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The electrical conductivity of the mantle beneath Europe derived from *C*-responses from 3 to 720 hr

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SUMMARY

The *C*-response, which connects the magnetic vertical component and the horizontal gradient of the horizontal components of electromagnetic fluctuations, forms the basis for estimating the conductivity–depth profile of the Earth. This paper describes new estimates of the *C*-response obtained from observatory hourly mean values. The $Z:\mathcal{Y}$-method is applied, which means that the vertical component is used *locally* whereas the horizontal components are used *globally* by expansion into terms of spherical harmonics. A special e ort is made to obtain unbiased estimates of *C*.

When applied to 90 months of hourly mean values from about 90 observatories, the method yields consistent results for European observatories in the entire period range from 3 to 720 hr, and for two di erent source mechanisms (*S* and D_{st}).

A good description of the source structure for individual time segments is essential. This was achieved by a separate spherical harmonic analysis of the data for each month (for D_{st}) or each day (for *S*), and by estimating a large number (120) of expansion coe cients.

The results are interpreted by means of 1-D conductivity models, which show that the upper mantle has remarkably little structure, with a monotonic decrease of resistivity from 100 Ω m near $z=200$ km to 0.7 Ω m below $z=1000$ km.

Key words: electromagnetic induction, geomagnetic variation, electrical conductivity, mantle.

1 INTRODUCTION

Several methods have been developed for inferring the electrical conductivity of the Earth from magnetic observations. Traditionally, a spherical harmonic analysis (SHA) is applied to the observed geomagnetic variations, and the ratio $Q(\omega)$ of internal (induced) to external (inducing) components is estimated (Schuster 1889; Chapman 1919; Chapman & Price 1930; Lahiri & Price 1939). If $Q(\omega)$ is known for several frequencies ω, a spherically symmetric model of the electrical conductivity in the Earth's interior can be constructed.

However, in contrast with the transfer function $Q(\omega)$, which represents the *globally* averaged conductivity, a *regional* transfer function $C(\omega)$ can be derived for a specific site (Banks 1969; Schmucker 1970; Schultz & Larsen 1987, for example). Schultz & Larsen (1987, 1990) have found some evidence for lateral conductivity heterogeneities in the Earth's mantle, and hence the assumption of a global spherically symmetrical conductivity is doubtful. The present study concerns mantle conductivity in a selected region, for example Europe, and therefore the *C*-response is used in preference to the *Q*-response.

The study of conductivity at depths between 400 and 1200 km requires periods of about 10 to 1000 hr. In this period range, two completely di erent sources are available for induction studies: (1) geomagnetic daily variations with periods of a few hours, caused by electric currents in the ionosphere and denoted as *S*; (2) irregular variations with periods of several days, caused by the decay of magnetic storms in the magnetosphere and denoted as D_{st}.

Determination of transfer functions like *Q* or *C* requires knowledge of the spatial structure of the external field variation. With *a priori* assumptions on the source structure, it is possible to derive the *C*-response for magnetic measurements at a single site. This technique is called the Z/H-method, and has been widely applied, especially in the D_{st} period range. Banks (1969) and Schultz & Larsen (1987) have used this technique assuming that the source magnetic potential for periods of a few days can be described by the single spherical harmonic function P_1^0. However, it has long been noted that the method yields *C*-responses that do not merge smoothly with those obtained from an analysis of *S* at periods near one day. This is presumably because other spherical harmonics besides P_1^0

contribute significantly to the source structure of D_{st}. Recent investigations have indeed shown that more terms are necessary to describe the source structure of magnetic storms.

Instead of using *a priori* information it is possible to derive the source structure from magnetic measurements at neighbouring sites. The technique is called the horizontal gradient method if applied to array data in a limited region (often in Cartesian geometry), and the $Z:\mathscr{Y}$-method if the source structure is determined from a worldwide distribution of sites by means of a spherical harmonic analysis. Examples of the former approach can be found in the works of Berdichevsky, Vanyan & Fainberg (1969); Schmucker (1970); Kuckes (1973a,b); Lilley & Sloane (1976) and Jones (1980); the latter is described in Schmucker (1979) and Olsen (1992).

In an earlier paper (Olsen 1992), the $Z:\mathscr{Y}$-method was used to estimate C-responses in the S period range between 4 and 24 hr. Small, but significant, regional di erences were found for Europe. In the D_{st} period range, Schmucker (1979) used the $Z:\mathscr{Y}$-method and data from 20 observatories for 16 months. The amount of digital data has increased considerably during recent years, and hence it is now possible to apply the $Z:\mathscr{Y}$-method to 90 months (1957.5–59, 1964–65, 1979–81) and about 90 observatories. The present paper is an extension of an earlier study (Olsen 1992) to the D_{st} period range between 40 and 720 hr. Here it is shown that consistent C-responses can be obtained for the entire period range from 3 to 720 hr and for two di erent source mechanisms (S and D_{st}). However, this requires a precise description of the source structure: the Z/H-method is not appropriate.

The content of the paper is as follows: Section 2 reviews the theory of determining the C-response from magnetic measurements as well as some properties of C. One problem in the estimation of transfer function arises if the squared coherency is low, which may lead to biased estimates. However, with the assumption of a 1-D conductivity structure and a frequency-independent noise-to-signal ratio it is possible to derive an unbiased estimate. This is the topic of Section 2.3. The algorithms and data are described in Section 3. An application of the method of Section 2.3 is presented in the first part of Section 4. The second part compares results obtained using the $Z:\mathscr{Y}$-method with those obtained using the Z/H-method (assuming a P_1^0 source), and the third part concerns regional C-response di erences in Europe. In Section 5, mean European C-responses are compared with the results of other authors, and 1-D conductivity models that are consistent with the mean European C-responses are presented.

2 THEORY

2.1 Definition and determination of $C(\omega)$

It is assumed that each time-harmonic (with time dependence $e^{i\omega t}$) of the observed magnetic field variation $\mathbf{B}(r, \vartheta, \lambda, \omega) = -\text{grad}\, V(r, \vartheta, \lambda, \omega)$ can be derived from a scalar magnetic potential V, which is approximated by an expansion in a series of spherical harmonics:

$$V = a \sum_{n=1}^{N} \sum_{m=-n}^{n} \left[\varepsilon_n^m \left(\frac{r}{a}\right)^n + \iota_n^m \left(\frac{a}{r}\right)^{n+1} \right] P_n^m(\cos\vartheta)\, e^{im\lambda}. \tag{1}$$

$\varepsilon_n^m(\omega)$ and $\iota_n^m(\omega)$ are the complex expansion coe cients of the primary (external) and secondary (internal) parts of the potential at frequency ω; (r, ϑ, λ) are spherical coordinates with $r = a = 6371$ km as the mean Earth's radius and ϑ and λ as geomagnetic colatitude and longitude; and P_n^m is the associated Legendre function.

The time-harmonics of the magnetic field components at the Earth's surface ($r = a$) are given by

$$B_\vartheta(\omega, \vartheta, \lambda) = -\sum_{n,m} v_n^m(\omega) \frac{dP_n^m(\cos\vartheta)}{d\vartheta} e^{im\lambda}, \tag{2a}$$

$$B_\lambda(\omega, \vartheta, \lambda) = -\sum_{n,m} v_n^m(\omega) \frac{im}{\sin\vartheta} P_n^m(\cos\vartheta)\, e^{im\lambda}, \tag{2b}$$

$$B_r(\omega, \vartheta, \lambda) = -\sum_{n,m} z_n^m(\omega) P_n^m(\cos\vartheta)\, e^{im\lambda}, \tag{2c}$$

with

$$v_n^m = \varepsilon_n^m + \iota_n^m \quad \text{and} \quad z_n^m = n\varepsilon_n^m - (n+1)\iota_n^m.$$

Expanding the horizontal components $\mathbf{B}_H = (B_\vartheta, B_\lambda)$ yields the *unseparated* potential coe cients, i.e. the sum $v_n^m = \varepsilon_n^m + \iota_n^m$ of external and internal parts.

