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The Evolution of Massive Stars. I

Shirô SAKASHITA*, Yôrô ÔNO* and Chushiro HAYASHI**

*Department of Physics, Hokkaido University, Sapporo
**Department of Nuclear Science, Kyoto University, Kyoto

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The evolution of stars with mass 15.6 times the solar mass is considered in their early stage. Our model star consists of three regions; a convective core consuming hydrogen, a radiative envelope, and an intermediate zone of continuously varying composition. Radiation pressure is taken into account throughout the three regions. The hydrogen content in the core ranges from 90 per cent to 6.3 per cent by weight. It is found that the convective core retreats as its hydrogen content decreases, setting up the intermediate zone of continuously varying composition and a thin convective unstable region between the radiative envelope and the intermediate zone. The evolutionary track in the $H-R$ diagram is compared with the stars of the $h+\chi$ Persei clusters.

§ 1. Introduction

It is known that many chemical elements observed in our galaxy have been synthesized by nuclear reactions occurring in the interior of stars in the course of their evolution.[1] The details of the synthesizing processes are intimately connected with the evolutionary changes in the stellar structures. Such evolutionary processes of stars have been studied by many authors applying various stellar models, especially by Hoyle and Schwarzschild[2] on the stars of the globular clusters. However, little has been known about the nuclear processes in massive stars which seem to play an important role in element sythesis on account of their rapid evolution.

It is known that stars with different masses evolve differently. Tayler[3] and Kushwaha[4] studied the evolution of massive stars in their early stages and proposed models consisting of a radiative envelope, an intermediate zone in which the mean molecular weight varies continuously, and a convective core. One of the characteristic features of massive stars is the effectiveness of radiation pressure. Although its effect was shown to be negligible for the mass ($\sim 10\,M_{\odot}$) chosen by Tayler and Kushwaha, it is expected that the radiation pressure becomes effective, as the mass of stars increases. Its effects appear particularly in the condition of convective stability, that is, they make the stellar configuration more unstable against convection. In this respect, Kushwaha's treatment, which neglects the effect of radiation pressure on convective instability, leaves something unsatisfactory.

The purpose of this paper is to investigate the structure of the more massive stars ($15.6\,M_{\odot}$) in which the effect of radiation pressure is no longer negligible.

Including the initial homogeneous model, five model sequences are obtained in the early stages of the evolution as the hydrogen content in the core decreases. The evolutionary path is compared with the $H-R$ diagram of the $h+\chi$ Persei stars obtained by Johnson and Hiltner[5] which includes the most massive stars observed. These results will give a basis for the study of the later stages of the evolution, the stages of gravitational contraction and helium reactions.

§ 2. Physical assumptions and basic equations

The model here considered consists of a helium enriched core in convective equilibrium, an envelope of almost pure hydrogen in a radiative equilibrium, and an intermediate zone of variable composition in radiative equilibrium. The following subscripts are used to denote the quantities in each part; e=envelope, i=core, c=center, 1=interface between the radiative envelope and the intermediate zone, and f=interface between the intermediate zone and the convective core.

Simpifying assumptions are made as follows.

1. There is no mixing of matter in the envelope, so that the envelope has a constant composition. In the convective core, the mixing caused by the convective turbulence is fully effective so that the composition is homogeneous.

2. The opacity is due to electron scattering only. Kushwaha's calculation showed that the main opacity is electron scattering for the model of ten solar masses. It seems reasonable, therefore, to assume that in the more massive stars the electron scattering opacity is predominant.

3. Steady ejection of mass from the surface of star is neglected, i. e., the total mass of star remains constant throughout the whole stage of evolution.

4. The initial hydrogen and metal contents have been taken as $X_e=X_i=0.900$, and $Z_e=Z_i=0.020$, which give as the mean molecular weight in the envelope, $\mu_e=4/(5X_e-Z_e+3)=0.535$.

The basic equations of mechanical and thermal equilibrium for the radiative zone are

$$\frac{dM_r}{dr}=4\pi r^2\rho,\quad \frac{dP}{dr}=-\frac{GM_r}{r^2}\rho,\quad \frac{dT}{dr}=-\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L_r}{4\pi r^2}. \tag{1}$$

Owing to the assumption 2, the opacity is taken as

$$\kappa=0.19(1+X). \tag{2}$$

The equation of state is given by

$$P=k\rho T/\mu H+aT^4/3. \tag{3}$$

Composition parameters are defined as

$$l\equiv\frac{\mu}{\mu_e},\quad j\equiv\frac{\mu}{\mu_e}\frac{1+X}{1+X_e}. \tag{4}$$

In the homogeneous envelope, the value of l and j are $l=1$, $j=1$. Composition parameters in the intermediate zone are not constant. Following the method of Härm and Schwarzschild,[6] we divide the intermediate zone into a number of thin shells and express l and j in each shell by the formula

$$l=(q/q_n)^{-\nu_n} l_n, \quad j=l^{0.285}, \tag{5}$$

where the suffix n represents the n-th shell and l_n and q_n are the values at its outer edge.

