf{H}_W$ is the total external torque on the atmosphere. In this case, the torque $\mathbf{L}$ on the solid earth and oceans from the atmosphere is

$$\mathbf{L} = -(\partial_t \mathbf{H}_W + \boldsymbol{\Omega} \times \mathbf{H}_W) = -\int_{S_a} \frac{\mathbf{r} \times \boldsymbol{\Omega}\, \mathbf{r} \cdot \boldsymbol{\Omega}}{g} P_1 - \int_{S_a} \mathbf{r} \times \boldsymbol{\delta}\mathbf{n} P_1 + \int_{S_a} \mathbf{r} \times (\hat{\mathbf{n}} \cdot \mathbf{T}). \tag{5.10}$$

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The first term on the right-hand side of (5.10) and (5.9) represents a partial cancellation between the gravitational and pressure torques. It is present in (5.10) because the surface normal for a hydrostatic earth is not exactly parallel to the gravitational acceleration (the normal is affected by centrifugal forces but **g** is not). This term can be loosely described as the torque on the Earth due to pressure against the Earth's elliptical bulge. Since the Earth's ellipticity (or, what is more pertinent, $|\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})/\mathbf{g}|$) is $\approx 1/300$, the tangential force per unit area which produces this torque has a magnitude of roughly $P_1/300$.

The second term on the right-hand side of (5.10) is the torque due to pressure against surface topography. This 'mountain torque' is potentially the same order as the first term, since $\boldsymbol{\delta}\mathbf{n}$ due to topography is the order of 1/300 over many portions of the Earth. (Strictly speaking, the importance of the mountain torque depends on whether there are important pressure variations with the same scale as the topographic gradient.) The last term in (5.10) represents the frictional torque. Although frictional stresses are probably small in the atmosphere and oceans, their contribution to the surface torque is not reduced by 1/300 as is the case for pressure. In fact, numerical studies indicate that for the atmosphere the frictional torque is the same order of magnitude as the mountain torque (see, e.g. Newton 1971; Manabe & Terpstra 1974; Oort & Bowman 1974).

The contribution to ϕ from the first 'ellipticity torque' in (5.10) reduces exactly to $1.44\,\phi_{\text{matter}}$ with ϕ_{matter} given by (3.4). Since we know from Section 3 that the response of the solid earth to atmospheric forcing gives a contribution to ϕ of $-0.44\,\phi_{\text{matter}}$ we can identify ϕ_{motion} in the angular momentum approach (3.5) with contributions from the second and third terms on the right-hand side of (5.10). Specifically, using (5.10) in (2.29) gives

$$\phi_{\text{motion}} = \phi_{\text{mtn}} + \phi_{\text{frict}} \tag{5.11}$$

where ϕ_{mtn} and ϕ_{frict} represent contributions from, respectively, the mountain torque and the frictional torque. Using

$$\boldsymbol{\delta}\mathbf{n} = -\boldsymbol{\nabla} H(\theta, \lambda) \tag{5.12}$$

where $H(\theta, \lambda)$ is the elevation of the Earth's surface above sea-level, ϕ_{mtn} becomes

$$\phi_{\text{mtn}} = \frac{-a^2\,1.61}{\Omega^2(C-A)} \int_{S_a} [\sin\theta\, \partial_\theta\, H + i\cos\theta\, \partial_\lambda\, H]\, \exp(i\lambda)\, P_1\, d\theta\, d\lambda. \tag{5.13}$$

The frictional torque depends on the poorly known frictional stress, T. Although we will not model T here, we can easily express ϕ_{frict} in terms of the frictional surface traction on the solid earth and oceans

$$\mathbf{F} = \hat{\mathbf{n}} \cdot \mathbf{T}. \tag{5.14}$$

We find

$$\phi_{\text{frict}} = \frac{-1.61 a^3}{\Omega^2(C-A)} \int_{S_a} [F_\theta + i\cos\theta\, F_\lambda]\, \sin\theta\, \exp(i\lambda)\, d\theta\, d\lambda \tag{5.15}$$

where F_θ and F_λ are local components of **F** in the southward and eastward directions, respectively.

