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THE EQUILIBRIUM AND OPTIMAL TIMING OF PRICE CHANGES

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ABSTRACT

This paper studies the welfare properties of the equilibrium timing of price changes. Staggered price-setting has the advantage that it permits rapid adjustment to firm-specific shocks but the disadvantage that it causes price level inertia and therefore increases aggregate fluctuations. Because each firm ignores its contribution to inertia, staggering can be a stable equilibrium even if it is highly inefficient. In addition, there can be multiple equilibria in the timing of price changes; indeed, whenever there is an inefficient staggered equilibrium, there is also an efficient equilibrium with synchronized price-setting.

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INTRODUCTION

The timing of wage and price changes is crucial to the real effects of nominal disturbances in an important set of recent Keynesian theories. Taylor (1979, 1980) and Blanchard (1983, 1986) show that if firms change their prices and wages often but different firms adjust at different times, the aggregate price level responds slowly to nominal shocks: staggered price-setting causes price level inertia. As a result, economic fluctuations may be much larger, and welfare much lower, when price changes are staggered rather than synchronized.

These results raise a puzzle: why do firms choose to change prices at different times if they would be better off under synchronization? One possible answer is that firms' decisions to stagger are evidence that staggering is in fact desirable. This paper investigates the alternative possibility that synchronization is socially optimal but that some market failure leads to staggering in a decentralized economy. This issue cannot be addressed in the Taylor and Blanchard models, because these authors treat the timing of price changes as exogenous. The purpose of this paper is thus to make the timing endogenous and then study the welfare properties of the equilibrium timing.

In the Taylor and Blanchard models, firms are identical and all shocks are aggregate, and so firms have no desire to change prices at different times. As a result, if timing is made endogenous, the outcome is synchronization--the Taylor and Blanchard models cannot explain staggering.[1] Therefore, in addition to making the timing of price changes endogenous, we introduce an incentive for staggering. Specifically, we add to the Blanchard model the most obvious reason that firms in actual economies do not change prices simultaneously: shocks that alter profit-maximizing prices occur at different times for different firms. For example, the gas station on the corner changes prices at different times from the grocery store next door because OPEC meetings occur at different times from weather changes in farm areas. Using a model with both firm-specific and aggregate shocks, we derive the condition that determines which of synchronization and staggering is optimal and the conditions for each to be an equilibrium.

Comparison of the equilibrium and optimal timing leads to two major conclusions. First, inefficient staggering is indeed possible. If there are firm-specific shocks of any size, then staggered price-setting, with each firm changing its price when it receives shocks, is a stable equilibrium. But if the variance of nominal shocks is large relative to the variance of idiosyncratic shocks, synchronization is preferable to staggering--it limits firms' ability to adjust to idiosyncratic shocks but, by eliminating price level inertia, it reduces the size of aggregate fluctuations. Moreover, if the variance of aggregate shocks is large, the increase in fluctuations and the reduction in welfare caused by staggering can be large. The reason that staggering can be an equilibrium despite being highly inefficient is that

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1. This result is a folk theorem among some economists. It is demonstrated formally for the Blanchard model in this paper and in Ball and Cecchetti (1987).

firms' choices of the timing of their price changes have externalities. By contributing to inertia, a firm's decision to adjust at different times than others hurts all firms. In a large economy, each price-setter ignores this effect, since it takes the behavior of the price level as given.[2]

Our second principal conclusion is that there can be multiple equilibria in the timing of price changes--synchronization can remain an equilibrium even when firm-specific shocks make staggering an equilibrium. Intuitively, adjusting its price at the same times as others allows a firm to reduce fluctuations in its relative price. If price changes are bunched, this is a strong force causing them to remain bunched. But if price changes are staggered, there is no force to bring them together, and so they remain staggered. The incentive to remain bunched is so strong that the condition for synchronization to be an equilibrium is weaker than the condition for it to be optimal. Thus, if staggering is an inefficient equilibrium, there must be a superior synchronized equilibrium as well. In addition, inefficient synchronization is possible.

We demonstrate these results in Blanchard's model modified in the simplest ways that allow us to address the subject of the paper. Specifically, we follow Blanchard in assuming that prices are fixed for two periods; thus a firm's only choice about timing is whether to change its price every even period or every odd period. Similarly, we introduce idiosyncratic shocks in the simplest way that gives firms incentives to change prices at

2. Just as we show that choices of the <u>timing</u> of price changes have externalities, recent analyses of "small menu cost" models (Mankiw, 1985, Akerlof and Yellen, 1985, and Ball and Romer, 1987) and of long-term contract models (Ball, 1986a, 1986b) show that choices of the <u>frequency</u> of price changes have externalities: less frequent price changes increase the variance of real aggregate demand, which harms all firms.

different times: we assume that half of the firms in the economy receive idiosyncratic shocks every even period and half every odd period. In the conclusion, we discuss generalizations such as stochastic arrival of firm-specific shocks and more complicated rules for when to change prices. We argue that our results about the possibility of inefficient staggering and the existence of multiple equilibria are likely to be robust.[3]

The remainder of the paper consists of five sections. Section II presents our model. In Section III, we solve for the behavior of the aggregate price level under synchronization and under staggering. This leads, in Section IV, to the condition that determines which regime is optimal. Section V derives the conditions under which synchronization and

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3. Several previous authors present models of endogenous staggering. The models closest to ours are those of Fethke and Policano (1984, 1986a), who study the timing of wage negotiations, and Parkin (1986). (See also Fethke and Policano [1986b, 1987] and Matsukawa [1986]. Maskin and Tirole [1985] and Gertner [1985] study staggering that results from strategic behavior within an oligopolistic industry.) Our analysis differs from this previous work in three major respects. First, we address a different question: while previous papers focus on the conditions under which staggering and synchronization are equilibria, we focus on the welfare properties of equilibria. (An exception in previous work is Fethke and Policano [1986a], who compare equilibrium and optimal timing in their model.) Second, the source of staggering in our model--heterogeneous times of firm-specific shocks--is clearly an important reason that firms in actual economies change prices at different times. In previous models, by contrast, staggering occurs only under unrealistic conditions. Fethke and Policano (1986a) show that staggering arises in their models only if the economy consists of a few large sectors containing firms that receive identical shocks. In Parkin's model, staggering arises only under unusual assumptions about monetary policy: an increase in the price level must lead to a decrease in the money supply. Third, in previous models of endogenous timing, staggering does not lead to price level inertia. In Fethke and Policano, the reason is that wages, while "predetermined," are not "fixed": firms set wages for several periods at once, but they can choose different wages for different periods. In Parkin, the reason is simply that the model contains no aggregate shocks. Without price level inertia, staggering does not lead to large output fluctuations; thus previous models ignore what we consider the key macroeconomic effect of staggering.

staggering are stable Nash equilibria. Finally, in Section VI we discuss robustness and offer conclusions.

II. THE MODEL

Our model is a simple extension of Blanchard's (see the version in Blanchard and Fischer, 1985, ch. 9). The economy contains a large number of price-setters who adjust their prices every two periods. Blanchard studies the behavior of the economy under two regimes: synchronization, in which by assumption all firms change prices in even periods, and staggering, in which half change in even periods and half in odd periods. As described above, we depart from Blanchard's work by allowing firms to choose whether to change prices in even or odd periods and by introducing idiosyncratic productivity shocks.

The specifics of the model are as follows. The economy consists of N farmers, where N is a large number. Each farmer uses his own labor to produce a differentiated product, then sells the product and purchases the products of all other farmers.[4] Farmers take each others' prices as given. Omitting time subscripts, farmer i's utility function is

$$(1) \qquad U_i = C_i - \frac{\varepsilon - 1}{\gamma\varepsilon} L_i^{\gamma}, \qquad C_i = N\left[\frac{1}{N} \sum_{j=1}^{N} C_{ij}^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)},$$

where C_{ij} is farmer i's consumption of farmer j's product, C_i is an index of farmer i's total consumption, ε is the elasticity of

4. This is an inessential simplification of Blanchard's model, which contains both goods and labor markets.

substitution between goods $(\varepsilon > 1)$, L_i is farmer i's labor supply, and γ measures the extent of increasing marginal disutility of labor $(\gamma > 1)$. The coefficient multiplying L_i is chosen for convenience.

