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THE EQUILIBRIUM AND STABILITY OF THE GASEOUS COMPONENT OF THE GALAXY. I.

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ABSTRACT

The distribution of gas satisfying hydrostatic and Poisson conditions with distance above the galactic plane is derived and compared with Schmidt's observations of $\rho_g(z)$. Equipartition magnetic $(B^2/8\pi)$ and cosmic-ray (P_{c-r}) components are assumed. $\rho_g(z)$ is calculated for two limiting cases: (i) the total mass density at the galactic plane $\rho_{go} + \rho_{*o}$ is equal to the sum of the observed gas and star densities in the solar neighborhood $(0.089\ M_\odot/pc^3)$, and (ii) Oort's data on the z component of the galactic gravitational acceleration K_z are used, in which case we implicitly assume that $\rho_{go}+\rho_{*o}= 0.15\ M_\odot/pc^3$. Using recently observed values for B_o, P_{c-ro}, and ρ_{go}, the rms z turbulent gas velocity dispersion $<v_{tz}^2>^{1/2}$ is found.

An expression is derived for the half-thickness $z_{1/2}$ of the equilibrium disk of galactic gas: $z_{1/2} \propto Q/(\rho_{go}+\rho_{*o})^{1/2}$ where $Q^2 = <v_{tz}^2> + B_o^2/8\pi\rho_{go} + P_{c-ro}/\rho_{go}$. Using the Innanen galactic mass model, the half-thickness is computed as a function of distance from the galactic center over the range $4kpc \leq R \leq 14kpc$ and compared to the observations of McGee and Milton. The observed increase in half-thickness at distances beyond the solar distance R_o is reproduced theoretically. A galactic mass model is derived, using the observed layer half-width and a constant value for Q. Comparison of the two mass models indicates that, because $z_{1/2}$ is observed to remain essentially constant in the region $4kpc \leq R \leq R_o$, Q increases like $(\rho_{go}+\rho_{*o})^{1/2}$ with decreasing R.

I. INTRODUCTION

In recent years observations of interstellar molecular absorption and emission lines (e.g. OH, H_2O, NH_3, CH, CN, CO etc.), interstellar extinction, and 21-cm radiation have aided in determining the distribution of the gaseous component in our galaxy. This paper discusses, among other topics, two observational findings relating to the equilibrium state of the galactic gas disk: (i) the run of gas density ρ_g with distance above and below the galactic plane (Schmidt 1956), and (ii) the rapid increase in half-thickness of the gas layer at distances from the galactic center greater than the solar distance (McGee and Milton 1964), and its relatively uniform thickness at distances between about 4 kpc and 10 kpc (Schmidt 1956; McGee and Milton 1964; Kerr 1969). To compute the equilibrium state of the gas disk, the Poisson equation and the hydrostatic equilibrium equations for the gaseous and stellar components of a two-fluid, static, plane-parallel mixture are used. Each component is assumed to be isothermal but each has its own characteristic 'temperature'. The analysis includes one-dimensional magnetic and cosmic-ray components, each constrained to vary with $\rho_g(z)$ according to equipartition of energy.

II. THE EQUILIBRIUM DISTRIBUTION OF GAS ABOVE THE GALACTIC PLANE

The equilbirum configuration of the galactic gaseous, stellar, magnetic, and cosmic-ray components may be pictured as follows. A plane-stratified (in z) infinitely-extended (in r) non-rotating gas slab is immersed in a star slab of similar description, the gas and star densities attaining maximum values at the mid-plane z=0. The weight of the gas layer confines a one-dimensional magnetic field and cosmic-ray gas, the direction of the field assumed parallel to the plane z=0. Early optical polarization measurements (Hiltner 1956) gave support to such a simple magnetic topology. However, more recent optical polarization measurements (Mathewson 1968; Mathewson and Nicholls 1968) on large numbers of stars indicate the presence of a helical component with a pitch angle of about 7°.

The Poisson and hydrostatic equilibrium equations expressed in cylindrical geometry may be written as follows:

$$\frac{d^2\varphi}{dz^2} = -4\pi G(\rho_g+\rho_*) \qquad (1)$$

$$\frac{d}{dz}(P_g + B^2/8\pi + P_{c-r}) = \rho_g\frac{d\varphi}{dz} \qquad (2)$$

$$\frac{dP_*}{dz} = \rho_*\frac{d\varphi}{dz} , \qquad (3)$$

where ρ_g and ρ_* are the gas and star densities, P_g, P_*, P_{c-r}, and $B^2/8\pi$ are, respectively, the gas, star, cosmic-ray, and magnetic pressures, and φ is the total gravitational potential, contributed to by both gas and stars. The terms containing derivatives with respect to r in the expressions for ∇ and ∇^2 are omitted. Their contribution is estimated to be less than 10% of the total at the sun's distance from the galactic center. The equation of state relating the pressure and density of the isothermal gaseous component can be written as

$$P_g = \langle v_{tz}^2\rangle\, \rho_g, \qquad (4)$$

where $\langle v_{tz}^2\rangle$ is the mean square z turbulent and thermal gas velocity dispersion. Following Parker (1966, 1970) we assume that $\langle v_{tz}^2 \rangle$ and the ratios of magnetic and cosmic-ray pressure to gas pressure are independent of z, allowing us to write that

$$B^2(z)/\rho_g(z) = B_o^{\,2}/\rho_{go}$$

and (5)

$$P_{c-r}(z)/\rho_g(z) = P_{c-ro}/\rho_{go},$$

where the subscript o designates that the quantities are evaluated at the galactic plane (z=0). With the aid of equations (5), equation (4), and a similar equation for the stellar component

