The evolution of spillovereffects between oil and stock marketsacross multi-scales using a wavelet-basedGARCH-BEKK model

Xueyong Liu[a], Haizhong An[a,b,c,*],Shupei Huang[a] and Shaobo Wen[a]

[a] School of Humanities and Economic Management, China University of Geosciences, Beijing 100083, China

[b] Key Laboratory of Carrying Capacity Assessment for Resource and Environment, Ministry of Land and Resources, Beijing 100083, China

[c] Open Lab of Talents Evaluation, Ministry of Land and Resources, Beijing 100083, China

Abstract

Aiming to investigate the evolution of mean and volatility spillovers between oil and stock marketsin the time and frequency dimensions,we employed WTI crude oil prices, the S&P 500 (USA) index and the MICEX index (Russia) for the period Jan. 2003-Dec. 2014 as sample data. We first applied a wavelet-based GARCH-BEKK method to examine the spillover features in frequency dimension.To consider the evolution of spillover effectsin time dimension at multiple-scales, we then divided the full sample period into three sub-periods, pre-crisis period, crisis period, and post-crisis period. The results indicate thatspillover effects varyacross wavelet scales in terms of strength and direction.By analysis the time-varying linkage, we found the different evolution features of spillover effects between the Oil-US stock market and Oil-Russia stock market. The spillover relationship between oil and US stock market is shifting to short-termwhile the spillover relationship between oil and Russia stock market is changing to all time scales. That result implies that the linkage between oil and US stock market is weakening in the long-term, and the linkage between oil and Russia stock market is getting close in all time scales. This may explain the phenomenon that the US stock index and the Russia stock index showed the opposite trend with the falling of oil price in the post-crisis period.

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Keywords: Multi-scale; volatility spillover; oil price; stock index; wavelet; GARCH-BEKK model

1 Introduction

[*]Correspond to: School of Humanities and Economic Management, China University of Geosciences, Beijing, China. Tel.: +861082323783; Fax: +861082321783. E-mail addresses: ahz369@163.com

The spillover effect refers to the information transmission between financial markets; its essence is the risk transfer between markets [1].The rapid development of globalization and dramatic improvement in trading technology make the linkage among markets increasingly close, which leads to faster information transmission between different financial markets[2].Even though investments can be done in different countries and financial markets, the close interactionsamong markets could spreadrisks quickly among countries' financial markets, leading to a "contagious financial crisis". Therefore, investigating the spillover effects between markets could be helpful to deepen the understanding of the financial markets fluctuations.

In recent decades, there has been increasing interest in modeling the mean and volatility spillovers that exists across different financial markets [3-7].Tse finds that there is a significant bidirectional information flow between the Dow Jones Industrial Average index and the index futures markets with a bivariate EGARCH model [3].Papapetrou studied the dynamic relationship among oil prices, real stock prices, interest rates, real economic activity and employment through a multivariate VAR approach [4]. Papapetrou made a conclusion that oil price changes affect real economic activity and employment while stock returns do not lead to changes in real activity or employment.Mensiexamined the return links and volatility spillovers between the S&P 500 and commodity price indices using a VAR-GARCH model [5].Lee applied the dynamic conditional correlation (DCC), constant conditional correlation (CCC) and BEKK models to investigate the volatility spillover between the stock prices of the Group of Seven and WTI crude oil prices [6]. Zhanginvestigated the volatility spillovers between stock and bond markets in the G7 and BRICS countries using a newly developed causality-in-variance test [7].

GARCH family models successfully portray the fluctuation characteristics and volatility spilloversbetween financial time series [8-11]. However, most previous researches mainly examinethe cross-market volatility spillover effects from the time dimension, and ignore the frequency dimension characteristicsthat exist in financial time series.The features in frequency dimension can help us to understand the market information from another perspective. Considering the frequency dimension characteristics, the wavelet methodis introducedto analyze spillovers,which extendvolatility spillovers between time series into joint time-frequency domain [12-15].Khalfaoui introduced a wavelet-based MGARCH method to study the linkage across the crude oil market (WTI) and the stock markets of the G7 countries [16]. Zhou used the maximal overlap discrete wavelet transform (MODWT) to examine the international linkage among REIT returns and volatilities among seven countries [17].The empirical results show that the market linkage could differ across time scales.Comparing the wavelet-based multi-resolution GARCH model with a traditional multivariate GARCH, Huang found that the former model performedbetter in capturing the complex pattern of return and volatility spillovers [18].

Moreover, previousliterature has examined the mean and volatility spillovers between crude oil markets and stock marketsover the whole sample period, which may mask the dynamic information over time existing in markets [19].Take the study of Khalfaoui for example, they captured the multiscale features of linkage by applying the wavelet method. However, they ignore the change of dynamic spillover effects over time.According to the recent researches, the transmission of information between these markets is not constantand the linkage is time varying [1, 20-24],so it is also necessary to consider the time varying change of thespillover effects during different time periods [20, 25]. Therefore, based on previous studies, to consider the dynamic feature of the spillover effects we take the international crude oil price and stock markets as example, the entire time period of the empirical data studied in this paper will be divided into several sub-periods, and then a wavelet-based MGARCH-BEKK model will be applied to study the spillovers across crude oil markets and stock markets.The main innovation of this paper is in examining the evolution of the unstable mean and volatility spillovers in thetime dimension andfrequency dimensions between crude oil market and stock markets.

