Mon. Not. R. Astron. Soc. **313**, 671–677 (2000)

The evolution of a rapidly accreting helium white dwarf to become a low-luminosity helium star

Hideyuki Saio[1] and C. Simon Jeffery[2★]

[1]*Astronomical Institute, School of Science, Tohoku University, Sendai 980, Japan*
[2]*Armagh Observatory, College Hill, Armagh BT61 9DG, Northern Ireland*

Accepted 1999 November 5. Received 1999 November 3; in original form 1999 April 16

ABSTRACT

We have examined the evolution of merged low-mass double white dwarfs which become low-luminosity (or high-gravity) extreme helium stars. We have approximated the merging process by the rapid accretion of matter, consisting mostly of helium, on to a helium white dwarf. After a certain mass is accumulated, a helium shell flash occurs, the radius and luminosity increase and the star becomes a yellow giant. Mass accretion is stopped artificially when the total mass reaches a pre-determined value. As the helium-burning shell moves inwards with repeating shell flashes, the effective temperature gradually increases as the star evolves towards the helium main sequence. When the mass interior to the helium-burning shell is approximately $0.25\,M_{\odot}$, the star enters a regime where it is pulsationally unstable. We have obtained radial pulsation periods for these models.

These models have properties very similar to those of the pulsating helium star V652 Her. We have compared the rate of period change of the theoretical models with that observed in V652 Her, as well as with its position on the Hertzsprung–Russell diagram. We conclude that the merger between two helium white dwarfs can produce a star with properties remarkably similar to those observed in at least one extreme helium star, and is a viable model for their evolutionary origin. Such helium stars will evolve to become hot subdwarfs close to the helium main sequence. We also discuss the number of low-luminosity helium stars in the Galaxy expected for our evolution scenario.

Key words: stars: chemically peculiar – stars: evolution – stars: individual: V652 Her – stars: oscillations.

1 INTRODUCTION

The major question concerning the evolutionary origin of extreme helium stars is whether they are the products of single star or binary star evolution. Extreme helium stars (EHes) are rare B- and A-type giant stars with extremely low surface abundances of hydrogen (Jeffery 1996). In most cases they are also characterized by enhancements of CNO-processed, 3α and α-capture products, and the majority have $\log L/M > 4$ (as indicated by their surface gravities). A few have significantly lower *L*/*M* ratios and do not show 3α and α-capture products in their atmospheres (e.g. V652 Her: Jeffery, Hill & Heber 1999).

The task of stellar evolution theory is to explain how these stars acquired their unusual characteristics. The task has been difficult from the outset because, in the normal evolution of single stars from the main sequence to the white dwarf phase, it seemed impossible to remove the hydrogen-rich surface. Two principal hypotheses emerged during the 1980s.

The 'merged binary white dwarf model' (MBWD: Webbink 1984; Iben & Tutukov 1984) considered the accretion of a white dwarf (WD) secondary on to a WD primary, resulting in the ignition of a helium shell in the accreted envelope which forces the star to expand to become a cool giant. Subsequent evolution would follow the canonical post-asymptotic giant branch (post-AGB) contraction to the white dwarf track, in the case of a CO+He WD merger, or contraction to the helium main sequence – possibly giving a subdwarf B star – in the case of He+He WD merger (Iben 1990).

The 'late thermal pulse' model (LTP: Iben et al. 1983) considered what would happen when the helium layer remaining near the surface of a star at the end of AGB evolution was of such a mass that a final thermal pulse would occur after the star had become a WD, also forcing the star to expand rapidly. Again, subsequent evolution would resemble the canonical post-AGB sequence.

The LTP model has been studied extensively in recent years (Iben & MacDonald 1995; Blöcker & Schönberner 1997) and used to discuss the origins of various hydrogen-deficient stars. Part of

★ E-mail: csj@star.arm.ac.uk

the success of LTP models has been due to the very large degree of freedom allowed in reproducing a wide range of surface compositions, from s-process elements in R CrB stars (Bond, Luck & Newman 1979; Lambert & Rao 1994) to very high C and O concentrations in PG 1159 and [WC] stars (Werner, Heber & Hunger 1991; Leuenhagen, Hamann & Jeffery 1996). The LTP model has also been supported by the rapid evolution from WD to cool giant observed in V605 Aql (Pollacco et al. 1992), FG Sge (Herbig & Boyarchuk 1968) and V4334 Sgr (Duerbeck & Benetti 1996), all of which are hydrogen-deficient to some extent. However, another part of the success of the LTP model may have been due to the absence of detailed numerical MBWD models. In particular, the LTP model cannot account for all EHes, especially the low-luminosity EHe V652 Her.

Methods to study the consequences of MBWD systems have been developed by considering evolutionary models for accreting white dwarfs (Saio & Nomoto 1998). In these models, once sufficient accretion has occurred, nuclear reactions are ignited in a shell and propagate into the degenerate core via a series of mild flashes. Whilst the general properties of such models for both CO and He WDs have been considered (Iben 1990; Saio & Nomoto 1998), it is necessary to examine their detailed properties within the context of observational constraints.

It is the purpose of this paper to present evolutionary models for accreting helium WDs with final masses in the range 0.5–0.8 $\mathrm{M}_\odot$. These models, including their pulsation properties, are compared with observational data for low-luminosity EHes. In doing so, it is demonstrated that the MBWD model for the origin of EHes is viable in at least one highly constrained case.

