Theoret. Appl. Mech., Vol.39, No.2, pp. 99–125, Belgrade 2012

The effects of thermal radiation and viscous dissipation on MHD heat and mass diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface conditions

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Abstract

This investigation is undertaken to study the hydromagnetic flow of a viscous incompressible fluid past an oscillating vertical plate embedded in a porous medium with radiation, viscous dissipation and variable heat and mass diffusion. Governing equations are solved by unconditionally stable explicit finite difference method of DuFort – Frankel's type for concentration, temperature, vertical velocity field and skin - friction and they are presented graphically for different values of physical parameters involved. It is observed that plate oscillation, variable mass diffusion, radiation, viscous dissipation and porous medium affect the flow pattern significantly.

Keywords: Oscillating Plate, Radiation, Variable Heat and Mass Diffusion, MHD, Finite Difference, Viscous Dissipation, Porous Medium

1 Introduction

Free convection flow is a significant factor in several practical applications that include, for example, cooling of electronic components, in designs related to thermal insulation, material processing, and geothermal systems etc.

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Transient natural convection is of fundamental interest in many industrial and environmental situations such as air conditioning systems, atmospheric flows, motors, thermal regulation process, cooling of electronic devices, and security of energy systems. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns. Magnetohydrodynamic has attracted the attention of a large number of scholars due to its diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering it finds its application in MHD pumps, MHD bearings etc. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. Convective heat transfer in porous media has received considerable attention in recent years owing to its importance in various technological applications such as fibre and granular insulation, electronic system cooling, cool combustors, and porous material regenerative heat exchangers. Books by Nield and Bejan [1], Bejan and Kraus [2] and Ingham et al. [3] excellently describe the extent of the research information in this area. The phenomena of mass transfer is also very common in theory of stellar structure and observable effects are detectable, at least on the solar surface. The study of effects of magnetic field on free convection flow is important in liquid-metals, electrolytes and ionized gases. The thermal physics of hydromagnetic problems with mass transfer is of interest in power engineering and metallurgy. Thermal radiation in fluid dynamics has become a significant branch of the engineering sciences and is an essential aspect of various scenarios in mechanical, aerospace, chemical, environmental, solar power, and hazards engineering. Viscous mechanical dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. In the literature, extensive research work is available to examine the effect of natural convection on flow past a plate.

Extensive research has been published on free convection flow past a vertical plate. Free convection at a vertical plate with transpiration was investigated by Kolar and Sastri [4]. Ramanaiah and Malarvizhi [5] considered natural convection adjacent to a surface with 3 thermal boundary conditions. A numerical study for natural convective cooling of a vertical plate was presented by Camargo et al.[6] with different boundary conditions. The more difficult problem of transient free convection flow past a semi-infinite isothermal vertical plate was first studied by Siegel [7] using an integral method. The experimental confirmation of these results was presented by Goldstein and Eckert [8]. Another review of transient natural convection was presented by Raithby and Hollands [9], wherein a large number of papers on this topic were referred to. In reference to transient convection, Gebhart et al.[10] introduced the idea of leading edge effect in their book. They explained that the transition from conduction to Convection begins only when some effects from the leading edge have propagated up the plate as a wave, to a particular point in question. Later on, numerous investigators considered transient convective flow past a vertical surface by applying different boundary conditions and techniques. Transient convective heat transfer was pioneered by Padet [11] The flow past a vertical plate with sudden change in surface temperature was examined by Harris et al. [12]. Das et al. [13] analysed transient free convection flow with periodic temperature variation of the plate by Laplace-transform technique. In all the studies cited above, the effects of magnetic field and porous medium on the flow are ignored.

Many studies have been carried out to investigate the magnetohydrodynamic transient free convective flow. Gupta [14] first discussed the transient natural convection flow from a plate in the presence of magnetic current. Chowdhury and Islam [15] investigated magnetohydrodynamic free convection flow past a vertical surface by Laplace-transform technique. Aldoss and Al-Nimr [16] analysed transient hydromagnetic free convection flow over a surface. All the above studies are concerned with the absence of porous medium in the flow.

Convective heat transfer through porous media has been a subject of great interest for the last three decades. In recent years, only a few studies have been performed on transient convective flows in porous media. A detailed review of the subject, including an exhaustive list of references, can be found in the papers by Bradean et al. [17] and Pop et al. [18] Magyari *et al.* [19] have discussed analytical solutions for unsteady free convection in porous media. The magnetic current in porous media considered by Geindreau *et al.* [20].

Fewer studies have been carried out to investigate the heat transfer by simultaneous radiation and convection. Hossain et.al. [21] studied the radiation effects on mixed convection along a vertical plate with uniform surface temperature using the Rosseland flux model. Abd-El-Naby et al. [22] studied the radiation effects on MHD unsteady free convection flow over a vertical plate with variable surface temperature. Pathak et. al. [23] studied the radiation effects on an unsteady free convective flow through a porous medium bounded by an oscillating plate with a variable wall temperature. Deka et. al. [24] studied the hydromagnetic flow of a viscous incompressible fluid past an oscillating vertical plate with radiation and variable mass diffusion.

In all the investigations mentioned above, viscous mechanical dissipation is neglected. A number of authors have considered viscous heating effects on Newtonian flows. Mahajan et al. [25] reported the influence of viscous heating dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Isreal-Cookey et al. [26] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Zueco [27] used network simulation method (NSM) to study the effects of viscous dissipation and radiation on unsteady MHD free convection flow past a vertical porous plate. Suneetha et al. [28] have analyzed the thermal radiation effects on hydromagnetic free convection flow past an impulsively started vertical plate with variable surface temperature and concentration by taking into account of the heat due to viscous dissipation. Recently Suneetha et al. [29] studied the effects of thermal radiation on the natural conductive heat and mass transfer of a viscous incompressible gray absorbing-emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation. Very recently Hiteesh [30] studied the boundary layer steady flow and heat transfer of a viscous incompressible fluid due to a stretching plate with viscous dissipation effect in the presence of a transverse magnetic field.