2 Methodology

2.1 Multi-resolution wavelet transform

Wavelet theory is a very effective toolfornonlinear time series analysis and provides a method for multi-resolution analysis, that is,different frequencies processed by different resolutions. There are two types of wavelet transform, namely continuous and discrete one. Based on our research purpose, feature extraction, we choose the later one.The discrete wavelet transform (DWT) decomposes a time seriessignal $X(t)$into a set of subsequences based on two types of filters called the wavelet filter and the scaling filter. We denote the two wavelet filter and the scaling filter by$h_l$and g_l,respectively, $(l = 0, \ldots, L-1)$. The wavelet and scaling coefficients, $W_{j,t}$and $V_{j,t}$, at the $j$thlevel are defined as [26]:

$$W_{j,t} = \sum_{l=0}^{L-1} h_{j,l} X(t-l) \quad (1)$$

$$V_{j,t} = \sum_{l=0}^{L-1} g_{j,l} X(t-l) \quad (2)$$

Althoughthe DWT method has been widely applied to time series analysis in many fields, it has two main drawbacks: the dyadic length requirement and the fact that the wavelet and scaling coefficients are not shift invariant due to their sensitivity to circular shifts because of the decimation operation [27, 28]. The MODWT is a modified version of the DWT, whose definition is obtained directly from the DWT method. The MODWT wavelet $\tilde{h}_l$and scaling $\tilde{g}_l$filters at$j$th level of decompositionare directly defined as:

$$\tilde{h}_{j,l} = h_{j,l}/2^{j/2} \text{ and } \tilde{g}_{j,l} = g_{j,l}/2^{j/2} \quad (3)$$

Similarly, the wavelet and scaling coefficients are obtained as follows:

$$\widetilde{W}_{j,t} = \frac{1}{2^{j/2}} \sum_{l=0}^{L-1} \tilde{h}_{j,l} X(t-l) \quad (4)$$

$$\tilde{V}_{j,t} = \frac{1}{2^{j/2}} \sum_{l=0}^{L-1} \tilde{g}_{j,l} X(t-l) \quad (5)$$

We can find that the MODWT wavelet coefficients at each scale will have the same length as the original signal $X$from the above expressions. According to the equations above, the wavelet and scaling coefficients can also be expressed in matrix notation as follows:

$$\widetilde{\mathrm{W}}_j = \widetilde{w}_j X \text{ and } \widetilde{\mathrm{V}}_j = \tilde{v}_j X \quad (6)$$

Then, we can finally recover the original time series X from its MODWT as follows:

$$\mathrm{X} = \sum_{j=1}^{J} \widetilde{w}_j^T \widetilde{W}_j + \tilde{v}_J^T \tilde{V}_J = \sum_{j=1}^{J} \widetilde{D}_j + \tilde{S}_J \quad (7)$$

where $\widetilde{D}_j$ represents the MODWT details of original time series Xat scale jand $\tilde{S}_J$ is the MODWT smooth of X at scale J. The above equationdefines a MODWT-based multi-resolution analysis (MRA).

2.2 Wavelet-based VAR–GARCH–BEKK model

The price volatility transmission literature commonly uses Multivariate Generalized

AutoRegressive Conditional Heteroskedasticity models (MGARCH) to investigate volatility spillover effects because they are able to explicitly parameterize the sources and magnitudes of the spillover effects [29, 30]. In the present study, we adopt abivariateGARCH-BEKK model developed by Engle and Kronerto investigate volatility spillover between two markets [31]. The advantage of the BEKK specification is that it does not impose any restriction on the correlation structure between the variables [32].

Usually, the AIC and SC information criteria can be used to choose the optimal lag length of GARCH process (i.e., the values of p and q). However, in the study of T. Bollerslev, they found an interesting result. With small numbers of parameters, GARCH (1, 1) process is sufficient to model the variance dynamics of financial time series [33]. So similar to the previous researches [10, 16, 19, 34-36], we select one lag for the mean and variance equations.

Generally, a bivariateGARCH (1, 1) model can be definedbythe following equations:

$$R_t = X_t\theta + \epsilon_t, \qquad \epsilon_t|_{\Omega_{t-1}} \sim N(0, h_t) \tag{8}$$

$$h_t = a_0 + a_1\epsilon_{t-1}^2 + \beta h_{t-1} \tag{9}$$

where$X_t$is an vector representing the explanatory variables, $\theta$is acoefficient vector, ϵ_t is an vectorrepresentingthe normally distributed condition residuals, and$h_t$is a vector representing the conditional variance.

Specially, the bivariate GARCH-BEKK modeladopted in this study is specified as follows:

Mean equation:

$$R_t(i) = \begin{bmatrix} R_{s,t}(i) \\ R_{o,t}(i) \end{bmatrix} = \begin{bmatrix} \mu_s(i) \\ \mu_o(i) \end{bmatrix} + \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix} \begin{bmatrix} R_{s,t-1}(i) \\ R_{o,t-1}(i) \end{bmatrix} + \begin{bmatrix} \epsilon_{s,t}(i) \\ \epsilon_{o,t}(i) \end{bmatrix} \tag{10}$$

Variance equation:

$$H_t(i) = C'C + A'\epsilon_{t-1}(i)\epsilon_{t-1}'(i)A + B'H_{t-1}(i)B \tag{11}$$

$$C = \begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix}, \ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

where$R_t(i)$is an (2×1) vector of stock $s$and oil $o$attime $t$for wavelet scale$i$,$\mu_s(i)$ and $\mu_o(i)$are collectively referred to as the $\mu(i)$. So,$\mu(i)$represents long-termdrift coefficients and is also an (2×1) vector, $\epsilon_t(i)$is the vector of random errors attime t for wavelet scale$i$, and $\epsilon_t(i)|_{\Omega_{t-1}} = [\epsilon_{s,t}(i), \epsilon_{o,t}(i)]' \sim N\{0, H_t(i)\}$. $H_t(i)$ is an (2×2) vector representing the conditional variance matrix at scale i, theelement$C$is a constant coefficient matrix, $A$is the coefficient of conditional residual matrix, and$B$is the coefficient of conditional covariance matrix. The off-diagonal elements of the matrices A and B (a_{12}, a_{21}, b_{12} and b_{21})can reflect the volatility transmission and spillover between the oil and stock markets [16].

