1977MNRAS.179..359R

Mon. Not. R. astr. Soc. (1977) **179**, 359–375

The evolution of W Ursae Majoris systems

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Received 1976 September 27; in original form 1976 July 23

Summary. In an attempt to understand the structure of W UMa binaries, the evolution of close binary models in and near contact has been investigated. Calculations have been performed for a total system mass of $1.8 M_{\odot}$, and for various orbital angular momenta and degrees of nuclear evolution. Cyclic behaviour on a thermal timescale, with alternating contact and semi-detached phases, is generally found although in some circumstances a stable equal-mass system is obtained. Both nuclear evolution and lower angular momentum cause the cycles to be centred on more extreme mass ratios, and the fraction of time spent in contact appears to increase considerably. One evolved model settles in an apparently stable state of shallow contact with $q \sim 0.4$. We suggest that this may represent a fairly typical A-type W UMa system. The drawback of the cyclic models is that during the semi-detached phase the systems would be expected to exhibit EB-type light curves, and it must be admitted that (so far) no short-period ($P \lesssim 0$, four days) EB binaries have been observed. We give in the text reasons for believing that this difficulty is not insurmountable.

1 Introduction

The earliest attempts to construct models of W Ursae Majoris stars as contact binary systems were based on the assumption of thermal equilibrium. The undoubted justification for this assumption was that the very high frequency of W Ursae Majoris stars points to their being young systems, still on or near the main sequence, and evolving on a nuclear timescale. Their presence in galactic open clusters, for example TX Cnc, which is a member of the young cluster Praesepe (Haffner 1937, 1938) is strong evidence for some at least being unevolved. As was shown by Kuiper (1941), it is not possible to construct contact binaries in thermal equilibrium simply by bringing two main-sequence stars into contact, since the mass–radius relation for main-sequence stars and contact configurations differ. But it is possible if it is assumed that a significant portion of the luminosity of the brighter component is transferred to the other star via a common envelope before being radiated. The radius and temperature of the first star are then decreased and of the second, increased. For the appropriate

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luminosity exchange, the surfaces of the two stars may be made to lie on the same equipotential surface.

Various approaches have been made to the problem of constructing equilibrium models. None have been in particularly good agreement with the observations. Following the early work of Lucy (1968a), the existence of a common envelope in convective equilibrium which serves to transfer the luminosity has been universally accepted. By insisting on the equality of the entropy in each star's envelope, Lucy was able to construct equilibrium models which possessed observable properties closely matching those of W UMa systems. The predicted light curves were particularly good. But he found it necessary to use extreme Pop I compositions or enhanced nuclear reaction rates for the CNO cycle to obtain models lying within the limits of Eggen's (1967) observed period–colour relation for contact binaries.

Biermann & Thomas (1972, 1973) argued that the presence of a considerable entropy difference between the two stellar envelopes is needed to drive the fluid circulation which carries the energy. Although they were able to construct models of contact binaries in thermal equilibrium, they were unable to obtain reasonable agreement with the observations. In particular, the large difference in surface temperature of their models meant that the theoretical light curves had very unequal depths of minima, in marked contrast to one of the most prominent features of W UMa light curves, which always have closely similar depths in the two minima. In many cases (Binnendijk's (1970) 'W-type' systems) the deeper minimum is associated with the eclipse of the less massive star, which is then (apparently) the hotter component. In all the models of Biermann & Thomas, the more massive star is several hundred degrees warmer than its companion.

Whelan (1972) has built models in which the luminosity is transferred in the superadiabatic region of the envelope, in contrast to the other models in which the transfer is effected at the base of the convective envelope. This causes the secondary's temperature to be higher and allows him to obtain W-type light curves. Various models requiring that one or other component be partially evolved have also been investigated (Hazlehurst 1970; Moss & Whelan 1970).

The failure of most of the above models to provide satisfactory agreement with the observations, together with the existence of very noticeable fluctuations in the light curves and period changes of these stars led some workers to the conclusion that W UMa systems cannot be in thermal equilibrium but must be undergoing cyclic evolution in marginal contact. The possibility of generating instabilities in these systems associated with the weak degree of contact is certainly very attractive. Ruciński (1973) suggested that W-type systems might be explained in this way.

Since then, models of non-equilibrium systems in marginal contact have been constructed by Lucy (1976) and Flannery (1976). Both have found that the evolution of such systems is likely to proceed in cycles consisting of two phases: a contact phase, during which mass is transferred from the secondary to the primary under the influence of energy transfer in the opposite direction, and a semi-detached phase, during which mass is returned to the secondary. No energy transfer is possible in this latter phase, apart from that associated with the internal energy of the matter falling on to the secondary. Both components deviate from equilibrium throughout the cycle, and it is the non-existence of contact equilibrium models together with the thermal timescale instability associated with mass loss from the primary in the semi-detached phase which maintains the cyclic behaviour.

It is, however, clear that these cyclic models of Lucy and Flannery do not satisfy the observational requirements. Although the light curves during the contact phase will closely resemble those of W UMa systems, the surface entropies and temperatures of the stars are very different during the semi-detached phase and the stars would not then exhibit W UMa-type light curves.

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In this paper we investigate such cyclic evolution more fully, and take into account the effects of nuclear evolution and loss of orbital angular momentum from the system.

2 Observational properties of W UMa systems

W UMa systems have been divided by Binnendijk (1970) into A- and W-type systems according as the primary minimum in the light curve is a transit, in which the smaller star partially eclipses the larger, or an occultation, when the larger star is in front. W-type systems generally exhibit unstable light curves and variable orbital periods, and work on light curve synthesis indicates that their common envelopes are very shallow. The classical approach to light curve analysis by Russell & Merrill (1952) has proved to be inadequate, since the components are very far from being ellipsoidal. Application of their method usually shows W UMa systems to be detached. A-type systems possess more stable light curves and less rapidly varying periods, and possibly rather deeper common envelopes. There is a general assumption now that the light curves of W-type systems are explained by the secondary's being hotter than the primary. Whelan (1972) has shown that this is possible if the energy transfer between the components occurs principally through the strongly superadiabatic outer part of the common envelope. It is not clear, however, that energy transfer in this region would always be effective enough to transfer the requisite luminosity.

On the basis of this model in which the secondary and primary differ in temperature, Ruciński (1974) obtained a very clear division of W UMa binaries into two classes which closely correspond to Binnendijk's A and W types. The absence of systems intermediate between the two types which Ruciński finds suggests that A- and W-type systems are not simply to be considered as different evolutionary stages of the same kind of object. It should be noted that Ruciński finds very poor correlation of the temperature excess, $\Delta T/T$, for the W-type systems with other parameters. This might make one suspicious of the meaningfulness of $\Delta T/T$ as a parameter describing these systems. It is nevertheless true that there is no other clear means of differentiating the types if radial velocity data are not available.

