THE EGG RECORDS OF LIMITED PERIODS AS CRITERIA FOR PREDICTING THE EGG PRODUCTION OF THE WHITE LEGHORN FOWL

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INTRODUCTION

This paper, which is one of a series dealing with various problems of fecundity in the domestic fowl, has for its purpose the treatment of one phase of the problem of the prediction of the egg production of a long period from the recorded performance of a relatively short period of time.

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From the purely scientific side the interrelationship of the egg production of different periods is a problem of great biological interest. From the economic side, two ends which may be either quite distinct or interdependent, are to be attained by the development and application of formulae for the prediction of the egg production of a bird during any period of the pullet year. The first is the determination of the probable future record of an individual bird from her past performance, as a basis for the decision as to whether she shall be kept for egg production or sold for meat. The second is the estimation of the annual record of a bird as a basis of decision as to whether or not she shall be kept until the following season to be used as a breeder.

It will be evident to those who have had to consider the problems with which we have to deal, that economic factors,—particularly the cost of trap-nesting,—and the purpose for which prediction is being made will have great weight in determining the period and the number of periods to be used in the prediction equations. In determining which birds shall be sold to the commission man and which may be fed with reasonable prospects of profitable returns for the remainder of the year, the breeder is not concerned primarily with the record which the bird makes for the year as a whole. Practically he requires to know what returns she will make for the remainder of the period over which she may be retained in the flock. He has already maintained her for n of the 12 months. The question which the poultryman would like to have answered is whether her record during this period has been such that he can afford to feed and house her for the remaining $12 - n$ months. It is evident that to be of the greatest value for this purpose the prediction should be made from periods as early as possible in the life of the bird. In other words, if birds are to be culled out of the flock and sold for their flesh because they are unprofitable as egg producers, this should be done at a time when the maximum saving in cost of maintenance can be coupled as closely as possible with the maximum sale price.

The correlations required for this purpose are, therefore, those between the record of any period which may be selected as a basis for judgment, and the record of later months.

Since in the selection of birds to be held over for breeders the total annual production is presumably the factor to be chiefly taken into consideration it is evident that the correlations to be determined are those between the record of the individual months and of the year as a whole. It would, of course, be better for this purpose if the records of the entire year were known, but as pointed out by CARD (1917, p. 66) many poultry-where E represents the annual egg production or the production of any period of months, and e_p denotes the production of any period used as a basis for prediction.

The reader may quite legitimately suggest that in certain cases better prediction might have been secured by the use of regression curves of a higher order. This may be true. Our plan has been to test not merely the linear equations but others as well. Considerable progress has been made toward this end. Comprehensive tests will, we hope, eventually be published. Since, however, a relatively high degree of accuracy of prediction may be attained in most cases by the use of the linear equation, it does not seem proper to withhold useful results until it is possible to determine whether additional refinement can be attained.

The essential characteristics of equations for the prediction of egg yield are two:

1. That the errors of prediction be distributed about the true numbers in such a manner that estimations will not in the long run be either too high or too low.

2. That the magnitude of the deviations of the predicted from the observed egg productions be as small as possible.

Thus in testing formulae by determining how efficiently they predict the production of birds whose record is actually known, we shall consider that formula the best which (a) shows the least error in the direction of consistently too high or too low prediction, and (b) gives the lowest deviation of the predicted from the observed record.

To test the first of these essentials we have merely to determine the average deviation with regard to sign of the predicted from the actually measured egg production. This is given by

$$\frac{\Sigma\,(E'_p - E_p)}{N}$$

where E_p is the actual egg production of a bird, E'_p the theoretical egg record of an individual bird for a period p, and N the number of birds considered. Here a negative sign indicates that the equation has predicted records which are on the average too low, whereas a positive sign indicates that it has predicted records which are on the average too high.

But, as noted above, a formula must do more than fail to consistently overestimate or underestimate. It must give predicted values which show the lowest possible deviation from those determined by trap-nesting. We have, therefore, to consider the test which shall be applied to deterof months and combinations of months in order (a) to determine the months which give the best results and (b) to enable those who wish to predict from any group of months.

In the investigations one phase of which is presented in this paper, we have sought among other things:

(1) To determine the best method of predicting the annual egg production of a bird from the known record of any individual month.

(2) To determine the best method of predicting the annual egg production of a bird from the combined records of two or more months.

(3) To determine the best method of predicting the egg record of a bird during a portion of the year from the record of a single antecedent month or a group of antecedent months.

(4) To compare the relative merits of these several methods of prediction among themselves and to determine thereby which of the methods makes possible the most exact prediction as a basis for determining which is likely to be of the greatest practical value.

We fully recognize, and desire to emphasize especially, the fact that the whole problem of the prediction of future egg production cannot be solved in a single investigation. The problem is exceedingly complex and a number of factors are not taken into consideration at all in the present paper. All that has been attempted is to indicate the possibility of an important line of advance and to lay the foundations, in a series of statistical constants, for wider investigations. Some of these are already in progress. In the meantime, the results presented here may prove useful both from the practical standpoint and in facilitating to some extent further and more adequate investigations.

The first definite step in the direction of the use of the egg record of a short recorded period for the prediction of the probable production during a subsequent or a longer period was, as far as we are aware, taken in 1917 when it was shown (HARRIS, BLAKESLEE, WARNER and KIRKPATRICK 1917) that in a heterogeneous series of birds such as are submitted by practical breeders in egg-laying contests, the October egg production is correlated with that of every other month of the year. The whole subject was carried much further in a second memoir (HARRIS, BLAKESLEE and KIRKPATRICK 1917, 1918) in which the correlations between the records of the individual months and the production of the whole year, between the records of the individual months and of the remaining 11 months of the year, and between the production of 5 of the individual months and the production of all the other individual months, were published for two series of birds. In this paper the equations for the prediction of total annual production from the record of the individual months were given.

The results given in our second paper (HARRIS, BLAKESLEE and KIRKPATRICK 1917, 1918) show clearly that it is possible to predict with a considerable degree of accuracy the annual egg production of a group of birds from their record for a given month. They also indicate that it is possible within limits to predict the egg production of any month, p, from the egg record of any other month, q.

Almost simultaneously CARD (1917) considered the correlation between the records of various periods as a basis for the prediction of annual egg production. Prediction equations were not, however, given.

While the determination of equations for the prediction of the egg production of a subsequent or a more extended period from the actually recorded production of a limited period must rest upon biometric theory, we have deemed it proper in the testing of these equations to proceed in a purely objective manner.

We have determined a series of prediction equations and have used these equations for estimating the egg production of a series of birds, the egg record of which is unknown as far as the development of the equations is concerned. We then determined the difference between the yield predicted by the equations and the actual yield in the case of each individual bird. The average of these deviations, or any other suitable mathematical constant based upon them, furnishes a criterion of the suitability of the equation for purposes of prediction. That equation is best which predicts most exactly the annual egg yield, or the egg production of any shorter period, for a bird of which the record of a limited period is known.

Since the birds entered in the INTERNATIONAL EGG-LAYING CONTEST at Storrs are drawn from a wide geographical area and are furnished by a large number of breeders, and since the conditions in the different years are maintained as nearly constant as possible, it seemed desirable to utilize records from this contest subsequent to those upon which the equations are based in testing the value of the equations. The problem is: How closely can the actual production of a bird entered in the contest in a given year be predicted from equations based on the records of previous years when one or more months' performance of this bird is known from observation? We have, therefore, as already noted, based the test of our series of equations first of all upon the records secured in Connecticut during the contest year 1917 and 1918.

The equations which we publish are based upon 1840 single-comb White Leghorn birds entered in the INTERNATIONAL EGG-LAYING CONTEST for the years 1911 to 1917. The prediction equations have been tested upon 415 birds whose records were obtained during the year extending from November 1, 1917, to October 31, 1918.

The justification for the course followed is found in the general principle that a theory should not be tested against the observations upon which it is based.

For practical reasons this paper is limited to a test of the accuracy with which the egg record of a series of 415 birds trap-nested at Storrs during 1917–1918 can be predicted by a series of linear equations based on the experience of the six preceding years, 1911–1917, at the same place. It may be urged that conditions at Storrs are not representative of those prevailing in different parts of the country. Recognizing, for the sake of argument at least, the validity of this objection we have been glad to avail ourselves of records taken elsewhere. These are now being used to test the accuracy with which the production of birds in any locality may be predicted by means of equations based primarily upon experience in another place or with another series of birds. The results of these studies will eventually be published.

NOTATION AND THEORY EMPLOYED

We shall find it convenient to have a simple and rigid notation. Let e represent the recorded egg production of a bird in any month, Σ denote a summation of monthly egg records for a given bird, 1, 2, 3, . . . 12 denote the twelve successive months of the pullet year, i.e., the November of the year in which the bird was hatched until and including the following October. Then $e_1, e_2, e_3, \ldots, e_{12}$ represent the November, December, January, . . ., October egg record of a bird with an annual record of $E = \sum_{1}^{12} (e)$ eggs. Further, E_n denotes the total number of eggs laid in any month or group of months subsequent to any given month or group of months used as a basis of prediction, i.e.,

$$E_{11} = E - e_1 = \sum_{2}^{12} (e),\ E_{10} = E - e_1 - e_2 = \sum_{3}^{12}(e),\ \ldots,\ E_1 = e_{12}.$$

In the present paper we have used only the linear prediction equations derived from the means, standard deviations and product-moment coefficients of correlation between the periods, or groups of periods, of egg production, i.e., with equations of the type

$$E = \left(\overline{E} - r_{e_pE}\frac{\sigma_E}{\sigma_{e_p}}\bar{e}_p\right) - r_{e_pE}\frac{\sigma_E}{\sigma_{e_p}}e_p$$

where E represents the annual egg production or the production of any period of months, and e_p denotes the production of any period used as a basis for prediction.

The reader may quite legitimately suggest that in certain cases better prediction might have been secured by the use of regression curves of a higher order. This may be true. Our plan has been to test not merely the linear equations but others as well. Considerable progress has been made toward this end. Comprehensive tests will, we hope, eventually be published. Since, however, a relatively high degree of accuracy of prediction may be attained in most cases by the use of the linear equation, it does not seem proper to withhold useful results until it is possible to determine whether additional refinement can be attained.

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Thus in testing formulae by determining how efficiently they predict the production of birds whose record is actually known, we shall consider that formula the best which (a) shows the least error in the direction of consistently too high or too low prediction, and (b) gives the lowest deviation of the predicted from the observed record.

To test the first of these essentials we have merely to determine the average deviation with regard to sign of the predicted from the actually measured egg production. This is given by

$$\frac{\Sigma\,(E'_p - E_p)}{N}$$

where E_p is the actual egg production of a bird, E'_p the theoretical egg record of an individual bird for a period p, and N the number of birds considered. Here a negative sign indicates that the equation has predicted records which are on the average too low, whereas a positive sign indicates that it has predicted records which are on the average too high.

But, as noted above, a formula must do more than fail to consistently overestimate or underestimate. It must give predicted values which show the lowest possible deviation from those determined by trap-nesting. We have, therefore, to consider the test which shall be applied to determine which formula gives the lowest deviation. Two methods may be suggested.

First, the deviations may be summed without regard to sign and divided by their number. This gives an average deviation without regard to sign of the predicted from the recorded production for any flock and period under consideration.

The disadvantages of this method are two: (a) It ignores mathematical convention with regard to signs. (b) It gives large and small deviations a weight proportional to their actual magnitudes. Thus 50 deviations of 3 eggs each and 50 deviations of 5 eggs each would give an average deviation of 4 eggs, while 50 deviations of 1 egg each, 25 deviations of 6 eggs each and 25 deviations of 8 eggs each would also give a general average deviation of 4 eggs. But since one of the ideals to be attained in the selection of a formula would seem to be to obtain one which will avoid the grosser errors it seems proper to weight the larger deviations. This can be most logically done by squaring. Then

$$\left\{\frac{\Sigma\,(E'_p - E_p)^2}{N}\right\}^{\frac{1}{2}}$$

gives a square root of mean square deviation, or a "root mean square deviation." This is probably the best available measure of the deviation of prediction from observation.

For completeness we shall employ all three methods in the tests of equations used in this paper.

The method of taking the difference has been so chosen that a positive sign, indicating larger error of estimating, shows an inferiority in the equation.

Two of the criteria are values without sign. In the case of the average deviation with regard to sign the criteria may be either positive or negative. In comparing two different methods of prediction we have considered that the *magnitude* of the error and not the *sign* is the critical point. In such comparisons, therefore, all of the criteria have been considered as alike in sign. Cases may possibly arise in which it is desirable to consider the question of over prediction or under prediction by two formulae which may be under consideration. If so our tables of criteria and not the differences as published should be consulted by the reader.

The characteristic equation given above is strictly valid only when applied to the population from which it is deduced. Its extension without modification to another population is justified only if the physical constants

TABLE 1

Comparison of constants for series of birds on which equations were based (1911 to 1917) and series on which they were tested (1917–1918).

