Canadian Journal of Physics
Revue canadienne de physique

The elastic wave motions for a Photothermal medium of a dual-phase-lag model with an internal heat source and gravitational field
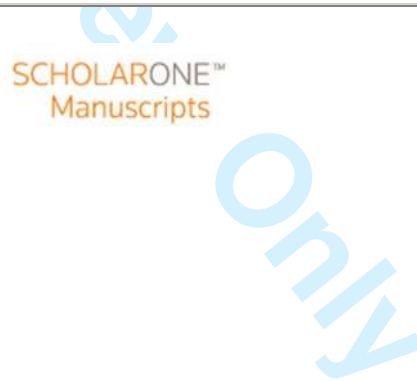
SCHOLARONE™
Manuscripts

The elastic wave motions for a Photothermal medium of a dual-phase-lag model with an internal heat source and gravitational field

Khaled Lotfy

Department of Mathematics, Faculty of Science, P.O. Box 44519, Zagazig University, Zagazig, Egypt.

Department of Mathematics, Faculty of Science and Arts, Al-mithnab, Qassim University,   P.O. Box 931, Buridah 51931, Al-mithnab, Kingdom of Saudi Arabia.

E-mail: khlotfy_1@yahoo.com

Abstract

In this work, The dual-phase-lag (DPL) heat transfer model is introduced to study the problem of isotropic generalized thermoelastic medium with an internal heat source that is moving with a constant speed. Thermal loading at the free surface of a semi-infinite semiconconducting medium coupled plasma waves with the effect of mechanical force during a photothermal process to study the effect of the gravity field. The harmonic wave analysis is used to obtain the exact expressions for the considered variables, also the carrier density coefficient were obtained analytically. The variations of the considered variables through the horizontal distance are illustrated graphically under the effects of some several parameters based on the DPL model. The results are discussed and depicted graphically.

Keywords: Dual-phase-lag model; gravity effect; internal heat source;

Photothermal; Thermoelasticity.

PACS No.: 46.25.Hf

1 Introduction

Thermoelasticity theories, which involve finite speed of thermal signals (second sound), have created much interest during the last three decades. The conventional coupled dynamic thermoelasticity theory (CTE) is based on the mixed parabolic hyperbolic governing equations of Biot [1]. Chadwick [2] predicts an infinite speed of propagation of thermoelastic disturbance. To remove the paradox of infinite speed for propagation of thermoelastic disturbance, several generalized thermoelasticity theories which involve hyperbolic governing equations have been developed. The first theory of generalized thermoelasticity is Lord and Şhulman's (LS) [3] which introduced the generalization of the thermoelasticity theory with one relaxation time parameter (single-phase-lag model) through postulating a new law of heat conduction to replace the classical Fourier's law. The Green and Lindsay (GL) [4] model developed a more general theory of thermoelasticity involving two thermal relaxation times known as temperature rate dependent thermoelasticity. Many researchers and many works have been done under these theories [5–6].

It is well known that the usual theory of heat conduction based on Fourier's law predicts an infinite heat propagation speed. Heat transmission at low temperature has been observed to propagate by means of waves. These aspects have caused intense activity in the field of heat propagation. Extensive reviews on the so-called second sound theories (hyperbolic heat conduction) are given by Chandrasekharaiah [7]. The propagation of surface waves in elastic media is of considerable importance in earth-quake engineering and seismology due to the stratification in the earth's crust. Tzou [8, 9] has developed a new model called dual phase-lag model (DPL), which describes the interactions between phonons

and electrons on the microscopic level as retarding sources causing a delayed response on the macroscopic scale. For a macroscopic formulation, it would be convenient to use the DPL model for the investigation of the micro-structural effect on the behavior of heat transfer. The physical meanings and the applicability of the DPL model have been supported by the experimental results (see Tzou [10]). The DPL proposed by Tzou [8] is such a modification of the classical thermoelastic model in which Fourier law is replaced by an approximation to modified Fourier law with two different time translations: a phase-lag of the heat flux τ_q and a phase-lag of temperature gradient τ_θ. The delay time τ_θ is interpreted as that caused by the micro-structural interactions (small-scale heat transport mechanisms occurring in micro scale) and is called the phase-lag of the temperature gradient. The other delay time is τ_q interpreted as the relaxation time due to the fast transient effects of thermal inertia (small scale effects of heat transport in time) and is called the phase-lag of the heat flux.

The photothermal method was discovered firstly by Gordon et al. [11] when they found an intracavity sample where a laser-based apparatus gave rise to photothermal blooming, the photothermal lens. Sometime later, Kreuzer showed that photoacoustic spectroscopy could be used for sensitive analysis when laser light sources were utilized [12]. Photothermal spectroscopy has been used to measure acoustic velocities, thermal diffusion coefficients, sample temperatures, bulk sample flow rates, specific heats, and volume expansion coefficients in solids [13–15]. The photothermal generation during a photothermal process was studied by many researchers. The thermoelastic (TE) deformation and electronic deformation (ED) are prominent deformations of semiconductors and the main driven mechanisms for micromechanical structures. The thermal waves in the sample cause elastic vibrations [16]. This is the thermoelastic mechanism of elastic

deformation. The TE effect is based on the heat generation in the sample and the elastic wave generation through thermal expansion and bending [16]. A general theoretical analysis of the TE and ED effects in a semiconducting medium during a photothermal process consists in modeling the complex systems by simultaneous analysis of the coupled plasma, thermal, and elastic wave equations [17]. With the development of technologies, the semiconducting materials were used widely in modern engineering. The study of wave propagation in a semiconducting medium will have important academic significance and application value. However, to the researcher's knowledge, the wave propagation problem in a semiconducting medium during a photothermal process was not recorded so far.

Problem of heat sources acting in an elastic body has got its mathematical interest and physical importance. Mukhopadhyay et al. [18] discussed the theory of two-temperature thermoelasticity with two-phase lags. Chakravorty [19] discussed the transient disturbances in a relaxing thermoelastic half space due to moving stable internal heat source. Kumar and Devi [20] studied thermomechanical interactions in porous generalized thermoelastic material permeated with heat source. Lotfy [21] studied the transient disturbance in a half-space under generalized magneto-thermoelasticity with a stable internal heat source. Lotfy [22] discussed the transient thermo-elastic disturbances in a visco-elastic semi-space due to moving internal heat source. Othman [23] studied the generalized thermoelastic problem with temperature-dependent elastic moduli and internal heat sources. Though the problem of instantaneous and moving heat sources in infinite and semi-infinite space has been investigated by many authors [18-27], only a few papers are seen to study two dimensional wave propagation under the theory of generalized thermo-elasticity with relaxation time where the dynamic heat source has been considered in a visco-elastic semi-space.

The gravity effect is generally neglected in the classical theory of elasticity. The gravity effect on the problem of wave propagations in solids is very important, particularly on an elastic globe, was first studied by Bromwich[28]. Love[29] considered the effect of gravity and showed that the velocity of Rayleigh waves increased to a significant extent by the gravitational field when wave lengths were large. De and Sengupta [30-32] studied many problems of elastic waves and vibrations under the effect of gravity field, on the propagation of waves in an elastic layer and Lamb's problem on a plane. Sengupta and Acharya[33] studied the interaction of gravity field on the propagation of waves in a thermoelastic layer. Das et al. [34] investigated surface waves under the gravity influence in a non-homogeneous Rotation effect in generalized thermoelastic solid under the gravity influence elastic solid medium. Abd-Alla et al. [35- 37] presented the influences of rotation, gravity, magnetic field, initial stress and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space. Othman and Lotfy [38] studied the influence of gravity on 2-D problem of two temperature generalized thermoelastic medium with thermal relaxation.

This paper examines the interaction of gravity field and elastic wave motions at the free surface of a semi-infinite semiconducting medium during a photothermal process under LS theory and DPL model with internal heat source. The harmonic wave method was used to obtain the two temperature parameter under stress, exact expression of Normal displacement, Normal force stress, carrier density and temperature distribution. The paper also discusses the influences of the thermal relaxation times, thermoelastic coupling parameter and thermoelectric parameter on the photothermal theory.