Flows past a vertical plate oscillating in its own plane has many industrial applications. The first exact solution of Navier-Stokes equation was given by Stokes [31] which is concerned with flow of viscous incompressible fluid past a horizontal plate oscillating in its own plane. Natural convection effects on Stokes problem was first studied by Soundalgekar [32]. The same problem was considered by Revankar [33] for an impulsively started or oscillating plate. Gupta et al. [34] have analyzed flow in the Ekman layer on an oscillating plate. An exact solution to the flow of a viscous incompressible unsteady flow past an infinite vertical oscillating plate with variable temperature and mass diffusion by taking into account of the homogeneous chemical reaction of first-order was investigated by Muthucumaraswamy et. al. [35] Chaudhary et.al. [36] have studied the MHD flow past an infinite vertical oscillating plate through porous medium, taking account of the presence of free convection and mass transfer. Free convection flow of a viscous incompressible flow past an oscillating infinite vertical plate with variable temperature and mass diffusion has been studied by Muthucumaraswamy et. al. [37]. The free convection flow of a viscous incompressible fluid past an infinite vertical oscillating plate with uniform heat flux in the presence of thermal radiation was studied by Chandrakala [38].

Although different authors studied mass transfer with or without radiation and viscous dissipation effects on the flow past oscillating vertical plate by considering different surface conditions but the study on the effects of magnetic field on free convection heat and mass transfer with thermal radiation viscous dissipation and variable surface conditions in flow through an oscillating plate has not been found in literature and hence the motivation to undertake this study. It is therefore proposed to study the effects of thermal radiation and variable surface conditions on hydromagnetic flow past an oscillating vertical plate embedded in a porous medium with viscous dissipation

2 Mathematical analysis

We consider a two – dimensional flow of an incompressible and electrically conducting viscous fluid along an infinite vertical plate that is embedded in a porous medium. The x' - axis is taken along the infinite plate and y' - axis normal to it. Initially, the plate and the fluid are at same temperature T'_∞ with concentration level C'_∞ at all points. At time $t' > 0$, the plate starts oscillating in its own plane with a velocity $U_R \cos w't'$, the plate temperature is raised to T'_w and the concentration level at the plate is raised to C'_w. A magnetic field of uniform strength is applied perpendicular to the plate and the magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected [39]. There is no applied electric field. Viscosity is taken into account with the constant permeability of porous medium. The MHD term is derived from an order-of-magnitude analysis of the full Navier-Stokes equations. We regard the porous medium as an assembly of small identical spherical particles fixed in space, following Yamamoto et.al. [40]. Under these conditions and assuming variation of density in the body force term (Boussinesq's approximation), the problem can be governed by the following set of equations:

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y' 2} + \left[g\beta(T' - T'_\infty) + g\beta_c(C' - C'_\infty \right] - \frac{\sigma B_0^2 u'}{\rho} - \frac{\upsilon u'}{k'} \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + \frac{\upsilon}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

with the following initial and boundary conditions:

$$\begin{aligned}
&u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad for\, all \quad y', t' \leq 0\\
&u' = U_R \cos w't', \quad T' = T'_\infty + (T'_w - T'_\infty)At',\\
&\qquad C' = C'_\infty + (C'_w - C'_\infty)At' \quad at\ y' = 0,\ t' > 0\\
&u' \to 0, \quad T' \to T'_\infty, \quad C' \to C_\infty{}' \quad as\ y' \to \infty,\ t' > 0
\end{aligned} \tag{4}$$

Where u' is the velocity component in x'- axis, t'- the time, B_0 is the magnetic field component along y'- axis, C' is concentration at any point in the flow field, C'_w is concentration at the plate, C'_∞ is concentration at the free stream, D is mass diffusivity, C_p is specific heat at constant pressure, g is gravitational acceleration, T' is temperature of the fluid near the plate, T'_w is the plate temperature, T'_∞ is temperature of the fluid far away from the plate, β is coefficient of volume of expansion, β_c is concentration expansion coefficient, ρ is density, σ is Electrical conductivity, $\in$ is amplitude (constant), k is thermal conductivity of fluid, υ is kinematic viscosity, q_r is the radiation heat flux and k' is the permeability of the porous medium.

The second term of R.H.S. of the momentum equation (1) denotes buoyancy effects, the third term is the MHD term, the fourth term is bulk matrix linear resistance, that is Darcy term. The second term of R.H.S. of the energy equation (2) denotes radiation term, the third term is viscous dissipation term. The heat due to viscous dissipation is taken into an account. Also, Darcy dissipation term is neglected for small velocities in equation (2).

Thermal radiation is assumed to be present in the form of a unidirectional flux in the y-direction i.e., q_r (Transverse to the vertical surface). By using the Rosseland approximation [41] the radiative heat flux q_r is given by:

$$q_r = -\frac{4\sigma_s}{3k_e}\frac{\partial T'^4}{\partial y} \tag{5}$$

Where σ_s is the Stefan – Boltzmann Constant and k_e - is the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficiently small, then equation (5) can be linearized by expanding T'^4 in Taylor series about T'_∞ which after neglecting higher order terms takes the form:

$$T'^4 \cong 4{T_\infty}'^3 T' - 3{T_\infty}'^4 \tag{6}$$

In view of equations (5) and (6), equation (2) reduces to :

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma_s}{3k_e\rho c_p}{T_\infty}'^3\frac{\partial^2 T'}{\partial y'^2} + \frac{\upsilon}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{7}$$

Skin – friction is given by

$$\tau'_s = -\mu\left(\frac{\partial u'}{\partial y'}\right)_{y=0} \tag{8}$$

We introduce the non-dimensional variables:

$$\begin{aligned}
t &= \frac{t'}{t_R},\quad y = \frac{y'}{L_R},\quad u = \frac{u'}{U_R},\quad w = w' t_R,\quad K = \frac{U_R^2 k'}{\upsilon^2},\quad \Pr = \frac{\mu C_p}{k},\\
M &= \frac{\sigma B_0^2 \upsilon}{\rho U_R^2},\quad Sc = \frac{\upsilon}{D},\quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\quad \Delta T = T'_w - T'_\infty,\\
Gc &= \frac{\upsilon g\beta_c\left(C'_w - C'_\infty\right)}{U_R^3},\ U_R = \left(\upsilon g\beta\Delta T\right)^{1/3},\ L_R = \left(\frac{g\beta\Delta T}{\upsilon^2}\right)^{-1/3},\ A = \frac{1}{t_R}\\
t_R &= \left(g\beta\Delta T\right)^{-2/3}\upsilon^{1/3},\ N = \frac{k_e k}{4\sigma_s {T_\infty}'^3},\ E_c = \frac{U_R^2}{C_P\Delta T},\ Gr = \frac{g\beta\upsilon(T'_w - T'_\infty)}{U_R^3}
\end{aligned} \tag{9}$$

Where K is permeability parameter, Pr is Prandtl number, Gm is modified Grashof number, M is magnetic parameter, Sc is Schmidt number, t

is time in dimensionless coordinate, N is radiation parameter, E_c is Eckertnumber, L_R is reference length, t_R is reference time, u is dimensionless velocity component, U_R is reference velocity, μ is viscosity of fluid, θ is the dimensionless temperature, C is dimensionless concentration, w is frequency of oscillation.