The C-response was introduced by Schmucker (1970) and refers in its original definition to a geomagnetic-variation field approximated by a single spherical harmonic. For a spherically symmetric 1-D (i.e. conductivity σ depends only on radius r) conductivity distribution it is connected to the Q-response by means of

$$C_n(\omega) = \frac{a}{n+1} \frac{1 - \frac{n+1}{n} Q_n(\omega)}{1 + Q_n(\omega)} = \frac{a}{n(n+1)} \frac{z_n^m(\omega)}{v_n^m(\omega)} \tag{3}$$

(Schmucker 1970, 1987), where $Q_n = \iota_n^m/\varepsilon_n^m$ is independent of the order m of the spherical harmonic for a 1-D conductivity. Inserting z_n^m of eq. (3) into (2c) yields, for the vertical component $Z = -B_r$ at a specific site with coordinates (ϑ, λ),

$$Z(\omega, \vartheta, \lambda) = \frac{1}{a} \sum_{n,m} C_n(\omega)\, n(n+1) v_n^m(\omega) P_n^m(\cos\vartheta)\, e^{im\lambda}. \tag{4}$$

Although this equation has been derived with the assumption of a 1-D conductivity, it has general validity, provided that the scalelength of lateral conductivity inhomogeneities beneath the site is large compared with $|C|$ (i.e. $\sigma/|\partial\sigma/\partial x|, \sigma/|\partial\sigma/\partial y| \gg |C|$).

C_n has the important property of depending only weakly on the source-field geometry (i.e. on the degree n of the spherical harmonic) and is therefore—to a first approximation—equal to the asymptotic value $C = C_0$. (Here, C_0 is the zero wave-number flat-earth response, which di ers only insignificantly from C_1.) This yields

$$Z = C \frac{1}{a} \sum_{n,m} n(n+1) v_n^m(\omega) P_n^m(\cos\vartheta)\, e^{im\lambda} = C\mathscr{G}, \tag{5}$$

with

$$\mathscr{G}(\omega, \vartheta, \lambda) = \frac{1}{a} \sum_{n,m} n(n+1) v_n^m(\omega) P_n^m(\cos\vartheta)\, e^{im\lambda} \tag{6a}$$

$$= \frac{1}{a\sin\vartheta} \left[\frac{\partial}{\partial\vartheta}(\sin\vartheta B_\vartheta) + \frac{\partial B_\lambda}{\partial\lambda} \right] \tag{6b}$$

$$= \left(\frac{\partial}{\partial x} B_x + \frac{\partial}{\partial y} B_y \right) \tag{6c}$$

$$= \nabla_H \cdot \mathbf{B}_H \tag{6d}$$

as the divergence of the horizontal components. Estimation of the C-response by means of eq. (5) is known as the horizontal gradient method (Berdichevsky *et al.* 1969; Schmucker 1970) if $\mathscr{G}$ is determined using eqs (6b) or (6c), and as the $Z:\mathscr{Y}$-method (Schmucker 1985) if $\mathscr{G}$ is determined via eq. (6a).

However, $C_n \approx C_0$ holds only if $|C_n| \ll a/n$, and this condition is not fulfilled for periods of several days (with their corresponding C-response of about 1000 km) and high-degree spherical harmonics. Schmucker (1990) and Olsen (1992) therefore introduced correction factors

$$\Gamma_n(\omega) = \frac{C_n(\omega)}{C_0(\omega)}, \tag{7}$$

which are calculated from a preliminary spherically symmetric conductivity model. Hence a better approximation of eq. (4) is given by

$$Z = C\frac{1}{a}\sum_{n,m} \Gamma_n n(n+1) v_n^m P_n^m(\cos\vartheta)\, e^{im\lambda}$$
$$= C\mathscr{Y}, \tag{8}$$

with

$$\mathscr{Y}(\omega, \vartheta, \lambda) = \frac{1}{a}\sum_{n,m} \Gamma_n(\omega) n(n+1) v_n^m(\omega) P_n^m(\cos\vartheta)\, e^{im\lambda}. \tag{9}$$

$\mathscr{Y}$ is asymptotically equal to $\mathscr{G}$ (i.e. $\Gamma_n \rightarrow 1$) for the case of negligible source e ects, which holds (1) for shorter periods and hence smaller C-responses; or (2) for small horizontal gradients of the source field (which means a source structure describable with low-degree spherical harmonics only). Table 1 lists $\Gamma_n(\omega)$, determined from the four-layer conductivity model of Fig. 6 by the method described in Chapter 5.2 of Schmucker (1970).

It is in general true that $|\Gamma_n| \leq 1$ and $\arg\{\Gamma_n\} \geq 0$. An approximation of Γ_n can be obtained by combining eqs (5) and (29) of Schmucker (1987):

$$\Gamma_n(\omega) \approx 1 - \frac{n(n+1)}{2}\left(\frac{C_0(\omega)}{a}\right)^2. \tag{10}$$

Instead of obtaining $\mathscr{G}$ or $\mathscr{Y}$ from the horizontal components of neighbouring observatories via the expansion coe cients v_n^m ($Z:\mathscr{Y}$-method), it is possible to use *a priori* assumptions about the source structure—a technique called the Z/H-method. A common assumption for the $D_{\rm st}$ period range is a source-field potential $V \propto P_1^0 = \cos\vartheta$, which yields $H = -B_\vartheta \propto -\sin\vartheta$, $B_\lambda = 0$, $Z = -B_r \propto \cos\vartheta$, *cf.* eq. (2). According to Table 1, $\Gamma_1 \approx 1$ and therefore $\mathscr{Y} \approx \mathscr{G} = -2H\cot\vartheta/a$. This allows us to determine C from the data of only one single observatory by means of the formula

$$Z = C\frac{-2\cot\vartheta}{a}H. \tag{11}$$

2.2 Properties of $C(\omega)$ for a 1-D conductivity structure

Weidelt (1972) and Parker (1980) discuss some properties of the C-response. For the case of a 1-D conductivity structure, it is possible to expand the frequency dependence of C according to

$$C(\omega) = \sum_{m=1}^{M} \frac{b_m}{\lambda_m + i\omega} + b_0 b_m,\ \lambda_m > 0 \tag{12a}$$

$$= h_0 + \cfrac{1}{i\omega\mu_0\tau_1 + \cfrac{1}{h_1 + \cfrac{1}{i\omega\mu_0\tau_2 + \cfrac{1}{h_2 + \cdots \cfrac{1}{h_M}}}}}. \tag{12b}$$

Parker (1980) shows that this corresponds to the response of a conductivity model consisting of a stack of thin layers of conductance τ_m separated by insulators of thickness h_m. He called this a D^+ model. The coe cients b_m, λ_m and the optimum number of layers M are determined from K observations of $C(\omega_k)$ with their corresponding rms errors $\delta C(\omega_k)$ by minimization of the misfit

$$\chi^2 = \sum_{k=1}^{K} \left|\frac{C^{\rm obs}(\omega_k) - C^{\rm mod}(\omega_k)}{\delta C(\omega_k)}\right|^2 \tag{13}$$

(Parker & Whaler 1981), where $C^{\rm mod}(\omega_k)$ are the model values of eq. (12). (Layer conductance τ_m and layer separation h_m can be derived from the coe cients b_m, λ_m.) It can be proved that of all possible classes of models, D^+ contains the 1-D inverse with the smallest possible misfit and is therefore suitable to test the assumption of a 1-D conductivity structure.