The above conditional variance equation (11) can be represented as an expanded form as follows:

$$H_t(i) = \begin{bmatrix} h_{s,t}(i) & h_{so,t}(i) \\ h_{os,t}(i) & h_{o,t}(i) \end{bmatrix} \tag{12}$$

where

$$\begin{aligned} h_{s,t}(i) = {} & c_{11}^2 + a_{11}^2\epsilon_{s,t-1}^2(i) + 2a_{11}a_{12}\epsilon_{s,t-1}(i)\epsilon_{o,t-1}(i) + a_{21}^2\epsilon_{o,t-1}^2(i) \\ & +b_{11}^2 h_{s,t-1}(i) + 2b_{11}b_{12}h_{so,t-1}(i) + b_{21}^2 h_{o,t-1}(i) \end{aligned} \tag{13}$$

$$\begin{aligned} h_{o,t}(i) = {} & c_{12}^2 + c_{22}^2 + a_{12}^2\epsilon_{s,t-1}^2(i) + 2a_{12}a_{22}\epsilon_{s,t-1}(i)\epsilon_{o,t-1}(i) + a_{22}^2\epsilon_{o,t-1}^2(i) \\ & +b_{12}^2 h_{s,t-1}(i) + 2b_{12}b_{22}h_{so,t-1}(i) + b_{22}^2 h_{o,t-1}(i) \end{aligned} \tag{14}$$

$$\begin{aligned} h_{so,t}(i) = h_{os,t}(i) = {} & c_{11}c_{12} + a_{11}a_{21}\epsilon_{s,t-1}^2(i) + a_{22}a_{21}\epsilon_{o,t-1}^2(i) \\ & +[a_{11}a_{22} + a_{21}a_{12}]\epsilon_{s,t-1}(i)\epsilon_{o,t-1}(i) + b_{11}b_{21}h_{s,t-1}(i) \\ & +[b_{21}b_{12} + b_{11}b_{22}]h_{so,t-1}(i) + b_{22}b_{12}h_{o,t-1}(i) \end{aligned} \tag{15}$$

Eqs. (13), (14) and (15) revealhow shocks and volatility are transmitted across markets and over wavelet scales.The model is estimated by the maximum likelihood estimation method optimized by the BHHH algorithmto obtain the final estimate of the variance–covariance matrixwith corresponding standard errors. The conditional log likelihood function $L(\theta)$ can be expressed as follows:

$$L(\theta) = -T log(2\pi) - 0.5 \sum_{t=1}^{T} log|H_t(\theta)| - 0.5 \sum_{t=1}^{T} \epsilon_t(\theta)' H_t^{-1} \epsilon_t(\theta) \tag{16}$$

where $T$is the number of observations and θrepresents the vector of all unknown parameters.

3Empirical Study

3.1 Data and Reprocessing

The empirical data sets used in this study consist of daily crude oil prices and the daily stock market index. The data for the oil market are the daily spot closing prices of West Texas Intermediate (WTI). The typicaldaily stock indicesarethe S&P 500 (United States) and the MICEX index (Russia) [7, 36].All the data are collected from Wind Economic Database, and encompass the period from January 2003 to December 2014.Because the threemarkets operate with different holidays, some daily observations are deleted so that the threetime series match.Finally, there are 3017total observations in the sampling period. Then, the data are divided into three parts to consider the time-varyingspilloverduringnon-crisis and crisis periods [20, 25].To provide a visual comprehension aid, Figure 1 shows the empirical data and the three sub-periods:period 1 (the pre-crisis period), period 2 (the crisis period) and period 3 (the post-crisis period).

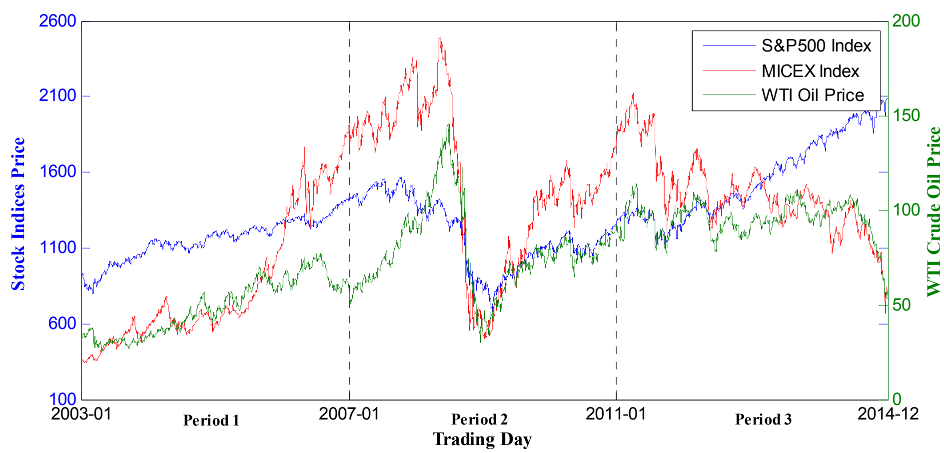

Figure 1 Daily price of the S&P500, MICEX index and spot oil priceof WTI from January 2003 to November 2014. The vertical dashed lines represent the subdivision into pre-crisis, crisis and post-crisis.

Wavelet analysis was applied to examine the complex multi-scale volatility transmissions between the crude oil market and stock market in the US. The raw data$S(t)$ were decomposed into five subsequences (D1, D2, D3, D4, and R4) using MODWT with Daubechies least asymmetric (LA) filters witha length of 8 [37].LA (8) is one of the wavelet filters which has been widely used in financial markets [38, 39]. Each detail (D1, D2, D3, and D4) represents the contribution of fluctuations of a specific time scale to the original price variations, while the smooth R4 represents its trend. As shown in Table 1, the various decomposition levels correspond to time scales: D1 (2 to 4 days), D2 (4 to 8 days), D3 (8 to 16 days) and D4 (16 to 32 days).As an example, Figure 2 shows the decomposition results of S&P 500 index.