$$P_* = \langle v_{*z}^2 \rangle \rho_*, \tag{6}$$

the two hydrostatic equilibrium equations assume the form

$$Q^2 \frac{d}{dz} \ell n \rho_g = \frac{d\varphi}{dz} \tag{7}$$

$$\langle v_{*z}^2 \rangle \frac{d}{dz} \ell n \rho_* = \frac{d\varphi}{dz}, \tag{8}$$

where Q^2 denotes the quantity $\langle v_{tz}^2 \rangle + B_o^{\,2}/8\pi\rho_{go} + P_{c-ro}/\rho_{go}$. We have explicitly assumed here that both the gaseous and stellar components of the two-fluid mixture are isothermal, i.e. that $\langle v_{tz}^2 \rangle$ and $\langle v_{*z}^2 \rangle$ are independent of z.

It is not necessary to use the Poisson equation and the stellar hydrostatic equilibrium equation explicitly to determine the distribution $\rho_g(z)/\rho_{go}$ if the gravitational acceleration $d\varphi/dz = K_z$ is known as a function of z. Oort (1960) determined this acceleration by analyzing star counts and radial velocities of K giants. Oort found the density of all matter in the solar neighborhood to be $0.15\,M_\odot/pc^3$, significantly greater than the combined observed density of stars and gas which is 0.089 $M_\odot/pc^3$ (Luyten 1968; Weaver 1970). More recently Woolley and Stewart (1967), noting that the velocity dispersion of A type stars increases near the galactic pole with increasing magnitude, concluded that the density of all matter in the solar vicinity is 0.11 $M_\odot/pc^3$, considerably closer to the observed mass density than found by Oort.

The question of the true density in the solar neighborhood and hence of the component of the gravitational acceleration perpendicular to the galactic plane is not yet fully settled. We choose therefore to follow two separate courses in calculating $\rho_g(z)$, keeping in mind that the 'correct' approach may lie between the following two limiting cases. (i) The Poisson equation and the two hydrostatic equilibrium equations are solved simultaneously for $\rho_g(z)$, assuming that the total mass density at the galactic plane is represented by the sum of the observed gas and star densities in the solar neighborhood (0.089 $M_\odot/pc^3$). Since the bulk of the stellar mass in the solar vicinity resides in the main sequence G, K, and M stars (Oort 1960) and since each type has nearly the same velocity dispersion of 18 km/sec in the z direction (Wehlau 1957; Woolley 1958), a unique value of 18 km/sec for $\langle v_{*z}^2 \rangle^{1/2}$ will be assumed for all stars. (ii) The gas hydrostatic equilibrium equation is solved for $\rho_g(z)$ using Oort's K_z, and thus implicitly assuming that $\rho_{go}+\rho_{*o} = 0.15\ M_\odot/pc^3$. This procedure differs from Parker's (1966, 1970) work in three ways. First, the simplifying approximation $K_z(z) = \langle K_z \rangle$ is not employed. Second, we take B_o, P_{c-ro}, and ρ_{go} as known from the observations and derive $\langle v_{tz}^2 \rangle^{1/2}$

by equating the observed half-thickness of the gas disk with the calculated half-thickness. Parker assumed that B_o, P_{c-ro}, and $<v_{tz}^2>^{1/2}$ are known and derived ρ_{go}. It is the opinion of the present writer that $<v_{tz}^2>^{1/2}$ is the least well-known of the four quantities for the following reason. 21-cm profiles have so far given information on only the component of the turbulent gas velocity dispersion in the galactic plane (6-7 km/sec) since most of the neutral hydrogren is confined to a thin disk, with the sun near the mid-plane. There is essentially no data for $<v_{tz}^2>^{1/2}$, although there may be some theoretical justification in taking $<v_{tz}^2>^{1/2} = <v_{tx}^2>^{1/2} = <v_{ty}^2>^{1/2}$. We do not choose to make such an assumption. Third, the distribution of gas above the galactic plane $\rho_g(z)$ is derived once the appropriate value of $<v_{tz}^2>^{1/2}$ is found.

We proceed with (i) by equating the left-hand sides of equations (7) and (8) and integrating, with the result that

$$\rho_* = \rho_{*o}(\rho_g/\rho_{go})^{Q^2/<v_{*z}^2>} . \qquad (9)$$

Combining this with the Poisson equation then gives

$$\frac{d^2\varphi}{dz^2} = -4\pi G[\rho_g+\rho_{*o}(\rho_g/\rho_{go})^{Q^2/<v_{*z}^2>}]. \qquad (10)$$

The gas hydrostatic equilibrium equation is differentiated and combined with equation (10):

$$\frac{d^2}{dz^2}\, \ell n\rho g/\rho go = -\frac{4\pi G}{Q^2}[\rho_g+\rho_{*o}(\rho_g/\rho_{go})^{Q^2/<v_{*z}^2>}]. \qquad (11)$$

Equation (11) may be reduced to a dimensionless form by defining four dimensionless parameters

$$\alpha = \rho_g(z)/\rho_{go}$$

$$\beta = z/H_g \quad \text{where } H_g = Q/(8\pi G\rho_{go})^{1/2} \tag{12}$$

$$\gamma = \rho_{*o}/\rho_{go}$$

$$\delta = Q^2/\langle v_{*z}^2\rangle,$$

in which case equation (11) becomes

$$\frac{d^2}{d\beta^2}\,\ell n\alpha = -\,1/2(\alpha + \gamma\alpha^{\delta}). \tag{13}$$

Equation (13) is solved numerically for $\alpha(\beta)$ with three values of δ and hence of Q. Q is chosen to be 5.0, 7.7, and 10.0 km/sec; $\langle v_{*z}^2\rangle^{1/2}$ is chosen to be 18 km/sec as mentioned above. The observed gas and star densities ρ_{go} = 1 H atom/cm^3 = 0.025 $M_\odot/pc^3$ (Weaver 1970) and ρ_{*o}= 0.064 $M_\odot/pc^3$ (Luyten 1968) are used. The resultant curves of $\rho_g(z)/\rho_{go}$ are presented in Figure 1 together with a smooth curve through Schmidt's (1956) data, the latter corrected by the change in galactic distance scale R_o(new)/R_o(old) = 10.0 kpc/8.2 kpc = 1.22.