Table 1. Correction factors $\Gamma_n(\omega) = C_n(\omega)/C_0(\omega)$, determined from the four-layer conductivity model of Fig. 6.

	$T = 48$ h		$T = 65$ h		$T = 120$ h		$T = 240$ h		$T = 360$ h		$T = 720$ h	
n	$\lvert\Gamma_n\rvert$	$\arg\{\Gamma_n\}$	$\lvert\Gamma_n\rvert$	$\arg\{\Gamma_n\}$	$\lvert\Gamma_n\rvert$	$\arg\{\Gamma_n\}$	$\lvert\Gamma_n\rvert$	$\arg\{\Gamma_n\}$	$\lvert\Gamma_n\rvert$	$\arg\{\Gamma_n\}$	$\lvert\Gamma_n\rvert$	$\arg\{\Gamma_n\}$
1	0.991	0°	0.990	0°	0.989	0°	0.987	0°	0.985	1°	0.983	1°
2	0.974	1°	0.972	1°	0.967	1°	0.961	1°	0.957	2°	0.950	2°
3	0.950	2°	0.945	2°	0.935	2°	0.925	2°	0.918	3°	0.904	4°
4	0.919	3°	0.911	3°	0.897	3°	0.881	4°	0.870	5°	0.847	7°
5	0.884	4°	0.873	4°	0.854	4°	0.829	5°	0.816	6°	0.785	9°
6	0.845	5°	0.831	5°	0.807	6°	0.780	7°	0.762	8°	0.722	11°
7	0.804	6°	0.788	6°	0.760	7°	0.729	8°	0.707	9°	0.662	13°
8	0.762	7°	0.745	7°	0.714	8°	0.679	9°	0.655	11°	0.605	14°
9	0.721	9°	0.702	9°	0.669	9°	0.631	10°	0.607	12°	0.554	15°
10	0.682	10°	0.661	10°	0.627	10°	0.588	11°	0.562	13°	0.509	16°

2.3 Bias in $C(\omega)$

There are two reasons for biased C-response estimates of a univariate linear system

$$Z = C\mathscr{Y} \tag{14}$$

like that of eqs (5), (8) or (11). First, C is biased if $\mathscr{Y}$ is biased (Z will be assumed to be unbiased). Since $\mathscr{Y}$ represents the geometry of the source field, this kind of bias could be denoted as *geometric bias*. For example: if the assumption of a P_1^0 source is not fulfilled because higher spherical harmonics are present, approximating the source by P_1^0 often results in an incorrect estimate of $\mathscr{Y}$ and hence a biased C-response. Second, there is a *statistical bias* in the presence of noise, which is the topic of this section. Estimating an unbiased transfer function C from L observations Z_l and $\mathscr{Y}_l$ requires assumptions about the signal-to-noise ratio of Z and $\mathscr{Y}$. Usually, both Z_l and $\mathscr{Y}_l$ contain errors, and thus eq. (14) should be written as

$$Z_l = Z_{0,l} + \delta Z_l = C(\mathscr{Y}_{0,l} + \delta\mathscr{Y}_l), \tag{15}$$

where $\mathscr{Y}_{0,l}$ and $Z_{0,l} = \mathrm{E}\{C\}\mathscr{Y}_{0,l}$ are assumed to be exactly correlated, whereas the noise part δZ_l and $\delta\mathscr{Y}_l$ are assumed to be uncorrelated, which means that $\langle \delta\mathscr{Y}\delta Z^* \rangle = 0$. $\mathrm{E}\{\ldots\}$ denotes the expectation value, δZ^* is the complex conjugate of δZ, and $\langle \ldots \rangle$ denotes summation over L realizations. Let us further assume that $\langle \delta Z Z_0^* \rangle = \langle \delta\mathscr{Y}\mathscr{Y}_0^* \rangle = \langle \delta Z\mathscr{Y}_0^* \rangle = \langle \delta\mathscr{Y} Z_0^* \rangle = 0$. The ratios of noise energy to signal energy in Z and $\mathscr{Y}$ are defined respectively as

$$\zeta_Z = \frac{\langle \delta Z \delta Z^* \rangle}{\langle Z_0 Z_0^* \rangle}, \tag{16a}$$

$$\zeta_{\mathscr{Y}} = \frac{\langle \delta\mathscr{Y}\delta\mathscr{Y}^* \rangle}{\langle \mathscr{Y}_0\mathscr{Y}_0^* \rangle}. \tag{16b}$$

In the case of the $Z{:}\mathscr{Y}$-method, the values $\mathscr{Y}_l$ have been estimated from the SHA of both horizontal components of many observatories, and it is therefore a common practice to assume that $\mathscr{Y}$ is noise-free (i.e. $\delta\mathscr{Y}_l = 0$) and to estimate C in a least-squares sense by minimization of $\langle |\delta Z|^2 \rangle = \langle \delta Z \delta Z^* \rangle$. This leads to

$$C = \frac{\langle Z\mathscr{Y}^* \rangle}{\langle \mathscr{Y}\mathscr{Y}^* \rangle} = \frac{\langle Z_0\mathscr{Y}_0^* \rangle}{\langle \mathscr{Y}_0\mathscr{Y}_0^* \rangle} \quad \text{if} \quad \delta\mathscr{Y}_l = 0, \tag{17}$$

which is an unbiased estimate only if $\mathscr{Y}$ is really noise-free. If this assumption fails and both Z_l and $\mathscr{Y}_l$ contain noise, the estimated value

$$C_\mathrm{D} = \frac{\langle Z\mathscr{Y}^* \rangle}{\langle \mathscr{Y}\mathscr{Y}^* \rangle} = \frac{\langle Z_0\mathscr{Y}_0^* \rangle}{\langle \mathscr{Y}_0\mathscr{Y}_0^* \rangle + \langle \delta\mathscr{Y}\delta\mathscr{Y}^* \rangle} = C\frac{1}{1 + \langle \delta\mathscr{Y}\delta\mathscr{Y}^* \rangle / \langle \mathscr{Y}_0\mathscr{Y}_0^* \rangle} = C\frac{1}{1+\zeta_{\mathscr{Y}}} \tag{18}$$

is downward-biased, as noise in $\mathscr{Y}$ does not influence the nominator because we assumed that $\delta\mathscr{Y}$ is not correlated with δZ. Note that the amplitude of the transfer function is a ected, but not the argument (since $\zeta_{\mathscr{Y}}$ is real); that is, $|C_\mathrm{D}| \le |C|$ and $\arg\{C_\mathrm{D}\} = \arg\{C\}$.

On the other hand, minimization of $\langle |\delta\mathscr{Y}|^2 \rangle$ in eq. (15) yields an upward-biased estimate:

$$C_\mathrm{U} = \frac{\langle ZZ^* \rangle}{\langle \mathscr{Y}Z^* \rangle} = \frac{\langle Z_0 Z_0^* \rangle + \langle \delta Z \delta Z^* \rangle}{\langle \mathscr{Y}_0 Z_0^* \rangle} = C(1 + \langle \delta Z\delta Z^* \rangle / \langle Z_0 Z_0^* \rangle) = C(1+\zeta_Z), \tag{19}$$

with $|C_\mathrm{U}| \ge |C|$ and $\arg\{C_\mathrm{U}\} = \arg\{C\}$.

