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The equilibrium form of a rotating earth with an elastic shell

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SUMMARY

The equilibrium form of the Earth is generally computed using a hydrostatic theory that assumes a rotating, inviscid planet. We compute the perturbation to this equilibrium form due to the presence of a thin elastic lithospheric shell using viscoelastic Love number theory. The thin shell acts to reduce the flattening of the equilibrium form relative to the value obtained from the traditional hydrostatic calculation. Our results indicate that current estimates of the excess non-hydrostatic flattening of the Earth, defined as the discrepancy between the observed and hydrostatic forms, may therefore be underestimating the actual departure of the observed form from its equilibrium state. This conclusion may be important for viscous flow models of mantle convection, which are commonly constrained to fit the non-hydrostatic flattening. For completeness, we also adopt the Love number formulation to estimate the excess flattening associated with the gradual slowing of the Earth's rotation. Our predictions of the fossil rotational bulge confirm the widespread view that this effect is small for reasonable mantle viscosity profiles.

Key words: Earth's rotation, figure of the Earth, geoid.

1 INTRODUCTION

Determining the equilibrium shape of a rotating earth is a classic problem in geophysics. The rotationally-induced ellipticity is generally predicted using a theory that treats the planet as an inviscid fluid (e.g. de Sitter 1924; Kopal 1960; Jeffreys 1963; Nakiboglu 1979, 1982). The 'hydrostatic form' derived in this manner neglects all long-term elastic lithospheric strength and assumes that any lag between slow changes in Earth rotation and the planetary shape (i.e. the fossil rotational bulge) is negligible.

The effective elastic thickness of the lithosphere is laterally heterogeneous and dependent upon the timescale of the applied forcing. Estimates of short-term elastic lithospheric thickness obtained from seismic studies are larger than estimates of the long-term elastic thickness determined from flexure and gravity anomaly measurements in regions of crustal loading (e.g. Watts & Daly 1981). The latter estimates fall within the range 10–40 km in oceanic settings and they can be greater than 100 km within continental interiors. Due to the presence of these lateral variations, and in particular the network of plate boundaries, defining an effective global value for the elastic lithospheric thickness may have limited utility; however, it is clear that the lithosphere does retain non-zero elastic strength over long timescales.

The perturbation to the equilibrium form associated with the presence of an elastic (lithospheric) shell has received little attention. In contrast, the present-day fossil rotational bulge arising from a slowing rotation rate on a viscous earth has played a prominent role in the history of mantle viscosity estimates. Early indications that the observed flattening of the Earth was significantly greater than the predicted hydrostatic form led to suggestions of a large fossil rotational signature and very high deep mantle viscosity (e.g. MacDonald 1965; McKenzie 1966); however, subsequent analyses by Goldreich & Toomre (1969) undermined this argument (for a discussion and analysis see Ricard *et al.* 1993).

The discrepancy between the satellite-derived flattening of the Earth and the form predicted from hydrostatic theory, the so-called excess non-hydrostatic flattening, is widely believed to reflect the dynamic effect of mantle convective flow (e.g. Forte *et al.* 1993, 1995; Panasyuk & Hager 2000). Indeed, predictions of long-wavelength geophysical observables derived from viscous flow theory routinely adopt the non-hydrostatic geoid coefficient at degree two and order zero, which is to first order proportional to the flattening, as a constraint on Earth structure. Furthermore, the prediction of other observables linked to convective flow, for example long-term true polar wander and Earth orbital variations, are commonly constrained to fit the present-day degree-two, order-zero, non-hydrostatic form (Steinberger & O'Connell 1997; Forte & Mitrovica 1997).

In the presence of a lithosphere with non-zero 'global' elastic strength, the equilibrium form of the planet will differ from the hydrostatic form computed assuming a purely inviscid body. The presence of a non-zero fossil rotational bulge would furthermore imply that the equilibrium form (whether derived from an earth model with or without an elastic lithospheric shell) has not been

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attained. If either effect is non-negligible, then the non-hydrostatic form may not be an appropriate constraint on convection simulations (of true polar wander, long-wavelength gravity, etc.).

In this research note we compute perturbations to the hydrostatic form that arise from a global elastic lithosphere of non-zero thickness, and demonstrate that these perturbations can be significant. For completeness, we also revisit the issue of a fossil rotational bulge using recently published constraints on the geological evolution of the Earth's length-of-day.