	MEAN					STANDARD DEVIATION				
	1911 to 1917	1917 to 1918	Difference	$\frac{\text{Diff.}}{E_{\text{Diff.}}}$	Percentage difference	1911 to 1917	1917 to 1918	Difference	$\frac{\text{Diff.}}{E_{\text{Diff.}}}$	Percentage difference
November	5.20±0.09	5.78±0.20	+0.58±0.21	2.76	11.11	6.00±0.06	6.34±0.14	+0.34±0.15	2.26	5.66
December	6.58±0.11	6.21±0.22	−0.37±0.24	1.54	5.62	7.10±0.07	6.80±0.15	−0.30±0.16	1.87	4.22
January	6.07±0.10	6.63±0.19	+0.56±0.21	2.66	9.22	6.63±0.07	5.83±0.13	−0.80±0.14	5.71	12.06
February	10.10±0.09	9.93±0.19	−0.17±0.21	0.80	1.68	5.91±0.06	5.94±0.13	+0.03±0.14	0.21	0.50
March	17.45±0.08	16.96±0.18	−0.49±0.19	2.58	2.80	5.31±0.05	5.52±0.13	+0.21±0.13	1.62	3.95
April	18.85±0.07	17.48±0.18	−1.37±0.19	7.21	7.26	4.67±0.05	5.44±0.12	+0.77±0.13	5.92	16.48
May	20.55±0.08	21.84±0.16	+1.29±0.17	7.58	6.27	5.24±0.05	5.06±0.11	−0.18±0.12	1.50	3.43
June	20.41±0.09	19.18±0.20	−1.23±0.21	5.85	6.02	5.84±0.06	6.13±0.14	+0.29±0.15	1.93	4.96
July	19.28±0.10	17.96±0.24	−1.32±0.26	5.07	6.84	6.43±0.07	7.43±0.17	+1.00±0.18	5.55	15.55
August	17.10±0.11	16.71±0.26	−0.39±0.28	1.39	2.28	7.30±0.08	7.96±0.18	+0.66±0.19	3.47	9.04
September	11.78±0.13	12.98±0.26	+1.20±0.29	4.13	10.18	8.36±0.09	7.93±0.18	−0.43±0.20	2.15	5.14
October	4.92±0.10	5.87±0.23	+0.95±0.25	3.80	19.30	6.63±0.07	7.07±0.16	+0.44±0.17	2.58	6.63
Annual	158.36±0.68	157.59±1.41	−0.77±1.56	0.49	0.48	43.34±0.48	42.63±0.99	−0.71±1.10	0.64	1.63

and the correlations of the variables in the two populations are essentially identical.

Because of the uniformity of care and the wide origin of the birds exhibited each year at the INTERNATIONAL EGG-LAYING CONTEST at Storrs the average productions do not differ widely in the different years. Thus the monthly and annual averages and standard deviations for the 1840 birds upon which the equations were based and the 415 birds upon which they were tested appear in table 1.[2]

While certain of the differences are significant in comparison with their probable errors it is quite clear that the averages for the two periods are in fair agreement.

Bird 997, Pen 100

	1	2	3	4	5	6	7	8
N.		143.1	−17.9	320.41				
D.		137.2	−23.8	566.44	161	143.1	−17.9	320.41
J.	3	148.1	−12.9	166.41	161	134.4	−26.6	707.56
F.	13	169.7	+8.7	75.69	158	135.6	−22.4	501.76
M.	24	189.1	+28.1	789.61	145	136.2	−8.8	77.44
A.	21	169.2	+8.2	67.24	121	129.2	+8.2	67.24
M.	29	199.6	+38.6	1489.96	100	99.4	−0.6	0.36
J.	25	180.4	+19.4	376.36	71	92.3	+21.3	453.69
J.	27	193.2	+32.2	1036.84	46	61.8	+15.8	249.64
A.	13	142.0	−19.0	361.00	19	44.9	+25.9	670.81
S.		118.5	−42.5	1806.25	6	12.8	+6.8	46.24
O.	6	162.0	+1.0	1.00	6	−0.7	−6.7	44.89
Year	161							

The method followed in the calculations may be illustrated by one of the calculation blanks for the individual bird—No. 997, pen 100. The first column shows the production for the month indicated by the letters on the stub. This serves as the basis of prediction. The second column shows the predicted number of eggs for the year, the third shows the deviation of this predicted number from the annual total of 161 eggs. The fourth column gives the squares of these deviations of prediction from observation. The fifth column shows the number of eggs in the remaining months of the year.[3] The sixth column shows the number of eggs predicted

[2] The percentage differences have been calculated by using the monthly averages for 1911 to 1917 as a base.

[3] The yields for the remaining months (columns 5 to 8) are dropped one space so as to coincide with the first month of the period. For example, bird 997 laid 161 eggs in the period from December to October; 161 in the period from January to October; 158 in the period from February to October, and so on.

for the remaining months. The seventh shows the deviations and the eighth the squares of the deviations of these predicted values from the actual record for the remaining months.

Calculation blanks for each individual bird were made on this principle for each of the equations used. The labor of testing the equation has, therefore, been very heavy, involving the calculation of 29,465 predicted values and the summations of the errors and squares of errors of the deviations of these predicted records from their true value.

The excessive arithmetical routine has been ably handled by Miss EDNA M. PECKHAM, Miss IDA M. PECKHAM, Miss RUTH T. CRAWSON, and Miss KATHLEEN GAVIN of the Biometric Laboratory of the STATION FOR EXPERIMENTAL EVOLUTION. We are indebted to Miss EDNA K. LOCKWOOD for the diagrams, as well as for much assistance in the computations.

TESTS OF EQUATIONS EMPLOYED

Prediction of annual production from the record of one month

Consider first of all the results of the attempts to predict the annual egg production of 415 White Leghorn birds observed at Storrs in 1917–1918 from the records of a single month's production. The equations based on the 1911 to 1917 experience are as follows:

Month from which prediction is made	*Prediction equation*
November	$E = +143.186 + 2.914\, e_1$
December	$E = +137.293 + 3.200\, e_2$
January	$E = +138.271 + 3.308\, e_3$
February	$E = +118.689 + 3.926\, e_4$
March	$E = +76.160 + 4.708\, e_5$
April	$E = +62.688 + 5.074\, e_6$
May	$E = +58.009 + 4.883\, e_7$
June	$E = +59.977 + 4.818\, e_8$
July	$E = +71.137 + 4.523\, e_9$
August	$E = +90.391 + 3.974\, e_{10}$
September	$E = +118.509 + 3.381\, e_{11}$
October	$E = +141.470 + 3.429\, e_{12}$

These are in good general agreement with the equations for two of the years, 1913–1914 and 1914–1915, published in a former paper (HARRIS, BLAKESLEE and KIRKPATRICK 1918, page 33, table 5). The graphical tests for linearity of regression (*loc. cit.*, diagrams 2–5, p. 34–39) for these two years, indicate a fairly close approximation to linearity throughout the greater part of the range of variation of monthly egg production. A critical test of linearity presents some difficulties because of the concentration of the bulk of the birds into a few of the classes, with the result that a rather large number of classes contain only a few birds each. A closer study of the fit of the regression line may, therefore, be deferred until more data are in hand.

The results of the tests of accuracy of prediction in the 415 White Leghorn birds of the 1917–1918 contest are given in tables 2 to 4. Since later

TABLE 2

Average deviation with regard to sign of predicted annual egg record from actual record. Prediction of annual production from one- and from two-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs, 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM TWO MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
For the whole year	November	+2.39	1.52	Nov. + Dec.	+1.16	0.74	+1.23	+0.78
	December	−0.49	0.31	Nov. + Dec.	+1.16	0.74	−0.67	−0.43
	December	−0.49	0.31	Dec. + Jan.	+1.15	0.73	−0.66	−0.42
	January	+2.58	1.64	Dec. + Jan.	+1.15	0.73	+1.43	+0.91
	January	+2.58	1.64	Jan. + Feb.	+1.75	1.11	+0.83	+0.53
	February	+0.06	0.04	Jan. + Feb.	+1.75	1.11	−1.69	−1.07
	February	+0.06	0.04	Feb. + Mar.	−1.22	0.77	−1.16	−0.73
	March	−1.63	1.03	Feb. + Mar.	−1.22	0.77	+0.41	+0.26
	March	−1.63	1.03	Mar. + Apr.	−4.94	3.13	−3.31	−2.10
	April	−6.23	3.95	Mar. + Apr.	−4.94	3.13	+1.29	+0.82
	April	−6.23	3.95	Apr. + May	+0.48	0.30	+5.75	+3.65
	May	+7.02	4.45	Apr. + May	+0.48	0.30	+6.54	+4.15
	May	+7.02	4.45	May + June	+0.82	0.52	+6.20	+3.93
	June	−5.21	3.31	May + June	+0.82	0.52	+4.39	+2.79
	June	−5.21	3.31	June + July	−6.60	4.19	−1.39	−0.88
	July	−5.27	3.34	June + July	−6.60	4.19	−1.33	−0.85
	July	−5.27	3.34	July + Aug.	−3.81	2.42	+1.46	+0.92
	August	−0.82	0.52	July + Aug.	−3.81	2.42	−2.99	−1.90
	August	−0.82	0.52	Aug. + Sept.	+2.60	1.65	−1.78	−1.13
	September	+4.78	3.03	Aug. + Sept.	+2.60	1.65	+2.18	+1.38
	September	+4.78	3.03	Sept. + Oct.	+5.34	3.39	−0.56	−0.36
	October	+3.95	2.51	Sept. + Oct.	+5.34	3.39	−1.39	−0.88

we shall have to compare the results for prediction from one month's performance with that from two- and from three-months record it has been desirable to give the results side by side in the same table. The reader need not, therefore, concern himself with the values for prediction from two-months production until later. Since the errors of prediction of the annual record from each individual month must be compared with the results for prediction from the combined production of two months, the constants for the single months have been given in duplicate.

The average errors with regard to sign are generally low, that for prediction from November and from January production gives on the average 2.4 eggs too many for the year. For December, February, March and August the prediction is in error by less than 2 eggs. The values predicted from April, May, June, July, September and October records are about 4 to 7 eggs in error.

TABLE 3

Average deviation without regard to sign of predicted annual egg record from actual record. Prediction of annual production from one- and from two-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs, 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM TWO MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
For the whole year	November	29.59	18.78	Nov. + Dec.	28.09	17.82	+1.50	+0.96
	December	29.26	18.57	Nov. + Dec.	28.09	17.82	+1.17	+0.75
	December	29.26	18.57	Dec. + Jan.	27.23	17.28	+2.03	+1.29
	January	30.09	19.09	Dec. + Jan.	27.23	17.28	+2.86	+1.81
	January	30.09	19.09	Jan. + Feb.	27.35	17.35	+2.74	+1.74
	February	27.28	17.31	Jan. + Feb.	27.35	17.35	−0.07	−0.04
	February	27.28	17.31	Feb. + Mar.	25.04	15.89	+2.24	+1.42
	March	27.95	17.73	Feb. + Mar.	25.04	15.89	+2.91	+1.84
	March	27.95	17.73	Mar. + Apr.	26.74	16.97	+1.21	+0.76
	April	28.72	18.22	Mar. + Apr.	26.74	16.97	+1.98	+1.25
	April	28.72	18.22	Apr. + May	26.68	16.93	+2.04	+1.29
	May	28.62	18.16	Apr. + May	26.68	16.93	+1.94	+1.23
	May	28.62	18.16	May + June	25.99	16.49	+2.63	+1.67
	June	29.03	18.42	May + June	25.99	16.49	+3.04	+1.93
	June	29.03	18.42	June + July	26.17	16.61	+2.86	+1.81
	July	28.35	17.99	June + July	26.17	16.61	+2.18	+1.38
	July	28.35	17.99	July + Aug.	24.88	15.79	+3.47	+2.20
	August	26.87	17.05	July + Aug.	24.88	15.79	+1.99	+1.26
	August	26.87	17.05	Aug. + Sept.	23.18	14.71	+3.69	+2.34
	September	24.78	15.72	Aug. + Sept.	23.18	14.71	+1.60	+1.01
	September	24.78	15.72	Sept. + Oct.	23.93	15.18	+0.85	+0.54
	October	27.37	17.37	Sept. + Oct.	23.93	15.18	+3.44	+2.19

The average deviations without regard to sign are of course much larger since they constitute a measure of the error of prediction of the records of individual birds. They range from 24.8 to 30.1 eggs. The significance of errors of this magnitude will be more clearly brought out later.

The square root of mean square deviation also shows considerable regularity from month to month. These measures are naturally considerably larger than the average deviation without regard to sign. They range from 32.9 to 38.8 eggs.

It is clear that the annual egg production of birds similar in origin to the series upon which the prediction equations were based and maintained under similar conditions may be predicted with a relatively high degree of accuracy providing their record for any month is definitely known.

The accuracy with which prediction may be made will be clear if the errors of prediction are expressed in terms of the actual average annual production of the group of birds upon which the test is made.

TABLE 4

Square root of mean square deviation of predicted annual egg record from actual record. Prediction of annual production from one- and from two-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs, 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM TWO MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
For the whole year	November	38.65	24.52	Nov. + Dec.	36.46	23.13	+2.19	+1.39
	December	37.61	23.86	Nov. + Dec.	36.46	23.13	+1.15	+0.73
	December	37.61	23.86	Dec. + Jan.	35.50	22.53	+2.11	+1.33
	January	38.77	24.60	Dec. + Jan.	35.50	22.53	+3.27	+2.07
	January	38.77	24.60	Jan. + Feb.	34.32	21.78	+4.45	+2.82
	February	34.70	22.02	Jan. + Feb.	34.32	21.78	+0.38	+0.24
	February	34.70	22.02	Feb. + Mar.	31.32	19.87	+3.38	+2.15
	March	34.28	21.75	Feb. + Mar.	31.32	19.87	+2.96	+1.88
	March	34.28	21.75	Mar. + Apr.	32.89	20.87	+1.39	+0.88
	April	35.31	22.40	Mar. + Apr.	32.89	20.87	+2.42	+1.53
	April	35.31	22.40	Apr. + May	32.76	20.79	+2.55	+1.61
	May	35.89	22.77	Apr. + May	32.76	20.79	+3.13	+1.98
	May	35.89	22.77	May + June	32.53	20.64	+3.36	+2.13
	June	36.53	23.18	May + June	32.53	20.64	+4.00	+2.54
	June	36.53	23.18	June + July	33.00	20.94	+3.53	+2.24
	July	35.89	22.77	June + July	33.00	20.94	+2.89	+1.83
	July	35.89	22.77	July + Aug.	31.83	20.20	+4.06	+2.57
	August	34.34	21.79	July + Aug.	31.83	20.20	+2.51	+1.59
	August	34.34	21.79	Aug. + Sept.	30.39	19.28	+3.95	+2.51
	September	32.94	20.90	Aug. + Sept.	30.39	19.28	+2.55	+1.62
	September	32.94	20.90	Sept. + Oct.	32.74	20.77	+0.20	+0.13
	October	36.47	23.14	Sept. + Oct.	32.74	20.77	+3.73	+2.37

Remembering that the average annual production of the 415 test birds is 157.573 eggs, we use this as a base to determine the percentage errors for the equations for each month. These are given in columns with the caption "percentage deviation" in the tables.