The smaller the squared coherency between Z and $\mathscr{Y}$, the greater the di erence between C_D and C_U, because of

$$\mathrm{coh}^2 = \frac{\langle Z\mathscr{Y}^* \rangle \langle \mathscr{Y}Z^* \rangle}{\langle ZZ^* \rangle \langle \mathscr{Y}\mathscr{Y}^* \rangle} = \frac{C_\mathrm{D}}{C_\mathrm{U}} = \frac{1}{(1+\zeta_{\mathscr{Y}})(1+\zeta_Z)}. \tag{20}$$

Usually both Z and $\mathscr{Y}$ contain noise, and therefore it follows from eqs (18) and (19) that

$$\eta = \frac{\zeta_{\mathscr{Y}}}{\zeta_Z} \tag{21}$$

$$= \frac{\langle \delta\mathscr{Y}\delta\mathscr{Y}^* \rangle / \langle \mathscr{Y}_0\mathscr{Y}_0^* \rangle}{\langle \delta Z\delta Z^* \rangle / \langle Z_0 Z_0^* \rangle} = \frac{C/C_\mathrm{D} - 1}{C_\mathrm{U}/C - 1}, \tag{22}$$

which can be solved for C if η is known:

$$C(\eta) = \frac{C_\mathrm{D}}{2}\left\{1 - \eta + \sqrt{(1-\eta)^2 + \frac{4\eta}{\mathrm{coh}^2}}\right\}. \tag{23}$$

η varies between $\eta = 0$ ($\mathscr{Y}$ noise-free), which yields $C = C_\mathrm{D}$, and $\eta = \infty$ (Z noise-free), which yields $C = C_\mathrm{U}$. The case $\eta = 1$ (equal relative noise in $\mathscr{Y}$ and Z) leads to $C = \sqrt{C_\mathrm{U} C_\mathrm{D}}$, i.e. the geometric mean.

Jones (1980) drew attention to the problem of upward- and downward-biased estimates of C. He stated incorrectly that the *arithmetic* mean $(C_\mathrm{D} + C_\mathrm{U})/2$ is unbiased 'if the input signal-to-noise ratio exactly equals the output signal-to-noise ratio'. But in that case ($\eta = 1$) the *geometric* mean $\sqrt{C_\mathrm{D} C_\mathrm{U}}$ is the unbiased estimate. However, this has only minor influence for Jones' analysis because of the high squared coherency ($\mathrm{coh}^2 \approx 0.9$).

An unbiased estimate of the transfer function requires knowledge of (or assumptions about) η. It is important to notice that it is always necessary to choose some value when estimating transfer functions. Even if one assumes $\mathscr{Y}$ (or H) to be error-free (as most authors do), the particular value $\eta = 0$ has been chosen, whether or not this is specifically stated. In Section 4.2 it is shown that a value di erent from the usual choice $\eta = 0$ yields in many cases more plausible estimates of C.

3 ALGORITHMS AND DATA

The following procedure is used for the estimation of C-responses in the D_st period range using the $Z{:}\mathscr{Y}$-method.

(1) 90 months of hourly mean values (1957.5–1959, 1964–1965, 1979–1981) of the three magnetic components from about 90 observatories between $\pm 60^\circ$ geomagnetic latitude are used.

(2) Gaps of fewer than six missing values are interpolated by splines, and secular variation is removed by subtracting a quadratic polynomial fitted to the annual mean values.

(3) For each 30-day data segment (hereafter called a month), 18 time-harmonics (1/30 cpd to 0.6 cpd) are determined for each observatory and each component.

(4) For each month and each period, a robust SHA of the horizontal components $\mathbf{B}_H$ is carried out and 120 coe cients v_n^m (degree $n=1, \ldots, N=10$ and order $m=-n, \ldots, n$) are estimated. Higher spherical harmonic degrees are 'damped' according to Marquardt (1970) by adding a damping constant $\alpha^2=\alpha_0^2 n^6$ to the diagonal elements of the normal equations. α_0^2 was chosen to be 10^{-3}. Details are given in Olsen (1992).

(5) $\mathcal{Y}$ of eq. (9) is synthesized for each month, for each of the 18 periods and for each observatory using the correction factors Γ_n of Table 1.

(6) Smoothed spectra $\langle Z\mathcal{Y}^*\rangle$, $\langle \mathcal{Y}\mathcal{Y}^*\rangle$ and $\langle ZZ^*\rangle$ are estimated for seven periods ($T=720$, 360, 240, 144, 90, 65 and 48 hr) using a Parzen filter (*cf.* item 9). A robust technique is used when estimating the spectra; details are described in Olsen (1992), eqs (7)–(9).

(7) Downward-biased C-responses and squared coherency are calculated for each observatory and each period by means of eqs (18) and (20).

(8) An unbiased estimate C is determined by means of eq. (23) assuming a frequency-independent value $\eta=0.25$, i.e. the noise-to-signal ration in $\mathcal{Y}$ is 25 per cent of that in Z. This value will be justified in Section 4.2.

(9) The statistical error δC is determined from

$$\left|\frac{\delta C}{C}\right|^2=\frac{1-\mathrm{coh}^2}{\mathrm{coh}^2}\frac{2}{2L-2}F^{-1}_{v:2L-2;1-\beta}, \tag{24}$$

where $F^{-1}_{v:2L-2;1-\beta}$ is the inverse of the F distribution function with v and $2L-2$ degrees of freedom. L is the e ective number of data segments (i.e. the sum of the weights used in the robust method of step (6), v depends on the length of the Parzen filter used and varies between $v=5.32$ (smoothing over seven periods) for $T=40$ hr and $v=2$ (no smoothing) for $T=720$ hr. The confidence level $(1-\beta)$ is chosen to be $(1-0.32)=0.68$.

Two remarks about this scheme might be helpful. First, instead of using the fixed window length of 30 days used in step 3 it would be better to have a data window length that depends on the period (shorter windows for shorter periods) to avoid a bias due to fluctuations either in the noise level or in the spatial geometry of the source fields which are taking place on a timescale shorter than 30 days (Banks, private communication, 1997). However, such an approach has not been applied since it would complicate the analysis. Second, one should keep in mind that a heavily regularized SHA is performed in step 4, and hence the SHA-coe cients v_n^m are not individually significant, and should not be used in isolation. Only the potential V (respectively $\mathcal{Y}$), synthesized from those coe cients for regions where su cient data are available (such as Europe), should be treated.

The C-responses for the D_{st} period range are augmented by C-responses obtained by extending the study of Olsen (1992) to shorter periods: the $Z:\mathcal{Y}$-method is applied to the same data set and the SHA is carried out with 120 coe cients for each of the ≈1650 days and each of the 10 daily harmonics (1 to 10 cpd) separately. Contrary to Olsen (1992), where $\mathcal{Y}$ was assumed to be noise-free, a value of $\eta=0.25$ is chosen in an attempt to obtain unbiased estimates of C, as for the D_{st} period range.

To compare results obtained with the $Z:\mathcal{Y}$-method with those gained with the Z/H-method for the D_{st} period range, 50 years (1945–1994) of hourly mean values of the observatory Fürstenfeldbruck (FUR) were taken. As for the $Z:\mathcal{Y}$-method, data are split up into 30-day segments, and 18 time-harmonics are determined. A rotational transformation is applied to the horizontal components such that variations in H were maximized. It turns out that a rotation angle of $-18°$ is suitable for all frequencies. This angle corresponds to transforming between the geographic and the geomagnetic coordinate systems. The remainder of the procedure is similar to that presented above in steps 4 to 7: smoothed spectra $\langle ZZ^*\rangle$, $\langle HH^*\rangle$ and $\langle ZH^*\rangle$ are estimated using a Parzen filter and eq. (11) is used to obtain downward- and upward-biased C-responses (and their statistical errors) according to eqs (18) and (19), respectively.