There are alternative mechanisms for the production of W-type systems. The temperature distribution on the surface of a star is approximately related to the variation of surface gravity by $T \propto g^{\beta}$. For stars in radiative equilibrium, $\beta = 0.25$, while for stars with convective envelopes, Lucy (1967) has found that $\beta = 0.08$. Wilson & Devinney (1973) and Wilson & Biermann (1976) have analysed two W-type light curves and have obtained satisfactory fits with $\beta = 0.28$ or 0.38 for RZ Com and between 0.02 and 0.25 for TX Cnc. They offer no explanation for the existence of such large values of β associated with what one would otherwise expect to be convective envelopes. They did not find any other parameter that could be varied to produce W-type light curves. Our own fits to the light curve of RZ Com using a slightly different procedure (unpublished) suggest that β must exceed 0.4 before W-type light curves can be obtained. The two most prominent effects of an increase in β are the production of deeper minima and the deepening of the occultation minimum relative to the transit. W-type curves can be obtained for smaller values of β, but only at much smaller angles of inclination of the orbit (Lucy 1968b). An alternative possibility which can produce W-type light curves is the existence of a region of enhanced temperature in the neck between the two components (Berthier 1973). The relation $T \propto g^{\beta}$ must be violated in this region, since the effective gravity vanishes at L_1, the inner Lagrangian point.

3 Evolution in contact

We present the evolution of a close binary of total mass $1.8\,M_\odot$ for various values of the orbital angular momentum and for various assumed efficiencies for the luminosity transfer

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process in the contact state. In common with other investigations of the evolution of close binaries, we make the following simplifying assumptions. Each component is assumed to be spherical, and the orbit is assumed to be circular. Rotational and tidal effects on the structure of the stars are neglected, as is the angular momentum associated with the spin of the components. The equipotential surfaces of the binary are assumed to be spheres of the same volume as the corresponding lobes given by the Roche model. Three distinct phases for a close binary system may be distinguished. In the detached phase, each component lies within its Roche lobe and the equipotentials corresponding to the surfaces of the stars are independent. In the semi-detached phase, the equilibrium radius of one star exceeds the volume radius of its lobe. In this case, the radius of the star must very nearly equal that of the lobe, since matter lying above the lobe would be lost from the star on a dynamical time-scale. We therefore assume that the radii of the star and lobe are equal in the semi-detached phase. In the contact phase, the radii of both stars exceed their Roche radii, and to maintain hydrostatic equilibrium it is necessary that the stellar surfaces lie on the same equipotential. We use the following approximate relation (Paczyński 1971) for the radius of the Roche lobe about the primary

$$R_{\mathrm{L}}(m_1,m_2,H) = \frac{H^2(m_1+m_2)}{Gm_1^2m_2^2}(0.38+0.2\log_{10}m_1/m_2), \tag{1}$$

where H is the angular momentum. Clearly the lobe radius for the secondary is $R_L(m_2, m_1, H)$..

When mass transfer is occurring, it is assumed that the matter exchanged is accreted spherically, directly on to the mass-gaining star, so that the possibility of formation of a disc is not allowed. We discuss this further in Section 4.

The effect of neglect of rotation on the stellar structure is to overestimate the nuclear luminosity of each component. Inclusion of spin angular momentum, although amounting to only a few per cent of the total angular momentum of the system, could have a significant influence on the structure of a contact binary, since the storage of angular momentum in spin leads to a reduction in the separation of the components. The importance of spin in semi-detached systems is even greater.

An important feature of the single-star evolution code (Eggleton 1971) which we modified for the present work is its treatment of the distribution of mesh points within a model. In contrast to the usual procedure, in which the mesh-point distribution is specified in advance, this code treats the mesh distribution as an unknown function which has to be solved for together with the structure equations. To do this, two extra equations are required for two unknown functions q and c

$$\frac{dq}{dm} = c\left(\frac{1}{m}+\frac{Gm}{4\pi r^4 p}\right) \tag{2}$$

and

$$\frac{dc}{dm} = 0. \tag{3}$$

with the boundary conditions

$q = 0$ at the centre (more precisely, at $m = 10^{-4}\,\mathrm{m}_*$, which is taken as the innermost mesh point)

and

$q = 1$ at the surface, i.e. the photosphere.

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In practice, q, rather than the mass variable, is used as the independent variable and the mesh points are chosen at equal intervals in q. Thus mesh points are effectively at equal intervals of the mesh-spacing function Q, where

$$Q \equiv \log m - \log p. \tag{4}$$

This dissection of the star automatically puts mesh points where they are most needed, although in practice a slightly more elaborate expression for Q is better still. It is not necessary to treat the surface layers separately, as is commonly done in stellar evolution codes. Incorporating the envelope in the regular stellar evolution code is particularly advantageous in mass transfer situations, since much of the thermal energy release is close to the surface.

Because the mesh is non-Lagrangian, we have to write for the thermal energy generation

$$\epsilon_{\text{thermal}} \equiv T\left(\frac{\partial s}{\partial t}\right)_{\text{m}} = T\left(\frac{\partial s}{\partial t}\right)_q - T\ \frac{\partial s}{\partial m}\left(\frac{\partial m}{\partial t}\right)_q, \tag{5}$$

and in a mass transfer situation this can approximate very closely the steady-state assumption of Paczyński (1967)

$$\epsilon_{\text{thermal}} = -T\frac{\partial s}{\partial m}\frac{dm_*}{dt}. \tag{6}$$

But equation (5) is still correct even if the mass transfer is not exactly in a steady state; one does not have to concern oneself with whether or not the steady-state approximation is reasonable.

So far as a single star is concerned, taking q as the independent variable means that

(a) the usual differential structure equations, in Lagrangian form, have to be divided through by equation (2);

(b) two further equations, (2) and (3) but written with q as independent variable, have to be added to them and two further dependent variables, m and c, have to be added to the usual four (say p, T, r, L) and solved for iteratively;

(c) two further boundary conditions, namely $m = m_*$ (given) at the surface and, say, $m = 10^{-4} m_*$ at the centre have to be added;

(d) the thermal energy term has to be written in the non-Langrangian form (5).