2 Formulation of the problem and Fundamental equations

We consider a homogeneous generalized thermoelastic half-space with gravity field, let theoretical analyses of the transport process in a semiconductor involve in the consideration of coupled plasma waves, thermal waves and elastic waves simultaneously. The carrier density $N(\vec{r}, t)$, temperature distribution $T(\vec{r}, t)$, and elastic displacement $\boldsymbol{u}(\vec{r}, t)$ are the main variable quantities. For a medium with internal heat source of a linear, homogeneous and isotropic properties of the medium whose state can be expressed in terms of the space variables x, z ($\vec{r}$ is the position vector) and the time variable t. The coupled plasma, thermal and elastic transport equations can be given below (with new model under gravity and DPL model with internal heat source) as a vector form of rectangular coordinate system [5, 22, 31, 32] as,

$$\frac{\partial N(\vec{r},t)}{\partial t} = D_E \nabla^2 N(\vec{r},t) - \frac{N(\vec{r},t)}{\tau} + \kappa\, T(\vec{r},t), \tag{1}$$

$$\rho C_e (1+\tau_q \frac{\partial}{\partial t})\frac{\partial T(\vec{r},t)}{\partial t} = k(1+\tau_\theta \frac{\partial}{\partial t})\nabla^2 T(\vec{r},t) - \frac{E_g}{\tau} N(\vec{r},t) -$$

$$\gamma T_0 (1+\tau_q \frac{\partial}{\partial t})\nabla.\vec{u}(\vec{r},t) + (1+\tau_q \frac{\partial}{\partial t})Q, \tag{2}$$

The equation of motion with carrier density without gravity takes the form [39]:

$$\rho\frac{\partial^2 \vec{u}(\vec{r},t)}{\partial t^2} = \mu\nabla^2 \vec{u}(\vec{r},t) + (\mu+\lambda)\nabla(\nabla.\vec{u}(\vec{r},t)) - \gamma(1+\tau_\theta \frac{\partial}{\partial t})\nabla T(\vec{r},t) - \delta_n \nabla N(\vec{r},t), \tag{3}$$

In this article, the general case, that is, the non-zero thermal activation coupling parameter (κ) was studied. Where, $\kappa = \frac{\partial N_0}{\partial T}\frac{T}{\tau}$, N_0 is equilibrium carrier concentration at temperature T [33]. D_E is the carrier diffusion coefficient, τ is the

photogenerated carrier lifetime, E_g is the energy gap of the semiconductor. μ and λ are the Lamé elastic constants, ρ is the density, k is the thermal conductivity of the sample, T_0 is the absolute temperature. $\gamma = (3\lambda + 2\mu)\alpha_T$ is the volume thermal expansion where α_T is the coefficient of linear thermal expansion and C_e is specific heat at constant strain of the solid plate, δ_n is the difference of deformation potential of conduction and valence band. τ_θ, τ_q ($0 \le \tau_\theta \prec \tau_q$) are the phase-lag of temperature gradient and the phase-lag of heat flux respectively, Q is an internal heat source. In equation (2), the second term on the right side characterizes the effect of heat generation by the carrier volume and surface de-excitations in the sample and third term describes the heat generated by stress waves respectively. In the elastic equation (3), the third and fourth terms describes source term and influence of the thermal wave, plasma wave on the elastic wave, respectively [34]. We restrict our analysis parallel to xz-plane, so the displacement is defined by $\vec{u} = (u, 0, w), \quad u(x, z, t), \quad w(x, z, t)$.

The constitutive relations take the form:

$$\sigma_{xx} = (2\mu + \lambda)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z} - (3\lambda + 2\mu)(\alpha_T(1 + \tau_\theta\frac{\partial}{\partial t})T + d_n N), \tag{4}$$

$$\sigma_{zz} = (2\mu + \lambda)\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x} - (3\lambda + 2\mu)(\alpha_T(1 + \tau_\theta\frac{\partial}{\partial t})T + d_n N), \tag{5}$$

$$\sigma_{xz} = \mu(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}), \tag{6}$$

By Helmholtz's theorem, the displacement vector $\boldsymbol{u}$ can be written in the displacement scalar potential functions $\Pi(x, z, t)$ and $\psi(x, z, t)$, defined by the relations in the non-dimensional form:

$$\boldsymbol{u} = grad\, \Pi + curl\, \vec{\Psi}, \quad \vec{\Psi} = (0, \psi, 0) \tag{7}$$

this reduces to

$$u = \frac{\partial \Pi}{\partial x} + \frac{\partial \psi}{\partial z}, \qquad w = \frac{\partial \Pi}{\partial z} - \frac{\partial \psi}{\partial x}. \tag{i}$$

The field equations and constitutive relations at the plane surface of (equation (3)) in generalized linear thermoelasticity with the influence of gravity and without body forces and heat sources are

$$\rho\left(\frac{\partial^2 u}{\partial t^2} + g\frac{\partial w}{\partial x}\right) = \mu\nabla^2 u + (\mu+\lambda)\frac{\partial e}{\partial x} - \gamma(1+\tau_\theta\frac{\partial}{\partial t})\frac{\partial T}{\partial x} + (3\lambda+2\mu)d_n\frac{\partial N}{\partial x}, \tag{ii}$$

$$\rho\left(\frac{\partial^2 w}{\partial t^2} - g\frac{\partial u}{\partial x}\right) = \mu\nabla^2 w + (\mu+\lambda)\frac{\partial e}{\partial z} - \gamma(1+\tau_\theta\frac{\partial}{\partial t})\frac{\partial T}{\partial z} + (3\lambda+2\mu)d_n\frac{\partial N}{\partial z}, \tag{iii}$$

where g is the acceleration due to the gravity.

For simplicity, we will use the following non-dimensional variables

$$(x',z',u',w') = \frac{(x,z,u,w)}{C_T t^*}, \quad (t',\tau_q',\tau_\theta') = \frac{(t,\tau_q,\tau_\theta)}{t^*}, \quad (T',\varphi') = \frac{\gamma(T,\varphi)}{2\mu+\lambda},$$

$$\sigma_{ij}' = \frac{\sigma_{ij}}{\mu}, \quad N' = \frac{\delta_n N}{2\mu+\lambda}, \quad (\Pi',\psi') = \frac{(\Pi,\psi)}{(C_T t^*)^2}, \quad Q' = \frac{\gamma t^{*2}}{\rho k}Q, \quad g' = \frac{t^*}{C_T}g. \tag{8}$$

Hence, using scalar function (i) and equation (8) in equations (1)-(4) and (ii), (iii), we have (dropping the dashed for convenience).

$$(\nabla^2 - q_1 - q_2\frac{\partial}{\partial t})N + \varepsilon_3 T = 0, \tag{9}$$

$$(1+\tau_\theta\frac{\partial}{\partial t})\nabla^2 T - (1+\tau_q\frac{\partial}{\partial t})\frac{\partial T}{\partial t} - \varepsilon_2 N - (1+\tau_q\frac{\partial}{\partial t})\{\varepsilon_1\frac{\partial}{\partial t}\nabla^2\Pi - Q\} = 0, \tag{10}$$

$$(\nabla^2 - \frac{\partial^2}{\partial t^2})\Pi + g\frac{\partial\psi}{\partial x} - (1+\tau_\theta\frac{\partial}{\partial t})T - N = 0, \tag{11}$$

$$(\nabla^2 - \beta^2\frac{\partial^2}{\partial t^2})\psi - g\frac{\partial\Pi}{\partial x} = 0, \tag{12}$$

where,

$$q_1 = \frac{kt^*}{D_E \rho \tau C_e} \quad , \quad q_2 = \frac{k}{D_E \rho C_e}, \quad \varepsilon_1 = \frac{\gamma^2 T_0 t^{*2}}{k\rho}, \quad \varepsilon_2 = \frac{\alpha_T E_g t^*}{d_n \rho \tau C_e}, \quad \varepsilon_3 = \frac{d_n k \kappa t^*}{\alpha_T \rho C_e D_E},$$

$$C_T^2 = \frac{2\mu + \lambda}{\rho}, \quad C_L^2 = \frac{\mu}{\rho}, \quad \beta^2 = \frac{C_T^2}{C_L^2}, \quad \delta_n = (2\mu + 3\lambda)d_n, \quad t^* = \frac{k}{\rho C_e C_T^2}. \tag{13}$$

There, ε_1 represents the thermoelastic coupling parameter and ε_3 denotes the thermoelectric coupling parameter.

Stresses components in non-dimensional form becomes

$$\sigma_{xx} = \frac{(2\mu + \lambda)}{\mu}\frac{\partial^2 \Pi}{\partial x^2} + \frac{\lambda}{\mu}\frac{\partial^2 \Pi}{\partial z^2} + 2\frac{\partial^2 \psi}{\partial x \partial z} - \frac{(2\mu + \lambda)}{\mu}((1+\tau_\theta \frac{\partial}{\partial t})T + N), \tag{14}$$

$$\sigma_{zz} = \frac{(2\mu + \lambda)}{\mu}\frac{\partial^2 \Pi}{\partial z^2} + \frac{\lambda}{\mu}\frac{\partial^2 \Pi}{\partial x^2} - 2\frac{\partial^2 \psi}{\partial x \partial z} - \frac{(2\mu + \lambda)}{\mu}((1+\tau_\theta \frac{\partial}{\partial t})T + N), \tag{15}$$

$$\sigma_{xz} = \frac{\partial^2 \psi}{\partial z^2} + 2\frac{\partial^2 \Pi}{\partial x \partial z} - \frac{\partial^2 \psi}{\partial x^2}. \tag{16}$$

3 Solution of the problem

For a harmonic wave propagated in the direction, where the wave normal lies in the *xz*-plane. The solution of the considered physical variable can be decomposed in terms of normal modes as the following form:

$$[\Pi, \psi, \varphi, T, \sigma_{ij}, N](x,z,t) = [\Pi^*(x), \psi^*(x), \varphi^*(x), \theta^*(x), \sigma_{ij}^*(x), N^*(x)]\exp(\omega t + ibz), \tag{17}$$

$$Q = Q_0^* \exp(\omega t + ibz),$$

where ω be a complex circular frequency, i is the imaginary value, b is a wave number in the z-direction, Π^*, ψ^*, $N^*(x), \varphi^*(x), \theta^*(x)$ and $\sigma_{ij}^*(x)$ are the amplitude of the physical functions and $Q_0^* = Q_0$ is the magnitude of stable internal heat source.