The equations (1), (2) and (7) reduce to following non-dimensional form :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC - \left(M + \frac{1}{K}\right) u \tag{10}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\left[1 + \frac{4}{3N}\right]\frac{\partial^2 \theta}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2 \tag{11}$$

$$Sc\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} \tag{12}$$

with the following initial and boundary conditions:

$$u = 0, \qquad \theta = 0, \qquad C = 0 \qquad for\,all\;\; y, t \leq 0 \tag{13}$$

$$u = \cos\omega t, \qquad \theta = t, \qquad C = t, \qquad at\;\; y = 0,\;\; t > 0$$

$$u \to 0, \qquad \theta \to 0, \qquad C \to 0 \qquad as\;\; y \to \infty,\;\; t > 0 \tag{14}$$

where ωt is phase angle.

3 Skin-Friction

In non-dimensional form, the skin-friction is given by

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{15}$$

4 Numerical technique

Equations (10)-(12) are coupled non-linear partial differential equations and are to be solved under the initial and boundary conditions of equations (13) and (14). However exact or approximate solutions are not possible for this set of equations and hence we solve these equations by the unconditionally stable explicit finite difference method of DuFort – Frankel's type as explained by Jain et. al. [42] The finite difference equations corresponding to equations (10)-(12) are as follows:

$$\begin{aligned}\left(\frac{u_{i,j+1}-u_{i,j-1}}{2\Delta t}\right) &= \left(\frac{u_{i-1,j}-u_{i,j+1}-u_{i,j-1}+u_{i+1,j}}{(\Delta y)^2}\right)+\\ &\frac{Gr}{2}\left(\theta_{i,j+1}+\theta_{i,j-1}\right)+\frac{Gm}{2}\left(C_{i,j+1}+C_{i,j}-1\right)-\\ &\frac{1}{2}\left(M+\frac{1}{K}\right)\left(u_{i,j+1}+u_{i,j-1}\right)\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\theta_{i,j+1}-\theta_{i,j-1}}{2\Delta t} &= \frac{1}{\Pr}\left(1+\frac{4}{3N}\right)\left(\frac{\theta_{i-1,j}-\theta_{i,j+1}-\theta_{i,j-1}+\theta_{i+1,j}}{(\Delta y)^2}\right)+\\ &E_c\left(\frac{u_{i+1,j}-u_{i,j}}{\Delta y}\right)^2\end{aligned} \tag{17}$$

$$Sc\left(\frac{C_{i,j+1}-C_{i,j-1}}{2\Delta t}\right)=\left(\frac{C_{i-1,j}-C_{i,j+1}-C_{i,j-1}+C_{i+1,j}}{(\Delta y)^2}\right) \tag{18}$$

Initial and boundary conditions take the following forms

$$\begin{aligned}u_{i,0} &= 0, \qquad \theta_{i,0}=0, \qquad C_{i,0}=0 \qquad for\ \ all\ \ i\\ u_{0,j} &= \cos wt, \qquad \theta_{0,j}=j\Delta t, \qquad C_{0,j}=j\Delta t\\ u_{L,j} &= 0, \qquad \theta_{L,j}=0, \qquad C_{L,j}=0\end{aligned} \tag{19}$$

where L corresponds to ∞.

Here the suffix $'i'$ corresponds to y and $'j'$ corresponds to t.

Also $\Delta t = t_{j+1} - t_j$ and $\Delta y = y_{i+1} - y_i$.

Initially, the heat is transferred through the plate by conduction. But a little later stage, convection currents start flowing near the plate. Hence, it is essential to know the position of a point on the plate where conduction mechanism changes to convection mechanism. The distance of this point of transition from conduction to convection is given by

$$X_p = \int_0^t u(y,t)dt \tag{20}$$

5 Discussion

Extensive computations were performed. Default values of the thermo physical parameters are specified as follows:

Radiation parameter $N = 3$ (strong thermal radiation compared with thermal conduction), magnetic parameter $M = 2$, Prandtl number $Pr = 0.71$(air), Eckert number $Ec = 0.5$, Schmidt number $Sc = 0.22$(hydrogen), phase angle $\omega t = \frac{\pi}{2}$, thermal Grashof number $Gr = 10$, mass Grashof number $Gc = 10$, permeability parameter $K = 0.5$ and time $t = 0.4$.

All graphs therefore correspond to these values unless otherwise indicated.