Table 1 Wavelet decomposition scales and the corresponding periods

Wavelet decomposition scales	Shocking time scales (day)	
Scale 1	2-4	Daily
Scale 2	4-8	Weekly
Scale 3	8-16	Bimonthly
Scale 4	16-32	Monthly

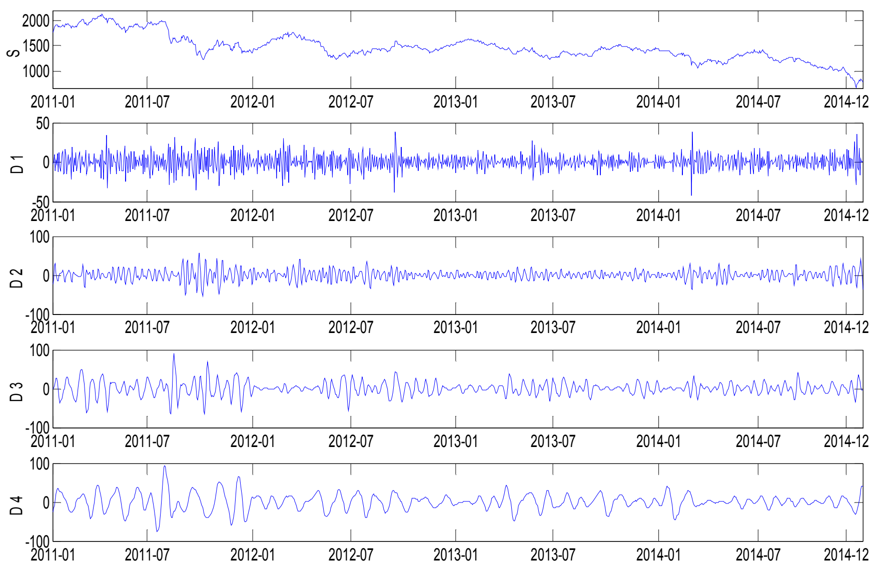

Figure 2 Wavelet decomposition of the S&P 500 (US) stock market index

Table 2 shows the selected descriptive statistics of the sample data at each wavelet scale. The sample mean, standard deviation, skewness, kurtosis, Jarque–Bera statistic and ADF test are reported for each scale. As shown in Table 2, the sample means of oil prices and the US stock index are equal to or approximately equal to zero. According to the standard deviations, the stock market has a greater volatility than the crude oil market. The Jarque–Bera (J–B) statistic test is consistently rejected at a high significance level, which means that all wavelet series are not Gaussian distributed. The kurtosis statistics are greater than 3, which suggest that the wavelet series tend to follow a leptokurtic distribution with higher peaks and fatter tails. The ADF test results reject the null hypothesis of a unit root with large negative values at the 1% level of significance, which suggests that the sample series used in this research can be regarded as stable, so econometric models can be established without a spurious regression problem.

Table 2 Descriptive statistics of wavelet components.

	D 1	D 2	D 3	D 4
WTI oil price				
Mean	0.000	0.000	0.000	-0.001
Std. Dev	0.702	0.779	1.005	1.407
Skewness	0.158	0.138	0.064	-0.127
Kurtosis	19.19	9.805	6.469	5.407
J-B (p-values)	0.000	0.000	0.000	0.000
ADF	-30.131***	-23.890***	-23.503***	-16.708***
Stationarity	Yes	Yes	Yes	Yes
USA stock index (S&P 500)				
Mean	0.000	-0.001	-0.003	-0.015
Std. Dev	5.650	6.483	8.347	11.392
Skewness	-0.046	0.188	0.014	0.013
Kurtosis	7.606	12.269	5.793	8.167
J-B (p-values)	0.000	0.000	0.000	0.000
ADF	-30.911***	-26.482***	-20.066***	-17.803***
Stationarity	Yes	Yes	Yes	Yes
Russia stock index (MICEX)				
Mean	0.001	0.009	0.012	0.000
Std. Dev	10.405	14.160	18.155	23.527
Skewness	0.064	0.077	0.173	0.220
Kurtosis	11.684	7.449	6.220	5.363
J-B (p-values)	0.000	0.000	0.000	0.000
ADF	-28.636***	-31.183***	-19.832***	-23.187***
Stationarity	Yes	Yes	Yes	Yes

***Significant at the 1% level.

3.2 Results and Discussion

In this section, we focused on the evolution of time-varying mean and volatility spillover between the crude oil market (WTI) and the stock markets (S&P 500, MICEX) across different time horizons based on a wavelet series (details). For analyzingthe spillovers between the oil market and the two stock markets, our research is based on a multivariate GARCH(1,1) model with BEKK parameterizations for the variance equation. The estimated results of the GARCH(1,1)–BEKK model on each time scale are reported in Tables 3 to 6 (Supplementary Tables in the end of paper).

Table 3 shows the estimation results based on the finest wavelet component, $D1$, which represents a short-term (daily fluctuations) variation due to the shocks occurringat time scales of 2 to4 days. Similarly, Table 4 and Table 5 give the estimation results based on the two wavelet components, $D2$ and $D3$, which represent mid-term (weekly to bimonthly fluctuations) variations at time scales of 4 to 8 days and 8 to 16 days, respectively. Moreover, estimation results based on thewavelet component, $D4$, which represent a long-term (monthly fluctuations) variation at the time scale of 16 to 32 days are shown in Table 6.

Tables 3-6 report the estimation results of the wavelet-based GARCH-BEKK model for the oil price and the two stockindices.We find that the mean and volatility spillovers are time varying and that the spillovers are unevenly spread across different wavelet scales.Compared with spillovers between Russia and WTI, there exist a significant evolution of the spillovers between USA and WTI which is changing from short term to long termover time. Table 7 are the summary of the time varying mean and volatility spillovers between the WTI crude oil market and two stock markets.

First, the spillovers or transmissions in the mean part are considered. The mean spillovers between the WTI oil market and thetwo stock markets(represent the US and Russia) are given by the significance of estimated coefficients φ_{12}andφ_{21}.From tables 3-6, we can see that there exist unidirectional, bidirectional or no mean spillovers between the WTI oil market and the two stock markets in differentperiodsacrossdifferent wavelet scales, i.e.,the price mean spillover varies across wavelet scales and time periods.For instance,the spillovers between WTI oil price and S&P500 index, in period 1 we note thatthe estimates reveal bidirectional linkages between the WTI oil market and US stock market at all wavelet scales except the short-term scale, D1. Only unidirectional mean spillovers (from the WTI oil market tothe US stock market) exist at scale D1.Then, in period 2, the unidirectional linkage becomesbidirectional at scale D1, while the bidirectional linkage becomesunidirectional (from the US stock market tothe WTI oil market) at scale D3. Finally, in period 3, there are no linkages at long-term scales D3 and D4. The bidirectional linkage at scale D1 becomes unidirectional (from the US stock market tothe WTI oil market).Similar to the linkage between Oil-US stock markets, the spillover effects between WTI oil price and the Russia MICEX index are time-varying, and the spillover relationship become increasingly closeover the observed sub-periods. All the mean spillover effects between WTI oil market and the two stock marketsare shown in Table 7.

