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The Elements of Probabilistic Time Geography

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Abstract Time geography uses *space-time volumes* to represent the possible locations of a mobile agent over time in a *x*-*y*-*t* space. A volume is a qualitative representation of the fact that the agent is at a particular time t_i inside of the volume's base at t_i. Space-time volumes enable qualitative analysis such as potential encounters between agents. In this paper the qualitative statements of time geography will be quantified. For this purpose an agent's possible locations are modeled from a stochastic perspective. It is shown that probability is not equally distributed in a space-time volume, i.e., a quantitative analysis cannot be based simply on proportions of intersections. The actual probability distribution depends on the degree of *a priori* knowledge about the agent's behavior. This paper starts with the standard assumption of time geography (no further knowledge), and develops the appropriate probability distribution by three equivalent approaches. With such a model any analysis of the location of an agent, or relations between the locations of two agents, can be improved in expressiveness as well as accuracy.

Keywords time geography · space-time-cone · topological relations

1 Introduction

Tracking systems log sequences of discrete locations of mobile agents, technically because observations are always discrete, but more practically due to limited bandwidth or database limitations. Locations of the mobile agents in between two consecutive logged locations are frequently modeled by interpolation [14]. This approach leaves

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some uncertainty about the location of agents between the logged locations, depending on the interpolation method, the degree of constraint movement, and the time interval between two observations. A more rigorous approach to model the possible locations of mobile agents is provided by time geography [15, 23, 41, 24], representing and analyzing all possible locations of an agent in space and time in form of *space-time volumes*. One of these volumes, the *space-time cone*, describes the potential locations of an agent if its location is known only at one time. The other volume, a *space-time prism*, describes the potential locations of an agent if its location is known at two times. The *space-time path* describes the (known) locations of an agent over time in form of a linear trajectory. Lifelines [18, 25] consist of any combination of these volumes for tracking an agent over time. This means time geography analyzes discrete (countable) geometries that can be volumetric.

Interesting operations in time geography are intersections of these volumetric geometries. Since these operations are computationally expensive [18, 8, 25], they are usually replaced by intersections of the projected geometries on the x-y-plane. By this way time geography facilitates qualitative statements such as about potential locations at a particular time, and potential encounters of agents. A quantification of the results was, to our knowledge, never tried. Quantifications based on the relative size of the intersections would be misleading, as such an approach ignores the likelihoods of finding the agent at particular locations at particular times within these intersections. We will demonstrate that these likelihoods do not follow an equal probability distribution, hence proportions of intersections are meaningless. Instead, the probability of finding an agent somewhere at a particular time t_i depends on factors such as their goal-orientation, or the regularity of their behavior. But even a completely undirected random movement does not lead to an equal probability distribution. As Pearson has summarized early findings on random walks: "The lesson of Lord Rayleigh's solution is that in open country the most probable place to find a drunken man who is at all capable of keeping on his feet is somewhere near his starting point!" [33, p. 342]. Thus, the hypothesis of this paper is three-fold: (I) A probability distribution of agent locations in space-time volumes is non-equal, (II) can be determined from *a priori* knowledge about the agent's behavior, and (III) knowing the probability distribution of an agent's location over time can facilitate quantitative analysis in time geography.

To prove this hypothesis this paper concentrates on space-time cones. Space-time cones represent the case of a space-time modeler knowing nothing more about an agent's movements than a start location and a maximum speed. Drawing on a discrete *space-time aquarium* [10], a *probabilistic space-time cone* will be introduced (Fig. 1). Without loss of generality, the discrete probabilistic space-time cone approximates the continuous probability distribution of the location of a mobile agent by any appropriate resolution. Within this discrete volumetric model probability distributions will be derived by three approaches: (a) from a random walk simulation, (b) from combinatorics, and (c) from convolution. All three approaches come to equivalent results, supporting the hypothesis, part I, for the limited case of space-time cones (part II). What is demonstrated here for a limited modeler's knowledge, who can only assume undirected movements, can be extended for directed movements, which will be demonstrated for *probabilistic space-time prisms* in another paper.

Addressing the hypothesis, part III, Section 4 develops the formula for quantitative analysis with probabilistic space-time cones, and discusses their behavior. Questions involving a quantification of likelihoods in time geography were not addressed before, except a suggestion of fuzzy space-time volumes [30], an idea that was not further de-

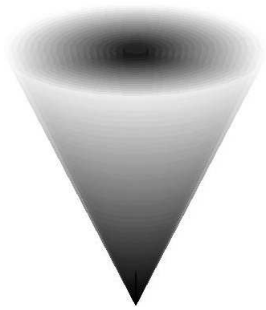

Fig. 1 The probabilistic space-time cone showing different probability densities of an agent's locations over time.

veloped in that paper. These questions would be of the kind: What is the most probable arrival time of an agent A at a particular location B? Or what is the probability that two agents A and B have met by t_i? Or, if A searches at t_0 for a collision-free path in a populated environment, what is the safest path alternative? These questions are relevant for applications such as search in rescue and criminology, collision avoidance, or the estimation of arrival time of individual agents.