By using the normal mode defined in the Eq. (17), equations (9)-(13), we arrive at a system of five homogeneous equations:

$$(D^2 - \alpha_1)N^* + \varepsilon_3\,\theta^* = 0\ ,\tag{18}$$

$$(D^2 - s_2)\theta^* + s_4\,N^* + \alpha_2(D^2 - b^2)\Pi^* = s_1 Q_0\ ,\tag{19}$$

$$(D^2 - \alpha_3)\Pi^* + gD\psi^* - s_5\,\theta^* - N^* = 0,\tag{20}$$

$$(D^2 - \alpha_4)\psi^* - gD\Pi^* = 0\ ,\tag{21}$$

Stress components equations (14)- (16) will take the following forms:

$$\sigma_{xx}^* = \alpha_5 D^2\Pi^* - \alpha_6\,b^2\Pi^* + 2ibD\psi^* - \alpha_5(\theta^* + N^*),\tag{22}$$

$$\sigma_{zz}^* = -\alpha_5 b^2\Pi^* + \alpha_6\,D^2\Pi^* - 2ibD\psi^* - \alpha_5(\theta^* + N^*)\ ,\tag{23}$$

$$\sigma_{xz}^* = 2ibD\Pi^* - (D^2 + b^2)\psi^*.\tag{24}$$

where

$$D = \frac{d}{dx},\ \alpha_1 = b^2 + q_1 + \omega q_2,\ \alpha_2 = \varepsilon_1\omega(1 + \tau_q\omega)\,,\ \alpha_3 = b^2 + \omega^2,\ \alpha_4 = b^2 + \omega^2\beta^2\ ,$$

$$s_1 = -\frac{s_3}{\omega}\ s_2 = b^2 + s_3\ ,\ s_3 = \frac{\omega(1 + \tau_q\omega)}{s_5},\ s_4 = \frac{\varepsilon_2}{s_5},\ s_5 = (1 + \tau_\theta\omega),\ \alpha_5 = \frac{(2\mu + \lambda)s_5}{\mu}\ ,\ \alpha_6 = \frac{\lambda}{\mu},$$

$$\Re = \alpha_1\xi_1 s_1 Q_0.\tag{25}$$

We get a sixth order equation by eliminating $\theta^*(x),\ \ \Pi^*(x),\ N^*(x)$ and $\psi^*(x)$ between Eqs. (18), (19), (20) and (21), we obtain the partial differential equation satisfied by $\Pi^*(x)$

$$[D^8 - ED^6 + FD^4 - GD^2 + H]\Pi^*(x) = \Re,\tag{26}$$

where

$$\xi_1 = s_5\alpha_4 + \varepsilon_3,\ \ \ \xi_2 = \alpha_4 + \alpha_3 g^2,\ \ \ \xi_3 = \alpha_3\alpha_4,$$

$$E = \xi_2 + \alpha_1 + s_2 - s_5\alpha_2,\tag{27}$$

$$F = \xi_3 + \alpha_1 s_2 + \xi_2 (\alpha_1 + s_2) - s_4 \varepsilon_3 - \alpha_2 (\xi_1 + s_5 (b^2 + \alpha_1)), \tag{28}$$

$$G = \xi_3 (\alpha_1 + s_2) + \xi_2 (\alpha_1 s_2 - s_4 \varepsilon_3) - \alpha_2 (\xi_1 \alpha_1 + b^2 (\xi_1 + s_5 \alpha_1)), \tag{29}$$

$$H = \xi_3 (\alpha_1 s_2 - s_4 \varepsilon_3) - \alpha_2 \alpha_1 b^2 \xi_1, \tag{30}$$

The above equation can be factorized

$$\left(D^2 - k_1^2 \right)\left(D^2 - k_2^2 \right)\left(D^2 - k_3^2 \right)\left(D^2 - k_4^2 \right)\Pi^*(x) = \Re, \tag{31}$$

where, $k_n^2 \ (n = 1,2,3,4)$ are the roots of the homogenous equation of (31) and the characteristic equation of the homogenous equation of (31), take the form:

$$k^8 - Ek^6 + Fk^4 - Gk^2 + H = 0. \tag{32}$$

The general solution of eq. (31), which is bounded as $x \to \infty$, is given by

$$\Pi^*(x) = \sum_{n=1}^{4} M_n(b, \omega) \exp(-k_n x) + L_1, \tag{33}$$

where L_1 is the particular solution of non homogenous equation (31), we obtain the value of L_1 is

$$L_1 = \frac{\Re}{H}. \tag{34}$$

In a similar way, we get

$$\theta^*(x) = \sum_{n=1}^{4} M_n'(b, \omega) \exp(-k_n x) + L_2, \tag{35}$$

$$N^*(x) = \sum_{n=1}^{4} M_n''(b, \omega) \exp(-k_n x) + L_3, \tag{36}$$

$$\psi^*(x) = \sum_{n=1}^{4} M_n'''(b, \omega) \exp(-k_n x) + L_4, \tag{37}$$

where, $L_2 = -\dfrac{\xi_3 \Re}{\xi_1 H}$, $L_3 = -\dfrac{\varepsilon_3 \xi_3 \Re}{\alpha_1 \xi_1 H}$, $L_4 = -\dfrac{g \Re}{\alpha_4 H}$. $\qquad$ (38)

So we can write the amplitude value of displacement components by using the amplitude of scalar potential functions as:

$$u^*(x) = D\Pi^* + ib\psi^*, \qquad (39)$$

$$w^*(x) = ib\Pi^* - D\psi^*, \qquad (40)$$

Using equations (33) and (37) in (39) and (40) we get the following relations,

$$u^*(x) = -\sum_{n=1}^{4} M_n(b,\omega) k_n e^{-k_n x} + ib(\sum_{n=1}^{4} M_n'''(b,\omega) e^{-k_n x} + L_4), \qquad (41)$$

$$w^*(x) = ib(\sum_{n=1}^{4} M_n(b,\omega) e^{-k_n x} + L_1) + \sum_{n=1}^{4} k_n M_n'''(b,\omega) e^{-k_n x}. \qquad (42)$$

Where M_n, M_n', M_n'' and M_n''' are some parameters depending on b and ω.

Substituting from equations (34)-(37) into equations (18)-(21), we have

$$M_n'(b,\omega) = H_{1n} M_n(b,\omega), \qquad n = 1,2,3,4, \qquad (43)$$

$$M_n''(b,\omega) = H_{2n} M_n(b,\omega), \qquad n = 1,2,3,4, \qquad (44)$$

$$M_n'''(b,\omega) = H_{3n} M_n(b,\omega), \qquad n = 1,2,3,4. \qquad (45)$$

Where

$$H_{1n} = \frac{(k_n^4 - \xi_2 k_n^2 + \xi_3)}{(s_5 k_n^2 - \xi_1)}, \qquad n = 1,2,3,4, \qquad (46)$$

$$H_{2n} = -\frac{\varepsilon_3 (k_n^4 - \xi_2 k_n^2 + \xi_3)}{(k_n^2 - \alpha_1)(s_5 k_n^2 - \xi_1)}, \qquad n = 1,2,3,4, \qquad (47)$$

$$H_{3n} = \frac{g k_n}{k_n^2 - \alpha_4}, \qquad n = 1,2,3,4. \qquad (48)$$

Thus, we have

$$\theta^*(x) = \sum_{n=1}^{4} H_{1n} M_n(b,\omega) \exp(-k_n x) + L_2, \tag{49}$$

$$N^*(x) = \sum_{n=1}^{4} H_{2n} M_n(b,\omega) \exp(-k_n x) + L_3, \tag{50}$$

$$\psi^*(x) = \sum_{n=1}^{4} H_{3n} M_n(b,\omega) \exp(-k_n x) + L_4 . \tag{51}$$