Second, the spillovers or transmissions in the volatilitypart are considered.The off-diagonal elements of matrices A and B (a_{12},a_{21}, b_{12}, and b_{21}) can capture the volatility spillovers between markets.Similar to mean spillover, the volatility spillover between the WTI oil market and thetwo stock marketsvaries across wavelet scales and time periods.Sometimes, we seeunidirectional or bidirectional volatility linkages, and at other times, we see no volatility linkages between the markets under study.For instance,in period 1,thevolatility spillovers from the WTI oil market to the US stock market are significant onlyat scale D2, while the volatility spillovers from the US stock market to the WTI oil market are significant at scale D2, D3, and D4, but with much smaller magnitudes.Then, in period 2, there exist bidirectionaltransmissions at scale D2 and D3, unidirectional transmissions from the US stock market to the WTI oil market at scale D1, and no linkages between the two markets under study at the long-term scale, D4. Comparing the volatility spillovers between periods 2 and 3, we find that the bidirectionalvolatility spillovers disappeared at mid-term scale,D3. Compared to the volatility spillover relationship between WTI and US stock market, there exist commonly bidirectionalor unidirectional volatility spillover effects between the WTI price and Russia stock market except D1 wavelet scale in period 1. All the volatility spillover effects between the WTI oil market and theUS stock market are shown in Table 7.

Table 7reports the time-varying spillovers effects across wavelet scales and time periods between the WTI oil market and the two stock index. As we can see from Table 7, the spillovers between WTI oil price and Russia MICEX stock index are changing at every wavelet scale over time. However, there is a significant evolution between the WTI oil market and Russia stock market that the linkage is getting close at all wavelet scales. But for the linkage between the US stock market and the WTI oil market, there exists a significant evolution which is from short term to long term. The linkages between the WTI oil market and theUS stock market are strong in period 1 and period 2, and there exist significantbidirectional or unidirectional transmissionsbetween the two marketsacrossall wavelet scales.However, in period 3, the linkages disappear at long-term scales, D3 and D4. The spillovers in the volatility part show a similar pattern with the mean part that the contacts between the two markets under study gradually disappearat the long-term scales. This result indicates that the transmission of information between the crude oil market and the US stock market were gradually weakened and mainly maintained in short-term scale at the present stage. This maybe a reason why the Russia MICEX stock index has been falling while the US S&P 500 stock index keep going up with the WTI oil price going down in the period 3, see Figure 1.

According to the results, we canconcludethat the spillover effects between markets are not constant;with the development of markets, spillover effects between markets may be flipped or disappear.The time-varying spillover effects give the investors and the policy makers an implication that the interaction between the stock markets and crude oil market can change over time. Investors' adjustments must be made timely based on the changes of relations. As observed in this paper, there exist different evolution of the spillovers between the Oil-US stock market and the Oil-Russia stock market. E.g.for Oil-US stock market, before crisis (in period 1), the volatility spillovers exist in the D2, D3 and D4 wavelet scales, while after crisis (in period 3), the spillovers shifted to D1 and D2 wavelet scales. That means there exists an evolution of spillover effectsfrom short term to long term. This result implies that there is a significant change in the US stock market. The interaction between oil market and US stock market is disappearing in the long-term. The investors in the oil and US stock market can pay more attention on the volatility in short-term while pay less attention on the long-term trend of each other than before.

4 Conclusions and Future Work

This paper identifiesthe evolution of mean and volatility spilloversacross wavelet scales between the WTI spot oil price and the twostock markets(S&P 500index, and MICEX index),combining the wavelet analysis with a bivariate GARCH-BEKK model in both the time and frequency perspectives. One innovation of this paper is applying the wavelet method to investigatethe underlying multi-scale spillovers between two markets.The other innovation is that the dynamic evolution of spillover relationship over time is analyzed across different wavelet scales.The daily sample data correspond to the entire period from January 2003 to December 2014, and it is divided into three sub-periods: pre-crisis period, crisis period and post-crisis period.

By applying the approach, we captured the two spillover relationships: the relationship between WTI oil market and US stock market, and the relationship between WTI oil market and Russia stock market. Generally, the proposed approach provides an obvious result that spillovers between WTI oil prices and the two stock markets are dynamic over pre-crisis, crisis, and post-crisis time periods across multiple wavelet scales.As observed from the empirical results, there exist different evolution of the time varying spillover effects between the WTI oil market and the two stock markets. An obviously evolution of spillover between oil market and US S&P

500 stock market is shifting to short term over the full observed period. Specifically, there exist strong mean spillovers at each wavelet scale in periods 1 and 2. However, in period 3, there are no linkages at long-term scales D3 and D4; spillover effects only exist at short-term scales.The spillovers in the volatility part are similar to those in the mean part. We can see that the mean and volatility spillovers are bidirectional, unidirectional or no linkage across wavelet scales. For the evolution between oil market and Russia MICEX stock market, the linkage is getting close at all time scales over the observed sub-periods. This may explain the phenomenon that with the falling of oil prices, the US stock price showed the opposite trend to Russia stock price.The results may give the investors some suggestions such as the investors in USA stock market should pay more attention on the volatility of oil market in short-term while pay less attention on the long-term trend than before. The investors in Russia and oil market need to pay attention on the volatility in both short-term and long-term.

In summary, based on the wavelet method and multivariate GARCH-BEKK model,a powerful framework was established in this study to analyze the time-varying volatility spillover effects at different scales between the WTI spot oil market and stock markets.The proposedapproach can be applied to other international markets. A challenge for future work will be to exploretheapproach's performance on hedgingratios.

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Appendix

Table 3 Estimated coefficients of the MGARCH-BEKK model for WTI oil price and stock indices data from scale D1.