2 Background

Probabilistic time geography is based on several areas, among them (classical) time geography and random walks.

2.1 Time geography

Time geography [15, 24] addresses questions like: Given a location of a mobile agent at t_0, where is the agent at a later time $t_i > t_0$, or where was the agent at a previous time $t_i < t_0$? Assuming the agent can move in any direction and is limited only by a maximum speed $v_{\max}$, time geography represents the reachable locations of this agent by a right cone in x-y-t-space. The cone apex represents the agent's location at t_0, and the aperture the maximum speed of the agent, such that a cone base b_i represents the set of locations the agent may settle at a time $t_i > t_0$ (Figure 2a), or may have settled at $t_i < t_0$ (Figure 2b). If the agent's location is known at t_0 and then again at t_n, the possibly reachable volume is described by a cone with apex in t_0, reduced by the intersection with an inverse cone with an apex at t_n. This *space-time prism* [15] (sometimes called *bead* [18]) is straight if the agent returns at t_n to the location of the origin at t_0 (Figure 2c); otherwise it is oblique (Figure 2d). The *space-time path* is a degenerated space-time prism in form of a linear trajectory, and a stationary space-time path as in Figure 3b is called a *space-time station*. A parallel projection of a space-time volume on the x-y-plane describes all places a moving agent can possibly have reached and is called their *potential path area*. If the fundamental assumption of isotropic space is limited by constraining movements along networks, Miller [24] has shown how to map space-time volumes to network spaces, and extended this later to anisotropic network space by including velocity fields [27].

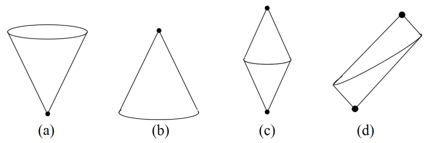

Fig. 2 Traditional space-time volumes.

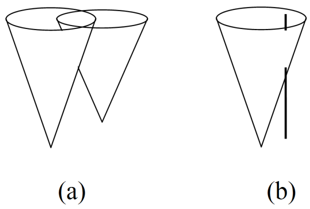

Fig. 3 Time geographic analysis: potential meetings.

To allow for temporal uncertainty—i.e., uncertainty on departure or arrival time—or spatial uncertainty—i.e., uncertainty on departure or arrival location—Neutens *et al.* [31] introduced rough space-time prisms. In this model based on rough set theory [32], uncertainty defines a lower volume $\underline{V}$ and upper volume $\overline{V}$ such that $\underline{V}$ is describing the space-time volume certainly accessible, and $\Delta V = \overline{V} - \underline{V}$ the volume possibly accessible. While this model is still qualitative, it already differentiates three values at a base b_i: *true*, *false*, and *maybe*. In this paper we will further differentiate, and do so based on probability rather than on logic. Note that Neutens *et al.* model the uncertainty of departure or arrival (time and location), while we model the uncertainty about the agent's movement decisions.

All such space-time volumes facilitate qualitative analysis, be it Boolean or three-valued logic. Time geography is typically interested in the possible topological relations of two agents a_1 and a_2 with known locations at t_0 and a_2 either a mobile agent or a static object. Questions are, for example, whether a_1 has possibly met by t_i the mobile agent a_2 (Figure 3a), or reached the static location a_2 (Figure 3b), respectively. Such qualitative questions can be answered by testing whether the intersection of their corresponding space-time volumes is empty or not. Computationally cheaper is the evaluation of projections, such as the analysis at the particular time t_i. The cone base b_i at t_i represents the uncertainty about an agent's location at t_i, hence, for the analysis at t_i one can apply the usual relational calculi for extended objects: the 4-intersection model [9], or the region connection calculus [36]. Even extensions of these calculi for uncertain objects exist [47, 7, 6, 31]. The qualitative statements derived from these calculi have to consider the point-like nature of the agents, such that 2-dimensional topological relationships of space-time volume bases have to be mapped onto qualitative likelihoods (*impossible*, *possible*) of the only distinguishable 0-dimensional relationship, *meet*.

