The Equilibrium Distribution of Income and the Market for Status*

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Abstract

This paper explores the implications for risk-taking behavior and the equilibrium distribution of income of assuming that the desire for status positions is a powerful motive and that it raises the marginal utility of consumption. In contrast to previous analyses, we consider the case where status positions are sold in an hedonic market. We show that such a complete hedonic market in status positions can be perfectly replicated by a simpler arrangement with a 'status good' and a social norm that assigns higher status to those that consume more of this good. The main result is that for a wide range of initial conditions the equilibrium distribution of income, status and consumption are the same, that this allocation requires inequality of income and consumption, and that this allocation coincides with optimum of a utilitarian planner.

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1 Introduction

For several centuries, economists, sociologists, and philosophers have been concerned with the magnitude and effects of inequality. Economists have concentrated on inequality in income and wealth, and have linked this inequality to social welfare, aggregate savings and investment, economic development, and other issues. They have explained the observed degree of inequality by the effect of random shocks, inherited position, and inequality in abilities and in access to human capital and assets. But none of these models provide any scope for individuals collectively "choosing" the shape and degree of inequality in the distribution of income and wealth.

Pareto believed that the personal distribution of incomes in different countries is much more similar than the underlying "functional" determinants of incomes based on factor prices and distributions of human and physical capital. He showed that his measure of inequality and skewness, the coefficient in the now called Pareto distribution, does not vary much across a sample of countries he considered, see Pareto (1896). But Pareto never developed a theory that would explain why personal distributions of incomes should be more similar than the underlying functional determinants.

Our analysis shows that Pareto's instincts were right, for under certain assumptions, personal distributions of income tend to be similar even when the underlying functional determinants are quite different. We prove that if the initial functional income distribution is sufficiently "compact"– in a sense we make precise – and if preferences are the same in different societies, then they will have exactly the same equilibrium personal distribution of incomes.

Participation in lotteries and other risky activities is the mechanism that converts different functional income distributions into the same equilibrium personal distribution. In our analysis, individuals are willing to participate in fair lotteries and other gambles even though they are assumed to have diminishing marginal utility of income. Their willingness to gamble is the result of the assumed importance of status in their preferences, and of the interaction between status and consumption.

With few exceptions, economists have paid little attention to status, whereas sociology has concentrated on the sizeable inequality in status, privilege, and opportunity found in most societies. Persons with higher status generally receive deference, esteem, and respect from those with lower status. James Coleman (1990) observed that:

> "Differential status is universal in social systems... status, or recognition from others, has long been regarded by psychologists as a primary source of satisfaction to the self. That is, an interest in status can be regarded as being held by every person." (1990, pg.130)

We follow the lead of sociologists, and assume that preferences depend in an important way on status as well as on consumption of other goods and services. When status is important, individuals would be willing to pay a lot in time, effort, and money for sufficiently high status. This is why many individuals make considerable sacrifices of effort and money as they strive to attain higher status.

Willingness to pay has an important role in our analysis because most of the paper assumes a market in some types of status, although it also considers the case where status depends only on rank in the distribution of income. Individuals are assumed to be able to buy larger quantities of some types of status at market-determined costs in terms of foregone consumption goods. This assumption of a market in status does not deny either that individuals are endowed with different amounts of status, or that some forms of status- such as those which depend on family background- are fixed, at least in the short run. In fact, we derive interesting results about the effects of unequal endowments of status on the equilibrium distribution of incomes.

However, striving for status would not be so common if status were completely outside the control of individual actions. Moreover, we believe that inequality in status may have evolved in all societies mainly to generate behavior by individuals striving for higher status that indirectly helps others. Whether or not that is true, the assumption of a complete market in status, where status is "for sale", is an abstract recognition of the possibility of gaining higher status by working hard to "pay" for it.

Status is important for the distribution of income in our analysis because we assume that higher status interacts with consumption by raising the marginal utility of consumption. Higher status raises the marginal utility of a given level of income partly because persons with high status often have access to clubs, friends, and other "goods" that are costly but are not available to those with low status. This type of marginal utility assumption was made in pioneering articles a half century ago by Friedman and Savage (1948) and Friedman (1953) in order to explain why the marginal utility of income might be rising in certain income intervals.

Like Friedman and Savage, we use the assumption that higher status raises the marginal utility of income to explain the demand for risky activities, even when utility is concave in income for a given level of status. A preference for risk could be explained without this assumption even if utility were separable in income and status in our setup because higher income is required to pay for higher status even if consumption is equalized. However, in that case, the optimal fair lottery would have equal consumption for everyone, regardless of their status, because the marginal utilities of consumption across persons with different status levels would then be equalized only with equal consumption.

Equal consumption is obviously violated by the evidence on the unequal distributions consumption in all countries. On the other hand, complementarity in utility between status and consumption not only implies a demand for lotteries and other risky activities, but it also has unequal consumption in the form of a positive relation between status and consumption. That is, with the assumption of complementarity, unequal status itself implies a corresponding degree of inequality in consumption at the equilibrium distribution of income.

Economists have recognized the positive relation between status and consumption, and have explained this by the concept of non-competing groups. This concept states that some persons have higher functional incomes than others due to unequal access to human and physical capital. They "buy" or otherwise get higher status with their higher functional incomes.

Although we believe that the inequality in functional incomes is important, we also believe the positive relation between consumption and status is not only due to this inequality. Indeed, by building on the assumption of complementarity in preferences between status and consumption, our analysis implies a strong equilibrium positive relation between status and consumption even when everyone has the same functional income.

Economists and philosophers have evaluated distributions of income from an ethical perspective given by a social welfare function, such as the utilitarian's maximization of the sum of individual utilities. If the welfare function is symmetrical in utilities of different individuals, and if all utilities are concave in income, then social welfare would be higher when income is less unequally distributed.

This analysis, however, typically ignores status and its unequal distribution. If utility depends on status as well as consumption goods, social welfare that is related to individual utilities would depends on status as well as consumption. Moreover, if status and consumption were complements in utility, then social welfare would be maximized when persons with higher consumption also have higher status.

Indeed, we show that if the initial functional distribution of income were sufficiently compact, the distributions of consumption and status that maximized social welfare would be exactly the same as the equilibrium distribution produced by the private sector with a full market in status. Moreover, a social planner would then increase rather than decrease inequality in income by redistributing initial income from the poor to the rich rather than visa versa.

The rest of the paper is organized as follows. Section 2 sets out the model of preferences and status and discusses an example with two status positions. Section 3 discusses implicit and explicit markets for status. Section 4 proves the main result on unique equilibrium distributions of consumption and status in a private lottery market. Section 5 shows that these equilibrium distributions and the optimal distributions chosen by utilitarian social planners are identical. Several important empirical implications of the analysis about observed income distributions are considered in Section 6. Sections 7 consider the equilibrium distribution of income when status depends only on income rank. Section 8 offers a few conclusions.

2 Status and Income

We assume that the utility of each person depends on his or her own consumption and status. This assumption implies that utility does not directly depend on the consumption or status of anyone else. Everyone is assumed to have the same utility function that is twice continuously differentiable and increasing and concave in consumption and increasing in status:

$$U = u(c, s), \text{ where } u_c > 0,\ u_{cc} < 0,\ u_s > 0.$$

A crucial assumption of the analysis is that a rise in status increases the marginal utility of consumption:

$$u_{cs} > 0.$$

Status also affects the marginal utility of leisure, as in Veblen's classic study of social influences on economic behavior with the title "The Theory of the Leisure Class" (1934, see especially chapter III). In the interest of simplicity, we ignore the relation between status and leisure.

