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THE EVOLUTION OF GAUGINO MASSES AND THE SUSY THRESHOLD

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Abstract

We propose a numerical iterative method to account for the evolutions of the gaugino masses (EGM). The effect of these evolutions on the most exhaustive model of SUSY breaking is presented. From above 21 TeV, the one-σ lower bound for SUSY breaking is brought down by more than two orders of magnitude. The model without EGM needed two-σ to reach the Z^0-mass range. The same model with EGM needs only one-σ to reach the same level. The conclusion is that the SUSY threshold can be anywhere, including within the mass range where LEP I, HERA, LEP II and other colliders are working or planning to work.

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1. Introduction

We first review the status of SUSY in order to focus the main problem of concern in this exciting field of particle physics. Then we choose what appears at present to be the "best model" for the SUSY threshold predictions. A numerical iterative method to account for the evolution of the gaugino masses is discussed, together with its consequences on the "best model" for SUSY threshold predictions. Finally, we present the conclusion on the new lower bound for the light SUSY threshold: it can be anywhere, including in the mass range where LEP I, HERA, LEP II and other colliders are working or planning to work in the near future. Claims based on a partial and restricted use of our present experimental and theoretical knowledge are not very meaningful.

2. Status of SUSY: a supersynthesis

One of the central problems in particle physics is to establish if SUSY exists. From the theoretical viewpoint, there are good basic motivations for its existence. Historically, supersymmetry [1]-[4] has given rise to supergravity [5]; this produced superstring theory, which has no-scale supergravity as infrared solution [6]-[9]. Thus supersymmetry breaking can be produced dynamically, via "radiative effects" [10] [11]. At this point, the link with electroweak breaking appears natural [7], if we want to avoid that the physics at energies near the Fermi scale is destabilized by the Planck energy scale [12]-[16]. On the other hand, if SUSY exists, the crucial problem is to understand if it is possible to predict where the threshold for SUSY particles lies. There are good reasons why this threshold should not be very high in energy. Already the 1 TeV level implies "fine tuning". If supersymmetry is the solution to "fine tuning", the superpartners at 1 TeV are already at the limit for "naturalness". From the theoretical viewpoint, a nice feature of SUSY is that it allows a better convergence [17] of α_1, α_2, α_3 towards a common point.

What about experimental data? They are of two distinct classes. The direct search for SUSY particles and the measurements of those quantities which are sensitive to the existence of SUSY. In the spring of last year, some authors decided to choose the three couplings α_1, α_2, α_3 as the parameters to work out consequences on the existence of SUSY. They worked out the evolution of α_1, α_2, α_3 using one of the possible solutions of the coupled equations for α_1, α_2, α_3. Moreover, for α_3 they adopted the result of a single experiment [18] instead of the world average; furthermore, the minimum confidence level (i.e. one σ) was taken to study the convergence

of α_1, α_2, α_3. These authors concluded that the SUSY threshold was expected to be in the TeV range [19]. There was no reason to choose only one experimental result for α_3. Nor the one-σ level. Moreover, the solution of the coupled equations for the evolution of α_1, α_2, α_3 was not the only possible one.

Using the world-average value for α_3, taking into account all known solutions of the coupled equations for α_1, α_2, α_3, introducing a new numerical solution (likely to be the best) and working out the two-σ level of confidence, we showed [20] that the goodness in the convergence of α_1, α_2, α_3 allowed the SUSY threshold to be in a very wide range: from GeV to PeV.

Following our work, the same authors later [21] adopted a world average value for α_3, a numerical solution for the coupled equations, as previously done by us [20], but unfortunately using one-σ level of confidence to keep claims on the SUSY threshold. These claims are still unjustified since they are derived from "compatibility" considerations and are far from being "predictions".

The first work where a model has been built in order to attempt a prediction for the SUSY threshold was published soon after our work [20] by Ellis et al. [22] [23]. The conclusion of this paper was that, taking into account all known data with their errors and making all possible reasonable estimates of the unknowns, the SUSY threshold is, at the one-σ level of confidence, predicted to be:

$$m_{1/2} \geq 21 \text{ TeV}. \qquad (1)$$

Furthermore, in order to get this threshold in the Z^0-mass range, two standard deviations are needed. This produced a lot of interest, because it was the result of the most exhaustive model ever attempted for SUSY threshold predictions: the best model. Discouragement affected a large physics community engaged in the search for SUSY particles [24]-[39]. For this reason, we decided to study this model [22] [23], trying to contribute to its improvement.

In the present work, we show that improvements are possible. In fact, using the same range of experimental and theoretical uncertainties, the above limit (1) can go below the Z^0 mass, for example

$$m_{1/2} \geq 45 \text{ GeV}, \qquad (2)$$

thus giving a good motivation for further searches in the low-energy domain of LEP I. And taking out of despair those physicists (including some of us) who are planning not only to continue the search for SUSY particles at LEP, but also to go on at HERA and LEP II.