For interacting binaries, we find it convenient to put the two stars 'side by side' in the computer, so that 12 diferential equations, in 12 variables, have to be solved, with six boundary conditions at each end. Ten of these boundary conditions are simply five of the normal single-star boundary conditions doubled up, while the remaining two, which apply at the surface, take various forms according as the binary is detached, semi-detached, or in contact. These situations are covered by

(i) detached: m_1 = given (7)

m_2 = given, as for single stars;

(ii) semi-detached: $m_1 + m_2$ = given (8)

$r_1 = R_L(m_1, m_2, H)$ (if star 1 fills its Roche lobe);

(iii) contact: $m_1 + m_2$ = given (9)

$r_1/R_L(m_1, m_2, H) = r_2/R_L(m_2, m_1, H)$.

In the present work, the only direct coupling between the sets of equations for the two components is via these boundary conditions. Although we include heat transfer (see later)

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we do so in an artificial but simple manner that avoids direct coupling between the equations at levels below the photosphere. However, if we had a believable statement for the rate of heat transfer at different levels (say, proportional to the entropy difference) we would hope to be able to incorporate it in a very straightforward manner. Equation (9), while not a precise statement of the condition that both stars fill the same equipotential surface, is nevertheless closely equivalent to this condition.

As was found by Webbink (1975), in the transition from the detached to the semi-detached phase, the direction of mass transfer is not determined by the structure equations or the boundary conditions. To ensure that mass transfer is initiated in the correct direction, we evaluate the separation of the components explicitly on the timestep which produces the semi-detached model, and keep it fixed during the iteration. This forces the primary component to overflow its Roche lobe and to lose mass. Once mass loss has begun, the thermal disequilibrium of the primary ensures that mass loss continues in the same direction.

Out initial model (Fig. 1) comprises two zero-age main-sequence stars of Population I ($X = 0.70$, $Z = 0.02$) with masses 1.2 and $0.6 M_\odot$ and orbital angular momentum, H, of 5.129×10^{51} erg-s ($\log H = 51.710$). The initial separation is 1.741×10^{11} cm and the orbital period is 0.3418 days. The initial model is a detached binary, but the surface of the primary lies only a short way inside its Roche lobe which it fills after only 10^7 yr of nuclear evolution. Thereafter it loses mass to its companion at a rather slow rate which rises approximately linearly to $1.7 \times 10^{-8} M_\odot$/yr. This slow mass-loss rate is essentially a consequence of the system being almost unevolved at the onset of mass transfer, being slower than the thermal timescale associated with mass loss from more evolved primaries (Morton 1960).

The addition of mass on to the secondary causes it to expand and its effective temperature and luminosity to increase, and after a total of 2.23×10^7 yr of evolution from the main sequence it has also swollen to fill its Roche lobe. Discussion of the semi-detached phase of

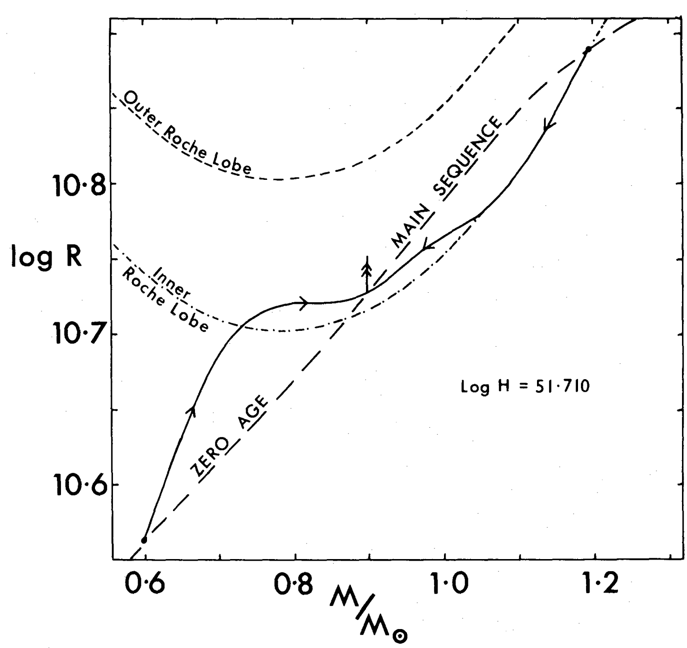

Figure 1. Evolution of a close binary system into contact with $\log H = 51.710$. The primary, starting at the upper right, reached its Roche lobe almost immediately after leaving the zero-age main-sequence. Accretion on to the secondary, starting from the lower left, caused it to swell until it also filled its lobe. The resulting contact model, or a very similar model, is the initial model for most subsequent calculations. Further evolution in the figure assumes (unrealistically) that no heat transfer takes place during contact. The system evolves to equal masses.

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evolution is given in Section 4 below. The contact binary formed at this point has masses of 1.06238 and $0.73762 M_\odot$ and a mass ratio of 0.69, and this model, or a very similar model, is used as the initial model for the subsequent examination of evolution in contact. We do not wish to suggest that all contact binaries arises from initially detached main-sequence binaries, but the majority of detached binaries which later undergo 'Case A' mass transfer, i.e. mass transfer during core hydrogen burning, must evolve into contact (Benson 1970; Yungel'son 1973).

We consider first the evolution of the contact system under the assumption that no heat transfer takes place. The behaviour of the system in this case is shown in Fig. 1. Also shown are the mass–radius relation for the main sequence and for the inner and outer Roche lobes. Nuclear evolution is included in these models, although the influence of the composition changes is negligible in the early stages. The differential equations describing the chemical composition in each star are solved separately before the simultaneous solution of the structure equations for both stars. The stars evolve steadily towards equal masses on a thermal timescale. This occupies a period of 3.75×10^7 yr. Thereafter the system evolves at equal masses into ever deeper contact on a nuclear timescale, and would eventually overflow its outer Roche lobe. The absence of contact binaries of low total mass and equal masses suggests that this course of evolution is not followed. We can anticipate that the transfer of significant amounts of energy through the common envelope surrounding the components in the direction from primary to secondary has the effect of preventing evolution towards equal masses. It has been shown by both Lucy (1976) and Flannery (1976) that a contact binary with equal components is unstable to mass transfer if it is required that the surface entropies be equal, a condition which requires such an energy transfer.