2 V652 HER

V652 Her is an exceptional helium star. It first attracted attention because of the discovery of radial pulsations with a period of 0.108 d (Hill et al. 1981). These pulsations have provided remarkably precise dimensions (M, L, R, T_{eff}) for V652 Her which tightly constrain evolutionary models.

The effective temperature $\log T_{\mathrm{eff}} = 4.370 \pm 0.025$ (Lynas-Gray et al. 1984) adopted here was based on ultraviolet spectrophotometry. Baade's method was used by the same authors to determine the radius and hence the luminosity $\log L/\mathrm{L}_\odot = 3.03 \pm 0.12$. We note that Jeffery et al. (1999) give a slightly higher effective temperature for V652 Her. With a spectroscopic measurement of the surface gravity, the stellar mass has then been obtained directly, as $0.7^{+0.4}_{-0.3}\,\mathrm{M}_\odot$ by Lynas-Gray et al. (1984) and $0.69^{+0.15}_{-0.12}\,\mathrm{M}_\odot$ by Jeffery et al. (1999).

The pulsations have provided yet more constraints on evolutionary models, since the period was found to be decreasing at a rate $\mathrm{d}P/\mathrm{d}n = -8.3 \times 10^{-9}$ d (Kilkenny & Lynas-Gray 1982; Kilkenny, Lynas-Gray & Roberts 1996) commensurate with a secular contraction. Kilkenny et al. (1996) have also measured the second derivative of the period, $\mathrm{d}^2P/\mathrm{d}n^2 = 1.7 \times 10^{-14}$ d.

Jeffery et al. (1999) have shown that the extremely helium-rich surface of V652 Her is >1 dex underabundant in carbon and oxygen and ~1 dex overabundant in nitrogen, indicating that it primarily comprises the residue of CNO-processed material. There is some residual hydrogen, ~1 per cent by numbers, but no evidence of any helium-burning products. The abundances of other elements are typically solar, and the surface composition may be characterized by mass fractions of hydrogen, helium and metals as $X = 0.0017$, $Y = 0.9825$, $Z = 0.0158$.

In an attempt to interpret some of these observations, Jeffery (1984) constructed models of a 0.7-$\mathrm{M}_\odot$ mass helium star contracting towards the helium main sequence. Whilst able to reproduce the observed properties of V652 Her successfully, it was difficult to account for the initial conditions adopted for the evolutionary sequence – a 'helium-rich horizontal branch' model – within single star evolution theory.

The LTP model introduced above provides an attractive alternative because it can, in principle, be fine-tuned to match many combinations of observables. However, its principal property is that of a helium-burning shell around a degenerate carbon–oxygen core. For such stars, the luminosity is determined principally by the core mass (Jeffery 1988). Detailed models of thermally pulsing post-AGB stars (Blöcker & Schönberner 1997) with masses of 0.625 and 0.836 $\mathrm{M}_\odot$ have luminosities $\log L/\mathrm{L}_\odot \sim$ 3.8 and 4.2 respectively. A somewhat older model for a 0.553-$\mathrm{M}_\odot$ post-AGB star exhibiting a late thermal pulse was given by Schönberner (1983). The pulse drives the star through a red loop in the Hertzsprung–Russell (HR) diagram and, in the range $\log T_{\mathrm{eff}} = 4.3$–4.4, the star achieves luminosities $\log L/\mathrm{L}_\odot \sim$ 3.15 and 3.4 on the expanding and contracting branches, respectively. The rates of period change at these locations would be, for $P = 0.108$ d, approximately $\mathrm{d}P/\mathrm{d}n = +3 \times 10^{-7}$ and -6×10^{-8} d.

None of these models is consistent with the observed dimensions of V652 Her. Following the most recent measurement of $\log g$ (Jeffery et al. 1999), the allowed mass and luminosity ranges ($\pm 1\sigma$) are $M/\mathrm{M}_\odot = 0.57$–0.84 and $\log L/\mathrm{L}_\odot = 2.91$–3.15. These are not fully independent, since either limit on the measured radius (from Baade's method: Lynas-Gray et al. 1984) leads to equivalent limits on $M \propto gR^2$ and $L \propto R^2T^4$. At best, if $M/\mathrm{M}_\odot = 0.57$, then $\log L/\mathrm{L}_\odot = 2.91$ is 0.5 dex lower than the luminosity of the 0.55-$\mathrm{M}_\odot$ LTP model on its contracting branch.

Pulsation properties place even tighter constraints upon the dimensions of V652 Her (Jeffery & Saio 1999). Thus, for example, if the luminosity were only as high as $\log L/\mathrm{L}_\odot = 3.15$, the observed pulsation period would require the mass to be greater than 0.9 $\mathrm{M}_\odot$, in direct confrontation with the spectroscopic properties. The implication is therefore that V652 Her cannot have a degenerate CO core and that an alternative evolutionary model must be found.