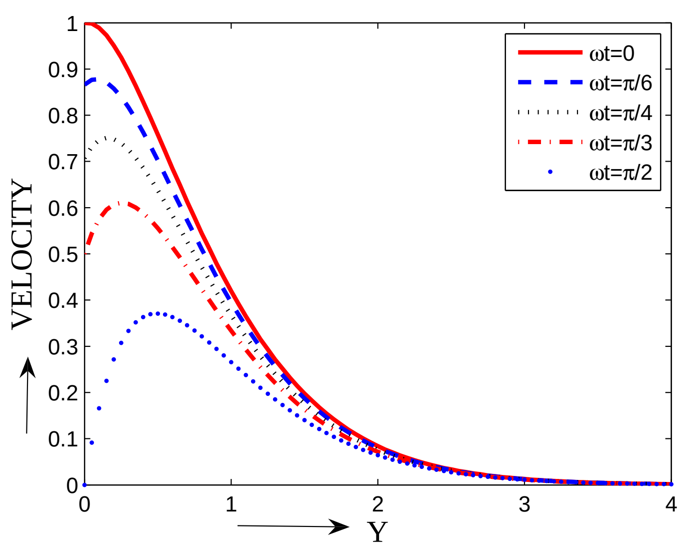

Figure 1: Velocity profile for different values of ωt

In order to point out the effects of various parameters on flow characteristic, the following discussion is set out. The values of the Prandtl number are chosen $Pr = 7$(water) and $Pr = 0.71$(air). The values of the Schmidt number are chosen to represent the presence of species by hydrogen (0.22), water vapour (0.60) and ammonia (0.78). Fig.1 represents the velocity profiles due to the variations in ωt. It is evident from figure that the velocity near the plate exceeds at the plate i.e. the velocity overshoot occurs. Furthermore, the magnitude of the velocity decreases with increasing phase angle (ωt) for air ($Pr = 0.71$). Figs.2 and 3 reveal the velocity variations with Gr and Gc in cases of cooling and heating of the surface respectively. It is observed that greater cooling of surface (an increase in Gr) and increase in Gc results in an increase in the velocity for air. It is due to the fact increase in the values of thermal Grashof number and mass Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow. The reverse effect is observed in case of heating of the plate ($Gr < 0$). Figs.4 and 5 illustrate the influences of M, K in cases of cooling and heating of the plate respectively. In case of cooling of the plate, the velocity near the plate is greater than at the plate. The maximum velocity attains near the plate and is in the neighbourhood of point $y = 0.5$. After $y > 0.5$ the velocity decreases and tends to zero as $y \to \infty$. Again it is found that the velocity decreases with increasing magnetic parameter for $Pr = 0.71$. It is because that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. The presence of a porous medium increases the resistance to flow resulting in decrease in the flow velocity. This behaviour is depicted by the decrease in the velocity as K decreases. In Fig.5, the opposite phenomenon is observed for heating of the plate. Figs.6 and 7 display the effects of Sc (Schmidt number), and t (time) on the velocity field for the cases $Gr > 0$, $Gc > 0$ and $Gr < 0$, $Gc < 0$ respectively. In case of cooling of the plate, the velocity near the plate increases owing to the presence of foreign gases (such as hydrogen, water vapour and ammonia) in the flow field. We again noticed that although there is a rise in the velocity due to presence of water vapour and ammonia, but it is not so high as in the case of hydrogen. The magnitude of the velocity for hydrogen increases with time for air. The reverse effect is observed in case of heating of the plate. Figs.8 and 9 illustrate the influences of N in cases of cooling and heating of the plate respectively. In case of cooling of the plate, the velocity near the plate is greater than at the plate. The maximum velocity attains near the plate and is in the neighbourhood of point $y = 0.5$ After $y > 0.5$ the velocity decreases and tends to zero as $y \to \infty$. Again it is found that the velocity decreases with increasing radiation parameter for $Pr = 0.71$. In Fig.9, the opposite phenomenon is observed for heating of the plate.

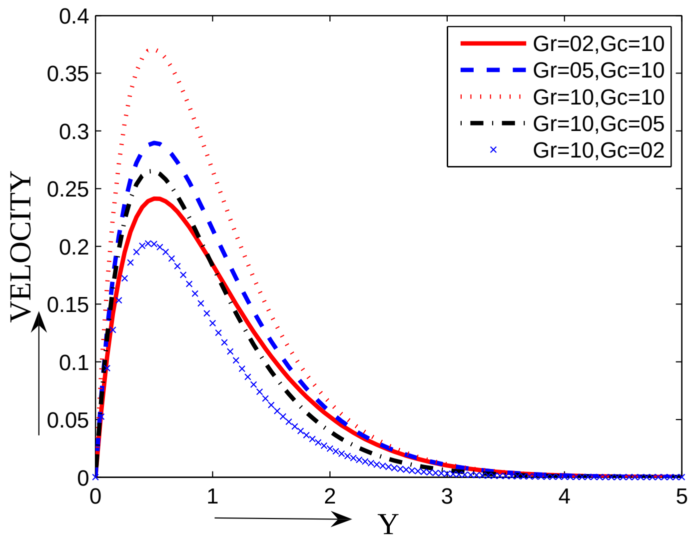

Figure 2: Velocity profile for different values of Gr and Gc

Fig.10 depicts the temperature profiles against y(distance from plate). The magnitude of temperature is maximum at the plate and then decays to zero asymptotically. Again it is found that the temperature decreases with increasing phase angle ωt for $Pr = 0.71$. The effect of radiation parameter N on the temperature variations is depicted in Fig.11. The radiation parameter N (i.e., Stark number) defines the relative contribution of conduction heat transfer to thermal radiation transfer. As 'N' increases, considerable reduction is observed in temperature profiles from the peak value at the plate ($y = 0$) across the boundary layer regime to free stream ($y \to \infty$), at which the temperature is negligible for any value of N. The effect of Eckert number 'E' on the temperature is shown in Fig.12. Eckert number is the ratio of the kinetic energy of the flow to the boundary layer enthalpy difference. The effect of viscous dissipation on flow field is to increase the energy, yielding a

