THE EINSTEIN SPACE OF AN ACCELERATED FRAME

AND

UNIFORM GRAVITY

C. Y. Lo

Applied and Pure Research Institute
17 Newcastle Drive, Nashua, NH 03060

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Abstract

Einstein's equivalence principle was first expressed in terms of the equivalence of uniform gravity and an accelerated frame of reference. Additionally based on the principle of general relativity, Einstein's equivalence principle for the curved Riemannian physical space was developed and three tests verified. Nevertheless, attempts to present the initial form of Einstein's equivalence principle in terms of a space-time metric had failed, and it was claimed that Einstein's principles are invalid. On the other hand, Einstein insisted on the fundamental importance of his equivalence principle to general relativity. It is shown that these failures are due to misconceptions and misinterpretation of Einstein's theory. To this end, the related misconceptions are clarified by analyzing the case of uniform rotation. It is found that, in Einstein's Riemannian space, the frame of reference has dual structures, a Riemannian structure and a Euclidean structure that emerge from different, but complementary, methods of measurements. A Riemannian space together with its Euclidean structure is called an *Einstein space* named after its creator. An Einstein space is a physical space in Einstein's theory if Einstein's equivalence principle is satisfied. After these theoretical clarifications, the space-time metric of an accelerated frame is calculated to support Einstein's equivalence principle.

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1. Introduction

It is generally agreed, as pointed out by Einstein [1], Eddington [2], Pauli [3], Weinberg [4], Misner, Thorne & Wheeler [5], Straumann [6], and Yu [7], that Einstein's equivalence principle is the theoretical foundation of general relativity. However, a surprising fact is, as Einstein [8] saw it, that few like Eddington [2] understand Einstein's equivalence principle in terms of physics adequately. This is confirmed, for instance, by the (invalid) calculations of Tolman [9] and Fock [10]. In addition, Synge [11] professed his misunderstandings on Einstein's equivalence principle as follows:

> "...I have never been able to understand this principle...Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute property; it has nothing to do with any observer's world line...The Principle of Equivalence performed the essential office of midwife at the birth of general relativity...I suggest that the midwife be now buried with appropriate honours and the facts of absolute spacetime be faced."

Currently, similar misunderstanding surprisingly persists. For instance, Thorne [12] criticized Einstein's principle as follows:

> "In deducing his principle of equivalence, Einstein ignored tidal gravitation forces; he pretended they do not exist. Einstein justified ignoring tidal forces by imagining that you are (and your reference frame) are very small."

However, these imagined problems have already been explained and answered satisfactorily by Einstein. For instance, the problem of tidal forces has been answered in Einstein's July 12, 1953 letter to A. Rehtz [13] as follows:

> "The equivalence principle does not assert that every gravitational field (e.g., the one associated with the Earth) can be produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field."

This is not an after thought because an accelerated frame also cannot account for the gravity of a rotating disk [1]. Einstein [8] explained to Laue, "What characterizes the existence of a gravitational field, from the empirical standpoint, is the non-vanishing of the Γ^l_{ik} (field strength), not the non-vanishing of the R_{iklm}." and no gravity is a special case of gravity. This view is crucial because it allows Einstein to conclude that the geodesic equation is also the equation of motion of a massive particle under gravity. In fact, Einstein insisted, throughout his life, on the fundamental importance of the principle to his general theory of relativity [8]. On the other hand, Einstein's insistence on this point has created a puzzle for philosophers and historians of science [8]. This shows also, how much was Einstein's equivalence principle being understood in terms of physics.

Einstein explained the initial form of his equivalence principle in terms of the uniform gravity and acceleration clearly in 1911 and in 1916 [14]. Einstein assumed that the mechanical equivalence of an inertial system K (x, y, z) under a uniform gravitational field, which generates a gravitational acceleration γ (but, system K is free from acceleration), and a system K' (x', y' z') accelerated by γ in the opposite direction, can be extended to other physical processes. Based on this assumed equivalence, Einstein [14] derived the gravitational red shifts. He found also that his equivalence principle is compatible with the Doppler effects and even the notion of photon. Thus, the equivalence principle has been firmly established on the ground of universality of physics, although the formula for light speeds under gravity needed improvement. This is independent of the need of a Lorentz Riemannian space, which is additionally due to the principle of general relativity and special relativity [1].