Farmer i's production function is

$$(2) \qquad Y_i = \frac{L_i}{\phi_i} ,$$

where Y_i is farmer i's output and ϕ_i is a productivity shock. Let θ_i denote $\ell n\ \phi_i$. If i is even, then θ_i changes every even period; otherwise, it changes every odd period. θ_i has mean zero and variance σ^2_θ . In periods in which it changes, θ_i is uncorrelated both with its own past values and with θ_j for all $j \neq i$.

A transactions technology determines the relation between real money balances and total spending on goods:[5]

$$(3) \qquad Y = \frac{M}{P} ,$$

where

$$Y = \frac{1}{N} \sum_{j=1}^{N} \frac{P_j Y_j}{P} ,$$

$$P = \left[\frac{1}{N} \sum_{j=1}^{N} P_j^{1-\varepsilon}\right]^{1/(1-\varepsilon)} .$$

M is the money supply, P_j the price of farmer j's product, and P the price index corresponding to the consumption measure C_i . Let m denote ℓn M . m follows a random walk; its innovations have mean zero and variance σ^2_m .

5. Our results would not change if, following Blanchard and Kiyotaki (1985), we added money to the utility function rather than introducing money as we do.

The utility function determines the demand for farmer i's product as a fraction of aggregate spending (see Blanchard and Fischer):

$$(4) \qquad \frac{Y_i^D}{Y} = \left(\frac{P_i}{P}\right)^{-\varepsilon} .$$

Combining (3) and (4) yields product demand:

$$(5) \qquad Y_i^D = \left(\frac{M}{P}\right)\left(\frac{P_i}{P}\right)^{-\varepsilon} .$$

Finally, farmer i's consumption is determined by his real revenues:

$$(6) \qquad C_i = \frac{P_i Y_i}{P}$$

(again, see Blanchard and Fischer).

If farmer i set his price every period, he would choose the price that maximizes utility, (1), subject to (2) - (6). In logs, this price is

$$(7) \qquad p_i^* = vm + (1-v)p + w\theta_i ,$$

where $p = \ln P$, and where

$$v = \frac{\gamma - 1}{1 + \varepsilon\gamma - \varepsilon} , \quad 0 < v < 1 ,$$

$$w = \frac{\gamma}{1 + \varepsilon\gamma - \varepsilon} , \quad 0 < w < 1 .$$

We assume, however, that farmer i fixes his price for two periods. If he sets a price at t , it is in effect at t and $t + 1$.

Finally, for expositional simplicity we make two standard approximations in our analysis in the text. First, we approximate farmer i's utility at t by $-(p_{it}-p_{it}^*)^2$. Since we neglect discounting, this means that when farmer i chooses a price for t and $t + 1$, he minimizes the loss function

$$(8) \qquad Z_i = \frac{1}{2}\left[(p_{it}-p_{it}^*)^2 + E_t(p_{it+1}-p_{it+1}^*)^2\right] .$$

Minimization of (8) implies a simple price-setting rule:

$$(9) \qquad x_{it} = \frac{1}{2}(p^*_{it}+E_t p^*_{it+1}) ,$$

where x_{it} is the log of the price set by farmer i for t and $t + 1$.

Our second approximation is

$$(10) \qquad p_t \cong \frac{1}{N}\sum_{j=1}^{N} p_{jt} .$$

This is a first order approximation of (3), and it simplifies aggregation.

Appendix A shows that (8) - (10) are not essential for our qualitative results by redoing our analysis without them. This exercise may be of more general interest, because assumptions like (8) - (10) appear in many papers and are often criticized as ad hoc (especially loss functions like (8)).[6]

III. THE BEHAVIOR OF THE PRICE LEVEL

This section derives the behavior of the aggregate price level under staggering and synchronization. This is the first step in determining when each is optimal and when each is an equilibrium. Because the idiosyncratic shocks that distinguish our model from Blanchard's average to zero, the results of this section are similar to Blanchard's.

A. Synchronization

Suppose that all farmers (or "firms") set prices in even periods. Substituting (7) into (9) yields the log price that firm i sets at an even t :

$$(11) \qquad x_{it} = \frac{1}{2}[vm_t + (1-v)p_t + w\theta_{it}] + \frac{1}{2}[vE_t m_{t+1} + (1-v)E_t p_{t+1} + wE_t\theta_{it+1}] .$$

6. The usual defense that loss functions like (8) are second order approximations to true objective functions is not valid (see Appendix A).

Since m follows a random walk, $E_t m_{t+1} = m_t$. Because all prices set at t are in effect at $t + 1$, $E_t p_{t+1} = p_t$. Thus (11) implies

$$(12) \qquad x_{it} = vm_t + (1-v)p_t + \frac{w}{2}[\theta_{it} + E_t\theta_{it+1}] .$$

For even firms (that is, firms that receive idiosyncratic shocks in even periods), $\theta_{it+1} = \theta_{it}$, so $E_t\theta_{it+1} = \theta_{it}$. For odd firms, θ_{it+1} is uncorrelated with previous shocks, so $E_t\theta_{it+1} = 0$.

Aggregating (12) leads to

$$(13) \qquad p_t = vm_t + (1-v)p_t \qquad (t \text{ even}) .$$

The average of the firm-specific shocks is zero because the shocks are uncorrelated across firms and the economy is large. (13) implies that the price level in an even period is

$$(14) \qquad p_t = m_t \qquad (t \text{ even}) .$$

The price level in an odd period is the same as the price level in the preceding even period, when all prices were set. Thus if t is odd,

$$(15) \qquad p_t = m_{t-1} \qquad (t \text{ odd}) .$$

B. Staggering

Now suppose that each firm changes its price when it receives real shocks. This implies that half of the firms change prices in even periods and half change in odd periods. Equation (11) again describes price-setting. In the staggered regime, (11) reduces to

$$(16) \qquad x_{it} = vm_t + \frac{1-v}{2}[p_t + E_t p_{t+1}] + w\theta_{it} .$$

The derivation of (16) uses $E_t\theta_{it+1} = \theta_{it}$, because θ_i does not change in the period after firm i sets its price. p_{t+1} does not equal p_t, since half of the firms change prices at $t + 1$.

To solve for the price level, note that

$$(17) \qquad p_t = \frac{1}{2}(x_t+x_{t-1}) ,$$

where x_t is the mean of the log prices set at t. Substituting (17) into (16), aggregating, and solving for x_t yields

$$(18) \qquad x_t = \frac{2v}{1+v}m_t + \frac{1-v}{2(1+v)}x_{t-1} + \frac{1-v}{2(1+v)}E_t x_{t+1} .$$

The method of undetermined coefficients leads to a solution for x_t:

$$(19) \qquad x_t = \lambda x_{t-1} + (1-\lambda)m_t ,$$

$$\lambda = \frac{1-\sqrt{v}}{1+\sqrt{v}} , \quad 0 < \lambda < 1 .$$

Finally, substituting (19) into (17) yields

$$(20) \qquad p_t = \lambda p_{t-1} + \frac{1-\lambda}{2}(m_t+m_{t-1}) .$$

Equations (14) - (15) show that, under synchronization, the price level adjusts fully to monetary shocks every two periods. Equation (20) shows that staggering leads to price level inertia--the price level adjusts slowly to shocks. The degree of inertia is greatest (λ is largest) when v is small. v is small when ε is large (firms face highly elastic product demand) and when γ is small (the marginal disutility of labor increases slowly).

IV. THE OPTIMAL TIMING OF PRICE CHANGES

This section compares firms' losses under synchronization and staggering to determine which regime is socially optimal. Consider synchronization first. If all firms set prices in even periods, then the price-setting rule, (12), and the equation for the price level, (14), imply

$$(21) \qquad x_{it} = m_t + \frac{w}{2}(\theta_{it}+E_t\theta_{it+1}) ,$$

where t is even. The deviation of firm i's price from the utility-maximizing level at t and $t + 1$ can be derived from (21), (14), and the formula for the utility-maximizing price, (7). The result is

$$(22) \qquad x_{it} - p^*_{it} = \frac{w}{2}(E_t\theta_{it+1}-\theta_{it}) ;$$

$$x_{it} - p^*_{it+1} = - v\Delta m_{t+1} + \frac{w}{2}(\theta_{it}+E_t\theta_{it+1}-2\theta_{it+1}) ,$$

where $\Delta m_{t+1} = m_{t+1} - m_t$. As noted above, $E_t\theta_{it+1} = \theta_{it+1} = \theta_{it}$ for even firms and $E_t\theta_{it+1} = 0$ for odd firms. Substituting these formulas and (22) into firm i's loss function and taking expectations yields

$$(23) \qquad Z_E^{SYNC} = \frac{1}{2}v^2\sigma_m^2 ,$$

$$Z_O^{SYNC} = \frac{1}{2}v^2\sigma_m^2 + \frac{3}{4}w^2\sigma_\theta^2 ,$$

where Z_E^{SYNC} and Z_O^{SYNC} are the expected losses under synchronization of an even and an odd firm respectively. Even firms are better off, because they change prices in the periods in which they receive idiosyncratic shocks.

