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# The Energy Balance of a Plasma in Partial Local Thermodynamic Equilibrium

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**Abstract**—The energy balance for electrons and heavy particles constituting a plasma in partial local thermodynamic equilibrium are derived. Much attention is given to the source term for collisional excitation and ionization, which is written in such a way that the flow pattern does not need to be known to be able to evaluate it. As an example, a flowing cascaded arc plasma is analyzed.

## I. INTRODUCTION

THE MODELING of thermal plasmas has been extensively used to describe several sources of plasma which are used in a multitude of applications. We recall the work in rocket exhaust problems [1], flow and kinetics of inductively coupled plasmas [2], and in expanding dc arcs used for plasma spraying [3] or plasma deposition [4]. In plasmas, the already complex gasdynamic problem is made more difficult by the presence of inelastic collisions, ionization, radiation, and radiation trapping. Also, the temperatures of electrons and heavy particles may be different, and as the electron temperature plays a crucial role in, for example, the ionization, it needs to be described accurately.

Several modeling approaches are applied. In many models the plasma is described as a fluid in local thermodynamic equilibrium (or possibly at different temperatures for electrons and heavy particles). Then, in principle, some of the complexities of partially trapped radiation (e.g., the free-bound radiation to the ground state) can be incorporated in the fluid transport properties as heat conductivity.

In recent years there has been a tendency to also take nonequilibrium effects into account, and therefore explicit formulation of the energy sources and losses is required. In this paper we will describe a new formulation of the energy balance of a plasma in *partial* local thermodynamic equilibrium (PLTE): The ground state of the neutrals is allowed not to be in balance with the continuum, and the temperatures ( $T_e$  and  $T_h$ ) for electrons and heavy

particles (ions and neutrals), respectively, may be different.

In this paper we will treat a one-ion-type plasma with charge number,  $z = 1$ . In view of quasi-neutrality, the electron density is equal to the ion density. Taking the electron density as a basis, the deviation from equilibrium can be characterized by a over- or underpopulation of the ground level with respect to the Saha-Boltzmann density.

The overpopulation will be represented by a dimensionless factor. This representation offers the possibility of classifying plasmas into either recombining or ionizing [5]. The consequences, which the difference between these two kinds of plasma has for the energy balance, are much more clear than in other conventional formulations such as the Heller-Elenbaas equation.

In the proposed formulation the major part of the transport of ions—ground and excited states as well as radiative recombination and resonant radiation—is reformulated as volume terms in the nonequilibrium term in the electron energy equation. This has considerable advantages over the conventional formulations, because now the flow field needs not be known in order to be able to calculate the source terms of the energy balance. Furthermore, the partially trapped radiation (free bound) to the ground state is also contained in the nonequilibrium term.

The objective of this paper is to present the theoretical framework of the proposed formulation. The energy equations (and specifically the source terms) are derived and written into their final form. As an illustration, some rate coefficients will be given for an argon plasma, and the evolution of the several terms of the electron energy equation in a flowing cascaded arc will be discussed.

## II. FORMULATION OF THE ENERGY BALANCE

The energy balance equation for a species  $a$  reads [6]:

$$\frac{\partial}{\partial t} \left( \frac{3}{2} n_a k T_a \right) + \nabla \cdot \left( \frac{3}{2} n_a k T_a \underline{w}_a \right) + n_a k T_a \nabla \cdot \underline{w}_a + \underline{\Pi}_a : \nabla \underline{w}_a + \nabla \cdot \underline{q}_a = Q^a \quad (1)$$

where  $n_a$ ,  $w_a$ , and  $T_a$  are the density, velocity, and temperature of species  $a$ ;  $k$  is the Boltzmann's constant;  $\underline{\Pi}_a$  is the viscosity tensor; and  $\underline{q}_a$  is the thermal heat flux of species  $a$ .  $Q^a$  represents the source term; i.e., all energy (excluding viscous dissipation and heat conduction) that is supplied to species  $a$  by collisions with other particles.