We note that in predicting from December, February and August record the average error with regard to sign *is less than one percent of the average annual yield of the flock.* In predicting from November, January and March the error lies between one and two percent. When April, May, June, July, September and October records are used as a basis the average errors of prediction are about 2.50 to 4.50 percent of the average annual yield.

The average deviations without regard to sign are less than 20 percent of the annual production. The values for the individual months range from 15.7 for September to 19.1 for January.

The square root of mean square deviations are less than 25 percent of the average annual production. The individual values range from 20.9 for September to 24.6 for January.

These two latter tests may at first seem to indicate very unsatisfactory prediction. Such, however, is not the case. These give the average errors *either above or below the true record made in the prediction of the results for an individual bird.* The thing which is required in practice is generally *the prediction for a group of birds of a particular class.* In a flock of 415 birds this has been shown above to be possible *with an error of less than 5 percent of the actual production for any month of the year and less than one percent for a number of the months.*

The closeness of prediction may be made clear by a set of diagrams.

In these the estimated production is shown by the straight line. The actual average production for the year or for the group of remaining months for which prediction is made is shown by solid dots for each group of birds as classified by monthly record. The shaded areas are determined as follows. The birds were first grouped into classes of five-eggs range with respect to number of eggs laid during the period of time used as a basis of prediction. The birds of these classes of five-eggs range were further subdivided into those in which actual egg production was greater than the predicted and those in which the actual number was less than the predicted number.[4] The average error of prediction was determined for each of these groups, and these averages represent the upper and the lower limits of the shaded areas. The upper limit represents, therefore, the average deviation (for the period for which prediction is made) of all birds which make a higher record than that predicted for their class. The lower limit

[4] A range of five eggs was used in order to obtain a number of birds sufficiently large to reduce somewhat the irregularities due to the errors of random sampling. The errors of prediction were in each case determined for classes of unit range. Grouping is used for graphic representation merely. The average deviations represented by the limits of the shaded zone are to be thought of as measured from a line perpendicular to the ordinates and intersecting the prediction line on the mid-ordinate of the 5-egg class.

of the shaded area marks the average deviation for all birds which show an egg record lower than that predicted.

The graphs representing the prediction of annual production from the individual-months production appear in diagram 1 for the first six months of the year and in diagram 2 for the last half of the year.

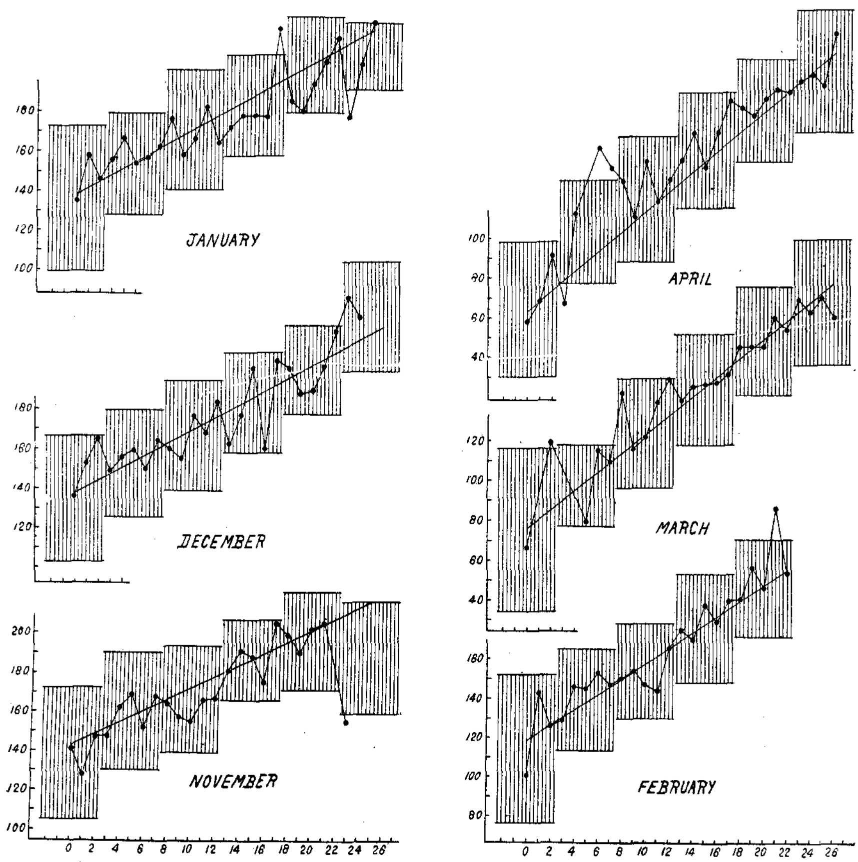

DIAGRAM 1.—Tests of prediction of annual production from single-month records. Montl. of November to April. For explanation see text.

Notwithstanding the irregularities which are inevitable in graphs based on such a highly variable character as annual egg production in a flock of only 415 birds, the most critical reader must admit that the prediction is excellent.

Prediction of the production of a group of remaining months from the record of any month

As noted above (pages 266–268) the worker may desire to predict either the total egg production for the year or the egg production for a group of subsequent months of the year.

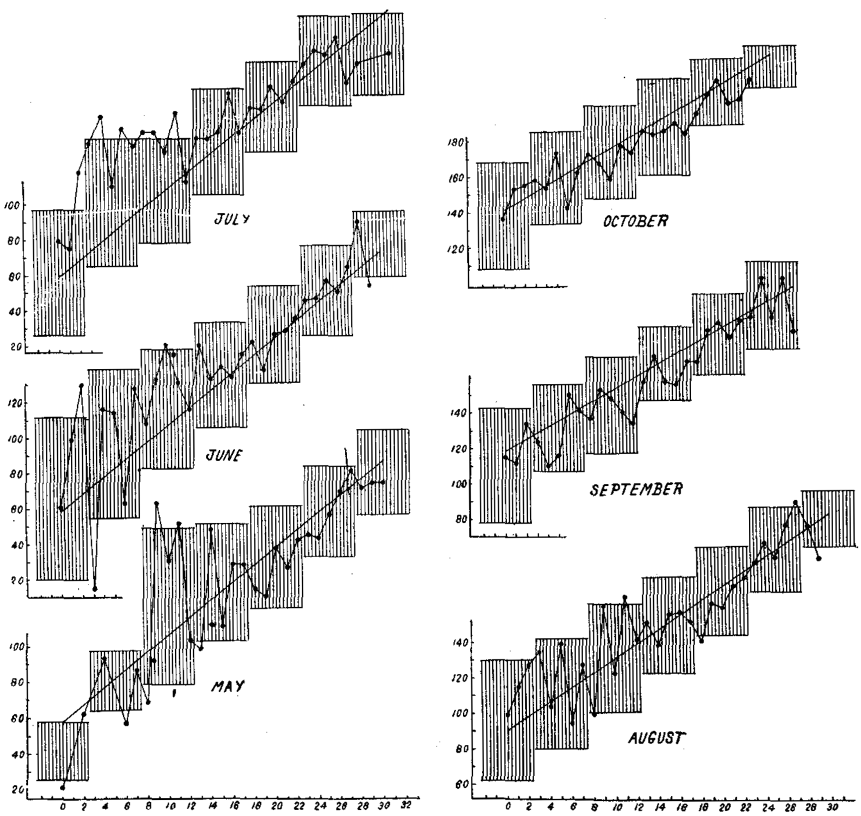

DIAGRAM 2.—Tests of prediction of annual production from single-month records. Tests for May to October.

In general the requirement will probably be the prediction of the total egg production of the remaining months of the year. Since, however, it is necessary to deal with other groups later, the errors of prediction of (a) the total egg production of the months of the year subsequent to the pth month, where p is the base of prediction and of (b) the months of the year subsequent to the $(p + 1)$th month will be considered in this place.[5]

The equations required are as follows:

Month from which prediction is made	*Period for which prediction is made*	*Prediction equation*
November	December to October	$E_{11} = +143.186 + 1.914\, e_1$
November	January to October	$E_{10} = +139.262 + 1.403\, e_1$
December	January to October	$E_{10} = +134.491 + 1.835\, e_2$
December	February to October	$E_9 = +131.461 + 1.373\, e_2$
January	February to October	$E_9 = +130.997 + 1.564\, e_3$
January	March to October	$E_8 = +123.011 + 1.215\, e_3$
February	March to October	$E_8 = +109.824 + 2.035\, e_4$
February	April to October	$E_7 = +96.619 + 1.614\, e_4$
March	April to October	$E_7 = +69.966 + 2.471\, e_5$
March	May to October	$E_6 = +60.338 + 1.932\, e_5$
April	May to October	$E_6 = +46.490 + 2.523\, e_6$
April	June to October	$E_5 = +39.849 + 1.786\, e_6$
May	June to October	$E_5 = +27.639 + 2.233\, e_7$
May	July to October	$E_4 = +20.623 + 1.581\, e_7$
June	July to October	$E_4 = +13.895 + 1.920\, e_8$
June	August to October	$E_3 = +8.740 + 1.228\, e_8$
July	August to October	$E_3 = +6.049 + 1.440\, e_9$
July	September to October	$E_2 = +2.323 + 0.746\, e_9$
August	September to October	$E_2 = +0.724 + 0.935\, e_{10}$
August	October	$E_1 = +0.407 + 0.264\, e_{10}$
September	October	$E_1 = -0.726 + 0.480\, e_{11}$

The test of accuracy of prediction of these equations when applied to the 415 White Leghorns of 1917–1918 is given in comparison with the results for the prediction from two-months production (to be discussed later) in tables 5 to 7.

Limiting our attention for the moment to the errors of predicting the production of the months of the year remaining after any given month used as a basis of prediction, we note that in general the average deviations

[5] In the comparison between the egg production of a period of two months and the egg production of a single month as a basis of prediction, it is necessary to base critical comparisons upon the results of predictions of the records of periods subsequent to the two months under consideration. Concretely, if we are to compare November-plus-December record with November record and with December record as bases for the prediction of the annual production, the two-month period will contribute more to the annual record than either of the two months individually considered. Neither will contribute to the January-to-October production. We must, therefore, in testing prediction equations, base the test upon the results secured in predicting January-to-October egg record.

For this purpose we must have equations which show the relation between the egg record of the individual months and the egg record of groups of remaining months. For example, we require for November the January-to-October production; for December, the February-to-October production; for January, the March-to-October production and so on. For convenience merely the equations are given here in comparison with the other one-month equations.

TABLE 5

Average deviation with regard to sign of egg record for a period of months from actual record. Prediction from one- and from two-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM TWO MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
Dec. to Oct.	November	+2.39	1.57	—	—	—	—	—
Jan. to Oct.	November	+1.73	1.19	Nov. + Dec.	+1.16	0.80	+0.57	+0.39
Jan. to Oct.	December	+0.24	0.16	Nov. + Dec.	+1.16	0.80	−0.92	−0.64
Feb. to Oct.	December	+0.98	0.71	Dec. + Jan.	+1.69	1.22	−0.71	−0.51
Feb. to Oct.	January	+2.37	1.71	Dec. + Jan.	+1.69	1.22	+0.68	+0.49
Mar. to Oct.	January	+2.02	1.57	Jan. + Feb.	+1.45	1.12	+0.57	+0.45
Mar. to Oct.	February	+0.09	0.77	Jan. + Feb.	+1.45	1.12	−1.36	−0.35
Apr. to Oct.	February	+0.56	0.50	Feb. + Mar.	−0.05	0.04	+0.51	+0.46
Apr. to Oct.	March	−0.24	0.21	Feb. + Mar.	−0.05	0.04	+0.19	+0.17
May to Oct.	March	−1.51	1.60	Mar. + Apr.	−3.09	3.27	−1.58	−1.67
May to Oct.	April	−4.00	4.23	Mar. + Apr.	−3.09	3.27	+0.91	+0.96
June to Oct.	April	−1.70	2.33	Apr. + May	+0.64	0.88	+1.06	+1.45
June to Oct.	May	+3.62	4.98	Apr. + May	+0.64	0.88	+2.98	+4.10
July to Oct.	May	+1.55	2.90	May + June	−0.45	0.84	+1.10	+2.06
July to Oct.	June	−2.86	5.34	May + June	−0.45	0.84	+2.41	+4.50
Aug. to Oct.	June	−3.32	9.33	June + July	−3.73	10.48	−0.41	−1.15
Aug. to Oct.	July	−3.71	10.43	June + July	−3.73	10.48	−0.02	−0.05
Sept. to Oct.	July	−3.19	16.89	July + Aug.	−3.32	17.60	−0.13	−0.71
Sept. to Oct.	August	−2.56	13.57	July + Aug.	−3.32	17.60	−0.76	−4.03
October	August	−1.10	18.74	Aug. + Sept.	−0.79	13.46	+0.31	+5.28
October	September	−0.45	7.67	Aug. + Sept.	−0.79	13.46	−0.34	−5.79