A connection between gravity and a curved Riemannian space was established in 1915 [15]. However, a space-time metric that corresponds to a uniformly accelerated frame of reference remains to be clarified (see Section 4). This incompleteness contributed to a speculation that the deficiency is intrinsic and thus added difficulty in understanding Einstein's theory. Moreover, such speculation is supported by both calculations of Tolman [9] and Fock [10] that showed the accelerated frame failed in relating to a metric for uniform gravity, as Einstein's principle requires.

Recently, such a speculation has been proven incorrect because the field equation is intimately related to Einstein's equivalence principle. The Maxwell-Newton Approximation [16.17] (the same as the linearized field equation for weak gravity due to massive matter) that produced an accurate bending of light has been derived with Einstein's equivalence principle together with the notion of a curved space if Newtonian theory is taken as a form of first order approximation. Thus, Einstein's equivalence principle is fully compatible with the notion of a curved Riemannian space-time.

However, the question of how the curved space is related to an accelerated frame remains a puzzle. In this paper, it will be shown that failures of Tolman and Fock are due to the conceptual error that identified a frame of reference with only a Euclidean subspace. Some related conceptual considerations are also addressed (see Sections 2 & 3). Finally, an appropriate metric form for an accelerated frame will be derived (see Section 4). To this end, let us first identify the existing misconceptions.

2. The Euclidean Structure and Measurements in a Riemannian Space-Time.

Many theorists believed, as Pauli [3] did, that in general relativity "it is necessary to abandon Euclidean geometry" because "Einstein showed for example of a rotating reference system, the time intervals and spatial distances in non-Galilean systems cannot just be determined by means of a clock and rigid standard measuring rod." However, these two statements need some clarifications. Euclidean geometry is "abandoned" only in the invariant line element [1]. Moreover, Einstein showed only one type of "measurements", but he explained inadequately another type of necessary measurements in general relativity [1].

In general relativity, the invariant line element is

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}, \tag{1}$$

where $g_{\mu\nu}$ is a general space-time metric in a Riemannian space. Since $g_{\mu\nu}$ is not a constant metric, one cannot hope to derive from (1) a simple distance formula as in Euclidean geometry that the spatial distance d (P_1, P_2) of two points P_1 and P_2 is still

$$d(P_1, P_2) = [(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{1/2}, \tag{2}$$

However, in a different way, the Euclidean structure (2) is actually preserved within the Riemannian geometry.

To illustrate this, let us examine the Schwarzschild solution [1] of Riemannian space (x, y, z, t),

$$ds^2 = (1 - 2M\kappa/\rho)dt^2 - (1 - 2M\kappa/\rho)^{-1}d\rho^2 - \rho^2 d\theta^2 - \rho^2 \sin^2\theta d\varphi^2, \tag{3}$$

where

$$\rho^2 = x^2 + y^2 + z^2, \qquad x = \rho \sin\theta\cos\varphi, \quad y = \rho \sin\theta\sin\varphi, \quad \text{and} \quad z = \rho \cos\theta \tag{4}$$

and κ is a coupling constant, and M is the total mass. (For simplicity, an interior solution is not presented here.) Since the metric is defined in terms of ρ, θ, and φ, the Riemannian space is actually defined in terms of the Euclidean characteristics (4). This illustrates that the Euclidean subspace (x, y, z) is necessarily included in Einstein's Riemannian space. In fact, Einstein [12] stated that the velocity of light is defined in the sense of Euclidean geometry. The subspace (x, y, z) is called the frame of reference. Thus, the notion of Riemannian space-time is compatible with the frame of reference having a Euclidean structure.