Uncertain locations gain interest in moving object databases research. Typically these uncertain locations are modeled by discrete bases of space-time volumes. For example, [34] calculates the elliptic base of a space-time prism to model uncertainty, and others formulate the constraints of time geography to enable queries to distinguish between conditions that must hold and those that may hold in movement uncertainty

[40, 17]. The latter is particularly useful to manage the uncertainty in updating policies [48]. With uncertain object locations analysis based on distances from moving objects is affected, such as nearest-neighbor queries [45, 19, 21]. We will introduce later basic analysis operators considering the quantification by probabilities to meet agents at certain locations.

2.2 Location prediction of moving agents

The basic argument in this paper is that unknown movements of agents can be modeled as random walks. Then a large number of random walks would provide probability distributions of an agent being at a particular time at a particular location. The related work is discussed here.

A random walk is a sequence of random steps. Unbiased random walks approximate for example continuous diffusion processes such as Brownian motion. Biased random walks follow a trend, a direction superimposed by random decisions. Biased random walks have not found much attention in the literature. Human agents move less randomly assuming rational behavior (Pearson's drunken person mentioned above might be an exception). Rational behavior typically comes with goal orientation, heuristics in wayfinding, and optimization of paths, although people's wayfinding behavior is also regular and habitual [38, 13, 4]. Human wayfinding may appear more random when people stroll, i.e., move without a particular goal in mind, and also as the average behavior of a large number of goal-oriented agents, or perhaps even the goal-directed behavior of an agent over large time spans.

This paper starts out to design a probabilistic time geography with unbiased random movements. In this respect, it does not assume a goal orientation (a bias) of the agents, or it simply does not know about a bias and cannot assume anything better than an unbiased movement. Goal orientation, or modeling the movement by biased random walks, will be developed in another paper. A bias can easily brought in by biased transition probabilities, say, towards the destination of a space-time prism.

It is broadly accepted to model the behavior of large numbers of mobile agents by random walks. For example, random walks can be used to simulate large numbers of mobile agents to study higher order patterns such as flocking behavior, crowding, queuing or congestions of agents [20, 3, 22]. Note that, depending on the amount of *a priori* knowledge about the individuals and their behavior, simulation can also go beyond random walking models; traffic simulation, for example, takes origin-destination matrices or activity-based destination models and assumes shortest path travels [28, 2, 46]. In turn, sensor network research deploys random walks to establish a population of mobile sensor nodes that then can be studied for their ad-hoc communication behavior [5].

In contrast to studying large numbers of agents, the present paper simulates by random walks the many wayfinding options a single agent has during a period of time. Large numbers of random walk simulations provide frequencies of visited places at particular times, which can be normalized to probabilities of finding an individual agent at a particular time at a particular location. An alternative approach to derive this spatial and temporal probability distribution would be by machine learning from collected travel data [16].

Random walks are treated in this paper in discrete space and time. Only then one can expect probabilities of agents at particular locations larger than zero (the integral of a probability distribution over a point is always zero). Polya has shown for infinite random walks in one- and two-dimensional discrete space that the likelihood of an agent returning to the origin (or, equivalently, reaching any other particular point in space) is one [35]. In contrast, this paper is interested in finite time spans only, as they occur in time geography, and hence, we will get probabilities of an agent reaching a particular location that are smaller than one.

While this paper concentrates on motion prediction over longer time periods (or larger time budgets), short term motion prediction is bound to physical constraints. A popular method for short term motion prediction is Kalman filtering [12]. The Kalman filter is a recursive estimation procedure based on the recent observations of location, bearing, and speed of the agent, and assuming that an agent is moving with linear change of speed. In contrast, in this paper we do not limit ourselves to short term predictions. Also, with coarser temporal resolution one can no longer assume linear extrapolation. Accordingly, we will allow any jump in direction and speed between two time instances for our agent.

3 Uncertainty of location

This section will introduce a random walk in a discrete space-time aquarium, and then develop three approaches of computing probabilistic space-time cones from random walks.

Assume an isotropic discrete space-time aquarium that is regularly partitioned into voxels of Δx, Δy and Δt. Assume an agent moving in this aquarium, starting at t_0 from a known location (x_0, y_0), and ending at t_n after n time steps of *a priori* unknown movements. The discrete steps of the agent are limited by its maximum speed $v_{\max}$. Assume a discrete $v_{\max}$, which can be realized for example by choosing a suited temporal resolution Δt. For simplicity, let us assume $v_{\max} = 1$, and a distance measure in the x-y-plane realized by 4-neighborhood (von Neumann neighborhood). Thus, in one step the agent can move to the South, North, West, or East neighbor of its actual location, or it can stay where it is (Fig. 4). Then a random walk consists of a sequence of a pair of actions: a move according to the actual heading, and a random assignment of a new heading. This random assignment should be unbiased because no prior knowledge of the agent's goal, preferences or behavior exists (Fig. 2a).