Substituting from equations. (49)- (51) and (33) into equations. (22)- (24), we get

$$\sigma_{xx}^* = \sum_{n=1}^{4} h_n M_n(b,\omega) \exp(-k_n x) - \varsigma_1, \tag{52}$$

$$\sigma_{zz}^* = \sum_{n=1}^{4} h_n' M_n(b,\omega) \exp(-k_n x) - \varsigma_2, \tag{53}$$

$$\sigma_{xz}^* = \sum_{n=1}^{4} h_n'' M_n(b,\omega) \exp(-k_n x) - \varsigma_3, \tag{54}$$

$$u^*(x) = -\sum_{n=1}^{4} (k_n + ibH_{3n}) M_n(b,\omega) e^{-k_n x} - ibL_4, \tag{55}$$

$$w^*(x) = \sum_{n=1}^{4} (ib + k_n H_{3n}) M_n(b,\omega) e^{-k_n x} + ibL_1. \tag{56}$$

where

$$h_n = \alpha_5 k_n^2 - \alpha_6 b^2 - 2ibk_n H_{3n} - \alpha_5 (H_{1n} + H_{2n}), \quad \varsigma_1 = \alpha_6 b^2 L_1 + \alpha_5 (L_2 + L_3), \tag{57}$$

$$h_n' = -\alpha_5 b^2 + \alpha_6 k_n^2 - 2ibk_n H_{3n} - \alpha_5 (H_{1n} + H_{2n}) \quad , \quad \varsigma_2 = \alpha_5 b^2 L_1 + \alpha_5 (L_2 + L_3), \tag{58}$$

$$h_n'' = 2ibk_n - (b^2 + k_n^2) H_{3n} \quad , \quad \varsigma_3 = -b^2 L_4. \tag{59}$$

4 Applications

In this section we determine the parameters $M_n (n = 1,2,3,4)$. In the physical

problem, we should suppress the positive exponentials that are unbounded at

infinity. The constants M_1, M_2, M_3, M_4 have to chosen such that the boundary conditions on the surface at $x = 0$ (The boundary conditions at the interface $x = 0$ is adjacent to vacuum) take the form:

1) A thermal boundary conditions that the surface of the half-space is subjected to

   thermal shock

$$T(x, z, t) = f(z, t), \qquad \frac{\partial T(0, z, t)}{\partial x} = 0. \tag{60}$$

2) A mechanical boundary condition that surface of the half-space is subjected to an arbitrary normal force p_1^* is

$$\sigma_{xx}(0, z, t) = -p_1^*, \tag{61}$$

where $p_1^* = p_1 \exp(\omega t + ibz)$, p_1 is the magnitude of mechanical force.

3) A mechanical boundary condition that surface of the half-space must be

   continuous at the boundary $x = 0$

$$\sigma_{xz}(0, z, t) = 0, \tag{62}$$

4) During the diffusion process, the carriers can reach the sample surface, with a finite probability of recombination. So the boundary condition for the carrier density can be given below:

$$\frac{\partial N(0, z, t)}{\partial x} = \frac{s}{D_e} N. \tag{63}$$

Substituting the expressions of the variables considered into the above boundary conditions, we can obtain the following equations satisfied by the parameters

$$\sum_{n=1}^{4} H_{1n} k_n M_n(b,\omega) \exp(-k_n x) = 0, \tag{64}$$

$$\sum_{n=1}^{4} h_n M_n(b,\omega) \exp(-k_n x) - \varsigma_1 = -p_1, \tag{65}$$

$$\sum_{n=1}^{4} h_n'' M_n(b,\omega) \exp(-k_n x) - \varsigma_3 = 0, \tag{66}$$

$$\sum_{n=1}^{4} H_{2n} k_n M_n(b,\omega) \exp(-k_n x) = -\frac{s}{D_e} N. \tag{67}$$

Invoking the boundary conditions (64)-(67) at the surface $x = 0$ of the plate, we obtain a system of four equations. After applying the inverse of matrix method (or Cramer's rule), we have the values of the four constants $M_j, j = 1,2,3,4$. Hence, we obtain the expressions of all physical quantities of the plate.

5 Particular cases

5.1. Neglecting gravitational effect (i.e. $g = 0$) in equations (i) and (ii), the expressions for displacements, force stresses, conductive temperature and temperature distribution reduces in a photothermal generalized thermoelastic medium with internal heat source.

5.2. Neglecting internal heat source (i.e. Q=0), we get the expressions for displacement, force stresses, conductive temperature and temperature distribution in a generalized thermoelastic medium with gravity.

5.3. When we put $\tau_\theta = 0$ in the above equations, it reduced to the equation of the LS theory.

6 Numerical Results and Discussions

The silicon (Si) material was chosen for these purposes of numerical evaluations. For which we have the physical constants as follows (parameters in SI unit) in

the table 1 (The MATLAB 2008a program was used on a personal computer. The accuracy main-trained was 7 digits for the numerical program)

$\lambda = 3.64 \mathrm{x} 10^{10}\, N/m^2$	$\mu = 5.46 \times 10^{10}\ N/m^2$	$\rho = 2330\, kg/m^3$	$\tau = 5\mathrm{x}10^{-5}\, s$
$T_0 = 800\ \mathrm{K}$	$d_n = -9\mathrm{x}10^{-31}\, m^3$	$D_E = 2.5 \times 10^{-3}\, m^2/s$	$E_g = 1.11\, eV$
$C_e = 695\, J/(kg\, K)$	$\alpha_t = 4.14\mathrm{x}10^{-6}\, K^{-1}$	$s = 2\, m/s$	$t = 2$
$\omega_0 = -0.3$	$P = 4$	$\xi = 0.1$	$z = 1$
$b = 0.9$	$\tau_\theta = 0.02\, s$	$\tau_q = 0.2\, s$	$p_1 = 2$

Table1: the physical constants in SI unit for the silicon (Si) material

Since we have $\omega = \omega_0 + i\xi$ where the imaginary unit is i, $e^{\omega t} = e^{\omega_0 t}(\cos \xi t + i \sin \xi t)$

and for small value of time, we can take $\omega = \omega_0$ (real). The numerical technique, outlined above, was used for the distribution of the real part of the thermal distribution T, the displacement components $u,\ w$, Carrier density N and the stress $(\sigma_{xx}, \sigma_{zz}, \sigma_{xz})$ distribution for the problem. The field quantities, temperature, displacement components and stress components depend not only on space x and time t but also on the thermal relaxation time τ_q, τ_θ. Here all the variables are taken in non dimensional forms and displayed graphically as 2D plots.

6.1. Comparison between LS theory and DPL model

The first group (figure 1), shows the curves predicted by two different theories of thermoelasticity. In this group, solid lines represent the solution in the LS theory, and the dashed lines represent the solution derived using the DPL model. The field quantities, temperature, displacement components and stress components depend not only on space x and time t but also on the thermal relaxation time τ_θ and τ_q. Here all the variables are taken in non dimensional forms. We notice that the results for the temperature, the displacement, stress distribution and Carrier density when the relaxation time is including in the heat equation are distinctly different from those when the relaxation time is not (phase-lag of temperature gradient $\tau_\theta = 0$) mentioned in heat equation, because the thermal waves in the Fourier's theory of heat equation travel with an infinite speed of propagation as opposed to finite speed in the non-Fourier case. This

demonstrates clearly the difference between the theory of thermoelasticity (LS) and DPL model.

6.2. Effect of gravitational field

The second group (figure 2), shows the variation of the temperature T, the displacements u, w, the stresses distribution σ_{xx}, σ_{xz} and the carrier density N distribution against horizontal distance x for different values of the gravity. It is seen from first shape that the distribution of the temperature T has a slight increase change with an increasing of the gravity. It is observed from this figure that T, starting from its positive values and fast increases to reach its highest value and smooth decreases to arrive the zero value as x tends to infinity. From the second shape, it is appear that the distribution of displacement u sharp decreases in the beginning to its minimum value and return arriving to the zero value with the increased values of the x axis; also, it decreases with an increasing of the gravity. It is depicted from the third shape that the displacement component w start from the positive values when $g = 5$ and $g = 9.8$, but start from negative value when $g = 0.0$, to reach a maximum value and the decreases tending to zero as x tends to infinity and it has the opposite behaviour of displacement u, increasing with an increasing of the earth gravity for small values of x axis. The fourth and fifth shapes, they are seen that the distribution of the stress (σ_{xx}, σ_{xz}) distribution does affect strongly by the difference values of the gravity while they have sharp decreases when start to arrive the minimum value then increases arriving to zero as x tends to infinity. The sixth shape, it is obvious that the Carrier density N has large variation with decreasing of g and tends to zero as x tends to infinity.