TABLE 6

Average deviation without regard to sign of predicted egg record for a period of months from actual record. Prediction from one- and from two-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs, 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM TWO MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
Dec. to Oct.	November	29.59	19.49	—	—	—	—	—
Jan. to Oct.	November	28.47	19.56	Nov. + Dec.	28.09	19.30	+0.38	+0.26
Jan. to Oct.	December	28.43	19.53	Nov. + Dec.	28.09	19.30	+0.34	+0.23
Feb. to Oct.	December	26.92	19.38	Dec. + Jan.	26.25	18.90	+0.67	+0.48
Feb. to Oct.	January	26.48	19.06	Dec. + Jan.	26.25	18.90	+0.23	+0.16
Mar. to Oct.	January	25.03	19.40	Jan. + Feb.	24.98	19.37	+0.05	+0.03
Mar. to Oct.	February	24.07	18.66	Jan. + Feb.	24.98	19.37	−0.91	−0.71
Apr. to Oct.	February	22.93	20.46	Feb. + Mar.	22.39	19.98	+0.54	+0.48
Apr. to Oct.	March	22.81	20.36	Feb. + Mar.	22.39	19.98	+0.42	+0.38
May to Oct.	March	21.78	23.03	Mar. + Apr.	21.31	22.54	+0.47	+0.49
May to Oct.	April	21.36	22.59	Mar. + Apr.	21.31	22.54	+0.05	+0.05
June to Oct.	April	20.22	27.80	Apr. + May	19.65	27.02	+0.57	+0.78
June to Oct.	May	19.89	27.35	Apr. + May	19.65	27.02	+0.24	+0.33
July to Oct.	May	18.09	33.79	May + June	17.30	32.31	+0.79	+1.48
July to Oct.	June	17.62	32.91	May + June	17.30	32.31	+0.32	+0.60
Aug. to Oct.	June	14.88	41.82	June + July	14.05	39.49	+0.83	+2.33
Aug. to Oct.	July	13.91	39.09	June + July	14.05	39.49	−0.14	−0.40
Sept. to Oct.	July	10.92	57.90	July + Aug.	10.36	54.93	+0.56	+2.97
Sept. to Oct.	August	9.76	51.75	July + Aug.	10.36	54.93	−0.60	−3.18
October	August	5.67	96.59	Aug. + Sept.	5.13	87.39	+0.54	+9.20
October	September	4.57	77.85	Aug. + Sept.	5.13	87.39	−0.56	−9.54

TABLE 7

Square root of mean square deviation of predicted egg record for a period of months from actual record. Prediction from one- and from two-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs, 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM TWO MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
Dec. to Oct.	November	38.65	25.46	—	—	—	—	—
Jan. to Oct.	November	37.05	25.45	Nov. + Dec.	36.46	25.05	+0.59	+0.40
Jan. to Oct.	December	36.63	25.16	Nov. + Dec.	36.46	25.05	+0.17	+0.11
Feb. to Oct.	December	34.94	25.15	Dec. + Jan.	34.15	24.58	+0.79	+0.57
Feb. to Oct.	January	34.61	24.91	Dec. + Jan.	34.15	24.58	+0.46	+0.33
Mar. to Oct.	January	32.52	25.22	Jan. + Feb.	32.28	25.03	+0.24	+0.19
Mar. to Oct.	February	30.86	23.92	Jan. + Feb.	32.28	25.03	−1.42	−1.11
Apr. to Oct.	February	29.17	26.03	Feb. + Mar.	28.12	25.09	+1.05	+0.94
Apr. to Oct.	March	28.40	25.34	Feb. + Mar.	28.12	25.09	+0.28	+0.25
May to Oct.	March	27.09	28.65	Mar. + Apr.	26.51	28.03	+0.58	+0.62
May to Oct.	April	26.50	28.02	Mar. + Apr.	26.51	28.03	−0.01	−0.01
June to Oct.	April	25.02	34.40	Apr. + May	24.48	33.66	+0.54	+0.74
June to Oct.	May	24.81	34.11	Apr. + May	24.48	33.66	+0.33	+0.45
July to Oct.	May	22.34	41.72	May + June	21.24	39.67	+1.10	+2.05
July to Oct.	June	21.44	40.04	May + June	21.24	39.67	+0.20	+0.37
Aug. to Oct.	June	18.18	51.09	June + July	17.40	48.90	+0.78	+2.19
Aug. to Oct.	July	17.14	48.17	June + July	17.40	48.90	−0.26	−0.73
Sept. to Oct.	July	13.21	70.02	July + Aug.	12.67	67.18	+0.54	+2.84
Sept. to Oct.	August	12.12	64.26	July + Aug.	12.67	67.18	−0.55	−2.92
October	August	6.92	117.84	Aug. + Sept.	6.24	106.30	+0.68	+11.54
October	September	5.71	97.27	Aug. + Sept.	6.24	106.30	−0.53	−9.03

with regard to sign are small. No one of the errors is over 4 eggs. The percentage values, in which the actual average yields of the remaining months in question are used as bases, range from 0.2 for the prediction of January-to-October production from December production to 13.6 percent in the case of the prediction of September-to-October production from the August record. The average deviations without regard to sign range from 4.6 to 29.6 eggs. The percentage values range from 18.7 to 77.9 percent of the actual production for the given remaining period.

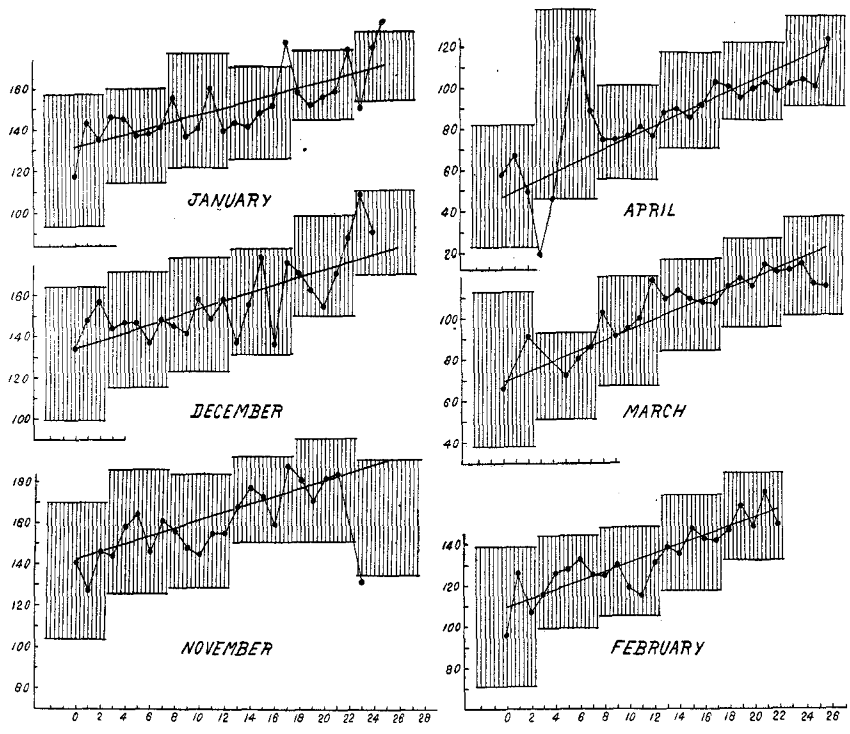

DIAGRAM 3.—Tests of prediction of production for a group of remaining months from single-month records. Tests for November to April. For explanation see text.

The square root of mean square deviations vary from 5.7 to 38.7 eggs, or from 23.9 to 97.3 percent of the actual yield.

The values of the average deviation without regard to sign and of square root of mean square deviation decrease from the earlier to the later months. This is, of course, due to the fact that in predicting the egg record of the remaining months of the year the total record decreases as the number of remaining months becomes smaller. It is to be expected

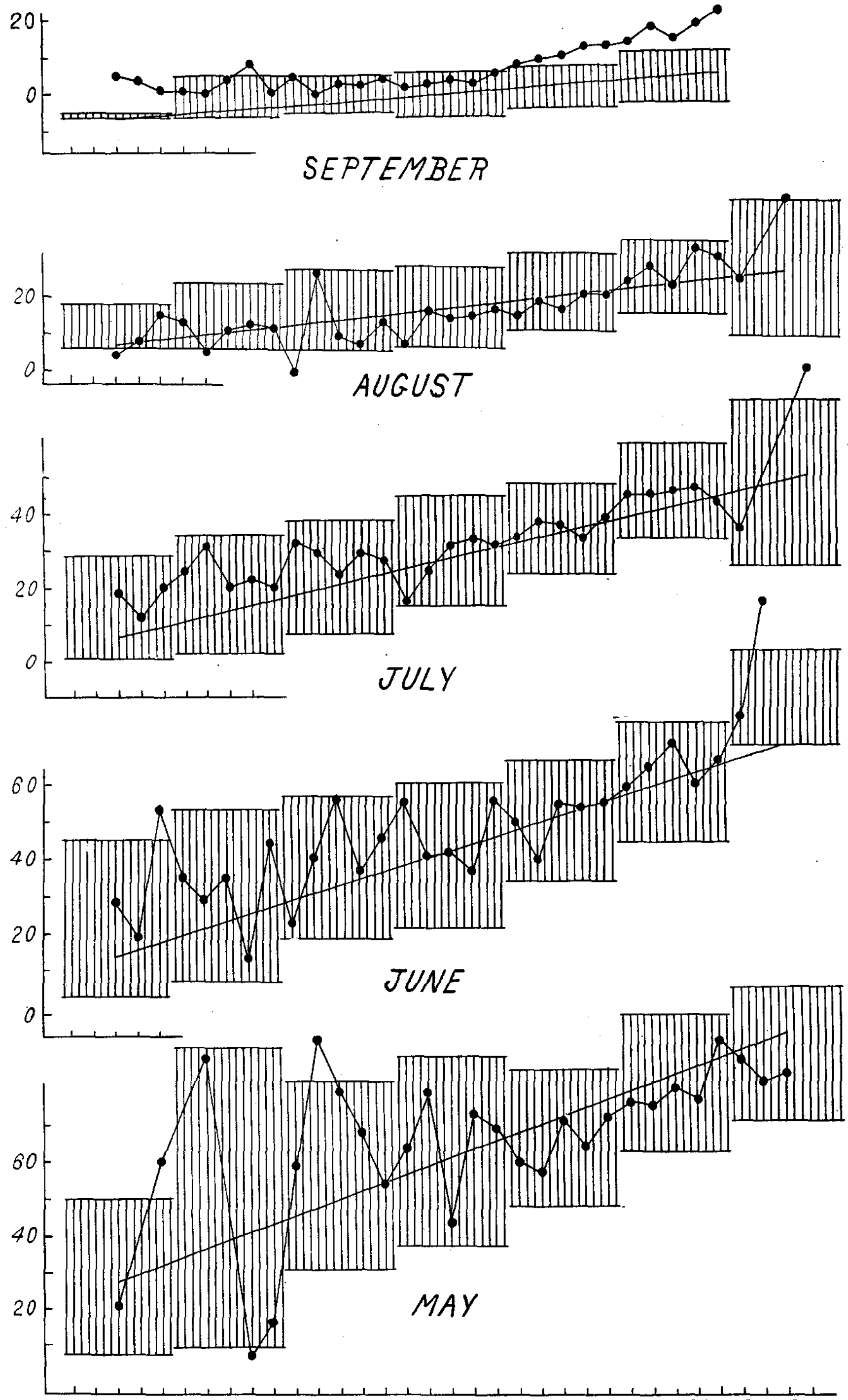

DIAGRAM 4.—Tests of prediction of the production of a group of remaining months from single-month production. Tests for May to September.

therefore, that the absolute error of prediction will be smaller than when the prediction is made for a longer period. The relative errors of prediction are conspicuously larger than those found when the prediction is made for the year as a whole. Furthermore, these relative (percentage) errors increase as the period for which prediction is made becomes shorter. The test shows clearly that prediction of the results of short remaining periods cannot be made,—at least by means of the linear equations for prediction from one month's record tested in this paper,—with a satisfactory degree of accuracy.

When prediction is made for the period subsequent to the $(p + 1)$th month the average deviations with regard to sign vary from 0.56 to 3.32 or from 0.50 to 18.74 percent of the actual production for the period. The average deviations without regard to sign vary from 28.47 eggs for the prediction of January-to-October production from November production to 5.67 eggs for the prediction of October production from August record. The percentage values range from 19.38 to 96.59 percent. Similar results are found in the case of the square root of mean square deviation which ranges from 37.05 eggs for the prediction of January-to-October production from November record to 6.92 eggs for the prediction of October production from August record. The percentage values range from 25.2 to 117.8 percent of the actual records.

The graphic representation of the errors of the prediction of the remaining months of the year is made in diagrams 3 and 4.

The slope of the lines and the moderate narrowness of the shaded areas as well as the fair agreement of the empirical and the predicted means for the remaining periods, evidence for fairly satisfactory prediction for the first six months of the year. As the end of the year is approached and as the period of remaining months becomes shorter the slopes of the lines are more moderate. The narrowness of the shaded areas, representing the difference between the averages of the errors of over-prediction and under-prediction, does not indicate great accuracy of prediction as compared with that attainable in the earlier months, but merely that (because of the smaller egg record made by birds in the latter months of the year) great deviations from prediction are improbable. It is evident, therefore, that for the prediction of the record of the later months of the year from the record of immediately preceding months the equations have relatively little value.

It is quite clear that while the prediction of a group of remaining months may be made with a relatively high degree of accuracy early in the year, the predictions are relatively poor toward the end of the year.

Prediction of annual production from the sum of two monthly records

Before considering the results of equations for the prediction of annual production from the combined record of two or more months, some general questions of theory must be considered.

If the egg production of each individual month be correlated with that of the whole year it would seem that a better prediction of the annual total may be made from the record of two or more individual monthly records than from one month's record only. This is a point emphasized by CARD (1917) who has correlated the total production of groups of months with the annual yield.