Now consider the staggered regime. Calculating E_tp_{t+1} from (20), substituting the result into the price-setting rule, (16), and simplifying by using the definition of λ leads to

$$(24) \qquad x_{it} = \sqrt{v}\, m_t + (1 - \sqrt{v})p_t + w\theta_{it} .$$

Combining this with (7) leads to expressions for $x_{it} - p^*_{it}$ and

$x_{it} - p^*_{it+1}$ as functions of real money:

$$(25) \qquad x_{it} - p^*_{it} = (\sqrt{v} - v)(m_t - p_t) ;$$

$$x_{it} - p^*_{it+1} = -(\sqrt{v} - v)(m_t - p_t) - \sqrt{v}\,\Delta m_{t+1} .$$

Manipulating (20) leads to an expression for real money as a function of current and past innovations in nominal money:

$$(26) \qquad m_t - p_t = \frac{1+\lambda}{2} \sum_{i=0}^{\infty} \lambda^i \Delta m_{t-i} .$$

Substituting (25) and (26) into the loss function and taking expectations yields

$$(27) \qquad Z^{STAG} = \frac{1}{4} \sqrt{v}\,(1+v)\sigma_m^2 ,$$

where Z^{STAG} is the loss for each firm in the staggered regime. The loss is the same for odd and even firms, since both change prices in the periods in which they receive shocks.

We now ask which pattern of price changes is socially optimal. We define the optimal regime as the one that minimizes the average of the losses of odd and even firms. Our qualitative results are the same if we adopt other reasonable definitions.[7]

Equations (23) and (27) determine when synchronization is optimal:

$$(28) \qquad \tfrac{1}{2}(Z_E^{SYNC} + Z_O^{SYNC}) < Z^{STAG} \quad \text{iff} \quad \frac{\sigma_\theta^2}{\sigma_m^2} < K_1 ,$$

$$K_1 = \frac{2\sqrt{v}(1+v) - 4v^2}{3w^2} , \qquad K_1 > 0 .$$

Thus synchronization is superior to staggering if the variance of

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7. For example, we could define the optimal regime as the one that minimizes the average of a convex function of the losses. This would place value on equity between the cohorts.

idiosyncratic shocks is sufficiently small compared to the variance of monetary shocks.

In the special case in which $\sigma_{\theta}^{2} = 0$ --the Blanchard model-- (28) shows that synchronization is always superior to staggering. In this case, staggering reduces welfare for two reasons: it causes price level inertia, and hence larger fluctuations in real aggregate demand; and it causes undesired fluctuations in relative prices, because some prices are fixed when others adjust. In the case of $\sigma_{\theta}^{2}>0$, staggering has these costs, but it also has a benefit: it allows odd as well as even firms to adjust fully to idiosyncratic shocks. If these shocks are large, the benefit outweighs the costs, and staggering is superior to synchronization.

V. THE EQUILIBRIUM TIMING OF PRICE CHANGES

This section derives the conditions under which synchronization and staggering are stable equilibria. Part A shows that synchronization is an equilibrium if the variance of the idiosyncratic shocks is sufficiently small compared to the variance of monetary shocks, and that this condition is weaker than the condition for synchronization to be optimal. Part B considers staggering. We first show that if $\sigma_{\theta}^{2} = 0$ (Blanchard's model), then staggering is an equilibrium, but an unstable one. We then show that if $\sigma_{\theta}^{2} > 0$ (that is, if idiosyncratic shocks of any size exist), staggering is a stable equilibrium. These results, along with those of the previous section, imply that staggering can be a stable equilibrium even if it is inefficient, but that synchronization is also an equilibrium in this case. In addition, synchronization can be an inefficient equilibrium.

A. Is Synchronization an Equilibrium?

Suppose that all firms change prices in even periods. Synchronization is a Nash equilibrium if no firm, taking the behavior of others as given, can gain by switching to odd periods. The incentive for switching is greatest for odd firms, since switching would allow them to adjust fully to idiosyncratic shocks. Thus, to see when synchronization is an equilibrium, we compare the loss of an odd firm in the synchronized regime to its loss if it switches.

The loss of an odd firm in the synchronized regime is Z_O^{SYNC} (equation (23)). To compute the firm's loss if it switches to odd periods, we first derive its price-setting rule for this situation. Since the firm is small relative to the economy, the behavior of the aggregate price level is the same as in the synchronized regime. If t is an odd period, then $p_t = m_{t-1}$ and $p_{t+1} = m_{t+1}$ (equations (14)-(15)). Substituting these formulas into the price-setting rule, (11), shows that the switcher, firm i, sets

$$(29) \qquad x_{it} = \frac{1}{2}[(1-v)m_{t-1} + (1+v)m_t] + w\theta_{it} .$$

The derivation of (29) uses the fact that $\theta_{it+1} = \theta_{it}$ if t is odd and firm i is an odd firm.

Combining (29), (7), and the formulas for the aggregate price level leads to

$$(30) \qquad \begin{aligned} x_{it} - p^*_{it} &= \frac{1-v}{2}\Delta m_t \; ; \\ x_{it} - p^*_{it+1} &= -\frac{1-v}{2}\Delta m_t - \Delta m_{t+1} . \end{aligned}$$

Substituting (30) into (8) and taking expectations yields firm i's loss if it switches:

$$(31) \qquad Z_O^{SWITCH} = \frac{1}{2}\left[\frac{(1-v)^2}{2} + 1\right]\sigma_m^2 .$$

Firm i chooses not to switch to odd periods, and hence synchronization is an equilibrium, if $Z_O^{SWITCH} > Z_O^{SYNC}$. Comparing (23) and (31) shows that

$$(32) \qquad Z_O^{SWITCH} > Z_O^{SYNC} \quad \text{iff} \quad \frac{\sigma_\theta^2}{\sigma_m^2} < K_2 ,$$

$$K_2 = \frac{3 - 2v - v^2}{3w^2} , \quad K_2 > 0 .$$

Switching to odd periods has the benefit for firm i that it makes possible full adjustment to idiosyncratic shocks, but the drawback that, since the firm now changes prices in different periods from other firms, it can no longer adjust fully to changes in the aggregate price level. The losses from switching outweigh the gains if the variance of θ_i is small compared to the variance of money, which determines the variance of the price level.

The most important implication of (32) can be derived by comparing it with (28). Straightforward algebra shows that

$$(33) \qquad K_2 - K_1 = \frac{1}{3w^2}(1-\sqrt{v})^2(3 + 4\sqrt{v} + 3v) > 0 .$$

Thus $K_2 > K_1$: the range of values of $\sigma_\theta^2/\sigma_m^2$ for which synchronization is an equilibrium is larger than the range for which synchronization is optimal. Synchronization must be an equilibrium if it is optimal, and it may be an equilibrium even if staggering is optimal.

This result is surprising. Staggering leads to price level inertia, and hence to large fluctuations in real aggregate demand, but each firm ignores this negative macroeconomic effect in deciding when to change its price. Intuitively, this suggests that synchronization is more likely to be socially optimal than to be a decentralized equilibrium.

To understand why we find the opposite, recall that synchronization is optimal if firms' losses are smaller under synchronization than under staggering, while synchronization is an equilibrium if each firm's loss is smaller under synchronization than it is if the firm switches out of synchronization.[8] Thus our result that synchronization is more likely to be an equilibrium than to be optimal means that a firm that switches from synchronization is worse off than a firm in the staggered regime. Equivalently, a switcher gains if half of the rest of the firms switch with him. We now explain why this is so.