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To determine the contribution of a certain process to the source term  $Q^a$  we will evaluate the increase of the total thermal energy of species  $a$  per unit volume if a process occurs. The phenomena which will be taken into account are: Collisional excitation and ionization, de-excitation and three-particle recombination, line radiation (called bb for bound-bound), recombination radiation (fb for free-bound), brehmsstrahlung (ff for free-free), elastic energy exchange, and ohmic heating. The particles that are considered are electrons and heavy particles (atoms and ions together). All velocity distributions are assumed to be Maxwellian.

### III. THE SOURCE TERMS

#### A. Ohmic Heating

The electrons dissipate the ohmic input during collisions with the heavy particles. The contribution  $Q_o^e$  of ohmic heating to the source term of the electrons equals:

$$Q_o^e = j \cdot E, \quad Q_o^h = 0 \quad (2)$$

where  $j$  and  $E$  represent the current density and the electrical field strength, respectively.

#### B. Elastic Relaxation

If the temperatures of the electrons and heavy particles (atoms and ions) are different, there will be a constant energy exchange caused by elastic collisions. The contributions  $Q_{el}^e$  and  $Q_{el}^h$  to the source terms of the energy equations of the electrons and heavy particles, respectively, due to this effect equal [6]:

$$Q_{el}^e = -Q_{el}^h = -3 \frac{m_e}{m_h} n_e \frac{1}{\tau_{eh}} k(T_e - T_h),$$

$$\frac{1}{\tau_{eh}} = \frac{1}{\tau_{ei}} + \frac{1}{\tau_{eo}} \quad (3)$$

where  $m_e$  and  $m_h$  are the masses of electrons and heavy particles. The characteristic time  $\tau_{eh}$  is the average collision time of an electron with a heavy particle, determined by the collision times between electrons and ions ( $\tau_{ei}$ ) and between electrons and neutral atoms ( $\tau_{eo}$ ).

#### C. Brehmsstrahlung

When electrons collide with ions they may be decelerated. During this process they emit a photon. The total electron energy loss due to this process can be expressed by [7]

$$Q_{ff}^e = -4\pi \int_0^\infty \epsilon_{ff}(\nu) d\nu$$

$$= -1.43 \times 10^{-40} g_{ff} n_e T_e^{1/2} \sum_z z^2 n_z$$

$$= -1.80 \cdot 10^{-40} n_e T_e^{1/2} \sum_z z^2 n_z \quad (4)$$

$$Q_{ff}^h = 0 \quad (5)$$

where  $\epsilon_{ff}$  is the spectral free-free continuum emissivity, and  $\nu$  is the frequency of the light. The temperature  $T_e$  is

in degrees Kelvin. The average Gaunt factor  $g_{ff} = 1.27$  as given by Karzas and Latter [8] has been substituted.

#### D. Radiative Recombination

When an electron encounters an ion, the two might recombine to an atom under emission of a photon. During such a process the total amount of thermal energy per unit volume *does not* change for the heavy particles, but it *does* change for the electrons, since a thermal electron is eliminated. The recombination energy itself, which is also contained in the escaping photon, does not lower the thermal energy. Therefore we arrive at:

$$Q_{rb}^e = - \sum_{p=1}^N n_+ A_{+p} (\frac{3}{2} k T_e), \quad Q_{rb}^h = 0. \quad (6)$$

Here the radiative transition probability  $A_{+p} = n_e k_{+p}^2$ , the product of the electron density and the rate coefficient  $k_{+p}^2$  for radiative recombination. The symbol  $A_{+p}$  has been chosen to show the similarity of radiative recombination to line radiation (also denoted by transition probabilities  $A_{pq}$ ), which is yet to be treated.