4 RESULTS

Although the analysis was performed with observatories distributed worldwide, only results for European observatories will be presented. Their location is shown in Fig. 1. The dense distribution of European observatories is optimal for an application of the $Z:\mathcal{Y}$-method, and allows for a study of lateral heterogeneities in mantle conductivity in a certain area, of, say, 1500 km radius.

4.1 Comparison of Z/H and $Z:\mathcal{Y}$-methods

Table 2 compares the two methods for determining C-responses in the D_{st} period range for Fürstenfeldbruck (FUR).

First of all the squared coherency is much larger for the $Z:\mathcal{Y}$-method ($\overline{\mathrm{coh}^2}\approx 0.92$ for the D_{st} periods) than for the Z/H-method ($\overline{\mathrm{coh}^2}\approx 0.62$), which indicates that the spatial source structure is better described by $Z:\mathcal{Y}$. Owing to the high coherency, the di erences between upward- and downward-biased estimates are much smaller than results obtained with Z/H.

However, the mean *statistical* error of the Z/H-method is smaller (mean error for the D_{st} period range: $\overline{\delta C}=37$ km). This is partly due to the longer time series (50 years compared with 8.5 years), from which one would expect a reduction by a factor of about $\sqrt{50/8.5}\approx 2.5$. The actual ratio, however, is only 1.3 as a consequence of the lower squared coherency.

Presumably, however, the *geometric bias* (*cf.* Section 2.3) is larger for the Z/H-method. This becomes evident from Fig. 2, which shows the Fürstenfeldbruck results as apparent resistivity $\rho_a=\omega\mu_0|C(\omega)|^2$ and phase $\phi(\omega)=\pi/2+\arg\{C(\omega)\}$. The upper and lower boundaries of the shadowed area represent upward- and downward-biased estimates of ρ_a, obtained with Z/H (dark shadowed) and $Z:\mathcal{Y}$ (light shadowed), respectively. Results obtained with $Z:\mathcal{Y}$ assuming $\eta=0.25$ are plotted with their error bars; the solid lines indicate the response of the D^+ model fitted to those data. It is remarkable that the $Z:\mathcal{Y}$-method yields reasonable C-responses even at periods of about 3 hr, at least in so far as being interpretable by a 1-D model.

Obviously the results obtained with Z/H are too high, and do not merge smoothly with the S responses at 1 cpd; the transition becomes much 'smoother' when using the $Z:\mathcal{Y}$-method. This could be due to an underestimation of the horizontal gradient of H when using the Z/H method, probably because the single P_1^0 term is not an appropriate description of the source structure, as mentioned at the beginning of Section 2.3. Curiously, the downward-biased values are closest to those obtained with $Z:\mathcal{Y}$. Obviously, the P_1^0 assumption

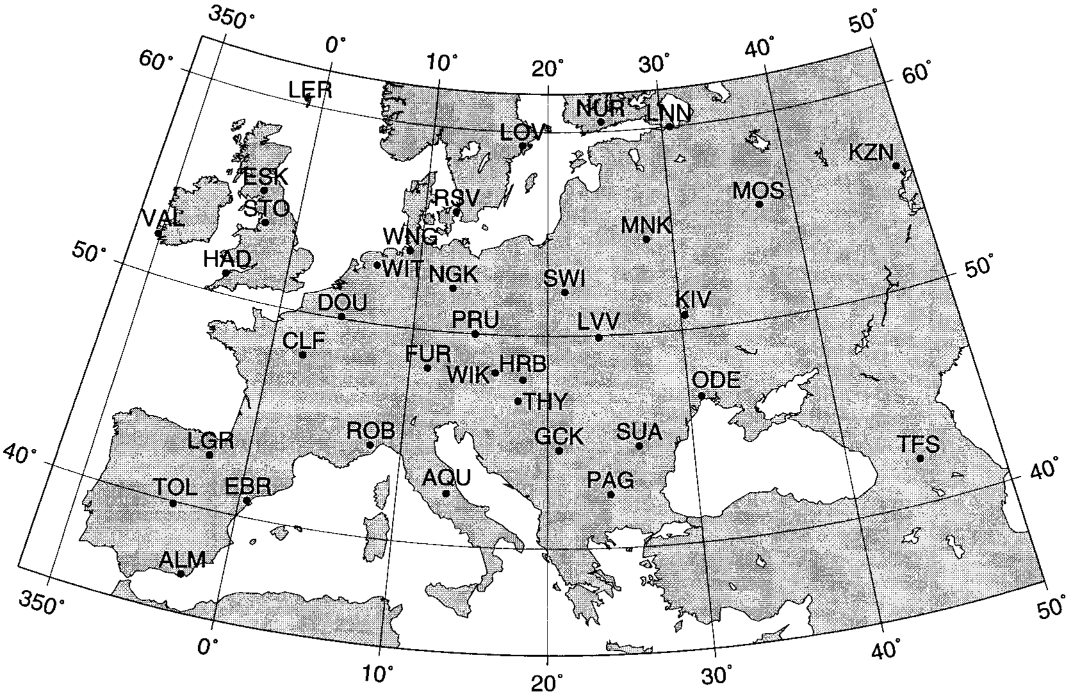

Figure 1. Location of the European observatories used in this study.

Table 2. C-responses (in km) and squared coherency of the observatory Fürstenfeldbruck for 15 periods T (in seconds), estimated with the Z/H-method and the $Z\!:\!\mathcal{Y}$-method, respectively. A frequency-independent value $\eta = 0.25$ was used when estimating the C-responses.

		Z/H				$Z:\mathcal{Y}$			
	T	Re$\{C\}$	Im$\{C\}$	δC	coh^2	Re$\{C\}$	Im$\{C\}$	δC	coh^2
D_{st}	2590000	1142	−325	35	0.75	1031	−310	64	0.89
	1300000	956	−256	30	0.73	961	−272	53	0.91
	864000	927	−230	25	0.74	904	−289	34	0.94
	518000	901	−184	27	0.69	796	−197	33	0.94
	324000	864	−129	26	0.56	734	−204	27	0.92
	236000	894	−82	26	0.49	709	−189	24	0.91
	173000	960	−85	32	0.41	666	−216	23	0.91
S	86400					567	−258	7	0.94
	43200					474	−223	6	0.92
	28800					425	−225	6	0.86
	21600					373	−218	9	0.71
	17300					328	−219	13	0.56
	14400					315	−191	14	0.47
	12340					287	−191	22	0.32
	10800					269	−156	21	0.24

yields resistivities that are too high, but this tendency is partly compensated if downward-biased estimates are considered, as is usually the case. However, this is probably a fortunate coincidence.

Phase angles obtained with Z/H are represented by triangles in Fig. 2. They are obviously too high—especially in the period range of a few days—although they are not a ected by biased estimates of the power spectra caused by an incorrect assumption about the signal-to-noise ratio (i.e. $\arg\{C_D\} = \arg\{C_U\}$). Hence the fact that Z/H yields phases that are too high is solely due to an inappropriate description of the source structure. All these results indicate the superiority of the $Z\!:\!\mathcal{Y}$-method.