2 METHODS AND RESULTS

Following Nakiboglu (1979, 1982), the radius of the geoid, r, for an inviscid, rotating earth model may be described by a Legendre polynomial expansion of the form

$$r(\theta) = r_{\mathrm{avg}}\left(1 + \sum_{n=0}^{\infty} f_n P_n(\cos\theta)\right), \tag{1}$$

where r_{avg} is the mean radius of the geoid, θ is co-latitude and f_n the coefficients associated with the Legendre polynomials, P_n, which are normalized such that

$$\int_0^{\pi} P_n(\cos\theta) P_{n'}(\cos\theta)\sin\theta\, d\theta = \frac{2}{2n+1}\delta_{nn'}. \tag{2}$$

The symbol $\delta_{nn'}$ represents the Kronecker-delta function. In the adopted normalization, $P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$.

The flattening of the geoid is generally quantified not by the Legendre polynomial coefficients of eq. (1) but rather by its ellipticity, ϵ, defined as

$$\epsilon \equiv \frac{r_{\mathrm{eq}} - r_{\mathrm{pole}}}{r_{\mathrm{eq}}}, \tag{3}$$

where r_{eq} and r_{pole} are the radius of the geoid at the equator and the pole, respectively. Nakiboglu (1979, 1982) has shown that the ellipticity can be related to the Legendre polynomial coefficients of eq. (1) by

$$\epsilon = -\frac{3}{2}f_2 - \frac{5}{8}f_4 - \frac{3}{4}f_2{}^2. \tag{4}$$

The value of ϵ is $\sim \frac{1}{300}$, f_2 is of order ϵ and f_4 is of order ϵ^2 (Nakiboglu 1979, 1982). We are concerned with perturbations to the ellipticity, $\Delta\epsilon$, associated with the presence of an elastic lithospheric shell and fossil rotational effects. In these cases, we need only consider the leading order term on the right-hand-side of eq. (4), and we can thus write

$$\Delta\epsilon = -\frac{3}{2}\Delta f_2. \tag{5}$$

The perturbation to the geoid induced by the centrifugal potential can be computed using viscoelastic tidal Love number theory (Peltier 1974; Milne & Mitrovica 1998). The viscoelastic tidal (or tidal-effective) k Love number at degree 2 can be expressed in the form (Peltier 1974)

$$k_2(t) = k_2^{\mathrm{E}}\delta(t) + \sum_{j=1}^{J} r_{2,j} e^{-s_{2,j}t}, \tag{6}$$

where the first term on the right represents the elastic response (hence the superscript E and the delta-function time dependence) and the second term is a non-elastic response that is comprised of a sum of J modes of pure exponential decay with amplitude, $r_{2,j}$, and inverse decay time, $s_{2,j}$.

If we adopt an expansion of the form (1) to describe the centrifugal potential associated with rotation, then the only non-zero coefficients are at degrees zero and two (e.g. Lambeck 1980). Since we are interested here in the flattening of the geoid we require only an expression for the latter. For the normalization (2) the time varying rotational potential at degree 2, $\Lambda_2(t)$, is given by (e.g. Lambeck 1980)

$$\Lambda_2(t) = -\frac{1}{3}a^2\Omega^2(t), \tag{7}$$

where a is the mean radius of the Earth and $\Omega(t)$ is the rotation rate as a function of time. The degree two perturbation in the geoid arising from this rotation history can be found by convolving the centrifugal potential with a Green's function for the potential perturbation constructed using the k Love number defined in (6) (e.g. Milne & Mitrovica 1998). In particular, we can write

$$f_2(t) = \frac{1}{ag}\int_{-\infty}^{t} \Lambda_2(t')[\delta(t-t') + k_2(t-t')]\,\mathrm{d}t'. \tag{8}$$

The term $\delta(t-t')$ in this equation incorporates the direct gravitational effect of the centrifugal potential load, while the term involving the k_2 Love number provides the indirect effect associated with planetary mass redistributions. g is the mean acceleration due to gravity at the Earth's surface.

The particular perturbations that we are considering can be derived from the general eq. (8). The solution of this equation can be mapped into a perturbation in ellipticity using eq. (5).