In terms of nondimensional variables

$$P=p\frac{GM^2}{4\pi R^4}, \quad T=t\frac{\mu_e H}{k}\frac{GM}{R}, \quad M_r=qM, \quad r=xR, \tag{6}$$

and the above composition parameters, the basic equations (1) for the radiative regions, the envelope and the intermediate zone, are expressed as

$$\frac{dp}{dx}=-\beta\frac{pq}{x^2 t}l, \quad \frac{dq}{dx}=\beta\frac{x^2 p}{t}l, \quad \frac{dt}{dx}=-C\beta\frac{p}{t^4 x^2}j, \tag{7}$$

with $$\beta=1-At^4/p,$$

$$A\equiv\frac{4\pi a}{3}\left(\frac{\mu_e H}{k}\right)^4 G^3 M^2, \quad C\equiv\frac{3}{4ac}\frac{0.19(1+X_e)}{(4\pi)^2}\left(\frac{k}{\mu_e HG}\right)^4\frac{L}{M^3}. \tag{8}$$

Homology variables in the envelope are given by

$$U_e=\frac{x^3 p}{qt}l\beta, \quad V_e=-\frac{q}{xt}l\beta, \quad (n+1)_e=\frac{q}{C}\frac{t^4}{p}. \tag{9}$$

For the convective core, the solutions including the radiation pressure have previously been obtained by Henrich.[7] In terms of Henrich's variables, the homology invariants in the convective core are expressed as

$$U_i=\frac{-\eta\zeta}{dZ/d\eta}, \quad V_i=-\frac{5}{2}\frac{\eta\zeta(dZ/d\eta)}{\pi}, \quad (n+1)_i=\frac{\theta}{\pi}\frac{d\pi}{d\theta}. \tag{10}$$

§ 3. Construction of models

The equations for the envelope have two parameters A and C. Taking the mass equal to 15.6 $M_\odot$ and the mean molecular weight $\mu_e=0.535$, we determine the value of A from the definition (8) as $A=16.0$. Then, we have a series of solutions which are specified by the value of C.

Solutions near the surface ($q\simeq 1$) are given by

$$p=t^4/4C, \quad t=\beta_0(1/x-1)/4, \quad \beta_0=1-4AC, \tag{11}$$

which are used as the starting values for the numerical integrations.

The homogeneous model, which represents the initial stage of evolution, is constructed as follows. With the above starting values, the integration over the envelope is carried out inward and is stopped at the point where the solution reaches the convective instability. The condition of the convective instability in the homogeneous region is given by

$$(n+1)_{rad} \leqq (n+1)_{ad}, \tag{12}$$

where

$$(n+1)_{rad} = \frac{GM_r \rho}{\kappa L_r} \frac{16\pi acT^4}{3P}, \quad (n+1)_{ad} = \frac{32-24\beta-3\beta^2}{8-6\beta}. \tag{13}$$

At the point of convective instability, the envelope solution is fitted to the core solution continuously. The continuity of physical quantities at the interface between the envelope and the core leads to the relations

$$\beta_{1e} = \beta_{1i}, \quad U_{1i}/U_{1e} = V_{1i}/V_{1e} = 1. \tag{14}$$

Among the families of the solutions for the radiative envelope and the convective core, the solution which satisfies the above fitting conditions is determined uniquely. In this way mathematical properties of the model and the values of q_1, x_1, t_1, β_1 and β_c, are completely determined.

The model sequence following the initial model is obtained as follows. In each step of the parameter C, the integration over the envelope is carried out inward as in the case of the initial model and is stopped at the point where $q=q_1^{(0)}$, $q_1^{(0)}$ being the mass fraction of the convective core of the initial model. From this point, the integration over the intermediate zone of variable composition is carried out inward.

It must be noted here that, if the radiative solution of the envelope reaches the point $q=q_1^{(0)}$ before the convective instability sets in, the envelope is surely in radiative equilibrium. Certainly, the models calculated by Kushwaha and Tayler satisfy these conditions. In our case, however, the effect of radiation pressure is more serious on account of the larger mass, and the envelope solution reaches convective instability before it reaches the point $q=q_1^{(0)}$ as shown in Fig. 1.