If half the firms join the switcher, then price-setting is staggered and the switcher is hurt by the resulting increase in aggregate demand fluctuations. This is outweighed, however, by a gain to the switcher: his relative price fluctuates less than it does if he switches alone. There are two reasons. First, the switcher changes his price at the same times as half the rest of the firms rather than by himself. Second, because of the price level inertia, even the firms that change prices at different times from the switcher do not deviate greatly from the prices set by the switcher's cohort; in contrast, if all other firms changed prices at different times from the switcher, they would respond fully to monetary shocks in periods in which he could not respond. The smaller fluctuations in the switcher's relative price that result when other firms switch with him lead to smaller deviations of the switcher's output from his utility-maximizing level.

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8. More precisely, optimality depends on the average of odd and even firms' losses in the two regimes, while the losses of odd firms alone determine whether synchronization is an equilibrium (see equations (28) and (32)). This distinction is unimportant for the explanation of our results.

B. Is Staggering a Stable Equilibrium?

We now ask when staggered price-setting is a stable equilibrium.[9] The algebra required for the answer is complicated. We therefore sketch our arguments here and present the details in Appendix B.

Consider first the special case of no idiosyncratic shocks ($\sigma_\theta^2 = 0$). Staggering is clearly a Nash equilibrium in this case. Since the economy is large, each firm takes the proportion of firms in each price-setting cohort as fixed at a half. When there are no idiosyncratic shocks, this means that each firm views the two cohorts as identical. Thus no firm has an incentive to change its timing.

Staggering is not stable, however. Our definition of stability follows Blanchard and Fischer (1985, ch. 9): staggering is stable if, given a small perturbation in the sizes of the cohorts, firms in the larger cohort have an incentive to switch to the smaller one. To see whether staggering is stable in the absence of idiosyncratic shocks, we calculate a firm's loss as a function of the proportion in its cohort. The derivative of this function evaluated at one half is negative. Thus if the sizes of the cohorts are perturbed, the firms in the larger group suffer smaller losses than those in the smaller group; all firms have an incentive to join the larger one.

The intuition for this result is simple. A firm minimizes unintended fluctuations in its relative price by changing its price along with as many other firms as possible. Thus a firm always wants to join the larger cohort.

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9. As throughout the paper, we consider only "uniform" staggering, that is, a regime with half the firms in each cohort. One can show that no unequal division of firms between cohorts is ever a stable equilibrium.

In the case in which there are idiosyncratic shocks $(\sigma_\theta^2 > 0)$, however, staggering is stable. If each firm changes its price when it receives real shocks, and so half the firms are in each cohort, then each firm strictly prefers to remain in its cohort rather than to switch. The reason is simply that switching increases the losses from idiosyncratic shocks while leaving the losses from monetary shocks unchanged. The loss of a firm in a cohort is a continuous function of the proportion in the cohort. Thus if a firm strictly prefers a cohort given that it contains half the firms, the firm continues to strictly prefer the cohort for a small perturbation away from a half. As a result, each firm will return to the cohort in which it can respond fully to idiosyncratic shocks after a small perturbation away from this equilibrium.[10]

Thus a small change in the Blanchard model--the addition of a small idiosyncratic shock--is sufficient to make staggering a stable equilibrium. Recall that the variance of the real shock must be large for staggering to be socially optimal. Thus for $\sigma_\theta^2/\sigma_m^2$ positive but sufficiently small, staggering is an inefficient stable equilibrium. Indeed, the welfare losses from inefficient staggering can be arbitrarily large: staggering is a stable equilibrium even if σ_m^2 is very large and σ_θ^2 very small, which implies large losses from the price level inertia caused by staggering and only small offsetting gains from adjustment to real shocks. The source of this market failure is the negative macroeconomic externality of price level inertia.

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10. Note that our definition of stability is local. The condition for staggering to be globally stable--that is, for each firm to return to the cohort in which it can respond fully to real shocks after an arbitrarily large perturbation--is the same as the condition for synchronization not to be an equilibrium.

Inertia would be reduced if firms moved toward synchronization, but each firm ignores this in deciding whether to switch cohorts.

Finally, recall that synchronization is an equilibrium if $\sigma_\theta^2/\sigma_m^2$ is sufficiently small. Thus, since staggering is a stable equilibrium as long as $\sigma_\theta^2>0$, there are multiple equilibria for a range of $\sigma_\theta^2/\sigma_m^2$. In this range, if price-setters are bunched each firm remains with the bunch to minimize fluctuations in its relative price. But if price-setting is staggered, there is no force to move firms toward synchronization--a bunch does not attract firms unless it already exists. Thus both synchronization and staggering can be self-sustaining. Indeed, our earlier result that synchronization is an equilibrium whenever it is optimal implies that whenever there is an inefficient staggered equilibrium, there is a superior synchronized equilibrium as well.

VI. DISCUSSION AND CONCLUSIONS

A. Robustness

Our analysis employs special assumptions about the timing of shocks and the choices available to firms concerning the timing of price changes: time is discrete, the timing of firm-specific shocks is deterministic, and firms can choose only whether to change prices in odd or even periods. In this section we discuss the effects of relaxing each of these assumptions. Our main conclusions are robust to relaxation of the assumptions of discrete time and of deterministic timing of shocks, and we believe that they are robust to relaxation of our assumption concerning firms' choices. Staggering is an inefficient equilibrium in a wide class of models because of the negative externalities from price level inertia. The result that complete synchronization can be a second equilibrium is not robust, but more general models possess equilibria with "near synchronization"--regular intervals at which most though not all prices change simultaneously. Multiple equilibria arise frequently because many models besides our simple one contain forces keeping synchronized price-setters together but not bringing staggered price-setters together.

Continuous Time. Consider first the implications of moving our model from discrete to continuous time. In the spirit of the discrete time specification, we assume that the money supply follows a Wiener process and that the idiosyncratic shocks affecting a given firm occur at a fixed interval--for concreteness, at a fixed time every month. Similarly, we assume that the time during the month that shocks arrive is distributed uniformly across firms. Finally, each firm changes its price at monthly intervals and can choose only the time during the month that it makes decisions.

Staggered price setting, with each firm changing its price when it receives real shocks, can be an inefficient equilibrium in this model. As in the discrete time model, staggering is a stable equilibrium as long as the variance of idiosyncratic shocks is positive, but staggering is inefficient if the variance of monetary shocks is sufficiently large relative to the variance of idiosyncratic shocks.

In contrast to the discrete time results, however, complete synchronization is never an equilibrium. If, for example, all firms change prices at noon on the first of each month, firms that receive shocks an instant after noon on the first can gain by breaking from synchronization. The cost to these firms of waiting until their shocks occur to change prices is infinitesimal, because waiting puts them only slightly out of step with other price setters, but the benefit from waiting--the ability to adjust to real shocks--is not infinitesimal.

While perfect synchronization cannot be an equilibrium, "near synchronization" can be. If price setters are bunched at noon on the first, the ones who receive shocks shortly after this time will move, but the bunch need not unravel entirely. To see this, suppose that firms whose shocks arrive between noon and 6:00 P.M. on the first leave the bunch. For a firm whose shock arrives after 6:00, the costs and benefits of leaving are of the same order: the cost of being out of step with the bunch by a discrete amount of time is not infinitesimal. As a result, for some parameter values there is an equilibrium in which most though not all firms are bunched at noon on the first. Since staggering is a stable equilibrium as long as there are idiosyncratic shocks, there can be multiple equilibria. Unfortunately, it is difficult to compare the condition for near synchronization to be an equilibrium with the condition for near synchronization or complete synchronization to be optimal.

Stochastic Arrival of Idiosyncratic Shocks. We now alter the continuous time version of our model by relaxing the assumption that firm-specific shocks arrive deterministically. Specifically, let the arrival be a Poisson process that is independent across firms, and let τ denote the mean time between a firm's shocks. We continue to allow very limited choices for the timing of price changes: a firm can change its price either at fixed intervals of length τ or whenever it receives an idiosyncratic shock. (Note that in either case the average frequency of price changes is $1/\tau$.) If each firm changes prices when it receives shocks, then price-setting is staggered, since an equal proportion of firms receives shocks at every instant. Synchronization arises if each firm adjusts at a fixed interval.