There are several points to be taken into consideration here. First, it should be clear that the superiority of a group of months for predicting the annual yield of a bird is to a considerable extent due to the fact that the records of these months are included in the annual total. Thus in predicting annual total from November performance, the November record is included in the annual total. In predicting from November, December and January production the records of these *three* months are included in the annual total. As far as their own contribution is concerned, prediction can be made with absolute certainty. The importance of this factor would be especially great during the spring months when the number of eggs laid by practically all birds is high. If the principle of an increase in the number of months upon which prediction is to be based be extended to its limit, it is clear that the annual total can be predicted with exactness from the record of twelve months. The importance of this factor was fully recognized in our second publication (HARRIS, BLAKESLEE and KIRKPATRICK 1918), in which we determined the correlation between the production of each individual month and that of the remaining eleven months of the year, as well as that between the production of the individual months and the annual record.

It is evident that it is impossible to compare directly and critically the errors made in predicting annual egg production from two-month periods and from single-month periods; in one case a single component only is included in the first and second variable of the pair whereas in the second case two components are involved. The problem of a direct comparison will be taken up in a subsequent section.

Second, from the economic standpoint it is clear that trap-nesting for two months or three months is (disregarding initial investment) twice or three times as expensive as trap-nesting for one month. In general it is important to utilize the shortest practicable period on which prediction may be based.

Third, the mathematical theory of multiple correlation shows that in dealing with correlated characters the gain in accuracy of prediction rapidly decreases with the number of characters employed. In our first detailed treatment of the problem of the correlation between the egg records of the individual months we showed by the constants for a series of selected months that the egg records of the individual months are correlated among themselves. This has since been demonstrated for the entire series of $\frac{1}{2}\ n(n - 1) = 66$ different combinations of the 12 months of the pullet year. It is evident, therefore, that very large gains in accuracy of prediction cannot be expected to result from an increase in the number of periods, except in so far as the gain is due directly to the contribution of the months included.

We now turn to the results of the test of equations for the prediction of annual record from two-consecutive-months production. The equations are as follows:

Months from which prediction is made	*Prediction equation*
November and December	$E = +132.887 + 2.160\ (e_1 + e_2)$
December and January	$E = +130.822 + 2.176\ (e_2 + e_3)$
January and February	$E = +146.040 + 2.579\ (e_3 + e_4)$
February and March	$E = +78.008 + 2.915\ (e_4 + e_5)$
March and April	$E = +48.374 + 3.029\ (e_5 + e_6)$
April and May	$E = +39.955 + 3.005\ (e_6 + e_7)$
May and June	$E = +32.783 + 3.065\ (e_7 + e_8)$
June and July	$E = +44.650 + 2.864\ (e_8 + e_9)$
July and August	$E = +62.861 + 2.625\ (e_9 + e_{10})$
August and September	$E = +91.865 + 2.302\ (e_{10} + e_{11})$
September and October	$E = +122.597 + 2.140\ (e_{11} + e_{12})$

Since a primary object of the present analyses is a comparison of equations based on two-months production with those based on a single month's record as a means of predicting the annual egg record of a bird, it is advantageous to place the results for the two methods side by side in the same tables. The results are given in tables 2 to 4.[6]

Table 2 shows the average errors with regard to sign of the egg records of the 415 White Leghorns studied at Storrs in 1917–1918, when prediction is made from two-months production using equations based on the Storrs experience of the preceding six years.

The average deviations with regard to sign are small. In 7 cases the equations have predicted values which are too large, whereas in 4 cases they have predicted values which are too small. The individual errors

[6] It has seemed conducive to clearness to duplicate entries in order to secure the 22 differences which serve as a basis of comparison.

are very small. Two are less than 1 egg, 4 are less than 2 eggs, while 5 are from 2 to 7 eggs. The percentage errors based on the mean annual production are less than 1 percent in 5 of the cases and less than 5 percent

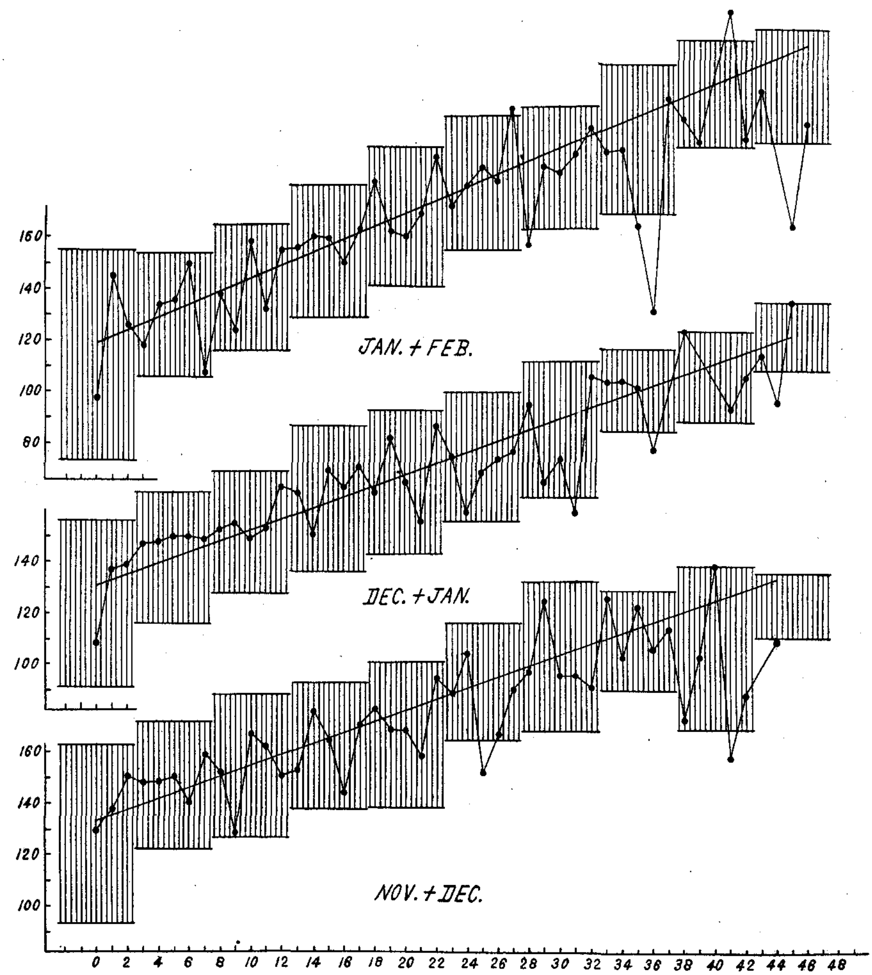

DIAGRAM 5.—Tests of prediction of annual production from combined record of two consecutive months. For explanation see text. Tests for November to February.

in the other 6 cases. The average error in actual number of eggs, disregarding the sign of the error, is 2.72 eggs while the average of percentage errors is 1.72 percent.

It seems unnecessary to discuss in detail the average deviations without regard to sign, of the predicted from the observed annual egg production.

The errors, shown in table 3, range from 23.2 to 28.1 eggs or from 14.7 to 17.8 percent.

Similar results for the square root of mean square deviation are given in table 4 which shows that prediction from the sum of two-consecutive-

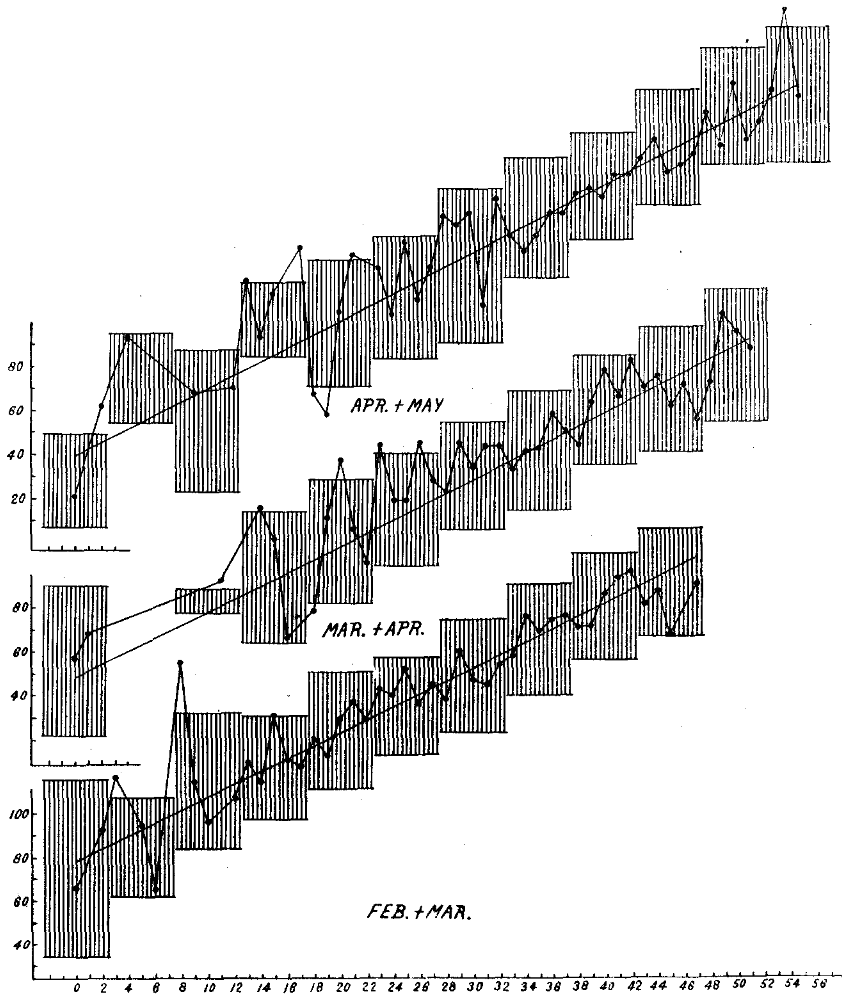

DIAGRAM 6.—Tests of prediction of annual production from the combined record of two consecutive months. Tests of February-to-May production.

months production gives a square root of mean square deviation ranging from 30.4 to 36.5 eggs or from 19.3 to 23.1 percent of the annual production.

Thus it is clear that the annual egg record of a bird may be predicted with a high degree of accuracy from the combined egg record of any two consecutive months.

These results may be represented graphically by diagrams 5–8, which have been prepared on the same principle as those for the results of prediction from a single month's production. The general excellence of the agreement (considering the fact that there are only 415 birds upon which equa-

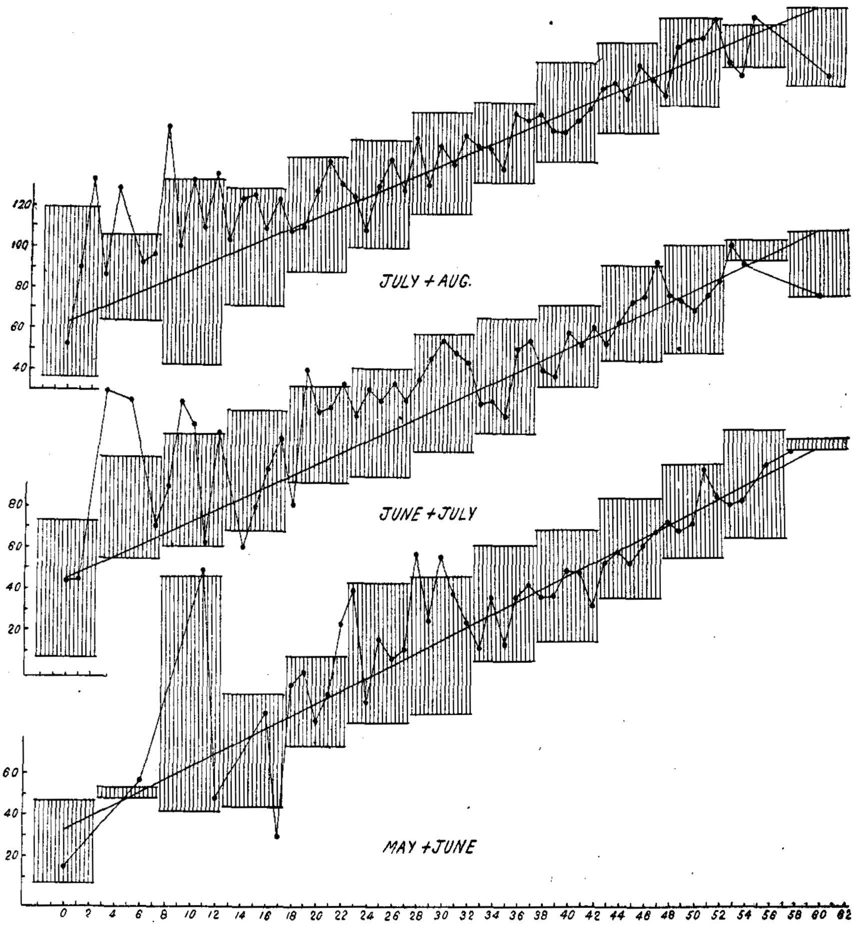

DIAGRAM 7.—Tests of the prediction of annual production from the combined record of two consecutive months. Tests for May to August.

tions based upon an entirely different series are being tested) renders detailed discussion of these diagrams superfluous.

The most interesting feature of these tables is, however, the comparison between the value of two-months observation and of single-month observation as bases for the estimation of the total (annual) egg-producing capacity of the organism.

The differences in the average deviations with regard to sign, as shown in the first of the two final columns of table 2, range from less than a single egg (5 comparisons) to a maximum of less than 7 eggs. The average difference is 2.21 eggs. If signs be considered the average difference is only + 0.67 eggs. The differences in the percentage deviation when prediction is made by single- and by two-month periods are shown in the final column of the table.