The analysis of social markets (Becker and Murphy, 2000) consider the complementarity between social forces and various kinds of behavior, including smoking, and purchasing jewelry and expensive watches. A natural extension is to complementarity between status - a particular form of social capital - and total consumption itself. Not only may higher status persons have access to consumer goods in limited supply that are not available to others, but also the general population expects persons with higher status to have larger homes with better views, to be more educated and knowledgeable, to be leaders in fashion, collect art and other objects, entertain well, travel extensively, and so forth.

A second major assumption of our analysis is that the distribution of available status categories in society is fixed and given, at least in the short run. Presumably, status positions are in more fixed supply than most goods – as in status due to higher rank in the income distribution. Otherwise, status would simply be another good in the utility function, and there would be less interest in distinguishing status and "goods".

Complementarity between consumption and status implies that individuals with greater income and status may have higher marginal utility of income than those with lower status and lower income. In that case, both richer and poorer individuals would be willing to take gambles, through lotteries or other risky activities, in which winners get *both* higher consumption and higher status, and losers get lower consumption and lower status. The result would be a possibly highly unequal distribution of income and utility, with status and consumption positively related.

We show in the next sections that the analysis applies to any number of individuals and status categories, but we first illustrate some of the main principles graphically with two individuals A and B, who have the same utility function, and two status categories, s_h and s_l with $s_h > s_l$. Consider Figure 1, where the utility of A is plotted along the horizontal axis, and that of B is plotted along the vertical axis. If B had the higher status, the utility possibility boundary would be given by the negatively sloped concave curve BB as consumption good c is reallocated between A and B. Similarly, the boundary would be the concave curve AA if A had the higher status. The slope of these boundaries at each point equals the marginal utility of consumption to A relative to that of B, given the distribution of status between A and B.

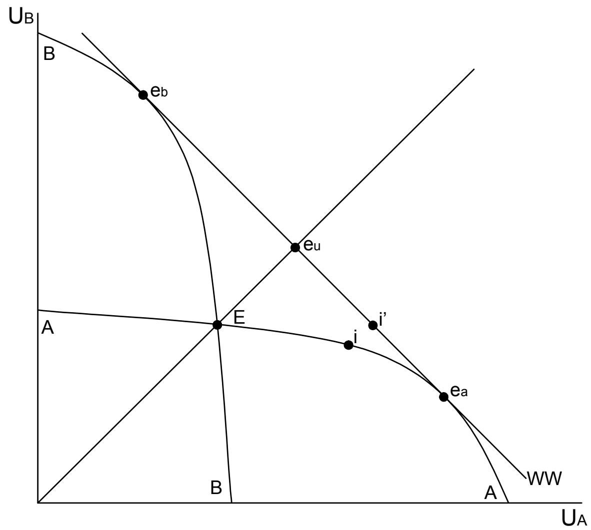

Figure 1: The utility possibility frontier with two individuals and two status positions.

The economy's boundary is the symmetrical curve BEA that is the envelope of the two curves AA and BB. This boundary has a kink at point E, and the assumption of equal utility functions means that E must lie on the 45 degree line. The economy's boundary is not everywhere concave – the utility possibility set is not everywhere convex – because there is a shift at point E of higher status from A to B as income and status is redistributed from A to B. The assumed complementarity between status and income raises the marginal utility of income to B and lowers that of A by discrete amounts when the high status position s_h switches from A to B.

The gains from engaging in lotteries can be shown by using this kinked utility-possibility frontier. With expected utility, lotteries can convexify the feasible set. The pair of prizes which maximizes the attainable set is given by the symmetrical points, e_a and e_b in Figure 1, where the slope of the utility frontier equals -1. Then between e_a and e_b the frontier expected utility of both A and B would lie along the chord WW with a slope of -1 that is tangent to these points. Because the slope at e_a and e_b equals -1 the marginal utility of consumption when holding the high and low status is equalized:

$$u_c(c_h^*, s_h) = u_c(c_l^*, s_l).$$

Given the assumed complementarity, $u_{cs} > 0$, this condition implies that $c_h^* > c_l^*$. With optimal lotteries, winners get higher utility, consumption, and status, and the marginal utility of consumption is the same to winners and losers. In effect, the lottery is over both consumption and status and winners get both higher consumption and higher status.

To be sure, even separability between consumption and status could induce optimal lotteries where winners get higher status and higher utility, but then they would not get higher consumption. If status and consumption were substitutes, winners would have higher status and higher utility, but lower consumption. Only if consumption and status are complements would winners get both higher status *and* higher consumption, which is the empirically important case.

Up to this point the discussion has been entirely about the Pareto optimality of introducing lotteries to expand the utility frontier. We now briefly discuss how one might expect equilibria outcomes in market arrangement with fair lotteries to reach this frontier and select a particular position on it. The actual description of the market arrangements and their equilibria are spelled out in the next sections.

We've seen that given complementarity between consumption and status, persons who win higher status also win greater consumption. In a market arrangement it may be necessary to pay more for higher status and lottery prizes are directly over income, but then the income prize for the winner would be sufficiently higher so that consumption would be greater net of the cost of higher status. By contrast, without lotteries, if both individuals start with the same income there would be a compensating differential in consumption for higher status, so that persons with higher status would have lower consumption and the same utility as others who start with the same incomes.

If the initial utility position of individuals A and B without lotteries is on BEA between points e_b and e_a both A and B would willingly participate in lotteries. For example, if they have the same initial income then the equilibrium without lotteries would yield equal utility at point E. In a market equilibrium with lotteries then they would obtain equal chances of winning the lottery, and their *expected* utility position would be on WW at point e_u along the 45 degree line. Note that although expected utility for both A and

B their actual ex-post utility positions would be at e_a and e_b, depending on the outcome of the lottery.

If the initial utility position without lotteries is higher to A than to B at point i, the expected utility position would be on WW to the right and above point i, say at i'. Although A gambles over the same pair of prizes, A has the higher probability of winning e_a because A starts with high income. As A's income increases relative to B's, the difference between their probabilities of winning also increases.

The extreme is reached when A's income is such that he would attain e_a without lotteries. It is clear from Figure 1 that neither A nor B would be interested in lotteries if A's income were sufficiently large to push the initial position to the right of e_a. A symmetrical analysis applies to B.

We conclude that the optimal lottery increases the ex-post inequality in utility, consumption, and income as long as initial incomes are within a range. Moreover, the equilibrium distribution of income, consumption, and ex-post utility is the same, *regardless* of the initial position, as long as the initial utility position is between e_a and e_b in Figure 1. To use Rosen's felicitous language (1997), an economy "manufactures" a unique degree of inequality through the desires of individuals to participate in lotteries in income, consumption, and status.

3 Explicit and Implicit Status Markets

We now consider an economy with a continuum measure one of agents, indexed by $i \in I = [0, 1]$, and a continuum of status positions. We assume that there are at least as many status positions as individuals. Of course, only the top measure one of status positions will be used in equilibrium. Note that by redefining preferences we can always re-normalize any continuous distribution of s so that the relevant status positions are distributed uniformly over $[0, 1]$. We adopt this convenient normalization for the rest of the paper.