To understand the Euclidean subspace (x, y, z) in terms of physical measurements, we must first clarify what "measure" means in relation to Einstein's equivalence principle. In Einstein's theory, the measuring instruments are resting but in a free fall state (see Section 3). From Einstein's equivalence principle, time dilation and space contraction are obtained. On the other hand, if the measuring instruments are resting and are attached to the frame of reference, since the measuring instruments and the coordinates being measured are under the same influence of gravity, a Euclidean space structure emerges as if gravity did not exist. Such a Euclidean structure would make a distinct class of Riemannian space (see also Section 3).

To make a useful distinction, such a class should be called the Einstein Spaces named after its creator. Then, an Einstein space would be a physical space that models reality only if Einstein's equivalence principle is satisfied.

The notion of frame of reference plays a crucial role in the theory of general relativity, as pointed out by Fock [10], in particular to Einstein's equivalence principle, but Fock incorrectly identified the frame of reference with a Euclidean subspace. A related problem is that Pauli [3] regards the equivalence principle essentially as just the existence of local Minkowski spaces. The popular version of the equivalence principle expressed by Pauli [3] is the following:

> "For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system K_0 (X_1, X_2, X_3, X_4) in which gravitation has no influence either in the motion of particles or any physical process."

Einstein strongly objected Pauli's version as reported in details by Norton [8]. The notion of acceleration with respect to a frame of reference is essential in Einstein's equivalence principle [10]. It has been shown that static acceleration may not exist for a non-constant metric, and this situation leads to inconsistent in physics [17].

Since other related physical considerations such as acceleration are ignored, Pauli's version is actually only a mathematical statement of the Lorentz manifold [11]. A noted difference from Pauli version is that Einstein requires additionally: i) "the special theory of relativity applies to the case of the absence of a gravitational field [14, p.115]" and ii) a local Minkowski space is obtained by choosing the acceleration. Einstein [14, p.118] wrote, "... we must choose the acceleration of the infinitely small ("local") system of coordinates so that no gravitational field occurs; this is possible for an infinitely small region." In fact, acceleration with respect to a frame of reference is crucial for measurements in Einstein's general relativity (see Section 3).

Nevertheless, the inadequacy of Pauli's version for a world region of a physical space was not a serious problem until it is incorrectly claimed that the existence of Local Minkowski space had replaced Einstein's equivalence principle such that the physical validity of any Lorentz manifold could be justified. Consequently, Einstein's theory is distorted and his notion of physical space [1,13,14] has been ignored to the point that professional relativists often ask what is a physical space.

3. The Principle of General Relativity, Uniform Rotation, and Riemannian Space-Time

Einstein considered a Galilean (inertial) system of reference K (x, y, z, t) and a system K' (x', y', z', t') in uniform rotation Ω relatively to K. The origins of both systems and their axes of Z permanently coincide. (The gravity of this example would be useful for the calculation of the gravity related to a uniformly accelerated frame.) For reason of symmetry, a circle around the origin in the x-y plane of K may at the same time be regarded as a circle in the x'-y' plane of K'. Then, *according to special relativity*, in the x-y plane and the x'-y' plane, the metrics of K and K' [1,18] are respectively the following:

$$ds^2 = c^2\,dt^2 - dr^2 - r^2\,d\phi^2 - dz^2 \qquad \text{where} \quad x = r\cos\phi, \qquad y = r\sin\phi, \tag{5}$$

and

$$ds^2 = (c^2 - \Omega^2 r'^2)\,dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2\,d\phi'^2 - dz'^2 \tag{6}$$

Then,

$$\oint ds = (1 - \Omega^2 r'^2/c^2)^{-1/2} r' \int_0^{2\pi} d\phi' = 2\pi r'(1 - \Omega^2 r'^2/c^2)^{-1/2} \qquad x' = r'\cos\phi', \qquad y' = r'\sin\phi'. \tag{7}$$

would be the circumstance of a circle of radius r' (= r) for an observer in K'. Moreover, as Einstein pointed out, "an observer at the common origin of co-ordinates, capable of observing the clock at the circumferences by means of light, would therefore see it lagging behind the clock beside him". *So, he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be* [14]. Thus, Einstein also defined a *physical space-time coordinate system together with its metric* that is related to local clock rates and local spatial measurements.