2.1 Influence of the lithosphere

We begin with the case of a perturbation to the equilibrium form of the planet arising from the presence of an elastic lithospheric shell in the absence of fossil rotational effects. Let us assume that the present rate of rotation was imposed instantaneously at some time, $t^\star$, in the distant past. In this case, the centrifugal potential in eq. (7) can be written as

$$\Lambda_2(t) = -\frac{1}{3}a^2\Omega^2 H(t - t^\star), \tag{9}$$

where H is the Heaviside step function. Substitution of this expression into eq. (8) yields (using eq. 6)

$$f_2(t) = \frac{\Lambda_2(t)}{ag}\left\{1 + k_2^{\mathrm{E}} + \sum_{j=1}^{J}\frac{r_{2,j}}{s_{2,j}}\left[1 - e^{-s_{2,j}(t-t^\star)}\right]\right\}. \tag{10}$$

For $t \gg t^\star$, this expression becomes

$$f_2 = -\frac{1}{3}\frac{a\Omega^2}{g}\left(1 + k_2^f\right), \tag{11}$$

where k_2^f is the degree two fluid Love number defined as

$$k_2^f \equiv k_2^{\mathrm{E}} + \sum_{j=1}^{J}\frac{r_{2,j}}{s_{2,j}}. \tag{12}$$

The fluid Love number incorporates the density and elastic structure of the earth model. Specifically, this number is a function of the thickness of the adopted elastic lithospheric shell. If we denote this thickness as LT, then the perturbation to the coefficient f_2 associated with the presence of an elastic outer shell is simply

$$\begin{aligned}\Delta f_2 &= f_2(LT) - f_2(LT=0)\\ &= -\frac{1}{3}\frac{a\Omega^2}{g}\left[k_2^f(LT) - k_2^f(LT=0)\right].\end{aligned} \tag{13}$$

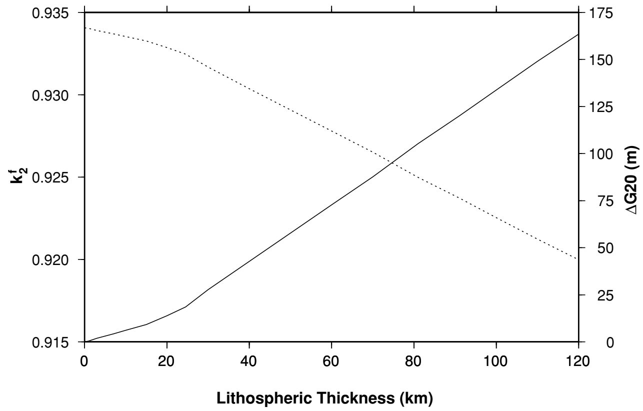

Figure 1 Dotted line and left ordinate scale: predictions of the *k* fluid Love number (see eq. 12) at degree two as a function of the elastic lithospheric thickness of the earth model. Solid line and right ordinate scale: prediction of the perturbation to the degree two zonal geoid coefficient of the Earth's equilibrium rotational form (see eq. 15) as a function of the thickness of the elastic lithosphere. All calculations adopt the radial density structure given by PREM (Dziewonski & Anderson 1981).

Substitution of this expression into eq. (5) gives the perturbation in the ellipticity due to the elastic lithosphere

$$\Delta\epsilon = \frac{1}{2}\frac{a\Omega^2}{g}\left[k_2^f(LT) - k_2^f(LT = 0)\right]. \quad (14)$$

We have computed the fluid Love number k_2^f as a function of the elastic lithospheric thickness, LT, for earth models constrained to have the density structure of PREM (Dziewonski & Anderson 1981). The results are shown in Fig. 1. As the lithospheric thickness is increased, the fluid Love number decreases, and thus, according to eq. (14), the ellipticity of the equilibrium form of the planet is reduced. As one would expect, the presence of an elastic lithospheric shell acts to limit the rotationally-induced equilibrium flattening.

Viscous flow models of mantle convection adopt a spherical harmonic, rather than a Legendre polynomial, expansion of the geoid. In this case, the degree two zonal harmonic of the non-hydrostatic geoid (that is, the satellite-derived value minus the value predicted using Nakiboglu's (1982) hydrostatic theory) is ∼−99 m. (The normalization is such that the negative sign refers to an excess ellipticity.) If we adopt the same expansion and spherical harmonic normalization, then the degree two zonal harmonic of the geoid perturbation due to an elastic lithospheric shell, which we will denote by $\Delta G_{2,0}$, is related to Δf_2 by

$$\Delta G_{2,0} = \sqrt{\frac{4\pi}{5}}a\Delta f_2$$
$$= -\frac{1}{3}\sqrt{\frac{4\pi}{5}}\frac{a^2\Omega^2}{g}\left[k_2^f(LT) - k_2^f(LT = 0)\right]. \quad (15)$$

Using eq. (15) we have computed $\Delta G_{2,0}$ as a function of lithospheric thickness (or fluid Love number). The results are also shown in Fig. 1 (note the right ordinate scale).