We can think of the equilibrium in two stages. In the first one agents engage in lotteries over income. At the start of the second stage these income prizes are realized, and agents then participate in the hedonic status market choosing s and c.

In this section we work backwards and describe the second stage. Indeed, we present two different arrangements for the second stage: an explicit market for status and an implicit one. We then show that these two arrangements yield equivalent outcomes. We first describe the explicit status market, and then the implicit status market and the equivalence result.

3.1 Explicit Hedonic Status Market

We first consider an explicit hedonic market for status. In this market arrangement we envision each status position s as being offered for sale at a price $P(s)$, expressed in units of the consumption goods. Individuals must select a single status position optimally trading off status and consumption taking as given the continuous price schedule $P(s)$.

An individual with income y solves,

$$v(y;P) \equiv \max_{c,s} u(c,s) \quad \text{s.t. } c + P(s) = y. \tag{1}$$

Let $\bar{c}(y;P)$ and $\bar{s}(y;P)$ denote the solution to this problem.

Our assumptions that $u_{cc} < 0$ and $u_{cs} > 0$ imply that status is a normal good in the sense that $\bar{s}(y;P)$ is non-decreasing in y for all possible price schedules $P(s)$. This assumption generates positive equilibrium sorting between income and status.

As is well known in these hedonic markets a boundary condition is required to pin down $P(0)$. In our model the value of $P(0)$ would have no effect in two interesting cases: when each agents is either endowed with one initial status position or with an equal fraction of all the status positions. In such cases, $P(0)$ plays the role of a constant on both sides of individuals budget constraints, since they must select one status position, and has no real effect on his budget constraint. Thus, setting $P(0) = 0$ in such cases would be a convenient normalization.

In fact, $P(0) = 0$ is a necessary equilibrium condition if there are slightly more status positions than individuals in the population. For example, if there is a supply of measure $1 + \varepsilon$ of status positions distributed with unit density over $[-\varepsilon, 1]$ then the lowest status position used in equilibrium, $s = 0$, must be free because there are arbitrarily close alternatives that are free. These considerations lead us to focus on equilibria where $P(0) = 0$.

For a given distribution of income $\Phi(y)$ we define a positively-sorted equilibrium in the explicit status market to be an allocation of consumption and status and a price function $\{c(y), s(y), P(s)\}$, with $P(0) = 0$ such that: (i) individuals taking P as given optimize so that $s(y) = \bar{s}(y,P)$ and

$c(y) = \bar{c}(y, P)$; (ii) the status market clears with positive sorting: $s(y) = F(y)$ (given that s is distributed uniform on $[0, 1]$).

Given a distribution over income $\Phi(y)$ at the second stage the hedonic market equilibrium generates a price function $P(s)$ as follows. Positive sorting, and the normalization that the distribution of status is uniform, implies that in equilibrium $\bar{s}(y; P) = \Phi(y)$. Equivalently, we can essentially invert to obtain $y(s) = \Phi^{-1}(s)$.[1] The individual's first order conditions yields the following ordinary differential equation for $P(s)$,

$$P'(s) = \frac{u_s(y(s) - P(s), s)}{u_c(y(s) - P(s), s)} \tag{2}$$

which can be solved with the initial condition $P(0) = 0$.

Given our assumptions on preferences a price function $P(s)$ constructed in this way, which imposes the first-order necessary conditions of all agents in the population, implies that agents do indeed find their global maximum at the proposed allocation, i.e. that $\bar{s}(y(s); P) = s$ and $\bar{c}(y(s); P) = y(s) - P(s)$. This follows because the normality of status implies a single-crossing property on preferences, that the marginal rate of substitution function $u_s(c, s)/u_c(c, s)$ is strictly increasing in c, for any s. This single-crossing property ensures that if the first-order conditions hold for all agents along a monotonic allocation then agents are at their global optimum. We discuss this in some more detail below in Section 4.2 in the proof of Proposition 2.

Equilibrium in the consumption good market is automatically guaranteed, by a version of Walras law, if the distribution of income is such that average income equals the value of aggregate endowments:

$$\int_0^\infty y d\Phi(y) = \int_0^1 y(s)\, ds = \bar{\omega}^c + \int_0^1 P(s)\, ds$$

where $\bar{\omega}^c$ is the aggregate endowment of the consumption good. This condition will be met automatically because in the first stage, described in more detail below, agents will bring their endowments and select fair lotteries over income. Fair lotteries and a law of large numbers ensure that average income remains equal to the value of endowments. As a consequence, we do not impose equilibrium in the consumption good until we introduce the first stage in section 4.

[1] Wherever Φ jumps we define $y(s)$ to be flat. Wherever Φ is flat over some region we take $y(s)$ to be the highest value of y.

Economic development increases income, but presumably development has a much smaller, if any, effect on the supply of status. For example, the distribution of income ranks is independent of average income level, and it is not obvious whether other forms of prestige and status are in significantly greater supply now than at the turn of this century, or several centuries ago.

If status becomes scarcer relative to consumption goods as economies develop then normality of status implies that the price of status would rise relative to that of consumption goods. In particular, from (2) and the fact that $c(s) = y(s) - P(s)$ we see that the marginal price for status $P'(s)$ at a given s must rise as long as $c(s)$ rises with development, since in such a case the numerator increases, given $u_{sc} > 0$, and the denominator decreases, given $u_{cc} < 0$.

3.2 Implicit Status Market

Although certain status positions have been explicitly traded throughout history, we do not believe trade generally takes place in a rich hedonic market. Rather, a subset of goods, such as diamonds and gold, may implicitly provide a market for social status, perhaps by the relative amounts consumed of these goods. In this subsection we investigate how much can be done with such a simpler market arrangement.

We find that under certain conditions such an implicit market organization can generate exactly the same allocations as an explicit market for status. This result suggests that explicit hedonic social market may not be observed because they are not required to generate the explicit market's allocation of status.

Suppose there are two physical goods, the consumption good c and a 'social good' z, and status, s. The consumption good, c, is valued for intrinsic personal-consumption purposes, whereas the social good, z, is intrinsically worthless and is valued solely for its indirect effect on status. We capture this by defining utility over c and s only, as before, so that z does not enter the utility function directly.

There are no initial individual status endowments, and no direct trade in status takes place. Instead, status is assigned according to the relative consumption of z in the population, the "social good". That is, people are *ranked* according to their consumption of z, the higher the z, the higher the status. In this sense, purchases of z indirectly buy status.

Agents take the price of z, p_z, and the distribution of consumption over

z in the population as given. The consumption of z across the population determines a cumulative distribution function $R(z)$. Given the uniform distribution of z, our assumption that the ranking on z determines the assignment of status implies that $s = R(z)$.[2]

For any continuous distribution $R(z)$ the problem that an individual with income y faces is,

$$\max_{c,z} u(c, R(z))$$
$$\text{s.t. } c + p_z z = y$$

Given some distribution of income $\Phi(y)$ an equilibrium is a price and a distribution of consumption for the z good p_z, $R(z)$, and an allocation of consumption and the social good, $c(y)$, $z(y)$, such that: (i) taking the schedule R as given individuals optimize so that $c(y)$ and $z(y)$ solve the maximization above; (ii) consistency: the decision rule $z(y)$ and the distribution of $\Phi(y)$ together induce the distribution $R(z)$; (iii) the market for z clears so that $\int_0^\infty z(y)\, d\Phi(y) = \bar{\omega}^z$.

