The electric vehicle routing problem with backhauls

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CHRONICLE	ABSTRACT
	In the classical vehicle routing problem with backhauls (VRPB) the customers are divided into two sets; the linehaul and backhaul customers, so that the distribution and collection services of goods are separated into different routes. This is justified by the need to avoid the reorganization of the loads inside the vehicles, to reduce the return of the vehicles with empty load and to give greater priority to the customers of the linehaul. Many logistics companies have special responsibility to make their operations greener, and electric vehicles (EVs) can be an efficient solution. Thus, when the fleet consists of electric vehicles (EVs), the driving range is limited due to their battery capacities and, therefore, it is necessary to visit recharging stations along their route. In this paper the electric vehicle routing problem with backhauls (EVRPB) is introduced and formulated as a mixed integer linear programming model. This formulation is based on the generalization of the open vehicle routing problem considering a set of new constraints focussed on maintaining the arborescence condition of the linehaul and backhaul paths. Different charging points for the EVs are considered in order to recharge the battery at the end of the linehaul route or during the course of the backhaul route. Finally, a heuristic initialization methodology is proposed, in which an auxiliary graph is used for the efficient coding of feasible solutions to the problem. The operation and effectiveness of the proposed formulation is tested on two VRPB instance datasets of literature which have been adapted to the EVRPB.

1. Introduction

The vehicle routing problem with backhauls (VRPB) can be defined as the problem of determining a set of vehicle routes to visit all customer vertices, which are divided into two subsets. The first subset contains the vertices of the linehaul customers, each requiring a given quantity of products to be delivered. The second subset contains the backhaul customers, where a given quantity of inbound products must be picked up and transported to the depot. The VRPB objective is to determine a set of vehicle routes to visit all customers in order to satisfy the demand of goods. In such a case, the vehicles must attend first the customers with delivery requirements before the customers with collection requirements. This customer division is extremely frequent in practical situations in which it is required to avoid the permanent reorganization of the goods transported and the linehaul customers have a higher priority.

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Traditional subtour-elimination constraints fit perfectly into VRPs modeled with a single set of vertices, where the evaluation of the flow conservation and degree constraints can be made in a general way on all vertices. In VRPB, the precedence constraint stipulates that in each circuit the linehaul vertices precede the backhaul vertices. This leads to consider some special cases, such as the vertices at the end of a lineahaul route, the vertices at the start of a backhaul route and the routes with lineahaul customers only. Because of the above, and considering that the problem is known to be NP-hard in the strong sense, most of the existing literature about VRPB is related to heuristic and metaheuristic methodologies (Ropke & Pisinger, 2006). Few jobs concerning the exact approaches have been proposed and all of them focused on the inclusion of bounding techniques or set-partitioning models (Toth & Vigo, 2014).

Thus, we have approached the problem from another point of view; considering a representation of each part of the VRPB based on a generalization of the open vehicle routing problem (OVRP). The OVRP was first proposed in the early 1980s (Schrage, 1981; Bodin et al., 1983) when there were cases where a delivery company did not own a vehicle fleet or its fleet was inadequate for fully satisfying the customers' demand. Therefore, the contractors who were not employees of the delivery company used their own vehicles for the deliveries. In these cases, the vehicles were not required to return to the central depot after their deliveries because the company was only concerned with reaching the last customer. Thus, the goal of the OVRP is to design a set of Hamiltonian paths to satisfying customers' demand.

In the VRPB, the linehaul routes constitute a subproblem that has an arborescent configuration formed by a minimum spanning tree; starting from the depot, spanning all the linehaul customers, and ending up at a linehaul customer (Toro et al., 2017a, 2017b; Lourenco et al., 2002). Note that a spanning tree becomes a subgraph formed only by Hamiltonian paths if each customer node has a degree less than or equal to two. Similarly, the backhaul routes also have an arborescent configuration, entering the depot and spanning all the backhaul customers. Thus, the VRPB structure can be seen as OVRPs of linehaul and backhaul routes connected by tie-arcs.

With the progress of technology and ecological concerns, electricity has become a solid option for fuel replacement. Electric vehicles (EVs) are considered an alternative to implement in the transport sector, some advantages of using EVs are: i) the decrease of greenhouse gas release, ii) the reduction in the dependence of fossil fuels and iii) the little noise generated. However, the EVs still have to overcome some problems associated with the battery's autonomy, since the technology still needs to grow, and with the infrastructure of the charging stations, which are not yet installed massively. Thus, integrated planning of routes and charging stations is a problem that has been gaining great importance in the transport industry in the last years: Ge et al. (2011); Dharmakeerthi et al. (2012); Liu et al. (2013); Wang et al. (2013); Paz et al. (2018); Arias et al. (2017).

Several companies have already deployed electric delivery truck fleets. Generally, the fleet is made up of the kind of medium-duty commercial delivery trucks often used to deliver supplies to customers within one locality. It is a job particularly well-suited to electric trucks for several reasons: daily routes are often exactly the same, meaning range needs are fixed and predictable, and the vehicles always return to a charging station at night, making recharging easier. Additionally, because its parcel delivery trucks are not in operation overnight, the companies do not rely on public charging infrastructure (Electrification Coalition, 2012). Some studies analyze the actual use of EVs in commercial fleets from the point of view of the maximum necessary range of autonomy of the battery to cover most of the trips. In Pfriem and Gauterin (2013), the data suggests that about 90% of the mobile days could be covered with an EV range of 60 km and night recharge. They show a daily mobility far below their maximum range with long parking hours at night. Likewise there is no need for fast-charging.

Thus, a topic of great interest for transport companies with EV fleets is the planning of routes considering: i) an electric truck fleet, ii) a higher priority in the linehaul customers, iii) a slow recharge at a charging

point (CP) which is owned by the company (where the driver can rest or perform other activities) and iv) the return of the vehicle to the central depot serving backhaul customers.

This paper proposes a VRPB with a fleet consisting exclusively of EVs, where the customers with delivery requirements should not be affected by the recharge time of the battery in the charging stations, because these have a higher priority. The EVs must be recharged at the end of the linehaul route or during the course of backhaul route. Additionally it is important that the recharge takes place after the EV has covered a predetermined minimum distance, in order to take greater advantage of the initial charge of the battery. We have named this problem as electric vehicle routing problem with backhaul (EVRPB) and it is formulated as a mixed integer linear programming (MILP). The main characteristic of the proposed model is that the topological configuration of the solution is taken into account to efficiently eliminate the possibility of generating solutions formed by subtours. In order to solve the cases of greater complexity and size, a heuristic initialization methodology is proposed in which an auxiliary graph is used for the efficient decoding of feasible solutions to the problem given by a permutation.

The rest of the paper is organized as follows. In Section 2 we describe the literature review, presenting the contributions found. In Section 3 we present the problem formulation, presenting the nomenclature for the variables and parameters used in the mathematical model, also, we describe the model conditions and introduce the new mixed integer linear programming (MILP) formulation. In section 4, the initialization methodology, based on ILS metaheuristic and an auxiliary graph is presented. In Section 5 we present a computational study performed on 40 new proposed instances for the EVRPB. Finally, the conclusions are presented.

2. Literature review

Because the VRPB is NP-hard in the strong sense (Toth and Vigo, 2014), a lot of heuristic processes are appropriate for its solution and, therefore, most existing literature on the VRPB is related to heuristic and metaheuristic methodologies with high quality results. Two comprehensive reviews of metaheuristic techniques for VRPB are found in Ropke and Pisinger (2006). Two literature reviews cover the main works about VRPB: the first, presented by Toth and Vigo (2002), presents the existing work up to 2002 and the second, by Irnich et al. (2014) focuses on complementing the review up to 2013.

Goetschalckx and Jacobs-Blecha (1989) developed an integer programming formulation for the VRPB by extending the formulation of Fisher et al. (1986) to include pickup points. They develop a heuristic solution algorithm for this problem which, in turn, is broken into three subproblems. The first two subproblems correspond to the clustering decisions for the delivery customers and the pickup customers, which are independent generalized assignment problems. The third subproblem consists of solving K independent Traveling Salesman Problem (TSP) conformed by delivery and pickup customers, considering the precedence constraints, which impose a dependency relationship on all the model components.

The first exact method is reported by Toth and Vigo (1997), in which an effective Lagrangian bound is introduced that extends the methods previously proposed for the capacitated VRP (CVRP). The resulting Branch-and-Bound algorithm is able to solve problems with up to 70 customers in total. The second exact method is proposed by Mingozzi et al. (1999), in which a set-partitioning-based approach is presented and the resulting mixed integer linear programming (MIP) is solved through a complex procedure. The results show that the approach is capable of solving undirected problems with up to 70 customers. Toth and Vigo state that no exact approaches have been proposed for VRPB during the last decade (Toth and Vigo, 2014). In our review, we have reached the same conclusion and new proposals for unified exact models of VRPB were not found, since the only two existing proposals are used to derive the relaxations on which the exact approaches are based (Toth and Vigo, 1997).

Ropke and Pisinger (2006) proposed a unified model that is capable of handling most of the variants of the VRPB, they use different metaheuristic techniques for VRPB. Chávez et al. (2018), present a Tabu search metaheuristic to solve the routing problem, they divided it into sub-routes, one for linehaul customers and one for backhaul customers, in order to obtain a global solution for the minimum cost. Chávez et al. (2016), present a multiobjective ant colony algorithm for the Multi-Depot Vehicle Routing Problem with Backhauls (MDVRPB) where three objectives are minimized:

i) the traveled distance, ii) the traveling times and iii) the total consumption of energy.

Other two problems in the literature commonly handled by exact methods, where the backhaul load is considered, are: i) the mixed vehicle routing problem with backhauls (MVRPB) and ii) simultaneous pickups and deliveries. In the first, deliveries after pickups are allowed where the linehaul and backhaul customers are mixed along the routes. In the second, the customers may simultaneously receive and send goods. Although the differences between these two problems and the VRPB appear to be subtle, they are very different; direct comparisons between the problems serving pickups and deliveries in a mixed order or simultaneously with problems where the delivery is first and the pickup second should not be performed, since they are addressing different requirements. The VRPB is a problem with a special structure of the routes that consist of two distinct parts; a delivery and a pickup segment. A complete review of these two types of problems can be found in (Ropke & Pisinger, 2006; Wade & Salhi, 2003; Parragh et al., 2008).