Our conclusion is that, on the basis of the present experimental and theoretical knowledge, there is no lower bound for the SUSY threshold, even at the one-σ level. Claims for the SUSY threshold based on a partial and restricted use [21] of our experimental and theoretical knowledge are not very meaningful.

3. The SUSY threshold: two problems

Predicting where the SUSY threshold could be corresponds to facing at least two problems: the structure of the SUSY spectrum and the evolution of the masses involved. Many examples exist where a unique SUSY-breaking effective mass has been chosen, thus neglecting the effect of the mass spectrum of the various sparticles. To assume that all sparticles have the same mass is unphysical. Moreover, the evolution of these masses cannot be neglected. In the next chapter we introduce a numerical method in order to deal with these two problems: the spectrum and the evolution of the gaugino ($\tilde{g}$ and $\tilde{W}$) masses involved in the light threshold for SUSY breaking.

4. The evolution of the gaugino mass spectrum and its effect on the lower bound for the SUSY threshold

We chose a model where the gauge couplings satisfy evolution equations with the strong Grand Unification boundary conditions:

$$\alpha_3(m_{GUT}) = \alpha_2(m_{GUT}) = \alpha_1(m_{GUT}) \equiv \alpha_{GUT} \tag{3}$$

supplemented by the unification condition for the gaugino masses:

$$m_{\tilde{g}}(m_{GUT}) = m_{\tilde{W}}(m_{GUT}) = m_{1/2}. \tag{4}$$

The evolution of the gaugino masses, $m_{\tilde{g}}$ and $m_{\tilde{W}}$, is given by the coupled equations [40]:

$$m_{\tilde{g}} = \frac{\alpha_3(m_{\tilde{g}})}{\alpha_{GUT}(m_{1/2})} m_{1/2} \tag{5a}$$

$$m_{\tilde{W}} = \frac{\alpha_2(m_{\tilde{W}})}{\alpha_{GUT}(m_{1/2})} m_{1/2} \tag{5b}$$

$$m_{GUT} = (m_{GUT})_0 \left(\frac{m_{1/2}}{m_Z}\right)^{-7/60}. \tag{5c}$$

For the evolution of α_3 and α_2 we use the one-loop approximation [41]-[43]:

$$\frac{1}{\alpha_3(m_{\tilde{g}})} = \frac{1}{\alpha_{GUT}(m_{1/2})} - \frac{3}{2\pi} \log\left(\frac{m_{GUT}}{m_{\tilde{g}}}\right) \tag{6a}$$

$$\frac{1}{\alpha_2(m_{\tilde{W}})} = \frac{1}{\alpha_{GUT}(m_{1/2})} - \frac{19}{12\pi} \log\left(\frac{m_{1/2}}{m_{\tilde{W}}}\right) + \frac{1}{2\pi} \log\left(\frac{m_{GUT}}{m_{1/2}}\right) \tag{6b}$$

$$\frac{1}{\alpha_{GUT}(m_{1/2})} = \frac{1}{(\alpha_{GUT})_0} + \frac{73}{40\pi} \log\left(\frac{m_{1/2}}{m_Z}\right). \tag{6c}$$

The above formulae (5a)-(6c) are based on an approximate treatment of the threshold. This is justified, since we are computing corrections to genuine threshold parameters, i.e. $m_{\tilde{g}}$ and $m_{\tilde{W}}$.

Note that renormalization group calculations give approximately [23]:

$$m_{\tilde{g}} \approx 2.7\, m_{1/2} \tag{7a}$$

$$m_{\tilde{W}} \approx 0.79\, m_{1/2}. \tag{7b}$$

But these proportionality factors cannot stay constant. When the gaugino masses increase, α_2 and α_3 evolve and the net effect of this evolution is a dumping on the expected increase. The mechanism of this self-dumping works as follows. Consider for example eq. (5a). For a large value of $m_{1/2}$, there will be an increase of $m_{\tilde{g}}$ which would scale with $m_{1/2}$ if $\alpha_3(m_{\tilde{g}})/\alpha_{GUT}(m_{1/2})$ would stay constant. This is not the case. When $m_{\tilde{g}}$ increases, $\alpha_3(m_{\tilde{g}})$ will decrease, thus dumping the increase of $m_{\tilde{g}}$. On the other hand, $m_{1/2}$ is proportional, as we will see later [eq. (8)], to a positive power of $\alpha_3(m_{\tilde{g}})$, therefore an increase of $m_{1/2}$, inducing a decrease of $\alpha_3(m_{\tilde{g}})$, will self-consistently dump the initial increase of $m_{1/2}$.