It is convenient to define the inverse relation of $s = R(z)$ by $z = r(s)$.[3] To obtain status s the consumer must purchase $z = r(s)$ of the social good, z so the cost of obtaining it is equal to $p_z r(s)$. Thus, the consumer's problem is equivalent to the explicit status market case with $P(s) = p_z r(s)$, or more compactly $P = p_z r$. The demand for consumption and status is then $\bar{c}(y; p_z r)$ and $\bar{s}(y; p_z r)$. The demand for status generates an implicit demand for the status good z given by composing $\bar{s}$ and r, i.e. $\bar{z}(y; p_z, r) \equiv r(\bar{s}(y; p_z r))$.

In equilibrium it must be true that $R(z)$, the perceived ranking used in stating the individual's problem above, actually represents the true distribution of purchases of z. Positive sorting implies that this condition is equivalent to:

$$\bar{s}(y; p_z r) = F(y). \tag{3}$$

[2]The distribution of z could conceivably have a mass of people consuming the same level of the status good, corresponding to a discontinuity in R. In such cases we assume that individuals in such a group are assigned a status position randomly within that corresponding to their group. It follows that such discontinuities cannot be an equilibrium. Individuals within such a group would seize on the opportunity of increasing their consumption of z infinitesimally to distinguish themselves, thus attaining a discretely higher, and certain, status position.

[3]If R is flat over some interval so that $R(z) = s_0$ for $z \in [z', z'']$ we take $r(s_0) = z'$, to be the lowest value z, i.e. the cheapest way of attaining status s_0.

To see this note that individuals with income y purchase $r(\bar{s}(y;p_z r))$ of the z good. By normality all individuals with income below y purchase less than that, implying that the measure of agents purchasing less than $r(\bar{s}(y;p_z r))$ is $F(y)$. However, by definition of R the measure of agents that consumes $r(\bar{s}(y;p_z r)$ or less is given by $R\left[r(\bar{s}(y;p_z r))\right]$. Consistency then requires $R\left[r(\bar{s}(y;p_z r))\right] = F(y)$ and the result follows since R and r are inverses of each other.

It is now easy to see that any equilibrium in the explicit status market can always be attained by an equilibrium in the implicit status market. Suppose that $\{c(y), s(y), P(s)\}$ is an equilibrium for the explicit status-market, so that in particular, $\bar{s}(y;P) = F(y)$. We need to find a price for the status good p_z and a distribution over z summarized by its inverse $r(s)$ that constitutes an equilibrium in the implicit market and replicates the explicit market.

If $p_z r(s) = P(s)$ then the consumer's problem is the same as in the explicit market equilibrium, implying the same allocation for c and s. The condition that $\bar{s}(y;P) = F(y)$ with $P = p_z r$ implies the consistency of $R(s)$ required by (3). Market clearing for z requires,

$$\int_0^\infty z(y)\, d\Phi(y) = \int_0^1 r(s)ds = \frac{1}{p_z}\int_0^1 P(s)ds = \bar{\omega}^z.$$

The last equality together with $p_z r(s) = P(s)$ uniquely define $r(z)$ and p_z. Thus, we have satisfied all the requirements for an equilibrium in the implicit status market.

The following proposition summarizes this result that implicit markets can achieve the same allocations that explicit ones do.

Proposition 1. For a given a distribution of income $\Phi(y)$, if $\{c(y),\ s(y),\ P(s)\}$ *is an equilibrium of the explicit status market then there exists a* p_z, $R(z)$ and $z(y)$ such that $\{c(y),\ z(y),\ R(z),\ p_z\}$ is *an equilibrium of the implicit status market.*

This proposition is important because it implies that society does not require a rich explicit hedonic market for status. Instead, people can be ranked by their relative consumption of "social goods".

In equilibrium the implicit price of a certain status position s is $p_z r(s)$. Thus, even though the marginal price of the z good is constant the implicit price of status is not linear. Proposition 1 shows that p_z and $R(z)$ together provide enough flexibility to generate the same price function and equilibrium allocation as with an hedonic market structure.

We believe this result makes the notion of a market for status much more palatable. Interestingly, the implicit market arrangement assigns status by rank, yet ranks on z are drastically different from ranks on income. Ranks on z are equivalent to perfect hedonic markets for status, while ranks on income have various externalities associated with them.

These equivalence observations apply to a broader set of problems than the risk-taking application which is the focus of this paper. For example, it applies to the leisure-work margins stressed by Frank (1999) and others. If income from work is used to purchase intrinsically worthless items in fixed supply (such as z) in pursuit of social status, then there is no inefficiency from society's 'rat-race'.

We have made various assumptions to reach this equivalence result, and in the rest of this section we discuss their significance.

3.3 Other Assumptions

Crucial to the equivalence argument is the assumption that the 'social good' z is available in fixed supply. If instead z were producible from c or labor and other resources, then equilibrium would entail a wasteful use of resources and an over-production of the z good.

This follows because by Proposition 1 any small supply of the social good z is enough to allocate social status, s. By assumption z is not directly valued, so everyone could be made better off by reducing z if its production comes at the expense of the consumption good c, or perhaps other valuable resources not modeled here such as labor. This is the type of concern expressed by Frank and other authors. This suggests a potential efficiency explanation for why social goods, such as diamonds and gold, are often goods in rather fixed supply.

We also assumed that the 'social good' z is intrinsically worthless. If instead, it has some intrinsic value, its role as status-assigner would generate a distorted allocation. The 'social good' z would then be forced to play two different roles, and its price would generally be higher than the price that would prevail in the status-market where z does not order social status. The higher price induces some people to consume less and others more relative to the status-market allocation. However, the welfare loss would be small if the intrinsic value of z, while positive, were small, such as for gold and diamonds. If a good is of little value it does little harm to allocate it incorrectly.

Our conclusion is that commonly observed properties of "social goods" such as their relative fixed supply and their low intrinsic value may not be an accident. These properties of social goods make the allocation achieved by ranking on a "social good" closer to that required by a fully operating hedonic market in status. These results are only suggestive, a theory of the selection of 'status goods' is beyond the scope of this paper.

4 The Private Lotteries Equilibrium

Given our results on the equivalence of the explicit and implicit market arrangements it does not matter which concept is used. In the formal discussion that follows we use the explicit market arrangement for simplicity.

We now describe the first stage where agents engage in fair lotteries over income. A similar two stage equilibrium concept is used by Cole and Prescott (1997) in the context of the theory of clubs.

4.1 The Fair Lottery Market

In the first stage, each individual $i \in I$ engages in an ex-ante fair lottery over wealth. Their problem can be viewed as choosing a distribution of income $\phi_i(y)$ that has the same expected value of income as their endowment so as to maximize expected utility using the indirect utility function over income from the second stage, $v(y; P)$:[4]

$$\max_{\phi_i} \int_0^\infty v(y;P)\phi_i(y)dy \tag{4}$$

where $\phi_i(y) \geq 0$ for all y and,

$$\int_0^\infty \phi_i(y)dy = 1 \tag{5}$$

and,

$$\int_0^\infty y\phi_i(y)dy = \omega_i + \int_0^1 P(s)dH_i(s). \tag{6}$$

Here ω_i is the consumption good endowment and $\int_0^1 P(s)dH_i(s)$ the status endowment income for person i, where $H_i(s)$ represents the cumulative holdings function.