To illustrate Einstein's equivalence principle and the notion of Einstein space, let us first derive metric (6). Consider the coordinate transformation to the uniformly rotating disk [18], in terms of Newton's notion of "absolute time" as follows:

$$x = x'\cos\Omega t - y'\sin\Omega t, \qquad y = x'\sin\Omega t + y'\cos\Omega t, \qquad \text{and} \qquad z = z' \tag{8a}$$

or

$$r = r', \quad z = z'. \quad \phi = \phi' + \Omega t. \tag{8b}$$

in cylindrical coordinate systems of K and K', where Ω is the angular velocity. Here, we take advantage of the fact that one can start with an arbitrary coordinate system of a Riemannian space. Then, from (5) the resulting metric has the following form,

$$ds^2 = (c^2 - \Omega^2 r'^2)\,dt^2 - 2\Omega r'^2\,d\phi' dt - dr'^2 - r'^2\,d\phi'^2 - dz'^2 \tag{6'}$$

However, the mathematical system K* (x', y', z', t) is not a physical space-time coordinate system for the uniformly rotating disk K' because what measured in a resting local clock is time *t'* but not time *t*. In other words, metric (6') together with its coordinates K* is not a space-time coordinate system, as Einstein defined, that can be used for physical measurement.

Nevertheless, metric (6') alone can be used to derive metric (6), which has been claimed as incompatible with (5) by some theorists. To obtain a physical coordinate system including the time t' of the rotating disk, a comparison of (6) and (6') leads to,

$$d\phi' = d\phi - \Omega dt\ ; \tag{9a}$$

and

$$cdt' = [cdt - (r\Omega/c)rd\phi][1 - (r\Omega/c)^2]^{-1}\ . \tag{9b}$$

Thus, it is necessary to modify the time coordinate t'. Relation (9) makes clear that metric (6) is related to the flat metric (5).

The time dilation and the spatial contraction in general relativity [1,14], are results due to comparisons with a clock and a measuring rod in relatively rest at the beginning of a free fall. To verify this, consider a particle P resting at (r', ϕ', z'). Then, P has the velocity of Ωr in the ϕ'-direction, which is denoted by dx''. It follows that the Lorentz coordinate transformation is,

$$rd\phi = [1 - (r\Omega/c)^2]^{-1/2}\ [dx'' + r\Omega dt'']\ ; \tag{10a}$$

and

$$cdt = [1 - (r\Omega/c)^2]^{-1/2}\ [cdt'' + (r\Omega/c)dx'']\ . \tag{10b}$$

Then,

$$rd\phi' = [1 - (r\Omega/c)^2]^{1/2}\ dx''\ ; \qquad \text{and} \qquad cdt' = [1 - (r\Omega/c)^2]^{-1/2}\ cdt'' \tag{11a}$$

and

$$ds^2 = c^2\ dt''^2 - dx''^2 - dr'^2 - dz'^2 \tag{11b}$$

These are exactly the time dilation and spatial contraction. This illustrates that a particle resting at K', can attached to a local Minkowski space. Thus, this is also an example of Einstein's version of infinitesimal equivalence principle.

However, for the coordinate system K* (x', y', z', t), the question of time dilation is complicated because Einstein's equivalence principle is not applicable. Nevertheless, let us assume the Einstein's equivalence principle could be applied to K*. Mathematically, for a particle P resting at K*, the state vector of P is (0, 0, 0, dt). According to (9), P is also resting at K' with a state vector (0. 0. 0. dt'). Then the local Minkowski space for P is identical to (11b). It thus follows that

$$dx'' = [1 - (r\Omega/c)^2]^{-1/2}\ rd\phi'\ , \tag{12a}$$

and

$$dt'' = [1 - (r\Omega/c)^2]^{1/2}\ dt - [1 - (r\Omega/c)^2]^{-1/2}(r\Omega/c^2)\ rd\phi'. \tag{12b}$$

Thus,

$$dt = [1 - (r\Omega/c)^2]^{-1/2}\ dt'' \tag{12c}$$

would be considered as the time dilation since a clock rest at K* has $d\phi' = 0$. The problem of this derivation is that the parameter "t" is not the local time for the frame K' (x', y' z').