#### E. Line Radiation

A photon may be emitted when an atom or ion decays from an excited state. The emission of this radiation, however, does not lower the total thermal energy of the heavy particles nor the thermal energy of the electrons. Therefore it does not lead to a source term for the energy balance,

$$Q_{bb}^e = Q_{bb}^h = 0. \quad (7)$$

It might seem strange that one of the few ways along which potential energy can leave the plasma (the escape of radiation) does not lead directly to a source term for the energy balances. However, as we will show in the next section, this energy actually is taken into account, but it appears to be contained in the electron energy loss during collisional excitation and ionization.

#### F. Collisional Excitation and Ionization

The energy loss  $Q_{ex}^e$  of the electrons due to collisional excitation/deexcitation and ionization/three-particle-recombination is expressed by:

$$-Q_{ex}^e = n_e \sum_{p=1}^{N-1} \sum_{q=p+1}^N (n_p k_{pq} - n_q k_{qp}) E_{pq}$$

$$+ n_e \sum_{p=1}^N (n_p k_{p+} - n_+ n_+ k_{+p}) E_{p+}^*. \quad (8)$$

Here the energy  $E_{pq}$  is the potential energy difference between the levels  $q$  and  $p$ , and  $E_{p+}^*$  is defined as  $E_{p+}^* = E_{p+} - \Delta E_+$ , where  $E_{p+}$  and  $\Delta E_+$  represent the potential energy difference between the ion level and level  $p$  and the ionization potential lowering [9]. The symbols  $k_{pq}$ ,  $k_{p+}$ , and  $k_{+p}$  represent the rate coefficients for collisional (de-)excitation, collisional ionization, and three-particle recombination, respectively.

Equation (8) cannot be evaluated in a straightforward manner because the densities  $n_p$  are unknown. Therefore we will transform it into a form that only contains parameters which are known or parameters which can be calculated from the electron density, the electron temperature, and the overpopulation factor of the ground level.

*Transformation of the Collisional Excitation/Ionization Electron Energy Loss:* The mass balance of energy level  $p$  reads [10]:

$$\begin{aligned} \frac{dn_p}{dt} &= \frac{\partial n_p}{\partial t} + \nabla \cdot (n_p \underline{w}_p) = n_e \sum_{q=1}^{p-1} (n_q k_{qp} - n_p k_{pq}) \\ &+ n_e \sum_{q=p+1}^N (n_q k_{qp} - n_p k_{pq}) - n_e n_p k_{p+} \\ &+ n_e^2 n_+ k_{+p} + n_+ A_{+p} - \sum_{q=1}^{p-1} n_p A_{pq} \\ &+ \sum_{q=p+1}^N n_q A_{qp} \end{aligned} \quad (9)$$

where  $A_{qp}$  is the transition probability for line radiation for the transition of state  $p$  to state  $q$ . The mass balance of the ions is expressed by

$$\frac{dn_+}{dt} = \sum_{p=1}^N (n_e n_p k_{p+} - n_e^2 n_+ k_{+p} - n_+ A_{+p}). \quad (10)$$

If we multiply all mass balances of the levels  $p$  with their energy  $E_{1p}$  and add them to the ion mass balance multiplied by  $E_{1+}^*$ , the result is:

$$\begin{aligned} &\sum_{p=1}^+ E_{1p} \frac{dn_p}{dt} \\ &\equiv \sum_{p=1}^N E_{1p} \frac{dn_p}{dt} + E_{1+}^* \frac{dn_+}{dt} \\ &= n_e \sum_{p=1}^N \sum_{q=1}^{p-1} (n_q k_{qp} - n_p k_{pq}) E_{1p} \\ &+ n_e \sum_{p=1}^N \sum_{q=p+1}^N (n_q k_{qp} - n_p k_{pq}) E_{1p} \\ &+ \sum_{p=1}^N (-n_e n_p k_{p+} + n_e^2 n_+ k_{+p} + n_+ A_{+p}) E_{1p} \\ &+ \sum_{p=1}^N \left\{ -\sum_{q=1}^{p-1} n_p A_{pq} + \sum_{q=p+1}^N n_q A_{qp} \right\} E_{1p} \\ &+ \sum_{p=1}^N (n_e n_p k_{p+} - n_e^2 n_+ k_{+p} - n_+ A_{+p}) E_{1+}^*. \end{aligned} \quad (11)$$