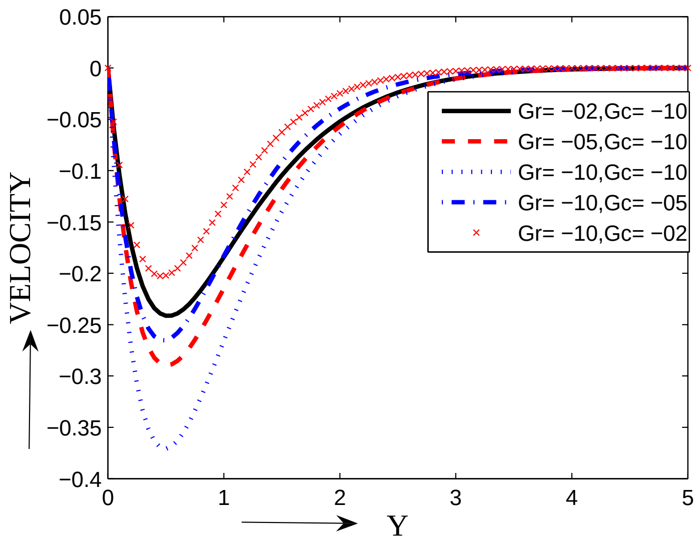

Figure 3: Velocity profile for different values of -Gr and -Gc

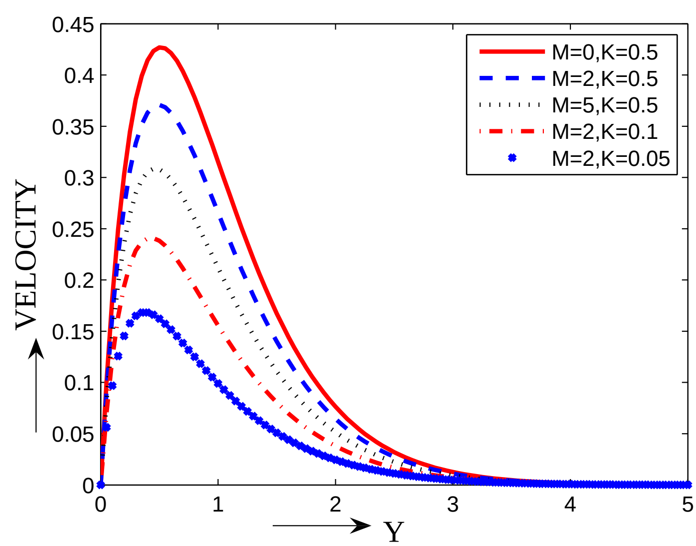

Figure 4: Velocity profile for different values of M and K

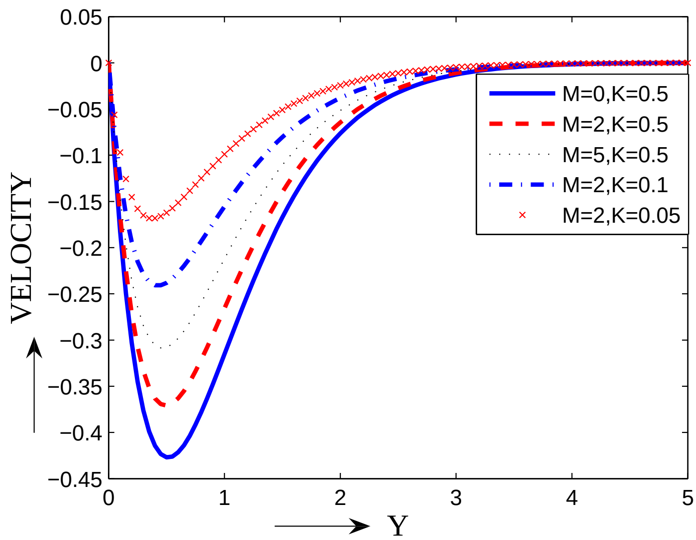

Figure 5: Velocity profile for different values of M and K with Gr=-10 and Gc=-10

greater fluid temperature and as a consequence greater buoyancy force. The increase in the buoyancy force due to an increase in the dissipation parameter enhances the temperature. Fig.13 illustrates the influence of t on the temperature. It is noted that the temperature is increasing with increasing values of t for both air and water. It is also observed that the magnitude of temperature for air $(Pr = 0.71)$ is greater than that of water $(Pr = 7)$. This is due to the fact that thermal conductivity of fluid decreases with increasing Pr, resulting a decrease in thermal boundary layer thickness.

Fig.14 concerns with the effect of Sc on the concentration. Like temperature, the concentration is maximum at the surface and falls exponentially. The Concentration decreases with an increase in Sc. Physically it is true, since the increase of Sc means decrease of molecular diffusivity. That results in decrease of concentration boundary layer. Hence, the concentration of species is higher for small values of Sc and lower for large values of Sc. Further, it is noted that concentration falls slowly and steadily for hydrogen in comparison to other gases.

Effects of variations in ωt and Sc on the penetration distance are presented in Fig.15. It is clear from the fig that the penetration near the plate increases

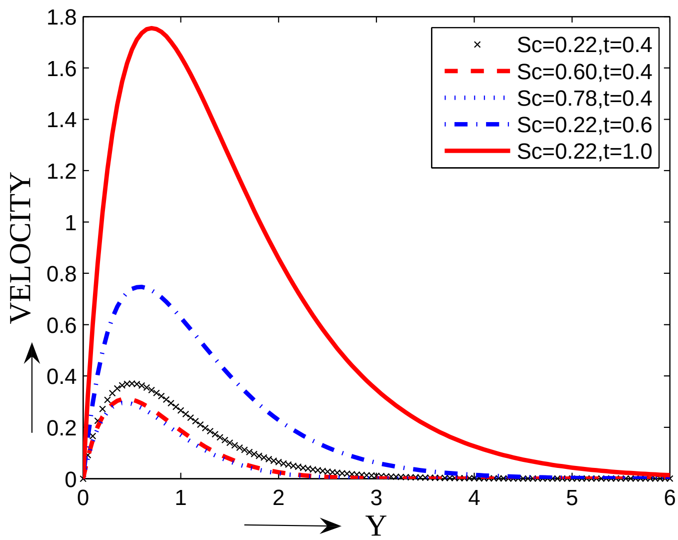

Figure 6: Velocity profile for different values of Sc and t

owing to the presence of the foreign gases such as hydrogen and water vapour. Further we noticed that it decreases with an increase in the value of 'Sc'. The penetration distance decreases on increasing '$\omega t'$ when hydrogen gas is presented in the flow for $Pr = 0.71$. Fig.16 shows the effects of the variations in M, K on the penetration. It is noted that the penetration falls owing to an increase in the magnetic parameter for both air and water. On the contrary, it increases with an increase in 'K'. The reason for them is same as that of explained for the velocity. Fig.17 concerns with the penetration against y for the various values of t, Gr and Gc. It is concluded from the figure that it increases with increase in t, Gr and Gc.