A recent survey paper with interesting conclusions and research perspectives on the VRPB, including models, exact and heuristic algorithms, variants, industrial applications and case studies, are identified in (Koç & Laporte, 2017). In this review, the authors highlight the importance of using matheuristic algorithms that allow the interoperation of metaheuristic and mathematical programming techniques. Additionally, they identify the need for new studies focused on developing effective and powerful exact methods to solve all available standard VRPB instances to optimality. The authors also conclude that no electric vehicle version has yet been studied for the VRPB. The OVRP has recently received increasing attention in the literature and has focused mainly on the development of heuristic methods to find good quality solutions quickly. Regarding the exact methods, a branch-and-cut algorithm for the open version of the CVRP, addressing the capacitated problem with no distance constraints is proposed by Letchford et al. (2007). Pessoa et al. (2008) present several branch-cut-and-price algorithms on a number of vehicle routing problem variants, among which is the capacitated OVRP, which is addressed by setting the cost of all arcs that have the depot as the endpoint to zero. Salari et al. (2010) proposed a heuristic improvement procedure for the OVRP based on integer linear programming techniques to improve a feasible solution of a combinatorial optimization problem. Alinaghian et al. (2016) proposed a mathematical model in which open paths are used into the problem of cross-docks. To model the open path, a dummy node is defined, whose distance to other nodes is considered zero, and from which the route starts. A comprehensive literature review on the OVRP is presented in (Li et al., 2007; Toro et al., 2017b,a).

In relation to EVs, in the context of the VRP, Yang and Sun (2015) present an electric vehicle battery swap station location routing problem (BSS-EV-LRP), which aims to determine the location strategy of battery swap stations (BSSs) and the routing plan of a fleet of electric vehicles (EVs) simultaneously under battery driving range limitations, a four-phase formulated heuristic technique, called SIGALNS, is proposed to solve the problem. Goeke and Schneider (2015) propose the Electric Vehicle Routing Problem with Time Windows and Mixed Fleet (E-VRPTWMF) to optimize the routing of a mixed fleet of electric commercial vehicles (ECVs) which assume energy consumption to be a linear function of the distance traveled and the recharging times at stations by time windows. Arias et al. (2017) present a probabilistic approach for the optimal charging of electric vehicles (EVs) in distribution systems, where the costs of both demand and energy losses in the system are minimised, subjected to a set of constraints that consider EVs smart charging characteristics and operative aspects of the electric network.

Finally, regarding the decoding of a permutation in the context of the VRP, Ochi et al. (1998) adopt a representation where depots are used as trip delimiters. A more straightforward way is to use a sequence of customers without trip delimiters, as has been done for the CVRP by Liu et al. (2008). When the vehicles are homogeneous, Prins (2004) developed a polynomial time procedure for deliveringa single product, using the shortest path problem on an auxiliary acyclic graph. In (Vidal et al., 2012; Cattaruzza et al., 2014), the authors present a procedure based on an adaptation of the procedure proposed by Prins, which also works on an auxiliary graph. Due to the existence of precedence constraints, the limited battery capacity and the different types of existing vertices on the EVRPB, the procedure proposed in Prins (2004) cannot be directly used and it is modified as explained in Section 4.2.

3. Proposed model for the EVRPB

3.1. Problem formulation

Fig. 1 shows the optimal solution of an VRPB with 25 customers; in which the first 20 customers (numbered from 1 to 20 and represented by circles) are linehaul customers and the other 5 (numbered from 21 to 25 and represented by squares) are backhaul customers. The depot is the vertex 0 and the dotted lines indicate the connecting arcs that connect the linehaul with the backhaul customers. For this instance, the capacity of all vehicles is equal to $Q = 1550$. The minimum number of vehicles needed to serve all the linehaul and backhaul customers is $K_L = 8$ and $K_B = 2$, respectively. These values can be obtained by solving the bin packing problem instances associated with the corresponding customer subset, which calls for the determination of the minimum number of bins, each with capacity Q, needed to serve all customers (Toth and Vigo, 2002). To ensure feasibility, we assume that the number of vehicles needed K V must be greater than or equal to the maximum value between K_L and K_B. The demand (delivered or collected) of each customer is shown in the figure with the notation ($\cdot$). Thus, the basic version of the VRPB must satisfy the following conditions:

- Each vertex must be visited exactly once by a single route. That is, each vertex has degree 2.
- Each route starts and finishes at the depot.
- Each customer must be fully attended when visited.
- All customers are serviced from a single depot.
- The vehicle capacity should never be exceeded in both the linehaul and backhaul route and all vehicles should have the same capacity.
- In each circuit the linehaul vertices precede the backhaul vertices (precedence constraint), if any. That is:
  - A circuit of only backhaul customers is not allowed.
  - The last customer of a linehaul route is always connected to the depot or to a backhaul customer (BC) who is starting a backhaul route.
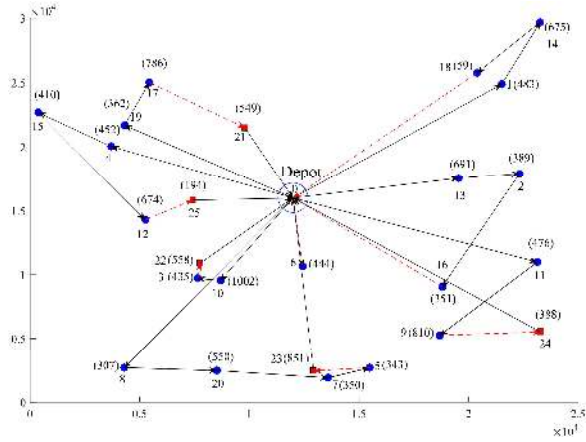 - The last BC of a backhaul route is always connected to the depot.

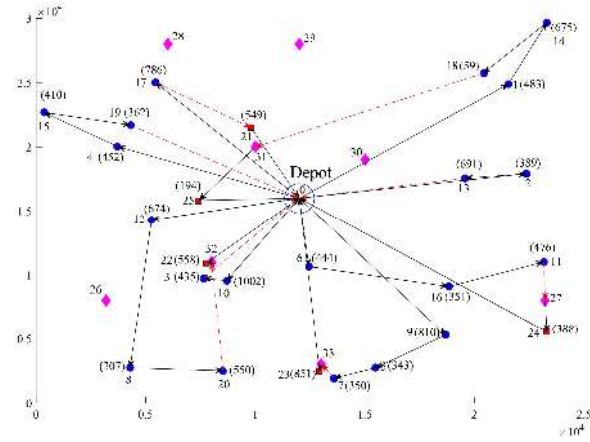**Fig. 1.** Vehicle routing problem with backhaul (VRPB)

Fig. 2. Electric vehicle routing problem with backhaul (EVRPB)

In the EVRPB, when the electric vehicle ends the linehaul route, the driver can follow several alternatives: i) start the backhaul route, ii) return directly to the depot, or iii) rest in the charging point and recharge the battery in slow mode until the next day. Figure 2 shows the optimal solution of an EVRPB where charging points are represented by diamonds, numbered from 26 to 33. Thus, when the charging points and battery life of the EV are considered, the EVRPB must, aditionally, satisfy the following conditions:

- Each charging point must be visited by one or more routes, or never be visited.
- The EVs are fully charged in the depot and in the charging points.
- The charging points are visited, only if it is necessary, at the end of the linehaul customers or during the course of the backhaul route.
- The charging stations are already built and their demand is equivalent to zero.

3.2. Nomenclature

The nomenclature for the sets, variables and parameters of the proposed model for the EVRPB is summarized next.

Sets:

L	Set of linehaul customers. $L = \{1, \ldots, n\}$.
B	Set of backhaul customers. $B = \{n+1, \ldots, n+m\}$.
K	Set of charging points. $K = \{n+m+1, \ldots, n+m+k\}$.
L_o	Set of linehaul customers and the depot $L_o = \{0\} \cup L$. Vertex 0 corresponds to the depot.
B_o	Set of backhaul customers, depot and charging points. $B_o = \{0\} \cup B \cup K$.
C_u	Set of linehaul and backhaul customers, including the charging points. $C_u = L \cup B \cup K$.
V	Set of Nodes $V = \{0\} \cup C_u$.

Parameters:

M_{ij}	Distance between nodes i and j.
C_{ij}	Cost of traveling between nodes i and j.
D_j	Nonnegative quantity of products to be delivered or collected (demand) of the customers $j \in C_u$.
K_V	Number of available vehicles (given in advance).
Q	Capacity in goods of the vehicles.
E^{max}	Electric capacity of the vehicles (identical vehicles).

Variables:

s_{ij}	Binary variable for the use of the path between nodes $i, j \in V$.
ξ_{ij}	Binary variable for the use of the path between nodes $i \in L$ and $j \in B_o$

l_{ij}	Continuous variable indicating the amount of goods transported between nodes i and j.
p_{ij}	Distance accumulated by the electric vehicle from the depot to the arc (i, j) nodes i and j.
p_j^L	Auxiliary variable that indicates the distance between the linehaul customers j (LC_j) and the depot. For a BC or a CP, this distance is denoted by the variables p_j^B and p_j^K, respectively.

3.3. Proposed Model for the EVRPB

The EVRPB can be defined as the following graph theoretic problem. Let $G = (V, A)$ be a complete and directed graph, where $V = 0 \cup C_u$ is the vertex set and A is the arc set. The vertex 0 denote the depot and vertex set C_u represents the feasible points that the EV can visit, once it leaves the depot. These feasible points are conformed by: the set of n linehaul customers (LCs), defined as $L = \{1, 2, \ldots, n\}$, the set of m backhaul customers (BCs), defined as $B = \{n + 1, , \ldots, n + m\}$ and the set of k charging points (CPs), defined as $K = \{n + m + 1, , \ldots, n + m + k\}$. Thus, $C_u = L \cup B \cup K$ where each vertex $j \in C_u$ is associated with a known nonnegative demand of goods D_j to be delivered or collected, considering that if $j \in K$ then $D_j = 0$. The depot has an unlimited fleet of identical vehicles with the same positive load capacity, denoted as Q, and the same electric capacity, denoted as E^{max}. The number K_V of vehicles for use is given in advance.

This mathematical formulation corresponds to a commodity flow model that uses two binary decision variables: s_{ij} that takes value 1 if arc $(i, j) \in A$ belongs to the optimal solution and ξ_{ij} that takes value 1 if the tie-arc between nodes $i \in L$ and $j \in B \cup 0$ is used. The tie-arcs connect the linehaul routes with the backhaul routes. The nonnegative flow variable l_{ij} is associated with the flow of goods transported by a vehicle through the arc $(i, j) \in A$. p_{ij} is a continuous variable indicating the EVs state of charge in distance units between nodes i and j. p_j^L is a continuous auxiliary variable that represents the distance between the linehaul node j and the depot.