To evaluate the energy exchange in the contact phase we first locate the mesh point which at the previous timestep lay just above the Roche lobe in each component. It is possible that in one of the components there is a mesh point still closer to, but still above, the lobe, since the Roche potential does not necessarily change at the same rate with respect to mesh-point number in each component. But in the contact phase the envelopes are so similar that the difference is slight (see Fig. 2). Let L_1 and L_2 be the luminosities and m_1 and m_2 the masses of the components at this mesh point. We assume that the effect of fully efficient energy exchange is to equalize the light-to-mass ratio of the stars, since this seems to obtain approximately among the observed systems. Thus if ΔL_0 is the luminosity lost by the primary and gained by the seconday, we require that

$$\frac{L_1 - \Delta L_0}{m_1} = \frac{L_2 + \Delta L_0}{m_2} \tag{10}$$

and therefore that

$$\Delta L_0 = \frac{m_2 L_1 - m_1 L_2}{m_1 + m_2}. \tag{11}$$

The luminosity transfer may not be fully efficient if there is only marginal contact between the components. We therefore write the transferred luminosity as $\Delta L = f \Delta L_0$, where we take

$$f = \min(1, \alpha[(R_*/R_\mathrm{L})^2 - 1]), \tag{12}$$

so that $0 \leqslant f \leqslant 1$ in contact. The parameter α is expected to be moderately large, so that heat transfer becomes fully efficient for stellar radii exceeding their Roche radii by some standard small amount. In a given model this energy transfer is evaluated explicitly from the previous model, as is the mesh point at which it is applied. The explicit character of this part

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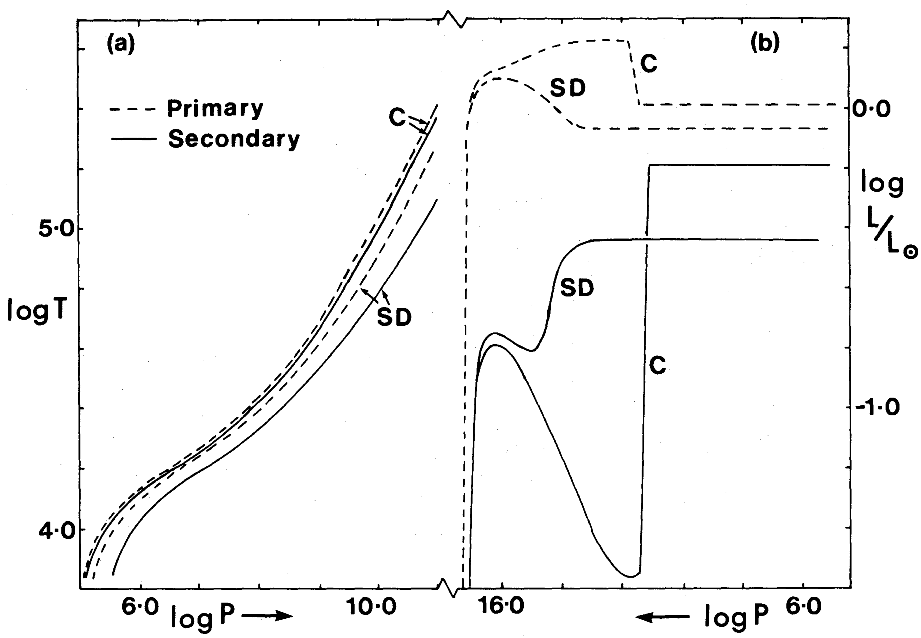

Figure 2. The thermal structure of the two components during a phase of semi-detached evolution (SD) and of contact evolution (C) with efficient energy transfer. In contact, luminosity is transferred abruptly at the inner Lagrangian surface, the amount transferred being approximately that necessary to keep both envelopes on the same adiabat.

of the calculation may be responsible for some numerical problems reported below. We hope to improve this in a later study.

The luminosity increment ΔL is applied to each component with appropriate sign at the first mesh point above the Roche lobe. This is achieved in practice by modifying the radiative temperature gradient above this level, writing

$$\nabla_r = \frac{3p\kappa(L \pm \Delta L)}{16\pi acGmT^4}. \tag{13}$$

Provided that the luminosity increment is applied within the adiabatic portion of the envelope of each star, the actual distribution of the luminosity is unimportant. It is not therefore necessary to consider a more sophisticated treatment of the energy exchange. The depth of the convective envelope was always found to be sufficient to ensure that energy transfer occurred within the convective region, rather than in the underlying radiative region. It was of course inevitable that at the points of making and breaking contact, energy should be transferred in the superadiabatic region, but these phases are short-lived. Anyway, the amount of energy transferred in these phases is very small, according to our assumptions. The requirement that the ratio L/m be the same for both stars in circumstances in which fully efficient energy transfer is occurring also ensures that the surface temperatures are very similar, since at the stellar surface

$$L/m = \pi acT^4R_*^2/m \tag{14}$$

and R_*^2/m is approximately the same for both components, by the Roche condition.

The evolution of the initial model with energy transfer included in this fashion is shown in Fig. 3 for $\alpha = 10$. The system undergoes cyclic evolution on a thermal timescale, with a period of about 10^7yr. The cycles consist essentially of a contact phase, during which mass is transferred from the secondary to the primary under the influence of the energy transfer

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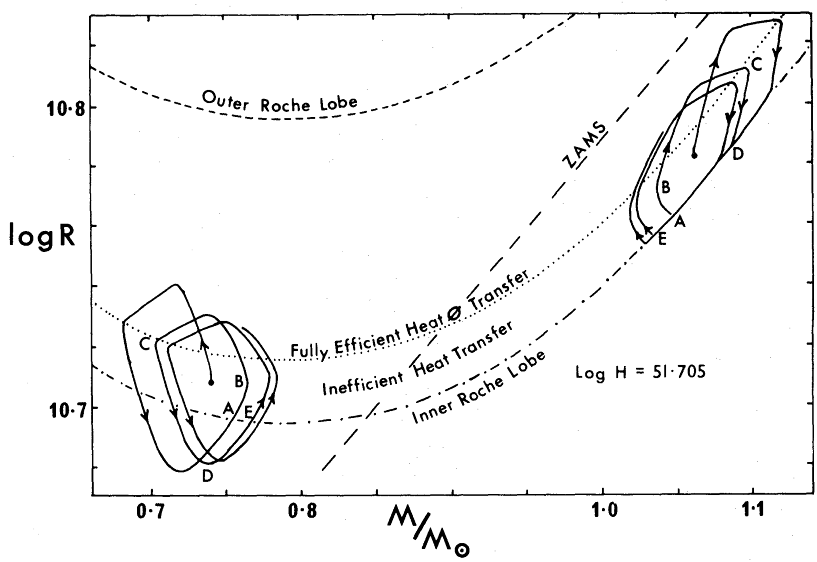

Figure 3. Cyclic evolution of a close binary ($\log H = 51.705$) with alternating contact and semi-detached phases. A steady cycle appears to be set up after two or three initial loops. Heat transfer is assumed to be fully efficient only when the depth of contact exceeds a certain fraction.