6.3. Effect of thermoelectric coupling parameter (photothermal effect)

The third group (figure 3), shows the thermoelectric coupling parameter ε_3 under DPL model and $g = 9.8$, (the case of different three values of thermoelectric coupling parameter) has a significant effect on all physical fields. With the decreases in the thermoelectric coupling parameter ε_3 causes increasing of the amplitude of T, also we have the inverse conclusion can be obtained for Carrier density N for DPL wave. The variations of physical quantities distribution are similar in nature for different value of thermoelectric coupling parameter. The value of these quantities approaches to zero when   the distance x increases (the effect of thermoelectric coupling parameter decreases with increases in horizontal distance). The displacement components u and the normal stress under different thermoelastic coupling parameters have the opposite behaviour, with the increase

in the thermoelastic coupling parameter a decrease in the displacement components u and inverse conclusion can be obtained for the normal stress.

6.4. Effect of Phase Lag of the Heat Flux (τ_q)

The fourth group (figure 4), displays the influence of the phase-lag of the heat flux τ_q (with fixed value of $\tau_\theta = 0.1$). First and fourth shapes shows the variations of temperature distribution T and Carrier density N with distance x. The pattern observed for $\tau_q = 1$, $\tau_q = 2$ and $\tau_q = 3$ are the same behaviour in nature with increases values which clearly reveals the effect of phase lag of the heat flux. The second shape depicts the variations of displacement component w with distance x. The behaviour of w with reference to x is same i.e. oscillatory for $\tau_q = 1$, $\tau_q = 2$ and $\tau_q = 3$ with difference in their magnitude. The variations of force stress σ_{xz} with distance x is depicted in three shape. The values of normal force stress σ_{xz} for $\tau_q = 1$, $\tau_q = 2$ and $\tau_q = 3$ show similar patterns with different degree of sharpness. i.e. the values for $\tau_q = 1$, $\tau_q = 2$ and $\tau_q = 3$ increases sharply in the beginning and then decreases alternately with distance x.

6.5. Effect of Phase-lag of Temperature Gradient (τ_θ)

Group 5 (figure 5), shows the comparison between the normal stress σ_{xx}, displacement component w, the stress σ_{xz} distribution and Carrier density N, the case of different three values of the phase-lag of temperature gradient τ_θ. The computations were carried out for a values of $\tau_q = 0.8$ and $g = 9.8$. Figure 5, exhibit the variation of all physical quantities with distance x for phase-lag of temperature gradient, in which we observe the following: significant difference in all physical quantities are noticed for different value of the non dimensional phase-lag of temperature gradient τ_θ and all curves similar oscillatory pattern with different degree of sharpness in magnitude. The value of all physical quantities converges to zero with increasing the distance x and satisfies the boundary conditions at $x = 0$.

6.6. Effect of internal heat source

The sixth group (figure 6), displays the variations of some physical variables with distance x in the presence and absence of internal heat source (i. e. with ($Q_0 = 10.0$) and without ($Q_0 = 0.0$) internal heat source) under DPL model and gravity. It is found that the presence of heat source has caused both decreasing and

increasing effects on the displacement and stress distributions. On the other hand, both thermodynamical and Carrier density N distributions are significantly increased due to presence of heat source, as can be seen from the plots as shown in figure 6 .

7 Conclusion

The dual-phase-lag (DPL) model plays a significant role on all the physical quantities, i.e. effect of phase-lag of heat flux (τ_q) and effect of phase-lag of temperature gradient (τ_θ) (effects of the thermal relaxation times) are observed and have strong effect on the harmonic functions. The influences of the coupling parameters (thermoelastic parameter and thermoelectric parameter) are evident under DPL theory. The presence of gravity is very sensitive in all the physical quantities. The method used in this work provides a quite successful in dealing with such problems. The effect of moving internal heat source has an essential role in changing the values of the some distributions. The method of normal mode gives exact solutions in the generalized thermoelastic medium without any assumed restrictions on the actual physical quantities that appear in the governing equations of the problem considered. This work can serve for the analysis and design of the thermal resistance coated materials, as well as many engineering practices related to interface analysis and design.

References

[1] M. A. Biot, Thermoclasticity and irreversible thermodynamics. J. Appl. Phys. 27, 240-253, (1956).

[2] P. Chadwick, In progress in solid mechanics, vol. I, edited by R. Hill and I.N.Sneddon, North Holland, Amsterdam, (1960).

[3] H.Lord, Y. Shulman, A generalized dynamical theory of thermoelasticity. J. Mech. Phys. Solids 15, 299–309, (1967).

[4] A.E. Green, K.A. Lindsay, Thermoelasticity. J. Elast. 2, 1–7 (1972) .

[5] D.S. Chandrasekharaiah, Thermoelasticity with second sound: a review. Appl. Mech. Rev. 39, 355–376 (1986)

[6] D.S. Chandrasekharaiah, Hyperbolic thermoelasicity: a review of recent literature. Appl. Mech. Rev. 51, 705–729 (1998)

[7] D. S. Chandrasekharaiah, Hyperbolic thermoelasticity: a review of recent literature, Appl. Mech. Rev. 51(12), 705–729, (1998).

[8] D. Y. Tzou, A unified approach for heat conduction from macro to microscales, J. Heat Transfer. 117, 8–16, (1995).

[9] D. Y. Tzou, The generalized lagging response in small-scale and high-rate heating, Int. J. Heat Mass Transfer. 38, 3231–3234, (1995).

[10] D. Y. Tzou, Macro-to microscale heat transfer: The lagging behavior, 1st Edition, Taylor & Francis, Washington (1996).

[11] J.P. Gordon, R.C.C. Leite, R.S. Moore, S.P.S.Porto, J.R.Whinnery, Long-transient effects in lasers with inserted liquid samples. Bull. Am. Phys. Soc. 119, 501 (1964).

[12] L.B. Kreuzer, Ultralow gas concentration infrared absorption spectroscopy. J. Appl. Phys. 42, 2934 (1971).

[13] A.C. Tam, Ultrasensitive Laser Spectroscopy, pp. 1–108. Academic Press, New York (1983).

[14] A.C. Tam, Applications of photoacoustic sensing techniques. Rev. Mod. Phys. 58, 381 (1986).

[15] A.C. Tam, Photothermal Investigations in Solids and Fluids, pp. 1–33. Academic Press, Boston (1989)

[16] D.M. Todorovic, P.M. Nikolic, A.I. Bojicic, Photoacoustic frequency transmission technique: electronic deformation mechanism in semiconductors. J. Appl. Phys. 85, 7716 (1999).

[17] Y.Q. Song, D.M. Todorovic, B. Cretin, P. Vairac, Study on the generalized thermoelastic vibration of the optically excited semiconducting microcantilevers. Int. J. Solids Struct. 47, 1871 (2010).

[18] SantwanaMukhopadhyay, Rajesh Prasad and Roushan Kumar, On the theory of Two-Temperature Thermoelasticity with Two phase-Lags. J. Thermal Stresses, 34 (4), 352-365, (2011).

[19] S. Chakravorty and A. Chakravorty,   Transient disturbances in a relaxing thermoelastic half space due to moving stable internal heat source. Int. J. Math. And Math. Sci, 21, pp. 595-602, (1998).

[20] R. Kumar and S. Devi, Thermomechanical interactions in porous generalized thermoelastic material permeated with heat source. Multidiscipline Modeling in Mat. and Str., 4, pp. 237-254, (2008)..

[21] Kh. Lotfy, Transient disturbance in a half-space under generalized magneto-thermoelasticity with a stable internal heat source under three theories. Mult., Mod., in Mat., Str., 7 No. 1, pp. 73-90, (2010).

[22] Kh. Lotfy, Transient thermo-elastic disturbances in a visco-elastic semi-space due to moving internal heat source. International Journal of Struc. Int., 2, pp. 264 – 280, (2011).

[23] M.I.A. Othman, State space approach to the generalized thermoelastic problem with temperature-dependent elastic moduli and internal heat sources. Journal of Appl. Mech. and Tech. Phys., 52, pp. 644-656, (2011).

[24] Nayfeh, A. and Nemat-Nasser, S. : Transient thermo-elastic waves in a half-space with thermal relaxation. J. Appl. Math. Phys., (ZAMP), 23, 50-68 (1972).

[25] I.A. Abbas, A Dual Phase Lag Model on Thermoelastic Interaction in an Infinite Fiber-Reinforced Anisotropic Medium with a Circular Hole, Mech. B. Des. Str. Mach., 43(4), 501-513, (2015).

[26] A.M. Zenkour and I.A. Abbas, Nonlinear Transient Thermal Stress Analysis of Temperature-Dependent Hollow Cylinders Using a Finite Element Model. Inter. J. Str. Sta. Dyn., 14(7), 1-17, (2014).

[27]I.A. Abbas, A GN model based upon two-temperature generalized thermoelastic theory in an unbounded medium with a spherical cavity. Applied Mathematics and Computation, 2014. 245: p. 108-115.