3 EVOLUTIONARY MODELS

We have calculated evolutionary models starting with a low-mass WD rapidly accreting helium-rich material ($Y = 0.98, Z = 0.02$). For the initial accretion phase, which is considered as a rough approximation of the merging process of a double WD system, we have adopted an accretion rate of $1 \times 10^{-5}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$, which is about a half of the Eddington limit accretion rate for WDs. Initial masses (M_{i}) of WDs considered are 0.3 and 0.4 $\mathrm{M}_\odot$. The accretion was stopped when the total mass increased to a pre-determined final mass. Considering that the final mass should be smaller than $2M_{\mathrm{i}}$, we have adopted final masses of $M_{\mathrm{f}} = 0.8$, 0.7, 0.6 and 0.5 $\mathrm{M}_\odot$ for $M_{\mathrm{i}} = 0.4\,\mathrm{M}_\odot$, and $M_{\mathrm{f}} = 0.6$ and 0.5 $\mathrm{M}_\odot$ for $M_{\mathrm{i}} = 0.3\,\mathrm{M}_\odot$. The computational method is the same as in Saio & Nomoto (1998) except that opacities have been obtained from OPAL95 tables (Iglesias & Rogers 1996).

Fig. 1 shows evolutionary tracks for the cases of $M_{\mathrm{f}} = 0.5$ and 0.7 $\mathrm{M}_\odot$ with $M_{\mathrm{i}} = 0.4\,\mathrm{M}_\odot$. The evolutionary tracks for the other cases are similar. The 3α reaction is ignited where the

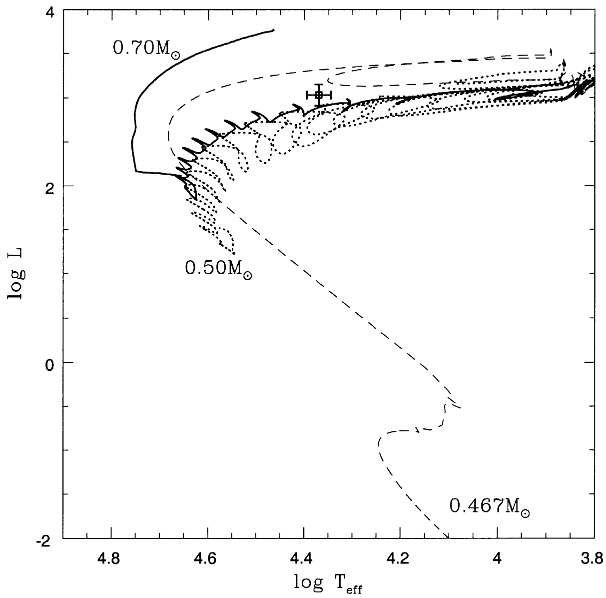

Figure 1. Evolutionary tracks starting with an accreting WD of 0.4 M⊙. The accretion was stopped when the total mass became 0.5 M⊙ (dotted line) or 0.7 M⊙ (solid line). The dashed line indicates the part in which accretion is switched on. The square with error bars shows the approximate position of V652 Her (Lynas-Gray et al. 1984).

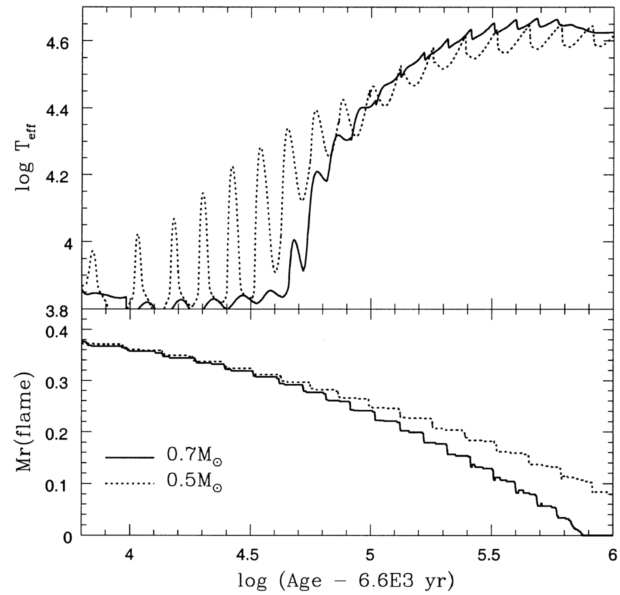

Figure 2. Evolutionary changes of the position of the helium-burning shell (lower panel) and the effective temperature (upper panel). The abscissa presents time after the first helium ignition. The initial mass is 0.4 M⊙ for both cases.

mass coordinate $M_r = 0.413\,M_\odot$ when the total mass has increased to $0.466\,M_\odot$ (for $M_i = 0.3\,M_\odot$ these quantities are 0.278 and $0.5\,M_\odot$, respectively). It leads to a shell flash with a peak nuclear luminosity of $7.7\times10^7\,L_\odot$ ($4.2\times10^5\,L_\odot$ for $M_i = 0.3\,M_\odot$). As the released energy migrates into the envelope, the radius as well as the luminosity increases so that the star becomes a yellow giant in $\sim10^3$ yr. Accretion is stopped when the total mass reaches a pre-determined final mass, which occurs during the yellow giant phase. Evolutionary tracks after the accretion phase are shown by solid ($M_f = 0.7\,M_\odot$) and dotted ($0.5\,M_\odot$) lines in Fig. 1, while the accretion phase is shown by a dashed line. The position of V652 Her is also shown.