One can show that staggering is an equilibrium if the variance of firm-specific shocks is sufficiently large compared to the variance of monetary shocks.[11] Synchronization is an equilibrium for $\sigma_\theta^2/\sigma_m^2$ sufficiently small, and, as in our main model, there are two equilibria for some values of $\sigma_\theta^2/\sigma_m^2$. Our intuition about the welfare properties of equilibria carries over to this model (for example, negative externalities from price level inertia should still produce inefficient staggering), but we have not investigated welfare formally.

More Complicated Rules for When to Change Prices. In both our main model and the generalizations discussed above, firms choose among a few simple rules for when to change prices. We now speculate about the implications of allowing more complicated rules. Specifically, in the model with Poisson arrival of idiosyncratic shocks, suppose that firms can change prices whenever they wish by paying a fixed adjustment cost. (To avoid the extreme result of Caplin and Spulber, 1987, assume that the money supply can both rise and fall, so that price changes are "two-sided"; see Blanchard and Fischer, ch. 9.) Although it is not possible to solve for the behavior of this economy, we believe that staggering can be a stable equilibrium. In the regime that we envision, each firm would follow something like an Ss rule, and differences in shocks would cause firms to

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11. In contrast to the results for our main model, staggering is not an equilibrium if the variance of idiosyncratic shocks is positive but very small. The reason is that breaking from staggering means changing prices at fixed rather than random intervals, which allows better adjustment to monetary shocks. This result is an artifact of our assumptions about firms' choices concerning the timing of price changes. We believe that in more general models with random arrival of idiosyncratic shocks, such as the one discussed below, a positive variance for these shocks assures that staggering is a stable equilibrium.

reach their Ss bounds at different times. As in our other models, the staggered regime could be inefficient because firms ignore their contributions to price level inertia in deciding when to change prices.

Complete synchronization would clearly not be an equilibrium; firms that received large idiosyncratic shocks at times other than when all prices were changed would adjust immediately, and firms whose desired adjustments were small when prices were changed would choose not to pay the cost of adjustment. But it appears plausible that once again near synchronization could be an equilibrium: if most prices changed at certain times, each firm would have a strong incentive to adjust its price at those times, and this might sustain the equilibrium. As a realistic example of possible near synchronization, consider labor contracts. It seems plausible to imagine an equilibrium in which most wages are set simultaneously at three-year intervals but in which each contract can be reopened at an irregular time if a firm experiences an unusually large shock.

B. Conclusions

In Keynesian macroeconomic models, staggered price-setting creates price level inertia, which increases cyclical fluctuations and reduces welfare. We ask whether firms could choose to change prices at different times even if staggering makes them worse off. More generally, we ask whether there is some market failure that causes the equilibrium timing of price changes in a decentralized economy to be inefficient. To address these issues, we alter previous models of staggering by making the timing of price changes endogenous, and by introducing an incentive for staggering: idiosyncratic shocks that arrive at different times for different firms.

We find that staggering is a stable equilibrium as long as there are firm-specific shocks of any size. Welfare is lower under staggering than under synchronization if nominal shocks are sufficiently large compared to the idiosyncratic shocks; if the nominal shocks are very large, welfare is much lower. Thus staggering can be a stable equilibrium even if it is highly inefficient.

Synchronization can be an equilibrium even when firm-specific shocks make staggering an equilibrium. Multiple equilibria are possible because there is an incentive for synchronized price-setters to remain bunched, but not for staggered price-setters to move toward synchronization. The condition for synchronization to be an equilibrium is weaker than the condition for it to be optimal. Thus if staggering is an inefficient equilibrium, there is a superior synchronized equilibrium as well.

The possibility of inefficient staggering suggests a role for government regulation of price-setting. Welfare might be raised, for example, by a requirement that firms sign labor contracts in the same years. The existence of multiple equilibria implies that regulation could be temporary: synchronization, once achieved, would be self-sustaining.

APPENDIX A

Overview. In the text, following Gray (1978), Blanchard (1983, 1986), Parkin (1986), Ball (1986a), and others, we make two important simplifying assumptions. First, we assume that the reduction in individual i's expected utility in period t relative to the situation in which all prices are flexible is proportional to $-E\left[\left(p_{it}-p_{it}^{*}\right)^{2}\right]$. We use this expression both to evaluate welfare under different timings of price changes and to derive individuals' price-setting rule,

$$(9) \qquad x_{it} = \frac{1}{2}\left(p_{it}^{*} + E_{t}p_{it+1}^{*}\right).$$

Second, we assume that the log of the price index equals the average of log prices; in other words, we assume

$$(10) \qquad p_{t} = \frac{1}{N}\sum_{i=1}^{N} p_{it}.$$

This Appendix relaxes these two assumptions. Our goal is to derive results about the behavior of the economy and about welfare that are correct up to second order. Since (10) is a first order approximation to the true price index (see equation (3)), we obviously must replace it with a second order approximation. For more subtle reasons, we must also alter the objective function and the price-setting rule. Quadratic loss functions like ours are often defended on the grounds that they are second order approximations to true objective functions (see, for example, Parkin). But they do not in fact lead to results that are correct up to second order. There are two difficulties.

First, an individual's utility depends on more than how successful he is in keeping his price close to the utility-maximizing level. Combining equations (1) - (7) in the text, we can write individual i's utility as a function of the real money stock, the ratio of his actual price to his utility-maximizing price, and his technology shock θ :

$$\text{(A-1)} \qquad U_{it} = \left(\frac{M_t}{P_t}\right)^{\gamma(1-\varepsilon v)} \Phi_{it}^{w(1-\varepsilon)} \left[\left(\frac{P_{it}}{P^*_{it}}\right)^{1-\varepsilon} - \frac{\varepsilon - 1}{\gamma\varepsilon}\left(\frac{P_{it}}{P^*_{it}}\right)^{-\varepsilon\gamma}\right]$$

$$\equiv V(m_t-p_t, p_{it}-p^*_{it}, \theta_{it}) \ ,$$

where v and w are as defined in the text. Using $-E[(p_{it}-p^*_{it})^2]$ to measure welfare is thus equivalent to taking a second order approximation of $V(\cdot)$ and then discarding some of the terms. As we show below, the most important of the neglected terms are the ones involving the mean and variance of real money; since the behavior of real money is different under staggering and synchronization, using $-E[(p_{it}-p^*_{it})^2]$ to measure utility leads to an inaccurate comparison of welfare in the two regimes.

The second difficulty is that assuming that agents maximize second order approximations to their objective functions, even if the right approximation is used, does <u>not</u> lead to correct second order approximations to behavior. Instead, the first order condition for maximizing a second order (i.e., quadratic) approximation to utility leads to a <u>first</u> order (linear) approximation to agents' true price-setting rule. Thus results using the price-setting rule in the text are accurate only up to first order--they do not, for example, correctly describe the (second order) effects of changes in the variances of shocks.

We now redo our analysis in a way that avoids these problems. To derive correct second order approximations to price-setting rules, we use second order approximations to exact first order conditions, not first order conditions for maximizing approximations to utility. To derive correct second order approximations to welfare, we use a second order approximation to (A-1) with no terms discarded. Finally, as mentioned above, we use a second order rather than first order approximation to the true log price level. Otherwise our analysis parallels that in the text. Our approach leads to specific results, such as the condition that determines the optimal regime, that differ from the ones in the text. We show, however, that our qualitative conclusions are unchanged. (The complexity of the analysis that follows makes clear why we employ the standard simplifying assumptions in the text despite the problems discussed above--they simplify the presentation dramatically.)

Second Order Approximations. Because they will be useful at several points below, we begin by taking second order approximations to the log price level and to expected utility.

Taking the log of the true price index (equation (3)) and approximating around the mean of log prices yields

$$\text{(A-2)} \qquad p_t \cong \bar{p}_t - \frac{1}{2}(\varepsilon-1)\frac{1}{N}\sum_{i=1}^{N}(p_{it}-\bar{p}_t)^2 ,$$

where $\bar{p}$ is the mean of log prices. (A-2) shows that dispersion in individual prices reduces the aggregate price index, making consumers better off. The benefit from price dispersion is increasing in ε, the elasticity of substitution between goods.