Using

$$E_{p+}^* = E_{1+}^* - E_{1p} \quad (12)$$

the two ionizational terms in (11) can be taken together:

$$\begin{aligned} &+ \sum_{p=1}^N (-n_e n_p k_{p+} + n_e^2 n_+ k_{+p} + n_+ A_{+p}) E_{1p} \\ &+ \sum_{p=1}^N (n_e n_p k_{p+} - n_e^2 n_+ k_{+p} - n_+ A_{+p}) E_{1+}^* \\ &= \sum_{p=1}^N (n_e n_p k_{p+} - n_e^2 n_+ k_{+p} - n_+ A_{+p}) E_{p+}^*. \end{aligned} \quad (13)$$

The two excitation terms can be taken together if in one of them the mathematical indices  $p$  and  $q$  are exchanged, followed by an inversion of the summation order. Taking into account that  $E_{1p}|_{p=1} = E_{11} = 0$ , and that  $p$  and  $q$  can never become larger than  $N$ ,

$$\begin{aligned} &n_e \sum_{p=1}^N \sum_{q=p+1}^N (n_q k_{qp} - n_p k_{pq}) E_{1p} \\ &+ n_e \sum_{p=1}^N \sum_{q=1}^{p-1} (n_q k_{qp} - n_p k_{pq}) E_{1p} \\ &= n_e \sum_{p=1}^{N-1} \sum_{q=p+1}^N (n_q k_{qp} - n_p k_{pq}) E_{1p} \\ &+ n_e \sum_{q=2}^N \sum_{p=1}^{q-1} (n_p k_{pq} - n_q k_{qp}) E_{1q} \\ &= n_e \sum_{p=1}^{N-1} \sum_{q=p+1}^N (n_q k_{qp} - n_p k_{pq}) E_{1p} \\ &+ n_e \sum_{p=1}^{N-1} \sum_{q=p+1}^N (n_p k_{pq} - n_q k_{qp}) E_{1q} \\ &= n_e \sum_{p=1}^{N-1} \sum_{q=p+1}^N (n_p k_{pq} - n_q k_{qp}) E_{pq} \end{aligned} \quad (14)$$

since  $E_{pq} = E_{1q} - E_{1p}$ .

The terms containing line radiation are transformed in a similar way:

$$\begin{aligned} &\sum_{p=1}^N \left\{ -\sum_{q=1}^{p-1} n_p A_{pq} + \sum_{q=p+1}^N n_q A_{qp} \right\} E_{1p} \\ &= -\sum_{p=1}^N \sum_{q=1}^{p-1} n_p A_{pq} E_{1p} + \sum_{p=1}^N \sum_{q=p+1}^N n_q A_{qp} E_{1p} \\ &= -\sum_{p=1}^{N-1} \sum_{q=p+1}^N n_q A_{qp} E_{pq}. \end{aligned} \quad (15)$$

Using (13)-(15), (11) transforms to:

$$\begin{aligned} &\sum_{p=1}^N E_{1p} \frac{dn_p}{dt} + E_{1+}^* \frac{dn_+}{dt} \\ &= n_e \sum_{p=1}^{N-1} \sum_{q=p+1}^N (n_p k_{pq} - n_q k_{qp}) E_{pq} \\ &+ n_e \sum_{p=1}^N (n_p k_{p+} - n_e n_+ k_{+p}) E_{p+}^* + \\ &- \sum_{p=1}^N n_+ A_{+p} E_{p+}^* - \sum_{p=1}^{N-1} \sum_{q=p+1}^N n_q A_{qp} E_{pq}. \end{aligned} \quad (16)$$

Taken together, the first two terms on the right-hand side of (16) equal exactly the electronic inelastic density loss given in (8). Therefore (16) can be regarded as a conservation equation of internal energy: The electronic inelastic loss due to collisional excitation/ionization is consumed by the in- or outflow of potential energy and radiation (fb and bb).