The helium-burning shell moves inward with repeating shell flashes as described by Saio & Nomoto (1998). Fig. 2 shows the temporal variation of the mass coordinate of the helium-burning shell, M_r(flame), and the effective temperature. Each flash phase corresponds to each sudden change in M_r(flame) in Fig. 2 (see also Fig. 3). As the flame moves inward, the effective temperature increases gradually, although it fluctuates owing to shell flashes. When $M_r(\text{flame}) \sim 0.25\,M_\odot$, the star enters the instability region on the HR diagram for radial pulsations. It takes about 10^5 yr for the flame to reach $M_r \sim 0.25\,M_\odot$, and about 10^6 yr to reach the centre. The evolution time-scale becomes longer as the star get closer to the helium zero-age main sequence. Since only 10 per cent or less of helium is burnt during a shell flash, the star has a structure similar to that of a helium main-sequence star when the flame reaches the centre.

We note that the accretion rate, $1\times10^{-5}\,M_\odot\,\text{yr}^{-1}$, is considerably higher than any considered for helium WDs by Saio & Nomoto (1998). Considerable mass is accumulated before the star leaves the yellow giant region. For the models of Saio & Nomoto (1998), significant mass accumulated only after the star approaches the helium zero-age main sequence.

Convective regions occur above the helium-burning shell during shell flashes and at the surface during the most redward excursion

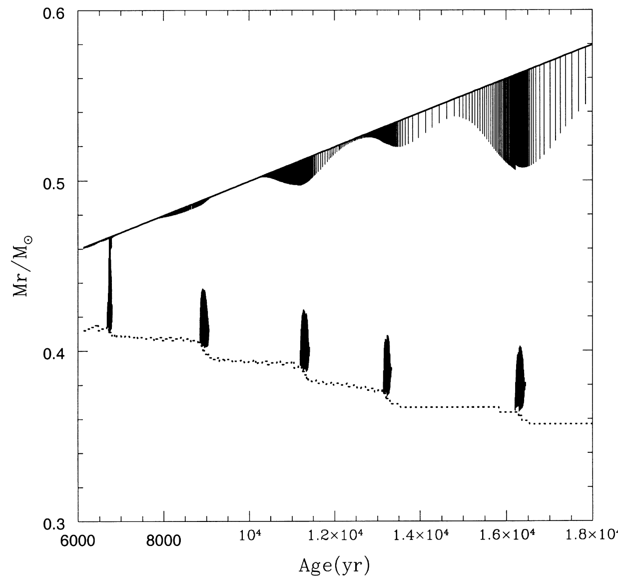

Figure 3. The location of convective regions during accretion of helium by a 0.4-M⊙ helium WD. The convective zones in the interior occur during helium shell flashes. The extent of the surface convection zone reflects the effective temperature at the surface. The zones are only shown for each time-step in the model sequence; the gaps between models are longer than they are during shell flashes.

of the evolutionary tracks (Fig. 3). Only the first flash was strong enough for the shell convection zone to reach close to the surface.

Fig. 1 shows that evolutionary tracks toward the helium main sequence pass near the position of V652 Her. The luminosity of models in inter-flash phases around the position of V652 Her is mainly determined by the mass interior to the helium-burning shell, i.e. M_r(flame). This is the reason why our models have a luminosity insensitive to the total mass and much lower than those of post-AGB models with the same total masses (Jeffery 1988; Saio 1988).

Thus the 'merged binary white dwarf' hypothesis for the progenitors of low-luminosity EHes satisfies the requirement for the position on the HR diagram. This scenario predicts that the luminosity of low-luminosity EHes is distinctly lower than that of normal EHes. This property seems to appear in the luminosity–frequency histogram for EHes shown by Jeffery (1996).

4 RADIAL PULSATIONS

Since many EHes show pulsations, and since the primary comparison target V652 Her shows well-defined radial pulsations with a secular period change, these can be used as a further strong constraint on the evolutionary models.

We have calculated envelope models along the evolutionary tracks and obtained complex eigenfrequencies using the linear non-adiabatic radial pulsation code described by Saio (1995). The real part of the eigenfrequency gives the period, and the imaginary part gives the stability of the mode. Helium-rich stellar envelopes around the location of V652 Her on the HR diagram are known to be unstable against pulsation owing to the Z-bump kappa-mechanism (Saio 1993). The fundamental radial mode is overstable in the range $4.26 \leq \log T_{\rm eff} \leq 4.43$ and the first overtone is overstable in the range $4.34 \leq \log T_{\rm eff} \leq 4.43$ for $M_{\rm f} = 0.7\,{\rm M}_{\odot}$. These ranges are almost independent of $M_{\rm f}$. Near the position of V652 Her, both fundamental and first-overtone pulsations are overstable in the linear analysis. However, the first-overtone component has not been detected in the observed light and velocity curves (Hill et al. 1981; Lynas-Gray et al. 1984; Lynas-Gray & Kilkenny 1986; Jeffery & Hill 1986), or in non-linear models by Fadeyev & Lynas-Gray (1996). This may mean that the amplitude of the first overtone is very small, or that the heavy element abundance of V652 Her is smaller than 0.02 so that the first overtone is stable.