[4]To simplify the exposition we represent lottery positions by a density.

Regarding $H_i(s)$ two special cases of are particular interest. In the first, each individual owns a single status position s_i so that $H_i(s)$ is a step function with $H_i(s) = 1$ for $s \geq s_i$ and $H_i(s) = 0$ for $s < s_i$. In the second case all individuals own equal shares of all the status positions so that $H_i(s) = H(s) = s$.

In any case, the $H_i(s)$ functions must add up to s given our normalization on the distribution of status:

$$\int_{i \in I} H_i(s) d\mu(i) = s,$$

where μ is the measure over individuals.

The first order condition of the above problem implies that for all income levels where the density is positive at y, $\phi_i(y) > 0$, we must have,

$$v(y; P) = \lambda_i y + \varphi_i.$$

where λ_i and φ_i are the multipliers on (5) and (6), respectively. Differentiating the above expression with respect to y in an interval where the density is positive gives:

$$u_c(\bar{c}(y; P), \bar{s}(y; P)) = \lambda_i.$$

where we are using the envelope condition that $v_y(y; P) = u_c(\bar{c}(y; P), \bar{s}(y; P))$.

We now turn to the formal definition of a competitive equilibrium with lotteries for our setup. The normality assumption on preferences greatly simplifies the equilibrium statement because of positive sorting.

Definition. *Given endowments $H_i(s)$ and ω_i, a positively sorted competitive lottery equilibrium is a price function, $P(s)$ with $P(0) = 0$, a distribution of final income, $\Phi(y)$, individual lottery positions $\{\phi_i(y)\}$ and an allocation for consumption and status as a function of final income $c(y)$ and $s(y)$, with $s(y)$ non-decreasing in y such that:*

1. *Individuals optimize:*
   (a) *given $P(s)$, $\phi_i(y)$ solves the individual i lottery problem using $v(y; P)$ for all $i \in I$*
   (b) *given $P(s)$ the demands $c(y)$ and $s(y)$ solve the consumer's second stage problem (1) for all y,* i.e. $c(y) = \bar{c}(y; P)$ and $s(y) = \bar{s}(y; P)$
2. *Markets clear:*

(a) *The lottery market clears so that the lottery density demands* $\phi_i(y)$ *generate the final distribution of income* $\Phi(y)$*:*

$$\int_{i \in I} \phi_i(y)\, d\mu(i) = \frac{\partial}{\partial y}\Phi(y)$$

(b) *The consumption market clears:*

$$\int_0^1 c(y) d\Phi(y) = \bar{\omega}^c = \int_{i \in I} \omega_i d\mu(i)$$

(c) *The status market clears with positive sorting:* $s(y) = \Phi(y)$.

We have stated the definition of a competitive lottery equilibrium in terms of the explicit status market. We sketch how to modify the definition above so that it is stated instead in terms of the implicit status market with 'social good' z.

With an implicit status market an individual i has endowments defined for the consumption and social goods. An equilibrium is then defined given these endowments as above with the obvious substitution of $P(s)$ for $\{p_z, R(z)\}$, $s(y)$ for $z(y)$, and the market clearing condition for status (c) must be replaced by market clearing and consistency of the 'social good' z.

Of course, given the equivalence result any competitive lottery equilibrium with explicit status markets is also a competitive lottery equilibrium with the implicit market arrangement. Indeed, as shown in the proof of Proposition 1 the total value of the endowment of z, $p_z\bar{\omega}^z$, is equal to the total value of the endowment of status, $\int_0^1 P(s)ds$. Thus, each endowment distribution for z in the implicit market arrangement corresponds to some endowment distribution for status in the explicit one, and vice-versa, in that they produce the same initial distribution of full income, y, and hence the same competitive lottery allocations. Consequently, we focus on the formal definition that uses the explicit status market arrangement.

4.2 The Main Result

We now construct a particular equilibrium which we denote by using the superscript $*$. We then show that $*$ has the property of being the equilibrium for a large set of initial conditions. The $*$-allocation is constructed by holding the marginal utility constant at a level that satisfies feasibility of consumption. That is,

$$u_c(c^*(s), s) = \lambda$$

and

$$\int_0^1 c^*(s) ds = \bar{\omega}^c.$$

The solution to these equations defines a strictly increasing consumption function $c^*(s)$ which can be used to define an implied price function using:

$$P^*(s) = \int_0^s \frac{u_s(c^*(\tilde{s}), \tilde{s})}{u_c(c^*(\tilde{s}), \tilde{s})} d\tilde{s} = \frac{1}{u_c(c^*(0), 0)} \int_0^s u_s(c^*(\tilde{s}), \tilde{s}) d\tilde{s}$$

where we are integrating (2) for the first expression and the second expression emphasizes the fact that $u_c(c^*(s), s)$ is constant.

Finally, with the price function $P^*(s)$ the implied full income is computed using:

$$y^*(s) = c^*(s) + P^*(s)$$

which is increasing in s, since both c^* and P^* are increasing functions. The associated distribution function, $\Phi^*(y)$, is simply the inverse of $y^*(s)$ since the distribution of s was normalized to be uniform on $[0, 1]$. Then $c(y)$ and $s(y)$ are simply $\Phi^*(y)$ and $c^*(\Phi^*(y))$, respectively.

For a large class of initial conditions this constructed allocation constitutes the final outcome of a competitive lottery equilibrium. That is, $\Phi^*(y)$ is the final distribution of full income for a large class of initial distributions of full income. The relevant initial conditions for which this result holds is best summarized in terms of the initial distribution of full income computed using the P^* price function:

$$F(y) \equiv \mu\left(\left\{i : \omega_i + \int_0^1 P^*(s) \, dH_i(s) \leq y\right\}\right).$$

Proposition 2. *If the initial distribution of full income F is such that Φ^* is a mean-preserving spread of F then $*$ is a competitive lottery equilibrium*

Proof. By construction the allocation satisfies market clearing of the consumption and status good. We first argue that optimality at the second stage is met. By the definition above,

$$P^{*\prime}(s) = \frac{u_s(c^*(s), s)}{u_c(c^*(s), s)}.$$

So that $c(y) = c^*(\Phi^*(y))$ and $s(y) = \Phi^*(y)$ satisfy the first order condition for optimality of the agent's problem with income y.

Imposing the first order condition for all y is enough to ensure optimality of an increasing $s(y)$ allocation given our assumption regarding the normality of s. To see this consider the indirect preferences over the price paid, p, and the status received, s, which depends on the level of income y: $U(p, s; y) \equiv u(y - p, s)$. When thinking about U income y is best thought of as a form of heterogeneity. Then normality of s is precisely the condition required for the single-crossing property that the marginal rate of substitution between status and the price paid:

$$\frac{U_s(p, s; y)}{U_p(p, s; y)} = \frac{u_s(y - P, s)}{u_c(y - P, s)},$$

is increasing in y. This reduces the question to whether or not $p^*(y) = y - c^*(y)$ and $s^*(y)$ are 'incentive compatible' in the sense that:

$$y \in \arg \max_{y' \in [y(0), y(1)]} U_s(y' - c^*(y'), s(y'); y).$$

That is, whether individuals making reports y' have the incentive to reveal their true type y. Given single-crossing, results from information economics (e.g. Mirrlees (1971)) imply that an allocation is incentive compatible if the first-order conditions for all agents are satisfied and the allocation is monotone. Of course, by construction this is the case for the $*$-allocation.