The above result seems to suggest a sequence of models such that the mass of the convective core grows up as its hydrogen content decreases, the mean molecular weight being discontinuous at the interface between the core and the radiative envelope. However, it is easily shown that this suggestion is not successful in the case in which the opacity is due to electron scattering. In this case the continuity of the radiation flux at the interface requires

$$(1+X_e)(n+1)_e = (1+X_i)(n+1)_i. \tag{15}$$

Since $X_i < X_e$, the above condition gives $(n+1)_i > (n+1)_e$=critical value of convective instability, which means that the core solution near the interface must be in radiative equilibrium contrary to the assumption.

On the basis of evolutionary considerations about the variation of the hydrogen content, it is possible to construct a consistent model as follows; an intermediate radiative zone with variable composition grows inward from the point $q=q_1^{(0)}$, and

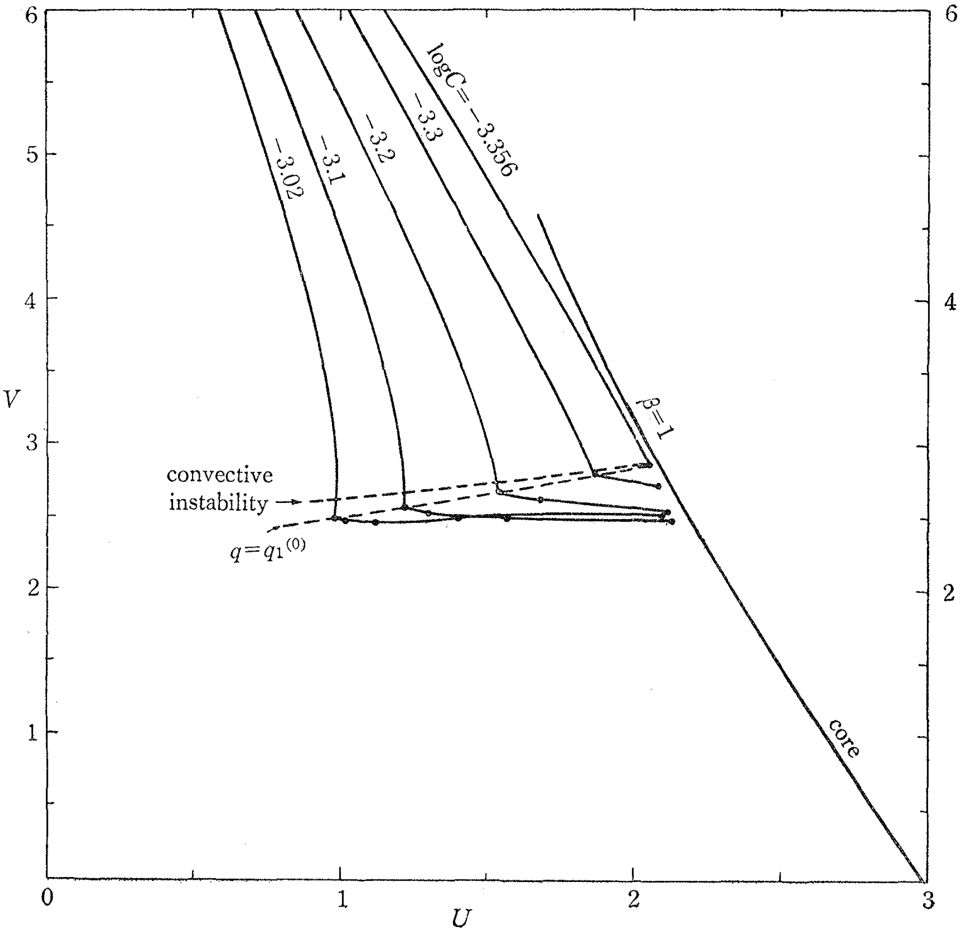

Fig. 1. Representations of model series in terms of the homolgy invariants U and V. The numbers on the envelope solutions denote the values of $\log C$. The fitting points are indicated by dots. The dashed line represents the locus of the points where each envelope solution reaches convective instability.

simultaneously a new convective zone is set up between this intermediate zone and the radiative envelope. In our model, however, the mass fraction of this convective zone is so small that the difference between the convective and radiative equilibrium can be neglected in a good approximation, that is, this region can be treated as an extension of the radiative envelope.