Since we now know the age and pulsation period at any point along an evolutionary track, we can obtain the rate of period change d*P*/d*t* by simple numerical differentiation. Fig. 4 shows the period change (in days) *per cycle* (*n*), $\mathrm{d}P/\mathrm{d}n = P(\mathrm{d}P/\mathrm{d}t)$, as a function of period *P* for the overstable fundamental mode. The cases of $M_{\rm f} = 0.6$ and $0.7\,{\rm M}_{\odot}$ with $M_{\rm i} = 0.4\,{\rm M}_{\odot}$ are shown.

As seen in this figure, d*P*/d*n* changes sign during the evolution in the Z-bump instability region. In an inter-flash phase the star contracts so that the period decreases ($\mathrm{d}P/\mathrm{d}n < 0$), while in a flash phase the envelope expands and hence $\mathrm{d}P/\mathrm{d}n > 0$.

The position of V652 Her in the *P*–d*P*/d*n* plane is shown in Fig. 4 by a crossed square. The observed rate of change agrees with the theoretical value for $M_{\rm f} = 0.6\,{\rm M}_{\odot}$ for $M_i = 0.4\,{\rm M}_{\odot}$. Table 1 shows the properties of those models that have a period of $P = 0.108$ d for all the cases considered in this paper. In this table *t* is the time that has passed since the mass accretion stopped, and $\mathrm{d}^2P/\mathrm{d}n^2$ is the second derivative of the period. In some cases, the period of V652 Her is realized more than once during its evolution because of expansion/contraction arising from helium shell flashes (see Fig. 4). Table 1 shows that all the models have surface gravities consistent with spectroscopic analysis by Jeffery et al. (1999). Although the first solution of $0.5\,{\rm M}_{\odot}$ and the third solution of $0.6\,{\rm M}_{\odot}$ with $M_{\rm i} = 0.4\,{\rm M}_{\odot}$ give values of d*P*/d*n* consistent with observation, the luminosities and the effective temperatures are slightly lower than those of V652 Her determined by Lynas-Gray et al. (1984). The luminosity and the effective temperature are consistent with the cases of 0.7 and $0.8\,{\rm M}_{\odot}$ with $M_{\rm i} = 0.4\,{\rm M}_{\odot}$.

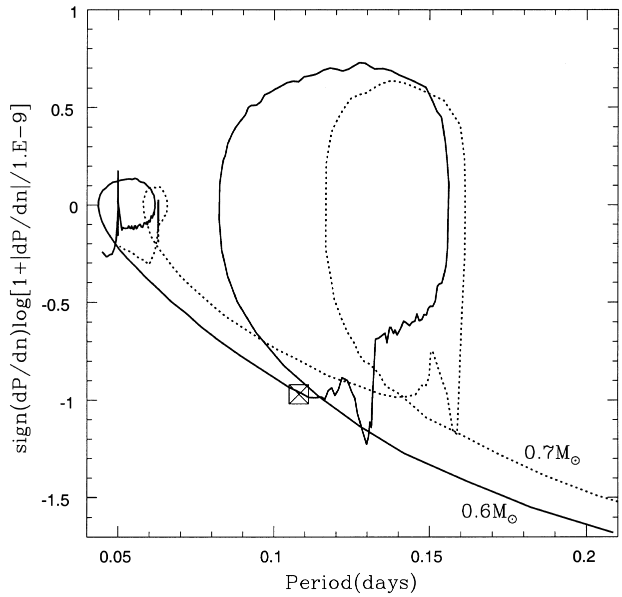

Figure 4. The rate of period change versus period for 0.7- and 0.6-M⊙ cases, where d*P*/d*n* is the period change per cycle in days. The crossed square indicates the observed period and the period change rate of V652 Her (Kilkenny et al. 1996).

Table 1. Period change rates when $P = 0.108$ d.

$M/{\rm M}_{\odot}$	$t(10^4\,{\rm yr})$	$\log L/{\rm L}_{\odot}$	$\log T_{\rm eff}$	$\log g$	$\frac{\mathrm{d}P}{\mathrm{d}n}$(d)	$\frac{\mathrm{d}^2P}{\mathrm{d}n^2}$(d)
$M_{\rm i} = 0.4\,{\rm M}_{\odot}$						
0.50	4.1	2.766	4.337	3.67	−1.26E-08	1.05E-14
0.50	4.3	2.679	4.315	3.67	9.70E-09	5.02E-15
0.50	5.4	2.834	4.355	3.68	−2.14E-08	1.31E-14
0.50	5.9	2.691	4.318	3.67	9.92E-09	2.38E-15
0.50	7.0	2.853	4.359	3.67	−1.61E-08	−4.14E-15
0.50	8.1	2.711	4.323	3.67	3.94E-09	−6.07*E*-17
0.50	8.8	2.836	4.355	3.67	−3.02E-09	7.39E-17
0.60	5.0	2.801	4.330	3.69	−6.89*E*-09	1.85E-15
0.60	6.7	2.751	4.317	3.69	3.31E-09	2.29E-16
0.60	7.8	2.849	4.341	3.68	−8.06*E*-09	1.39E-15
0.7	6.1	2.921	4.347	3.70	−5.24E-09	6.37E-16
0.8	4.6	3.006	4.358	3.72	−3.43*E*-09	2.62E-16
$M_{\rm i} = 0.3\,{\rm M}_{\odot}$						
0.50	0.3	3.012	4.401	3.68	−1.47E-08	5.39E-15
0.60	1.3	2.929	4.362	3.69	−2.53E-09	8.24E-17
V652 Her: Lynas-Gray et al. (1984), Kilkenny et al. (1996), Jeffery et al. (1999)						
0.7		3.03	4.370	3.68	−8.3*E*-09	1.7E-14