(a) Without charging point   (b) CP in the backhaul route   (c) CP at the end of the linehaul route   (d) Only linehaul customers
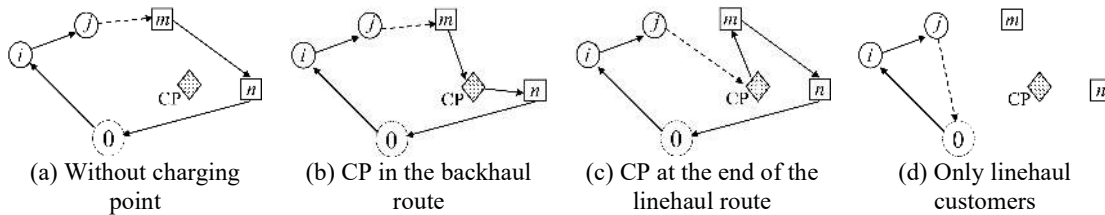
Fig. 3. Types of routes for the EVRPB

The commodity flow model (1)-(34) is an integer linear programming formulation of the EVRPB proposed. Figure 3, described the types of routes that can be found in the solution of EVRPB, where circles represent LCs, squares represent BCs and diamonds represent CPs. The EVRPB objetive is to minimize the total cost of routes needed to visit all customers or charging points.

$$\min = \sum_{i \in V} C_{ij} \times s_{ij} + \sum_{\substack{i \in L \\ j \in B_o}} C_{ij} \times \xi_{ij} \tag{1}$$

subject to

$$\sum_{\substack{i \in L_o \\ j \in L}} s_{ij} = |L| \tag{2}$$

$$\sum_{i \in L_o} l_{ij} - \sum_{k \in L} l_{jk} = D_j \qquad \forall j \in L \tag{3}$$

$$\sum_{i \in L_o} s_{ij} = 1 \qquad \forall j \in L \tag{4}$$

$$\sum_{k \in L_o} s_{jk} + \sum_{k \in B_o} \xi_{jk} = \sum_{i \in L_o} s_{ij} \qquad \forall j \in L \tag{5}$$

$$l_{ij} \leq Q \times s_{ij} \qquad \forall i \in L_o, \quad \forall j \in L \tag{6}$$

$$\sum_{j \in L_o} s_{0j} \geq \frac{\sum_{j \in L} D_j}{Q} \tag{7}$$

$$\sum_{i \in L} s_{0j} = K_V \tag{8}$$

$$p_j^L = \sum_{i \in L_o} M_{ij} * s_{ij} \qquad \forall j \in L \tag{9}$$

$$\sum_{i \in L_o} p_{ij} - \sum_{k \in V} p_{jk} = p_j^L \qquad \forall j \in L \tag{10}$$

$$p_{ij} \leq E^{max} \times s_{ij} \qquad \forall i \in L_o, \qquad \forall j \in L \tag{11}$$

$$p_{ij} \leq E^{max} \times \xi_{ij} \qquad \forall i \in L, \qquad \forall j \in B \tag{12}$$

$$p_{0j} = E^{max} \times s_{0j} \qquad \forall j \in L \tag{13}$$

$$p_{j0} \geq D_{j0} \times \xi_{j0} \qquad \forall j \in L \tag{14}$$

$$\sum_{\substack{i \in B \\ j \in B_o}} s_{ij} = |B| \tag{15}$$

$$\sum_{i \in B} l_{ij} - \sum_{k \in B_o} l_{jk} = -D_j \qquad \forall j \in B \tag{16}$$

$$\sum_{j \in B_o} s_{ij} = 1 \forall i \in B \tag{17}$$

$$\sum_{k \in B} s_{ki} + \sum_{j \in L} \xi_{ji} + \sum_{m \in K} s_{mi} = \sum_{j \in B_o} s_{ij} \quad \forall i \in B \tag{18}$$

$$l_{ij} \leq Q \times s_{ij} \qquad \forall i \in B_o, \qquad \forall j \in B_o \tag{19}$$

$$\sum_{i \in B} s_{i0} \geq \frac{\sum_{j \in B} D_j}{Q} \tag{20}$$

$$\sum_{i \in B} s_{i0} + \sum_{i \in L} \xi_{i0} = \sum_{j \in L} s_{0j} \tag{21}$$

$$p_j^B = \sum_{i \in B} M_{ij} s_{ij} + \sum_{i \in L} M_{ij} \xi_{ij} + \sum_{i \in K} M_{ij} s_{ij} \qquad \forall j \in B \tag{22}$$

$$\sum_{i \in C_u} p_{ij} - \sum_{k \in B_o} p_{jk} = p_j^B \qquad \forall j \in B \tag{23}$$

$$p_{ij} \leq E^{max} \times s_{ij} \qquad \forall i \in B, \qquad \forall j \in B_o \tag{24}$$

$$p_{j0} \geq D_{j0} \times s_{j0} \qquad \forall j \in B \tag{25}$$

$$\sum_{k \in B} s_{ki} + \sum_{j \in L} \xi_{ji} = \sum_{j \in B} s_{ij} \qquad \forall i \in K \tag{26}$$

$$\sum_{i \in B} l_{ij} - \sum_{k \in B_o} l_{jk} = 0 \qquad \forall j \in K \tag{27}$$

$$p_j^K = \sum_{i \in B} M_{ij} \times s_{ij} + \sum_{i \in L} M_{ij} \times \xi_{ij} \qquad \forall j \in K \tag{28}$$

$$\sum_{i \in C_u} p_{ij} - \sum_{k \in K_o} p_{jk} = p_j^K \qquad \forall j \in K \tag{29}$$

$$p_{ij} \leq E^{max} s_{ij} \qquad \forall i \in K, \qquad \forall j \in B \tag{30}$$

$$s_{ij} + s_{ji} \leq 1 \qquad \forall i \in V, \qquad \forall j \in V \tag{31}$$

$$\xi_{ij} \in \{0,1\} \qquad \forall i \in L, \qquad \forall j \in B_0 \tag{32}$$

$$s_{ij} \in \{0,1\} \qquad \forall i,j \in V \tag{33}$$

$$l_{ij} \in R \qquad\qquad \forall i,j \in V \qquad\qquad\qquad (34)$$

The objective function (1) minimises the operating costs and consist of 2 terms. The first corresponds to the sum of the total travelling cost of the routes used to deliver and collect the goods and to visit the charging points. The second corresponds to the use of the tie-arcs connecting the last customer of a linehaul route to the backhaul customer, to the charging point or to the depot.

The sets of constraints (2)-(8) allow modelling the OVRP for linehaul routes, where (2) and (3) impose the connectivity requirements. In the optimal solution of the OVRP, each route has an arborescent configuration formed by a minimum spanning tree; starting from the depot, spanning all the nodes, and ending at a customer. We have named this subproblem the linehaul open vehicle routing problem (LOVRP). In the vehicle routing problem context, the necessary condition for obtaining a minimum spanning tree is that the number of arcs be equal to the number of customer nodes. This necessary condition is guaranteed by the equality constraint (2), where the number of customer nodes is given by the cardinality of the set L. However, this constraint is necessary but not sufficient because there may be customer nodes with a degree greater than two and disconnected solutions can be obtained.

A spanning tree becomes a subgraph formed only by Hamiltonian paths if each customer node has a degree less than or equal to two. Therefore, another necessary condition is given by the sets of degree constraints (4) and (5). The indegree constraints (4) impose that exactly one arc enters each customer node and, consequently, the outdegree constraints (5) impose that exactly one arc leaves each LC, considering two situations: i) a tie-arc can only go from a LC towards a BC or towards the depot and ii) only a arc coming from a LC or from the depot can arrive at a LC. However, the addition of these degree constraints in directed graphs may not represent a spanning tree, because a disconnected graph can be obtained.

The addition of the flow balance constraint by each customer node avoids getting disconnected solutions, since an infeasible solution is obtained when the goods leaving the depot can not reach the LCs. Thus, the set of constraints reported in (3) guarantees network connectivity through the flow conservation constraint for each LC so that they are fully served when visited. Similarly, the constraints (16) and (27) guarantees network connectivity through the balance of the demand flow by each BC and charging point, respectively. Note that in the constraints (27) the demand for the CP is considered to be 0.

The constraints (6) and (7) impose both the vehicle and depot capacity requirements, respectively. The first is an upper limit defined by the capacity of the vehicle to transport a quantity of products on any linehaul-arc, while the second is a lower limit to the number of routes out of the depot to supply linehaul customers, which is determined by the ratio between the total demand to be collected and the vehicle capacity. Constraint (8) limits the minimum number of vehicles used on linehaul routes.

Similarly to the sets of constraints (2)-(8), are established the sets of constraints (15)-(20) for modeling the OVRP for backhaul routes (BOVRP). The set of constraints (21) ensures that the number of arcs leaving the depot is equal to the number of arcs coming to depot. Comparing (21) and (8) one can see that the number of linehaul arcs leaving the depot may be different from the number of backhaul arcs arriving at the depot. This case occurs when there are tie-arcs between a linehaul route and the depot.

The sets of constraints (9)-(14) represent the limitations of EVs when crossing a route of LCs. The constraints (9) and (10) guarantee the fulfillment of the distance balance constraint on a LCs route, which is necessary for the calculation of the accumulated distance at the moment of crossing every arc (i, j) of the optimal solution. Similarly, constraints (22) and (23) guarantee the fulfillment of the distance balance constraint on a BCs route, and (28) and (29) do the same for the set of vertices that are CPs.

The sets of constraints (11) and (12), ensure that when an arc between LCs or a tie-arc is crossed, respectively, the maximum capacity of the vehicles battery, in terms of distance, is not exceeded.

Similarly, the sets of constraints (24) and (30) verify compliance with this same electrical capacity constraint when an arc between BCs or between a CP and a BC is crossed, respectively.

The set of equations (13) ensures that the EV leaves the depot with the battery fully charged. The return to the depot is always done through a tie-arc or an arc coming out of a backhaul node. Therefore, the constraints (14) ensure that the battery charge is sufficient to return to the depot through a tie-arc. The constraints (25) do this same verification when the EV returns to the depot through an arc that leaves a node backhaul.

The set of equations (26) imposes that the number of arcs arriving and leaving a CP is the same, considering two situations: i) that a tie-arc from an LC or BC can arrive at a CP and ii) that from a CP an arc can only be connected to a BC. The direct return from a CP to the depot is not allowed since the objective is to take advantage of the total charge of the EV to make a backhaul route, and not only to return to the depot. Note that this constraints are similar to (18), which impose that exactly one arc leaves each BC visited. In (18) two situations are considered: i) that an arc that arrives at a BC can only come from another BC, from a tie-arc that leaves an LC or from a CP, and ii) that an arc from a BC can only be connected to another BC or to the depot.

Finally, the constraints (31) ensure that only one of the two variables s_{ij} or s_{ji} be used. Constraints (32) and (33) define all binary decision variables, and constraints (34) define the real variables. The mathematical model (1)-(34) can represent the classic VRPB, when the capacity of the battery is not considered ($E^{\max}$ large enough).