	S&P 500 index (USA)						MICEX index (Russia)					
	Period 1 (Pre-crisis)		Period 2 (Crisis)		Period 3 (Post-crisis)		Period 1 (Pre-crisis)		Period 2 (Crisis)		Period 3 (Post-crisis)	
Estimation results of mean equations												
μ_s	-0.0006	(0.0086)	0.0034	(0.0151)	-0.0096	(0.0119)	0.0078	(0.0088)	-0.0177	(0.0127)	0.0025	(0.0112)
μ_o	-0.0311	(0.0753)	0.0739	(0.1131)	-0.0487	(0.0946)	-0.0566	(0.0784)	0.0043	(0.1884)	-0.0085	(0.1456)
φ_{11}	-0.5056***	(0.0337)	-0.4350***	(0.0264)	-0.4154***	(0.0272)	-0.5075***	(0.0302)	-0.5356***	(0.0275)	-0.6018***	(0.0311)
φ_{12}	-0.0087**	(0.0035)	0.0071**	(0.0033)	-0.0003	(0.0022)	-0.0022	(0.0019)	0.0093***	(0.0020)	-0.0074***	(0.0020)
φ_{21}	-0.1185	(0.1996)	0.9945***	(0.1722)	1.0663***	(0.1785)	-0.1336	(0.1762)	-1.7325***	(0.4048)	2.5194***	(0.4352)
φ_{22}	-0.5206***	(0.0270)	-0.4419***	(0.0257)	-0.3826***	(0.0261)	-0.5208***	(0.0253)	-0.5639***	(0.0305)	-0.5019***	(0.0305)
Estimation results of conditional variance–covariance equations												
c_{11}	0.1712***	(0.0142)	0.1406***	(0.0192)	0.2274***	(0.0205)	0.1873***	(0.0146)	-0.1298**	(0.0511)	0.2295***	(0.0199)
c_{21}	-0.1843	(0.2259)	0.5466***	(0.1253)	-1.9148***	(0.2251)	-0.1461	(0.1202}	1.9585	(1.3409)	0.1271	(0.5575)
c_{22}	1.2756***	(0.1804)	0.0001	(3.2568)	-0.00002	(4.0310)	0.5850***	(0.1115)	2.2088**	(0.9826)	2.8943***	(0.3559)
a_{11}	0.5508***	(0.0390)	0.3997***	(0.0495)	0.5664***	(0.0401)	0.5558***	(0.0448)	0.5513***	(0.0434)	0.5716***	(0.0448)
a_{12}	0.5264*	(0.2782)	0.7216***	(0.1916)	0.4827	(0.3148)	0.0166	(0.2936)	-3.1765***	(0.5647)	3.5242***	(0.7198)
a_{21}	-0.0032	(0.0050)	0.0060	(0.0050)	0.0023	(0.0034)	-0.0006	(0.0027)	0.0220***	(0.0027)	-0.0189***	(0.0033)
a_{22}	0.4645***	(0.0406)	0.3486***	(0.0317)	0.5471***	(0.0387)	0.4723***	(0.0409)	0.4689***	(0.0352)	0.6717***	(0.0406)
b_{11}	0.7300***	(0.0338)	0.8896***	(0.0333)	-0.6983***	(0.0357)	0.7155***	(0.0350)	0.5581***	(0.0385)	0.6481***	(0.0402)
b_{12}	-0.4307	(0.3260)	-0.8251	(0.8925)	0.2703	(0.5923)	0.1248	(0.2555)	7.4441***	(0.5793)	-4.5396***	(0.7698)
b_{21}	0.0081	(0.0079)	-0.0381***	(0.0121)	0.0151*	(0.0080)	-0.0005	(0.0014)	-0.0339***	(0.0024)	0.0164***	(0.0023)
b_{22}	0.7852***	(0.0414)	-0.9080***	(0.0302)	0.7334***	(0.0462)	0.8960***	(0.0163)	0.7085***	(0.0360)	0.6173***	(0.0416)

Notes: The transformed coefficients on each term in the variance and covariance equations and their standard errors (in parentheses) are presented. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 4 Estimated coefficients of the MGARCH-BEKK model for WTI oil price and stock indices data from scale D2.

	S&P 500 index (USA)						MICEX index (Russia)					
	Period 1 (Pre-crisis)		Period 2 (Crisis)		Period 3 (Post-crisis)		Period 1 (Pre-crisis)		Period 2 (Crisis)		Period 3 (Post-crisis)	
Estimation results of mean equations												
μ_s	-0.0060	(0.0081)	0.0088	(0.0109)	-0.0051	(0.0103)	-0.0075	(0.0062)	-0.0050**	(0.0127)	0.0021***	(0.0092)
μ_o	-0.0271	(0.0669)	0.0457	(0.1000)	-0.0266	(0.0866)	-0.0097	(0.0687)	-0.1216	(0.2524)	-0.1716	(0.1759)
φ_{11}	0.2892***	(0.0258)	0.3493***	(0.0264)	0.3683***	(0.0254)	0.3016***	(0.0248)	0.2512**	(0.0294)	0.2993**	(0.0259)
φ_{12}	-0.0037*	(0.0022)	-0.0147***	(0.0023)	-0.0101***	(0.0026)	0.0020**	(0.0010)	-0.0001***	(0.0010)	-0.0072***	(0.0011)
φ_{21}	0.3063**	(0.1380)	1.6238***	(0.2016)	0.6531***	(0.1347)	-0.0807	(0.1718)	0.1583	(0.3983)	3.3669	(0.3457)
φ_{22}	0.2234***	(0.0262)	0.1884***	(0.0277)	0.2059***	(0.0245)	0.2830***	(0.0254)	0.3227**	(0.0268)	0.2324**	(0.0245)
Estimation results of conditional variance–covariance equations												
c_{11}	0.1240***	(0.0125)	0.0927***	(0.0353)	0.1482***	(0.0166)	0.1296***	(0.0091)	0.2190**	(0.0179)	0.0921**	(0.0254)
c_{21}	-0.1222	(0.1606)	0.7607	(0.5152)	-0.1123	(0.2249)	-0.1907*	(0.1135)	1.1070	(0.4868)	-0.7695	(0.7300)
c_{22}	1.1374***	(0.1025)	-1.4882***	(0.2270)	1.5555***	(0.1369)	0.8750***	(0.1204)	3.0997	(0.3609)	2.7174	(0.3862)
a_{11}	0.7149***	(0.0363)	0.5878***	(0.0320)	0.6274***	(0.0332)	0.8609***	(0.0399)	0.8219**	(0.0394)	0.6282**	(0.0302)
a_{12}	-0.4112*	(0.2115)	-2.5645***	(0.2563)	-0.6331***	(0.1787)	0.2098	(0.2746)	0.8076	(0.5564)	-3.2385	(0.4916)
a_{21}	0.0060*	(0.0036)	0.0242***	(0.0036)	0.0118***	(0.0042)	-0.0035**	(0.0016)	0.0002***	(0.0014)	0.0089***	(0.0013)
a_{22}	0.7354***	(0.0410)	0.7683***	(0.0379)	0.7652***	(0.0377)	0.8699***	(0.0367)	0.6526**	(0.0417)	0.7026**	(0.0339)
b_{11}	0.7270***	(0.0237)	0.8042***	(0.0193)	0.7891***	(0.0203)	0.6557***	(0.0196)	0.6817**	(0.0185)	0.7823**	(0.0189)
b_{12}	0.4562***	(0.1580)	2.4319***	(0.1998)	0.5509***	(0.1468)	-0.1075	(0.1922)	-1.1200	(0.3652)	3.5635	(0.3581)
b_{21}	-0.0062**	(0.0031)	-0.0349***	(0.0034)	-0.0088***	(0.0030)	0.0032***	(0.0010)	0.0001***	(0.0009)	-0.0100***	(0.0010)
b_{22}	0.6943***	(0.0283)	0.5417***	(0.0264)	0.6694***	(0.0228)	0.7201***	(0.0163)	0.7914**	(0.0201)	0.6646**	(0.0224)