In the following sections three approaches will be discussed to calculate the probability distribution of the location of an agent at any time $t_0 \leq t_i \leq t_n$: by simulation, by combinatorics, and by convolution. All these approaches provide equivalent models of a probabilistic space-time volume (such as Fig. 1), but differ in their computational complexity.

3.1 Simulation approach

Random walks can easily be constructed in the discrete space and time of cellular automata [39], essentially forming a discrete Markov chain with state space and transition probabilities. Cellular automata are nonlinear dynamical systems [29]. According to [44], a cellular automaton a can be represented by a set of states S, a set of transition rules T, and a set of neighboring automata N, $a \sim (S, T, N)$. The transition rules define the automaton's state s_i^a at a time instance t_i based on the previous states of itself,

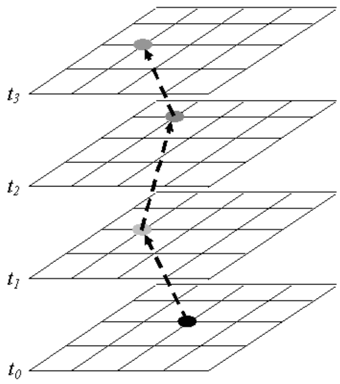

Fig. 4 One random walk of $v_{\max} = 1$ in discrete space-time between t_0 and t_3.

s^a_{i-1}, and its neighbors, s^n_{i-1}, $n \in N$. Basic cellular automata are defined on a regular square partition of space, and choose a fix neighborhood, such as 4-neighborhood (von Neumann) or 8-neighborhood (Moore).

Adopting [39, 11] for our purpose, basic cellular automata applying the 4-neighborhood realize a random walk by a set of states $S = \{0 \ldots 5\}$, such that $s = 0$ means the cell is empty; $s = 1$ means the cell is settled by an agent that is heading towards North; $s = 2$ means the cell is settled by an agent that is heading towards East; and so on, until $s = 5$ means the cell is settled by an agent that is planning to stay in the next time interval. The corresponding set of transition rules T is: (1) If at t_i a cell is empty ($s = 0$) do nothing; (2) if at t_i a cell is settled with an agent of state $s = 5$, then update its status by assigning randomly a new heading from $\{1 \ldots 5\}$; (3) if at t_i a cell is settled with an agent of state $1 - 4$, then relocate the agent to the cell it is heading to and update its status by assigning randomly a new heading from $\{1 \ldots 5\}$. To initialize a random walk simulation at t_0, all cells are set empty except the one in the center of the space that has a random status between 1 and 5.

For example, Figure 5 presents the base of a space-time cone created by 100 concurrent random walkers (black spots) over 200 iterations, who all started in the apex of the cone. The total explored area is shown in grey. The figure was computed by a Swarm simulation provided online by the Complexity Virtual Lab, Monash University. This tool allows to set a probability of the agents changing their directions. This probability was set here to 80%, according to the five possible states of which one is the current one. Unfortunately the simulation shows us only visited places, but no frequencies of visits.—Note that the radius of the cone base after 200 iterations is 200, and is much larger than the total explored area in Figure 5. This means it is unlikely that an agent reaches outer areas of the base. Even in the explored area the density of agents varies, with the highest probability to find an agent in the center.

To find frequencies, let us again consider m random walks of length n of an agent in this space-time aquarium, $m \gg n$. Now for each cell in the aquarium, register the frequency of being visited. These frequencies provide a discrete probability distribution for each time t_i by norming the frequencies in the base $b_i = b(t_i)$. For continuous movements (Brownian motion, Wiener process) the distribution is expected to be normal. In the present discrete version the distribution is 2-dimensional multinomial with $k = 5$ possible outcomes.

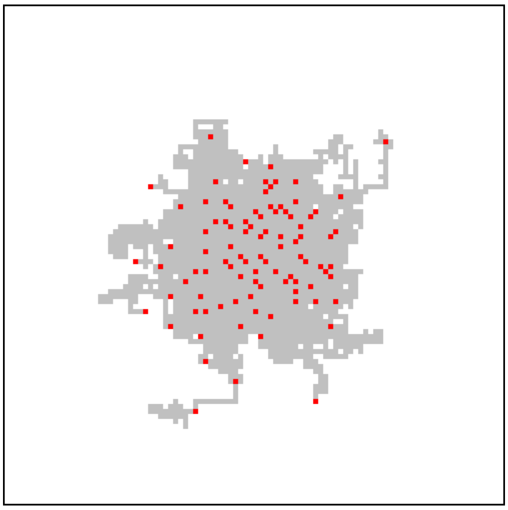

Fig. 5 100 random walks of 200 steps; black cells show the end locations of the agents, gray cells show previously visited locations.