We have elaborated a program which, by iteration, calculates sequentially the set of SUSY masses ($m_{1/2}$, $m_{\tilde{g}}$, $m_{\tilde{W}}$) until it reaches a set of convergent solutions. We proceed as follows. The starting points are the values of $(\alpha_{GUT})_0$, $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$, $\alpha_{em}(m_Z)$, F(X), and $(m_{GUT})_0$, $m_{\tilde{g}}$, $m_{\tilde{W}}$, $m_{1/2}$ [see table 1 and formula (11)]. Using the one-loop approximation (6a), (6b) and (6c), we evaluate $\alpha_3(m_{\tilde{g}})$, $\alpha_2(m_{\tilde{W}})$ and $\alpha_{GUT}(m_{1/2})$. The next step is the use of the coupled equations (5a), (5b) and (5c) to work out $m_{\tilde{g}}$ and $m_{\tilde{W}}$. The last equation to be used is [23]:

$$\log\left(\frac{m_{1/2}}{m_Z}\right) = \frac{15\pi}{\alpha_{em}(m_Z)}\left[0.2029 + \frac{7\alpha_{em}(m_Z)}{15\alpha_3(m_Z)} - \sin^2\theta_{\overline{MS}}(m_Z) + \Delta_T(\text{heavy})\right] +$$
$$+ f(y,w) - \frac{9}{4}\log\left(\frac{m_t}{m_Z}\right) - \frac{3}{4}\log\left(\frac{m_h}{m_Z}\right) - 3\log\left(\frac{\mu}{m_Z}\right) +$$
$$+ 7\log\left(\frac{\alpha_3(m_{\tilde{g}})}{\alpha_{GUT}(m_{1/2})}\right) - 8\log\left(\frac{\alpha_2(m_{\tilde{W}})}{\alpha_{GUT}(m_{1/2})}\right) \qquad (8)$$

which allows us to work out the value of $m_{1/2}$. These values for $m_{\tilde{g}}$, $m_{\tilde{W}}$, $m_{1/2}$ are the new inputs for the next iteration. We find that after less than 100 iterations the values for $m_{\tilde{g}}$, $m_{\tilde{W}}$, $m_{1/2}$ converge towards a stable solution: this is the result of the evolutions of the gaugino masses (EGM). The details of this work will be published elsewhere [44].

Here we will limit ourselves to the main steps which allow the derivation of (8). This will illustrate the exhaustive nature of the model [23] chosen for SUSY-breaking predictions.

Let us start with $\sin^2\theta_{\overline{MS}}(m_Z)$, which can be written as follows [23]:

$$\sin^2\theta_{\overline{MS}}(m_Z) = 0.2 + \frac{7\alpha_{em}(m_Z)}{15\alpha_s(m_Z)} + 0.0029 +$$
$$+ \Delta_T(\text{heavy}) + \delta_{sch} + \Delta_T(\text{light}). \qquad (9)$$

The first two terms in (9) are the well-known one-loop formula. The third matches numerically the two loops [22] [23]. The sum of these two numbers, 0.2029, is the first term in (8). The term Δ_T(heavy) stands for the contribution of the GUT superheavy mass fields [45]. Following Ellis et al. [23], it will be neglected: in fact any contribution from Δ_T(heavy) can only increase the lower bound for $m_{1/2}$. The effects of the conversion of the renormalization schemes are contained in δ_{sch}: this term depends on the way the threshold matching is treated. At the foreseen level of precision for

$\sin^2\theta$, this term can be neglected, even if it is straightforward to account for it numerically. The term Δ_T(light) accounts for the threshold effects and has a lot of physics in it. Different, contradictory formulations exist in the literature. We have worked on a few of them [22] [23] [46], using the old-fashioned method of determinants, and checking their β-function structure. We agree with only one of the various formulations for Δ_T, namely that given in [23]. This reads:

$$\Delta_T(\text{light}) = \frac{\alpha_{em}(m_Z)}{20\pi}\left\{\frac{28}{3}\log\left(\frac{\alpha_3(m_{\tilde{g}})}{\alpha_{GUT}(m_{1/2})}\cdot\frac{(m_{1/2})}{m_Z}\right) - \right.$$
$$- \frac{32}{3}\log\left(\frac{\alpha_2(m_{\tilde{w}})}{\alpha_{GUT}(m_{1/2})}\cdot\frac{(m_{1/2})}{m_Z}\right) - \log\left(\frac{m_h}{m_Z}\right) - 4\log\left(\frac{\mu}{m_Z}\right) -$$
$$- 3\log\left(\frac{m_t}{m_Z}\right) + \frac{5}{2}\log\left(\frac{m_{\tilde{q}}}{m_Z}\right) - 3\log\left(\frac{m_{\tilde{l}_L}}{m_Z}\right) + 2\log\left(\frac{m_{\tilde{l}_R}}{m_Z}\right) -$$
$$\left. - \frac{19}{36}\log\left(\frac{m_{\tilde{\tau}1}}{m_Z}\right) - \frac{35}{36}\log\left(\frac{m_{\tilde{\tau}2}}{m_Z}\right)\right\}. \qquad (10)$$