Now a more convenient notation will be introduced. The relative ( $\delta b_p$ ) and absolute ( $b_p$ ) overpopulation factors of level  $p$  are defined with the aid of the Saha-density  $n_{ps}$  [11] of level  $p$ :

$$b_p \equiv \frac{n_p}{n_{ps}}, \quad n_{ps} \equiv n_e n_+ \frac{g_p}{2g_+} \left\{ \frac{h^2}{2\pi m_e k T_e} \right\}^{3/2} \cdot \exp\left(\frac{E_{p+}^*}{k T_e}\right), \quad \delta b_p \equiv b_p - 1. \quad (17)$$

Now,

$$n_p k_{pq} - n_q k_{qp} = n_{ps} k_{pq} (1 + \delta b_p) - n_{qs} k_{qp} (1 + \delta b_q) = n_{ps} k_{pq} (\delta b_p - \delta b_q). \quad (18)$$

Here detailed balancing has been used:

$$n_{qs} k_{qp} \equiv n_{ps} k_{pq}. \quad (19)$$

Similarly,

$$n_p k_{p+} - n_e n_+ k_{+p} = n_{ps} k_{p+} (1 + \delta b_p) - n_e n_+ k_{+p} = n_{ps} k_{p+} \delta b_p \quad (20)$$

since  $n_{ps} k_{p+} \equiv n_e n_+ k_{+p}$ . Equation (16) now simplifies to:

$$E_{1+}^* \frac{dn_+}{dt} + \sum_{p=1}^N E_{1p} \frac{dn_p}{dt} = n_e \sum_{p=1}^{N-1} \sum_{q=p+1}^N \{n_{ps} k_{pq} E_{pq} (\delta b_p - \delta b_q)\} + n_e \sum_{p=1}^N n_{ps} k_{p+} E_{p+}^* \delta b_p - \sum_{p=1}^N n_+ A_{+p} E_{p+}^* - \sum_{p=1}^{N-1} \sum_{q=p+1}^N n_q A_{qp} E_{pq}. \quad (21)$$

To eliminate the term containing the collisional processes that do not come from or lead to the ground state, the ground-state mass balance is used:

$$\frac{dn_1}{dt} = -n_e \sum_{p=2}^N n_{1s} k_{1p} (\delta b_1 - \delta b_p) - n_e n_{1s} k_{1+} \delta b_1 + \sum_{q=2}^N n_q A_{q1} + n_+ A_{+1}. \quad (22)$$

If we assume that

$$\delta b_p \Big|_{p \neq 1} \ll \delta b_1 \quad (23)$$

then we obtain:

$$\frac{dn_1}{dt} = -n_e n_{1s} \delta b_1 \left\{ k_{1+} + \sum_{p=2}^N k_{1p} \right\} + \sum_{q=2}^N n_q A_{q1} + n_+ A_{+1}. \quad (24)$$

Equation (23) is valid in most plasmas, and the condition is certainly met in ionizing plasmas. Within the concept of PLTE (on which this paper focuses), condition (23) is met automatically. With the definition of the total excitation/ionization rate coefficient  $K_1$  according to:

$$K_1 \equiv k_{1+} + \sum_{p=2}^N k_{1p} \quad (25)$$

and after multiplication with  $E_{1+}^*$ , the ground-state mass balance transforms to:

$$E_{1+}^* \frac{dn_1}{dt} = -n_e n_{1s} K_1 \delta b_1 E_{1+}^* + \sum_{q=2}^N n_q A_{q1} E_{1+}^* + n_+ A_{+1} E_{1+}^*. \quad (26)$$