The difficulty in using the angular momentum result (3.5) to compute ϕ_{motion} for the atmosphere, is that (3.5) requires global knowledge of the winds at different levels in the atmosphere and so is likely to be sensitive to inadequate wind data over the oceans. The torque approach allows at least a partial check on the results of (3.5) since the mountain torque contribution is readily calculable from the existing pressure observations (there is

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no mountain torque over the oceans). On the other hand, a complete calculation using the torque approach clearly requires some model for the frictional stress at the atmosphere–solid earth and atmosphere–ocean surfaces. This stress is not well understood at the solid earth interface.

5.2 THE OCEANS

The torque approach is potentially useful for the atmosphere because it offers an alternative method of including the meteorological data. For the oceans there are virtually no data, and so oceanic displacements must be modelled. In this case there is probably little advantage to a torque approach, and the contributions to ϕ_{motion} from the ocean are best computed using (3.5), taking V in (3.5) to be the volume of the ocean.

However, for completeness and for an increased understanding of the interactions between the atmosphere, the oceans and the solid earth, we now describe a torque approach for the oceans. Here, we must extend the atmospheric results to take into account the atmospheric torque on the oceans and the effects of load-induced deformation in the solid earth. The non-gravitating atmospheric effects on the ocean can be included by extending the surface integrals in (5.9) over the air–sea interface and by noting that the pressure at the Earth's surface is $\rho_0 g\eta + P_{\mathrm{a}}$ where η is the sea-level height and P_{a} is the applied atmospheric pressure.

The gravitational and deformational effects are more difficult. They were not important in determining the atmospheric torque on the Earth because of the low atmospheric density. To include them for the oceans we add a term $\rho_0 \mathbf{g}_1$ to the right-hand side of (5.3), where $\mathbf{g}_1$ represents the perturbed gravitational acceleration, and we add a perturbation $-\boldsymbol{\nabla} U$ to the surface normal (5.7) where U is the load-induced radial deformation of the Earth. Then, repeating the steps which led to (5.10) for the atmosphere, we find that the total oceanic torque on the solid earth plus atmosphere is

$$
\begin{aligned}
L = &- \int_{S_{\mathrm{f}}} \mathbf{r} \times \boldsymbol{\Omega}\, \boldsymbol{\Omega}\cdot\mathbf{r}\, \rho_0 \eta - \int_{S_{\mathrm{f}}} \mathbf{r} \times \boldsymbol{\delta}\mathbf{n}\, \rho_0 g \\
&\times \left[\eta + \frac{P_{\mathrm{a}}}{\rho_0 g} - \sum_{l,m} \frac{3\rho_0}{(2l+1)\rho_{\mathrm{E}}} \gamma_l' \left[\eta_l^m + \frac{a_l^m}{\rho_0 g}\right] Y_l^m \right] + \int_{S_{\mathrm{f}}} \mathbf{r} \times (\hat{\mathbf{n}}\cdot\mathbf{T}) \\
&+ \int_{S_{\mathrm{a}}} \mathbf{r} \times (\hat{\mathbf{n}}\cdot\mathbf{T}_{\mathrm{a}})
\end{aligned}
\tag{5.16}
$$

where S_{f} and S_{a} are the ocean floor and the air–sea interface, respectively, $\mathbf{T}_{\mathrm{a}}$ is the applied frictional stress at the air–sea interface, the a_l^m and η_l^m are spherical harmonic coefficients of the surface pressure (4.1) and sea-level height (4.3), the surface normal, $\hat{\mathbf{n}}$, is directed into the ocean, $\boldsymbol{\delta}\mathbf{n}$ represents the effects of topography on the normal to the seafloor, and γ_l', ρ_{E} and the Y_l^m are defined in Section 4. We use this result for $\mathbf{L}$ to find oceanic contributions to the excitation function which are equivalent to ϕ_{mtn} and ϕ_{frict} in the atmospheric case. The reason we include in (5.16) the torque from the oceans on the *atmosphere* is that our results for the atmosphere included the atmospheric torque on the oceans. When combined, these two contributions will cancel, and we will be left with the net atmospheric and oceanic torque on the solid earth.