To extract useful information from (10), it is necessary to include a spectrum for the gauginos [40] and the squarks–sleptons [47] which can be expressed in terms of two universal SUSY-breaking parameters: m_0 and $m_{1/2}$. The part concerning the squarks and sleptons, i.e. the last five terms in (10), is a complete representation of SUSY-SU(5) and therefore enjoys, at one-loop, a scale-invariance property. Owing to this approximate scale invariance, the contribution of the last five terms in (9) is small. It is synthesized by f(y,w) in (8) and a plausible estimate has been worked out [23]: (0.2 ± 0.2). Compared with other uncertainties in the accepted ranges of unknown masses and parameters, this contribution is small. The first two terms in (10) allow us to extract $\log(m_{1/2}/m_Z)$ versus $m_{\tilde{g}}$ and $m_{\tilde{w}}$, i.e. the basic equation (8), which can be rewritten as follows:

$$\log\left(\frac{m_{1/2}}{m_Z}\right) = \left[\,(F(X) \pm \Delta) + 7\log\left(\frac{\alpha_3(m_{\tilde{g}})}{\alpha_{GUT}}\right) - 8\log\left(\frac{\alpha_2(m_{\tilde{w}})}{\alpha_{GUT}}\right)\right]. \qquad (11)$$

Notice that in formula (11) and from now on $\alpha_{GUT} \equiv \alpha_{GUT}(m_{1/2})$.

The function F(X), with its error $\pm\Delta$, synthesizes, in this field of physics, all our knowledge (experimental data and theoretical estimates with their limits), except the effects due to the EGM.

In order to proceed further, we need the numerical expression for (11). Following Ellis et al. [23], this is:

$$\log\left(\frac{m_{1/2}}{m_Z}\right) = \left[\ 10.96\ \pm 7.3 \pm 6.9 \pm 0.3\ {}^{-\ 3.3}_{+\ 0.3} \pm 0.2 \pm 0.8\ \right] +$$

$$+\left[\ 7\log\left(\frac{\alpha_3(m_{\tilde{g}})}{\alpha_{GUT}}\right) - 8\log\left(\frac{\alpha_2(m_{\tilde{W}})}{\alpha_{GUT}}\right)\ \right]. \qquad (12)$$

The value 10.96 corresponds to the "central value" of F(X), from experimental data and theoretical estimates of the unknown quantities entering in (8). The first three errors are related to the experimental measurements of $\sin^2\theta$, α_3 and α_{em} at the Z^0 mass. The last three correspond to theoretical uncertainties. The focal point of our work is to compute the last two terms in (12). They represent the contribution to $m_{1/2}$ of the evolutions of the gaugino masses, i.e. the EGM effect.

So far, only the evolution of the couplings (α_1, α_2, α_3) has been accounted for in dealing with SUSY threshold problems, not the evolution of the masses. In order to see the effect of this evolution, we proceed to a straightforward comparison with the "best model" [23], so far the most exhaustive. To this end, we take exactly the same input values, errors, treatment of errors, plausible ranges of unknown parameters such as the $m_0/m_{1/2}$ ratio, the off-diagonal elements in the stop mass matrix, the treatment of the two Higgs doublets and the theoretical ranges of μ and m_h with their central values: all this in order to parallel the predictions of [23]. This is why we start with formula (8) above and use its numerical expression (12).

In order to calculate the evolutions of the gaugino masses, the value of F(X) is needed as an input to our iterative process (see table 1).

As the experimental and theoretical knowledge will improve, $\pm\Delta$ in (11) will become negligible. At present it is so large that it can shift the "central value" quite a lot.

In order to estimate the range of variation for F(X) we follow Ellis et al. [23], and proceed accordingly.

The theoretical uncertainties are added all together with their positive or negative signs, respectively; thus we have:

$$\sigma^+ \text{ (theoretical)} = +1.3$$
$$\sigma^- \text{ (theoretical)} = -4.3.$$

The experimental errors are added in quadrature. The result is:

$$1\sigma \text{ (experimental)} = \pm 10.05.$$

The one-σ level is obtained, combining experimental errors and theoretical uncertainties, as follows:

$$1\sigma \Rightarrow \begin{cases} 1\sigma^{+} = \sigma^{+} \text{ (theoretical)} + \sigma \text{ (experimental)} = +11.35 \\ 1\sigma^{-} = \sigma^{-} \text{ (theoretical)} - \sigma \text{ (experimental)} = -14.35. \end{cases}$$

For the two-σ level, the theoretical uncertainties are taken to be the same. The results are summarized in table 2. Within 1σ, the "central value" of F(X) = +10.96 in (11) and (12), can be as high as +22.31 and as low as −3.39.