4.2 Inferences from the noise-to-signal ratio

The squared coherency decreases with decreasing period, as can be seen from Table 2, and therefore the di erence between C_U and C_D (and hence between $\rho_{a,U}$ and $\rho_{a,D}$) increases: coh^2 is about 0.5 at 6 cpd, and therefore $\rho_{a,U}$ is almost four times

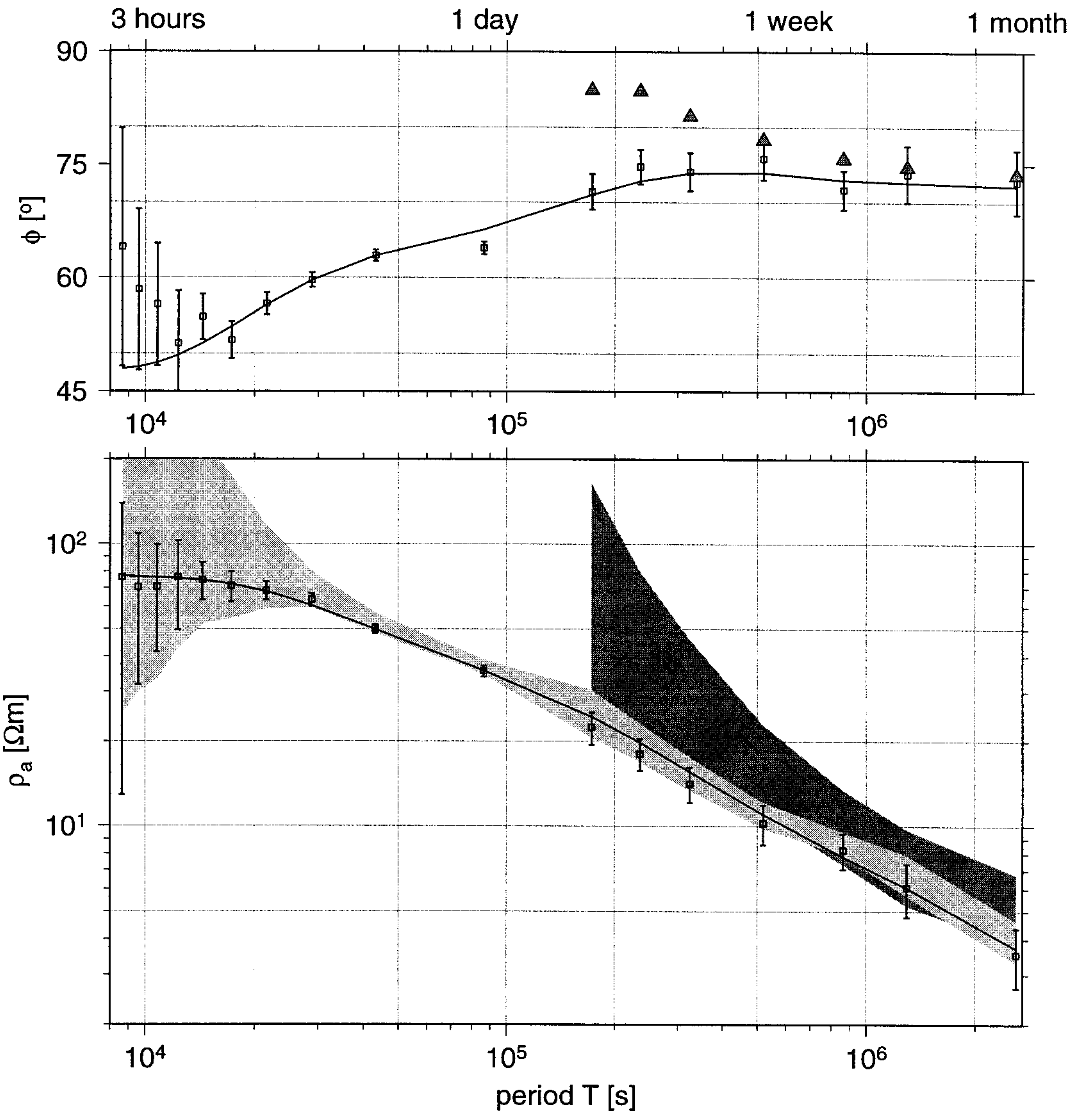

Figure 2. Apparent resistivity ρ_a and phase ϕ for Fürstenfeldbruck (FUR). Upward- and downward-biased values of ρ_a are shown as boundaries of the shadowed regions. The dark shadowed region refers to the Z/H-method, and the light shadowed region refers to the $Z:\mathscr{Y}$-method. Phase angles obtained with Z/H are represented by triangles. Values plotted with errorbars are obtained using the $Z:\mathscr{Y}$-method assuming $\eta = 0.25$; the solid line shows the responses of the D^+ model fitted to those values.

larger than $\rho_{a,D}$. In order to reduce the statistical bias of the C-response, a global, frequency-independent value $\eta = 0.25$ has been chosen, i.e. the noise-to-signal ratio in $\mathscr{Y}$ is assumed to be 25 per cent of that in Z. This value will be justified in the following.

For the case of a 1-D conductivity structure, Weidelt (1972) has shown that

$$\phi(T) \approx \frac{\pi}{4}\left\{1 - \frac{d \ln \rho_a(T)}{d \ln T}\right\}. \tag{25}$$

This condition is obviously not fulfilled for the downward-biased estimates $\rho_{a,D}$ for periods below 30 000 s at Fürstenfeldbruck (*cf.* the lower boundary of the light shadowed area in Fig. 2), because $\phi > 45^\circ$ is not consistent with the decrease of $\rho_{a,D}$ at short periods. If one wants to stick to the assumption of a 1-D conductivity, the assumption of noise-free $\mathscr{Y}$ has to be rejected.

The amplitude and phase of C are related for a 1-D conductivity structure (Weidelt 1972), and therefore knowledge of $\arg\{C\}$ can be used to infer $|C(\omega)|$ (apart from a scaling constant). Such a reconstruction of $C(\omega)$ has the advantage of not being a ected by a statistical bias due to an inappropriate assumption about the noise-to-signal ratio.

An alternative is to choose that noise-to-signal ratio (that value of η) which is most consistent with the assumption of a 1-D conductivity structure. Let us assume that we have smoothed spectra $\langle ZZ^* \rangle$, $\langle Z\mathscr{Y}^* \rangle$ and $\langle \mathscr{Y}\mathscr{Y}^* \rangle$ for K frequencies ω_k from which $C_D(\omega_k)$ and coh^2 are estimated according to eqs (18) and (20). Next, several values of η are chosen for which $C(\omega_k)$ are determined using eq. (23), and the χ^2 misfit to the D^+ model is calculated from those values according to eq. (13).

Fig. 3 shows χ^2 as a function of η for some European observatories. Despite the di erences, a value near $\eta = 0.25$

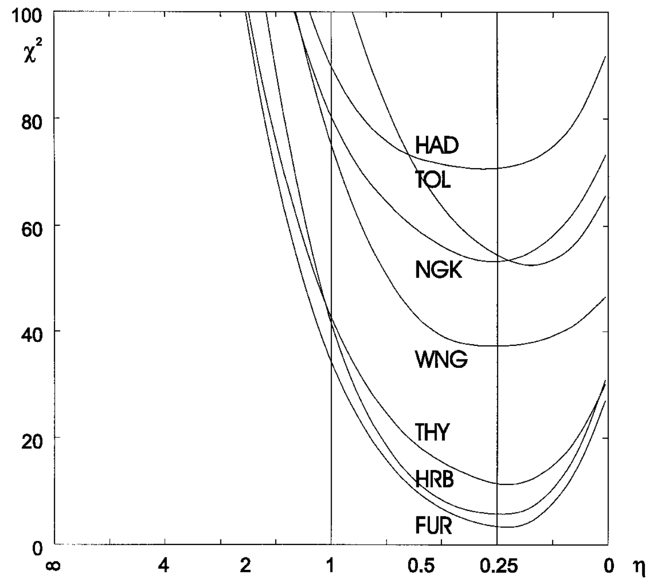

Figure 3. χ^2 as a function of the relative noise-to-signal ratio η for some European observatories. Note the minimum χ^2-misfit near $\eta = 0.25$.

yields a minimum misfit for almost all observatories, and hence this value has been taken globally.