in the opposite direction, and a semi-detached phase, during which no energy transfer can occur and the direction of mass transfer is reversed. For this value of α, the efficiency of energy transfer is complete for about half of the time spent in contact. After an initial rather wide loop in the radius–mass diagram, the subsequent evolutionary cycles are quite similar. The mass ratio varies between 0.64 and 0.76 through the cycles. Starting at point A there is a short period during which mass is being transferred to the secondary, in spite of the existence of a common envelope, and the depth of contact is increasing. Eventually, when the depth of contact has increased sufficiently, the mass transfer is reversed (point B) and the envelopes of the two stars becomes progressively more similar as the transfer of energy approaches full efficiency. There is then a long period (B to C) of rather slow and steady mass transfer (about 3.2×10^6 yr at an average rate of about $1.5 \times 10^{-8} M_\odot$/yr) which terminates abruptly when the depth of contact is no longer sufficient to maintain fully efficient energy transfer. This is a consequence of the increasing separation of the stars as their masses become progressively less equal. The direction of mass transfer again reverses and grows rapidly to about $10^{-7} M_\odot$/yr. The secondary contracts rapidly and its temperature and luminosity fall as its energy support is withdrawn. It loses contact with its Roche lobe about half way through this phase, and continues to contract towards thermal equilibrium (C to D) until the addition of mass from the primary causes it to expand once more into contact (D to E). The times involved are, in millions of years: A to B, 1.6; B to C, 5.0; C to D, 0.9; D to E, 1.6.

The size and location of these cycles which the model exhibits are presumably fully determined by the total mass and angular momentum of the system, together with the energy transfer parameter, α. The earlier history of the system has only a transient effect. We are encouraged by the fact that Flannery (1976) has obtained similar behaviour with $M = 1.97 M_\odot$, $\log H = 51.854$ and $\alpha = 100$.

The structure of the envelope of each star at two points in the cycle is shown in Fig. 2(a), where $\log T$ is plotted against $\log p$. As we would expect, the atmospheres of the two stars during an interval of good thermal contact are almost indistinguishable. In Fig. 2(b) we illustrate typical luminosity profiles during good contact, and in the semi-detached state.

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The major objection to the evolutionary picture presented here as a model for the evolution of W UMa stars is without doubt the existence of an extended interval of time (about half the cycle or 5×10^6 yr) during which the system is semi-detached, and the envelopes of the stars are quite different. Whereas the observable characteristics of the model during the contact phase are compatible with those of W UMa systems, the light curves predicted by the model in the semi-detached state resemble those of β Lyrae, with rather unequal minima. But β Lyrae systems are generally of longer period and there is a serious deficiency of systems of short period which are close to, but not actually in, contact. In an attempt to resolve this dilemma, we have considered the roles of the efficiency of energy transfer and of the angular momentum of the system.

The influence of the parameter α was investigated. If, starting from the same initial model as in Fig. 3, α is reduced to 5, the energy transfer is never sufficient to halt the mass flow from primary to secondary, and the system evolves unchecked towards equal masses. However, the system will not be permanently stable on reaching equal masses, since nuclear evolution will increase the depth of contact and the efficiency of heat transfer. We find that equal mass systems tend to be stable if the efficiency parameter f is less than ~ 0.7, and unstable, so that the system evolves towards unequal masses, if $f \gtrsim 0.7$. We also investigated the case $\alpha = 7.5$: this exhibited cycles similar to the case $\alpha = 10$ (Fig. 3).

An increase in the angular momentum of the system also has the effect of decreasing the heat transfer, because the contact is shallower. In Fig. 4 we show cases with $\log H =$ 51.710, 51.715. In the latter case the system evolves in rather shallow contact towards equal masses, which are reached after 3.6×10^7 yr. To check our expectation that the result of nuclear evolution would be evolution away from equal masses, we then set $\alpha = 10^6$, to ensure that the luminosity transfer was fully efficient. As expected, the equal mass system immediately became unstable. Fig. 4 also shows that for a slightly lower $\log H$ the system executes one loop before evolving to equal masses.

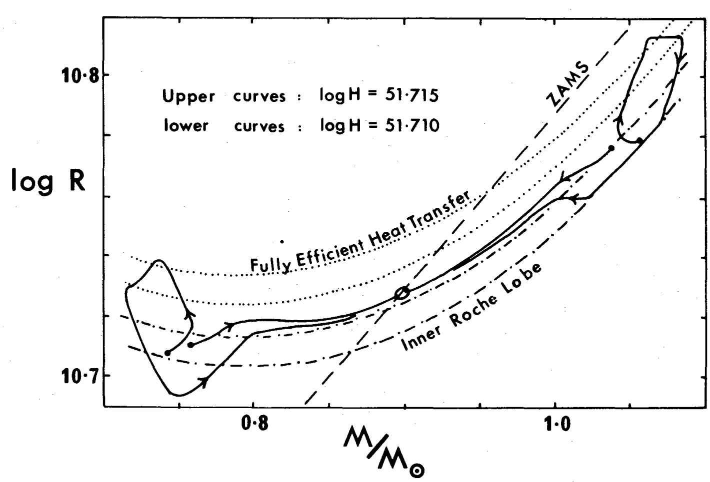

Figure 4. Non-cyclic evolution of a close binary when the depth of contact is insufficient to give fully efficient heat transfer. The models evolve to equal masses, although for the lower value of angular momentum one loop is executed first.

The subsequent evolution of these last two systems has not been followed. It seems unlikely that they can provide satisfactory models for W UMa stars, since the time spent at equal masses is likely to be a significant fraction of the nuclear lifetime. We may conclude either that in real systems the efficiency of luminosity transfer grows much more rapidly with depth of contact than we have generally assumed, or that although for a given total

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mass there is a range of angular momentum which allows stable equal mass contact systems to exist, systems are not commonly formed with that amount of angular momentum.

In Fig. 5 we compare the evolution of three systems of angular momenta $\log H = 51.705$, 51.700 and 51.675, each with $\alpha = 10$. These systems all exhibit cyclic behaviour, but the increased efficiency of luminosity transfer due to the greater depth of contact causes the cycles to lie at more extreme mass ratios for lower angular momenta. We note that the secondary in the system with $\log H = 51.675$ reaches a minimum mass of about $0.53 M_\odot$, but because its surface entropy is high due to the luminosity transfer, its convective envelope at this stage is only $3 \times 10^{-4} M_\odot$ in contrast with $0.07 M_\odot$ for a star of the same mass in thermal equilibrium. This means that the secondary can, and does, expand when accreting matter in the semi-detached phase, whereas a ZAMS star of the same mass would contract when accreting.