4. Proposed model for the EVRPB

The computational results obtained on several test instances, show that some cases configure a highly restricted problem, where obtaining a feasible integer initial solution requires high computational times. This can be evidenced in Tables 1 and 2, in Section 5. Thus, the purpose of the initialization phase is to quickly find a feasible integer initial solution through an efficient heuristic algorithm in order to provide an initial upper bound to the exact algorithm used by the commercial solver.

An iterated local search (ILS) algorithm is used as an initialization methodology, whose main characteristic is to apply inter-route and intra-route movements to explore the search space generated by the solution encoding strategy. One of the key aspects in the implementation of an efficient ILS is to properly define the solution encoding strategy. In the TSP, for example, a sequence or permutation of customers turns out to be a natural and efficient representation of a feasible solution to the problem, which provides the order in which the customers (cities) should be visited and does not require additional processes of feasibility or split. In the context of the VRP, Prins (2004) presents an optimal splitting procedure (OSP) of a permutation, which in a simple and efficient way allows to obtain a solution conformed by feasible routes that leave and arrive at the depot. The algorithm consists of transforming the VRP into the shortest path problem (SPP) using an auxiliary graph constructed from the evaluation of all possible routes resulting from following the sequence given by the permutation. Therefore, a permutation generates multiple feasible solutions to the VRP, but only the best of them is chosen through the optimal solution of the SPP. Finally, the feasibility of each route in the VRP is determined by compliance with two constraints: i) vehicle capacity and ii) maximum distance traveled.

In this paper, the encoding of a solution of the EVRPB is done through a sequence of customers without trip delimiters and a modified OSP is used to decode it.

4.1. Initial solution

In the context of the EVRPB, a randomly generated permutation (solution) can cause a conflict with the existing precedence constraint; e.g., if the first element of the permutation corresponds to a BC then the

solution is infeasible because trips leaving the depot directly to BC are not allowed. The same happens when the first element of the permutation corresponds to a CP.

To improve the robustness of the initial solution, a greedy strategy is proposed, where the objective is to obtain a quick and simple solution that prioritizes the following conditions: i) the trip starts with an LC, ii) from a vertex i, the nearest vertex j must be chosen as destination, where $j \in C_u : j = \min\{M_{i,j} \ \forall i \in Cu\}$, and iii) the compliance of the battery autonomy constraint must be guaranteed. The general structure of the strategy is shown in Algorithm 1.

Algorithm 1. Greeddy strategy pseudo code

Input: EVRPB	10: **else** $\{j \in L\}$
	From the vertex j, identify the
Output: Initial solution Π	11: nearest vertex $NV \in V$
1: $n \leftarrow |C_u|$	12: **end if**
2: $j \leftarrow$ pick a random linehaul customer from L	13: $AD \leftarrow AD + M_{j,NV}$
3: Set the accumulated distance $AD := 0$	14: $j \leftarrow NV$
4: Set $\Pi := \phi$	15: **if** $(AD \geq E^{max})$ and $(V \neq \phi)$ **then**
5: **for** i = 1 to n **do**	16: $AD \leftarrow 0$
6: $\Pi_i \leftarrow j$	17: $j \leftarrow$ pick a random vertex from V
7: $V = V - \{j\}$	18: **end if**
8: **if** $j \in B \cup K$ **then**	19: **end for**
From the vertex j, identify the	20: return Π
9: nearest vertex $NV \in$ $B \cup K$	

4.2. Optimal splitting procedure with backhaul (OSPB)

The main characteristic of the proposed OSPB is that, in addition to feasibility criteria based on the vehicle capacity, the following are also considered: i) the autonomy of the battery, ii) the precedence constraint, and iii) the minimum number of vehicles given in advance.

Given a solution $\Pi = permutation(C_u)$, then obtaining the value of the fitness function requires the construction of an auxiliary graph $H = (V, A', T)$. V is the set of vertices indexed from 0 to $|C_u|$. A' is the arc set, where each $arc(i, j)$ represents a feasible trip in which the EV departs from node 0 (depot) and visits nodes $i + 1, i + 2, i + 3, \dots, j - 1$, and j, consecutively. Thus, the feasible trip visiting vertices $v = \Pi_{i+1}$ to $w = \Pi_j$, in the order they are in Π, is denoted as $T^{v,w} \in T$. The set A' can contain a maximum number of trips $nt = \frac{1}{2}|C_u|(|C_u| - 1)$. Thus, T is the set of trips, where a trip $T^{v,w}$ is conformed, in turn, by u vertices, which can be LCs, BCs and/or CPs, arranged in any order. The trip distance associated with $arc(i, j)$, $z_{i,j}$, is calculated according to the following equation:

$$z_{i,j} = M_{0,v} + \sum_{k=i+1}^{i+u} M_{\Pi_k, \Pi_{k+1}} + M_{w,0} \tag{35}$$

However, there are different conditions that must be fulfilled so that a trip of the auxiliary graph is feasible and so that, in turn, there is a feasible solution to the SPP.

4.2.1. Trip feasibility

Constraints of precedence and vehicle capacity: according to the precedence constraint, a necessary condition for a trip $T^{v,w}$ to be feasible is that the first vertex corresponds to an LC. Additionally, a trip is still feasible if the first $t \leq u$ customers of the trip $T^{v,w}$ are LCs (see condition (36)) and the sum of their

demands, $DL_{i,j}$, does not exceed the capacity of the vehicle (see constraint (38)). Another necessary conditions for trip feasibility is that after visiting the LCs only BCs or CPs can be visited (see condition (37)) and the sum of their demands, $DB_{i,j}$, does not exceed the capacity of the vehicle (see constraint (39)).

$$\{v, \Pi_{i+2}, \Pi_{i+3}, \dots, \Pi_{i+t}\} \subseteq L \tag{36}$$

$$\{\Pi_{i+t+1}, \Pi_{i+t+2}, \dots, w\} \subseteq \{B \cup K\} \tag{37}$$

$$DL_{i,j} = \sum_{k=i+1}^{i+t} D_{\Pi_k} \leq Q \tag{38}$$

$$DB_{i,j} = \sum_{k=i+t+1}^{i+t+u} D_{\Pi_k} \leq Q \tag{39}$$

Battery autonomy constraint: the trip $T^{v,w}$ may contain c charging points in positions $\{q_1, q_2, \dots q_c\}$, $q_1 < q_2 < \dots < q_c$. When an EV visits a CP its battery is fully charged, which implies that the battery consumption, in terms of distance, is equivalent to the maximum distance, $ED_{i,j}$, traveled between CPs (including the depot). Therefore, the autonomy of the battery is guaranteed with the constraint (40).

$$ED_{i,j} = \max\left\{M_{0,v} + \sum_{k=i+1}^{i+q_1-1} M_{\Pi_k,\Pi_{k+1}}; \sum_{k=i+q_1}^{i+q_2-1} M_{\Pi_k,\Pi_{k+1}}; \dots \sum_{k=q_{c-1}}^{q_c-1} M_{\Pi_k,\Pi_{k+1}} + M_{w,0}\right\} \leq E^{max} \tag{40}$$

4.2.2. Construction of the auxiliary graph

A simple example with three LCs, $L = \{1,2,3\}$, three BCs, $B = \{4,5,6\}$, and three CPs, $K = \{7,8,9\}$ is given in Fig. 4, which shows different types of solutions, in the EVRPB context, that can be obtained from the decoding of a permutation. Fig. 4a shows a grand tour, given by the permutation $\Pi = \{3,2,7,6,1,8,4,9,5\}$, that starts and ends at the depot. This solution can be obtained directly from the sequence given by the permutation without the need for a split procedure. Note that this initial permutation can be obtained using Algorithm 1. It is assumed that all the arcs have a distance of 20 km, except $M_{4,9} = 50$ km. Additionally, it is considered that $M_{0,j} = 20km \; \forall j \in V$, $E^{max} = 60$ and $Q = 15$. Fig. 4a shows in parentheses the quantity of products that will be delivered in the LCs or that will be collected in the BCs. Fig. 4b shows a feasible solution of EVRPB consisting of two routes. This solution is feasible because the load capacity of the vehicle is not exceeded and the autonomy of the battery is guaranteed with the charge points 7 and 8. However, despite complying with the two previous capacity constraints, an infeasible solution can be obtained when there is a trip from the depot to a BC, as shown in Fig. 4c. The step-by-step construction of the auxiliary graph is shown in Fig. 5.
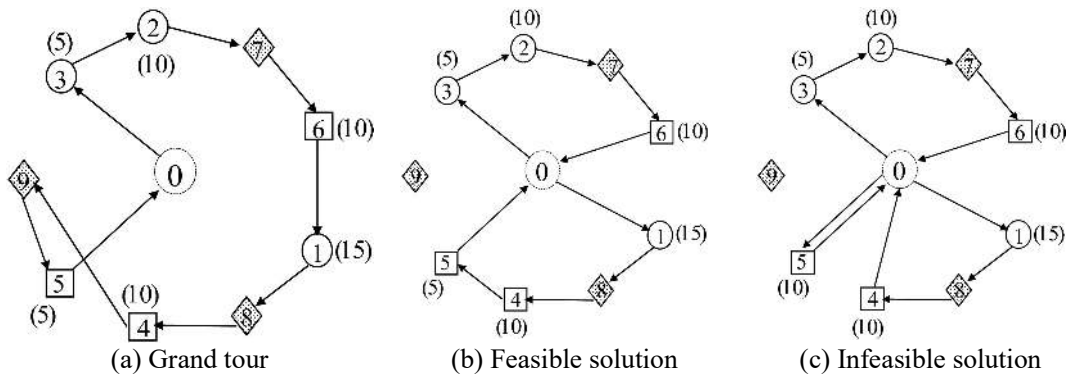

(a) Grand tour   (b) Feasible solution   (c) Infeasible solution

Fig. 4. Types of solution

(a) Step One: express delivery to a LC

(b) Step Two: feasible trips from the permutation

(c) Step three: infeasibility reduction by shifting a CP

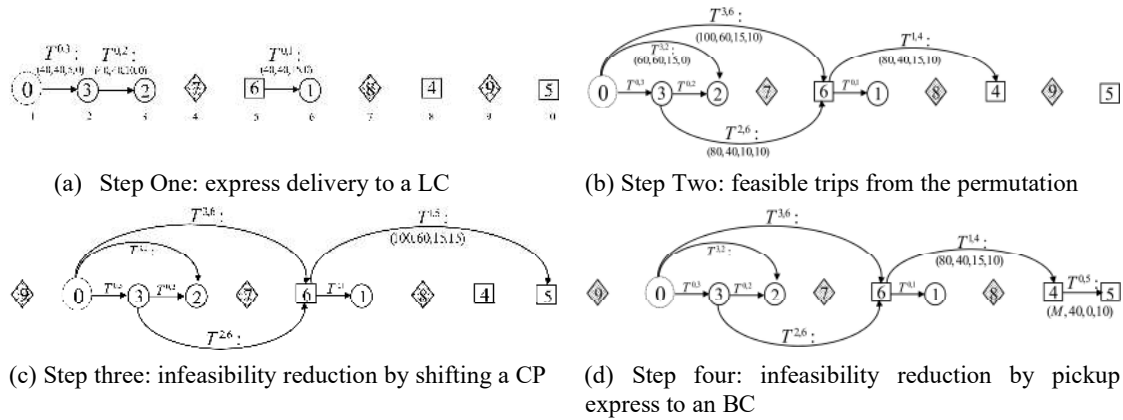(d) Step four: infeasibility reduction by pickup express to an BC

Fig. 5. Construction of the auxiliary graph

Initially, in step one shown in Figure 5a, all feasible trips that leave the depot, go to a LC and return to the depot are considered and stored in A'. For example, the trip $T^{0,1}$ of the auxiliary graph corresponds to an EV that leaves the depot, travels 20 km to the LC, delivers 15 units of product and travels another 20 km back to the depot. In this step, the depot is the origin vertex of the SPP and corresponds to the vertex i+1 in the auxiliary graph. Thus, the trip is characterized by a total distance of 40 km, a battery consumption of 40 km , 15 units delivered and 0 collected ($T^{0,1}$: (40,40,15,0)). This trip is feasible in terms of: battery life span compared to the distance traveled (see (40)), vehicle capacity (see (39) and (38)) and precedence constraint (see (36) and (37)).