Simulation with basic cellular automata yields a physical explanation for a probability distribution. For example, it is impossible ($p = 0$) that an agent is outside its space-time cone base at any time, and for $t \gg t_0$ it is unlikely that an agent is somewhere on the boundary of the base.

Such a simulation is a rigid abstraction of reality. It is restricted in the number of directions to move, in the speed to move, in the space to move, and it assumes that the directions are all alike (isotropy). The first restriction can be overcome by introducing more states and transition rules. The second one is interesting only if the agent can move with various speeds, and this again can be included by more states and rules. Movement can actually be constrained to network space, which again can be reflected in a modification of the model. The last point can be addressed by a modification of the random function, bringing in a bias, a case we referred to another paper.

3.2 Combinatorial approach

Another approach to think about random walks and probabilities of settling at particular places comes from combinatorics, studying the number of possible paths to reach each cell at t_n. The number of possible paths provides a frequency distribution.

Figure 6 shows an agent at a particular location at time t_0. This agent is able to move maximally one grid position within one time interval ($v_{\max} = 1$). This means the agent can take only one path from its actual location to p_1 at t_2. In contrast, a total of five paths exist to p_2 at t_2, i.e., the probability of finding the agent at t_2 at p_2 if five times higher than at p_1.

For a formalization, it is useful to start with one-dimensional space over time (Fig. 7). Here the number of possible paths is given by the trinomial triangle: For level i, the values represent the coefficients of $(1 + x + x^2)^i$. The triangle can be determined by summing the values of the direct predecessor, and its left and right neighbor.

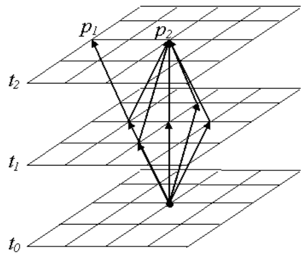

Fig. 6 Over two time steps, a mobile agent can travel only one route to p_1, but five routes to p_2.

Among the multiple ways of computing the trinomial coefficients there are direct and recursive formulas [1].

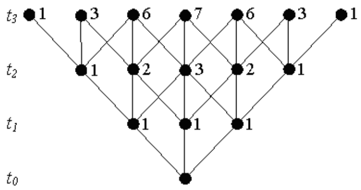

Fig. 7 The number of possible paths in a one-dimensional discrete space over three time intervals.

Extended to the two-dimensional discrete space-time aquarium, the result is a pyramid. A value at level t_i of the pyramid is determined by summing the values of five predecessors at t_{i-1}: the value itself and its four neighbors. Figure 8 shows quarters of the pyramid levels of t_0 to t_3. To our knowledge this pyramid has not been considered in combinatorics so far, although we expect that again a recursive computation can be formulated.

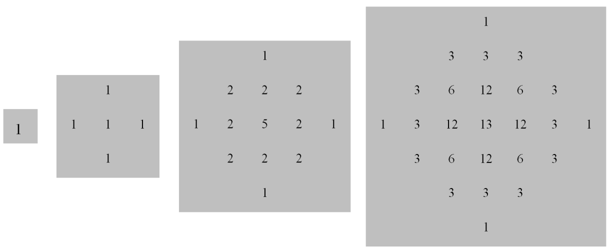

Fig. 8 The levels of the possible route pyramid from t_0 (left) until t_3 (right).

The values, normalized for each level, are equivalent the probability that a random walk passes this location. A recursive computation is of order n^3 for the frequencies, and linear for normalization, hence, $\mathcal{O}(n^3)$.

3.3 Computation by convolution

Another approach that computes only the distribution, but not the individual random walks, is by convolution. Convolution, or discrete linear spatial filtering of a field a by a kernel h, can be expressed by:

$$b_{x,y} = a_{x,y} \otimes h = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} h_{i,j}\, a_{x-i,y-j} \tag{1}$$

with b the resulting filtered spatial field. In this most general expression, the neighborhood of a focal point (x, y) is unlimited $(-\infty \ldots \infty)$, but in practical applications its area of influence can be limited to a small neighborhood, supported by Tobler's law: "Everything is related to everything else, but near things are more related than distant things." [42, p. 236]. A typical size of a kernel is only 3×3, describing the neighborhood from $(x-1, y-1)$ to $(x+1, y+1)$. This kernel has a radius of $d = 1$. In the present context the neighborhood is even physically limited to $d = 1$, by the maximum speed $v_{\max}$ of the agent. The kernel reflecting the above described discrete random walk is:

$$h = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{2}$$

Applied recursively on a unit impulse function at (x_0, y_0) (the agent's location at t_0) one gets directly the results of Figure 8: frequencies of visits. Using instead a normalized kernel:

$$h' = \frac{1}{5} h = \begin{bmatrix} 0.0 & 0.2 & 0.0 \\ 0.2 & 0.2 & 0.2 \\ 0.0 & 0.2 & 0.0 \end{bmatrix} \tag{3}$$

provides probabilities directly. For example, the probability for an agent to stay between t_0 and t_1 in (x_0, y_0) is 0.2. Figure 9 shows the first two convolution steps, where the intensity in grey is proportional to the probability value.