Thus, the radiative envelope is fitted to the intermediate zone at the point where $q=q_1^{(0)}$, and the intermediate zone is fitted to the convective core where it reaches the convective instability. The fitting conditions are the same as in the initial homogeneous case; i.e., the continuity of U, V and β, and in addition the convective instability at the interface are required. Since the mean molecular weight varies continuously in the intermediate zone, the condition of convective instability is slightly altered on account of a gradient of the mean molecular weight. This condition is expressed as

Table 1. Mathematical and physical characteristics of models.

Model	0	1	2	3	4
$\log C$	-3.356	-3.300	-3.200	-3.100	-3.020
U_1	2.06	1.87	1.54	1.22	0.975
V_1	2.87	2.78	2.67	2.57	2.50
$(n+1)_1$	2.80	2.83	2.90	2.98	3.05
$\log x_1$	-0.487	-0.518	-0.575	-0.649	-0.721
$\log p_1$	$+1.053$	$+1.136$	$+1.295$	$+1.509$	$+1.709$
$\log t_1$	-0.368	-0.332	-0.265	-0.184	-0.111
$\log q_1$	-0.382	-0.382	-0.382	-0.382	-0.382
β_1	0.952	0.945	0.929	0.909	0.888
U_f		2.08	2.12	2.13	2.09
V_f		2.72	2.55	2.48	2.55
$(n+1)_f$		2.87	2.99	3.14	3.31
$\log x_f$		-0.539	-0.648	-0.803	-1.026
$\log p_f$		$+1.193$	$+1.485$	$+1.894$	$+2.470$
$\log t_f$		-0.313	-0.201	-0.059	$+0.123$
$\log q_f$		-0.422	-0.512	-0.627	-0.774
β_f		0.942	0.918	0.881	0.832
ν		0.796	0.830	0.936	0.963
X_i	0.900	0.794	0.575	0.319	0.063
$\log L/L_\odot$	4.203	4.259	4.359	4.459	4.539
$\log R/R_\odot$	0.599	0.642	0.731	0.829	0.937
M_{bol}	-5.89	-6.03	-6.28	-6.53	-6.73
$\log T_e$	4.514	4.506	4.487	4.462	4.429
$\log T_c$	7.499	7.508	7.528	7.555	7.612
B. C.	-3.32	-3.28	-3.18	-3.04	-2.86
$B-V$	-0.42	-0.41	-0.39	-0.37	-0.35
life (year)	0	3.92×10^6	9.79×10^6	1.41×10^7	1.67×10^7

$$\frac{1}{(n+1)_{rad}} \geqq \frac{1}{(n+1)_{ad}} + \frac{\beta}{4-3\beta}\,\frac{d\log\mu}{d\log P}, \qquad (16)$$

where $(n+1)_{rad}$ and $(n+1)_{ad}$ are defined by equations (13). The gradient of mean molecular weight tends to suppress the convection, and consequent-

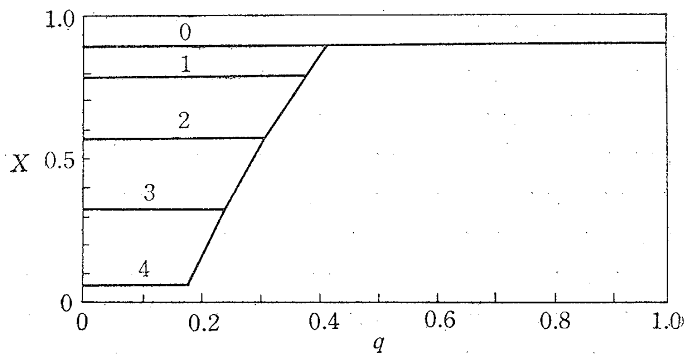

Fig. 2. The variation of the hydrogen content from the center to the surface.

ly guarantees that the intermediate zone is in radiative equilibrium.

The values of the exponent ν_n which appears in equation (5) are determined successively in such a way that the solutions satisfy the above fitting conditions. The mathematical characteristics of these solutions are listed in Table 1, and their $U-V$ curves are shown in Fig. 1. The distribution of the hydrogen content throughout the envelope, the intermediate zone and the core is plotted in Fig. 2.

