The Royal Society

The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis

By Norden E. Huang[1], Zheng Shen[2], Steven R. Long[3], Manli C. Wu[4], Hsing H. Shih[5], Quanan Zheng[6], Nai-Chyuan Yen[7], Chi Chao Tung[8] and Henry H. Liu[9]

[1] *Laboratory for Hydrospheric Processes/Oceans and Ice Branch, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA*

[2] *Department of Earth and Planetary Sciences, The John Hopkins University, Baltimore, MD 21218, USA*

[3] *Laboratory for Hydrospheric Processes/Observational Science Branch, NASA Wallops Flight Facility, Wallops Island, VA 23337, USA*

[4] *Laboratory for Atmospheres, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA*

[5] *NOAA National Ocean Service, Silver Spring, MD 20910, USA*

[6] *College of Marine Studies, University of Delaware, DE 19716, USA*

[7] *Naval Research Laboratory, Washington, DC, 20375-5000, USA*

[8] *Department of Civil Engineering, North Carolina State University, Raleigh, NC 27695-7908, USA*

[9] *Naval Surface Warfare Center, Carderock Division, Bethesda, MD 20084-5000, USA*

Received 3 June 1996; accepted 4 November 1996

Contents

Printed in Great Britain

TEX Paper

A new method for analysing nonlinear and non-stationary data has been developed. The key part of the method is the 'empirical mode decomposition' method with which any complicated data set can be decomposed into a finite and often small number of 'intrinsic mode functions' that admit well-behaved Hilbert transforms. This decomposition method is adaptive, and, therefore, highly efficient. Since the decomposition is based on the local characteristic time scale of the data, it is applicable to nonlinear and non-stationary processes. With the Hilbert transform, the 'instrinic mode functions' yield instantaneous frequencies as functions of time that give sharp identifications of imbedded structures. The final presentation of the results is an energy–frequency–time distribution, designated as the Hilbert spectrum. In this method, the main conceptual innovations are the introduction of 'intrinsic mode functions' based on local properties of the signal, which makes the instantaneous frequency meaningful; and the introduction of the instantaneous frequencies for complicated data sets, which eliminate the need for spurious harmonics to represent nonlinear and non-stationary signals. Examples from the numerical results of the classical nonlinear equation systems and data representing natural phenomena are given to demonstrate the power of this new method. Classical nonlinear system data are especially interesting, for they serve to illustrate the roles played by the nonlinear and non-stationary effects in the energy–frequency–time distribution.

Keywords: non-stationary time series; nonlinear differential equations; frequency–time spectrum; Hilbert spectral analysis; intrinsic time scale; empirical mode decomposition

1. Introduction

Data analysis is a necessary part in pure research and practical applications. Imperfect as some data might be, they represent the reality sensed by us; consequently, data analysis serves two purposes: to determine the parameters needed to construct the necessary model, and to confirm the model we constructed to represent the phenomenon. Unfortunately, the data, whether from physical measurements or numerical modelling, most likely will have one or more of the following problems: (*a*) the total data span is too short; (*b*) the data are non-stationary; and (*c*) the data represent nonlinear processes. Although each of the above problems can be real by itself, the first two are related, for a data section shorter than the longest time scale of a stationary process can appear to be non-stationary. Facing such data, we have limited options to use in the analysis.

Historically, Fourier spectral analysis has provided a general method for examining the global energy–frequency distributions. As a result, the term 'spectrum' has become almost synonymous with the Fourier transform of the data. Partially because of its prowess and partially because of its simplicity, Fourier analysis has dominated the data analysis efforts since soon after its introduction, and has been applied to all kinds of data. Although the Fourier transform is valid under extremely general conditions (see, for example, Titchmarsh 1948), there are some crucial restrictions of