Turning to the first stage, the marginal utility of the indirect utility function,

$$\frac{\partial}{\partial y} v(y; P) = u_c(c^*(\Phi^*(y)), \Phi^*(y)) = \lambda,$$

is constant by the definition of $c^*(s)$. This implies that all agents are indifferent to taking any lottery with income prizes that lie in the support of Φ^*, i.e. the interval $[y^*(0), y^*(1)]$. It remains to be shown that there exists a set of lottery demands $\{\phi_i(y)\}$ that generate $\Phi^*(y)$. This is guaranteed by the mean-preserving spread condition.

Since y^* is a mean-preserving spread of y then for each individual $i \in I$ with income y there must exist a random variable ε_y such that

$$y^* = y + \varepsilon_y \text{ and } E[\varepsilon_y|y] = 0$$

This implies that there does exist a fair lottery that produces the distribution y^* from y with net-income prizes of the lottery demands are given by ε_y. The lotteries defined in this way are fair by construction since $E[\varepsilon|y] = 0$. ■

In a sense, this result implies that there is a minimum feasible level of equilibrium inequality. Any initial distribution with less inequality, in the sense of second-order stochastic dominance, will spread out from the use of lotteries. The minimum level of inequality results from the inevitable inequality of status, a direct consequence of the heterogeneous supply of status positions. Inequality of status then requires inequality of consumption because status affects the marginal utility of consumption and creates the incentive to engage in lotteries.

To gain insight into the need for lotteries suppose we ignored the possibilities of lotteries and consider the equilibrium in the special case where all individuals have the same income $\bar{y}$. Without lotteries, the equilibrium price function is simply a compensating differential, that is, it makes all agents are indifferent so that $u(\bar{y} - P(s), s)$ does not vary with s. Although utility is equalized across status choices, for income $\bar{y}$, there is a strong desire for engaging in lotteries over income.

To make this point we show that in such a compensating differential case utility as a function of income $v(y; P) = \max_{s \in [0,1]} u(y - P(s), s)$ is not globally concave because of a kink at $\bar{y}$. Indeed, by normality of status we must have that $s = 0$ and $s = 1$ are strictly optimal for $y < \bar{y}$ and $y > \bar{y}$, respectively. The indirect utility functions are thus $v(y; P) = u(y - P(0), 0)$ for $y < \bar{y}$ and $v(y; P) = u(y - P(1), 1)$ for $y > \bar{y}$. Thus, $v(y; P)$ is locally concave, in that $v''(y; P) < 0$, for both the $y < \bar{y}$ and $y > \bar{y}$ regions. However, at $\bar{y}$ we have a convex kink:

$$\lim_{y \uparrow \bar{y}} v'(y; P) = u_c(\bar{y} - P(0), 0) < u_c(\bar{y} - P(1), 1) = \lim_{y \downarrow \bar{y}} v'(y; P)$$

since both $u_{cs} > 0$ and $u_{cc} < 0$ and $P(s)$ is increasing.

Thus, the compensating differential equilibrium allocation that results without lotteries would generate a strong desire for lotteries. Indeed, the argument here shows why in equilibrium there can be no mass points in the final distribution of income (i.e. Φ cannot have jumps), the case where everyone has the same income $\bar{y}$ being a special case.

Note that both c^* and P^* are determined solely from preferences. Thus, for any given preference specification that satisfies our assumptions the proposition should be seen as identifying a non-trivial range of initial endowments for which $*$ is the final distribution of income, consumption and status. Of course, in general this range will depend on c^* and P^* and thus, on the assumed preferences, yet in all cases it is non-trivial in the sense that there are many endowment specifications which satisfy the proposition.

As a simple example consider the case where all agents have identical status endowments, so that $H_i(s) = s$, for all $i \in I$ and all $s \in [0,1]$, and where endowments of the consumption good ω_i are distributed in such a way that $c^*(s)$ is a mean preserving spread of ω_i. For such a case the conditions of the proposition are easily verified without consulting $P^*(s)$. The final distribution of income must accommodate the higher spread, relative to the endowments, in both consumption and status, implying that final income must be more spread out than initial income.

Two extreme cases may help illustrate the logic behind Proposition 2. Consider first the case where there is full equality in the initial income distribution – such a case will always satisfy the hypothesis of Proposition 2. In equilibrium all individuals take out the same lottery over income given by the density $\phi^*(y) = \Phi^{*\prime}(y)$ and this produces the final distribution of income Φ^* in a straightforward fashion. This lottery is feasible for individuals because it is simply a mean preserving spread of their own initial income $\bar{y}$. Individuals find it optimal to take this particular lottery because given the price function P^* their indirect utility function $v(y; P^*)$ is linear over the support of Φ^*, and is strictly concave outside of it.

In the second extreme case the initial distribution of income is already Φ^*. In this case individuals do not engage in lotteries and the initial and final distribution of income coincide. Individuals find it optimal not to engage in lotteries because $v(y; P^*)$ is weakly concave for all y.

Proposition 2 identifies these two extreme cases and also shows that in all the intermediate cases $*$ is also an equilibrium. Thus, a wide range of initial conditions share a common final outcome for the distribution of consumption, status and income. To be sure, even distributions outside the range identified by Proposition 2 may lead to equilibria requiring some individuals to take lotteries. In general, in such cases there will be more inequality than in the $*$ allocation we identify.

4.3 An Example with a Power Law

This section works out an example that is meant to be illustrative of the possibilities of our model for the distribution of income, consumption and status.[5] Suppose we have a Cobb-Douglas utility function $u(c, s) = c^\alpha s^\beta$ for $\alpha \in (0, 1)$ and $\beta > 0$. We continue to assume the normalization that s is uniformly distributed on $[0, 1]$.

Below we verify that in this case:

$$\begin{aligned} P^*(s) &= \frac{1-\alpha}{\alpha}\ \kappa\ s^\delta \bar{\omega}^c \\ c^*(s) &= \kappa\ s^\delta\ \bar{\omega}^c \\ F^*(y) &= \left(\frac{\alpha}{\bar{\omega}^c \kappa}\right)^\delta\ y^{\frac{1}{\delta}} \end{aligned}$$

where the constants are given by $\kappa = (1 - \alpha + \beta) / (1 - \alpha)$ and $\delta = \beta/(1-\alpha)$ and the support of y is $[0, \bar{\omega}^c \kappa]$. The density of $F(y)$ is therefore,

$$f^*(y) = \delta \left(\frac{\alpha}{\bar{\omega}^c \kappa}\right)^\delta\ y^{\frac{1}{\delta}-1}.$$

Note that for β large δ tends to zero and the density $f^*(y)$ behaves approximately as $1/y$.

We now verify these claims. Setting $u_c(c^*(s), s) = \lambda$ and solving for $c^*(s)$ one obtains:

$$c^*(s) = \left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} s^\delta$$

Using the resource constraint to solve for λ yields $c^*(s) = \bar{\omega}^c\ (1 + \delta)\ s^\delta$. The price schedule can be computed as follows:

$$\begin{aligned} P^*(s) &= \int_0^s \frac{u_s(c^*(\tilde{s}), \tilde{s})}{u_c(c^*(\tilde{s}), \tilde{s})} d\tilde{s} = \frac{\beta}{\alpha} \int_0^s \frac{c^*(\tilde{s})}{\tilde{s}} d\tilde{s} \\ &= \frac{\beta}{\alpha} \bar{\omega}^c\ (1 + \delta) \int_0^s \tilde{s}^{\delta-1} d\tilde{s} = \frac{\beta \bar{\omega}^c (1 + \delta)}{\alpha\delta}\ s^\delta \end{aligned}$$

In equilibrium $s(y) = F^*(y)$ and its inverse is given by

$$y^*(s) = P^*(s) + c^*(s) = \left(\frac{\beta}{\alpha\delta} + 1\right)\ \bar{\omega}^c (1 + \delta)\ s^\delta$$

inverting $y^*(s)$ and rearranging the constants then yields $F^*(y)$.