Then, in step two (see Figure 5b), all feasible trips are obtained, following the sequence given by the solution Π. For example, the trip $T^{3,2}$ corresponds to a feasible trip consisting exclusively of LCs; the trip $T^{3,7}$ is not considered because a return to the depot from a CP is not allowed; the trip $T^{3,6}$ (see Figure 4b) corresponds to a feasible trip with a path $z^{3,6} = 100$, a maximum battery consumption $ED = \max\{60,40\} = 60 \le E^{\max}$, and 15 units delivered and 10 collected ($T^{3,6}$: (100,60,15,10)). Note that a trip leaving the depot to a vertex $j \in B \cup K$ is infeasible since it is not allowed to go directly from the depot to a BC or a CP. All feasible trips found in this step are added to A'.

4.2.3. Feasibility of the SPP

Of all the possible trips of a Π solution, only a percentage is feasible and they make up the arc set A' of the auxiliary graph. However, in the EVRPB, obtaining a feasible solution from the SPP may not be possible since, eventually, the auxiliary graph may be disconnected. Note that, in Figure 5b, the trip $T^{1,5}$ is infeasible by battery autonomy, which produces an infeasible graph for the SPP, since there is not at least one path between the depot and the last vertex of the permutation. Therefore, the next two steps consist of a process of reduction of infeasibility, which are applied sequentially until obtaining a feasible auxiliary graph for the SPP.

Step three: if there is a CP in the position k of the permutation that does not belong to a trip, in concordance with the condition (41), then this CP is shifted to the first position of the permutation.

$$\Pi_k \in K: \sum_{\forall i \in V} A_{i,\Pi_k} = 0 \tag{41}$$

Thus, the original permutation is modified and denoted as $\Pi^{new} = \{9,3,2,7,6,1,8,4,5\}$.

It is important to note that this step, implicitly, corresponds to a particular case of a well-known neighborhood structures so-called INSERT, which consists of removing the customer at the ith position from the permutation and then inserting it into the jth position, $i \ne j$. This case is illustrated in Figure

5c. Note that the depot is still the origin vertex of the SPP and becomes the vertex $i + 2$ in the auxiliary graph after the CP is shifted.

More generally, the EV departs from node 0 and visits nodes $i + Origin, i + Origin + 1, i + Origin + 2, \ldots, j - 1$, and j, consecutively, considering that $Origin = 0$ for the initial permutation Π and $Origin = Origin + 1$ each time a CP is shifted. Therefore, A' contain one arc (i,j), $i < j$, if a trip from $v = \Pi_{i+Origin+1}$ to $w = \Pi_j$ is feasible.

Step four: if there is still a vertex b of the permutation that does not belong to a trip, in concordance with the condition (42), then, exclusively for this BC, an *express pickup* with a very high distance traveled value is allowed (big M), as shown in Figures 4c and 5d with the trip $T^{0,5}$. The value of M is defined as the total distance traveled by the grand tour.

$$\Pi_b \in B: \sum_{\forall i \in V} A_{i,\Pi_b} = 0 \qquad (42)$$

4.2.4. Integrated algorithm for the optimal split with backhaul

The integration of all the steps, described above, allows building a connected auxiliary graph enabled to generate a feasible solution to the SPP. However, this solution could eventually be unfeasible for the EVRPB due to the need of creating a highly penalized arc in the auxiliary graph, corresponding to a direct trip from the depot to a BC (*express pickup*). The general structure of the OSPB strategy is shown in Algorithm 2. A binary variable F = 0 is used to indicate that the SPP solution is infeasible for the EVRPB (see line 13). When this happens, it is necessary to implement an infeasibility improvement process, for which, we propose a simple iterated local search technique.

Algorithm 2. OSPB pseudo code

Input: EVRPB, Initial solution Π	6: $\quad\quad \Pi^{new} \leftarrow 1$ apply step three to Π % Section (4.2.3)
Output:	7: $\quad\quad Origin \leftarrow Origin + 1$
$\quad\quad T^{SPP}$: set of solution trips to the SPP	8: $\quad\quad [A', T] \leftarrow$ apply steps one and two to Π^{new}
$\quad\quad F^{OBJ}$: value of the SPP objective function	9: $\quad\quad \Pi \leftarrow \Pi^{new}$
$\quad\quad F$: binary variable, where $F = 1$ if the solution is feasible	10: **end while**
$\quad\quad \Pi$: permutation that can eventually be modified in step three	11: **while** there are BCs disconnected according to (42) **do**
1: $\quad F \leftarrow 1$ %Initially, feasibility is assumed	12: $\quad\quad [A', T] \leftarrow$ apply step four to Π % Section (4.2.3)
2: $\quad Origin \leftarrow 1$ %SPP origin vertex	13: $\quad\quad F \leftarrow 0$ %Infeasible solution (express pickup to a BC)
3: $\quad [A', T] \leftarrow$ apply steps one and two to Π %Section (4.2.2)	14: **end while**
4: $\quad$ **return** Π	15: $\quad [T^{SPP}, F^{OBJ}] \leftarrow SPP(V, A', T,)$ % SPP Solution can be computed in $O(n^2)$ ((See Cormen et al. (2001)).).
5: $\quad$ **while** there are CPs disconnected according to (41) **do**	16: **return** T^{SPP}, F^{OBJ}, F, Π

4.3. Iterated local search

The OSPB proposed makes it possible for the EVRPB to be coded through a permutation and, therefore, many heuristic and metaheuristic strategies can be implemented easily to obtain efficient solutions. However, our objective is to present a simple technique based on local search that allows to obtain quickly a feasible integer solution of the EVRPB, that can be used as the upper limit in the solution of the larger and more complex instances when they are resolved through the proposed MIP model.

The iterated local search technique *(ILS)* is a method that uses a two-phase search approaches. In the first phase, denoted as *exploration*, a sequence of solutions is generated by applying perturbations to the current best solution. In the second phase, denoted as *intensification*, the current best solution is refined using an embedded heuristic strategy that allows to generate quality neighbors. The main objective of

the ILS is to obtain better results than the ones obtained using repeated random trials of that heuristic strategy. Interested readers are directed to Stützle (1999); Martin et al. (2003).

The pseudo code of the ILS proposed is illustrated in the Algorithm 3. In the line 4 of the algorithm it is established that while the solution is infeasible, for the EVRPB, typical and simple neighborhood structures will be applied to achieve feasibility. There are a large number of neighborhood structures developed by several authors (Taillard, 1993). However, all of them are basically generated by two types of moves, so-called SWAP and INSERT (Van Breedam, 1994). In lines 5 and 6 of the algorithm these two types of movements are considered. In line 5, of the Algorithm 3, the SWAP movement is applied as a disturbance strategy. The SWAP is swapping the customers at the ith position and the jth position in the permutation, $i \neq j$.

In step three of the OSPB shown in the Algorithm 2, the INSERT movement was applied implicitly, seeking an infeasibility improvement in the current solution. Therefore, in line 6 of the Algorithm 3 the INSERT movement is also used.

Algorithm 3. ILS pseudo code

Input: EVRPB	7: **if** $F_{new}^{OBJ} < F^{OBJ}$ **then** {acceptance criteria}
Output: S: set of arcs of the feasible integer solution for the EVRPB	8: $\Pi \leftarrow \Pi^{new}$
1: $F \leftarrow 1$ %Initially, feasibility is assumed	9: $F^{OBJ} \leftarrow F_{new}^{OBJ}$
2: $\Pi \leftarrow$ Greddy strategy (see Algorithm 1)	10: **end if**
3: $[T^{SPP}, F^{OBJ}, F, \Pi] \leftarrow \text{OSPB}(\Pi)$ (see Algorithm 2)	11: **end while**
4: **while** F==0 **do** {the solution is unfeasible}	12: $S \leftarrow$ Arcs that make up the trips of T^{SPP}
5: $\Pi^{new} \leftarrow \text{SWAP}(\Pi)$	13: **return** S
6: $\left[T^{SPP}, F_{new}^{OBJ}, F, \Pi^{new}\right] \leftarrow \text{OSPB}(\Pi^{new})$	

5. Computational results

The proposed model corresponds to a MILP formulation and was implemented in AMPL (Foureretal,1990) and solved with GUROBI 6.5 (called with the optimal gap option equal to 0%), with a time limit of 14400 seconds, on a computer intel core i5-4210 2,4 Ghz, 4 GB of RAM. Two datasets of scenarios are used in order to show the operation and effectiveness of the proposed formulation. The first dataset, denoted as *GJ dataset*, was proposed by Goetschalckx and Jacobs-Blecha (1989) and contains 62 instances with a range between 20 and 150 customers. Details on how these scenarios were generated can be consulted in (Toth and Vigo, 2002). The second dataset, denoted as *TV dataset*, was proposed by Toth and Vigo (1997) and contains 33 instances between 21 and 100 customers. All the instances were kept with their original data, only the locations of the charging points of the set K were added. The set of test instances for the EVRPB can be downloaded from http://academia.utp.edu.co/planeamiento/sistemas-de-prueba/.