When z < 200 pc the solution with Q = 7.7 km/sec agrees closely with Schmidt's observations. When z > 200 pc the observed curve lies above the theoretical solution, the discrepancy increasing with increasing z. In fact, the curve with Q = 10.0 km/sec fits the observed curve at z = 400 pc, an increase of 2.3 km/sec. Some possible causes of this discrepancy will be discussed shortly.

Since Q is composed of turbulent, thermal, magnetic and cosmic-ray components, i.e.

$$Q^2 = \langle v_{tz}^2\rangle + B_o^{\,2}/8\pi\rho_{go} + P_{c-ro}/\rho_{go}, \tag{14}$$

$<v_{tz}^2>^{1/2}$ can be determined by employing the Q found from Figure 1 and the observed values of $B_o^2/8\pi$, P_{c-ro}, and ρ_{go}. Using a cosmic-ray pressure of 0.50×10^{-12} dynes/cm^2 (Parker 1970), a magnetic pressure of 0.36×10^{-12} dynes/cm^2 corresponding to a field strength of 3.0×10^{-6} gauss (Verschuur 1970; Manchester 1971), and $\rho_{go} = 0.025\ M_\odot/pc^3$, we find that $<v_{tz}^2>^{1/2} = 3.00$ km/sec.

Recent evidence from RR Lyrae star counts near the galactic center (Plaut 1970) indicates that R_o may be significantly less than 10 kpc. If we choose $R_o = 8.2$ kpc and repeat the above procedure, $<v_{tz}^2>^{1/2}$ turns out to be 0 km/sec, i.e. there is no room for z gas turbulence. This result can clearly be ruled out.

The second approach consists in solving the gas hydrostatic equilibrium equation using Oort's K_z. We recall equation (7):

$$Q^2 \frac{d}{dz} \ln \rho_g/\rho_{go} = \frac{d\varphi}{dz} = K_z. \tag{15}$$

Integrating between the limits z=0 and z=z we can solve for $\rho_g(z)/\rho_{go}$:

$$\rho_g(z)/\rho_{go} = e^{1/Q^2 \int_0^z K_z dz} \quad . \tag{16}$$

Note that neither ρ_{*o} nor $<v_{*z}^2>$ appear here. Rather, they are contained implicitly in K_z. The computed distribution $\rho_g(z)/\rho_{go}$ is shown in Figure 2 together with a smooth curve through Schmidt's data, once again corrected for the change in galactic scale length. $Q = 9.84$ km/sec gives the closest fit to the observed curve for $z < 200$ pc. This solution follows closely the simultaneous solution of the Poisson equation and the two hydrostatic equilibrium equations computed above (procedure (i)) with $Q = 7.7$ km/sec, as a comparison of Figures 1 and 2 shows. The higher Q arising from the use of Oort's K_z is directly related to the higher density of all matter found by Oort (0.15 vs. 0.089 $M_\odot/pc^3$) at the galactic plane. Using Q = 9.84 km/sec with the above quoted values for $B_o^2/8\pi$, P_{c-ro}, and ρ_{go}, we find that $<v_{tz}^2>^{1/2}$ = 6.82 km/sec. We should emphasize that $<v_{tz}^2>^{1/2}$ is a function of the value chosen for R_o, since a change in R_o is reflected in the observed half-thickness of the gas layer. A decrease in R_o causes a corresponding decrease in half-thickness, and a smaller gas turbulent velocity is necessary to support the gas layer against gravity. Specifically, if R_o = 8.2 kpc rather than 10.0 kpc, $<v_{tz}^2>^{1/2}$ becomes 3.51 km/sec.

Let us comment briefly here as to the outcome of approaches (i) and (ii). Only (ii) using Oort's K_z (implicitly assuming that $\rho_{go} + \rho_{*o}$ = 0.15 $M_\odot/pc^3$) and R_o = 10 kpc yields $<v_{tz}^2>^{1/2}$ in the neighborhood of 7 km/sec. All other assumptions result in z turbulent gas velocity dispersions at least a factor of two smaller. Indeed, if $<v_{tz}^2>^{1/2}$ = $<v_{tx}^2>^{1/2}$ = $<v_{ty}^2>^{1/2}$ as some authors have suggested, and if $<v_{tx}^2>^{1/2}$ and $<v_{ty}^2>^{1/2}$ equal 6-7 km/sec as observed by Westerhout (1956) and others, we may conclude that the evidence presented above favors the Oort mass limit as being substantially correct and R_o close to 10 kpc. Table 1 summarizes the results of the present discussion, giving $<v_{tz}^2>^{1/2}$ as a function of $\rho_{go}+\rho_{*o}$ and R_o.