[5] We thank Fernando Alvarez for suggesting this example.

5 A Planner's Problem

We now show that the $*$-allocation identified by Proposition 2 corresponds to the allocation of the following social planner problem:

$$\max_{c^p(\cdot)} \int_0^1 u(c^p(s), s)ds$$

subject to,

$$\int_0^1 c^p(s)ds = \bar{\omega}^c.$$

The above problem can be interpreted in any of two extreme ways: (1) the planner is utilitarian (either in control of the distribution of the assignment of status across individuals or taking as given the assignments of s) that by choosing $c^p(s)$ chooses deterministic allocations across agents indexed by their assigned status s. In this case, agents are treated differently in equilibrium; (2) the planner maximizes the ex-ante expected-utility of a single lottery over consumption and social status assignments. In this case the planner treats agents equally ex-ante, assigning the same expected-utility level. Of course, intermediate interpretations are also valid where a lottery is held with some agents favored over others in their odds.

The first order condition, which is necessary and sufficient, for this problem is,

$$u_c(c^p(s), s) = \lambda^p,$$

for all $s \in [0, 1]$. This condition together with the resource constraint determine $c^p(s)$. Since these are exactly the same conditions that determined $c^*(s)$ we have the following result.

Proposition 3. *The competitive lottery equilibrium $*$ achieves the same ex-post allocation of consumption and status as the utilitarian planner, i.e.* $c^p(s) = c^*(s)$.

In the conventional problem without status, a utilitarian faced with diminishing marginal utility of incomes, and using lump-sum taxes and subsidies, would redistribute sufficient income from rich to poor to equalize everyone's marginal utility of income. If they have the same utility function, this implies equal consumptions, incomes, and utilities as well. The introduction of status, however, implies that consumption will generally not be equalized. In fact, with the complementarity assumption, consumption rises with income and so a planner widens differences in utility.

In the usual analysis without status and lotteries there is a major conflict between the income distribution proposed by a utilitarian planner and that generated by the market as long as individual do not have the same initial income. However, the conclusion is radically different if utility also depends on status, and if consumption and status are complements. Then the utilitarian and the market may arrive at the same ex-post distribution of consumption and status.

6 Some Empirical Implications

The crucial feature of our analysis (and of Friedman (1953), Robson (1992), and Rosen (1997)) is that the distribution of income is generated endogenously from behavior. In our analysis, behavior determines the equilibrium income distribution by matching the distribution of consumption to the given distribution of status. Most models of income distribution have stressed the effects of exogenous forcing processes, such as differences in market luck or genetic make-up, to explain the observed degree of income inequality and of mobility. These models do not incorporate decisions that offset or magnify these exogenous risks. Our analysis is just the opposite, for we ignore all forcing shocks, and consider only uncertainty created by markets in response to demands from individuals.

The most important result of the analysis is that the equilibrium income distribution would be the same for a range of initial functional distributions. This may help to explain Pareto's observation that income distributions tend to be relatively similar among countries and over time. Our analysis implies that if tastes and status distributions were the same, then, indeed, personal income distributions could be the same in different countries and over time, even though functional income distributions were different.

Figure 2 gives a graphical portrayal of this conclusion, where the horizontal axis plots a particular measure of the functional inequality of income – perhaps the standard deviation of the logarithm of income – and the vertical axis plots the equilibrium degree of income inequality. If the initial functional inequality is less than I then lotteries lead to the same equilibrium inequality E, with the degree of equilibrium inequality determined by the conditions in Proposition 2. When initial functional inequality exceeds I, lotteries are no

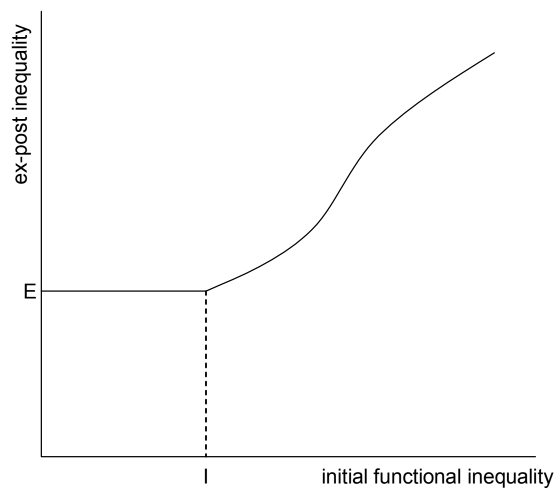

Figure 2: Initial and final equilibrium distribution of income

longer demanded by everyone, and the equilibrium income inequality would tend to rise along with functional inequality.

This analysis also implies that public policies which seek to reduce inequality by redistributing income may be rendered ineffective by compensating increases in self-generated inequality. Indeed, in our model, all redistributive policies that attempt to reduce initial income inequality below an initial equilibrium level of inequality at I will generally fail because lotteries and other gambles would restore the inequality towards E. However, policies that reduce the initial "functional" degree of inequality would reduce the inequality in ex-ante expected utilities even if they do not change the ex-post income and utility distributions.

The analysis also has interesting empirical implications about differences in measured distributions of income due to differences in the degree of equality of opportunity. In societies where status is not determined by family background, race, religion, etc., yet is up for "sale", the initial "endowment" of status would be much more equally distributed than in more highly structures societies, where family background and the like determine status endowments.

If there were a potential market for status in both types of societies, the equilibrium distributions of consumption and status would be the same in both types, if the conditions in Proposition 2 held in both cases. However, the distributions of measured income could be quite different. "Open" societies with more equal distributions of status endowments could have greater observed inequality in incomes because equilibrium incomes in these societies would include the cost of status. By contrast, since the equilibrium inequality in status would be more "endowed" and less purchased and sold in highly structured societies, the distribution of income there would mainly reflect only the distribution of consumption.

This may contribute to explaining the greater inequality in observed incomes in the United States than in Europe. The U.S. has generally had a more open and looser social structure than European nations. Therefore, the cost of buying status would be included in the United States' distribution of income, whereas in Europe the equilibrium distribution of status would be much more similar to the distribution of endowed status. As a result, European incomes would be largely measuring consumption alone, while American incomes would measure both consumption and status. Since in equilibrium, consumption and income increase and decrease together, American incomes are stretched out compared to incomes that mainly only measure consumption.

Higher status in open societies is mainly acquired by the winners of lotteries and other risky activities, while it is mainly endowed in more rigid societies. This would be consistent with the greater appeal of entrepreneurial and other risky activities in more open countries like the United States than in more rigid nations.

Although actual lotteries are popular and are highly profitable to the usual government monopolies, only lower income families typically spend more than a small fraction of their incomes on lottery tickets. Some persons have concluded from the unimportance of lotteries that most persons are risk averse, and that they are reluctant to gamble more than a small fraction of their wealth. However, lotteries may be unimportant to many groups not because they are risk averse, but because they have more efficient ways to gamble.