Since metric (6') satisfies Pauli's "equivalence principle", Pauli's version is clearly inadequate in physics. This calculation confirms that Einstein's equivalence principle is applicable only to a physical space. *Thus, in spite of general covariance, the freedom toward the physical space-time coordinate systems that can be used for physical interpretation is severely limited.*

The directional spatial contraction as indicated in metric (6), is measured with a resting measuring rod in the state of free fall. However, if a spatial measurement is performed with a measuring rod which is attached to the frame K' (x', y', z'), it would appear as Euclidean. In fact, it is based on this implicit assumption that the cylindrical coordinate system is well defined in K'. Thus, as shown in examples (3) and (6), *the distance in terms of the Euclidean structure is necessary and complementary to the metric for the Riemannian space that produces the local distance.* The system K* (x', y', z', t) has a Euclidean subspace, but the time t is not associated with the frame K' (x', y', z'). Although K* is diffeomorphic to K, K* is not a physical space-time since it fails the physical requirement of local time. Thus, diffeomorphic manifolds may not be equivalent in general relativity as Wald [19] and Logunov & Mestvirishvilli [20] believed.

Thus, an Einstein space is a Riemannian space with a Euclidean structure, and it is a physical space if it sufficiently satisfies all physical requirements, including Einstein's equivalence principle. However, as in special relativity, the Euclidean characteristics, on which a physical coordinate system is based, are not invariants.

4. Uniform Acceleration and Einstein's Equivalence Principle in Riemannian Space

The analysis of a rotating disk suggests that there are similarities with respect to the case of a uniform acceleration. Based on similarity to the case of the rotating disk, the metric for the case of uniform acceleration would be

$$ds^2 = (c^2 - v^2)\, dt'^2 - (1 - v^2/c^2)^{-1} dx'^2 - dy'^2 - dz'^2 , \qquad (13)$$

where v(t') is the relative velocity of the coordinate systems in the x'-direction. Metric (13) has a Euclidean structure as if v were zero. In other words, for the acceleration in the x-direction, the metric would have the following form,

$$ds^2 = (c^2 - 2U)\, dt'^2 - (1 - 2U/c^2)^{-1} dx'^2 - (dy'^2 + dz'^2), \quad \text{and} \quad c^2/2 > U(x', t') \geq 0 . \qquad (14)$$

Note that a uniform acceleration cannot exist forever, otherwise the resulting speed would exceed the velocity of light.

It follows that a uniform acceleration must be started at some time, for instance, $t = t_0 < 0$, and then decreased some time afterward. Moreover, a uniform gravity must be confined in a finite region; otherwise the light speed as the maximum velocity would be violated. Thus, uniform gravity like an electromagnetic plane wave, also does not really exist in nature. Thus, the equivalence of acceleration and uniform gravity is best described, as Einstein did, in terms of an elevator. In practice, the uniform gravity is essentially a local idealization of a non-uniform gravity.

Consider a system K' accelerated with an acceleration a relative to an inertial system K_0. Then, if the coordinates of the origin of K' in system K_0 is $(X_{0'}(t), 0, 0, t)$, we have

$$\frac{d^2 X_{0'}}{dt^2} = a , \qquad (15a)$$

and

$$X_{0'}(t) = X_{0'}(0) + at^2/2 ; \quad y = y' ; \quad z = z' , \quad \text{if} \quad v(0) = 0 . \qquad (15b)$$

where $X_{0'}(0)$ is arbitrary. Thus, to obtain a transformation compatible with form (14), we may assume similar to (11) that

$$dx' = dx - v(x,t)\, dt. \qquad x = x' - X_{0'}(0) + at^2/2 \qquad (16a)$$

and

$$cdt' = [1 - F(x., t)]^{-1}[cdt - f(x, t)c^{-1}dx] , \qquad (16b)$$

where v(x, t) is a relative velocity of the two systems, and f (x, t) is an unknown function. Note that (15a) is equivalent to

$$dx' \equiv 0 ; \qquad \text{and} \qquad \frac{d^2 x}{dt^2} = a \quad (\frac{dx}{dt} = v \;\text{ and }\; \frac{dv}{dt} = a). \qquad (16c)$$