The evolution of the gaugino masses depends on $\alpha_3(m_{\tilde{g}})$ and $\alpha_2(m_{\tilde{W}})$. The variation of these couplings versus the input values for F(X) is reported in fig. 1. For example, if we compute, with our iterative method, the values of α_3 and α_2, using as fixed input the "central value" of F(X), the results are:

$$\frac{\alpha_3(m_{\tilde{g}})}{\alpha_{GUT}} = 1.340 \qquad (13a)$$

$$\frac{\alpha_2(m_{\tilde{W}})}{\alpha_{GUT}} = 0.922. \qquad (13b)$$

As a consequence, the EGM effect, i.e. the contribution of the EGM to $m_{1/2}$, is:

$$(EGM)_{F(X) \text{ central value}} = 7 \log\left(\frac{\alpha_3(m_{\tilde{g}})}{\alpha_{GUT}}\right) - 8 \log\left(\frac{\alpha_2(m_{\tilde{W}})}{\alpha_{GUT}}\right) = 2.70. \qquad (14)$$

This value (14) should be compared with the result [23]:

$$7 \log\left(\frac{\alpha_3(m_{\tilde{g}})}{\alpha_{GUT}}\right) - 8 \log\left(\frac{\alpha_2(m_{\tilde{W}})}{\alpha_{GUT}}\right) = 8.84. \qquad (15)$$

It follows that the estimates for $m_{1/2}$ with and without the corrections for the evolutions of gaugino masses (EGM) are in the ratio:

$$\frac{(m_{1/2})_{\text{without EGM}}}{(m_{1/2})_{\text{with EGM}}} = \exp(8.84 - 2.70) = 4.64 \times 10^2. \qquad (16)$$

The result of Ellis et al. [23] for the one-σ lower bound of SUSY threshold was:

$$(m_{1/2})^{\text{without EGM}}_{\text{lower bound}} \simeq 21 \text{ TeV}. \qquad (17)$$

Using our correction, the new value is:

$$(m_{1/2})^{\text{with EGM}}_{\text{lower bound}} \simeq 45 \text{ GeV}. \qquad (18)$$

The EGM effect, as mentioned above, depends on the experimental and theoretical knowledge, synthesized in the function F(X). The smaller $\pm\Delta$, the better we can estimate the EGM effect. At present, $\pm\Delta$ is quite large, as shown in table 2. We report in fig. 2 the EGM effect versus the allowed input values for F(X). The higher are the masses of the gauginos, the smaller is the correction for the evolutions of these masses. If these masses are very high, there is not much to evolve before reaching the unification limit. This trend is present in fig. 2, where at high F(X) values the EGM effect is minimum. In fact, high F(X) corresponds to high energy.

Apart from the EGM effect, the result (18) followed the method, adopted by Ellis et al. [23], of using the "central value" for F(X) and then subtracting one standard deviation. The introduction of the EGM effect allows different possibilities to reach the $-1\sigma^-$ level for the $m_{1/2}$ lower bound.

For example, if the input $[F(X)-\Delta]$ is taken to compute the gaugino evolution and no subtraction of $1\sigma^-$ is applied to $m_{1/2}$, the $m_{1/2}$ lower bound is found to be at 1.9 TeV. On the other hand, if the evolution is computed taking as input $[F(X)+\Delta]$ and the subtraction of $2\sigma^-$ is applied to $m_{1/2}$, the $m_{1/2}$ lower bound goes again below the Z^0-mass range. So if the search is for a lower bound, the result is that within 1σ, this is below the Z^0 mass. Without the EGM effect [23], in order to go below 21 TeV and reach the Z^0-mass range, two standard deviations are needed.

Figure 3 shows the range where the new lower bound of the SUSY threshold is predicted to be.

5. Conclusions

The evolution of the gaugino masses should be taken into account when dealing with the problem of predicting where the lower bound for SUSY breaking is expected to be. We have introduced a numerical method of iteration to deal with the evolution of the involved masses. We have shown that this correction is not negligible. When applied to the "best model" for SUSY-breaking predictions, it produces a decrease of more than two orders of magnitude in the lower bound of the SUSY breaking threshold. Already at the one standard deviation level of confidence, the predicted lower bound for SUSY breaking is below the Z^0 mass.

We emphasize that all experimental and theoretical knowledge in SUSY physics is included in the model adopted. Models based on a partial and restricted use of experimental and theoretical knowledge are not very meaningful. Now a comment. The real problem is the discovery of the first example of a SUSY particle or of a clear SUSY effect. Experimental searches should be encouraged, incorrect claims for a high SUSY threshold [19] stopped, and attention concentrated on the solution of the real problems, which are very numerous: both in the technological and in the theoretical areas.

TABLE 1

The inputs of the numerical iterative solution of the coupled equations for the evolutions of the gaugino masses.
For details, see Ref. [20].

ARBITRARY INPUTS

$m_{\tilde{g}}$, $m_{\tilde{W}}$, $m_{1/2}$

INPUTS FROM DATA (EXPERIMENT AND THEORY)

$[\alpha_3(m_Z)]^{-1} = 8.85$

$[\alpha_{em}(m_Z)]^{-1} = 127.9$

$\sin^2\theta\,(m_Z) = 0.2331$

$$[(\alpha_{GUT})_0]^{-1} = \frac{1}{20}\left[\frac{3}{\alpha_{em}(m_Z)} + \frac{12}{\alpha_3(m_Z)}\right] = 24.5$$

$$(m_{GUT})_0 = m_Z \exp\left[\frac{\pi}{10}\left(\frac{1}{\alpha_{em}(m_Z)} - \frac{8}{3\alpha_3(m_Z)}\right)\right] =$$

$$= 1.55 \times 10^{16}\ \text{GeV}$$

Allowed range for F(X)

TABLE 2

Experimental errors combined with theoretical uncertainties and the values of $[F(X) \pm \Delta]$ and $[F(X) \pm \Delta']$ corresponding to 1σ and 2σ, respectively.