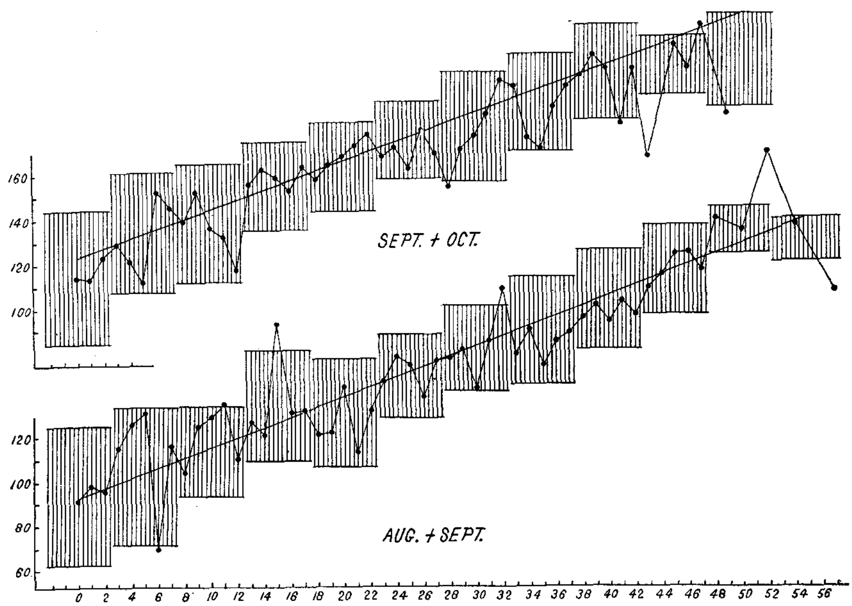

DIAGRAM 8.—Tests of prediction of annual production from the combined record of two consecutive months. Tests of August to October.

It is clear from these results that the results of prediction from two-months production are not materially better from the practical standpoint than those for single-month's production although the labor entailed in recording the performance of a bird for two months must be approximately twice as great as that for a single month.

The reader who cares to do so may verify these statements by a study of the results for average deviation without regard to sign and for square root of mean square deviation as shown in the two final columns of tables 3 and 4.

Prediction of the production of a group of remaining months from the sum of two monthly records

We now have to consider the problem of the accuracy with which the egg production of a group of subsequent months may be predicted from the sum of two consecutive monthly records.

The equations are the following:

Period from which prediction is made	*Period for which prediction is made*	*Prediction equation*
November and December	January to October	$E = +132.887 + 1.160\ (e_1 + e_2)$
December and January	February to October	$E = +128.112 + 0.979\ (e_2 + e_3)$
January and February	March to October	$E = +124.959 + 0.336\ (e_3 + e_4)$
February and March	April to October	$E = +76.542 + 1.320\ (e_4 + e_5)$
March and April	May to October	$E = +44.586 + 1.363\ (e_5 + e_6)$
April and May	June to October	$E = +25.022 + 1.231\ (e_6 + e_7)$
May and June	July to October	$E = +7.280 + 1.118\ (e_7 + e_8)$
June and July	August to October	$E = +0.994 + 0.827\ (e_8 + e_9)$
July and August	September to October	$E = -7.281 + 0.660\ (e_9 + e_{10})$
August and September	October	$E = -2.137 + 0.245\ (e_{10} + e_{11})$

The results appear in the second section of tables 5 to 7. Here they are laid beside the errors obtained for the prediction of the production of these same periods from the record of the two months individually considered, as given by the equations shown on page 282.

Table 5, giving the average deviation with regard to sign of the predicted from the observed values, shows that the actual deviations have a numerical range of 0.05 to 3.73 eggs or from 0.04 to 17.6 percent. The largest relative (percentage) deviations are, of course, in the final months of the year.

The average deviations without regard to sign appear in the second column of table 6. These vary from as low as 5.13 eggs in October to 28.09 eggs for the period January to October. Since the average production decreases as the number of remaining months becomes smaller we find the largest percentage errors in the later groups of months. These percentage values range from 18.9 for the period February to October to 87.4 for the month of October. Similar results with somewhat different numerical values are found in table 7 which shows the square root of mean square deviation of the predicted from the observed values.

These results show that when the number of remaining months is large, prediction of egg production can be made with relatively high accuracy from the combined record of two months. As the number of months becomes smaller the error of prediction is, as compared with the average production, relatively large.

Turning now to the problem of the comparison of periods of one month and of two months as bases of prediction, and testing the efficiency of these two periods on the egg production of comparable remaining periods of time, we note that the differences in the two final columns of tables 5 to 7, expressed either in number of eggs or in percentages of the total production, are small. Thus the differences for the average deviation with regard to sign are all less than 3 eggs and all less than 6 percent. Most of the differences are far smaller than this. In some cases the prediction from a single month gives the better result; in others prediction from two months gives the better result. The differences in the errors without regard to sign as obtained by the two methods are even smaller. No difference amounts to as much as a single egg per year. The large differences in the percentage errors by the two methods are found exclusively in the later months of the year where the total production is low. Comparable, but numerically somewhat different, results are found for the square root of mean square deviation.

Thus it is clear that there is little practical difference between single-month and two-months production as bases of the prediction of the egg record of a subsequent period.

Prediction of annual production from the sum of three monthly records

The equations required for the prediction of annual production from the combined record of three consecutive months are the following:

Months from which prediction is made	*Prediction equation*
November, December and January	$E = +126.742 + 1.770\,(e_1 + e_2 + e_3)$
December, January and February	$E = +113.940 + 1.951\,(e_2 + e_3 + e_4)$
January, February and March	$E = +82.129 + 2.266\,(e_3 + e_4 + e_5)$
February, March and April	$E = +50.502 + 2.323\,(e_4 + e_5 + e_6)$
March, April and May	$E = +29.450 + 2.267\,(e_5 + e_6 + e_7)$
April, May and June	$E = +19.349 + 2.324\,(e_6 + e_7 + e_8)$
May, June and July	$E = +23.786 + 2.233\,(e_7 + e_8 + e_9)$
June, July and August	$E = +41.079 + 2.065\,(e_8 + e_9 + e_{10})$
July, August and September	$E = +67.078 + 1.895\,(e_9 + e_{10} + e_{11})$
August, September and October	$E = +97.699 + 1.794\,(e_{10} + e_{11} + e_{12})$

The second section of table 8 shows the average deviation with regard to sign of the annual egg production predicted from the combined record of 3 consecutive months from the performance of the 415 White Leghorn birds studied at Storrs in 1917–1918.

The results show that the trimonthly totals, like the monthly records and bimonthly totals considered in preceding sections, give excellent predictions. December to February, January to March, March to May, and

July to September give average errors of prediction of less than 1 egg. November to January, April to June, and May to July give errors of prediction of between 2 and 3 eggs. August to October gives an error of prediction of between 3 and 4 eggs. February to April gives an error of prediction of between 4 and 5 eggs. Finally June to August gives an error of prediction of between 5 and 6 eggs.

TABLE 8

Average deviation with regard to sign of predicted annual egg record from actual record. Prediction of annual production from one- and from three-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs, 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM THREE MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
For the whole year	November	+2.39	1.52	Nov.–Jan.	+2.09	1.33	+0.30	+0.19
	December	−0.49	0.31	Nov.–Jan.	+2.09	1.33	−1.60	−1.02
	January	+2.58	1.64	Nov.–Jan.	+2.09	1.33	+0.49	+0.31
	December	−0.49	0.31	Dec.–Feb.	+0.78	0.49	−0.29	−0.18
	January	+2.58	1.64	Dec.–Feb.	+0.78	0.49	+1.80	+1.15
	February	+0.06	0.04	Dec.–Feb.	+0.78	0.49	−0.72	−0.45
	January	+2.58	1.64	Jan.–Mar.	+0.49	0.31	+2.09	+1.33
	February	+0.06	0.04	Jan.–Mar.	+0.49	0.31	−0.43	−0.27
	March	−1.63	1.03	Jan.–Mar.	+0.49	0.31	+1.14	+0.72
	February	+0.06	0.04	Feb.–Apr.	−4.07	2.58	−4.01	−2.54
	March	−1.63	1.03	Feb.–Apr.	−4.07	2.58	−2.44	−1.55
	April	−6.23	3.95	Feb.–Apr.	−4.07	2.58	+2.16	+1.37
	March	−1.63	1.03	Mar.–May	−0.73	0.46	+0.90	+0.57
	April	−6.23	3.95	Mar.–May	−0.73	0.46	+5.50	+3.49
	May	+7.02	4.45	Mar.–May	−0.73	0.46	+6.29	+3.99
	April	−6.23	3.95	Apr.–June	−2.31	1.47	+3.92	+2.48
	May	+7.02	4.45	Apr.–June	−2.31	1.47	+4.71	+2.98
	June	−5.21	3.31	Apr.–June	−2.31	1.47	+2.90	+1.84
	May	+7.02	4.45	May–July	−2.12	1.35	+4.90	+3.10
	June	−5.21	3.31	May–July	−2.12	1.35	+3.09	+1.96
	July	−5.27	3.34	May–July	−2.12	1.35	+3.15	+1.99
	June	−5.21	3.31	June–Aug.	−5.35	3.39	−0.14	−0.08
	July	−5.27	3.34	June–Aug.	−5.35	3.39	−0.08	−0.05
	August	−0.82	0.52	June–Aug.	−5.35	3.39	−4.53	−2.87
	July	−5.27	3.34	July–Sept.	−0.20	0.13	+5.07	+3.21
	August	−0.82	0.52	July–Sept.	−0.20	0.13	+0.62	+0.39
	September	+4.78	3.03	July–Sept.	−0.20	0.13	+4.58	+2.90
	August	−0.82	0.52	Aug.–Oct.	+3.91	2.48	−3.09	−1.96
	September	+4.78	3.03	Aug.–Oct.	+3.91	2.48	+0.87	+0.55
	October	+3.95	2.51	Aug.–Oct.	+3.91	2.48	+0.04	+0.03

Considered in their relation to the average annual production these values range from 0.13 to 3.39 percent. These results certainly show remarkable accuracy of prediction.

The average errors without regard to sign, given in table 9 need not be considered in detail. They range from 21.4 to 25.9 eggs per year or from 13.6 to 16.5 percent of the annual total.

TABLE 9

Average deviation without regard to sign of predicted annual egg record from actual record. Prediction of annual production from one- and from three-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs, 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM THREE MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
For the whole year	November	29.59	18.78	Nov.–Jan.	25.93	16.45	+3.66	+2.33
	December	29.26	18.57	Nov.–Jan.	25.93	16.45	+3.33	+2.12
	January	30.09	19.09	Nov.–Jan.	25.93	16.45	+4.16	+2.64
	December	29.26	18.57	Dec.–Feb.	25.31	16.06	+3.95	+2.51
	January	30.09	19.09	Dec.–Feb.	25.31	16.06	+4.78	+3.03
	February	27.28	17.31	Dec.–Feb.	25.31	16.06	+1.97	+1.25
	January	30.09	19.09	Jan. –Mar.	25.29	16.05	+4.80	+3.04
	February	27.28	17.31	Jan. –Mar.	25.29	16.05	+1.99	+1.26
	March	27.95	17.73	Jan. –Mar.	25.29	16.05	+2.66	+1.68
	February	27.28	17.31	Feb.–Apr.	24.16	15.33	+3.12	+1.98
	March	27.95	17.73	Feb.–Apr.	24.16	15.33	+3.79	+2.40
	April	28.72	18.22	Feb.–Apr.	24.16	15.33	+4.56	+2.89
	March	27.95	17.73	Mar.–May	25.42	16.13	+2.53	+1.60
	April	28.72	18.22	Mar.–May	25.42	16.13	+3.30	+2.09
	May	28.62	18.16	Mar.–May	25.42	16.13	+3.20	+2.03
	April	28.72	18.22	Apr.–June	24.33	15.44	+4.39	+2.78
	May	28.62	18.16	Apr.–June	24.33	15.44	+4.29	+2.72
	June	29.03	18.42	Apr.–June	24.33	15.44	+4.70	+2.98
	May	28.62	18.16	May–July	24.20	15.36	+4.42	+2.80
	June	29.03	18.42	May–July	24.20	15.36	+4.83	+3.06
	July	28.35	17.99	May–July	24.20	15.36	+4.15	+2.63
	June	29.03	18.42	June–Aug.	23.49	14.90	+5.54	+3.52
	July	28.35	17.99	June–Aug.	23.49	14.90	+4.86	+3.09
	August	26.87	17.05	June–Aug.	23.49	14.90	+3.38	+2.15
	July	28.35	17.99	July–Sept.	21.36	13.55	+6.99	+4.44
	August	26.87	17.05	July–Sept.	21.36	13.55	+5.51	+3.50
	September	24.78	15.72	July–Sept.	21.36	13.55	+3.42	+2.17
	August	26.87	17.05	Aug.–Oct.	21.59	13.70	+5.28	+3.35
	September	24.78	15.72	Aug.–Oct.	21.59	13.70	+3.19	+2.02
	October	27.37	17.37	Aug.–Oct.	21.59	13.70	+5.78	+3.67

The square root of mean square deviation of errors of prediction given in table 10 are, of course, larger than the average deviations without regard to sign. They vary from 28.1 to 33.8 eggs or from 17.8 to 21.5 percent of the annual production.

The range of variation in the egg production of three-month periods is so wide that it is impossible because of the limitations of space to represent the errors of prediction from three-month periods graphically for each of the equations.