We will isolate in the potential energy conservation equation (21) all terms which represent processes that come from or lead to the ground state:

$$E_{1+}^* \frac{dn_+}{dt} + \sum_{p=1}^N E_{1p} \frac{dn_p}{dt} = n_e n_{1s} \sum_{q=2}^N k_{1q} E_{1q} (\delta b_1 - \delta b_q) + n_e n_{1s} k_{1+} E_{1+}^* \delta b_1 - n_+ A_{+1} E_{1+}^* - \sum_{q=2}^N n_q A_{q1} E_{1q} + n_e \sum_{p=2}^{N-1} \sum_{q=p+1}^N \{n_{ps} k_{pq} E_{pq} (\delta b_p - \delta b_q)\} + n_e \sum_{p=2}^N n_{ps} k_{p+} E_{p+}^* \delta b_p - \sum_{p=2}^N n_+ A_{+p} E_{p+}^* - \sum_{p=2}^{N-1} \sum_{q=p+1}^N n_q A_{qp} E_{pq}. \quad (27)$$

With the definition of the mean excitation energy  $\langle E_1 \rangle$ :

$$\langle E_1 \rangle \equiv \left\{ \sum_{p=2}^N k_{1p} E_{1p} + k_{1+} E_{1+}^* \right\} / K_1 \quad (28)$$

the result is

$$E_{1+}^* \frac{dn_+}{dt} + \sum_{p=1}^N E_{1p} \frac{dn_p}{dt} = n_e n_{1s} K_1 \delta b_1 \langle E_1 \rangle - n_+ A_{+1} E_{1+}^* - \sum_{q=2}^N n_q A_{q1} E_{1q} + n_e \sum_{p=2}^{N-1} \sum_{q=p+1}^N \{n_{ps} k_{pq} E_{pq} (\delta b_p - \delta b_q)\} + n_e \sum_{p=2}^N n_{ps} k_{p+} E_{p+}^* \delta b_p + \sum_{p=2}^N n_+ A_{+p} E_{p+}^* - \sum_{p=2}^{N-1} \sum_{q=p+1}^N n_q A_{qp} E_{pq}. \quad (29)$$

The potential energy equation (29) and the transformed ground-state mass balance (26) are added. Using,

$$\frac{dn_1}{dt} + \frac{dn_+}{dt} + \sum_{p=2}^N \frac{dn_p}{dt} = 0 \quad (30)$$

the result can be expressed by

$$\begin{aligned} & n_e \sum_{p=2}^{N-1} \sum_{q=p+1}^N \{n_{ps}k_{pq}E_{pq}(\delta b_p - \delta b_q)\} \\ & + n_e \sum_{p=2}^N n_{ps}k_{p+}E_{p+}^* \delta b_p \\ & = \sum_{p=2}^N n_+ A_{+p} E_{p+}^* + \sum_{p=2}^{N-1} \sum_{q=p+1}^N n_q A_{qp} E_{pq} + \\ & - n_e n_{1s} K_1 \delta b_1 (\langle E_1 \rangle - E_{1+}^*) \\ & - \sum_{q=2}^N \left\{ n_q A_{q1} + \frac{dn_q}{dt} \right\} (E_{1+}^* - E_{1q}) \end{aligned} \quad (31)$$

where all collisional processes not leading to the ground state are brought to the left-hand side.

Now the energy loss  $Q_{ex}$  of the electrons due to collisional excitation/deexcitation and ionization/3-particle-recombination can be calculated:

$$\begin{aligned} -Q_{ex}^e & = n_e \sum_{p=1}^{N-1} \sum_{q=p+1}^N (n_p k_{pq} - n_q k_{qp}) E_{pq} \\ & + n_e \sum_{p=1}^N (n_p k_{p+} - n_e n_+ k_{+p}) E_{p+}^* \\ & = n_e n_{1s} K_1 \delta b_1 \langle E_1 \rangle + n_e \sum_{p=2}^{N-1} \sum_{q=p+1}^N \\ & \cdot \{n_{ps}k_{pq}E_{pq}(\delta b_p - \delta b_q)\} \\ & + n_e \sum_{p=2}^N n_{ps}k_{p+}E_{p+}^* \delta b_p \\ & = n_e n_{1s} K_1 \delta b_1 E_{1+}^* + \sum_{p=2}^N n_+ A_{+p} E_{p+}^* \\ & + \sum_{p=2}^{N-1} \sum_{q=p+1}^N n_q A_{qp} E_{pq} + \\ & - \sum_{q=2}^N \left\{ n_q A_{q1} + \frac{dn_q}{dt} \right\} E_{q+}^*. \end{aligned} \quad (32)$$