Additionally, two types of results are presented. In order to show the efficiency of the proposed model, the first type of results corresponds to the direct solution of the MILP model using a commercial solver only. Because by making the battery life very long the proposed model can represent the standard VRPB, then a comparison of the our results versus the best-known solution (BKS) reported in the literature for the VRPB is presented too. The second type of results shows the performance of the ILS technique in those cases in which optimality was not reached in the direct EVRPB solution. In Table 1 and Table 2, the results for the VRPB using the GJ and TV datasets, respectively, are compared with those obtained by earlier methods, which are the stateof-the-art methods for VRPB instances from the literature (Toth and Vigo (1997); Mingozzi et al. (1999); Ropke and Pisinger (2006)). In Table 1, we give in columns 1-7: the problem name, the number of LCs, the number of BCs, the number of CPs, the capacity of the vehicle and the autonomy of the battery, respectively. The BKS obtained by earlier methods are presented

in column 8. These methods correspond to the heuristic algorithm (HA) proposed by Ropke and Pisinger (2006), to the exact algorithm (TV) proposed by Toth and Vigo (1997) and to the exact algorithm (EHP) proposed by Mingozzi et al. (1999). The Euclidean distances were rounded to one decimal and the final result was rounded to an integer.

Table 1
Computational results for the VRPB cases from Goetschalckx and Jacobs-Blecha (1989)

Id	n	m	k	K_V	Q	E^{max}	BKS	Sol.	K_{LB}	Gap	Time	Sol.	K_{LB}	K_{CP}	Gap	t_1	t_2
				Instance					MILP-VRPB					MILP-EVRPB			
A1	20	5	8	8	1,550	32,000	TV †: 229,886	229,886	5	0.00	1.85	238,725	5	3	0.00	1.61	45
A2	20	5	7	5	2,550	35,000	TV †: 180,119	180,119	4	0.00	2.70	196,755	5	2	0.00	0.02	212
A3	20	5	3	4	4,050	38,000	TV †: 163,405	163,405	4	0.00	0.02	188,439	5	1	0.00	0.03	19
A4	20	5	5	3	4,050	50,000	TV †: 155,796	155,796	3	0.00	1.24	161,438	3	1	0.00	0.05	8
B1	20	10	3	7	1,600	40,000	TV †: 239,080	239,080	6	0.00	16.35	249,239	7	2	0.00	0.06	350
B2	20	10	6	5	2,600	30,000	TV †: 198,048	198,048	5	0.00	12.14	210,792	5	3	0.00	204.25	506
B3	20	10	7	3	4,000	45,000	TV †: 169,372	169,372	3	0.00	0.05	169,634	3	2	0.00	0.35	1
C1	20	20	5	7	1,800	35,000	TV †: 249,448	250,557	7	0.00	22.00	253,276	7	3	0.00	32.61	642
C2	20	20	7	5	2,600	34,000	TV †: 215,020	215,020	5	0.00	24.82	219,450	5	4	0.00	101.02	699
C3	20	20	7	5	4,150	30,000	TV †: 199,346	199,346	4	0.00	0.37	220,125	5	5	0.00	48.05	487
C4	20	20	8	4	4,150	35,000	TV †: 195,366	195,366	4	0.00	0.35	205,365	4	3	0.00	101.03	351
D1	30	8	9	12	1,700	30,000	TV †: 322,530	322,530	6	0.00	28.04	333,623	5	4	0.00	400.57	11,250
D2	30	8	5	11	1,700	38,000	TV †: 316,709	316,709	6	0.00	407.00	318,253	6	2	0.00	890.95	4,500
D3	30	8	6	7	2,750	35,000	TV †: 239,479	239,479	4	0.00	7.04	264,332	7	4	0.00	10.62	316
D4	30	8	7	5	4,075	38,000	TV †: 205,832	205,832	4	0.00	28.01	214,203	5	3	0.00	42.61	61.05
E1	30	15	5	7	2,650	45,000	TV †: 238,880	238,880	4	0.00	11.00	241,088	4	2	0.00	122.32	9,240
E2	30	15	5	4	4,300	48,000	TV †: 212,263	212,263	4	0.00	3.11	214,072	4	2	0.00	1.08	46.52
E3	30	15	8	4	5,225	40,000	TV †: 206,659	206,659	3	0.00	24.17	216,350	4	4	0.00	634.26	1,290
F1	30	30	5	6	3,000	58,000	TV †: 263,173	263,173	6	0.00	539.00	264,565	6	0	0.00	2,233.20	9,800
F2	30	30	7	7	3,000	35,000	TV †: 265,213	265,213	7	0.00	47.00	268,600	7	4	0.00	2,312.59	4,350
F3	30	30	10	5	4,400	40,000	TV †: 241,120	241,120	5	0.00	12.00	246,297	5	4	0.00	1,709.05	4,000
F4	30	30	10	4	5,500	48,000	TV †: 233,861	233,861	4	0.00	15.00	235,224	4	3	0.00	150.02	2,900
G1	45	12	7	10	2,700	50,000	EHP †: 306,305	306,305	6	0.00	6,268	308,779	6	0	2.98	1,628.21	14,400
G2	45	12	3	6	4,300	46,000	TV †: 245,441	245,441	4	0.00	72.00	265,498	4	1	8.78	2,304.00	14,000
G3	45	12	5	5	5,300	45,000	TV †: 229,507	229,507	4	0.00	60.64	252,495	4	1	11.68	2,498.62	14,000
G4 †	45	12	2	6	5,300	50,000	HA: 232,521	232,521	4	0.00 †	14.56	262,976	6	0	8.87	2,265.32	14,400
G5	45	12	7	5	6,400	58,000	TV †: 221,730	221,730	4	0.00	24.13	225,600	4	1	0.00	1,120.21	13,954
G6	45	12	3	4	8,000	55,000	TV †: 213,457	213,457	4	0.00	5.89	225,829	4	2	1.33	1,278.00	14,400
H1	45	23	4	6	4,000	54,000	TV †: 268,933	268,933	4	0.00	158.00	269,652	5	0	0.00	140.00	13,900
H2	45	23	4	5	5,100	60,000	TV †: 253,365	253,365	4	0.00	4.02	255,964	5	0	0.00	21.04	38.06
H3	45	23	4	4	6,100	60,000	TV †: 247,449	247,449	4	0.00	1.06	253,316	4	1	0.00	64.01	6,225
H4	45	23	6	5	6,100	50,000	TV †: 250,221	250,221	4	0.00	1.95	257,848	5	1	0.00	18.03	915.01
H5	45	23	5	4	7,100	60,000	TV †: 246,121	246,121	4	0.00	0.17	253,294	4	1	0.00	140.01	1,091
H6	45	23	5	5	7,100	50,000	TV †: 249,135	249,135	4	0.00	0.67	257,670	5	1	0.00	14.00	1,760
I1	45	45	5	10	3,000	56,000	HA: 350,246	350,246	10	1.05	14,400	351,553	10	0	2.60	2,332.21	14,400
I2	45	45	9	7	4,000	58,000	EHP †: 309,943	309,943	7	0.00	2,857	309,943	7	0	0.00	180.00	12,961
I3 †	45	45	10	5	5,700	55,000	HA: 294,507	294,507	5	0.00 †	3,897	296,676	5	2	0.00	105.08	12,684
I4 †	45	45	8	6	5,700	55,000	EHP: 295,988	295,988	6	0.00 †	137.89	297,067	6	1	0.00	1,221.00	4,710
I5 †	45	45	8	7	5,700	45,000	HA: 301,236	301,236	7	0.00 †	44.97	301,738	7	1	0.00	1,676.05	3,328
J1	75	19	7	10	4,400	45,000	EHP: 335,006	335,006	8	2.44	14,400	349,639	8	0	9.43	1,232.00	14,400
J2	75	19	8	8	5,600	70,000	HA: 310,417	310,801	8	2.11	14,400	315,408	8	0	4.95	2,300.01	14,400
J3 †	75	19	11	6	8,200	55,000	HA: 279,219	279,219	6	0.00 †	123.15	279,272	6	1	0.00	0.09	346.32
J4	75	19	6	7	6,600	65,000	HA: 296,533	296,945	7	3.03	14,400	299,376	6	0	5.94	1,260.32	14,400
K1	75	38	12	10	4,100	45,000	HA: 394,376	394,071 ††	9	1.38	14,400	452,086	9	3	15.40	1,800.21	14,400
K2 †	75	38	10	8	5,200	55,000	HA: 362,130	362,130	7	0.00 †	5,617	381,504	8	0	8.31	2,804.65	14,400
K3 †	75	38	14	9	5,200	48,000	EHP: 365,694	365,694	7	0.00 †	4,985	378,576	6	1	6.98	3,000.00	14,400
K4 †	75	38	6	7	6,200	62,000	HA: 348,950	348,950	6	0.00 †	6,530	378,049	6	3	10.30	1,200.54	14,400

BKS: best known solution values reported by first time.
MILP-VRPB: results for the MILP proposed model, considering an unlimited autonomy of the battery.
MILP-EVRPB: results for the EVRPB using the MILP proposed model.
K_V: number of available vehicles to use (given in advance)
K_{LB}: number of vehicles performing routes conformed by linehaul and backhaul customers.
K_{CP}: number of charge points visited by the electric vehicles.
Gap (%): percentage gap is calculated as $(z - LB)/LB$.
t_1: computing time until finding the first feasible integer solution. t_2: overall computing time.
†: optimality proven for the first time. ††: new BKS.