 [28] T. J. J. A. Bromwich, On the influence of gravity on elastic waves and in particular on the vibrations of an elastic globe. Proc. London Math. Soc. 30(1), 98–120, (1898).

[29] A. E. H. Love, Some Problems of Geodynamics, Dover Publishing Inc., New York (1911).

[30] S. N. De and P. R. Sengupta, Plane Lamb's problem under the influence of Gravity, Garlands. Beitr.Geophys. 82, 421–426, (1973).

[31] S. N. De and P. R. Sengupta, Influence of gravity on wave propagation in an elastic layer. J. Acoust. Soc. Am. 55(5), 919–921, (1974).

[32] S. N. De and P. R. Sengupta, Surface waves under the influence of gravity. Garlands. Beitr. Geophys. 85, 311–318, (1976).

[33] Y. Q. Song · J. T. Bai · Z. Y. Ren, Study on the reflection of photothermal waves in a semiconducting medium under generalized thermoelastic theory, Acta Mech, 223, 1545–1557 (2012)

[33] P. R. Sengupta and D. Acharya, The influence of gravity on the propagation of waves in a thermoelastic layer. Rev. Roum. Sci. Technol. Mech. Appl., Tome 24, 395–406, (1979).

[34] S. C. Das, D. P.Acharya and P. R. Sengupta, Surface waves in an inhomogeneous elastic medium under the influence of gravity. Rev. Roum. Des. Sci. Tech. 37(5), 539–551, (1992) .

[35] A. M. Abd-alla and S. M. Ahmed, Stonley and Rayleigh waves in a non-homogeneous orthotropic elastic medium under the influence of gravity, Appl. Math. Comput., 135, 187–200, (2003).

[36] M. Othman and Kh. Lotfy, The effect of magnetic field and rotation of the 2-D problem of a fiber-reinforced thermoelastic under three theories with influence of gravity, Mechanics of Materials, 60, pp. 120-143, (2013).

[37] A. M. Abd-alla, Influences of rotation, magnetic field, initial stress and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space, Appl. Math. Sci., 4(2), 91–108, (2010).

[38] M. Othman and Kh. Lotfy, The Influence of Gravity on 2-D Problem of Two Temperature Generalized Thermoelastic Medium with Thermal Relaxation, J., of Comp., and Theo., Nanoscience , 12,no 9, 2587-2600 (2015).