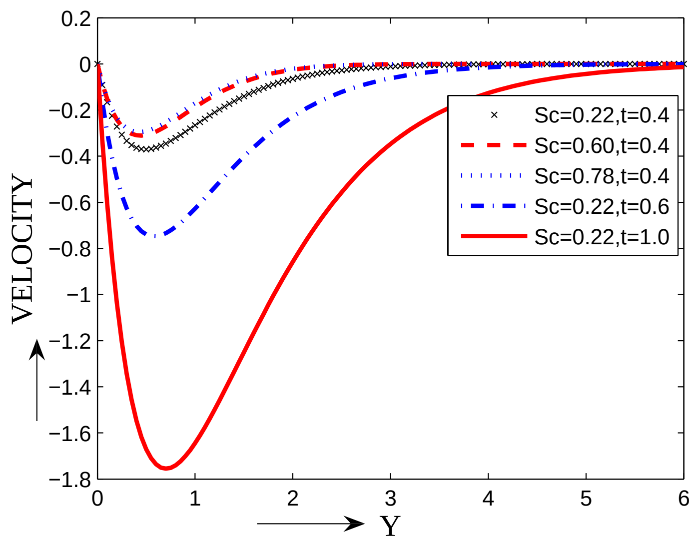

Figure 7: Velocity profile for different values of Sc and t with Gr=-10 and Gc=-10