Table 2
Computational results for the VRPB cases from Toth and Vigo (1997)

	Instance						BKS		MILP-VRPB ($E^{max} \rightarrow \infty$)				MILP-EVRPB				
Id	n	m	k	K_V	Q	E^{max}			Sol.	K_{LB}	Gap	Time	Sol.	K_{LB}	K_{CP}	Gap	t_1	t_2
EIL2250A	11	10	2	3	6,000	120	TV † :	371	371	3	0.00	0.01	379	3	1	0.00	0.02	1
EIL2266A	14	7	2	3	6,000	100	TV † :	366	366	3	0.00	0.01	430	3	2	0.00	0.04	15
EIL2280A	17	4	4	3	6,000	100	TV † :	375	375	3	0.00	0.01	394	2	2	0.00	5.04	30
EIL2350A	11	11	5	2	4,500	250	TV † :	682	682	2	0.00	0.01	706	2	2	0.00	0.01	2
EIL2366A	15	7	4	2	4,500	250	TV † :	649	649	2	0.00	0.01	667	2	1	0.00	0.03	5
EIL2380A	18	4	3	2	4,500	200	TV † :	623	623	2	0.00	0.01	671	2	1	0.00	0.01	2
EIL3050A	15	14	4	2	4,500	250	TV † :	501	501	2	0.00	0.01	506	2	1	0.00	4.91	50
EIL3066A	20	9	7	3	4,500	200	TV † :	537	537	3	0.00	0.30	537	3	0	0.00	1.87	25
EIL3080A	24	5	4	3	4,500	180	TV † :	514	514	3	0.00	0.30	518	3	1	0.00	1.82	20
EIL3350A	16	16	5	3	8,000	300	TV † :	738	738	2	0.00	0.14	742	2	1	0.00	0.01	1
EIL3366A	22	10	3	3	8,000	250	TV † :	750	750	2	0.00	0.38	758	2	2	0.00	1.02	2
EIL3380A	26	6	4	3	8,000	300	TV † :	736	736	3	0.00	7.31	859	3	0	0.00	0.03	1
EIL5150A	25	25	5	3	160	250	TV † :	559	559	3	0.00	1.80	559	3	0	0.00	2.01	5
EIL5166A	34	16	7	4	160	150	TV † :	548	548	4	0.00	2.01	553	4	1	0.00	10.56	58
EIL5180A	40	10	6	4	160	200	TV † :	565	565	3	0.00	26.11	569	3	1	0.00	2.51	3,250
EILA7650A	37	38	4	6	140	350	TV † :	739	739	6	0.00	64.21	760	6	0	0.00	18.52	747
EILA7666A	50	25	5	7	140	350	TV † :	768	768	6	0.00	743.00	768	6	0	0.00	2.09	10,820
EILA7680A	60	15	4	8	140	200	TV:	781	781	5	0.99	14,400	802	5	1	5.80	328.21	14,400
EILB7650A	37	38	5	8	100	120	TV † :	801	801	7	0.00	40.96	812	7	2	0.00	2.08	12,850
EILB7666A	50	25	5	10	100	450	TV † :	873	873	8	0.94	14,400	873	8	0	2.50	3.01	14,400
EILB7680A	60	15	8	12	100	450	TV † :	919	933	6	3.92	14,400	934	5	1	5.40	2.10	14,400
EILC7650A	37	38	6	5	180	150	TV † :	713	713	5	0.00	8.64	720	5	0	0.00	22.00	13,521
EILC7666A	50	25	2	6	180	120	EHP †	734	734	6	0.00	185.00	763	6	1	0.00	10.01	9,800
EILC7680A	60	15	5	7	180	350	TV:	733	733	5	1.50	14,400	747	4	1	4.62	2.01	14,400
EILD7650A	37	38	6	4	220	150	TV † :	690	690	4	0.00	6.03	703	4	3	0.00	3.03	653
EILD7666A †	50	25	10	5	220	150	TV:	715	715	5	0.00 †	32.54	721	5	1	0.00	12.09	4,015
EILD7680A †	60	15	5	6	210	170	EHP:	694	694	4	0.00 †	845.00	701	5	0	0.00	4.54	12,800
EILA10150A †	50	50	6	4	200	450	HA:	831	831	4	0.00 †	938.00	844	5	1	3.18	3.63	14,400
EILA10166A	67	33	5	6	200	180	TV † :	846	846	6	0.00	6.00	851	6	1	0.00	2.32	1,750
EILA10180A	80	20	7	7	200	100	HA:	857	859	6	0.82	14,400	938	5	4	9.41	6.38	14,400
EILB10150A †	50	50	8	7	112	240	HA:	925	923 ††	7	0.00 †	792	983	7	0	10.60	3,900	14,400
EILB10166A	67	33	3	10	112	200	HA:	989	971 ††	8	2.99	14,400	984	8	0	5.53	10.91	14,400
EILB10180A	80	20	7	11	112	850	HA:	1,008	1013	9	1.46	14,400	1,023	8	0	5.20	28.00	14,400

The nomenclature of this table is the same as that presented in Table 1.

In columns 9-12 the characteristics of the solution for the standard VRPB are presented. The solution obtained by solving the MILP model are presented in column 9. Column 10 shows the number of vehicles K_{LB} performing routes conformed by linehaul and backhaul customers. Thus, $K_V - K_{LB}$ is the number of vehicles performing routes conformed by linehaul customers exclusively (see Figure 6). Columns 11 and 12 show the percentage gap and computing time (expressed in CPU seconds), respectively. In columns 13-18 the results obtained by solving the MILP model for the EVRPB are presented. The number of recharging points (K_{CP}), used to recharge the EV batteries, are presented in column 15. Columns 17 and 18 show, respectively, the time it takes for the solver to find the first feasible integer solution and the total computation time until reaching the reported solution.

Regarding the standard VRPB, when E^{max} is considered large enough, the optimality for the GJ dataset was proved for the first time for 8 instances. One new best-known solutions was found for the instance k1. Optimality was achieved in 42 of the 47 scenarios (see Table 1). For the TV dataset the optimality of 4 TV instances is proven for first time. Two new best-known solutions were found considering both heuristic methods and exact methods. Optimality was achieved in 27 of the 33 cases (see Table 2). As can be seen from the computational results, the proposed model produces high quality results, obtaining equal or better upper bounds in all instances, and the final lower bounds prove stronger than those obtained by earlier methods.

Regarding the EVRPB, the optimality for the GJ dataset was achieved in 34 of the 47 intances and for the TV dataset, the optimality was achieved in 24 of the 33 intances. It is interesting to note that in some instances the number K_{CP} is 0, which implies that the EVs did not have the need to visit CPs. However, in some of these cases, such as the instance H1, the optimal solution found for the EVRPB differs from that found for the VRPB. This happens because, in the case of the EVRPB, the autonomy of the battery forces the model to generate a different solution, even using a different number of vehicles K_{LB}, with an objective function of greater value.

Note that among the instances for which it was not possible to achieve optimality, the instances G1, G2, G3, G4, G6, I1, J1, J2, J4, K1, K2, K3, K4, EILA7680A and EILB10150A present the longest times t_1. Therefore this set of instances is quite interesting to be addressed with the initialization methodology based on ILS. Table 3 shows

a comparison of the computational results obtained with the proposed mathematical model, the ILS technique and the model with initialization methodology. It can be seen that the proposed ILS technique is quite efficient to find a feasible integer solution in a short time, as shown in columns 6 and 7 of the table. However, in 14400 seconds the ILS technique, by itself, fails to achieve the results obtained with the exact model, as observed when comparing columns 2 and 5 of the table. Finally, when the first feasible integer solution (FFIS) found efficiently by the ILS method is used, the results obtained in 14400 seconds surpass all the previous ones, as can be seen in columns 8 and 9 of the table. Note that for the G6 instance the methodology reached optimality. The detailed routes of the optimal solution of this instance are shown in Figure 6. In Table 4, the detailed routes of the solutions of the aforementioned instances for the EVRPB are shown.

Table 3

Comparison of the computational results obtained with the proposed mathematical model, the ILS technique and the model with initialization methodology

Instance	MILP-EVRPB from Tables 1 and 2			ILS		MILP-EVRPB with initialization methodology			
	Sol [a].	Gap (%)	t_1	Sol [a].	FFIS	t_3	Sol [b].	Gap (%)	t_2
G1	308,779	2.98	1,628.21	356,854	401,562	0.02	307,965	1.63	14,400
G2	265,498	8.78	2,304.00	368,951	427,632	0.01	262,126	7.20	14,400
G3	252,495	11.68	2,498.62	321,458	456,554	0.01	248,214	6.67	14,400
G4	262,976	8.87	2,265.32	314,284	434,842	0.08	261,306	8.12	14,400
G6	225,829	1.33	1,278.00	327,773	417,266	0.95	224,966	0.00	8,997
I1	351,553	2.60	2,332.21	424,681	586,442	3.53	350,814	1.10	14,400
J1	349,639	9.43	1,232.00	382,459	526,168	1.04	344,192	5.80	14,400
J2	315,408	4.95	2,300.01	428,624	501,451	1.23	313,486	3.82	14,400
J4	299,376	5.94	1,260.32	379,915	482,714	0.09	297,766	2.43	14,400
K1	452,086	15.40	1,800.21	488,922	612,456	14.02	421,546	7.60	14,400
K2	381,504	8.31	2,804.65	428,561	594,821	8.65	378,744	7.53	14,400
K3	378,576	6.98	3,000.00	394,522	602,451	10.02	366,134	3.91	14,400
K4	378,049	10.30	1,200.54	398,205	598,428	16.32	355,109	3.30	14,400
EILA7680A	802	5.80	328.21	1,088	1,355	3.04	799	4.89	14,400
EILB10150A	983	10.60	3,900.21	1,224	1,550	2.04	934	4.76	14,400

Sol [a]: solution obtained in 14,400 seconds.   Sol [b]: solution obtained in t_2 seconds (overall computing time).
t_1: computing time until finding the first feasible integer solution.
FFIS: first feasible integer solution found by the ILS method in t_3 seconds.
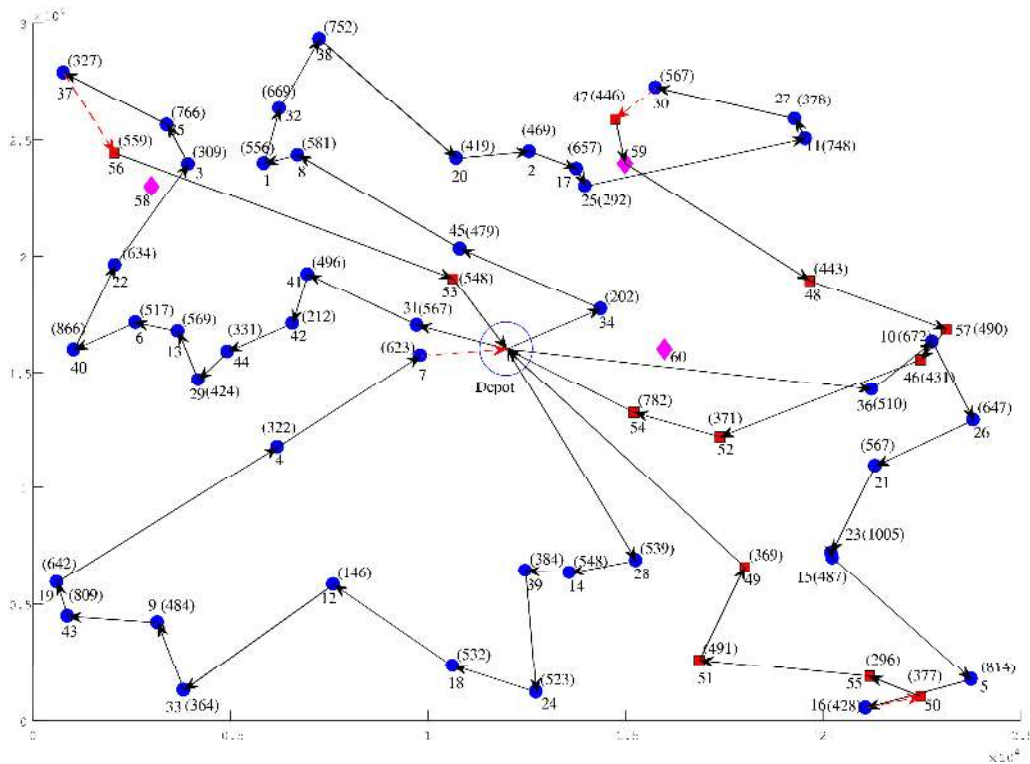