§ 4. Physical properties and discussions

For each stellar model the physical quantities of interest are obtained from the mathematical ones by equating the rate of the energy production to the stellar luminosity. The energy production in the core results approximately, by replacing the solution near the center by the polytropic one, in the luminosity

$$L \simeq \mathcal{E}_0 \left\{ \frac{3n_1+3}{2n_1+n_1/n_3} \cdot \frac{k}{2G\mu_i H\beta_c} \right\}^{3/2} \rho_c^{1/2} T_c^{s+3/2}, \tag{17}$$

where n_1 and n_3 are the polytropic indices

$$1+1/n_1 \equiv (d \log P/d \log \rho)_c, \quad 1/n_3 \equiv (d \log T/d \log \rho)_c, \tag{18}$$

and $\mathcal{E}_0$ and s are constants in the formula of the energy production

$$\mathcal{E} = \mathcal{E}_0 \rho T^s. \tag{19}$$

Noting that the main nuclear energy source is due to the $C-N$ cycle, we take as the appropriate values of $\mathcal{E}_0$ and s

$$\mathcal{E}_0 = 8 \times 10^{-114} \cdot X X_{CN} \,\mathrm{erg\, cm^3\, g^{-1}\, deg^{-16}}, \quad s=16, \tag{20}$$

with $X_{CN} = (1/3)Z$. The central temperature is determined in such a way that the total energy production obtained from (17) should be equal to the luminosity appeared in the envelope solution. The results of these calculation are shown in Table 1.

We are now in a position to check the validity of the assumptions. The first check is concerned with the stability of the new convective zone which is set up between the radiative envelope and the intermediate zone. If a certain perturbation stirs up hydrogen at the interface between this convective zone and the intermediate zone, mixing may occur throughout both regions, and then the gradient of mean molecular weight may be set up. This gradient suppresses the convection and leads the state to the radiative equilibrium. It is found that the mixing of only a fraction of per cent of hydrogen is enough to lead the state to the radiative one. Thus, the assumption that there is no mixing between the convective zone and the intermediate zone is shown to be valid.

The second check is made on the energy production in the last model. Since the hydrogen content in the last model decreases to six per cent, the assumption that the energy generation is confined to the core may break down. The result of the integration of energy production throughout the intermediate zone shows that the energy generation outside the core does not exceed five per cent of the core.

The evolutionary path in the $H-R$ diagram is shown in Fig. 3. The star leaving the main sequence evolves rather slowly towards right, i.e., the luminosity increases slightly and the effective temperature decreases. The lifetime of the evolution along the path is longer than that might have been expected. This is because the mass of the convective core relative to the total mass becomes large on account of the effect of radiation pressure. The theoretical evolutionary path and the observed data for the $h+\chi$ Persei stars obtained by Johnson and Hiltner[5] are compared in the $H-R$ diagram. It seems that the upper part of the $h+\chi$ Persei is now in a stage of gravitational contraction, having passed through an isothermal core stage. It is inferred that the time spent in the gravitational contraction stage may be rather short for these massive stars on account of the large mass fraction contained in the core. This might explains the gap in the diagram which is found between the early and late type supergiants.

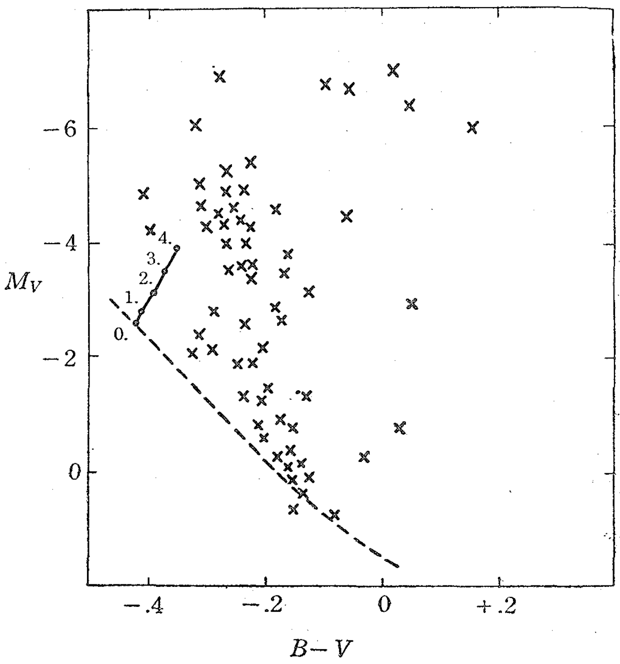

Fig. 3. The evolutionary track in the $H-R$ diagram. Successive evolutionary models are indicated by dots. Crosses represent the observed positions for the $h+\chi$ Persei stars.

Finally, it is noted that we have assumed no steady ejection of mass from the surface, i. e., the total mass remains constant. Nevertheless, it is found by Slettebak[8] that the line broadening in the supergiant stars of early type is due almost entirely to large scale atmospheric motions. Basing on this evidence, we speculate that the steady ejection from the surface might occur in massive stars. A rough estimate shows that the *loss of about one solar* mass for initial 15.6 $M_\odot$ in last 10^7 years is enough to remove the unstable convection zone in our models.

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