The last term represents a small correction for the overestimation of line radiation to the ground level and transport of excited states. The correction is of the order of 10% of the total energy loss term, which itself is contained implicitly in the term with  $\delta b_1$ . Furthermore, in most plasmas the actual term is small with respect to other contributions. Therefore it can usually be neglected.

All terms in (32) can be calculated if the electron density and temperature and the relative overpopulation factor of the ground state are known; since only the ground level is assumed to show substantial deviations from the

Saha density (concept of PLTE), one can safely use the Saha density  $n_{ps}$  to calculate the cascade radiation term.

#### IV. RECAPITULATION

The source terms  $Q^e$  and  $Q^h$  of the energy balances for electrons and heavy particles can now be constructed:

$$Q^e = Q_o^e + Q_{el}^e + Q_{ff}^e + Q_{fb}^e + Q_{ex}^e \quad (33)$$

$$Q^h = Q_{cl}^h. \quad (34)$$

In the electron energy balance  $Q_{fb}^e$  and  $Q_{ex}^e$  can be taken together. The final result is:

$$\begin{aligned} & \frac{\partial}{\partial t} \left( \frac{3}{2} n_e k T_e \right) + \nabla \cdot \left( \frac{3}{2} n_e k T_e \underline{w}_e \right) \\ & + n_e k T_e \nabla \cdot \underline{w}_e + \underline{\Pi}_e : \nabla \underline{w}_e + \nabla \cdot \underline{q}_e \\ & = \underline{j} \cdot \underline{E} - 3 \frac{m_e}{m_h} n_e \frac{1}{\tau_{eh}} k (T_e - T_h) \\ & - 1.80 \cdot 10^{-40} n_e T_e^{1/2} \sum_{\underline{z}} z^2 n_{\underline{z}} + \\ & - n_e n_{1s} K_1 \delta b_1 E_{1+}^* - \sum_{p=2}^N n_+ A_{+p} \left( \frac{3}{2} k T_e + E_{p+}^* \right) \\ & - n_+ A_{+1} \frac{3}{2} k T_e - \sum_{p=2}^{N-1} \sum_{q=p+1}^N n_q A_{qp} E_{pq} \\ & + \sum_{q=2}^N \left\{ n_q A_{q1} + \frac{dn_q}{dt} \right\} E_{q+}^*. \end{aligned} \quad (35)$$

The heavy particle energy balance reads:

$$\begin{aligned} & \frac{\partial}{\partial t} \left( \frac{3}{2} n_h k T_h \right) + \nabla \cdot \left( \frac{3}{2} n_h k T_h \underline{w}_h \right) \\ & + n_h k T_h \nabla \cdot \underline{w}_h + \underline{\Pi}_h : \nabla \underline{w}_h + \nabla \cdot \underline{q}_h \\ & = 3 \frac{m_e}{m_h} n_e \frac{1}{\tau_{eh}} k (T_e - T_h). \end{aligned} \quad (36)$$

As has been already mentioned, the last (correction) term of the electron energy balance can be neglected in most cases. The same is valid for  $n_+ A_{+1} (3/2) k T_e$ ; it is a small correction (usually around 7%) of the total radiative recombination to the ground level, which itself is contained implicitly in the term with  $\delta b_1$ .