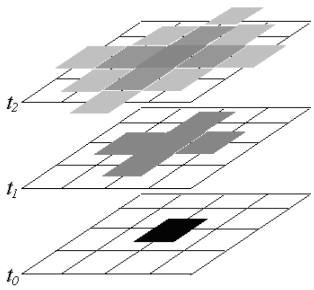

Fig. 9 The probability distribution of locations of a mobile agent at two sequential time steps.

In contrast to typical applications in signal processing, the convolution has to be computed here only locally, in the neighborhood of the moving agent, and not for the full space considered. For this purpose time geography allows to compute *a priori* the volume to be convoluted [37]. The space-time volume of a moving agent can be approximated by a discrete cylinder of n temporal layers and a radius of $n\,v_{\max}$. In fact, this cylinder approximating a space-time cone contains still about 75% empty cells. Thus, a recursive formulation of the convolution is applied to avoid computations for empty cells. For an agent at t_0 being located at (x_0, y_0) the space-time cone between t_0 and t_n consists of $n+1$ layers computed by:

$$c_{x,y,k} = \begin{cases} 0 & \text{if } |x - x_0| > k \vee |y - y_0| > k \\ c_{x,y,k-1} \otimes h' & \text{else} \end{cases} \tag{4}$$

This means computational complexity is a function of time only, more precisely, of the n time intervals for which a cone is computed. For each time step k the convolution computes $(2k+1)^2$ new values, i.e., $\mathcal{O}(n^2)$. Also biased convolutions can be formulated, which requires adaptations to Equations 3 and 4; we refer to the other paper for details. Overall, compared to the simulation or combinatorial approach, convolution turns out to be more transparent with respect to normalization and biasing.

3.4 Learning from collected travel data

While the theoretically derivations of probability distributions for the locations of a mobile agent (Sections 3.1-3.3) all lead to equivalent results, the applicability of the assumptions made, especially the random movement behavior, can only be tested from large scale travel data. Since such a test is always dependent on the context of the collected travel data, no general insights can be found from testing other than that remaining differences must be the result of the particular context of the data. The most relevant aspect of context is intentionality: (most) collected travel data concerns directed movements, and hence, should be discussed together with biased random walks.

4 Reasoning with uncertain location

Up to now time geography makes binary statements. For example, at a time $t_i > t_0$ a moving agent is *within* a space-time cone originating in its position at t_0; it is not outside. Or if space-time cones of two moving agents intersect from $t_i < t_n$ these two agents *can* have met between t_i and t_n; otherwise they cannot [26]. With rough space-time volumes, Neutens *et al.* [31] have extended the binary to a three-valued reasoning ('certainly accessible', 'possibly accessible', 'certainly not accessible'). However, the probabilistic extension of time geography enables to revisit the analysis operations of time geography, facilitating a refinement of queries as well as results. This section will explore this potential exemplarily; a thorough design of an algebra with probabilistic space-time cones is beyond the scope of this paper.

4.1 Meetings at time t_k

Queries regarding the likelihood that two agents meet at a time t_k, $0 \leq k \leq n$, have to be based on an agreed semantics of 'meeting'. We say two agents meet if they are both found at a time t_k at the same location $l_{x,y}$, which can be written in short as a cell in the discrete space-time aquarium: $l_{x,y,k}$. I.e., the semantics of 'meeting' depends on the granularity of locations, or the spatial and temporal extensions of the cells. We

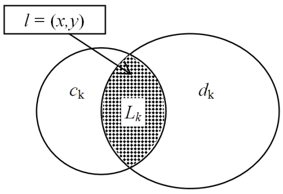

Fig. 10 Two agents, shown by their cone bases at t_k, can meet in the cells of the intersecting area.