However, the validity of a frequency-independent noise-to-signal ratio is dubious, but, as already stated in Section 2.3, it is always necessary to choose a value for the assumed noise-to-signal ratio, and $\eta = 0.25$ seems to be a better choice than the usual assumption of noise-free H or $\mathscr{Y}$ (i.e. $\eta = 0$).

4.3 Regional differences of *C*-responses in Europe

Olsen (1992) found a systematic trend in the *C*-responses in the *S* period range of a few hours: $\mathscr{Re}\{C\}$ is larger in the northwest than the southeast, which indicates increasing mean conductivity to the southeast.

To test this tendency for longer periods, four groups of European observatories were considered: a northwest group (DOU, WIT, WNG, NGK, HAD, STO); a southwest group (LGR, TOL, EBR, ALM, ROB, AQU); a central group (PRU, FUR, WIK, HRB, THY, GCK, PAG); and an east group (SWI, LVV, KIV, ODE, SUA). The mean *C*-responses for each group and their theoretical D^+ values are shown in Fig. 4. There is a tendency for more negative imaginary parts of *C* in northwestern Europe at periods below 1 day. At longer periods, however, the di erences become smaller, indicating lower heterogeneity in the deeper mantle. Note, however, the particularly large values of $\mathscr{Re}\{C\}$ for the northwest group in the period range between one and three days, which is probably due to the influence of the polar electrojets: their occurrence has a diurnal modulation, and it is di cult to model their small-scale source structure using spherical harmonics.

This is confirmed from an investigation of $\mathscr{Re}\{C\}$ for individual observatories with respect to geomagnetic latitude: there is a general increase of $\mathscr{Re}\{C\}$ to the north for periods between 6 and 300 hr, which probably indicates lower conductivity in the north. However, the increase of $\mathscr{Re}\{C\}$ is especially large for a period of 48 hr, which is probably caused by an insu cient determination of the source structure of the polar electrojets.

Data from all 24 European observatories have been merged both to reduce statistical errors and to remove e ects due to lateral conductivity distortions, and mean *C*-responses were estimated using the $Z:\mathscr{Y}$-method. They are listed together with apparent resistivities and phase angles in Table 3.

The mean *C*-responses have a D^+ misfit of only $\chi^2 = 17$, which is considerably lower than those of the individual observatories. From a statistical point of view, the expectation value $\mathrm{E}(\chi^2) \approx 2K = 30$ should be equal to the number of data (real and imaginary parts of *C* for $K = 15$ frequencies).

5 DISCUSSION

Fig. 5 shows the mean European *C*-responses of Table 3 in comparison with the results by other authors [a compilation of additional published *C*-responses in the *S* and D_{st} period range can be found on pages 111–112 of Schmucker (1985)]. The $Z:\mathscr{Y}$-method has been applied to European observatories

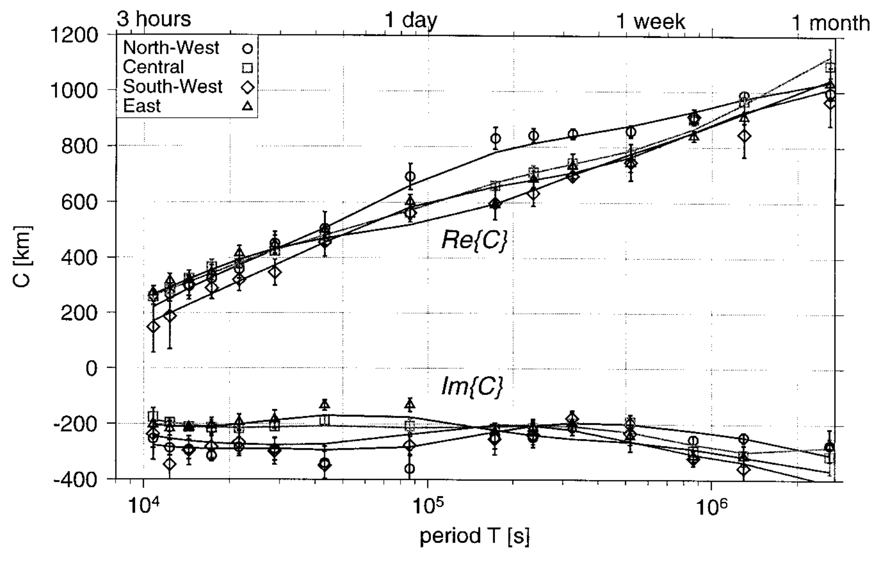

Figure 4. *C*-responses for groups of European observatories.

Table 3. Mean European *C*-responses (in km) and corresponding apparent resistivities ρ_a (in Ω m) and phases (in degrees) for 15 periods *T* (in s). A frequency-independent value $\eta = 0.25$ was used when estimating the *C*-responses.

	T	Re{C}	Im{C}	δC	ρ_a	ϕ
D_{st}	2592000	1012	−311	22	3.41 ± 0.29	72.4 ± 1.5
	1296000	949	−321	20	6.11 ± 0.50	70.6 ± 1.5
	864000	898	−291	11	8.15 ± 0.39	71.4 ± 0.9
	518400	826	−232	20	11.20 ± 1.06	73.9 ± 1.7
	324000	795	−194	20	16.32 ± 1.59	76.1 ± 1.7
	235636	763	−234	25	21.36 ± 2.67	72.4 ± 2.2
	172800	740	−231	31	27.45 ± 4.42	72.1 ± 2.9
S	86400	608	−247	23	39.33 ± 5.49	66.7 ± 2.6
	43200	500	−239	25	56.2 ± 10.3	62.6 ± 3.5
	28800	432	−244	20	67.5 ± 11.1	57.5 ± 3.2
	21600	389	−240	17	76.4 ± 11.1	54.7 ± 2.9
	17280	348	−247	18	83.4 ± 14.0	49.4 ± 3.3
	14400	325	−246	19	91.3 ± 16.6	46.6 ± 3.7
	12343	304	−235	20	94.4 ± 19.5	45.6 ± 4.2
	10800	272	−209	20	86.1 ± 20.2	46.1 ± 4.7

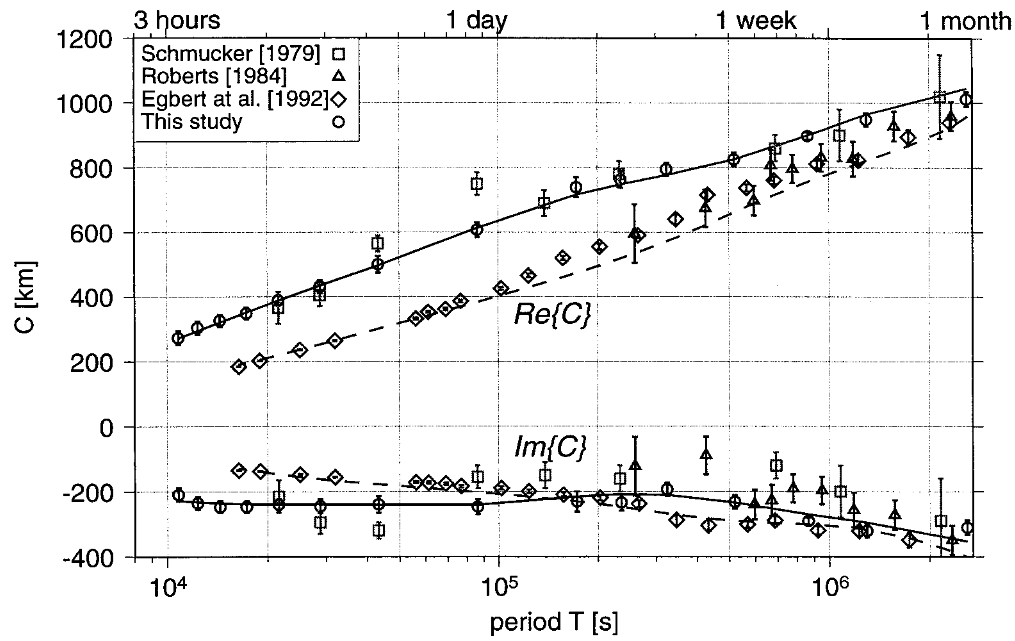

Figure 5. Comparison of *C*-response estimates by other authors.

by Schmucker (1979); his results are—within their error bars—in agreement with those of the present study for periods longer than one day, but disagree for the *S*-periods, especially at a period of one day. The present results, however, have much smaller statistical errors, presumably due to the increased amount of data and the use of more SHA coe cients, which certainly results in a higher coherency.