In addition to the cases $M_{\rm f} = 0.5$ and $0.6\,{\rm M}_{\odot}$ with $M_{\rm i} = 0.4\,{\rm M}_{\odot}$, a model having a mass slightly larger than $0.5\,{\rm M}_{\odot}$ with $M_{\rm i} = 0.3\,{\rm M}_{\odot}$ would also have the observed d*P*/d*n* (Table 1). We should note, however, that the time since the termination of accretion is very short $\lesssim 10^4$ yr) for $M_{\rm i} = 0.3\,{\rm M}_{\odot}$. This is because the first helium ignition occurs relatively deep at $M_r = 0.28\,{\rm M}_{\odot}$ and consequently when the effective temperature is higher. Such a

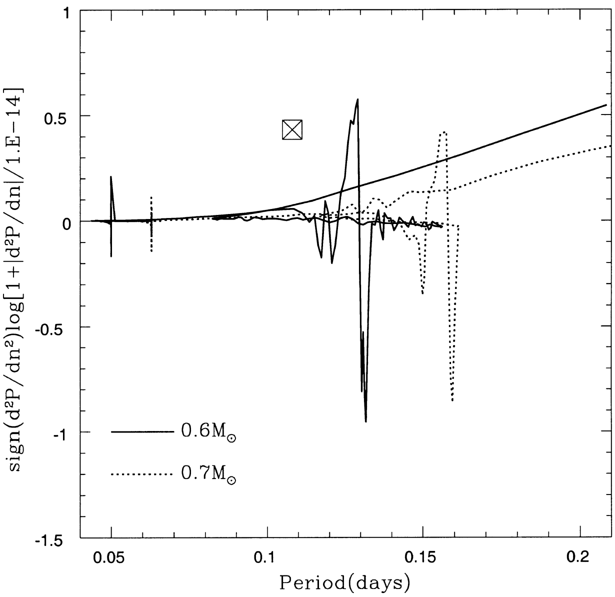

Figure 5. The second time derivative of the period versus period for 0.7- and 0.6-$M_\odot$($M_i = 0.4\,M_\odot$) cases. The crossed square indicates the observed period and second derivative for V652 Her (Kilkenny et al. 1996).

short interval after the accretion phase might conflict with the absence of any infrared excess around V652 Her (Lynas-Gray & Kilkenny 1986).

As Table 1 indicates, the effective temperature and the luminosity for $P = 0.108$ d tend to be higher for smaller M_i. An initial mass slightly less than $0.4\,M_\odot$ may be better for the progenitor of V652 Her. We infer that the best model, in terms of dP/dn and the position on the HR diagram, would be obtained by adopting an initial mass slightly less than $0.4\,M_\odot$ and a present mass between 0.6 and $0.7\,M_\odot$, which is consistent with spectroscopic estimates.

Table 1 shows that the measured second derivative of the period d^2P/dn^2 is much higher (by a factor of 10^2) than indicated by those theoretical models that do reproduce dP/dn, although the sign is right. The cause of the discrepancy is not clear. One possible explanation may be that the location of the helium shell flash in the P–d^2P/dn^2 diagram is sensitive to the initial conditions and final mass. It appears that d^2P/dn^2 varies sufficiently both during and between helium shell flashes that a closer agreement is possible (Fig. 5).

5 DISCUSSION

We have calculated rapidly accreting WD models as a rough approximation for what would be expected when a low-mass double WD binary system coalesces. Several short-period low-mass double WD systems are known to exist (Saffer, Liebert & Olszewski 1988; Bragaglia et al. 1990; Holberg et al. 1995; Marsh, Dhillon & Duck 1995; Marsh 1995; Moran, Marsh & Bragaglia 1997). The mass of the primary component of such a system is estimated to be similar to or less than $0.4\,M_\odot$, and the secondary mass is comparable to it. These WDs should consist of mostly helium because the core helium flash does not occur unless the helium core mass becomes about $0.45\,M_\odot$. Such a binary system loses angular momentum as a result of gravitational wave emission so that the separation decreases gradually. If the secondary mass is comparable to the primary mass, when the secondary fills its critical Roche lobe, a runaway mass transfer to the primary is expected to lead to the coalescence of the binary system. Among the known double WD systems, three systems have periods short enough to coalesce within the Hubble time (Marsh 1995; Marsh et al. 1995; Moran et al. 1997). These systems are candidates for the progenitors of our models which make the merging scenario to produce low-luminosity EHes viable.

The merger frequency of double helium WD systems in the Galaxy is theoretically estimated to be $\sim 0.006\,\mathrm{yr}^{-1}$ by Han (1998), and $\sim 0.02\,\mathrm{yr}^{-1}$ by Iben, Tutukov & Yungelson (1997). The known low-luminosity helium stars have effective temperatures in the range $4.3 \le \log T_{\rm eff} \le 4.5$ (Jeffery 1996). It takes $\sim 6 \times 10^4$ yr for a merged star to evolve in this temperature range (Fig. 2). Combining the evolution time and the above estimates for the merger frequency, we obtain $\sim 4 \times 10^2$–10^3 for the number of the low-luminosity helium stars in the Galaxy.