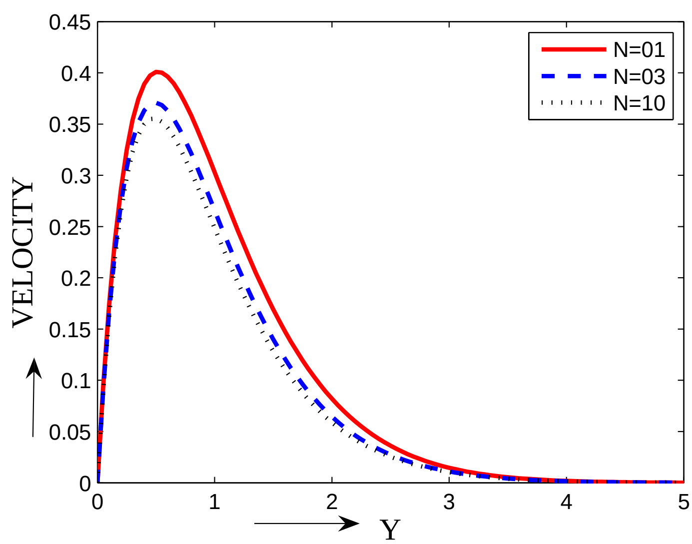

Figure 8: Velocity profile for different values of N

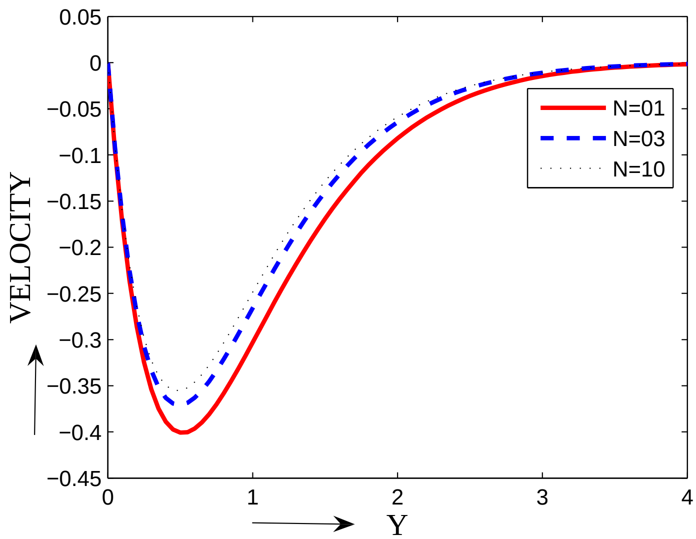

Figure 9: Velocity profile for different values of N with Gr=-10 and Gc=-10

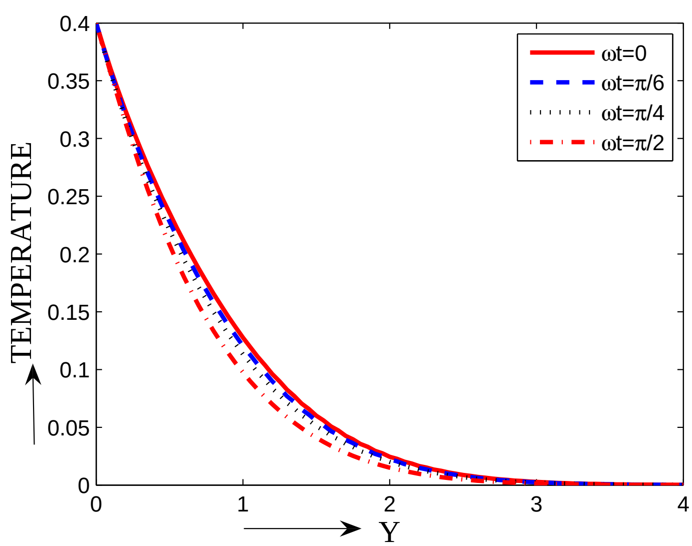

Figure 10: Temperature profile for different values of ωt

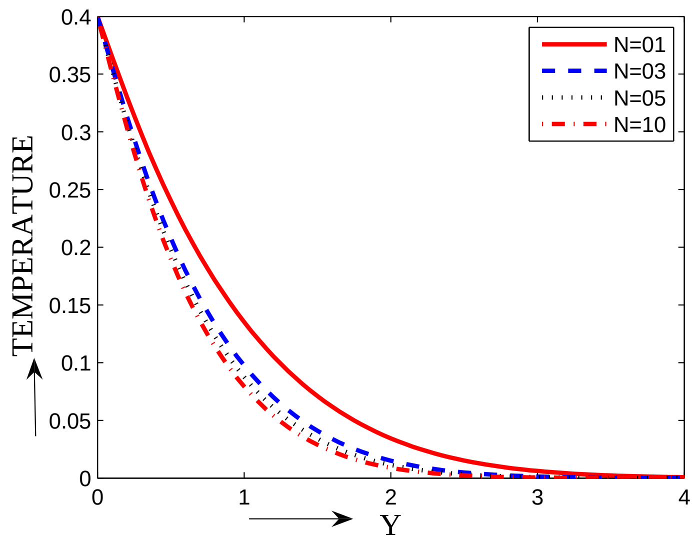

Figure 11: Temperature profile for different values of N

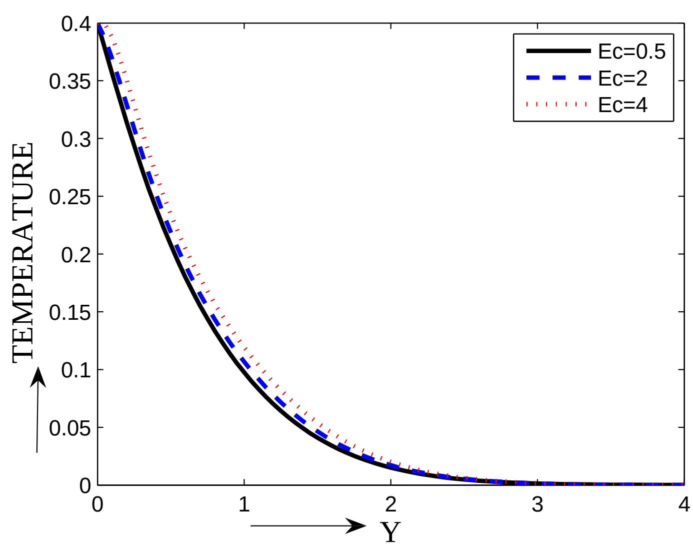

Figure 12: Temperature profile for different values of Ec

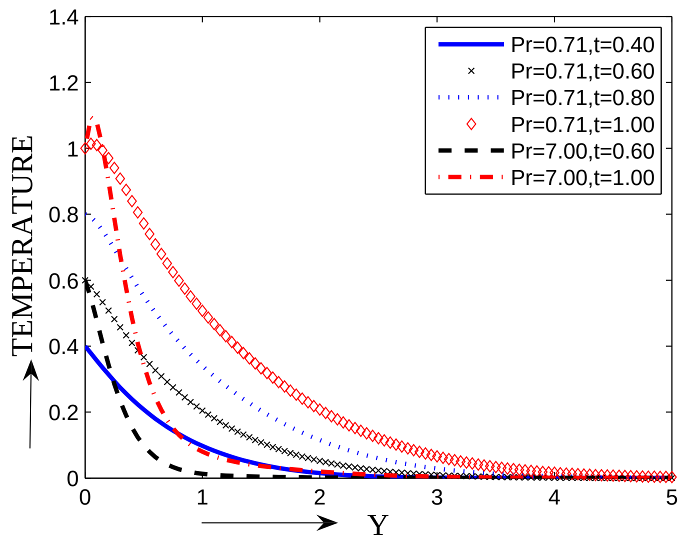

Figure 13: Temperature profile for different values of Pr and t

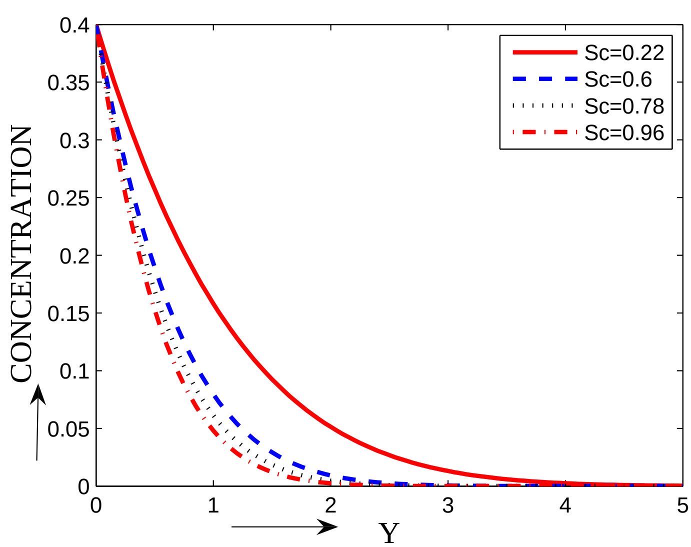

Figure 14: Consentration profile for different values of Sc

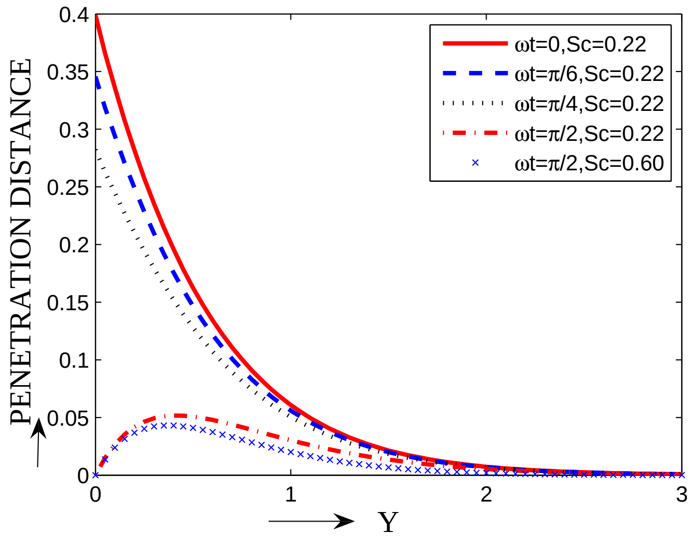

Figure 15: Penetration distance profile for different values of ωt and Sc

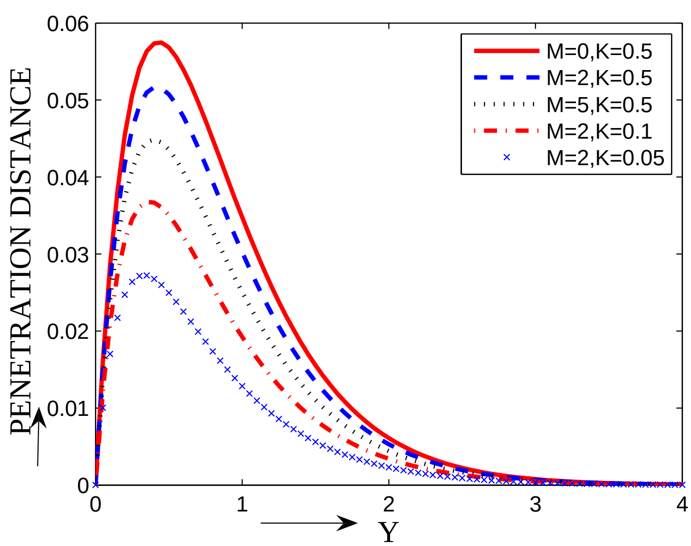

Figure 16: Penetration distance profile for different values of M and K

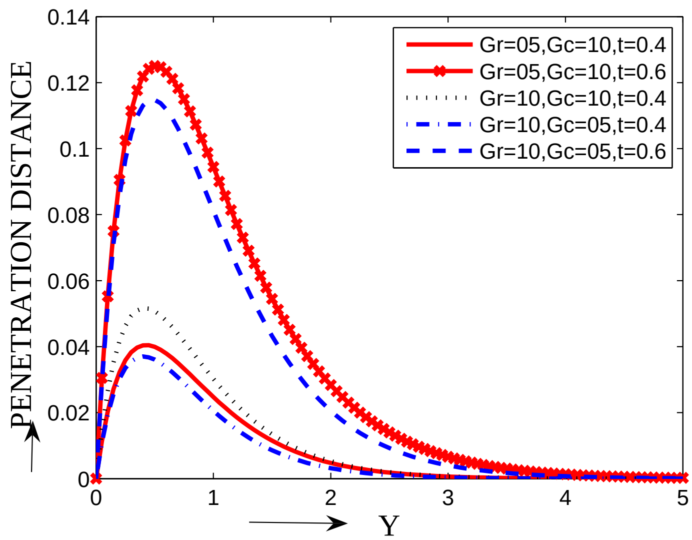

Figure 17: Penetration distance profile for different values of Gr, Gc and t

Figs.18 and 19 depicts skin-fiction against time t for different values of parameters. The Skin-fiction increases with an increase in Sc, N. Further, the skin-friction increases with M due to enhanced Lorentz force which imports additional momentum in the boundary layer. On the other hand, the skin-fiction decreases with increasing K, Gm, Gr, Ec and ωt. The magnitude of skin-friction for $Pr = 0.71$ is less than that of $Pr = 7$.

6 Conclusions

In this paper the effects of thermal radiation and viscous dissipation on MHD heat and mass diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface conditions have been studied numerically. Explicit finite difference method is employed to solve the equations governing the flow.

The present investigation brings out the following interesting features of physical interest on the flow velocity, temperature and concentration:

- Velocity decreases with increase in the phase angle (ωt) for air.

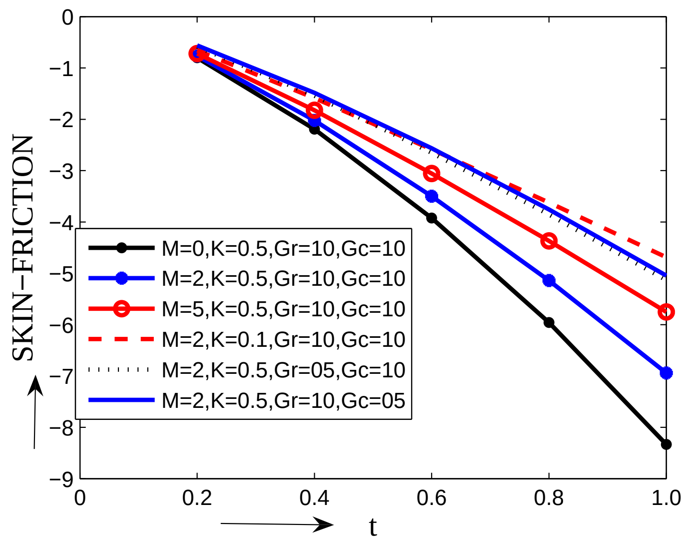