Notes: The transformed coefficients on each term in the variance and covariance equations and their standard errors (in parentheses) are presented. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 5 Estimated coefficients of the MGARCH-BEKK model for WTI oil price and stock indices data from scale D3.

	S&P 500 index (USA)						MICEX index (Russia)					
	Period 1 (Pre-crisis)		Period 2 (Crisis)		Period 3 (Post-crisis)		Period 1 (Pre-crisis)		Period 2 (Crisis)		Period 3 (Post-crisis)	
Estimation results of mean equations												
μ_s	0.0090*	(0.0047)	0.0173	(0.0119)	-0.0104	(0.0080)	-0.0077***	(0.0042)	0.0103**	(0.0116)	-0.0078***	(0.0080)
μ_o	-0.0357	(0.0532)	0.0780	(0.0642)	0.0536	(0.0678)	0.0156*	(0.0643)	0.2039	(0.2391)	0.1430	(0.1055)
φ_{11}	0.9091***	(0.0110)	0.8663***	(0.0102)	0.9090***	(0.0119)	0.9288***	(0.0108)	0.9303**	(0.0111)	0.9360**	(0.0109)
φ_{12}	0.0011*	(0.0006)	-0.0011	(0.0013)	-0.0010	(0.0008)	-0.0010***	(0.0002)	-0.0019***	(0.0005)	-0.0036***	(0.0006)
φ_{21}	0.3400***	(0.0550)	0.1007**	(0.0476)	0.0113	(0.0575)	-0.4401	(0.1114)	0.6683	(0.1312)	0.5444*	(0.0987)
φ_{22}	0.9310***	(0.0108)	0.8268***	(0.0101)	0.9013***	(0.0116)	0.8946**	(0.0108)	0.9137**	(0.0108)	0.9046***	(0.0100)
Estimation results of conditional variance–covariance equations												
c_{11}	0.0392***	(0.0043)	0.1365***	(0.0121)	0.0714***	(0.0065)	0.0454***	(0.0043)	0.1225**	(0.0100)	0.0939***	(0.0082)
c_{21}	0.1064	(0.0709)	0.3014***	(0.1010)	0.0579	(0.0776)	-0.2483*	(0.0762)	0.9141	(0.2842)	0.0713	(0.1240)
c_{22}	0.5608***	(0.0424)	0.5203***	(0.0885)	-0.6039***	(0.0545)	-0.5456*	(0.0518)	2.0734	(0.1596)	1.0442	(0.1144)
a_{11}	1.2087***	(0.0398)	1.0666***	(0.0330)	1.1971***	(0.0341)	1.2066**	(0.0374)	1.1796**	(0.0394)	1.1993**	(0.0386)
a_{12}	-0.3149**	(0.1304)	-0.2747***	(0.0931)	0.0390	(0.1203)	0.3999	(0.2180)	-0.3102	(0.2790)	-0.7370	(0.2609)
a_{21}	0.0009	(0.0014)	-0.0011	(0.0028)	0.0018	(0.0017)	0.0011***	(0.0007)	0.0015***	(0.0011)	0.0053***	(0.0015)
a_{22}	1.1964***	(0.0378)	1.1181***	(0.0342)	1.2078***	(0.0370)	1.2209**	(0.0375)	1.1886**	(0.0384)	1.2676**	(0.0385)
b_{11}	0.4832***	(0.0157)	-0.3492***	(0.0265)	0.4780***	(0.0147)	0.4769**	(0.0165)	0.4352**	(0.0180)	0.4608**	(0.0157)
b_{12}	-0.0706	(0.0630)	0.7344***	(0.1187)	-0.0299	(0.0537)	0.1049*	(0.0870)	-0.5001	(0.1497)	-0.0170	(0.1073)
b_{21}	-0.0015**	(0.0007)	0.0480***	(0.0030)	0.0007	(0.0007)	0.0003***	(0.0004)	0.0004***	(0.0005)	0.0004***	(0.0006)
b_{22}	0.4508***	(0.0159)	0.4030***	(0.0195)	0.4690***	(0.0145)	0.4882**	(0.0156)	0.4431**	(0.0181)	0.4589**	(0.0160)

Notes: The transformed coefficients on each term in the variance and covariance equations and their standard errors (in parentheses) are presented. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 6 Estimated coefficients of the MGARCH-BEKK model for WTI oil price and stock indices data from scale D4.