In the Introduction we quoted a range of 10–40 km for the long-term elastic thickness of the oceanic lithosphere and significantly higher values for the continental lithosphere. Let us consider the case of a 40 km globally averaged long-term elastic lithospheric thickness. From Fig. 1, $\Delta G_{2,0}$ would then be 44 m, where the sign of the perturbation represents a reduction of the equilibrium ellipticity relative to the value obtained using hydrostatic (inviscid earth) theory. That is, the traditional hydrostatic calculation would be overestimating the equilibrium flattening of the Earth and be significantly underestimating (by roughly 50 per cent in this case) the discrepancy between the observed and equilibrium forms. This latter underestimation is clearly relevant to models of convective flow that are constrained to fit the non-hydrostatic form.

As we have discussed, it is difficult to estimate a characteristic global elastic lithospheric thickness, particularly given the global network of plate boundaries. However, the results in Fig. 1 indicate that the presence of even a thin elastic outer shell has a non-negligible impact on the equilibrium form of the planet.

2.2 Fossil bulge

As a consequence of tidal dissipation, the Earth's rotation rate has been gradually slowing through time. Since the sub-lithospheric mantle behaves as a viscous fluid, there will be a lag between its response and any change in the centrifugal potential, and therefore the present flattening of the geoid will be greater than expected from equilibrium theory. This excess flattening will depend on the viscous structure of the planet and on the time history of the Earth's rotation rate. Previous estimates of the fossil bulge amplitude have found that it is small compared to the net non-hydrostatic flattening of the geoid (e.g. Ricard *et al.* 1993); for completeness we will revisit this issue. Our analysis of the present-day fossil rotational bulge can proceed from eq. (8), with the k_2 viscoelastic Love number defined by eq. (6), once the time variation of the degree two zonal harmonic of the centrifugal potential, $\Lambda_2(t)$, is specified. The latter is related to variations in the rotation rate through eq. (7).

Fig. 2 plots constraints on palaeorotation rates at 620 Ma, 900 Ma and 2500 Ma derived by Williams (1997) on the basis of his analysis of tidal rythmites and banded iron formations. These rotation rates refer to 401 ± 7, 420 (no uncertainty specified) and 466 ± 16 sidereal days per year, respectively. A final constraint is, of course, the present-day rotation rate. Williams (1997) noted that the relatively rapid changes in rotation rate evident over the last ∼1000 Myr (or high tidal dissipation rates) did not extend into the earlier Precambrian, and this was important for avoiding well-known problems involving the evolution of the Earth–Moon distance. Although detailed constraints on the long-term evolution of length-of-day are clearly lacking, the data in Fig. 2 suffice for a simple numerical estimate of the fossil bulge amplitude.

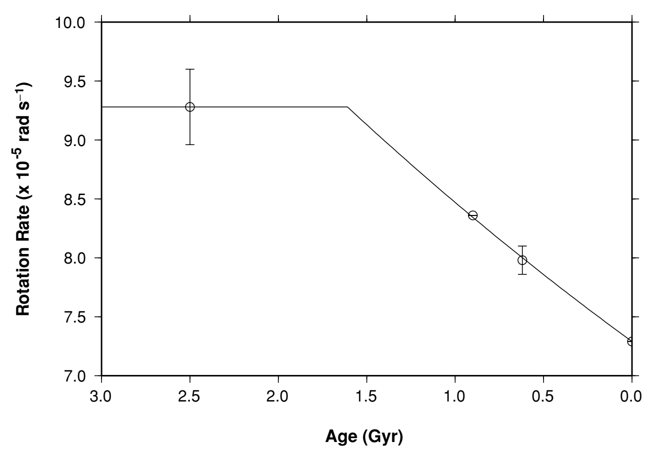

Figure 2 Observational constraints on the palaeorotation rate of the Earth derived by Williams (1997) together with the present-day rotation rate. The solid line is the simple model of the time history of the rotation rate adopted in our numerical predictions of the fossil rotational bulge (see eq. 16).