Recent high-resolution 21-cm observations (Kerr 1969) indicate that a smaller value for the layer half-thickness may be appropriate (200 pc), which is in better agreement with the layer width found for

Cepheids and HII regions (Kerr 1964). It is clear from our discussion above that $\langle v_{tz}^2\rangle^{1/2}$ less than 3 km/sec would be needed to support the gas layer to the observed width, even if R_o=10 kpc.

The theoretical distribution $\rho_g(z)/\rho_{go}$ falls off too rapidly with distances above the galactic plane greater than about 200 pc using either approach. We consider five possible explanations. (a) The gas is not in hydrostatic equilibrium above about 200 pc from the mid-plane. (b) The equipartition assumptions that $B^2/\rho_g\langle v_{tz}^2\rangle$ and $P_{c-r}/\rho_g\langle v_{tz}^2\rangle$ are independent of z break down when $z > 200$ pc. (c) The observations at $z > 200$ pc are incorrect. (d) Schmidt's data for $z > 200$ pc probably refers to tangential points significantly closer to the galactic center than R_o, where increased 21-cm line profile widths (Schmidt 1956) may indicate an increase in $\langle v_{tz}^2\rangle^{1/2}$. From this fact alone we would expect more gas at large z. However, this is not the whole story (see Section III), since $\rho_{go}+\rho_{*o}$ and thus K_z is also expected to increase with decreasing R, which would tend to reduce the amount of gas at large distances from the mid-plane. We will show below that these two efforts cancel, so that this explanation is probably not the correct one. (e) According to Field, Goldsmith and Habing (1969) a critical pressure P_c exists such that if $P_g(z) > P_c$, the neutral gas is essentially in a cool dense phase and if $P_g(z) < P_c$ the gas is in a hot rarefied phase. In addition they showed that ρ_{go} is such that the gas near the mid-plane should predominantly be in the cool phase. Therefore, since $dP_g/dz < 0$ from hydrostatic considerations, there will be a certain distance z_c above the mid-plane at which $P_g = P_c$. When $|z| < z_c$ the gas will be predominantly in the cool dense phase; when $|z| > z_c$ the gas will be mainly in the hot phase. From our calculations $\rho_g = 0.22$ atoms/cm^3 at z = 200 pc (assuming $\rho_{go} = 1.0$ atoms/cm^3), which agrees closely with the ρ_c found by Field et al. (1969) = 0.21 atoms/cm^3, by Spitzer and Scott (1969) = 0.21 atoms/cm^3, and by Hjellming, Gordon, and Gordon (1969) = 0.19 atoms/cm^3. The divergence between the observed and theoretically derived distributions $\rho_g(z)/\rho_{go}$ above z=200 pc may therefore be caused by the presence of the high temperature (8000°K) phase with large $\langle v_{tz}^2\rangle^{1/2}$ that should predominate above z=200 pc. It is of some interest to note here that Kepner (1970) observed gas in spiral arms extending 1-2 kpc from the galactic plane. We suggest that $\rho_g(z)/\rho_{go}$ be derived from the more recent 21-cm data so that these points may be clarified.

III. THE HALF-THICKNESS OF THE EQUILIBRIUM GAS DISK

Schmidt (1956) observed the thickness of the neutral hydrogren layer at several galactic tangential points and found that between about 3 kpc from the galactic center and R_o, the layer half-thickness is surprisingly constant, with a value of 220 pc (268 pc on the new distance scale with R_o=10 kpc). More recently Kerr (1969) has found a somewhat smaller constant half-thickness of about 200 pc over the region 4 kpc $\leq R \leq R_o$. Westerhout's (1956) isodensity contours of neutral hydrogen indicate that the half-thickness of the gas layer increases substantially with increasing distance from the galactic center beyond the sun's position R_o. McGee and Milton (1964) confirmed the constant half-thickness at distances between about 3 kpc and R_o (on the old distance scale with R_o = 8.2 kpc). Beyond R_o they found the half-thickness to increase rapidly, reaching more than 1000 pc 14 kpc from the galactic center. We shall attempt to explain these observations from a theoretical point of view.

We first recall equation (13):

$$\frac{d^2}{d\beta^2} \ln\alpha = -\ 1/2(\alpha+\gamma\alpha^{\delta}). \qquad (13)$$

δ is typically on the order of 0.2-0.3 from our discussion above and can be set equal to zero with little resulting loss of accuracy. Further, α is equal to $\rho_g(z)/\rho_{go}$, and since the latter may be approximated by $e^{-\beta^2}$ with good accuracy, equation (13) becomes

$$\frac{d^2}{d\beta^2} \ln\alpha \approx -\ 1/2(e^{-\beta^2}+\gamma). \qquad (17)$$

Approximating $e^{-\beta^2}$ by $1-\beta^2$ and twice integrating, equation (17) becomes

$$\ln\alpha \approx -\ 1/4(1+\gamma)\beta^2. \qquad (18)$$

Setting $\alpha = \rho_g(z)/\rho_{go} = 1/2$, equation (18) can be solved for $\beta_{1/2}$:

$$\beta_{1/2} \approx \left(\frac{2.773}{1+\gamma}\right)^{1/2}. \qquad (19)$$

The half-thickness, i.e. the distance between the points above and below the galactic plane where the gas density is one-half its value at the plane, expressed in dimensional form, is written $z_{1/2} = 2\beta_{1/2}H_g$:

$$z_{1/2} \approx \left(\frac{1.386}{\pi G}\right)^{1/2} \frac{Q}{(\rho_{go}+\rho_{*o})^{1/2}}\ . \qquad (20)$$

Equation (20) reveals that the half-thickness is directly proportional to Q, where we recall that $Q^2 = \langle v_{tz}^2 \rangle + B_o^2/8\pi\rho_{go} + P_{c-ro}/\rho_{go}$, and is inversely proportional to the square root of the total mass density at the galactic plane.