We approximate $V(\cdot)$ around $(0,0,0)$ ($m-p=p_i-p_i^*=0$ is the equilibrium if all prices are flexible, and $E[\theta]=0$). Note that $V_2(m-p,0,\theta)=0$ for all $m-p$ and θ ($p_i-p_i^*=0$ is always optimal). This implies that $V_2(0,0,0)$, $V_{12}(0,0,0)$, and $V_{23}(0,0,0)$ are all zero. Note also that θ has mean zero and is uncorrelated with $m - p$ by our assumptions in the text. Using these results, the Taylor approximation of $V(\cdot)$ simplifies to

$$\text{(A-3)} \qquad E[U_{it}] \cong V(0,0,0) + V_1 E\left[m_t - p_t\right] + \frac{1}{2}V_{11}E\left[\left(m_t - p_t\right)^2\right] + \frac{1}{2}V_{22}E\left[\left(p_{it} - p_{it}^*\right)^2\right] + \frac{1}{2}V_{33}E\left[\theta_{it}^2\right] ,$$

where all partial derivatives are evaluated at $(0,0,0)$.

$V(0,0,0)$ and $\frac{1}{2}V_{33}E[\theta_{it}^2]$ are constant across regimes. Thus our evaluation of welfare here departs from the analysis in the text by including terms in the mean and variance of real money. We can ignore these terms in asking whether an individual chooses to set his price in even or odd periods, because the choice of one agent does not affect the behavior of real money. But we cannot ignore these terms in comparing welfare under synchronization and staggering, because the behavior of real money is different in the two regimes.

Taking the appropriate derivatives of (A-1) and substituting into (A-3) yields

$$\text{(A-4)} \qquad E[U_{it}] - \left[V(0,0,0) + \frac{1}{2}V_{33}\sigma_\theta^2\right] \cong \frac{1}{\varepsilon} E\left[m_t - p_t\right] + \frac{1}{2}\frac{\gamma}{\varepsilon}\frac{1}{1 + \varepsilon\gamma - \varepsilon} E\left[(m_t - p_t)^2\right] - \frac{1}{2}(\varepsilon - 1)(1 + \varepsilon\gamma - \varepsilon)E\left[\left(p_{it} - p_{it}^*\right)^2\right] .$$

<u>Synchronization</u>. We now derive the behavior of prices and the level of welfare under synchronization. A farmer chooses his price for t and $t+1$ to maximize his expected utility in these periods, $U_{it} + E_t U_{it+1}$. Manipulating (A-1), we can write this objective function as

$$\text{(A-5)} \qquad \frac{1}{2}\left(U_{it}+E_tU_{it+1}\right) = \left(\frac{M_t}{P_t}\right)^{\gamma(1-\varepsilon v)} \Phi_{it}^{(1-\varepsilon)w/2} \; {}_t\Phi_{it+1}^{(1-\varepsilon)w/2}$$

$$\left\{ \frac{1}{2}\left[\left(\frac{X_{it}}{\hat{X}_{it}}\right)^{1-\varepsilon} - \frac{\varepsilon-1}{\gamma\varepsilon}\left(\frac{X_{it}}{\hat{X}_{it}}\right)^{-\gamma\varepsilon}\left(\frac{\Phi_{it}}{{}_t\Phi_{it+1}}\right)^{\gamma/2}\right] + E_t\,\frac{1}{2}\left[\left(\frac{X_{it}}{\hat{X}_{it}}\right)^{(1-\varepsilon)}\left(\frac{M_{t+1}}{M_t}\right)\right.\right.$$

$$\left.\left. - \frac{\varepsilon-1}{\gamma\varepsilon}\left(\frac{X_{it}}{\hat{X}_{it}}\right)^{-\gamma\varepsilon}\left(\frac{M_{t+1}}{M_t}\right)^{\gamma}\left(\frac{{}_t\Phi_{it+1}}{\Phi_{it}}\right)^{\gamma/2}\left(\frac{\Phi_{it+1}}{{}_t\Phi_{it+1}}\right)^{\gamma}\right]\right\},$$

where ${}_t\Phi_{it+1} = \exp\{E_t\theta_{it+1}\}$, $X_{it} = \exp(x_{it})$, $\hat{X}_{it} = \exp(\hat{x}_{it})$, and $\hat{x}_{it} = \frac{1}{2}(p^*_{it} + E_tp^*_{it+1})$ (that is, $\hat{x}_{it}$ is the expression for x_{it} in the text). The first order condition for $X_{it}/\hat{X}_{it}$ simplifies to

$$\text{(A-6)} \qquad E_t\left\{\left(\frac{X_{it}}{\hat{X}_{it}}\right)^{1+\varepsilon\gamma-\varepsilon}\frac{1}{2}\left[1+\frac{M_{t+1}}{M_t}\right]\right.$$

$$\left. - \frac{1}{2}\left[\left(\frac{\Phi_{it}}{{}_t\Phi_{it+1}}\right)^{\gamma/2} + \left(\frac{M_{t+1}}{M_t}\right)^{\gamma}\left(\frac{{}_t\Phi_{it+1}}{\Phi_{it}}\right)^{\gamma/2}\left(\frac{\Phi_{it+1}}{{}_t\Phi_{it+1}}\right)^{\gamma}\right]\right\} = 0 .$$

Taking a second order approximation yields

$$\text{(A-7)} \qquad x_{it} \cong \begin{cases} \hat{x}_{it} + \dfrac{1}{1+\varepsilon\gamma-\varepsilon}\left[\dfrac{1}{4}\left(\gamma^2-1\right)\sigma_m^2 + \dfrac{1}{8}\gamma^2\theta_{it}^2 + \dfrac{1}{4}\gamma^2\sigma_\theta^2\right] \\ \qquad \text{if } i \text{ is odd (that is, if farmer } i \text{ will receive an idiosyncratic shock in period } t+1\text{);} \\ \hat{x}_{it} + \dfrac{1}{1+\varepsilon\gamma-\varepsilon}\left[\dfrac{1}{4}\left(\gamma^2-1\right)\sigma_m^2\right] \\ \qquad \text{if } i \text{ is even.} \end{cases}$$

(A-7) shows that the correct second order approximation to x_{it} is larger than the formula in the text. Greater uncertainty (a larger value of

σ_m^2 or σ_θ^2) leads to higher prices. (Uncertainty raises prices, and thus lowers output, because the third derivative of utility with respect to $p_i - p_i^*$ is positive.)

Equation (12) gives $\hat{x}_{it}$. Substituting this into (A-7) and aggregating across farmers yields the mean of log prices:

$$\text{(A-8)} \qquad \bar{p}_t \cong vm_t + (1-v)p_t + \frac{1}{1+\varepsilon\gamma-\varepsilon}\,\frac{1}{4}\left(\gamma^2-1\right)\sigma_m^2 + \frac{1}{1+\varepsilon\gamma-\varepsilon}\,\frac{3}{16}\,\gamma^2\sigma_\theta^2$$

$$\equiv vm_t + (1-v)p_t + C^{SYNC} .$$

Similarly, we can substitute (12) into (A-7) and calculate the variance of x_{it} across farmers:

$$\text{(A-9)} \qquad \frac{1}{N}\sum_{i=1}^{N}(p_{it}-\bar{p}_t)^2 \cong \frac{5}{8}w^2\sigma_\theta^2 ,$$

where we neglect terms of higher than second order. (To derive (A-9), note that the coefficient on θ_{it} in the expression for x_{it} is $w/2$ for i odd and w for i even; see (12).)

Substituting (A-8) and (A-9) into (A-2) and solving for the price level yields

$$\text{(A-10)} \qquad p_t = m_t + \frac{1}{v}\left[C^{SYNC} - \frac{5}{16}(\varepsilon-1)w^2\sigma_\theta^2\right] .$$

Rearranging (A-10) and using the definition of C^{SYNC}, the log real money stock in an even period is given by:

$$\text{(A-11)} \qquad m_t - p_t \cong \frac{1}{v}\,\frac{5}{16}(\varepsilon-1)w^2\sigma_\theta^2 - (\gamma+1)\,\frac{1}{4}\,\sigma_m^2 - \frac{\gamma^2}{(\gamma-1)}\,\frac{3}{16}\,\sigma_\theta^2$$

$$\equiv \mu^{SYNC} .$$

Since no prices change in odd periods, the log real money stock in an odd period equals the expression in (A-11) plus the innovation in log nominal money in that period.