Roberts (1984) estimated *C*-responses for periods between 3 and 100 days using the *Z*/*H*-method with the P_1^0 assumption. Mean values, obtained by averaging his *C*-responses for 16 observatories, are presented in Fig. 5, too. The real parts of *C* are lower compared with those of the present study, especially for a period of a few days, whereas the imaginary parts are higher (less negative). However, Roberts assumed that *H* is error-free, and, if this assumption fails, the estimated transfer functions are downward-biased. This could explain the di erence from the present results. Fortunately, Roberts published values of the squared coherency, and hence it was possible to apply the scheme of Section 2.3 to his results. An analysis similar to that of Fig. 3 indicates a minimum of the χ^2 misfit for a relative noise-to-signal ratio of $\eta \approx 0.25$ for the majority of his observatories, which is in agreement with the present study. If $\eta \approx 0.25$ is applied to his transfer functions in order to obtain corrected estimates according to eq. (23), the corresponding *C*-responses agree much better with the present ones.

Egbert, Booker & Schultz (1992) used data from the Tucson observatory and a combination of the magnetotelluric method (for periods $T \le 5$ days) and of the *Z*/*H*-method (for $T > 5$ days) for estimating transfer functions. The results presented as ρ_a and phase in their Table 2 have been converted to

C-responses and are shown in Fig. 5 as diamonds; the response of the D^+ model fitted to those values is shown with dashed lines. These results di er considerably from those of the present study: $\mathscr{Re}\{C\}$ is 100–200 km higher in the present study. This could be a result of the di erent regions (southwest US and Europe, respectively). However, their responses systematically deviate from a 1-D response, especially at shorter periods: the $D+$ model fitted to the whole data set (24 responses between 4.5 hr and 91 days) yields a misfit of $\chi^2 = 566$, whereas excluding the shorter periods below 1 day reduces the misfit to $\chi^2 = 44$. Egbert *et al.* (1992) present three possible explanations for the large misfit in their data set: statistical bias due to incorrect assumptions about the signal-to-noise ratio; source structure e ects; and lateral variations of conductivity.

Although the main topic of this paper concerns estimation of unbiased C-responses, their interpretation by means of 1-D conductivity models is now briefly discussed.

Models with various numbers of layers were calculated using Schmucker's ψ-algorithm (Schmucker 1985; Larsen 1975). Fig. 6 presents a four-layer model with misfit $\chi^2 = 38$, which is higher than the expected value $\chi^2 = 30$. It turns out that it was not possible to decrease χ^2 by increasing the number of layers, probably because $d/\sqrt{\rho}$ is assumed to be constant in the ψ-algorithm (d is layer thickness and ρ is resistivity). Therefore Occam's inversion (Constable *et al.* 1987) has been applied and a smooth model (in the sense of minimum second derivative of log(conductivity) with log(depth) beneath a top layer of 100 km thickness) was constructed. The stepped curve of Fig. 6 presents a model with a misfit of $\chi^2 = 30$ corresponding to the expectation value. From these models it follows that the upper mantle has remarkably little structure: the Occam model has a monotonic decrease of resistivity from 100 Ω m near $z = 200$ km to 0.7 Ω m below $z = 1000$ km. The four-layer ψ-algorithm model contains conductivity increases at about 520 and 790 km, respectively, depths which coincide roughly with the good conducting layers of the D^+ model (609 and 853 km, respectively).

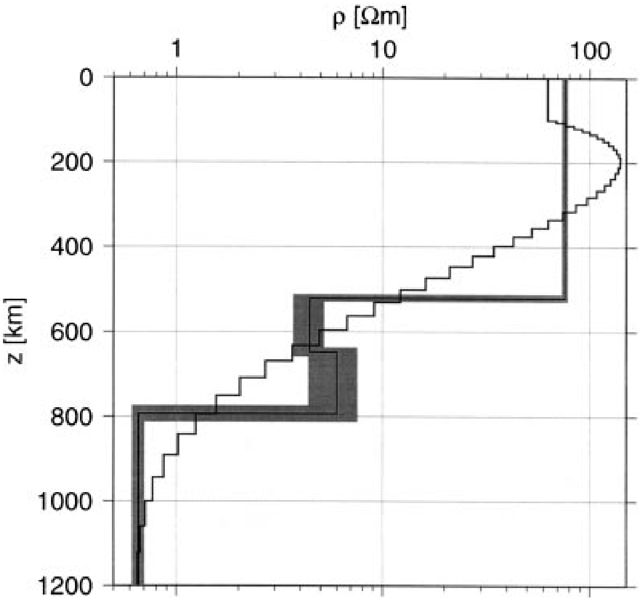

Figure 6. Mean 1-D-conductivity models for Europe. The four-layer model ($\chi^2 = 38$) was obtained using Schmucker's ψ-algorithm (shadowed area is the 68 per cent confidence region); the stepped line presents a model obtained using Occam's inversion ($\chi^2 = 28.5$).

Table 4. (a) Layer resistivity ρ and thickness d of the four-layer 1-D model, obtained with Schmucker's ψ-algorithm. (b) Conductance τ and depth z of the D^+-model.

(a)		(b)	
ρ [Ωm]	d [km]	τ [S]	z [km]
76 ±2.4	520 ± 8	$1.65 \cdot 10^3$	0
4.5 ±0.8	126 ± 11	$7.42 \cdot 10^3$	351
5.9 ±1.6	145 ± 19	$39.8 \cdot 10^3$	609
0.662 ±0.04		$318.4 \cdot 10^3$	853
		$1040 \cdot 10^3$	1322
$\chi^2 = 38$		$\chi^2 = 17$	

6 CONCLUSIONS

Application of the $Z\!:\!\mathscr{Y}$-method to observatory hourly mean values yields consistent C-responses for the entire period range between 3 hr and 30 days, and for two di erent source mechanisms (S and D_{st}). A comparison of the $Z\!:\!\mathscr{Y}$-method and the commonly used Z/H-method indicates the superiority of the former, provided that su cient data at neighbouring sites are available when applying the $Z\!:\!\mathscr{Y}$-method.

When interpreting the results in terms of mantle conductivity, it turns out that the upper mantle has remarkably little structure with a monotonic decrease of resistivity from 100 Ω m near $z = 200$ km to 0.7 Ω m below $z = 1000$ km.

The results presented obtained with the $Z\!:\!\mathscr{Y}$-method are encouraging for an analysis of even longer periods to probe the mantle conductivity below 1000 km. A study with periods between 30 days and 1 year is in preparation.

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