Now, let us estimate from observational data the number of low-luminosity helium stars in the Galaxy. Combining the known number of RCrB and hydrogen-deficient carbon (HdC) stars ($\sim$30) with the distribution in the Galaxy, Lawson et al. (1990) have estimated that $\sim$200–300 RCrB and HdC stars exist in the Galaxy. That is, multiplying the observed number with a factor of $\sim$10 yields the actual number of RCrB and HdC stars in the Galaxy. Compared with RCrB/HdC stars, the low-luminosity helium stars are about 10 times fainter in bolometric luminosity and $\sim$30 times fainter in visual luminosity because of the bolometric correction. This means that the volume in which low-luminosity helium stars are observed in the Galaxy is $\sim$30 times smaller than in the case of RCrB/HdC stars, if both distributions are more or less planar. Therefore multiplying the observed number of low-luminosity helium stars by $\sim$300 yields a rough estimate for the number in the Galaxy. We know four low-luminosity helium stars in the above temperature range: V652 Her, LSS 3184 (Drilling, Jeffery & Heber 1998), LS IV+6°2 (Jeffery 1998) and HD 144941 (Harrison & Jeffery 1997). Combining this number with the above multiplying factor yields $\sim 10^3$ for the number of low-luminosity helium stars in the Galaxy, which is surprisingly consistent with the number predicted from the merged WD scenario.

The surface composition of V652 Her, consisting of CNO-processed helium and no helium-burning products (Jeffery et al. 1999), is consistent with our models. Since the first flash was so strong, the outer boundary of the convective shell reached close to the surface. In the convective shell, 3 per cent (by mass) of the helium was converted to carbon. However, the enhanced carbon abundance has been covered by further accretion of helium. Since the subsequent shell flashes were weak and hence the convective shell was thin, helium-burning products were not mixed into the atmosphere (Fig. 3).

Another low-luminosity extreme star, LSS 3184, has very similar properties ($T_{\rm eff}$ and $\log g$) to V652 Her (Drilling et al. 1998) and pulsates with a period of 0.106 d (Kilkenny et al. 1999). Drilling et al. (1998) have shown that its atmosphere contains CNO-processed matter *and* carbon from helium burning. Such a chemical composition would result from a merger model if the mass accreted after the end of the initial helium flash is small, i.e. if the mass of LSS 3184 is smaller than that of V652 Her and around $0.5\,M_\odot$. There is observational support for this conjecture. The surface gravity of LSS 3184 ($\log g = 3.35$) determined by Drilling et al. (1998) is smaller than that of V652 Her ($\log g = 3.68 \pm 0.05$)

determined by Jeffery et al. (1999). This may indicate that LSS 3184 has a mass smaller than that of V652 Her.

A more puzzling problem is that V652 Her retains a small concentration of hydrogen (~0.2 per cent by mass: Jeffery et al. 1999). In a non-turbulent spherically symmetric merging process, the accreted material would settle on top of any residual hydrogen envelope possessed by the progenitor white dwarf. The real merging process is turbulent and three-dimensional, so that substantial mixing will occur. For example, some of the hydrogen could be expelled outwards during the initial dynamical phase to settle later on the surface of the merged product, or substantial mixing could occur throughout the surface layers during the merger process. Only $\sim 10^{-3}\,M_{\odot}$ of hydrogen-rich material in the progenitor system, mixed through the product envelope, would be required to explain the hydrogen observed in V652 Her. Two-dimensional calculations that include some surface hydrogen on the WDs, consider what mixing processes occur during initial mass transfer, and follow the surface hydrogen abundance through the complete accretion/shell-flash process are required before the surface abundances can be used as final arbiters of the evolution question.

Following their passage through the Z-bump instability zone, the subsequent evolution of our mass-accreted helium WD models will bring them to the helium main sequence where they will appear as hot subdwarfs. With masses slightly greater than $0.5\,M_{\odot}$, and hydrogen-poor surfaces, they might be expected to appear as subdwarf O or B stars.

Subdwarf B stars are generally recognized to be helium main-sequence stars with masses in the region of 0.5 to $0.6\,M_{\odot}$. However, they have very hydrogen-rich atmospheres and are extremely numerous. The scarcity of He+He WD binaries and the helium-rich surfaces of their descendants probably exclude them as normal sdB progenitors.

On the other hand, a small number of helium-rich subdwarf B stars have been detected in low-dispersion surveys (e.g. Green, Schmidt & Liebert 1986). Practically nothing is known about these stars at present beyond their general spectral characteristics (Jeffery et al. 1997). If they are also the products of mass-accreted WD evolution then, because the contraction time between helium shell ignition and the helium main sequence ($\sim 10^6$ yr) is very short compared with the helium main-sequence lifetime ($\sim 10^8$ yr), there should be many more such subdwarfs than EHes like V652 Her and LSS 3184, as appears to be the case.