Taking $z_{1/2}$ as known from the observations of McGee and Milton (1964), we may use equation (20) (i) to compare the observed and predicted distributions $z_{1/2}(R)$, using a galactic mass model for $\rho_{To}(R) = \rho_{go}(R) + \rho_{*o}(R)$ and choosing that value of Q which gives the best fit to the observations at $R = R_o$ (where Q is assumed to be independent of distance from the galactic center), (ii) to derive a galactic mass model assuming an appropriate value for Q independent of R, and (iii) to derive Q(R) on the basis of a galactic mass model. Schmidt (1956) has explored (iii) for $R < R_o$ using his galactic mass model. We shall pursue (i) and (ii) and comment briefly on (iii).

To pursue (i) the quantity $\rho_{To} = \rho_{go} + \rho_{*o}$ (the total density of matter at the galactic plane) must be known as a function of distance from the galactic center. Interstellar obscuration by dust severely limits our knowledge of $\rho_{*o}(R)$ at distances beyond about 1 kpc from the sun. Theoretically determined densities must therefore be used. We have selected the Innanen (1966) galactic mass model with R_o = 10.0 kpc and $V_\odot$ = 2.52 km/sec (Table 2). Q is chosen to be 7.25, 10.25, and 13.25 km/sec. The three curves for $z_{1/2}(R)$ that result are compared with the observations in Figure 3.

At distances less than about 8 kpc from the galactic center, all three of the theoretically derived curves fall below the mean curve through the observed points, the discrepancy increasing with decreasing R. This suggests that either a larger value is required for Q or that the total mass density at the galactic plane ρ_{To} has been overestimated. The curve with Q = 10.25 km/sec exhibits the best agreement with the observed half-thickness of 268 pc at the sun's position (assuming R_o = 10 kpc). This relatively high value for Q is in part due to the high mass density found by Innanen at R = R_o and z=0 (0.149 $M_{\odot}/pc^3$) and in part to the approximations employed in deriving equation (20). All three curves exhibit the sharp rise in $z_{1/2}(R)$ found by McGee and Milton beyond R_o. These results are consistent with the following interpretation: the increase in the half-thickness of the gas layer is due to a systematic decrease of ρ_{To} at distances beyond R_o, with Q remaining essentially constant.

We have implicitly assumed here that the plane of maximum gas density is perfectly flat. 21-cm observations (Kerr 1957; Gum, Kerr, and Westerhout 1960) indicate, however, that the layer of interstellar hydrogen is warped. The points of maximum hydrogen density in the region $R < R_o$ define a plane to within 50 pc. However, for $R > R_o$ the layer curves upward on one side of the galaxy and downward on the opposite side, the deviation from the principal plane reaching 1 kpc in the very outer rim of the galaxy. Theoretical discussions concerning the physical mechanism responsible for the distortion include interaction of the galactic halo with intergalactic gas (Kahn and Woltjer 1959), interaction of the galactic gas disk with the Magellanic clouds at or near their present distance (Avner and King 1967), and interaction with the Magellanic Clouds during a single close passage (Habing and Visser 1967; Hunter and Toomre 1969). If the increase in half-thickness is causally related to the warpage, our derivation may not be applicable.

The question naturally arises as to whether the increase in half-thickness of the gas layer is a general feature of extragalactic systems, as it should be if caused by a decrease in $\rho_{To}(R)$ at large R. Low resolution 21-cm studies of extragalactic system (Roberts 1968, 1969) at present offer no answer to this question. We can only hope to use the interstellar dust of the external galaxy as a gas tracer. Further, the system must be observed almost exactly edge on so as to minimize projection effects which confuse the issue. One favorable such galaxy, NGC 5866, has been observed by Burbidge and Burbidge (1960). Its dust layer extends very far from the center of the galaxy and remains remarkably flat and coplaner within a certain distance of the galactic center. Beyond this distance, both the half-thickness and warpage increase considerably with increasing R.

To pursue (ii), i.e. to construct a galactic mass model, we are faced with the problem of choosing appropriate values of Q(R). What makes this problem exceptionally difficult is that Q can change if either $\langle v_{tz}^2 \rangle$, B_o^2/ρ_{go}, or P_{c-ro}/ρ_{go} vary with R. Because we quite obviously lack this information, Q is taken to be independent of R for purposes of simplicity, and is determined by the constraints that $\rho_{To} = 0.15\ M_\odot/pc^3$ and $z_{1/2}$ = 268 pc in the solar vicinity. Under these restrictions, Q becomes 10.25 km/sec.