The system in Fig. 5 with $\log H = 51.675$ cannot yet be considered reliable, since it gave substantial numerical difficulty and ultimately broke down shortly after contact was re-established on the first loop. However, there are already some features of this model which make it a potentially better model for W UMa stars than the two of higher angular momentum. In particular, the system spent much more time in the contact phase, and especially in the shallow contact phase just before the heat transfer efficiency began to drop, while on the other hand, evolution in the semi-detached phase was slightly more rapid. The numerical problems may be associated with the explicit character of the way in which we compute the luminosity transfer, and perhaps also with the very different thermal time-scales in the interiors of the two components when their masses are very different. It is clearly important to improve the numerical technique and to investigate further the possibility that systems with mass ratios as extreme as 0.5 or less spend much the greatest fraction of their life in the phase of shallow contact.

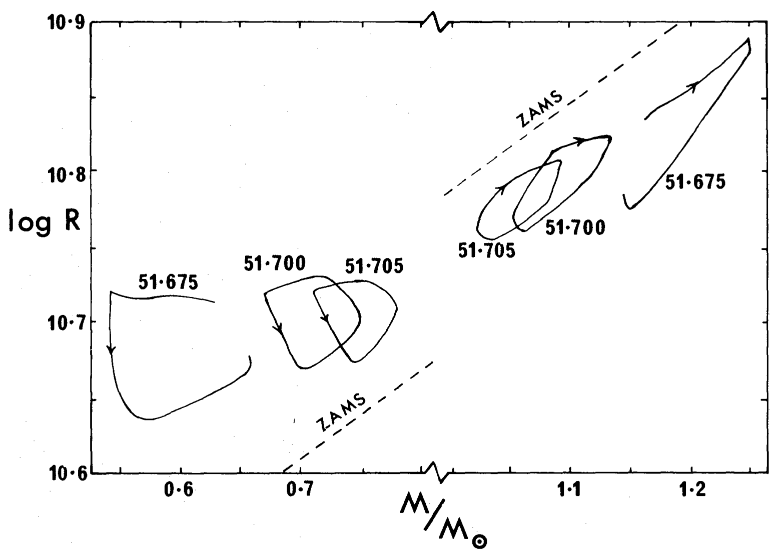

Figure 5. Approximate location of cycles for three values of angular momentum. For the lowest angular momentum numerical problems ended the sequence and so the position of the steady cycle is very uncertain.

We considered the effect of a steady loss of angular momentum on the evolution of a contact binary. We followed the evolution of a system with $\log H$ initially 51.705, losing angular momentum at different rates. The mass was conserved in these calculations, although it is probable that some mass loss would accompany angular momentum loss. We find that a

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loss rate $d\log_{10}H/dt = -10^{-8}$/yr is sufficient to maintain contact and drive the system towards more extreme mass ratios. With such a rapid loss rate however the degree of contact grows rather quickly and the system soon overflows its outer Roche lobe. By contrast, the effect of a rate of -10^{-9}/yr is simply to cause a drift of the loops towards more extreme mass ratios. However, the time spent in the semi-detached phase is somewhat reduced relative to the time spent in contact.

The rate of loss of angular momentum associated with gravitational radiation (Webbink 1975) is

$$\frac{d\ln H}{dt} = -8.1\times10^{-10}\,\frac{m_1 m_2(m_1+m_2)}{A^4} \qquad (15)$$

where A is the binary separation in solar radii. For the system we are considering this is initially 7×10^{-11}/yr. Though small, this is not negligible, since in $\sim10^9$ yr the system would cover the range in $\log H$ shown in Fig. 5. Angular momentum loss due to braking by stellar winds, a mechanism which certainly seems to remove angular momentum from single stars on a timescale of $\sim10^9$ yr (Kraft 1968) would presumably affect contact binaries of spectral type similar to the Sun or later. This process might be the dominant agent of long-term evolution, since nuclear evolution is even slower in low-mass systems.

We have made a preliminary investigation of the influence of nuclear evolution on the behaviour of contact binaries. To avoid the necessity of following many thermal cycles, we artificially increased the rate at which hydrogen in consumed in nuclear reactions by a factor of 4000. Two models with different degrees of hydrogen depletion were then followed on a thermal timescale, with the hydrogen consumption rate returned to normal. The thermal evolution of these models is shown in Fig. 6. The values for the central hydrogen abundance in the primary are $X_c = 0.37$ and $X_c = 0.15$ for the two cases. After an initial phase during which the depth of contact increases (not shown in the figure), the subsequent evolution is towards more extreme mass ratios and shallower contact. The model with $X_c = 0.37$ breaks contact after 3.8×10^7 yr, most of this time being spent near point A, and it appears that its subsequent behaviour would be similar to the unevolved system but at a more extreme mass

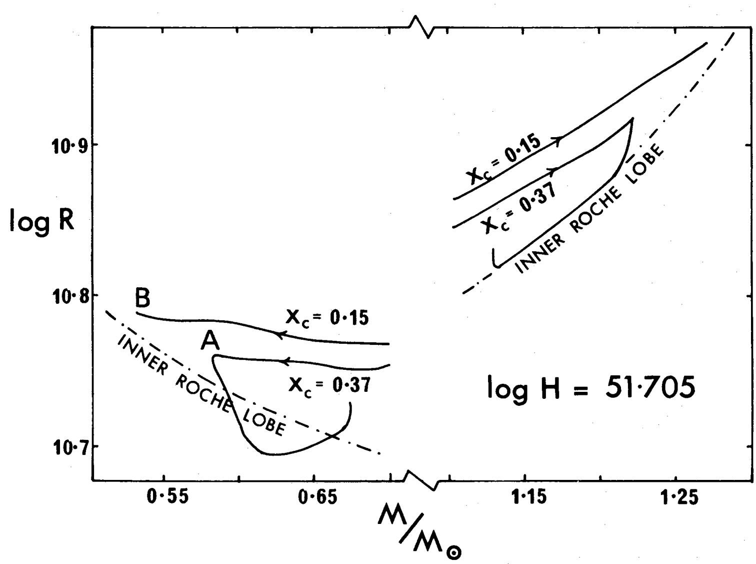

Figure 6. The effect of nuclear evolution on a contact binary. The curves are labelled by the central hydrogen abundance in the primary. The less evolved system appears to be cycling, as for the unevolved system in Fig. 3, but at a more extreme mass ratio. The more evolved system seems to settle in a state of shallow contact. As both sequences were terminated by numerical problems, they are not definitive.