The crude time history shown in Fig. 2 (solid line) was constructed by fitting an exponential through the youngest three data points (including the present-day value) and extending this exponential back in time until the rotation rate of the oldest data point ($\sim 9.3 \times 10^{-5}$ rad^{-1}) is first achieved. For all times prior to this date the rotation rate is held fixed. The model rotation history can thus be written

$$\Omega(t) = \Omega\left[H(t - t^A) + \left(e^{-\lambda(t-t^\star)} - 1\right)H(t - t^\star)\right], \quad (16)$$

where t^A is the time of Earth formation, $t^\star$ is the onset time of the exponential decay in the rotation rate, and Ω is the rotation rate prior to $t^\star$. In constructing the solid line in Fig. 2 we used $\lambda = 1.5 \times 10^{-10}$ yr^{-1} and $t^\star = -1.61 \times 10^9$ yr. Using eq. (16) in (7) yields

$$\Lambda_2(t) = -\frac{a^2\Omega^2}{3}\left[H(t - t^A) + \left(e^{-2\lambda(t-t^\star)} - 1\right)H(t - t^\star)\right]. \quad (17)$$

Substitution of this expression into eq. (8), and making use of the relation $G_{2,0} = \sqrt{4\pi/5}af_2$, gives

$$G_{2,0}(t) = -\frac{1}{3}\sqrt{\frac{4\pi}{5}}\frac{a^2\Omega^2}{g} \times \left\{\left(1 + k_2^{\mathrm{E}}\right)e^{-2\lambda(t-t^\star)} + \sum_{j=1}^{J}\frac{r_{2,j}}{s_{2,j}}\left[e^{-s_{2,j}(t-t^\star)}\right] + \sum_{j=1}^{J}\frac{r_{2,j}}{s_{2,j} - 2\lambda}\left[e^{-2\lambda(t-t^\star)} - e^{-s_{2,j}(t-t^\star)}\right]\right\}, \quad (18)$$

where we have assumed that at $t = t^\star$ the planet is at the equilibrium form associated with the original rotation rate Ω.

The fossil bulge will be the difference between the value of $G_{2,0}$ obtained using eq. (18) and the prediction based on an equivalent earth model with the exception that the mantle (not including any elastic lithosphere) is assumed to be inviscid. This 'inviscid' response, which we denote by $G_{2,0}^{\mathrm{inv}}$, can be found by assuming that $2\lambda \ll s_{2,j}$ for all j in eq. (18). This gives

$$G_{2,0}^{\mathrm{inv}}(t) = -\frac{1}{3}\sqrt{\frac{4\pi}{5}}\frac{a^2\Omega^2}{g}\left(1 + k_2^f\right)e^{-2\lambda(t-t^\star)}. \quad (19)$$

The geoid signal associated with the fossil rotational bulge is then

$$\Delta G_{2,0}^{\mathrm{FB}}(t) = G_{2,0}(t) - G_{2,0}^{\mathrm{inv}}(t). \quad (20)$$

We computed the viscoelastic tidal k_2 Love number using a model characterized by an upper-mantle viscosity of 10^{21} Pa s and a lower mantle viscosity of 10^{23} Pa s. (We will assume that the viscosity structure is fixed over the time interval of the calculation.) These values represent upper bounds on the mean viscosity within either of these two regions obtained in recent analyses of post-glacial rebound and convection observables (e.g. Mitrovica & Forte 1997). With the rotation history shown in Fig. 2 we computed a value of $\Delta G_{2,0}^{\mathrm{FB}}(t) = -2.5$ m. This value is small compared to the total non-hydrostatic signal. It is also smaller than the amplitude of the perturbation in the equilibrium form due to an elastic lithospheric shell for thicknesses larger than $\sim$5 km.

The amplitude of the fossil bulge depends on the ongoing rate of tidal deceleration. The palaeorotation history shown in Fig. 2 underestimates by approximately 2/3 the present day rate of secular deceleration (e.g. Christodoulidis *et al.* 1988). We therefore also considerd a tidal deceleration history of the same form as that shown in Fig. 2 in which the present day deceleration rate was scaled upwards by a factor of 1.5. This scaling also brings the model's rate of deceleration over the past 500 Myr into general agreement with that determined from studies of fossil palaeorotation indicators (e.g. Lambeck 1978). Adoption of this revised time history leads to a fossil bulge amplitude of $\Delta G_{2,0}^{\mathrm{FB}}(t) = -5.1$ m, which is $\sim$5 per cent of the present-day non-hydrostatic flattening of the geoid.

3 FINAL REMARKS

The calculations described here indicate that the presence of an elastic lithosphere impacts the equilibrium rotational form and may significantly affect estimates of the departure of the Earth's observed flattening from this equilibrium form. Viscous flow calculations which adopt the non-hydrostatic form as a measure of this departure may therefore be underestimating the dynamic effects of convective flow on the ellipticity. Our calculations also confirm the long-standing view that the departure from the equilibrium form due to the fossil rotational bulge is relatively small (e.g. Ricard *et al.* 1993).

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