The resultant galactic mass model is given in Table 2 along with Innanen's galactic mass model for purposes of comparison. That ρ_{To} is independent of R over the range 4 kpc $\leq$ R $\leq$ 10 kpc is a result of the constancy of the observed layer half-width and the assumed constancy of Q. Our mass densities are systematically less than those found by Innanen over this region, the discrepancy becoming more substantial as R decreases. The agreement at $R=R_o$ is forced by our choice of Q. Uncertainties of at least a factor of 1.5 in Q introduce uncertainties greater than a factor of 2 in the mass model. To this must be added the uncertainties in the observed values of $z_{1/2}(R)$. The importance of our mass model at $R < R_o$ rests not in the exact numerical values derived for $\rho_{To}(R)$ (most certainly incorrect) but in demonstrating that present-day galactic mass models (e.g. Innanen 1966) cannot be reproduced by simply assuming that the gas and star disks obey hydro-static and Poisson considerations and that $z_{1/2}(R)$ and Q(R) remain constant over the range 4 kpc $\leq$ R $\leq$ 10 kpc. If the mass densities quoted by Innanen over this range are even near correct, it is clear that we must part with at least one of our assumptions. The restriction Q(R) = constant appears to be the weakest link in the argument. Indeed, if Q is determined by the conditions $z_{1/2} = z_{1/2\ \text{observed}}$ and $\rho_{To} = \rho_{To\ \text{Innanen}}$, we find that Q = 24.8 km/sec at R = 4 kpc, significantly greater than its value at $R=R_o$. The suggested increase in Q(R) with decreasing R may be due to (a) larger gas turbulent velocities, possibly caused by an increase in the rate of star formation (Schmidt 1970) and or supernovae activity, (b) an increase in the term $B_o^2(R)/8\pi\rho_{go}(R)$, or (c) an increase in the term $P_{c-ro}(R)/\rho_{go}(R)$. It is interesting in this respect that 21-cm profile widths are observed to increase with decreasing R (Schmidt 1956), possibly indicating enhanced gas turbulence. The significant point to emerge from this discussion is simply that since $z_{1/2}(R)$ is observed to remain essentially constant over the range 4 kpc $\leq$ R $\leq$ 10 kpc, then from equation (20) it follows that $Q(R) \propto \rho_{To}^{1/2}(R)$. If $\rho_{To}(R)$ increases with decreasing R, as seems reasonable, then Q must also increase. Finally, it should be emphasized that the above analysis should not be applied within 4 kpc of the galactic center because the nuclear component, which has been neglected, will begin to dominate the disk component in its gravitational effect on the gas layer, and because $\delta = Q^2/\langle v_{*z}^2\rangle$ may become $\geq$ 1 so that the approximations employed in deriving equation (20) no longer are valid.

IV. SUMMARY

The distribution of gas density with distance above the galactic plane $\rho_g(z)$ is computed in Section II on the assumption that the gas and star disks obey hydrostatic and Poisson considerations and that the gas turbulent, magnetic, and cosmic-ray pressures are in equipartition. Comparing the theoretical gas distribution with Schmidt's observations of $\rho_g(z)$ at the galactic tangential points enables us to derive the rms z turbulent gas velocity dispersion. This dispersion is near 7 km/sec only if we accept the Oort mass limit and choose R_o = 10 kpc.

The same theoretical considerations are employed in Section III to derive an expression for the half-thickness of the galactic gas disk. Employing the Innanen galactic mass model and the observations of McGee and Milton relating to the hydrogen layer half-width in the region 4 kpc $\leq$ R $\leq$ 14 kpc, it seems likely that (i) the increase in half-width for $R > R_o$ is due to a decrease in gas plus star density at the galactic plane with increasing R, and (ii) since the half-width remains essentially constant when 4 kpc $\leq$ R $\leq$ 10 kpc, Q must scale as $(\rho_{go}+\rho_{*o})^{1/2}$ and therefore increase with decreasing R.

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TABLE 1

z TURBULENT GAS VELOCITY DISPERSION

$\rho_{go}+\rho_{*o}$ ($M_\odot/pc^3$)	R_o (kpc)	$<v_{tz}^2>^{1/2}$ (km/sec)
0.089	8.2	0
0.089	10.0	3.00
0.150	8.2	3.51
0.150	10.0	6.82

TABLE 2

GALACTIC MASS MODELS

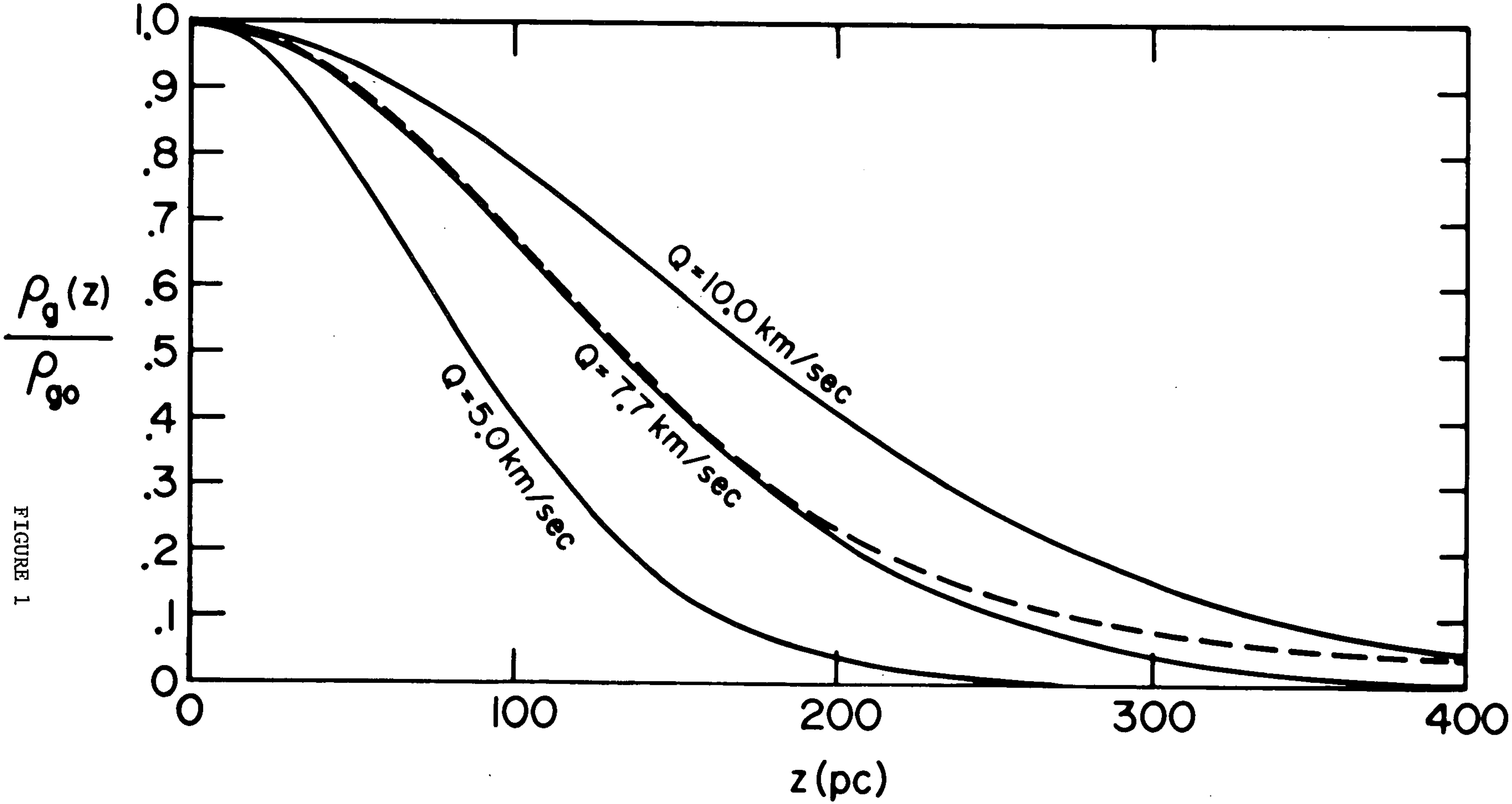
R (kpc)	ρ_{To}(Innanen) ($M_\odot/pc^3$)	ρ_{To}(Kellman) ($M_\odot/pc^3$)
4.0	0.872	0.149
6.0	0.470	0.149
8.2	0.269	0.149
10.0	0.149	0.149
12.0	0.062	0.0251
14.0	0.006	0.010
16.0		0.0056