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Setting $\mathbf{T} = \mathbf{T}_a = 0$ in (5.16) and using the result in (2.29), we find the oceanic equivalent of the effects of mountain torques:

$$\phi_{\text{mtn}} = \frac{a^2\,1.61}{\Omega^2(C-A)} \int_{S_f} [\sin\theta\ \partial_\theta\ D + i\cos\theta\ \partial_\lambda\ D]\ \exp(i\lambda)$$

$$\times \left[\rho_0 g\eta + P_a - \sum_{l,m} \frac{3\rho_0}{(2l+1)\rho_E}\ (1 + k_l' - h_l')(\eta_l^m \rho_0 g + a_l^m)\, Y_l^m\right] d\theta\ d\lambda \tag{5.17}$$

where D is the depth of the ocean. It is important to include in D the effects of continental boundaries.

From a physical viewpoint ϕ_{mtn} given by (5.17) is no longer just a mountain torque. The terms proportional to h_l' represent the torque due to the equilibrium oceanic pressure against the deformed seafloor. The terms proportional to k_l' represent the torque due to the equilibrium gravitational attraction of the ocean on the deformed density field within the solid earth.The interpretation of the terms in (5.17) proportional to the '1' in $1 + k_l' - h_l'$ is more difficult. The $1 \times \eta_l^m$ terms represent the torque due to the gravitational interaction between the perturbed sea-level and the non-spherical portion of the solid earth's equilibrium density field caused by its hydrostatic adjustment to the equilibrium gravitational attraction of the ocean. The $1 \times a_l^m$ terms represent the torque on the perturbed atmospheric mass from the equilibrium gravitational field of the ocean. All four of these torques vanish unless the ocean is of variable thickness, which is why their contribution to ϕ_{mtn} depends on ∇D. These terms modify ϕ_{mtn} by at most 20 per cent (see Section 4).

Using (4.8) in (5.17) together with the fact that

$$\int_{S_f} [\sin\theta\ \partial_\theta\ D + i\cos\theta\ \partial_\lambda D]\ \exp(i\lambda)\, d\theta\ d\lambda = 0$$

we see that for an equilibrium response to an applied pressure perturbation there is no contribution to ϕ_{mtn} from the ocean. This is what we should expect, since an equilibrium ocean has no induced currents.

The frictional terms in (5.16) can be included in (2.29) to give a frictional contribution to ϕ_{motion} of

$$\phi_{\text{frict}} = \frac{-1.61a^3}{\Omega^2(C-A)} \left[\int_{S_f} (F_\theta + i\cos\theta\, F_\lambda) \sin\theta\ \exp(i\lambda)\, d\theta\ d\lambda \right.$$

$$\left. - \int_{S_a} (F_\theta^a + i\cos\theta\, F_\lambda^a) \sin\theta\ \exp(i\lambda)\, d\theta\ d\lambda\right] \tag{5.18}$$

where $\mathbf{F} = \hat{\mathbf{n}} \cdot \mathbf{T}$ is the frictional traction on the solid earth at the ocean floor, and $\mathbf{F}^a = -\hat{\mathbf{n}} \cdot \mathbf{T}^a$ is the applied atmospheric frictional traction on the ocean at the air–sea interface. In each case, $\hat{\mathbf{n}}$ is directed into the ocean.

The integral over S_a in (5.18) represents the frictional torque from the ocean on the atmosphere. This term is offset by a corresponding contribution to ϕ_{frict} from the atmosphere (5.15). If we combine (5.15) for the atmosphere with (5.18) for the ocean we get a net frictional torque on the solid earth from the atmosphere and oceans of

$$\phi_{\text{frict}} = \frac{-1.61a^3}{\Omega^2(C-A)} \int [F_\theta + i\cos\theta\, F_\lambda]\ \sin\theta\ \exp(i\lambda)\, d\theta\ d\lambda$$

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where now **F** is the frictional traction on the solid earth from the atmosphere over land and from the ocean on the seafloor.