Substituting (16) to metric form (14), a comparison with the flat metric leads to three relations as follows:

$$-(1 - 2U/c^2)^{-1} + (1 - F)^{-2}(1 - 2U/c^2)(f/c)^2 = -1 ; \qquad (17a)$$

$$-(1 - 2U/c^2)^{-1}(v/c)^2 + (1 - F)^{-2}(1 - 2U/c^2) = 1 ; \qquad (17b)$$

$$(1 - 2U/c^2)^{-1}v = (1 - F)^{-2}(1 - 2U/c^2)f . \qquad (17c)$$

It follows that

$$v = (1 - F)^{-2}(1 - 2U/c^2)^2 f ; \qquad 1 - vf/c^2 = (1 - 2U/c^2) ; \qquad \text{and } v = f \qquad (18a)$$

Thus

$$U = (v)^2/2 \geq 0; \quad \text{and} \quad F = 2U/c^2 = (v/c)^2. \tag{18b}$$

Since v is the relative velocity of the two systems of coordinates K' and K_0, as expected, metric (14) is in the form of (13).

Also, in terms of physics, (16) is in complete agreement with (11) as shown be the following relations:

$$dx' = dx - v\,dt\,; \quad . \tag{19a}$$

and

$$cdt' = [1 - (v/c)^2]^{-1}[cdt - vc^{-1}dx]\,; \tag{19b}$$

or

$$dx = [1 - (v/c)^2]^{-1}dx' + vc^{-1}cdt'\,. \tag{19c}$$

and

$$cdt = cdt' + [1 - (v/c)^2]^{-1}vc^{-1}dx'\,. \tag{19d}$$

The limitation on the velocity of light and the definite sign of ds^2 for the time line element, also requires

$$c^2/2 > U(x', t') \geq 0\,, \qquad \text{and} \qquad \frac{d^2x'}{ds^2} = \frac{1}{c^2}\frac{\partial U}{\partial x'}\,, \tag{20}$$

from the geodesic equation for the case of dx'/ds = 0.

On the other hand, the gravitational acceleration of a particle in K' is equivalent to

$$dx \equiv 0\,; \tag{21a}$$

and

$$ds = cdt\,, \qquad dx' = -v\,dt\,, \qquad .\text{and} \qquad dt' = [1 - (v/c)^2]^{-1}dt\,. \tag{21b}$$

Thus,

$$\frac{d^2x'}{ds^2} = -\frac{dv}{c^2dt} \qquad \text{and} \qquad -a = \frac{\partial U}{\partial x'}\,. \tag{21c}$$

where a is the constant acceleration by assumption. Thus, if U is independent of the time t',

$$U(x') = -ax' + C\,, \qquad \text{or} \qquad U(x') = -a\,(x' - x_0) \tag{22}$$

where C is a constant. Since U(x') is bounded according to (20), we must also have a range for x'. From (16a), we have

$$U = -a\,(x' - x - X_{0'}(0)\,). \tag{23}$$

According to (21a) $dx \equiv 0$, $x + X_{0'}(0)$ should be considered as just an arbitrary constant. Thus, (23) agrees with (22).

To verify time dilation and spatial contraction, one should consider a system in relative rest at the beginning of a free fall. Let us consider the following Lorentz coordinate transformation,

$$dx = [1 - (v/c)^2]^{-1/2} [dx'' + vdt''] , \tag{24a}$$

and

$$cdt = [1 - (v/c)^2]^{-1/2} [cdt'' + (v/c) dx'']. \tag{24b}$$

where $v = [2U(x')]^{1/2}$. Then, we have the expected relation

$$dx' = [1 - (v/c)^2]^{1/2} dx'' , \quad \text{and} \qquad cdt' = [1 - (v/c)^2]^{-1/2} cdt'' . \tag{24c}$$

and

$$ds^2 = c^2 dt''^2 - dx''^2 - (dy'^2 + dz'^2). \tag{24d}$$

Thus, the initial form of Einstein's equivalence principle is indeed compatible with the notion of Riemannian space.