FIGURE CAPTIONS

1. The distribution $\rho_g(z)/\rho_{go}$ (———) for three values of Q, obtained by solving the Poisson and two hydrostatic equilibrium equations. A smooth curve through Schmidt's observed data (----) is shown.

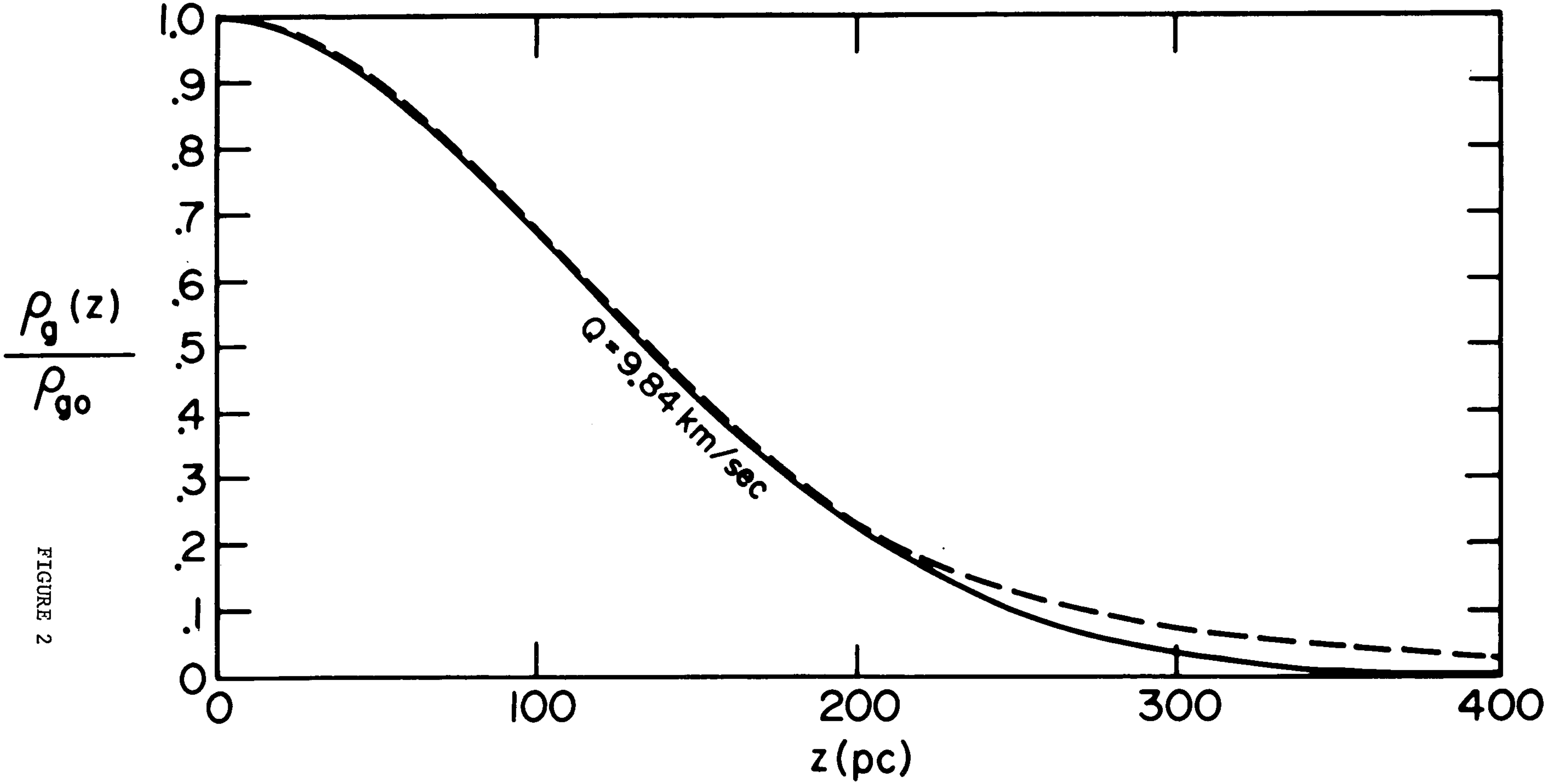
2. The distribution $\rho_g(z)/\rho_{go}$ (———) obtained by solving the gas hydrostatic equilibrium equation with Oort's K_z. A smooth curve through Schmidt's observed data (----) is shown.