Fig. 6. Optimal solution of the instance G6 found using the MILP-EVRPB with initialization methodology

6. Conclusions and future works

In this paper, we have presented a new general-purpose model for modeling the EVRPB by mixed integer programming. The results show that the proposed commodity flow model produces good quality solutions and allows to resolve efficiently instances proposed in the literature with up to 150 customers. The EVRPB is a problem of current interest that occurs in transport companies with fleets of electric vehicles that collect and deliver goods to customers. In relation to the standard VRPB it is appreciated that the EVRPB is a problem of greater complexity and much more restricted when it comes to finding a feasible integer solution. Thus, an iterated local search algorithm is presented for to obtain quickly and effectively a feasible integer solution of the EVRPB. When this solution is used as the upper limit in the solution of larger and more complex instances, the results obtained are much better than those obtained independently by the ILS technique or by the MIP model. We believe that from the point of view of the exact model, the results could further improve with the inclusion of bounding techniques or set-partitioning models, which could significantly increase the size and number of the instances solvable by exact techniques. From the point of view of heuristic techniques, the optimal splitting procedure of a permutation that we propose can be widely adapted and improved to much more elaborated heuristic techniques that allow to reach better solutions in instances of greater size.

Table 4
Detailed routes for the EVRPB

Instance	Sequence (Blue points = LCs; red points = BCs; magenta points = CPs)
G1:	r_1 =42 44 22 3 35 37 **56** ; r_2 =39 18 24 16 5 50 55 51 ; r_3 =34 36 21 26 10 46 57 48 ; r_4 =31 41 2 20 45 ; r_5 =25 17 11 27 30 47 ; r_6 =14 28 23 15 49 52 54 ; r_7 =12 33 9 43 19 ; r_8 =7 ; r_9 =4 29 40 6 13 ; r_{10} =1 8 32 38 53
G2:	r_1 = 45 20 2 17 25 11 27 30 **47** ; r_2 = 42 44 29 13 6 40 22 41 ; r_3 = 39 18 24 14 28 ; r_4 = 34 36 21 26 10 46 57 48 ; r_5 = 23 15 5 16 50 55 51 49 52 54 ; r_6 = 12 33 9 43 19 4 7 31 ; r_7 = 1 8 32 38 37 35 3 **56** 53
G3:	r_1 = 45 20 38 32 8 1 3 35 37 **56** 53 ; r_2 = 39 18 24 14 28 21 26 36 10 57 46 **62** 52 54 ; r_3 = 34 25 17 2 11 27 30 47 48 ; r_4 = 23 15 5 16 50 55 51 49 ; r_5 = 7 42 44 29 13 6 40 22 41 31 ; r_6 = 4 19 43 9 33 12
G4:	r_1 = 45 20 38 32 8 1 3 35 37 **56** ; r_2 = 39 18 24 14 28 **54** ; r_3 = 34 25 17 2 11 27 30 47 ; r_4 = 31 42 44 29 13 6 40 22 41 53 ; r_5 = 23 15 5 16 50 55 51 49 52 ; r_6 = 21 26 36 10 46 57 48 ; r_7 = 7 4 19 43 9 33 12
G6:	r_1 = 36 10 26 21 23 15 5 16 50 55 51 49 ; r_2 = 34 45 8 1 32 38 20 2 17 25 11 27 30 47 **59** 48 57 46 52 54 ; r_3 = 31 41 42 44 29 13 6 40 22 3 35 37 56 53 ; r_4 = 28 14 39 24 18 12 33 9 43 19 4 7
I1:	r_1 = 39 22 8 32 38 58 89 57 85 61 **92** 56 ; r_2 = 36 45 41 31 74 47 79 90 ; r_3 = 34 5 37 35 3 46 75 81 65 64 51 ; r_4 = 33 63 ; r_5 = 30 29 24 13 16 7 82 88 67 76 ; r_6 = 19 2 20 21 23 15 87 71 53 50 66 69 ; r_7 = 10 40 26 17 25 60 55 84 49 59 ; r_8 = 9 43 83 73 68 77 ; r_9 = 6 44 42 4 11 27 86 70 **93** 62 72 ; r_{10} = 1 28 14 18 12 54 52 78 48 80
J1:	r_1 = 74 39 32 66 8 49 14 63 93 83 ; r_2 = 55 75 72 56 38 36 2 89 ; r_3 = 37 12 69 54 ; r_4 = 34 60 70 47 43 25 17 16 48 79 88 ; r_5 = 33 46 52 73 64 28 22 90 ; r_6 = 31 30 62 40 18 67 35 3 7 ; r_7 = 10 23 15 53 45 19 42 6 92 ; r_8 = 9 58 65 71 68 11 27 81 78 80 87 76 ; r_9 = 5 61 4 59 51 44 21 77 94 91 ; r_{10} = 1 24 57 26 41 50 20 29 13 **96** 85 86 82 84
J2	r_1 = 74 34 60 70 47 43 25 17 16 48 79 88 ; r_2 = 55 75 72 56 38 36 2 89 ; r_3 = 37 54 69 39 32 66 8 49 14 63 93 83 ; r_4 = 33 9 58 51 44 65 71 68 11 27 81 78 80 87 76 ; r_5 = 12 22 6 42 19 45 53 15 23 10 ; r_6 = 7 31 30 40 3 35 67 18 62 21 77 94 91 ; r_7 = 5 61 4 59 46 52 73 64 28 90 92 ; r_8 = 1 24 57 26 41 50 20 13 29 85 86 82 84
J4	r_1 = 72 56 38 36 3 35 67 18 40 62 21 77 94 91 ; r_2 = 60 16 17 25 48 43 47 70 63 14 49 8 32 66 83 93 79 88 ; r_3 = 55 74 34 75 2 89 ; r_4 = 37 54 69 39 26 41 50 20 13 29 85 86 82 84 ; r_5 = 10 1 24 57 23 15 53 45 19 42 6 22 90 92 ; r_6 = 7 31 30 59 58 51 44 65 71 68 11 27 81 78 80 87 76 ; r_7 = 5 61 4 9 46 52 73 64 28 33 12
K1	r_1 = 70 53 45 71 14 88 101 ; r_2 = 60 13 29 19 64 62 20 42 83 93 102 **115** 99 97 79 111 ; r_3 = 58 69 63 2 73 74 34 84 92 90 108 ; r_4 = 49 23 16 48 68 57 1 110 77 ; r_5 = 44 55 7 31 38 43 106 82 86 81 78 ; r_6 = 39 6 47 37 ; r_7 = 35 67 50 30 72 56 10 26 105 ; r_8 = 24 25 17 66 46 21 85 112 80 104 107 ; r_9 = 15 54 28 59 18 51 52 41 103 94 113 91 109 ; r_{10} = 12 65 75 9 40 33 100 95 96 98 89 ; r_{11} = 11 27 36 61 5 22 32 8 76 87 ; r_{12} = 3 4
K2	r_1 = 60 13 29 19 64 62 20 42 33 9 40 96 95 100 83 93 102 **115** 99 97 79 88 101 ; r_2 = 55 7 31 38 43 10 26 72 56 30 77 110 94 ; r_3 = 49 48 16 68 1 57 41 103 113 91 109 98 89 ; r_4 = 39 58 69 63 2 73 74 34 84 92 90 108 ; r_5 = 24 25 17 66 46 21 85 112 86 106 82 81 78 80 104 107 ; r_6 = 15 54 28 59 18 51 52 23 ; r_7 = 12 65 75 14 71 45 53 70 111 ; r_8 = 11 27 36 61 5 22 32 6 37 47 8 76 87 ; r_9 = 4 3 35 44 67 50 105
K3	r_1 = 60 13 29 19 64 62 20 42 33 9 40 96 95 **125** 100 83 93 88 101 ; r_2 = 55 24 7 17 25 66 46 21 85 112 86 106 82 81 78 **122** 107 ; r_3 = 49 23 15 54 28 59 18 51 52 41 103 94 113 91 109 98 89 ; r_4 = 44 50 26 72 56 10 43 38 31 80 104 87 76 92 90 **119** 108 ; r_5 = 39 58 69 63 2 73 74 34 84 99 **115** 102 97 79 111 ; r_6 = 37 6 47 8 32 22 5 61 36 27 11 ; r_7 = 12 65 75 14 71 45 53 70 ; r_8 = 4 35 67 16 48 30 68 57 1 110 77 105 ; r_9 = 3
K4	r_1 = 70 53 45 71 14 88 101 79 97 99 102 93 83 100 95 96 98 89 ; r_2 = 58 37 6 47 69 63 2 73 74 34 84 92 90 108 ; r_3 = 39 55 7 38 31 17 25 66 46 21 85 112 86 106 82 81 78 80 104 107 ; r_4 = 23 15 54 28 59 18 51 52 41 68 57 1 110 94 103 113 91 109 ; r_5 = 12 65 75 9 40 33 42 20 62 64 19 29 13 60 111 ; r_6 = 4 35 67 44 24 27 11 36 61 5 22 32 8 76 87 ; r_7 = 3 49 48 16 50 26 43 10 56 72 30 77 105
EILB10150A:	r_1 = 48 30 50 47 31 58 100 99 96 98 97 ; r_2 = 45 42 3 9 23 10 6 32 95 66 55 81 94 76 ; r_3 = 39 2 15 40 17 41 89 67 62 90 84 56 ; r_4 = 28 13 20 34 12 38 61 87 86 78 52 77 63 ; r_5 = 27 11 37 21 29 8 22 71 51 70 79 ; r_6 = 16 4 24 25 82 68 73 54 74 91 59 ; r_7 = 14 35 1 26 5 18 33 36 83 60 65 85 75 88 64 ; r_8 = 7 44 49 19 43 46 72 57 69 93 92 80 53
EILA7680A:	r_2 = 55 2 50 23 49 56 29 30 57 72 74 64 63 61 69 r_1 = 37 7 16 44 11 46 22 42 ; r_2 = 28 54 ; r_3 = 27 59 1 35 33 34 52 18 ; r_4 = 21 47 58 25 31 53 73 62 ; r_5 = 14 3 36 26 8 32 10 ; r_6 = 6 43 9 48 12 67 ; r_7 = 5 41 13 51 19 45 40 20 15 71 65 70 68 ; r_8 = 4 24 39 38 17 60 66 75

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