[39] Y. Q. Song · J. T. Bai · Z. Y. Ren, Study on the reflection of photothermal waves in a semiconducting medium under generalized thermoelastic theory, Acta Mech, 223, 1545–1557 (2012)
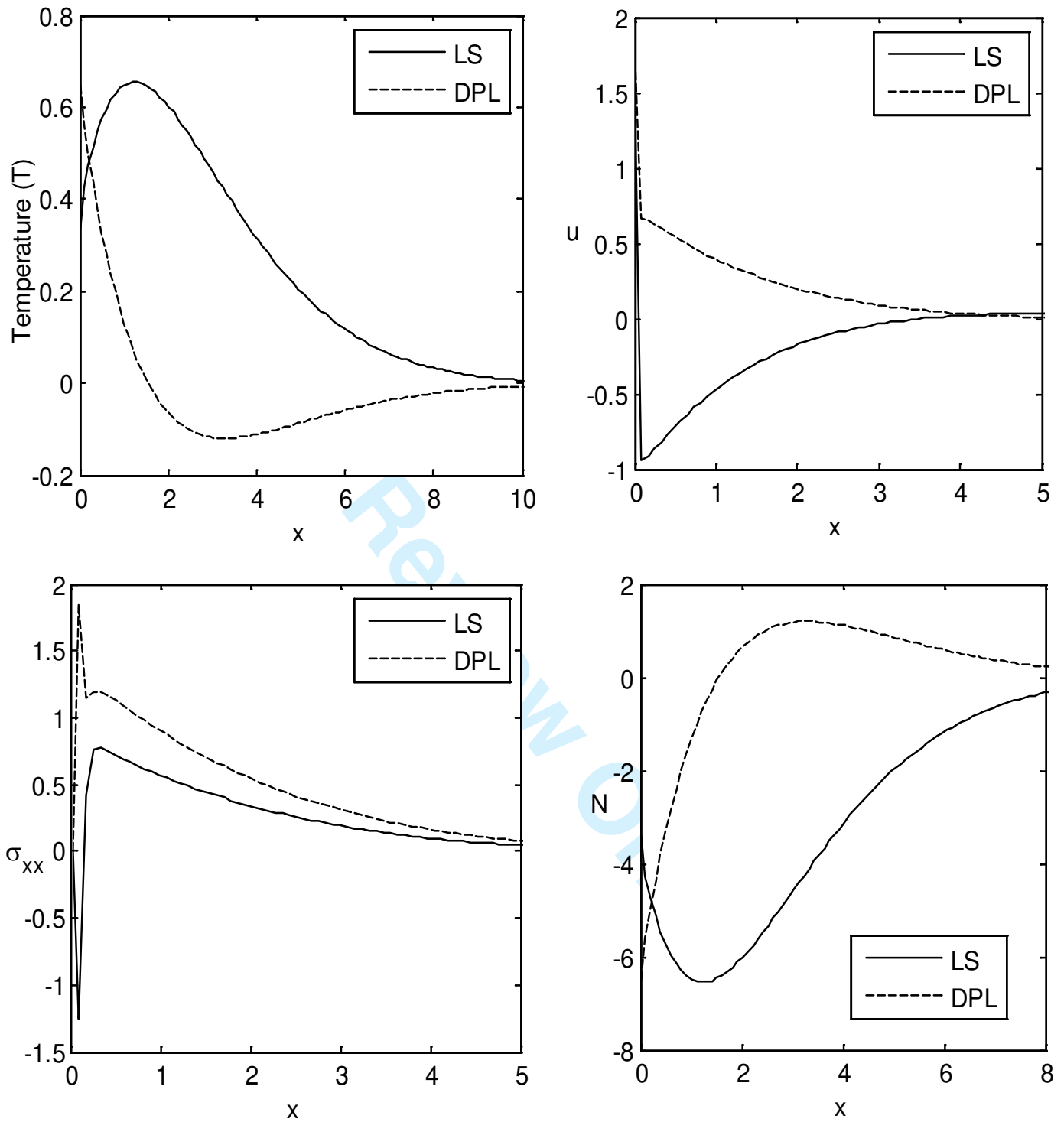
Figure 1 The thermal temperature, the displacement u, the stresses distribution σ_{xx} and the carrier density N distribution under LS theory and DPL model.
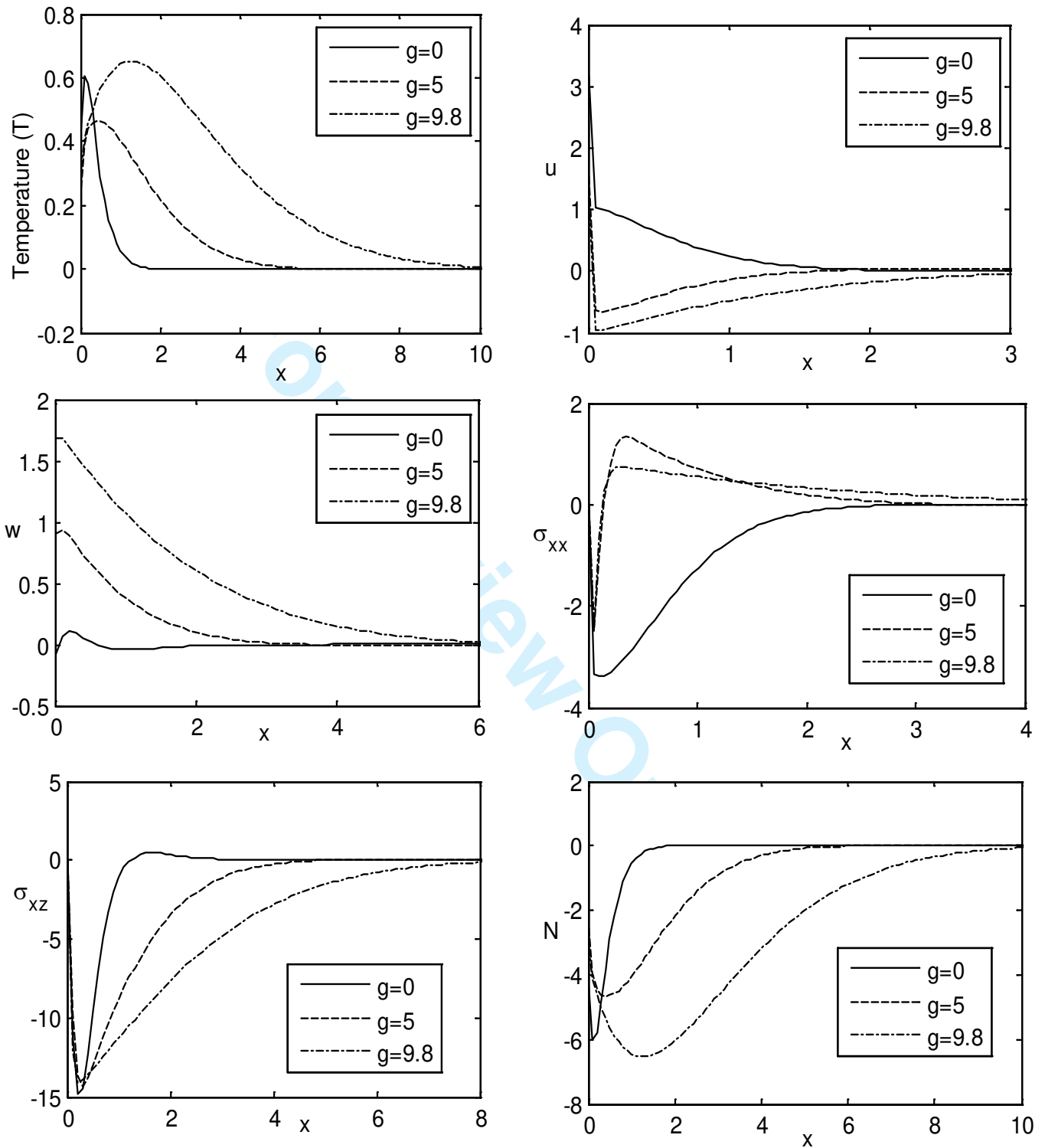
Figure 2 The thermal temperature, the displacement u, w, the stresses distribution σ_{xx}, σ_{xz} and the carrier density N distribution with different values of gravity.
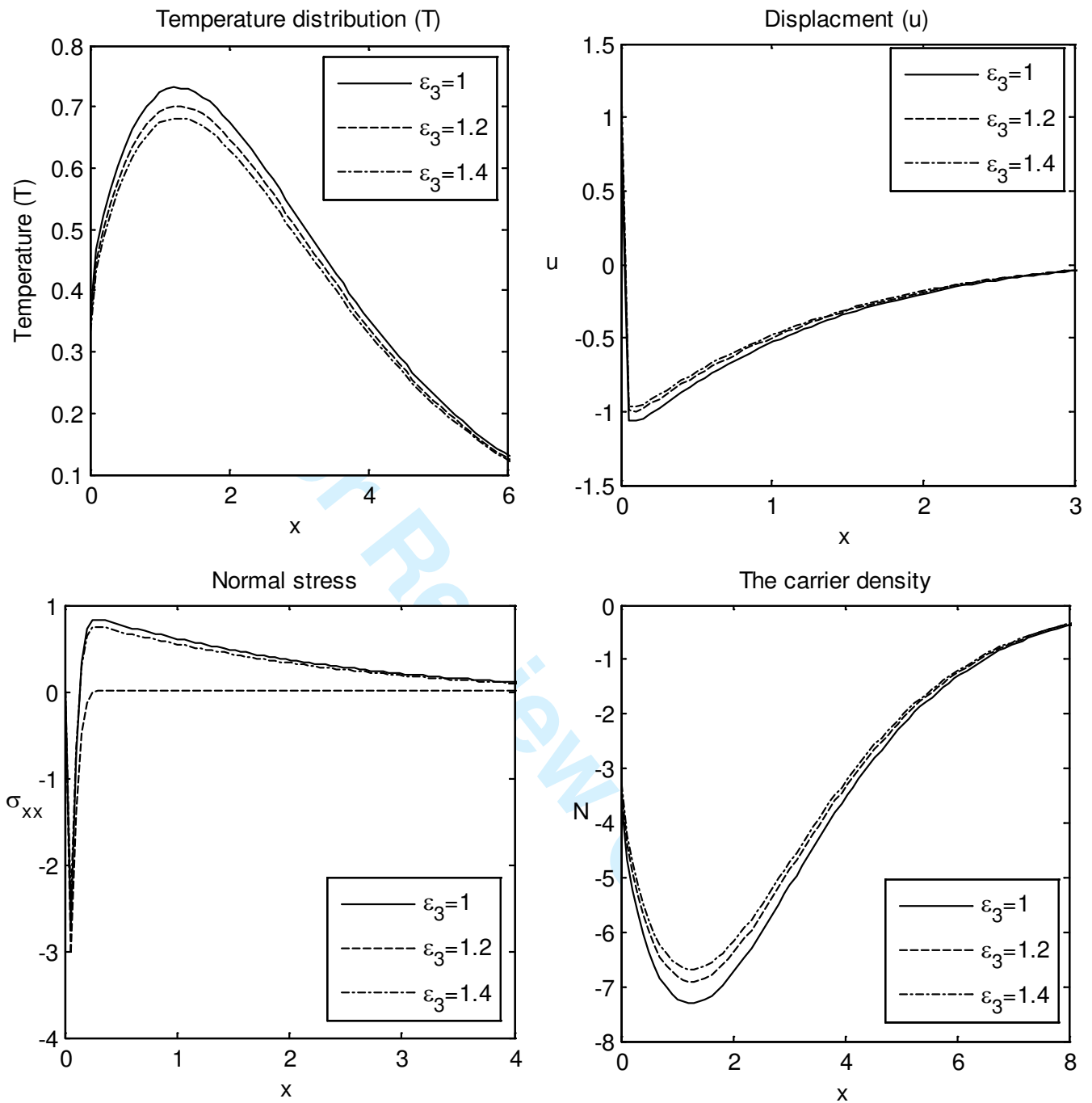