We can now evaluate welfare under synchronization. We compute the terms in (A-4), the approximation to an agent's expected utility, averaging each term over odd and even periods. The mean of real money is μ^{SYNC} in all periods. In even periods, the square of real money is $(\mu^{SYNC})^2$, which equals zero to second order; in odd periods, the expected square is $(\mu^{SYNC})^2+\sigma_m^2 \cong \sigma_m^2$. Thus the average square of real money is $\frac{1}{2}\sigma_m^2$. Note that the mean of real money differs from its level in the text (zero), but that the variance is the same to second order. Finally, because (A-7) implies that x_{it} differs from $\frac{1}{2}(p^*_{it} + E_t p^*_{it+1})$ only by second order terms, $E[(p_{it} - p^*_{it})^2]$ differs from its value under the assumption that firms use the simple rule (9) only by terms of fourth order and higher. Thus, averaging over the two kinds of firms, $E[(p_{it} - p^*_{it})^2] \cong \frac{1}{2}(Z_E^{SYNC} + Z_0^{SYNC}) = \frac{1}{2}v^2\sigma_m^2 + \frac{3}{8}w^2\sigma_\theta^2$. Substituting these results into (A-4) yields welfare under synchronization:

$$\text{(A-12)}\qquad E[U^{SYNC}] - [V(0,0,0) + \tfrac{1}{2}V_{33}\sigma_\theta^2] \cong \frac{1}{\varepsilon}\mu^{SYNC} + \frac{1}{2}\frac{\gamma}{\varepsilon}\frac{1}{1+\varepsilon\gamma-\varepsilon}\frac{1}{2}\sigma_m^2$$
$$- \frac{1}{2}(\varepsilon-1)(1+\varepsilon\gamma-\varepsilon)\left[\frac{1}{2}v^2\sigma_m^2 + \frac{3}{8}w^2\sigma_\theta^2\right].$$

<u>Staggering</u>. Under asynchronization, as under synchronization, one can show that relaxing the simplifying assumptions in the text has negligible effects on $E[(p_{it}-p^*_{it})^2]$ and $E[(m_t-p_t)^2]$; thus $E[(p_{it}-p^*_{it})^2]$ is again given by Z^{STAG} (equation (27)), and $E[(m_t-p_t)^2]$ equals $\frac{1}{4}\frac{1+\lambda}{1-\lambda}\sigma_m^2$ (the variance of (26)). Thus to determine welfare under staggering we need only find the mean of real money under staggering. To do this, we begin by guessing

$$\text{(A-13)} \qquad m_t - p_t \cong \mu^{STAG} + \lambda\left(m_{t-1}-p_{t-1}-\mu^{STAG}\right) + \frac{1+\lambda}{2}\Delta m_t ,$$

where μ^{STAG} is the mean of real money under staggering. We now solve for this term.

Calculations similar to those used to derive (A-7) show that when m_t-p_t obeys (A-13), individual i's price-setting rule is:

$$\text{(A-14)} \qquad x_{it} \cong m_t - (1-v)\left(\frac{1+\lambda}{2}\right)(m_t-p_t-\mu^{STAG}) + w\theta_{it} + C_1 - (1-v)\mu^{STAG} ,$$

$$C_1 = \frac{1}{4}\frac{1}{1+\gamma\varepsilon-\varepsilon}\left\{\left[(1-\lambda)\gamma(1-\varepsilon v)(1+\gamma\varepsilon-\varepsilon) + v(\gamma^2\varepsilon^2-(\varepsilon-1)^2)\right]\right.$$

$$\left. + \left[\frac{1}{2}(1-v)^2(1-\lambda)^2(\gamma^2\varepsilon^2-(\varepsilon-1)^2) - (1-\lambda)^2(1-v)\gamma(1-\varepsilon v)(1+\gamma\varepsilon-\varepsilon)\right]\frac{1}{4}\frac{1+\lambda}{1-\lambda}\right\}\sigma_m^2 .$$

The variance of prices across farmers is approximately

$$\text{(A-15)} \qquad \frac{1}{N}\sum_{i=1}^{N}(p_{it}-\bar{p}_t)^2 \cong w^2\sigma_\theta^2 + \frac{1}{4}\frac{1-\lambda}{1+\lambda}\sigma_m^2 ,$$

where the first term reflects idiosyncratic shocks and the second the fact that different prices are set at different times.

Aggregating (A-14) yields an expression for $\bar{x}_t$, and hence (using $\bar{p}_t = \frac{1}{2}(\bar{x}_t + \bar{x}_{t-1})$) for $\bar{p}_t$; substituting this result and (A-15) into (A-2) yields a formula for the aggregate price level:

$$\text{(A-16)} \qquad p_t = \frac{1}{2}(m_t+m_{t-1}) - (1-v)\left(\frac{1+\lambda}{2}\right)\frac{1}{2}\left[(m_t-p_t-\mu^{STAG}) + (m_{t-1}-p_{t-1}-\mu^{STAG})\right]$$

$$+ C_1 - (1-v)\mu^{STAG} + C_2 + \mu^{STAG} ,$$

$$C_2 = -\frac{1}{2}(\varepsilon-1)\left[w^2\sigma_\theta^2 + \frac{1}{4}\frac{1-\lambda}{1+\lambda}\sigma_m^2\right] .$$

Manipulating (A-16) and using the definition of λ (equation (19)), we obtain

$$\text{(A-17)} \qquad m_t - p_t = \mu^{STAG} + \lambda(m_{t-1} - p_{t-1} - \mu^{STAG}) + \frac{1+\lambda}{2}\Delta m_t$$

$$- \; v(1+\lambda)\mu^{STAG} - (1+\lambda)(C_1 + C_2) \; .$$

Finally, combining (A-13) and (A-17) yields

$$\text{(A-18)} \qquad \mu^{STAG} = - \frac{C_1 + C_2}{v} \; .$$

To find welfare under staggering, we now substitute μ^{STAG} for $E[m_t - p_t]$, $\frac{1}{4}\frac{1+\lambda}{1-\lambda}\sigma_m^2$ for $E[(m_t - p_t)^2]$, and Z^{STAG} for $E[(p_{it} - p_{it}^*)^2]$ into (A-4):

$$\text{(A-19)} \qquad E\left[U^{STAG}\right] - \left[V(0,0,0) + \frac{1}{2}V_{33}\sigma_\theta^2\right] \cong \frac{1}{\varepsilon}\,\mu^{STAG}$$

$$+ \frac{1}{2}\,\frac{\gamma}{\varepsilon}\,\frac{1}{1+\gamma\varepsilon-\varepsilon}\,\frac{1}{4}\,\frac{1+\lambda}{1-\lambda}\,\sigma_m^2 - \frac{1}{2}(\varepsilon-1)(1+\varepsilon\gamma-\varepsilon)\,\frac{1}{4}\sqrt{v}\,(1+v)\sigma_m^2 \; .$$

<u>Equilibrium and optimal timing</u>. As noted above, the formulas for $E[(p_{it} - p_{it}^*)^2]$ in the text are correct to second order. In addition, since an individual treats $E[(m_t - p_t)]$ and $E[(m_t - p_t)^2]$ as given, it is correct to assume that an individual chooses whether to change prices in odd or even periods to minimize $E[(p_{it} - p_{it}^*)^2]$. Thus the approach in the text to analyzing individual choices of the timing of price changes is correct to second order. As a consequence, the results about the equilibrium timing are correct: staggering is stable if $\sigma_\theta^2 > 0$ and unstable if $\sigma_\theta^2 = 0$, and synchronization is an equilibrium if $\sigma_\theta^2/\sigma_m^2 < K_2$.

The optimal timing is determined by comparing welfare under synchronization and staggering--that is, by determining which of (A-12) and (A-19) is larger. Examination of (A-12) and (A-19) shows that which regime is optimal depends on the ratio of σ_θ^2 to σ_m^2, as in the text, but that the critical ratio differs from the ratio in the text, K_1 (see equation (28)). Thus the condition for synchronization to be optimal is different from the one in the text. Comparison of the condition for synchronization to be optimal with the condition for it to be an equilibrium is too complex to perform analytically. We therefore proceed by evaluating (A-12), (A-19), and (32) numerically for a wide range of parameter values. We find that the result in the text that whenever synchronization is efficient it is an equilibrium holds without change.

APPENDIX B

This Appendix shows that staggering is unstable in the absence of idiosyncratic shocks but stable if idiosyncratic shocks of any size are present. This requires that we solve for the behavior of the economy when the proportions of firms setting prices in even and odd periods differ from one half.