However, there is an arbitrary constant in the potential U. This uncertainty is actually a necessary feature that a uniform gravity can be considered as a local idealization of a non-uniform gravity. Without this arbitrary constant, one cannot adjust the metric for uniform gravity to have the same local value of a changeable non-uniform gravity. Note also that metric (14) for a uniform gravity has the form of the Schwarzschild exterior solution far from the source. Just as the plane wave is a local idealization of an electromagnetic wave, the uniform gravity can be considered as a local idealization of the Schwarzschild solution. As a local idealization, one may not expect that metric (14) satisfies the normal boundary condition at infinite, just as an electromagnetic plane-wave has the same amplitude everywhere, and appears as having no source.

5. Discussions and Conclusions

An interesting result of this calculation is that it confirms Einstein's 1911 calculation on the gravitational red shifts is valid in terms of the Euclidean structure and this explains why the notion of a curved space was not used. The notion of Einstein space clarifies that it is valid to consider the frame of reference as if a three-dimensional Euclidean structure. Since a Euclidean structure is independent of gravity, it makes sense to define the Euclidean structure first to obtain the Schwarzschild solution.

To calculate the space-time metric of uniform gravity related to an accelerated frame, it is crucial to recognize the dual characteristics of the spatial subspace of Einstein's Riemannian space-time. Thus, the equivalence of gravitational force and acceleration is accurately valid. In other words, the criticisms of Fock and his followers [21] on Einstein's general relativity are baseless. This further strengthens the status of Einstein's equivalence principle that has been further established due to its role in the derivation of the Maxwell-Newton Approximation [16,17]. Weinberg [4, p.3] declared, "In my view, it much more useful to regard general relativity above all as a theory of gravitation, whose connection with geometry arises from the peculiar empirical properties of gravitation, properties summarized by Einstein's Principle of the Equivalence of Gravitation and Inertia."

In Einstein's theory [1], the reality is modeled with a physical space (-time) that has a frame of reference for the descriptions of the interested physics. In such a physical space, all physical requirements are sufficiently satisfied. From Einstein's simple example of uniform rotation, we have learned also that a change of sign of ds^2 in a coordinate would manifest the invalidity of a physical requirement beyond that region. Einstein [1,14] has shown that such a physical space is a Riemannian manifold with a Lorentz metric and a physically valid space-time coordinate system that time is measured by a local clock.

Thus, a different coordinate system can fail as a space-time coordinate system in physics although most calculations can be carried out with an arbitrary mathematical coordinate system (see Sections 2 & 3) Moreover, it has been illustrated that the local Minkowski space at a point is obtained by means of choosing the appropriate acceleration. This requires that the coordinates of a physical space must have physical meanings. As illustrated in Sections 3, Einstein's equivalence principle is applicable only in a physical space. Otherwise, the so-calculated local time rate and local spatial contraction would be incorrect in physics. Since the conditions for a physical space must be taken into consideration, Einstein's equivalence principle is not really a local principle in a manifold as Fock [10] believed.

Now, it is clear that Einstein's equivalence principle cannot be replaced with merely the existence of local Minkowski spaces, since Pauli's "equivalence principle" has been proven as inadequate in physics. (There is Lorentz manifold that is not diffeomorphic to a physical space [17].) Otherwise, not only this will surely end up in theoretical disagreement with Einstein, but also against the weighty fact that there are non-scalars in physics. Moreover, the existence of definitive gravitational red shifts testifies that a valid space-time coordinate system cannot be arbitrary.

In conclusion, a uniform acceleration, as an idealization, is valid for an illustration of Einstein's equivalence principle as a foundation of general relativity. It is hope that this paper would help clarifying the confusions on this subject in the literature. In particular, the author wishes that the damages done by Fock's erroneous approach would be rectified. On this basis, one may expect that general relativity would be infused with new life and be even more fruitful in the future.

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