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ratio. The model with $X_c = 0.15$ evolves to an even more extreme mass ratio, and at a slower rate, taking 6.6×10^7 yr to reach its minimum mass ratio. Most of this time was spent near point *B*. We have followed this model for a further 2.1×10^7 yr, and there is no indication that contact will eventually be broken. What we find is that the direction of mass transfer changes erratically and the fraction *f* of the energy transfer varies between 0.8 and 1.0. The model may have reached a point of stable equilibrium, as we suspect that the erratic behaviour is due, once again, to the explicit character of our luminosity transfer calculation. We tentatively suggest that this final model, with a mass ratio of 0.42 and a period of 0.40 day, may represent a typical A-type W UMa system.

In Table 1 we list some values derived from our computations. Column 1 gives the system's angular momentum and column 2 the number of cycles which have been computed. Columns 3, 4, 6 and 7 give crude averages of the mass ratio, period, colour and effective temperature during the interval of good thermal contact, and column 5, the central hydrogen abundance of the primary. Columns 9 to 12 give the lifetimes in various phases: contact, secondary expanding; contact, secondary contracting; semi-detached, secondary contracting; and semi-detached, secondary expanding.

Table 1. Mean parameters of cycling models.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\log_{10} H$	No. of cycles	q	P (days)	X_c	$B-V$	$\log T_e$	$\Delta L/L_1$	Lifetimes (10^6 yr) contact		semi-detached	
51.705	4.5	0.70	0.25	0.70	0.64	3.773	0.51	3.2	3.8	0.6	1.6
51.700	2.75	0.65	0.26	0.70	0.62	3.778	0.52	1.5	5.5	0.6	1.6
51.675	~0.75	0.50	0.27	0.70	0.56	3.796	0.34	29:		0.9	1.3
51.705	~1	0.48	0.35	0.35	0.58	3.790	0.27	38:		0.5	0.9
51.705	–	0.42	0.40	0.15	0.55	3.798	0.30	>87		–	

Notes:

(i) For each system, the total mass is $1.8 M_\odot$ and $\alpha = 10$. The zero-age composition is $X = 0.70$, $Z = 0.02$.
(ii) The values in columns 3, 4, 6 and 7 are crude averages over the phase of good thermal contact. Columns 5 to 7 refer to the primary, but the difference in $\log T_e$ between the stars never exceeds 0.005, during this phase.
(iii) Periods would be a little longer, and colours a little redder, if allowance were made for the effect of the rapid rotation of the components on their internal structure.
(iv) Lifetimes are given for four stages, where these can be distinguished (see text).

4 The semi-detached phase

The behaviour of the mass-accreting secondary component in a semi-detached binary system has recently been examined by several authors (Benson 1970; Yungel'son 1973; Webbink 1975). All have found that the star expands rapidly as it receives mass on the thermal time-scale of the envelope of the primary. This will occur provided that the secondary's structure is predominantly radiative, and therefore that its mass exceeds about $0.55 M_\odot$ (Webbink 1975). Contact is obtained after the transfer of as little as $0.1 M_\odot$.

Webbink has found that the inclusion of the effects of the infalling matter on the surface properties of the secondary may be appreciable. He finds that if these effects are included, the expansion of the secondary towards contact may be significantly more rapid, with the result that even less mass may be transferred during the semi-detached phase. A decision as to which of the two extreme cases – neglect of the energy of the incoming stream or inclusion of all the energy in the stream – is closer to the real situation is important. We

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shall see later that the subsequent behaviour in contact may be determined by the mass ratio of the system when it first reaches contact. Cyclic behaviour may be avoided if the mass ratio at the moment of contact is sufficiently close to one.

If the energy of the infalling matter can be dissipated above the photosphere of the secondary then we expect that there will be no significant effect on the stellar surface. If however the matter penetrates the photosphere before losing its energy then the energy will contribute to the already rapid expansion of the secondary. The location of the shock in the envelope of the secondary associated with the stream is therefore crucial to the semi-detached phase of evolution. This is not only important during the initial semi-detached phase, but also during the semi-detached portions of the cycles discussed above. The inclusion of the stream energy will reduce the time spent in the semi-detached phase and will also maintain rather more similar conditions in the atmospheres of the two components. We believe that if the fraction of the time which is spent in the semi-detached and poor thermal contact phases can be reduced to not more than 5 per cent then there would no longer be any incompatibility between the models and the observed systems. We note also that there are two other effects which operate in the same way to reduce the proportion of the cycle which is spent in the semi-detached state, namely angular momentum loss from the system and absorption of orbital angular momentum in the spin of the components. Since a change of only 0.03 in $\log H$ alters the cyclic behaviour dramatically, it is only reasonable to expect that the interchange of only a few per cent of the angular momentum of the system between orbit and spin will be sufficient to have a similar effect.

In the initial semi-detached phase of our system we assumed that the kinetic energy of the infalling matter was all absorbed by the secondary's atmosphere, and it therefore contributed to the rapid expansion of the secondary. It was not however included in any of the subsequent calculations. The times given in Table 1 are therefore an upper limit on the duration of the semi-detached phase.

5 Discussion

It is apparent from the models discussed in the previous sections that there is a fairly sharp transition between cyclic and non-cyclic behaviour as the angular momentum of a contact binary is increased with a fixed efficiency of luminosity transfer. The same is true at fixed angular momentum when the efficiency is allowed to vary. The onset of cyclic behaviour is only possible when the secondary is sufficiently perturbed from thermal equilibrium (oversized) as a result of energy being deposited in its atmosphere for it to begin losing some of its mass to the primary. As we have seen, the reversal of the net flow of mass in the system is not an immediate consequence of geometrical contact of the two stars. Good thermal contact must be established. Since the maximum energy which can be transferred decreases as the mass ratio tends to unity, it is essential that the depth of contact increase rapidly after contact is first achieved. Failure to achieve good thermal contact leads to equalization of the masses. We would like to predict the conditions under which cyclic behaviour may be induced.

In Tables 2 and 3 we list various details of the different models at the point in the contact phase at which the direction of mass transfer reverses and the mass of the secondary begins to decrease, or, if reversal does not occur, at which the luminosity transfer is a maximum. We see that for reversal of mass transfer it is necessary that the luminosity transferred to the secondary reach about 50 per cent of that generated by nuclear reactions within the core of the secondary. When this value has been reached, the fraction 'f' of the maximum transferrable luminosity is about 0.5 or 0.6. If we therefore assume that these figures are generally applicable to contact binaries of low mass, we can estimate an upper limit on the

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Table 2. Model details at maximum mass of secondary (cyclic behaviour).