Figure 18: Skin Friction Profile for different values of M, K, Gr, Gc

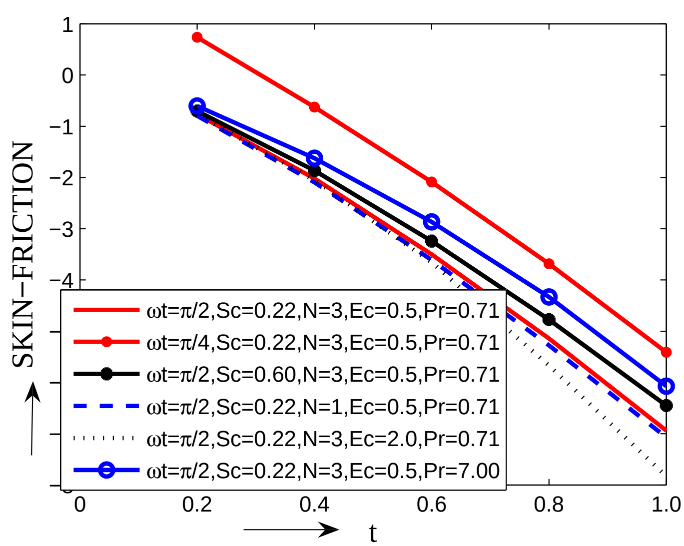

Figure 19: Skin Friction Profile for different values of ωt, Sc, N, Ec,Pr

- Velocity increases with increase in the thermal Grashof number (Gr¿0) and mass Grashof number (Gc) for air and the reverse effect is noticed for heating of the plate (Gr¡0).

- Velocity decreases with increasing magnetic parameter (M), permeability of the porous medium (K), for air in the cooling of the plate and reverse effect is noticed for heating of the plate.

- Velocity decreases with increasing Schmidt number 'Sc' and increases with time't' for air in the cooling of the plate and reversed effect is noticed for heating of the plate.

- Velocity decreases with increase in radiation parameter 'N' for air in the cooling of the plate and reversed effect is noticed for the heating of the plate.

- Temperature increases with increase in Eckert number 'E_c' and time't' while it decreases with increase in radiation parameter 'N'.

- Concentration decreases with increase in 'Sc'.

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Submitted in January 2011.

Uticaji termičkog zračenja i viskozne disipacije na MHD toplotno-maseno difuzno tečenje preko oscilirajuće vertikalne ploče potopljene u poroznu sredinu sa promenljivim uslovima na površi

Ovo istraživanje je izvedeno radi proučavanja hidromagnetskog tečenja viskoznog nestišljivog fluida preko oscilirajuće vertikalne ploče potopljene u poroznu sredinu sa zračenjem, viskozne disipacije i promenljive toplotno-masene difuzije. Vodeće jednačine su rešene bezuslovno stabilnim metodom DuFort–Frankel tipa za koncentraciju, temperaturu, polje vertikalne brzine i trenje na zidu i, potom, prikazane grafički za različite vrednosti značajnih fizičkih parametara. Primećuje se da oscilovanje ploče, promenljivo maseno difuziono zračenje, viskozna disipacija i porozna sredina značajno utiču na raspored slike strujanja.

doi:10.2298/TAM1202099K Math.Subj.Class.: 76S05, 76E25, 76W99