6 A combination of the angular momentum and torque approaches

It would be potentially useful for computing the effects of the atmosphere on ϕ if we could combine the angular momentum and torque approaches. For example, the angular momentum approach is probably more useful over land where wind observations are reliable. The torque approach, however, is possibly more useful over the ocean since the atmosphere–ocean coupling is reasonably well understood (see, e.g. Hellerman 1967; Bunker 1976).

We can develop this sort of combined approach by using the hybrid result (2.30) for the excitation function, ϕ, and taking V and W to be any two disjoint and complementary subsets of the atmosphere (the boundaries of V and W are assumed to be fixed relative to the solid earth). We then compute the contributions to ϕ from V and W using the angular momentum approach and the torque approach, respectively. For simplicity we include the oceans in the angular momentum volume, V; the results can easily be extended to the more general case.

The results in Section 5 show that ϕ_{matter} is given by (3.4) for both V and W. The contribution from V to ϕ_{motion} reduces to (3.5) plus a term proportional to the surface integral of $\mathbf{n}\cdot\mathbf{v}$ over the boundary between V and W. This surface integral is exactly the negative of the contribution from the $\mathbf{n}\cdot\mathbf{v}$ angular momentum flux term in (5.9). The contribution from W is found using (5.9) but ignoring the first term on the right-hand side, since this term contributes only to ϕ_{matter}. Consequently, we find ϕ_{motion} by integrating (3.5) over V and by using

$$\phi_{\text{motion}} = \phi_{\text{mtn}} + \phi_{\text{frict}} + \frac{1.61}{\Omega^2(C-A)}(-i\hat{x}+\hat{y})\cdot\int_{S_\perp}[\mathbf{r}\cdot\hat{\mathbf{n}}P_1 - \mathbf{r}\times(\hat{\mathbf{n}}\cdot\mathbf{T})]\,dS \tag{6.1}$$

for W, where $S_\perp$ is the vertical surface between W and V, $\hat{\mathbf{n}}$ is directed into W, $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors, and ϕ_{mtn} and ϕ_{frict} are given by (5.13) and (5.15) integrated over the lower surface of W. This result (6.1) for W includes, in order, a mountain torque effect, a skin friction effect, and two terms representing the pressure and frictional torques from the neighbouring volume, V. If $S_\perp$ is vertical with normal $\hat{\mathbf{n}} = \hat{\mathbf{e}}_\theta n_\theta + \hat{\mathbf{e}}_\lambda n_\lambda$, then the contribution to (6.1) from the pressure torque on $S_\perp$ reduces to

$$\phi^{\perp}_{\text{press}} = \frac{1.61}{\Omega^2(C-A)}\int_{S_\perp}[n_\theta + i\cos\theta\, n_\lambda]\sin\theta\exp(i\lambda)\,rP_1\,dS \tag{6.2}$$

where dS is an element of surface area and is proportional to $r^2\sin\theta$. Similarly, if $\mathbf{F} = \hat{\mathbf{n}}\cdot\mathbf{T}$ on $S_\perp$, then the contribution from the friction torque on $S_\perp$ is

$$\phi^{\perp}_{\text{frict}} = \frac{-1.61}{\Omega^2(C-A)}\int_{S_\perp}[F_\theta + i\cos\theta\, F_\lambda]\exp(i\lambda)\,r\,dS. \tag{6.3}$$

As one conceivable application, suppose W and V are the atmospheric volumes over the ocean and land, respectively. Then, we compute ϕ_{motion} for V using (3.5) together with wind data over land. We compute ϕ_{motion} for W using (6.1) and noting that since there is no topography over the oceans, $\phi_{\text{mtn}} = 0$, and so

$$\phi_{\text{motion}} = \phi_{\text{frict}} + \phi^{\perp}_{\text{press}} + \phi^{\perp}_{\text{frict}}. \tag{6.4}$$

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In this case, $\phi_{\text{press}}^{\perp}$ depends on pressure over the world's coastline where the data coverage is relatively good; $\phi_{\text{frict}}^{\perp}$ is probably unimportant since the shear stress across an air–air interface is small; and ϕ_{frict} depends on the frictional stress between the atmosphere and the ocean. As an extreme case, note that ignoring ϕ_{frict} in (6.4) combined with ignoring the effects of wind-induced circulation in the ocean is equivalent to assuming that the angular momentum imparted to the ocean from the winds is entirely absorbed by the ocean and not transmitted to the solid earth.