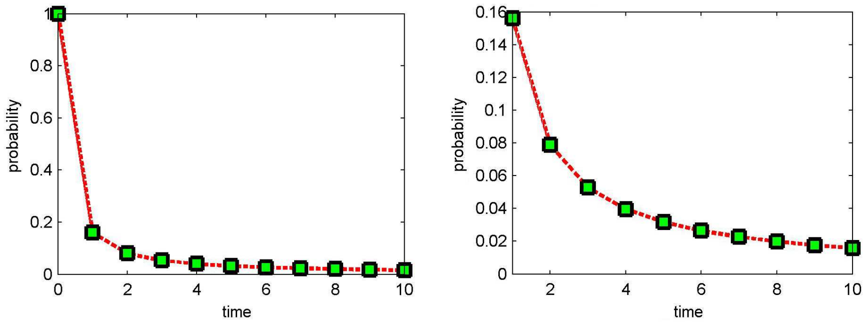

Fig. 11 Probabilities over time to be at the space-time station (x_0, y_0) (left, starting with 1 because this is the known origin of the agent), and to be at an ex-center space-time station (right).

also do know that two agents C and D can only meet in the overlapping area of their two space-time cones c and d (Figure 12). These areas cover multiple possible meeting locations at t_k, L_k, such that:

$$L_k = \bigcup (l_{x,y,k} \ \in \ c_k \cap d_k) \tag{5}$$

The trivial query with a probabilistic cone is: "What is the probability of an agent C being at the location $l_{x,y}$ at time t_k?" In terms of map algebra [43], this query can be answered by a *local* operation, extracting $c_{x,y,k}$ from the cone:

$$P(C \in (x, y, k)) = c_{x,y,k} \tag{6}$$

For example, Figure 11 (left) illustrates the probability of the agent to be at (x_0, y_0) over time, from t_0 to $t_1 0$ (at t_0 the probability is 1 since we know that agent started from here). Note that the values were actually computed by a continuous normal distribution, hence, small deviations from the discrete convolution are visible, but the overall behavior is present. On the right, the probability is shown for an ex-center space-time station. In both cases, values converge quickly towards 0, according to expectations.

Generalizing to a *zonal* query, we can also ask: "What is the probability of an agent C being in locations Z at time t_k, with Z_k being any subset of the cone base at t_k?" This query can be answered by a zonal operation, the integral over all locations $l \in Z_k$:

$$P(C \in (Z_k)) = \sum_{l \in Z} c_{x,y,k} \tag{7}$$

Also reasoning between the movements of two agents can be considered, e.g.: "What is the probability of the mobile agents C and D meeting in $l_{x,y}$ at t_k?" This query corresponds to the joint probability $P(C \in (x, y, k) \cap D \in (x, y, k))$, which for independent movements of C and D can be computed by the product $P(C \in (x, y, k)) \cdot P(D \in (x, y, k))$, or simply $c_{x,y,k} \cdot d_{x,y,k}$ if c represents the space-time cone for C and d for D:

$$P(C \in (x, y, k) \cap D \in (x, y, k)) = c_{x,y,k} \cdot d_{x,y,k} \tag{8}$$

Similarly, a zonal query based on the movements of two agents can be considered, e.g.: "What is the probability of the mobile agents C and D meeting somewhere in Z, $Z \subseteq L_k$ at time t_k?" The zone L_k where the two agents can meet at t_k is shown in Figure 10 (Equation 5). Let us denote the number of locations in Z by m, $m = |Z|$, and call the event of a meeting E, such that:

$$E(C \in (x_i, y_i, k) \cap D \in (x_i, y_i, k),\ 1 \le i \le m) = \bigcup_{i=1 \ldots m} E(x_i, y_i, k) \tag{9}$$

According to probability theory, the probability of the meeting event for two agents follows just as:

$$P(E(C \in (x_i, y_i, k) \cap D \in (x_i, y_i, k),\ 1 \le i \le m)) = P(\bigcup_{i=1 \ldots m} E(x_i, y_i, k)) \tag{10}$$

which in turn computes as:

$$P(\bigcup_{i=1 \ldots m} E(x_i, y_i, k)) = \sum_{i=1}^{m} P(E(x_i, y_i, k)) \tag{11}$$

$$= \sum_{i=1}^{m} (c_{x_i,y_i,k} \cdot d_{x_i,y_i,k}) \tag{12}$$

The following example is based on two agents' movements represented by identical kernels with 4-neighborhood according to Equation 3, over 200 time steps, and with an initial situation of the two agents being 20 units apart. Figure 12 illustrates the probability of the agents meeting at each time step. Since the agents, traveling with a maximum speed of 1 unit per time interval, and initially being 20 units apart, cannot meet the first 10 time intervals, the probability is 0 in the beginning, but then rises quickly. However, after some time the probability decreases again.

4.2 Meetings by time t_k

Questions about temporal ranges, such as: "What is the probability of an agent having reached location $l_{x,y}$ by t_k?", can be answered by cumulative probabilities, or $\sum_{t=0 \ldots k} c_{x,y,t}$. Figure 13 shows the cumulative probability of Figure 12, i.e., of the probability of a meeting at each time step. Basically it computes the sum of the meeting probabilities over the whole intersection of the two space-time cones.