TABLE 10

Square root of mean square deviation of predicted annual egg record from actual record. Prediction of annual production from one- and from three-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs, 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM THREE MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
For the whole year	November	38.65	24.52	Nov.–Jan.	33.84	21.47	+4.81	+3.05
	December	37.61	23.86	Nov.–Jan.	33.84	21.47	+3.77	+2.39
	January	38.77	24.60	Nov.–Jan.	33.84	21.47	+4.93	+3.13
	December	37.61	23.86	Dec.–Feb.	32.65	20.72	+4.96	+3.14
	January	38.77	24.60	Dec.–Feb.	32.65	20.72	+6.12	+3.88
	February	34.70	22.02	Dec.–Feb.	32.65	20.72	+2.05	+1.30
	January	38.77	24.60	Jan.–Mar.	31.58	20.04	+7.19	+4.56
	February	34.70	22.02	Jan.–Mar.	31.58	20.04	+3.12	+1.98
	March	34.28	21.75	Jan.–Mar.	31.58	20.04	+2.70	+1.71
	February	34.70	22.02	Feb.–Apr.	29.77	18.89	+4.93	+3.13
	March	34.28	21.75	Feb –Apr.	29.77	18.89	+4.51	+2.86
	April	35.31	22.40	Feb.–Apr.	29.77	18.89	+5.54	+3.51
	March	34.28	21.75	Mar.–May	31.14	19.76	+3.14	+1.99
	April	35.31	22.40	Mar.–May	31.14	19.76	+4.17	+2.64
	May	35.89	22.77	Mar.–May	31.14	19.76	+4.75	+3.01
	April	35.31	22.40	Apr.–June	30.59	19.41	+4.72	+2.99
	May	35.89	22.77	Apr.–June	30.59	19.41	+5.30	+3.36
	June	36.53	23.18	Apr.–June	30.59	19.41	+5.94	+3.77
	May	35.89	22.77	May–July	29.40	18.65	+6.49	+4.12
	June	36.53	23.18	May–July	29.40	18.65	+7.13	+4.53
	July	35.89	22.77	May–July	29.40	18.65	+6.49	+4.12
	June	36.53	23.18	June–Aug.	29.80	18.91	+6.73	+4.27
	July	35.89	22.77	June–Aug.	29.80	18.91	+6.09	+3.86
	August	34.34	21.79	June–Aug.	29.80	18.91	+4.54	+2.88
	July	35.89	22.77	July–Sept.	28.10	17.83	+7.79	+4.94
	August	34.34	21.79	July–Sept.	28.10	17.83	+6.24	+3.96
	September	32.94	20.90	July Sept.	28.10	17.83	+4.84	+3.07
	August	34.34	21.79	Aug.–Oct.	29.23	18.55	+5.11	+3.24
	September	32.94	20.90	Aug.–Oct.	29.23	18.55	+3.71	+2.35
	October	36.47	23.14	Aug.–Oct.	29.23	18.55	+7.24	+4.59

Two series, that for November to January and for March to May, have been selected at random to represent the goodness of fit of prediction in these cases. The results for prediction from November to January record are shown in diagram 9. Those for prediction from March to May pro-

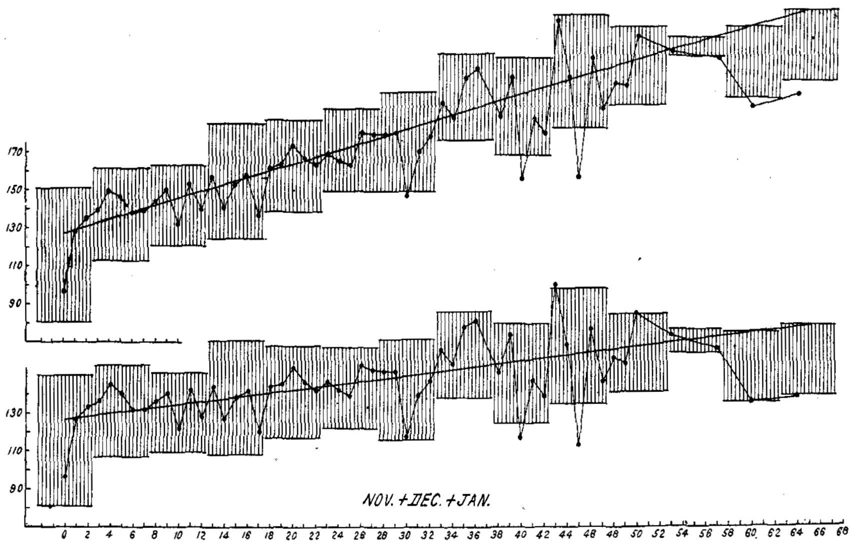

DIAGRAM 9.—Tests of the prediction of annual production (upper figure) and of the production of a group of remaining months (lower figure) from the combined record of three consecutive months. Tests for the period November to January.

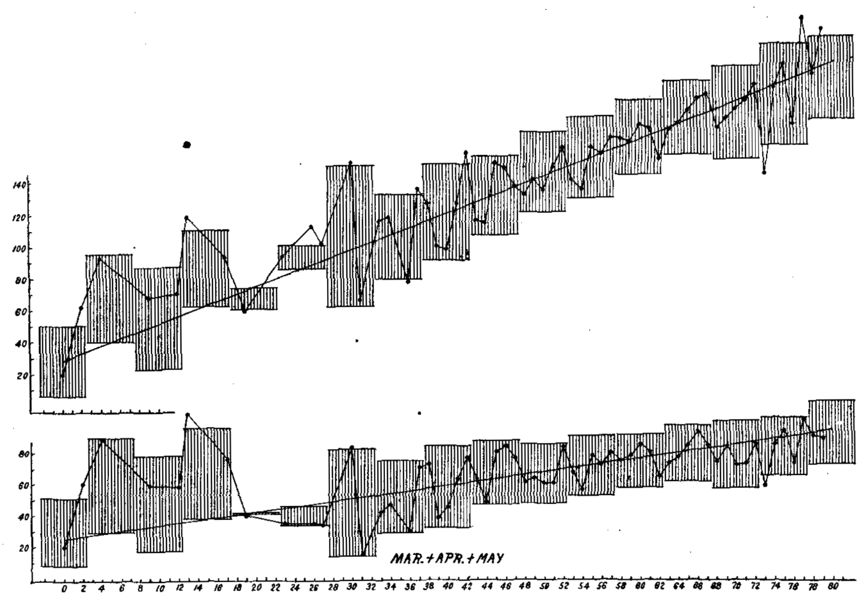

DIAGRAM 10.—Tests of the prediction of annual production (upper figure) and of the production of a group of remaining months (lower figure) from the combined record of three consecutive months. Tests for the period March to May.

duction are shown in diagram 10. In both cases the upper figure represents the prediction of annual production. The lower figure shows the prediction of the groups of remaining months and will be discussed in a subsequent section.

After the discussion of the preceding diagrams these graphs are self-explanatory.

When these results are compared, as in the last two columns of the tables, with those for prediction from a single one of the three months the differences are surprisingly small. For example the most important test,—that of the average deviation with regard to sign,—shows that 11 of the 30 differences are less than 1 egg per year; 3 are less than 2 eggs per year; while 16 are 2 eggs or more per year. In no case is the difference as much as 7 eggs per year. The difference in percentage deviation is in no case as large as 4 percent.

Turning to the comparison of average deviation without regard to sign when prediction is made from trimonthly periods and from the records of individual months we note that the differences are without exception positive in sign. Thus they show a greater error when prediction is made from a single monthly record. The differences are, however, always less than 7 eggs per year and are generally less than 5 eggs. The percentage differences vary from 1.3 to 4.4 percent when both percentages are based on the annual total.

Similar results are obtained for the square root of mean square deviation. The deviations are larger throughout when prediction is made from single-months records than when made from three-months records. The differences are not, however, large. They range from 2.05 to 7.79 eggs, or from 1.30 to 4.94 percent of the annual average production.

Thus while practically without exception a closer prediction of the annual egg record of individual birds can be made from three-months production the difference between a three-month period and a single-month period is by no means so large as one unacquainted with statistical theory might have assumed.

Prediction of the production of a subsequent period from the sum of three monthly records

The equations required are the following:

Months from which prediction is made	*Period for which prediction is made*	*Prediction equation*
November, Dec. and Jan.	February to October	$E_9 = +126.742 + 0.770\ (e_1 + e_2 + e_3)$
Dec., Jan. and Feb.	March to October	$E_8 = +112.051 + 0.806\ (e_2 + e_3 + e_4)$
Jan., Feb. and March	April to October	$E_7 = +81.464 + 0.935\ (e_3 + e_4 + e_5)$
February, March and April	May to October	$E_6 = +\ 49.753 + 0.955\ (e_4 + e_5 + e_6)$
March, April and May	June to October	$E_5 = +25.210 + 0.850\ (e_5 + e_6 + e_7)$
April, May and June	July to October	$E_4 = +7.063 + 0.770\ (e_6 + e_7 + e_8)$
May, June and July	August to October	$E_3 = -1.975 + 0.594\ (e_7 + e_8 + e_9)$
June, July and August	September to October	$E_2 = -4.701 + 0.377\ (e_8 + e_9 + e_{10})$
July, August and September	October	$E_1 = -3.343 + 0.172\ (e_9 + e_{10} + e_{11})$

Table 11 contains the average deviations with regard to sign, of the predicted yield of remaining months, from the actual productions, when prediction is made from the total yield of three consecutive months.

The deviations range from 0.20 to 3.30 eggs or from 0.27 to 18.22 percent of the actually observed yield. As far as this criterion shows, predictions are excellent for all periods from that including February to October to that for August to October. The September-to-October record and the October record, however, cannot be predicted with a high degree of accuracy, the errors being over 17 percent of the mean value for these months.

The average deviations without regard to sign, shown in table 12, range from 5.24 to 25.93 eggs, the values decreasing as the length of the period for which prediction is made becomes smaller. The reverse is true of the percentage values which increase from 18.66 percent for the period February to October to 89.23 percent of the actual yield for the month of October.

Similar results are obtained when the formulae are judged by the square root of mean square deviation of the predicted from the actually observed egg record as shown in table 13. These root mean square deviations range from 33.84 for February to October to 6.44 for the month of October alone, or from 24.30 percent for the group of 8 remaining months of the year to 109.67 percent for the last (single) month.

The results for the prediction of two of the groups of remaining months from the combined records of three-months production are represented graphically for the three months November to January in diagram 9 and for the three months March to May in diagram 10. It is the lower figure which is to be consulted in each case.

The gentle slope of the lines and the considerable irregularities of the means show that prediction of the record of a period of remaining months

TABLE 11

Average deviation with regard to sign of predicted egg record for a period of months from actual record. Prediction from one- and from three-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs, 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM THREE MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
Feb. to Oct.	November	+2.16	1.55	Nov.–Jan.	+2.09	1.50	+0.07	+0.05
Feb. to Oct.	December	+0.99	0.71	Nov.–Jan.	+2.09	1.50	−1.10	−0.79
Feb. to Oct.	January	+2.37	1.71	Nov.–Jan.	+2.09	1.50	+0.28	+0.21
Mar. to Oct.	December	+0.91	0.71	Dec.–Feb.	+1.35	1.05	−0.44	−0.34
Mar. to Oct.	January	+2.02	1.57	Dec.–Feb.	+1.35	1.05	+0.67	+0.52
Mar. to Oct.	February	+0.99	0.77	Dec.–Feb.	+1.35	1.05	−0.36	−0.28
Apr. to Oct.	January	+1.40	1.25	Jan.–Mar.	+0.73	0.65	+0.67	+0.60
Apr. to Oct.	February	+0.56	0.50	Jan.–Mar.	+0.73	0.65	−0.17	−0.15
Apr. to Oct.	March	−0.24	0.21	Jan.–Mar.	+0.73	0.65	−0.49	−0.44
May to Oct.	February	−0.77	0.81	Feb.–Apr.	−2.51	2.65	−1.74	−1.84
May to Oct.	March	−1.51	1.60	Feb.–Apr.	−2.51	2.65	−1.00	−1.05
May to Oct.	April	−4.00	4.23	Feb.–Apr.	−2.51	2.65	+1.49	+1.58
June to Oct.	March	−0.01	0.01	Mar.–May	+0.20	0.27	−0.19	−0.26
June to Oct.	April	−1.70	2.33	Mar.–May	+0.20	0.27	+1.50	+2.06
June to Oct.	May	+3.62	4.98	Mar.–May	+0.20	0.27	+3.42	+4.71
July to Oct.	April	−2.16	4.03	Apr.–June	−1.49	2.78	+0.67	+1.25
July to Oct.	May	+1.55	2.90	Apr.–June	−1.49	2.78	+0.06	+0.12
July to Oct.	June	−2.86	5.34	Apr.–June	−1.49	2.78	+1.37	+2.56
Aug. to Oct.	May	−0.51	1.43	May–July	−2.56	7.20	−2.05	−5.77
Aug. to Oct.	June	−3.32	9.33	May–July	−2.56	7.20	+0.76	+2.13
Aug. to Oct.	July	−3.71	10.43	May–July	−2.56	7.20	+1.15	+3.23
Sept. to Oct.	June	−3.00	15.91	June–Aug.	−3.30	17.50	−0.30	−1.59
Sept. to Oct.	July	−3.19	16.89	June–Aug.	−3.30	17.50	−0.11	−0.61
Sept. to Oct.	August	−2.56	13.57	June–Aug.	−3.30	17.50	−0.74	−3.93
October	July	−1.33	22.64	July–Sept.	−1.07	18.22	+0.26	+4.42
October	August	−1.10	18.74	July–Sept.	−1.07	18.22	+0.03	+0.52
October	September	−0.45	7.67	July–Sept.	−1.07	18.22	−0.62	−10.55