3. The theoretically determined half-thickness of the gas layer (———) for three values of Q. A mean curve through the observations of McGee and Milton (-----) is shown.



1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
$\rho_g(z) / \rho_{go}$
Q=10.0 km/sec
Q=7.7 km/sec
Q=5.0 km/sec
0
100
200
300
400
z (pc)

FIGURE 1



1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
100
200
300
400
z (pc)
$\frac{\rho_g(z)}{\rho_{go}}$
Q = 9.84 km/sec

FIGURE 2

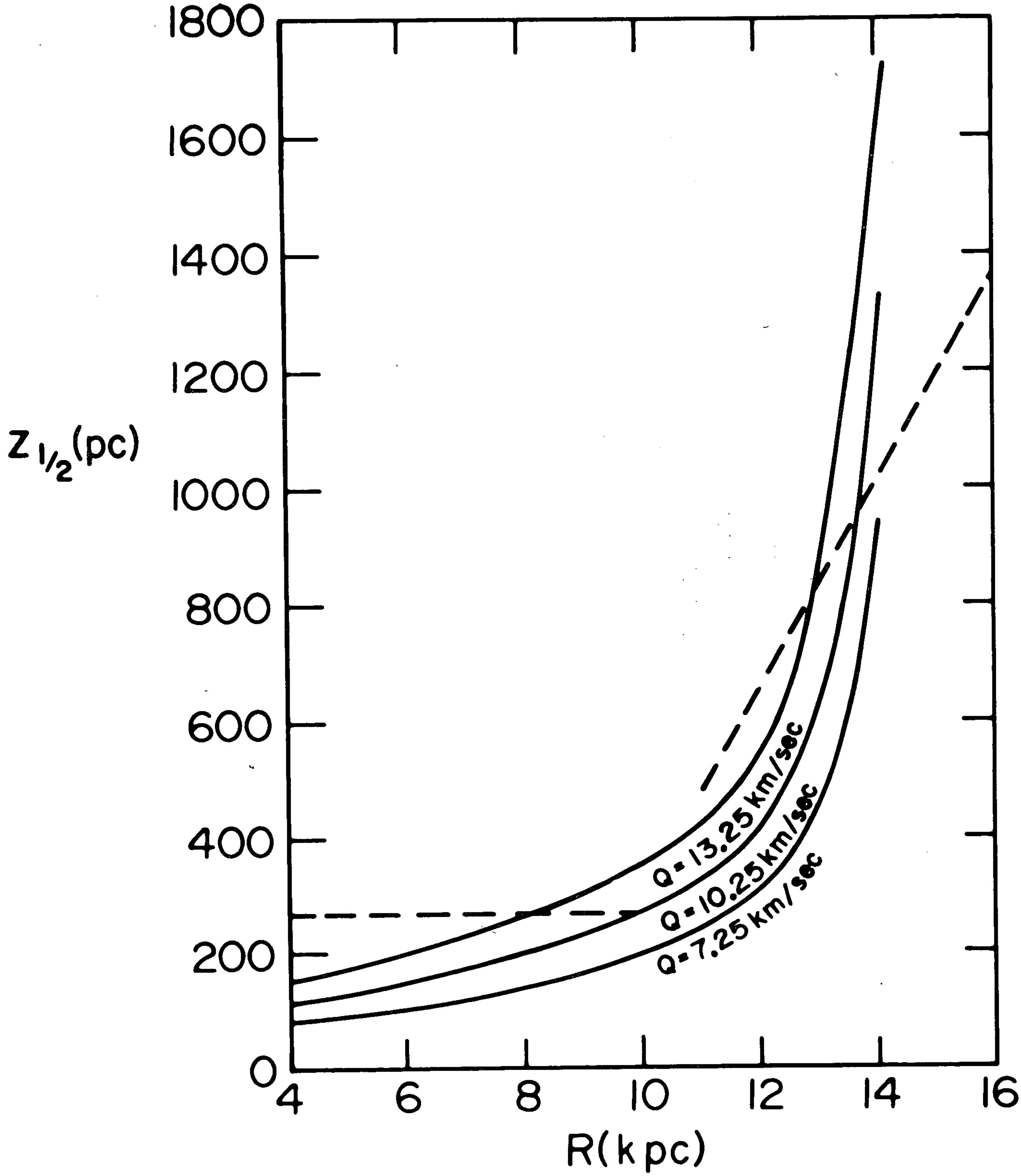

1800
1600
1400
1200
1000
800
600
400
200
0
Z 1/2 (pc)
4
6
8
10
12
14
16
R(kpc)
Q=13.25 km/sec
Q=10.25 km/sec
Q=7.25 km/sec

FIGURE 3