6 CONCLUSION

We have examined the merged binary white dwarf hypothesis for the origin of low-luminosity (or high-gravity) extreme helium stars. We have approximated the merging process by spherical rapid accretion on to a low-mass helium white dwarf. We have found that the evolutionary path of such a model passes close to the position of the low-luminosity helium star V652 Her. We have obtained the pulsation periods and their time derivatives for models along the evolutionary tracks. We have found that the observed period and period change rate for V652 Her, as well as its position on the HR diagram, would be reproduced by a model with an initial mass slightly less than $0.4\,M_{\odot}$ and a final mass between 0.6 and $0.7\,M_{\odot}$. The observed second derivative of the period and the surface hydrogen abundance in V652 Her are both larger than the values predicted in our models; further detailed modelling should indicate that these discrepancies can be resolved. We have also found that the predicted number of low-luminosity helium stars in the Galaxy is consistent with observation.

We conclude that the merged binary white dwarf hypothesis, as represented by an accreting helium white dwarf model, provides a viable explanation for the evolutionary origin of at least some extreme helium stars. These helium stars will evolve to become hot subdwarfs close to the helium main sequence.

ACKNOWLEDGMENTS

This research has been supported by the British Council through Collaborative Research Grant TOK/880/41/4, by the Department of Education in Northern Ireland through a grant to the Armagh Observatory, and by the Japan Society for the Promotion of Science.

REFERENCES

Blöcker T., Schönberner D., 1997, A&A, 324, 991
Bond H. E., Luck R. E., Newman M. J., 1979, ApJ, 233, 205
Bragaglia A., Greggio L., Renzini A., D'Odorico S., 1990, ApJ, 365, L13
Drilling J. S., Jeffery C. S., Heber U., 1998, A&A, 329, 1019
Duerbeck H. W., Benetti S., 1996, ApJ, 468, L111
Fadeyev Yu. A., Lynas-Gray A. E., 1996, MNRAS, 280, 427
Green R. F., Schmidt M., Liebert J., 1986, ApJS, 61, 305
Han Z., 1998, MNRAS, 296, 1019
Harrison P. M., Jeffery C. S., 1997, A&A, 323, 177
Herbig G. H., Boyarchuk A. A., 1968, ApJ, 153, 397
Hill P. W., Kilkenny D., Schönberner D., Walker H. J., 1981, MNRAS, 197, 81
Holberg J. B., Saffer R. A., Tweedy R. W., Barstow M. A., 1995, ApJ, 452, L133
Iben I. Jr, 1990, ApJ, 352, 215
Iben I. Jr, McDonald J., 1995, in Koester D., Werner K., eds, White Dwarfs. Springer, Berlin, p. 48
Iben I. Jr, Tutukov A., 1984, ApJS, 55, 335
Iben I. Jr, Kaler J. B., Truran J. W., Renzini A., 1983, ApJ, 264, 605
Iben I. Jr, Tutukov A., Yungelson L. R., 1997, ApJ, 475, 291
Iglesias C. A., Rogers F. J., 1996, ApJ, 464, 943
Jeffery C. S., 1984, MNRAS, 210, 731
Jeffery C. S., 1988, MNRAS, 235, 1287
Jeffery C. S., 1996, in Jeffery C. S., Heber U., eds, ASP Conf. Ser. Vol. 96, Hydrogen Deficient Stars. Astron. Soc. Pac., San Francisco, p. 152
Jeffery C. S., 1998, MNRAS, 294, 391
Jeffery C. S., Hill P. W., 1986, MNRAS, 221, 975
Jeffery C. S., Saio H., 1999, MNRAS, 308, 221
Jeffery C. S., Drilling J. S., Harrison P. M., Mohler S., Heber U., 1997, A&AS, 125, 501
Jeffery C. S., Hill P. W., Heber U., 1999, A&A, 346, 491
Kilkenny D., Lynas-Gray A. E., 1982, MNRAS, 198, 873
Kilkenny D., Lynas-Gray A. E., Roberts G., 1996, MNRAS, 283, 1349
Kilkenny D., Koen C., Jeffery C. S., Hill N. C., O'Donoghue D., 1999, MNRAS, 310, 1119
Lambert D. L., Rao N. K., 1994, JA&A, 15, 47
Lawson W. A., Cottrell P. L., Kilmartin P. M., Gilmore A. C., 1990, MNRAS, 247, 91
Leuenhagen U., Hamann W.-R., Jeffery C. S., 1996, A&A, 312, 167
Lynas-Gray A. K., Kilkenny D., 1986, in Hunger K., Schönberner D., Rao N. K., eds, Hydrogen Deficient Stars and Related Objects. Reidel, Dordrecht, p. 117
Lynas-Gray A. E., Schönberner D., Hill P. W., Heber U., 1984, MNRAS, 209, 387
Marsh T. R., 1995, MNRAS, 275, L1
Marsh T. R., Dhillon V. S., Duck S. R., 1995, MNRAS, 275, 828

Moran C., Marsh T. R., Bragaglia A., 1997, MNRAS, 288, 538
Pollacco D. L., Lawson W. A., Clegg R. E. S., Hill P. W., 1992, MNRAS, 257, 33P
Saffer R. A., Liebert J., Olszewski E. W., 1988, ApJ, 334, 947
Saio H., 1988, MNRAS, 235, 203
Saio H., 1993, MNRAS, 260, 465
Saio H., 1995, MNRAS, 277, 1393
Saio H., Nomoto K., 1998, ApJ, 500, 388
Schönberner D., 1983, ApJ, 272, 708
Webbink R. F., 1984, ApJ, 277, 355
Werner K., Heber U., Hunger K., 1991, A&A, 244, 437

This paper has been typeset from a TEX/LATEX file prepared by the author.