$1\sigma = \begin{matrix} +11.3 \\ -14.3 \end{matrix}$	$F(X) \pm \Delta = \begin{matrix} +22.31 \\ -\ 3.39 \end{matrix}$
$2\sigma = \begin{matrix} +21.4 \\ -24.4 \end{matrix}$	$F(X) \pm \Delta' = \begin{matrix} +32.36 \\ -13.44 \end{matrix}$
$[F(X)]_{\text{central value}} = +10.96$	

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Please note that for the notations, formulae and other details not discussed in the present paper, we refer the reader to this work.

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Figure captions

Fig. 1: The values of $\alpha_3(m_{\tilde{g}})/\alpha_{GUT}$ and $\alpha_2(m_{\tilde{W}})/\alpha_{GUT}$ versus F(X). The function F(X) represents all our knowledge in SUSY physics, i.e. experimental data and theoretical estimates of the unknowns. The range where F(X) values lie depends on the experimental errors and the theoretical uncertainties. At present, within one σ, F(X) can be in the range from +22.31 to −3.39. The two-σ limits are from +32.36 down to −13.44. However for very negative values of F(X) the formula allowing predictions for the lower bound of the SUSY threshold needs to be reconsidered.

Fig. 2: Showing the contribution to $\log(m_{1/2}/m_Z)$ from the evolutions of the gaugino masses, i.e. the so-called EGM effect, as computed with our numerical iterative method. The EGM effect depends on the input value for F(X), the function which represents all knowledge on SUSY (experimental data and theoretical estimates) as discussed in the text.

Fig. 3: Predictions for SUSY-threshold lower bound, with and without corrections for the evolutions of the gaugino masses (EGM). The grey range indicates the physics troubles when the lower bound for SUSY threshold is too low, as discussed in the text.

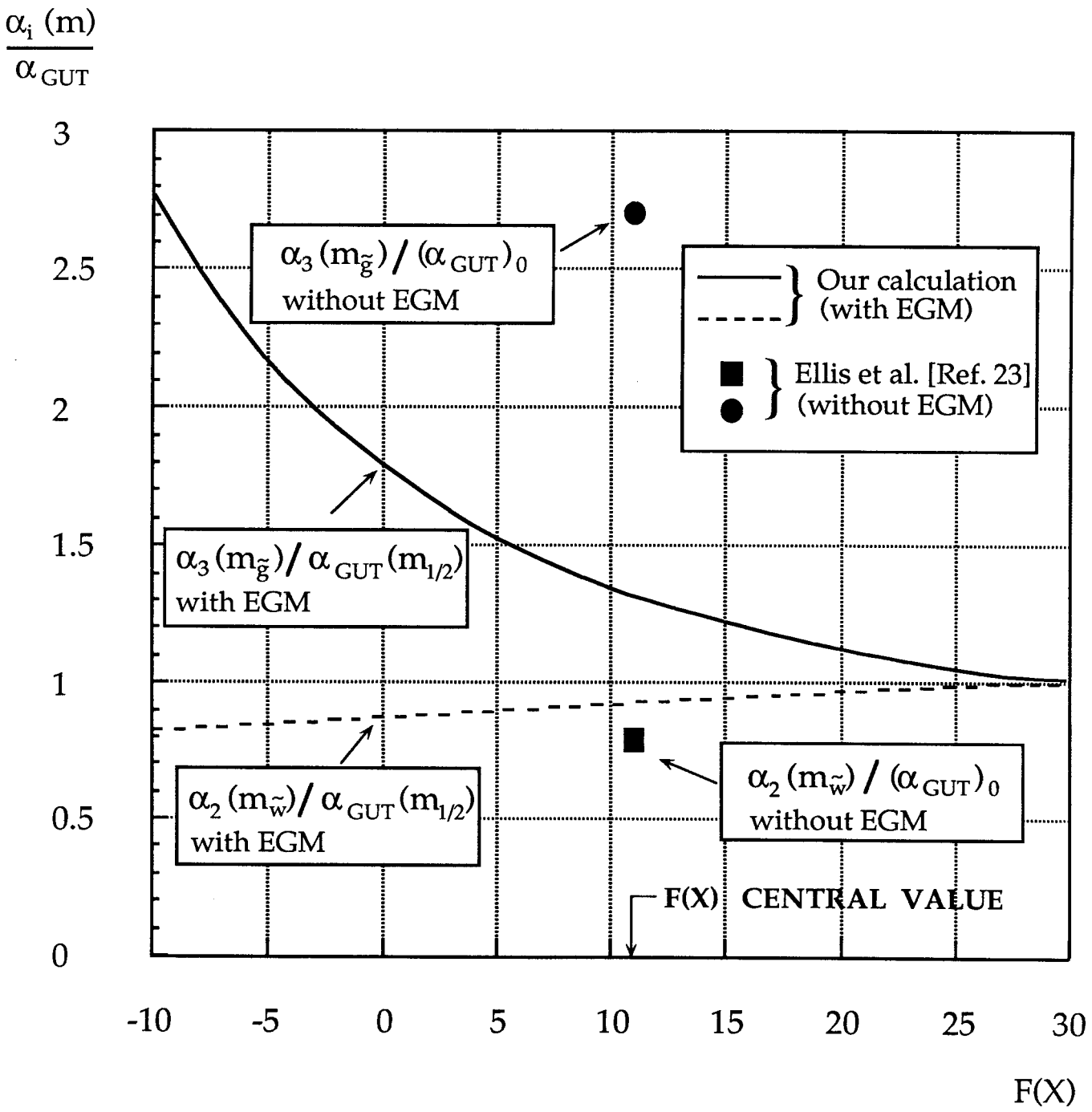
$\frac{\alpha_i\,(m)}{\alpha_{GUT}}$
3
2.5
2
1.5
1
0.5
0
-10
-5
0
5
10
15
20
25
30
F(X)
$\alpha_3(m_{\tilde{g}})/(\alpha_{GUT})_0$ without EGM
Our calculation (with EGM)
Ellis et al. [Ref. 23] (without EGM)
$\alpha_3(m_{\tilde{g}})/\alpha_{GUT}(m_{1/2})$ with EGM
$\alpha_2(m_{\tilde{w}})/\alpha_{GUT}(m_{1/2})$ with EGM
$\alpha_2(m_{\tilde{w}})/(\alpha_{GUT})_0$ without EGM
F(X) CENTRAL VALUE