	S&P 500 index (USA)						MICEX index (Russia)					
	Period 1 (Pre-crisis)		Period 2 (Crisis)		Period 3 (Post-crisis)		Period 1 (Pre-crisis)		Period 2 (Crisis)		Period 3 (Post-crisis)	
Estimation results of mean equations												
μ_s	-0.0082**	(0.0035)	0.0083	(0.0091)	-0.0125**	(0.0062)	0.0120***	(0.0079)	-0.0086***	(0.0062)	-0.0297***	(0.0068)
μ_o	0.0128	(0.0209)	-0.0431	(0.0406)	-0.1598***	(0.0558)	0.2267*	(0.0648)	-0.4935	(0.1386)	0.3415*	(0.0823)
φ_{11}	0.9544***	(0.0080)	0.9828***	(0.0063)	0.9644***	(0.0080)	0.9544***	(0.0083)	0.9717***	(0.0050)	0.9801***	(0.0063)
φ_{12}	0.0036***	(0.0005)	-0.0022***	(0.0004)	-0.0001	(0.0004)	-0.0005***	(0.0003)	0.0006***	(0.0001)	-0.0023***	(0.0003)
φ_{21}	0.1583***	(0.0211)	0.0470*	(0.0273)	-0.0214	(0.0427)	-0.1683*	(0.0525)	0.0382**	(0.0430)	0.1544**	(0.0430)
φ_{22}	0.9626***	(0.0075)	0.9719***	(0.0065)	0.9965***	(0.0063)	0.9740***	(0.0054)	0.9886***	(0.0053)	0.9652***	(0.0060)
Estimation results of conditional variance–covariance equations												
c_{11}	-0.0162***	(0.0029)	0.0387***	(0.0044)	0.0191***	(0.0036)	0.0391***	(0.0054)	-0.0256***	(0.0035)	0.0439***	(0.0038)
c_{21}	-0.0567**	(0.0225)	-0.0201	(0.0322)	-0.0112	(0.0506)	0.1246	(0.1119)	-0.3821	(0.1002)	-0.1366*	(0.0583)
c_{22}	0.0980***	(0.0141)	-0.1710***	(0.0296)	0.2972***	(0.0283)	0.2604*	(0.0625)	-0.4847*	(0.0539)	-0.3943*	(0.0580)
a_{11}	1.0671***	(0.0268)	1.0479***	(0.0303)	1.0818***	(0.0315)	0.9258**	(0.0252)	1.0732**	(0.0313)	1.0283**	(0.0295)
a_{12}	-0.1491***	(0.0433)	-0.0758	(0.0520)	-0.0084	(0.0888)	0.0921	(0.1075)	0.1976	(0.1110)	-0.0978	(0.1204)
a_{21}	-0.0032***	(0.0010)	0.0025**	(0.0010)	-0.0006	(0.0008)	0.0001***	(0.0007)	0.0003***	(0.0004)	0.0028***	(0.0007)
a_{22}	1.0392***	(0.0266)	1.0745***	(0.0305)	1.0511***	(0.0316)	0.9345**	(0.0271)	1.0567**	(0.0310)	1.0412**	(0.0300)
b_{11}	-0.5838***	(0.0103)	0.5720***	(0.0120)	-0.5628***	(0.0133)	0.4898**	(0.0211)	0.5755**	(0.0126)	0.5529**	(0.0137)
b_{12}	0.0171	(0.0241)	0.0154	(0.0247)	-0.0453	(0.0447)	0.3889	(0.2325)	-0.1785*	(0.0569)	-0.0266*	(0.0709)
b_{21}	0.0013***	(0.0005)	0.0003	(0.0005)	-0.0004	(0.0004)	-0.0100***	(0.0016)	-0.0007***	(0.0002)	0.0006***	(0.0004)
b_{22}	-0.5894***	(0.0099)	0.5725***	(0.0120)	-0.5538***	(0.0126)	-0.5110**	(0.0198)	0.5714**	(0.0122)	0.5664**	(0.0134)

Notes: The transformed coefficients on each term in the variance and covariance equations and their standard errors (in parentheses) are presented. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 7 The time varying mean and volatility spillovers between WTI oil market and the stock markets over scales based on the GARCH-BEKK model.

		S&P 500 Index (USA)			MICEX Index (Russia)		
	Wavelet scale	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3
	D 1	1 (WTI → Stock)	2 (WTI ↔ Stock)	1 (WTI ← Stock)	No	2 (WTI ↔ Stock)	2 (WTI ↔ Stock)
Mean spillovers	D 2	2 (WTI ↔ Stock)	2 (WTI ↔ Stock)	2 (WTI ↔ Stock)	1 (WTI → Stock)	No	2 (WTI ↔ Stock)
(WTI-Stock)	D 3	2 (WTI ↔ Stock)	1 (WTI ← Stock)	No	2 (WTI ↔ Stock)	2 (WTI ↔ Stock)	2 (WTI ↔ Stock)
	D 4	2 (WTI ↔ Stock)	2 (WTI ↔ Stock)	No	1 (WTI ← Stock)	1 (WTI → Stock)	2 (WTI ↔ Stock)
	D 1	No	1 (WTI ← Stock)	1 (WTI ← Stock)	No	2 (WTI ↔ Stock)	2 (WTI ↔ Stock)
Volatility spillovers	D 2	2 (WTI ↔ Stock)	2 (WTI ↔ Stock)	2 (WTI ↔ Stock)	1 (WTI ← Stock)	1 (WTI → Stock)	2 (WTI ↔ Stock)
(WTI-Stock)	D 3	1 (WTI ← Stock)	2 (WTI ↔ Stock)	No	1 (WTI → Stock)	1 (WTI → Stock)	2 (WTI ↔ Stock)
	D 4	1 (WTI ← Stock)	No	No	2 (WTI ↔ Stock)	2 (WTI ↔ Stock)	1 (WTI ← Stock)

Notes: The symbols → (←) indicates the direction of mean and volatility spillovers from the WTI market to the stock markets (from the stock marketsto the WTI market), ↔ indicates interdependence between the WTI market and stock markets, and"No"indicates there are no spilloversbetween the markets.