7 Geostrophic winds and currents

Following an initial discussion in Munk & MacDonald (1960, section 9.4) it has usually been assumed that geostrophic winds and ocean currents cannot effectively excite wobble. Since the atmosphere and oceans are believed to be nearly in geostrophic balance at long periods, the obvious conclusion is that ϕ_{motion} is probably much smaller than ϕ_{matter} for both the atmosphere and oceans. We show in this section that the effects of geostrophic motion on wobble may be important.

Suppose the atmosphere (or ocean) is in geostrophic balance. Then, the horizontal velocities u and v are related to the fluid pressure P_1 by

$$\rho_0 u = \left(\frac{1}{2a\Omega\cos\theta}\right)\partial_\theta P_1$$

$$\rho_0 v = \left(\frac{1}{2a\Omega\cos\theta\sin\theta}\right)\partial_\lambda P_1. \tag{7.1}$$

Using these results in the angular momentum result for ϕ_{motion} (3.5) we find

$$\phi_{\text{motion}} = \frac{-1.61a^2}{\Omega^2(C-A)}\int [\![\partial_\lambda[i\cos\theta\exp(i\lambda)P_1] + \partial_\theta[\sin\theta\exp(i\lambda)P_1]]\!]\, d\theta\, d\lambda\, dr. \tag{7.2}$$

For a constant thickness atmosphere $\phi_{\text{motion}} = 0$. This is the basis for the argument that geostrophic winds do not excite wobble. However, for a variable thickness atmosphere $\phi_{\text{motion}} \neq 0$ since the limits of integration for dr depend on θ and λ, and (7.2) reduces to

$$\phi_{\text{motion}} = \phi_{\text{mtn}} \tag{7.3}$$

where ϕ_{mtn} is the mountain torque contribution given exactly by (5.13). (For the ocean we add load-induced gravitational forces to the right-hand side of (7.1) and then our result for ϕ_{motion} reduces exactly to ϕ_{mtn} given by (5.17).) Consequently, geostrophic motion only allows us to ignore the effects of frictional torques. This indicates that the effects of geostrophic winds and currents need not be small. In fact, the discussion below equation (5.10) indicates that ϕ_{mtn} should be roughly the same order of magnitude as the contribution from the elliptical pressure torque, ϕ_{matter}.

8 Discussion

The effects of the atmosphere and oceans on the Earth's wobble can be assessed by computing their contributions to the wobble excitation function ϕ (2.21), and then comparing with the astrometrically observed ϕ found from (2.23). Most important are the amplitude and phase of the annual component and the continuous spectrum at periods

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near 435 day. The results above suggest a number of strategies for computing ϕ, each requiring a different set of meteorological or oceanographic data. This offers a chance to assess the quality of different types of data and, in the absence of data, to test the consequences of various assumptions concerning the atmosphere and oceans.

The excitation function, ϕ, is usefully separated into a term, ϕ_{matter}, dependent on density perturbations in the atmosphere and oceans, and a term, ϕ_{motion}, dependent on winds and currents. The more readily computed of these two terms is ϕ_{matter}. We can think of this term as representing either the incremental rotational angular momentum of the solid earth in response to a change in the inertia tensor of the atmosphere and oceans, or the torque on the solid earth due to pressure against the Earth's elliptical bulge. In either case, ϕ_{matter} is most usefully computed using (3.4) together with observations of atmospheric and oceanic pressure at the surface of the solid earth. Although there is ample surface pressure data over land, there is no way of directly inferring the pressure at the seafloor from existing data. This is partly because there is only limited atmospheric pressure data over the ocean, but, more importantly, is due to the fact that there are no long period observations of open ocean sea-level.