Figure 1

EGM effect

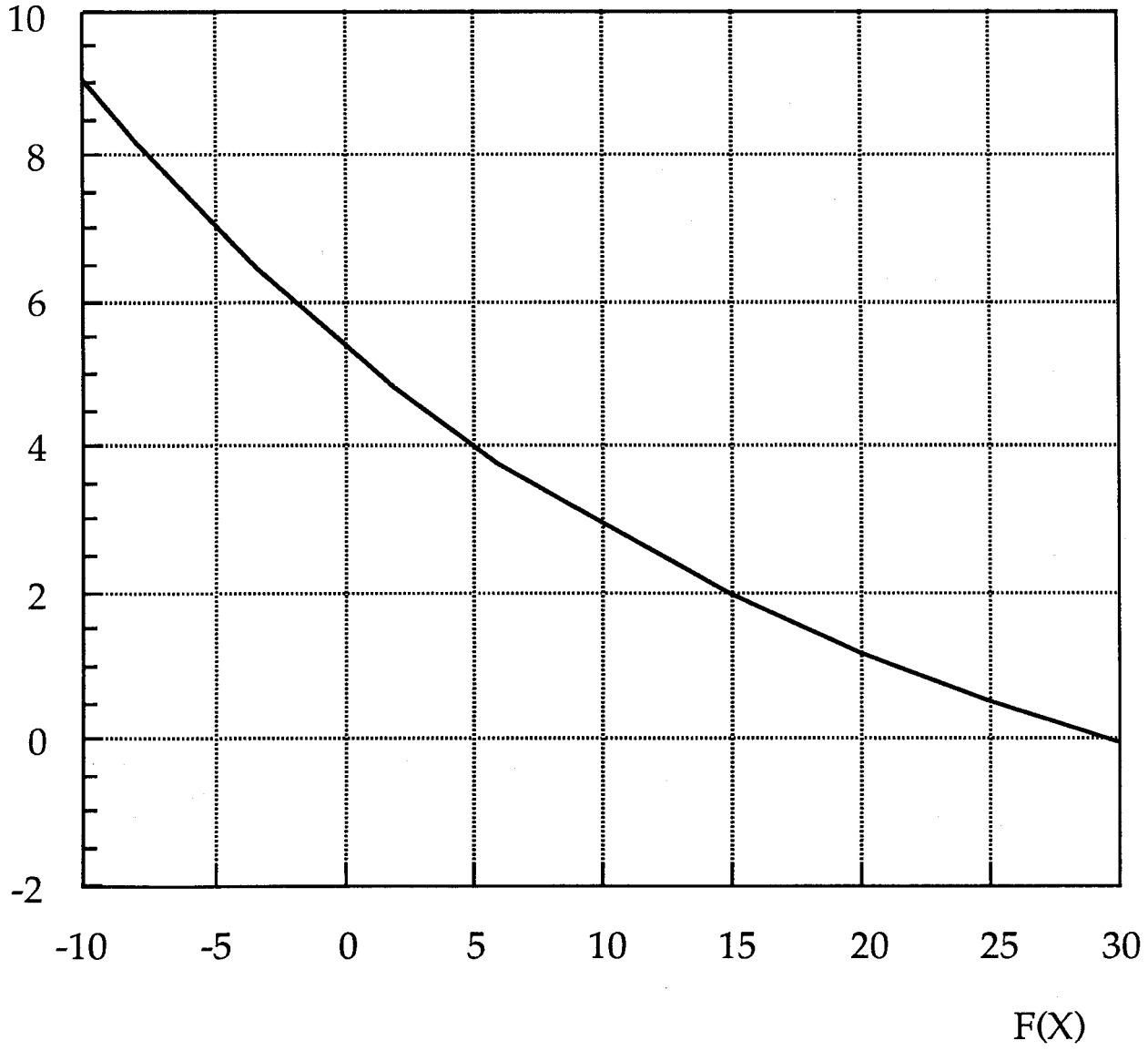

Figure 2

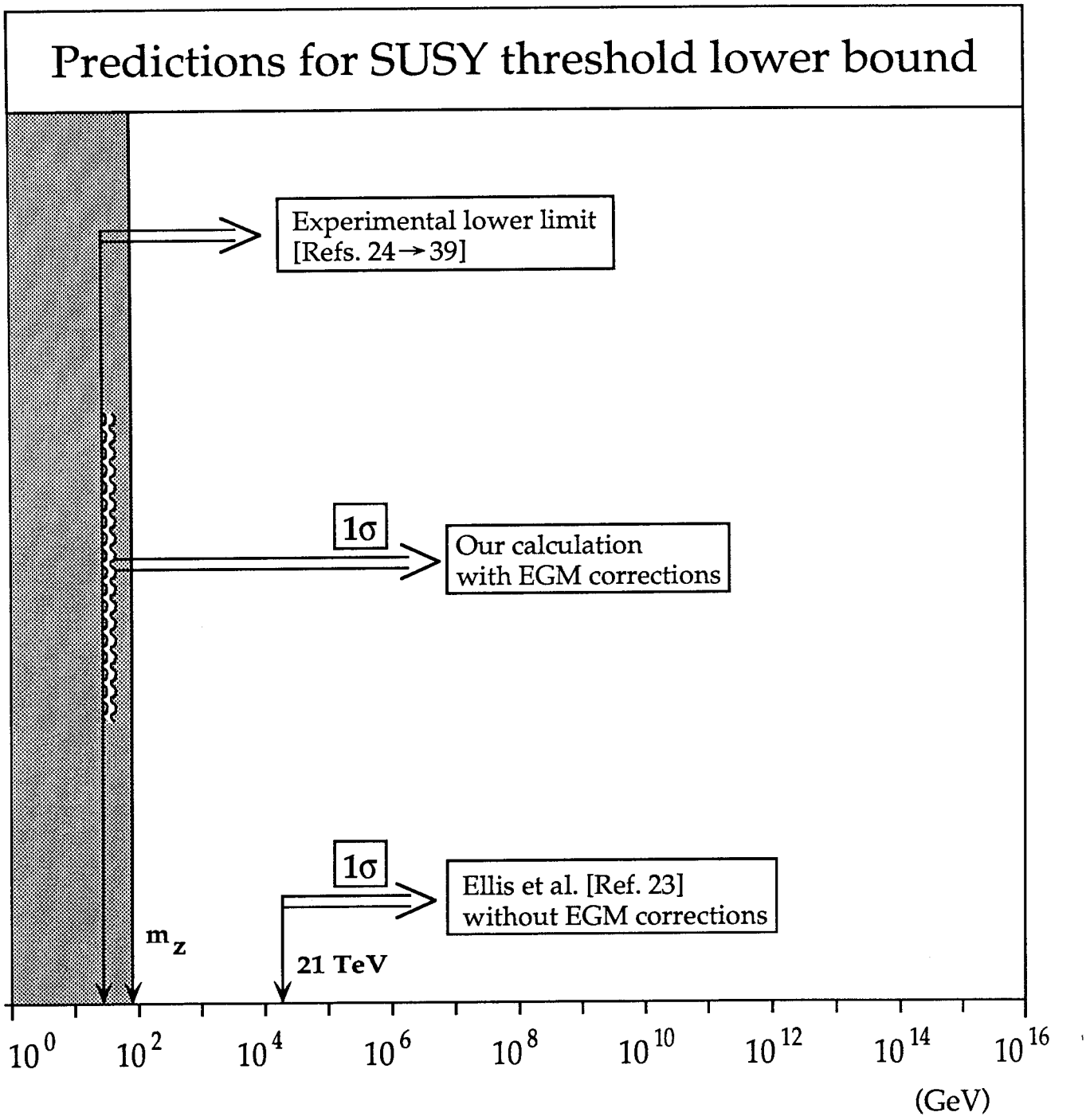
Predictions for SUSY threshold lower bound
Experimental lower limit
[Refs. 24→39]
1σ
Our calculation
with EGM corrections
1σ
Ellis et al. [Ref. 23]
without EGM corrections
m z
21 TeV
10 0
10 2
10 4
10 6
10 8
10 10
10 12
10 14
10 16
(GeV)

Figure 3