#### V. EXAMPLE

As an example we present the evolution of the several contributions to the energy balance when a thermal plasma flows through the central channel of a cascaded arc. The arc assembly has been described elsewhere [4]. In principle, it consists of a stack of electrically isolated copper plates, with a central bore that forms the central channel. In this channel an argon plasma is created at typical pressures of 0.1–0.7 bar. The arc current is 50 A and the channel diameter is 4 mm. Argon gas is flowing through the arc at a rate of 100 scc/s. The values of the electron density, electron temperature, gas pressure, and ohmic input

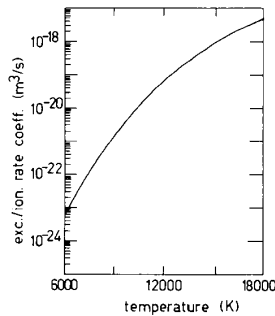


Fig. 1. The total excitation/ionization rate coefficient  $K_1$  for the neutral argon system as a function of the electron temperature.

( $j \cdot E$ ) have been measured as a function of the axial position in the arc [12]. The electron density was obtained from the Stark broadening of the hydrogen  $H_\beta$  spectral line (486.1 nm), the electron temperature was obtained from the electrical conductivity, and the gas pressure was determined with MKS Baratrans. The value of the heavy particle temperature and the overpopulation factor  $\delta b_1$  as a function of the axial position has been calculated from the heavy particle energy balance [12]. To give an impression: The electron density is about  $1.2 \cdot 10^{22} \text{ m}^{-3}$ , the electron temperature is about 12 200 K, the gas pressure decreases from 0.5 to 0.2 bar, and the heavy particle temperature increases from 10 000 to 12 200 K. The overpopulation factor  $\delta b_1$  associated with these conditions ranges from 7 to 40. The total rate coefficient for excitation/recombination  $K_1$  is, as well as are the other rate coefficients in (35), accumulated from the literature [10]. In Fig. 1,  $K_1$  is given for the neutral argon system as a function of the electron temperature.

Now all the quantities needed to calculate the source term of the electron energy balance are known. In Fig. 2 the relative fractions of the several contributions are shown as a function of the axial position in the arc. All negative contributions are scaled to the only positive contribution, the (almost constant) ohmic input  $j \cdot E$  ( $\sim 5 \cdot 10^9 \text{ W m}^{-3}$ ).

Due to the low value of the electron density (compared with stationary arcs) the radiative contributions are a factor of 100 smaller than the contributions of collisional ionization/recombination and elastical energy exchange. In Fig. 2 one can see very clearly the amount of energy that is left for transport terms like heat conduction, expansion, etc. In the beginning of the channel, near the cathodes, a large fraction of the ohmic input is used to heat the heavy particles (atoms and ions) and to ionize the neutral gas. After about 20 mm the plasma reaches a more or less stable situation, and then most of the energy is available for heat conduction and other transport effects.

## VI. CONCLUSION

The formulation of the energy balance as proposed here (equations (35) and (36)) allows for evaluation of the

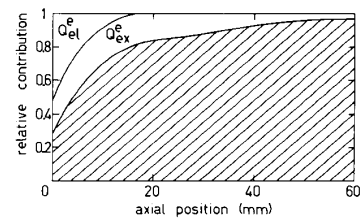


Fig. 2. Axial profile of the various contributions to the electron energy balance in a flowing cascade arc plasma in argon. The plasma flows from left to right (cathode to anode). The shaded part of the graph represents the amount of energy available for transport processes. The radiative contributions are all negligible due to the low value of the electron density. This figure is an example of the possibility presented by the formulation to calculate the source terms of the electron energy balance without knowing the flow pattern.

source terms without knowledge of the particle and radiation transport situation, since most of the contributions that arise from that situation (diffusion, local capture of radiation, etc.) are taken into account in the nonequilibrium term ( $-n_e n_{1s} k_1 \delta b_1 E_{1+}^*$ ). The small correction for the contribution of the loss of excited states, which is slightly overestimated in this approach, can usually be neglected. The overpopulation factor  $\delta b_1$ , which can often be evaluated from measurements of the pressure, electron density, and temperature, becomes one of the most important plasma parameters; in the energy balance it accounts for much of the phenomena which may be difficult to evaluate otherwise.

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