TABLE 12

Average deviation without regard to sign of predicted egg record for a period of months from actual record. Prediction from one- and from three-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs, 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM THREE MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
Feb. to Oct.	November	26.67	19.20	Nov.–Jan.	25.93	18.66	+0.74	+0.54
Feb. to Oct.	December	26.92	19.38	Nov.–Jan.	25.93	18.66	+0.99	+0.72
Feb. to Oct.	January	26.48	19.06	Nov.–Jan.	25.93	18.66	+0.55	+0.40
Mar. to Oct.	December	25.33	19.61	Dec.–Feb.	24.19	18.75	+1.14	+0.86
Mar. to Oct.	January	25.03	19.40	Dec.–Feb.	24.19	18.75	+0.84	+0.65
Mar. to Oct.	February	24.07	18.66	Dec.–Feb.	24.19	18.75	−0.12	−0.09
Apr. to Oct.	January	23.68	21.13	Jan.–Mar.	22.63	20.20	+1.05	+0.93
Apr. to Oct.	February	22.93	20.46	Jan.–Mar.	22.63	20.20	+0.30	+0.26
Apr. to Oct.	March	22.81	20.36	Jan.–Mar.	22.63	20.20	+0.18	+0.16
May to Oct.	February	21.73	22.98	Feb.–Apr.	21.03	22.24	+0.70	+0.74
May to Oct.	March	21.78	23.03	Feb.–Apr.	21.03	22.24	+0.75	+0.79
May to Oct.	April	21.36	22.59	Feb.–Apr.	21.03	22.24	+0.33	+0.35
June to Oct.	March	20.57	28.28	Mar.–May	19.75	27.16	+0.82	+1.12
June to Oct.	April	20.22	27.80	Mar.–May	19.75	27.16	+0.47	+0.64
June to Oct.	May	19.89	27.35	Mar.–May	19.75	27.16	+0.14	+0.19
July to Oct.	April	18.49	34.54	Apr.–June	17.44	32.58	+1.05	+1.96
July to Oct.	May	18.09	33.79	Apr.–June	17.44	32.58	+0.65	+1.21
July to Oct.	June	17.62	32.91	Apr.–June	17.44	32.58	+0.18	+0.33
Aug. to Oct.	May	15.03	42.25	May–July	14.03	39.44	+1.00	+2.81
Aug. to Oct.	June	14.88	41.83	May–July	14.03	39.44	+0.85	+2.39
Aug. to Oct.	July	13.91	39.09	May–July	14.03	39.44	−0.12	−0.35
Sept. to Oct.	June	11.27	59.76	June–Aug.	10.26	54.40	+1.01	+5.36
Sept. to Oct.	July	10.92	57.90	June–Aug.	10.26	54.40	+0.66	+3.50
Sept. to Oct.	August	9.76	51.75	June–Aug.	10.26	54.40	−0.50	−2.65
October	July	5.76	98.09	July–Sept.	5.24	89.23	+0.52	+8.86
October	August	5.67	96.59	July–Sept.	5.24	89.23	+0.43	+7.36
October	September	4.57	77.85	July–Sept.	5.24	89.23	−0.67	−11.38

TABLE 13

Square root of mean square deviation of predicted egg record for a period of months from actual record. Prediction from one- and from three-months performance. Equations based on Storrs experience, 1911 to 1917. Test of equations on 415 White Leghorns, Storrs, 1917–1918.

PERIOD FOR WHICH PREDICTION IS MADE	PREDICTION FROM ONE MONTH			PREDICTION FROM THREE MONTHS			DIFFERENCE IN ACTUAL DEVIATION	DIFFERENCE IN PERCENTAGE DEVIATION
	Base of prediction	Actual deviation	Percentage deviation	Base of prediction	Actual deviation	Percentage deviation		
Feb. to Oct.	November	35.01	25.20	Nov.–Jan.	33.84	24.36	+1.17	+0.84
Feb. to Oct.	December	34.94	25.15	Nov.–Jan.	33.84	24.36	+1.10	+0.79
Feb. to Oct.	January	34.61	24.91	Nov.–Jan.	33.84	24.36	+0.77	+0.55
Mar. to Oct.	December	32.74	25.38	Dec.–Feb.	31.35	24.30	+1.39	+1.08
Mar. to Oct.	January	32.52	25.22	Dec.–Feb.	31.35	24.30	+1.17	+0.92
Mar. to Oct.	February	30.86	23.92	Dec.–Feb.	31.35	24.30	−0.49	−0.38
Apr. to Oct.	January	30.30	27.04	Jan.–Mar.	28.62	25.54	+1.68	+1.50
Apr. to Oct.	February	29.17	26.03	Jan.–Mar.	28.62	25.54	+0.55	+0.49
Apr. to Oct.	March	28.40	25.34	Jan.–Mar.	28.62	25.54	−0.22	−0.20
May to Oct.	February	27.10	28.66	Feb.–Apr.	26.14	27.64	+0.96	+1.02
May to Oct.	March	27.09	28.65	Feb.–Apr.	26.14	27.64	+0.95	+1.01
May to Oct.	April	26.50	28.02	Feb.–Apr.	26.14	27.64	+0.36	+0.38
June to Oct.	March	25.49	35.05	Mar.–May	24.70	33.96	+0.79	+1.09
June to Oct.	April	25.02	34.40	Mar.–May	24.70	33.96	+0.32	+0.44
June to Oct.	May	24.81	34.11	Mar.–May	24.70	33.96	+0.11	+0.15
July to Oct.	April	22.51	42.05	Apr.–June	21.42	40.01	+1.09	+2.04
July to Oct.	May	22.34	41.72	Apr.–June	21.42	40.01	+0.92	+1.71
July to Oct.	June	21.44	40.04	Apr.–June	21.42	40.01	+0.02	+0.03
Aug. to Oct.	May	18.28	51.38	May–July	17.05	47.93	+1.23	+3.45
Aug. to Oct.	June	18.18	51.09	May–July	17.05	47.93	+1.13	+3.16
Aug. to Oct.	July	17.14	48.17	May–July	17.05	47.93	+0.09	+0.24
Sept. to Oct.	June	13.48	71.47	June–Aug.	12.52	66.38	+0.96	+5.09
Sept. to Oct.	July	13.20	70.00	June–Aug.	12.52	66.38	+0.68	+3.62
Sept. to Oct.	August	12.12	64.26	June–Aug.	12.52	66.38	−0.40	−2.12
October	July	7.06	120.23	July–Sept.	6.44	109.67	+0.62	+10.56
October	August	6.92	117.84	July–Sept.	6.44	109.67	+0.48	+8.17
October	September	5.71	97.27	July–Sept.	6.44	109.67	−0.73	−12.40

is not so satisfactory as might be desired. Great irregularities are to be expected when a flock of only 415 birds is divided up into such a large number of classes. The results indicate that if applied to larger flocks the prediction equations for the production of a group of remaining months from three-months recorded production might be made with greater precision.

In comparing the error of prediction for a group of subsequent months from three-month periods with the errors of prediction for the same period when prediction is made from single months we require three sets of equations for the prediction of the yields of a group of months from a monthly record. Two sets of these have been given on page 282 and used in comparison with the results of prediction from bimonthly periods.

The additional equations required are:

Month from which prediction is made	*Period for which prediction is made*	*Prediction equation*
November	February to October	$E_9 = +134.546 + 1.143\ e_1$
December	March to October	$E_8 = +122.721 + 1.165\ e_2$
January	April to October	$E_7 = +106.659 + 1.033\ e_3$
February	May to October	$E_6 = +80.512 + 1.342\ e_4$
March	June to October	$E_5 = +47.994 + 1.462\ e_5$
April	July to October	$E_4 = +30.111 + 1.219\ e_6$
May	August to October	$E_3 = +13.094 + 1.008\ e_7$
June	September to October	$E_2 = +3.462 + 0.649\ e_8$
July	October	$E_1 = +0.297 + 0.240\ e_9$

When we compare the results for the prediction of the yield of a group of subsequent months from single monthly records and from trimonthly records of production we find that the differences in errors of prediction are surprisingly small. Specifically we note that in the case of the average deviation with regard to sign, shown in the two last columns of table 11, the differences in actual errors range from 0.03 to 3.42 eggs while the differences in percentage values range from 0.05 to 10.55. In some cases the three-month period gives a numerically larger error of prediction while in other cases the one-month period gives the larger error.

When the comparison is made on the basis of average deviation without regard to sign (table 12) the single-month period gives a slightly larger average deviation in most cases, 23 out of 27 cases. The differences are, however, very small, varying from 0.12 to 1.14 eggs.

Similar results are obtained when the comparisons (between the single component months and the three-months record as bases of prediction) are based upon square root of mean square deviation (table 13). In 23 of the 27 cases prediction from a monthly record gives slightly more variable errors than prediction from the combined record of three months. The differences are, however, insignificant, varying from 0.02 to 1.68 eggs. It is clear, therefore, that if the linear equation be used for the purpose of predicting the yield of a group of remaining months, about as good results for practical purposes may be obtained from single month records as from the sum of three months records.

It is quite possible that with equations other than the linear this will not be the case. Such equations will be investigated in future work.

Comparison of the two- and three-month periods as bases for the prediction of the egg record of the subsequent months

In the foregoing discussion comparisons between the value of single-month periods and two-month periods and between single-month and three-month periods as bases for prediction have been made. It will be of some interest to compare two- and three-month periods in the same way. Certain of the data may be rearranged from preceding tables. Special calculations would, however, be necessary to complete all of the possible comparisons. It is evident that for a critical comparison between the two groups it is necessary to deal with the egg record of a group of remaining months. Thus in comparing November-to-January production with November-and-December or December-and-January production as bases of prediction it is necessary to determine the accuracy with which the egg production of February to October may be predicted since none of the months included in the base of prediction should also occur in the period for which prediction is made.

Limiting our attention to the comparisons which can be made from the data in the preceding tables[7] we note that in some cases there is a larger average deviation with regard to sign in predicting from two-months and in some cases a larger error in predicting from three-months production.

The same may be shown to be true for the average deviation without regard to sign and for the square root of mean square deviation of the predicted from the actual values. Thus there is little practical advantage in dealing with three-months production as compared with two-months production as bases for the prediction of the record of a group of subsequent months.

[7] The subsidiary tables upon which the following conclusions were based may be formed from tables 5 to 7 and 11 to 13. It seems unnecessary to publish these tables here.

Comparison of the four periods as bases for the prediction of the egg record of the year

In the introductory sections of this paper we called attention to the so-called periods or cycles of egg production which have been recognized by a number of students of fecundity in the domestic fowl. It might at first seem desirable to compare the results of predicting from these periods.

Since these periods are consecutive and together make up the entire laying year it is impossible to obtain any common basis for testing their efficiency such as has been found in periods of subsequent months in preceding tests.

In view of this fact it does not seem desirable in this place to go into the question of the comparison of these conventional periods as bases of prediction. Practically all of the data required for such comparison as can be made appear in the foregoing tables 2 to 13. The reader who desires to do so may abstract the constants from these tables.

SUMMARY AND CONCLUSIONS

The specific purpose of the present paper, which is one of a series dealing with the general problem of variation and correlation of egg production in the domestic fowl, is to consider the possibility of predicting the future egg production or the total annual egg production of White Leghorn birds from the record of an individual month or a group of consecutive months.

The investigation has been carried out because of two convictions: First, factors underlying the distribution, inheritance and interrelationships of fecundity in birds present a problem of first-rate biological importance. Second, that it is one of the functions of the biologist to provide the agricultural economist with the quantitative constants and formulae upon which the scientific agriculture of the future must largely rest.

The method followed has been to determine a series of prediction equations based on the experience of six years (1911 to 1917) of the INTERNATIONAL EGG-LAYING CONTEST at Storrs and to test these equations upon an additional series of 415 birds studied at Storrs in 1917–1918. Thus the equations have been tested upon a different series of birds from that upon which they were based, but upon birds maintained under conditions comparable with those upon whose record the fundamental equations were based.

The results show that the annual egg record of a series of birds may be predicted with a reasonably high degree of accuracy when their performance for a single month is known. Somewhat higher accuracy may be obtained when the record of two or more months is taken into consideration, but the improvement due to an increase in the number of months upon which prediction is based is not great.

Prediction of the egg record which will be made by groups of birds subsequently to the month or group of months chosen as a basis of prediction can also be made, but the accuracy of prediction decreases rapidly as the period for which prediction is made becomes shorter.

The results show that in the case of a flock of White Leghorn fowl, which is essentially identical in genetic composition and maintained under essentially uniform conditions from year to year, it is quite possible to estimate annual egg production from the record of either a single month or of two or three consecutive months with a high degree of accuracy. The same is presumably true of other breeds as well. This point is now under investigation.

It is probably not feasible to use the equations given in this paper for flocks differing greatly in genetic composition or in conditions of maintenance from that upon which these equations were based. The problem of the determination of corrective terms by which the equations may be applied to flocks other than that upon which they are based is now under investigation.

LITERATURE CITED

ALDER, B., and EGBERT, A. D., 1918 A quick method of obtaining individual egg records without the trap nest. Bull. Utah Agric. Exp. Sta. 162.

BLAKESLEE, A. F., HARRIS, J. ARTHUR, WARNER, D. E., and KIRKPATRICK, W. F., 1917 Pigmentation and other criteria for the selection of laying hens. Bull. Storrs Agric. Exp. Sta. 92, pp. 95–194.

CARD, L. E., 1917 A study of egg production in the White Leghorns. Bull. Storrs Agric. Exp. Sta. 91, pp. 41–90.

HARRIS, J. ARTHUR, BLAKESLEE, A. F., and WARNER, D. E., 1917 Body pigmentation and egg production in the fowl. Proc. Nation. Acad. Sci. **3**: 237–241. *Fig. 1–3.*

HARRIS, J. ARTHUR, BLAKESLEE, A. F., and KIRKPATRICK, W. F., 1917 Interperiodic correlation in the egg production of the domestic fowl. Proc. Nation. Acad. Sci. **3**: 565–569. *Fig. 1–2.*

HARRIS, J. ARTHUR. BLAKESLEE, A. F., and KIRKPATRICK, W. F., 1918 The correlation between egg production during various periods of the year in the domestic fowl. Genetics **3**: 27–72. *Fig. 1–11.*

HARRIS, J. ARTHUR, BLAKESLEE, A. F., WARNER, D. E., and KIRKPATRICK, W. F., 1917 The correlation between body pigmentation and egg production in the domestic fowl. Genetics **2**: 36–77. *Fig. 1–16.*