Figure 3 The temperature, the displacement components, normal force stress and the carrier density with the distance under different thermoelectric coupling parameters when $\varepsilon_1 = 1$.
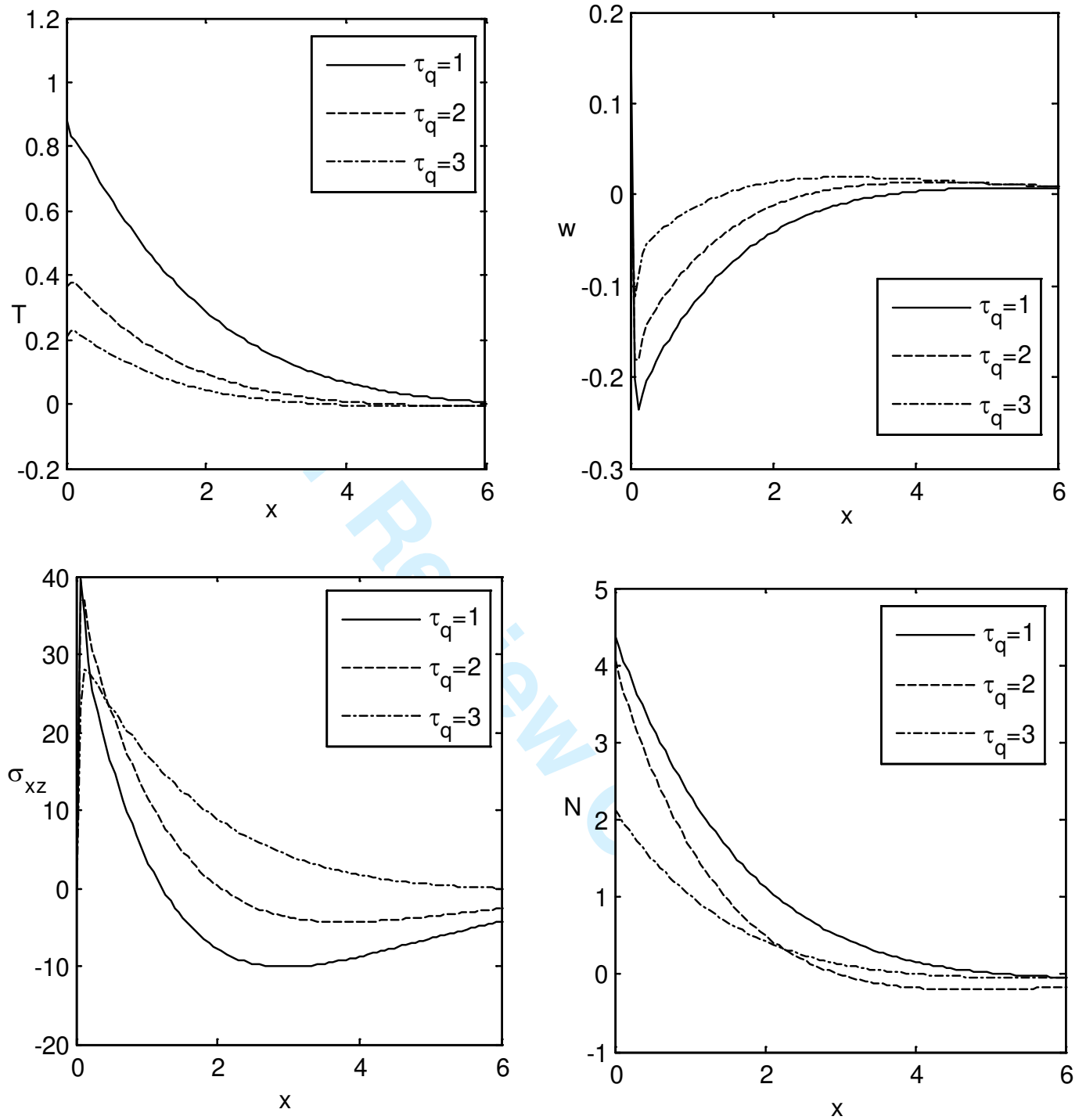
Figure 4 Variations of some physical quantities with distance x for phase-lag of heat flux at $\tau_\theta = 0.1$
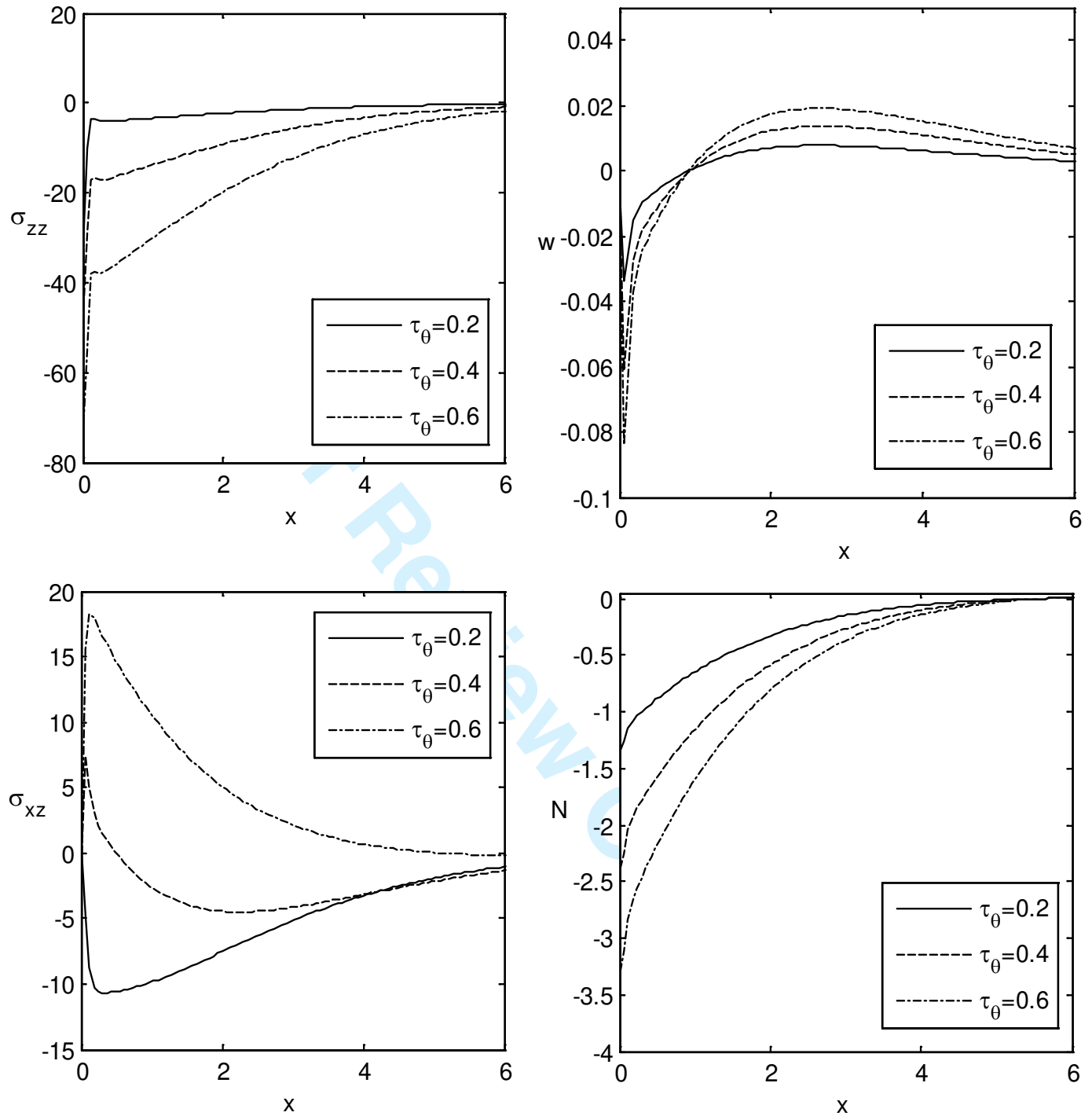
Figure 5 Variations of some physical quantities with distance x for phase-lag of temperature gradient at $\tau_q = 0.8$
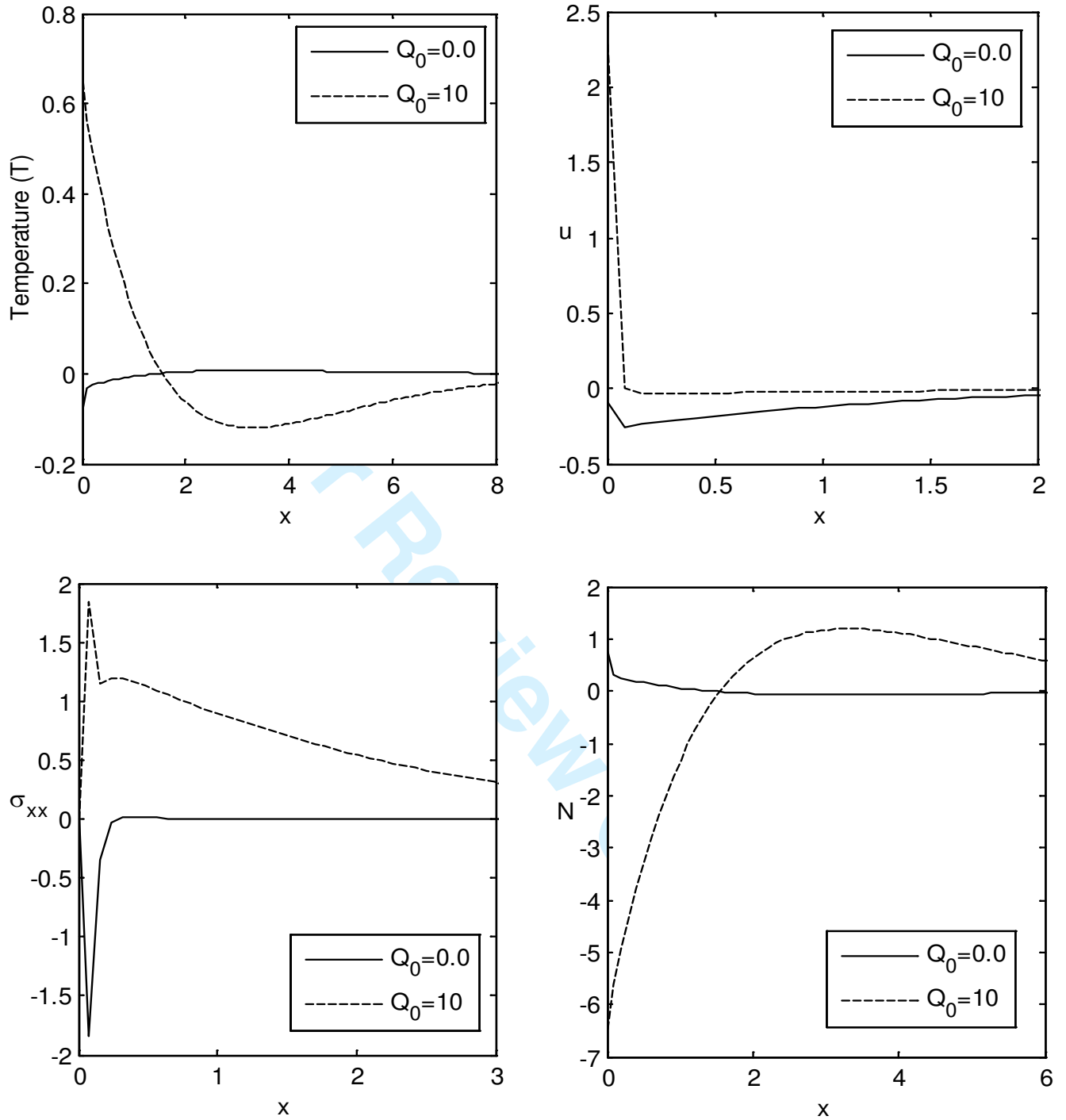
Figure 6 Variations of some physical quantities with distance x with and without internal heat source under DPL model.