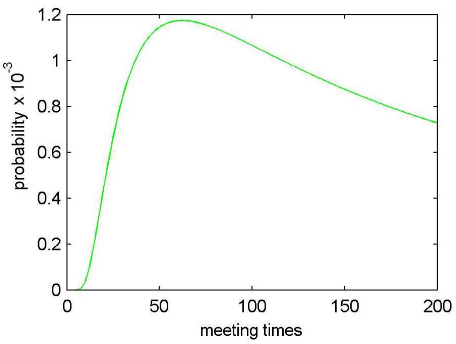

Fig. 12 Meeting probabilities over time. The agents have a maximum speed of 1 unit per time interval, and are initially 20 units apart, so the first 10 seconds they do not meet at all. Small values are a result of joint probabilities.

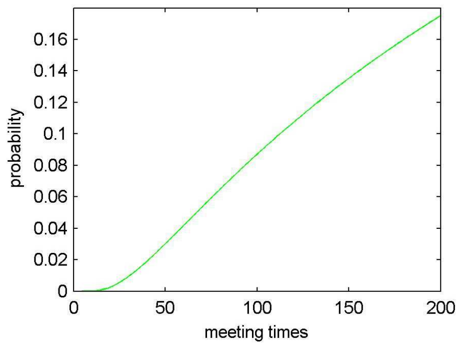

Fig. 13 The (cumulative) probability of two agents meeting by time t_k.

5 Discussion and conclusions

This paper has presented first steps into a probabilistic time geography, or more generally into a quantitative time geography. It addresses questions such as how likely it is to find an agent in a particular area, or how likely it is that two agents meet. The paper has clearly shown that the probability distribution of finding an agent within a space-time volume is non-equal. This is an important result, because it disqualifies any attempt to quantify by relative sizes of intersections.

The paper also discusses the importance of sufficient *a priori* knowledge about the agent's behavior. Different behavior forms significantly different probability distributions. For this paper, an agent's movement behavior was assumed to be unbiased. Unbiased movement can be assumed for example for an average behavior from a large number of independently moving agents, but also for a strolling agent without a particular goal, or perhaps even for an agent with goal-directed movements over longer time spans. Unbiased movement is well captured by the *a priori* knowledge expressed by the parameters of classic space-time cones: known is only the location of an agent at a time t_0, and their maximal speed $v_{\max}$ (and nothing else needs to be known).

In general, however, goal-directed behavior violates these assumptions, and hence, goal-directed movements are not well described by the presented probability distribution. If the modeler knows about the goals, this additional *a priori* knowledge can be brought in by biasing the transition probabilities and changing by that way the probability distribution. These extensions are mentioned, but their development are beyond the scope of the paper. One particular additional knowledge well-known in classical time geography is the space-time prism, adding to the knowledge of a modeler a (at least intermediate) goal of an agent, and the time when the agent was observed at this goal. In another paper we will study the form of the probability distribution for such a biased movement. It is clear that any of these extensions leads also to non-equal probability distributions.

The presented method is derived from theory. While it does not need an experimental validation, the paper refers to some experimental evidence of its applicability.

The presented model lays the foundations for a large number of future questions. Among these are a further refinement of the probabilistic cone model for different types of environments or different agent behaviors, the completion of the cone to a probabilistic space-time prism, and the further development of reasoning mechanisms with these probabilistic space-time volumes.

Refinement of the probabilistic cone model: The current model is based on a regular isotropic grid-shaped network. It is clearly worth to overcome these simplifying assumptions. First, the grid can be anisotropic. With clutter in the environment, some nodes may be impossible to be visited. Other nodes ('space-time stations') may form attractions and have higher probabilities of being visited. Second, the network to be modeled can have irregular shape instead of a regular grid. Then the combinatorial node-based computation of probabilities has to be changed to one that considers true lengths of edges, and is capable of dynamically segmenting the irregular network.

Completion of the cone to a space-time prism: Up to now all directions are considered equally likely for an agent's move (Equation 2), due to no better previous knowledge. Thus, the computations so far are sufficient to replace traditional space-time *cones* by probabilistic ones. But if a general heading of the agent is known, i.e., if an adequate tracking frequency reveals a (traditional) space-time prism, the assignment of assignment of values to the convolution kernel k has to be adapted to this knowledge. The reasoning behind, and the theory for probabilistic space-time prisms will be developed in a future paper.

Development of probabilistic reasoning mechanisms: The above demonstrated potential for probabilistic reasoning needs to be explored in depth. Especially the reasoning methods Neutens *et al.* apply [30] for fuzzy space-time volumes can be investigated. With fuzzy representations being rather a (semantic) generalization of probabilistic representations, their algorithm should be applicable in a probabilistic environment, and get more meaningful.

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