$\log H$	Cycle	α	q	f	$\Delta L/L_2$ (nuclear)	$\log T_1$	$\log T_2$
51.700	1	10	0.72	0.57	0.45	3.7722	3.7285
51.700	2	10	0.72	0.55	0.47	3.7720	3.7269
51.705	1	10	0.70	0.53	0.46	3.7769	3.7307
51.705	2	10	0.74	0.58	0.40	3.7715	3.7349
51.705	3	10	0.76	0.68	0.43	3.7690	3.7374
51.705	4	10	0.77	0.67	0.45	3.7686	3.7365
51.705	1	7.5	0.70	0.61	0.61	3.7770	3.7352
51.710	1	10	0.73	0.61	0.48	3.7748	3.7389

Table 3. Model details at maximum luminosity transfer (non-cyclic behaviour).

$\log H$		α	q	f	$\Delta L/L_2$ (nuclear)	$\log T_1$	$\log T_2$
51.705		5	0.73	0.61	0.50	3.7743	3.7418
51.710	(2nd cycle)	10	0.82	0.53	0.17	3.7651	3.7438
51.715		10	0.80	0.24	0.06	3.7677	3.7426

mass ratio, q ($\leqslant 1$) for such stars. If we take the mass–luminosity relation on the lower main sequence to be $L/L_\odot = (M/M_\odot)^{4.5}$, then the energy transfer is given by

$$\Delta L/L_0 = \frac{f m_1 m_2}{m_1 + m_2}(m_1^{3.5} - m_2^{3.5}) \tag{16}$$

and if $f = 0.5$ and $\Delta L = 0.5\,L_2$ (nuclear), then we have

$$m_2^{3.5} = \frac{m_1}{m_1 + m_2}(m_1^{3.5} - m_2^{3.5}) \tag{17}$$

which reduces to

$$(q + 2) = q^{-3.5}. \tag{18}$$

Thus we may crudely estimate the upper limit on q as being about 0.75, independently of the total system mass.

The termination of mass transfer from the secondary to the primary is extremely abrupt, and fully efficient luminosity transfer is maintained almost to the point of reversal of the mass flow. Throughout the contact phase the Roche lobes of both components are expanding, but the slowly increasing luminosity transfer made possible by the growing disparity in the masses permits the depth of contact to decrease slowly. The expression (16) for the energy transfer does not reach a maximum until a mass ratio of 0.19. The secondary is maintained at relatively constant radius throughout the contact phase. Eventually the rate of growth of the radius of the secondary's lobe is greater than the star can match and thermal contact is lost, leading to the immediate reversal of mass transfer and the collapse of the secondary towards equilibrium. As α increases, the points of loss of thermal and loss of geometrical contact approach one another. In Flannery's (1976) model, the two points are very close. The maximum of $\Delta L/L_1$ occurs at $q = 0.65$. If the mass ratio falls below this value during evolution in contact, the primary component will be retaining an increasing fraction of its total luminosity and will therefore attempt to expand. This may be the essential factor determining the lower limit on the mass ratio in the cycles.

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If we examine the observed systems, we find that while W-type systems have mass ratios up to 0.88, the largest mass ratio for the A-type systems is 0.54 (Ruciński 1973). This is compatible with A-type systems having lower angular momentum but it may equally be a consequence of evolution.

In Section 4 we noted that the treatmernt of the semi-detached phase was important for determining the mass ratio when contact was reached. If we set $f = 1$ and $\Delta L/L_2 = 0.5$ then we can estimate the maximum mass ratio at which the direction of mass transfer can be reversed. If contact is first obtained at a larger mass ratio, the system must continue to evolve towards equal masses. The critical value is about 0.83 with the given assumptions.

6 Conclusions

Contact binary systems of sufficiently low angular momentum for good thermal contact to be established between the components exhibit cyclic behaviour. Part of the cycle is spent in contact and part in a semi-detached state. The upper bound on the mass ratio in the cycles is determined by the rapid expansion of the secondary in response to mass accretion. The lower bound may be determined by the expansion of the primary component when the luminosity which it is required to supply as the secondary falls to an insignificant fraction of its nuclear luminosity. During the contact phase of the cycle the models are in reasonably good agreement with the observed properties of A-type W UMa systems. Although we cannot yet be certain, we find that for progressively lower angular momentum the mass ratio, averaged over a cycle, becomes progressively more extreme, and at a rather rapid rate: according to Table 1, a 7 per cent drop in angular momentum may change the mass ratio from ~0.7 to ~0.5. Perhaps more significantly, the fraction of time spent out of contact seems to drop very rapidly. Although mass ratios obtained from observations, either of light curves or of radial velocities, must be treated with considerable caution until an entirely satisfactory model of the surface layers of the common envelope has been established, the present evidence suggests that mass ratios less than 0.6 are substantially commoner than those greater than 0.6. On the basis of our calculations this implies that angular momenta are generally at least 10 per cent less than the maximum which would in principle be necessary for an equal-mass system in marginal contact (e.g. $\log H = 51.72$ for a total mass of $1.8\,M_\odot$). We do not believe that the angular momentum of any system is known to 10 per cent. Note, however, that all our values of angular momentum would be increased by a similar fraction if we were to allow for the effect of rotation on the internal structure of the components; this effect is of course to increase their radii and hence the angular momentum necessary for marginal contact. We therefore believe that it is reasonable to assume that most systems have sufficiently low angular momentum that their mass ratios are less than about 0.6 and hence, on the basis of the calculations we present, that they may spend only a few per cent of their lifetime out of contact. It may therefore be less of a problem than is usually supposed that no binaries are observed with periods in the same range as W UMa stars but with markedly different surface temperatures.

Nuclear evolution appears to act in a qualitatively similar fashion to the reduction of angular momentum, except that it gives longer periods for the same effective temperature. Long-term evolution of a contact binary may be determined by nuclear evolution, but steady angular momentum loss may be competitive or even dominant. Gravitational radiation may be more important than is usually thought, if the sensitivity of mass ratio to angular momentum really is as large as we tentatively conclude. However, a braking torque due to a stellar wind coupled to magnetic fields may be even more important. Whichever of nuclear evolution and angular momentum loss dominates, the progression towards more

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extreme mass ratios suggests that these systems end up as single stars. If there exists a small but significant population of rapidly rotating K subgiants, it would suggest that nuclear evolution is the more important effect.

Acknowledgments

We gratefully acknowledge stimulating discussions with Drs B. Flannery, L. Lucy, J. A. J. Whelan, R. F. Webbink and J. Faulkner. One of us (PPE) is grateful to the Lick Observatory for its hospitality while part of this work was in progress; JAR thanks the SRC for financial assistance. This work was supported in part by NSF grant GP-32337X.

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