Fortunately, we can partially resolve this problem by conceptually separating the effects of the ocean on both ϕ_{matter} and ϕ_{motion} into contributions from pressure-driven and wind-driven oceanic displacements. We showed in Section 4 that since the ocean probably responds to long-period pressure variations as though it were in a continual state of equilibrium, the pressure-driven displacements do not contribute to ϕ_{motion}, and the corresponding pressure at the seafloor is completely determined by the atmospheric pressure over land and by the change in the total mass of the atmosphere. This means that in practice we can confidently compute all contributions to ϕ_{matter} except those caused by wind-driven changes in sea-level height. These can be included by modelling the effects of winds on the ocean, as was done by O'Connor (1980), and as we must do to find ϕ_{motion} as described below.

The computation of ϕ_{motion} is considerably more difficult because of the lack of necessary data. We have discussed a number of possible methods for finding ϕ_{motion}. The most direct method is the angular momentum approach discussed in Section 3. Here, ϕ_{motion} is computed using (3.5) together with observed winds and currents. Unfortunately, wind data are scarce over the ocean and current data are virtually non-existent. And, unlike for the case of ϕ_{matter}, there is no *a priori* reason to expect the effects of the ocean and of the atmosphere above the ocean to cancel in ϕ_{motion}. As might be expected, the serious lack of any sort of oceanic data will be a problem for estimating the effects of the ocean no matter what method is used to compute ϕ_{motion}. Probably the best we can hope to do is to model the wind-induced oceanic response. Once this response has been modelled, the oceanic contribution to ϕ_{motion} would probably be most conveniently computed using the angular momentum result (3.5).

The reason, then, for introducing the torque and the hybrid methods described in Sections 5 and 6 is to handle the atmosphere and, in particular, the paucity of wind data over the ocean. Of course, any model of the wind-driven ocean would be affected by uncertainties in the winds over the ocean. However, to model the ocean response we would need to know only the winds at the surface of the ocean. These are considerably better known than the winds at different levels over the ocean, which are what we would need to compute ϕ_{motion} using (3.5).

The torque result (5.11) for ϕ_{motion} contains contributions from the mountain torque (5.13) and from the friction torque (5.15). The mountain torque can be readily computed from existing pressure data, since it requires pressure data over only the land. The contribu-

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tion from the mountain torque was included by Wilson & Haubrich (1976a). The friction contribution is more difficult since it requires knowledge of frictional stresses which are poorly known over land.

The hybrid approach described in Section 6 offers another strategy for computing ϕ_{motion}. In this case, we use the angular momentum result (3.5) over land where wind data are relatively good. Over the ocean we use the modified torque result (6.4) which includes contributions from frictional torques on the ocean (5.15) and from pressure torques (6.2) and frictional torques (6.3) acting between the atmosphere above land and the atmosphere above the ocean. The latter two torques depend on the pressure and frictional stresses vertically above the continental boundaries. The atmospheric frictional stress on the ocean can probably be estimated reasonably well from surface wind data (see references given in Section 6). The vertical pressure integral (6.2) can be determined from pressure data over the coastline. The vertical friction integral (6.3) is likely to be less important since there is little shear stress across an air–air interface. Note that the hybrid approach is independent of the frictional stress over land and of the winds at different levels over the ocean, which are the poorest determined contributions to the torque and angular momentum results respectively.

Most studies of the atmospheric and oceanic effects on wobble ignore ϕ_{motion} entirely, due in part to the lack of appropriate data and in part to the belief that ϕ_{motion} is unimportant for geostrophic motion. Probably the most useful exception is Wilson & Haubrich (1976a, b) where the contribution from the mountain torque, ϕ_{mtn}, was included. We showed in Section 7 that for geostrophic motion in either the atmosphere or oceans ϕ_{motion} is exactly equal to ϕ_{mtn}. And as discussed in Section 5, ϕ_{mtn} is likely to be the same order of magnitude as ϕ_{matter}. (I have found using real data that for the atmosphere ϕ_{mtn} is typically from 10 to 50 per cent of ϕ_{matter}.) Furthermore, most meteorological studies indicate that for the atmosphere the frictional torques are roughly the same order as the mountain torques (see the references given in Section 5). This does not imply that the atmosphere is non-geostrophic. Rather it implies that even for a geostrophic atmosphere all large-scale tangential tractions at the Earth's surface are small.

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