Suppose that higher income persons can gamble through occupational choices and entrepreneurial activities. Then lotteries might be of little value to them because they have superior ways to gamble through utilizing the more productive risks in an economy. Even an actuarially fair lottery has only a zero expected return, and most government lotteries are far from "fair" since they impose a heavy tax on lottery tickets. By contrast, the returns to various risky alternatives, such as entrepreneurial activities, usually yields positive expected excess returns.

Therefore, a desire to gamble may be more productively satisfied through the positive-sum gambles provided by human, physical, and financial capital investments than through negative-sum or even zero-sum lotteries (see the discussion of entrepreneurial activities and lotteries in Brenner, 1983). This may explain why start-ups and other entrepreneurial efforts, attempts to discover new goods, better production processes, and medical treatments, and various other risky activities are much more common and less well rewarded than would be expected from the usual assumptions of risk aversion and diminishing marginal utility of income.

7 Income Rank

Instead of assuming a market for status, the small economic literature on status usually assumes that status is automatically conveyed by rank in the distribution of income or of other "position" goods. This is the approach taken by Frank (1999), and by Robson (1992) and others when interpreting Friedman (1953) and Friedman and Savage (1948).

Our main result, summarized in Proposition 2, carries over here to the case where status is not bought, but is instead automatically related to income rank. In other words, the assumption of a market in status is not at all necessary to produce a unique equilibrium distribution of income when the initial distribution is sufficiently compact. A proof of such a result parallels closely a proof used by Robson (1992) to prove existence of what he calls a 'stable welfare distribution'.

The income-rank lottery model is described as follows. Consumption equals income and status assigned directly by rank in the income distribution (ignoring ties for simplicity), i.e. $s = \Phi(y)$. Thus, an individual with final income y obtains ex-post utility $v^r(y;\Phi) = u(y, \Phi(y))$ if Φ is the final distribution of income in the population. Individual $i \in I$ has initial income given by his consumption endowment ω^i and can engage in fair lotteries to maximize expected utility as in (4)-(6) but with $v^r(y;\Phi)$ replacing $v(y;P)$.

Given an initial distribution of income, $F(y)$, a rank-lottery equilibrium is a final distribution of income, $\Phi(y)$, such that: (i) individuals, taking Φ as given, optimally choose their lottery over final income; (ii) the individual lotteries generate the final distribution of income Φ from the initial one F.

As before we characterize a particular allocation that holds the marginal utility of income constant and denote this allocation by the superscript #. We then show that this allocation is an equilibrium for a wide range of initial distributions of income.

Differentiating $v^r(y;\Phi)$ within the support of Φ one obtains,

$$v_y^r(y;\Phi) = u_c(y, \Phi(y)) + \Phi'(y)u_s(y, \Phi(y)). \tag{7}$$

Outside the support we have that the marginal utility of income is equal to the partial u_c. That is, letting $y(0)$ and $y(1)$ denote the lowest and upper bound of the support, respectively, we have $v^r(y;\Phi) = u_c(y, 0)$ for $y < y(0)$ and $v^r(y;\Phi) = u_c(y, 1)$ for $y > y(1)$.

Note that in order to avoid a convex kink in the indirect utility function we require either $y(0) = 0$ or $\Phi'(y(0)) = 0$. If Φ does not satisfy these conditions then it cannot be can equilibrium because individuals would exploit the resulting non-convexity with lotteries and $y(0)$ would not be the lower bound.

Given this we proceed as follows constructing the allocation. Given $\lambda > 0$, define $y^{\#}(0;\lambda)$ to be highest value of y such that $u_c(y, 0) \leq \lambda$. Optimal lotteries imply as before that the indirect utility function lies on a straight line over any interval with positive density:

$$v^r(y;\Phi) = u(y, \Phi^{\#}(y;\lambda)) = \lambda(y - y^{\#}(0;\lambda)) + u(y^{\#}(0;\lambda), 0).$$

Given λ this equation can be solved for $\Phi^{\#}(y;\lambda)$. Finally, λ and $\Phi^{\#}(y)$ are determined so that total income equals the average endowment $\bar{\omega}^c$.

By the same logic used in the case with status markets, if the initial distribution of income, $F(y)$ is such that $\Phi^{\#}(y)$ is a mean-preserving spread of it then the ex-post distribution $\Phi^{\#}(y)$ is an equilibrium. This result is summarized in the following proposition.

Proposition 4. *If the distribution of income given by $\Phi^{\#}(y)$ is a mean-preserving spread of the initial distribution of income $F(y)$ then there exists an income rank lottery equilibrium with final distribution of income given by $\Phi^{\#}(y)$.*

Although the result is similar to Proposition 2, lotteries induced by status determined by income rank produces various real externalities, whereas lotteries induced by a market for status are Pareto optimal. A person who gambles to raise his rank ipso facto lowers the ranks of others when he wins, and raises the ranks of others when he loses. This imposes real, not simply pecuniary, positive and negative externalities on others.

Therefore, it no longer follows that the equilibrium inequality is the same as the inequality produced by a social planner. However, the observation that lotteries produce both positive and negative real externalities when status depends on rank suggests that the market's equilibrium degree of income inequality can be smaller or greater than the income inequality preferred by a planner, although the ordering generally depends on the measure of inequality.

8 Conclusions

This paper assumes, along with most commentators on social arrangements, that the desire for status is a powerful motive in any society where members interact with each other. It also assumes that status and consumption generally are strong complements in the sense that greater status raises the marginal utility of other consumption.

Several significant results are derived from these two rather simple basic assumptions, and from several auxiliary assumptions, especially a fixed distribution of status and the existence of fair lotteries. The most extraordinary result is that if the initial functional distribution of income is sufficiently compact, then the equilibrium distribution of income, and the equilibrium covariance of consumption and status, are the same for all initial income distributions within the "compact" range. By sufficiently compact is meant that the equilibrium distribution of income is a mean-preserving spread of the initial distribution.

This principal result is proven when there are complete markets in status, with higher and lower status levels sold at an equilibrium set of hedonic prices. The paper shows, however, that the result also follows when status is only acquired indirectly through the purchase of status goods.

Moreover, our principal result on an equilibrium distribution of income does not require that status is sold either directly or indirectly. Although sociologists have identified many determinants of status, most economic analyses relate status to relative position, as in rank in the distribution of income. We show that the principal result holds when status is automatically related to income rank. Then all initial distributions within a range have the same equilibrium distribution of income.

When status is either directly or indirectly bought and sold, the equilibrium distributions of income and status would be exactly the same as that produced by a utilitarian social planner who can fully allocate consumption and status. However, these equilibrium are not generally identical when status depends on ranks because changes in a person's rank, say through winning or losing lotteries, imposes positive and negative externalities on others by lowering or raising their ranks. In the rank case the equilibrium distribution of income produced by lotteries differs from that desired by a planner.

With a few prominent exceptions, economists have not assumed that behavior is important in directly "choosing" the observed distribution of income. Building on some of these exceptions, our paper shows that the assumption of risk-taking and lotteries to acquire higher status and higher incomes leads to predictions about the distribution of income that might explain both differences and similarities in income distributions among societies, and over time within the same society. This suggests that choice and risk-taking are crucial ingredients of the observed distributions of income and status.

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