We first solve for the behavior of the price level. Let π denote the fraction of firms that set their prices in even periods. Letting t be an even period, the price level is given by

$$\text{(B-1)} \qquad p_t^E = \pi x_t^E + (1-\pi)x_{t-1}^O ,$$

$$p_{t+1}^O = \pi x_t^E + (1-\pi)x_{t+1}^O ,$$

where (as in the text) x_t is the average of prices set at t, and where E and O superscripts denote odd and even periods respectively. Substituting (B-1) into equation (11) and aggregating, we obtain

$$\text{(B-2)} \qquad x_t^E = vm_t + \frac{1-v}{2}\left[(1-\pi)x_{t-1}^O + 2\pi x_t^E + (1-\pi)E_t x_{t+1}^O\right] .$$

As in the text, we solve for the behavior of x using the method of undetermined coefficients; that is, we posit a solution of the form

$$\text{(B-3)} \qquad x_t^E = \lambda^E x_{t-1}^O + (1-\lambda^E)m_t ,$$

$$x_{t+1}^O = \lambda^O x_t^E + (1-\lambda^O)m_{t+1} ,$$

and solve for λ^E and λ^O.

Calculating $E_t x^O_{t+1}$ from (B-3), substituting the result into (B-2), and rearranging terms yields

$$\text{(B-4)} \qquad x^E_t = \frac{\left[v + \frac{1-v}{2}(1-\pi)(1-\lambda^O)\right]m_t + \frac{1-v}{2}(1-\pi)x^O_{t-1}}{1 - \frac{1-v}{2}(2\pi + (1-\pi)\lambda^O)} .$$

Comparing (B-3) and (B-4) shows that λ^E equals the coefficient on x^O_{t-1} in (B-4):

$$\text{(B-5)} \qquad \lambda^E = \frac{\frac{1-v}{2}(1-\pi)}{1 - \frac{1-v}{2}(2\pi+(1-\pi)\lambda^O)} .$$

By symmetry, λ^O equals the same expression with λ^O replaced by λ^E and with π replaced by $1-\pi$:

$$\text{(B-6)} \qquad \lambda^O = \frac{\frac{1-v}{2}\pi}{1 - \frac{1-v}{2}(2(1-\pi)+\pi\lambda^E)} .$$

(B-5) and (B-6) yield

$$\text{(B-7)} \qquad \lambda^E = K_O - \sqrt{K_O^2 - K_O/K_E} ,$$

$$\lambda^O = K_E - \sqrt{K_E^2 - K_E/K_O} ,$$

where

$$K_E = \frac{1-(1-v)\pi}{(1-v)(1-\pi)} ,$$

$$K_O = \frac{1-(1-v)(1-\pi)}{(1-v)\pi} .$$

(As in the text, we have chosen the stable solutions: $|\lambda^E| < 1$, $|\lambda^O| < 1$.)

As the first step toward computing firms' loss functions, one can show that (B-3) implies

$$\text{(B-8)} \qquad x^E_t - x^O_{t-1} = \lambda^E\lambda^O(x^E_{t-2} - x^O_{t-3}) + (1-\lambda^E)\Delta m_t + (1-\lambda^E)\lambda^O\Delta m_{t-1} ,$$

$$x^E_t - x^O_{t+1} = \lambda^E\lambda^O(x^E_{t-2} - x^O_{t-1}) - (1-\lambda^O)\Delta m_{t+1} - (1-\lambda^O)\lambda^E\Delta m_t .$$

Similarly, (B-1) and (B-3) imply

$$\text{(B-9)} \qquad m_t - p^E_t = \lambda^E\lambda^O(m_{t-2}-p^E_{t-2}) + (1-\pi(1-\lambda^E))\Delta m_t + (1-\pi(1-\lambda^E))\lambda^O\Delta m_{t-1} ,$$

$$m_{t+1} - p^O_{t+1} = \lambda^E\lambda^O(m_{t-1}-p^O_{t-1}) + (1-(1-\pi)(1-\lambda^O))\Delta m_{t+1} + (1-(1-\pi)(1-\lambda^O))\lambda^E\Delta m_t .$$

Equations (7), (9), and (B-1) imply

$$\text{(B-10)} \qquad x^E_{it} - p^*_{it} = x^E_t - vm_t - (1-v)p^E_t$$

$$= (x^E_t - p^E_t) - v(m_t - p^E_t)$$

$$= (1-\pi)(x^E_t-x^O_{t-1}) - v(m_t-p^E_t) ,$$

$$x^E_{it} - p^*_{it+1} = (1-\pi)(x^E_t-x^O_{t+1}) - v(m_{t+1}-p^O_{t+1}) ,$$

where x^E_{it} is the price set by an even firm (at an even t). Combining (B-8)-(B-10) yields

$$\text{(B-11)} \qquad x^E_{it} - p^*_{it} = \lambda^O\lambda^E(x^E_{it-2}-p^*_{it-2}) + c^E\Delta m_t + \lambda^O c^E\Delta m_{t-1} ,$$

$$x^E_{it} - p^*_{it+1} = \lambda^O\lambda^E(x^E_{it-2}-p^*_{it-1}) + d^E\Delta m_{t+1} + \lambda^E d^E\Delta m_t ,$$

where

$$c^E = (1-\pi)(1-\lambda^E) - v(1-\pi(1-\lambda^E)) ,$$

$$d^E = -(1-\pi)(1-\lambda^O) - v(1-(1-\pi)(1-\lambda^O)) .$$

Using (B-11), the expected loss of an even firm changing prices in even periods is

$$\text{(B-12)} \qquad Z^E(\pi) = \frac{1}{2} \frac{[1+(\lambda^O)^2](c^E)^2 + [1+(\lambda^E)^2](d^E)^2}{1-(\lambda^O\lambda^E)^2} \sigma_m^2 .$$

By symmetry, the loss of an odd firm that changes prices in odd periods is

$$\text{(B-13)} \qquad Z^O(\pi) = Z^E(1-\pi) .$$

If an <u>even</u> firm switches to <u>odd</u> periods, its loss equals the loss of an odd firm that changes in odd periods plus the loss from its inability to adjust fully to idiosyncratic shocks. As in the text, this additional loss is $\frac{3}{4}w^2\sigma_\theta^2$ (see (23)). Thus the total loss for an "even switcher" is

$$\text{(B-14)} \qquad Z^{ESWITCH}(\pi) = Z^E(1-\pi) + \frac{3}{4}w^2\sigma_\theta^2 .$$

The gain to an even firm from switching to odd periods (the reduction in its loss) is therefore

$$\text{(B-15)} \qquad G^E(\pi) = Z^E(\pi) - Z^{ESWITCH}(\pi) = Z^E(\pi) - Z^E(1-\pi) - \frac{3}{4}w^2\sigma_\theta^2 .$$

By symmetry, the gain to an <u>odd</u> firm from switching to even periods is

$$\text{(B-16)} \qquad G^O(\pi) = Z^E(1-\pi) - Z^E(\pi) - \frac{3}{4}w^2\sigma_\theta^2 .$$

Note that $G^E(\frac{1}{2}) = G^O(\frac{1}{2}) = -\frac{3}{4}w^2\sigma_\theta^2$.

We can now determine when staggering is a stable equilibrium. First suppose that σ_θ^2 is zero. In this case $G^E(\frac{1}{2}) = G^O(\frac{1}{2}) = 0$. Thus, when

$\pi = \frac{1}{2}$ no firm can gain by switching cohorts, and staggering is an equilibrium. But one can show that $G^{E'}(\frac{1}{2}) < 0$ and $G^{O'}(\frac{1}{2}) > 0$. It follows that after a small perturbation away from $\pi = \frac{1}{2}$ firms in the larger cohort are better off than firms in the smaller cohort. Since all firms have an incentive to join the larger cohort, the staggered equilibrium is unstable.

Now suppose that $\sigma_\theta^2 > 0$. In this case $G^E(\frac{1}{2})$ and $G^O(\frac{1}{2})$ are negative, and so staggering is again an equilibrium. Since $G^E(\cdot)$ and $G^O(\cdot)$ are continuous, there is a neighborhood of $\pi = \frac{1}{2}$ in which $G^E(\pi)$ and $G^O(\pi)$ are negative. Thus if π departs slightly from one half, even firms still prefer the even cohort and odd firms the odd cohort. Staggering is therefore stable.

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