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The Exact MSSM Spectrum from String Theory

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Abstract

We show the existence of realistic vacua in string theory whose observable sector has exactly the matter content of the MSSM. This is achieved by compactifying the $E_8 \times E_8$ heterotic superstring on a smooth Calabi-Yau threefold with an $SU(4)$ gauge instanton and a $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson line. Specifically, the observable sector is $N = 1$ supersymmetric with gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, three families of quarks and leptons, each family with a right-handed neutrino, and *one* Higgs–Higgs conjugate pair. Importantly, there are no extra vector-like pairs and no exotic matter in the zero mode spectrum. There are, in addition, 6 geometric moduli and 13 gauge instanton moduli in the observable sector. The holomorphic $SU(4)$ vector bundle of the observable sector is slope-stable.

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In a number of conference talks [1], we introduced a minimal heterotic standard model whose observable sector has exactly the matter spectrum of the MSSM. This was motivated and constructed as follows.

The gauge group $Spin(10)$ is very compelling from the point of view of grand unification and string theory since a complete family of quarks and leptons plus a right-handed neutrino fits exactly into its **16** spin representation. Non-vanishing neutrino masses indicate that, in supersymmetric theories without exotic multiplets, a right-handed neutrino must be added to each family of quarks and leptons [2]. Within the context [3] of $N = 1$ supersymmetric $E_8 \times E_8$ heterotic string vacua, a $Spin(10)$ group can arise from the spontaneous breaking of the observable sector E_8 group by an $SU(4)$ gauge instanton on an internal Calabi-Yau threefold [4]. The $Spin(10)$ group is then broken by discrete Wilson lines to a gauge group containing $SU(3)_C \times SU(2)_L \times U(1)_Y$ as a factor [5]. To achieve this, the Calabi-Yau manifold must have, minimally, a fundamental group $\mathbb{Z}_3 \times \mathbb{Z}_3$.

Until recently, such vacua could not be constructed since Calabi-Yau threefolds with fundamental group $\mathbb{Z}_3 \times \mathbb{Z}_3$ and a method for building appropriate $SU(4)$ gauge instantons on them were not known. The problem of finding elliptic Calabi-Yau threefolds with $\mathbb{Z}_3 \times \mathbb{Z}_3$ fundamental group was rectified in [6]. That of constructing $SU(4)$ instantons was solved in a series of papers [7], where a class of $SU(4)$ gauge instantons on these Calabi-Yau manifolds was presented. Generalizing the results in [8, 9], these instantons were obtained as connections on rank 4 holomorphic vector bundles. In order for such connections to exist, it is necessary for the corresponding bundles to be slope-stable. A number of non-trivial checks of the stability of these bundles was presented in [7]. A rigorous proof of the conjectured slope-stability recently appeared in [10]. The complete low energy spectra were computed in this context. The observable sectors were found to be almost that of the minimal supersymmetric standard model (MSSM). Specifically, the matter content of the most economical of these vacua consisted of three families of quarks/leptons, each family with a right-handed neutrino, and *two* Higgs–Higgs conjugate pairs. Apart from these, there were no other vector-like pairs, and no exotic particles. That is, the observable sector is almost that of the MSSM, but contains an extra pair of Higgs–Higgs conjugate fields. Additionally, there are 6 geometric moduli [6] and 19 vector bundle moduli [11]. In [12], it was shown that non-vanishing μ-terms can arise from cubic moduli-Higgs–Higgs conjugate interactions. Despite the extra Higgs–Higgs conjugate fields, the vacua presented in [7] are so close to realistic particle physics that we refer to them as "heterotic standard models".

These results were very encouraging. However, an obvious question is whether one can, by refining these vector bundles, obtain compactifications of the $E_8 \times E_8$ heterotic string whose matter content in the observable sector is *exactly* that of the MSSM. The answer to this question is affirmative. In this paper, we present models with an $N = 1$ supersymmetric observable sector which, for both the weakly and strongly coupled heterotic string, has the following properties:

Observable Sector

- $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ gauge group
- Matter spectrum:
    - 3 families of quarks and leptons, each with a *right-handed neutrino*
    - 1 Higgs–Higgs conjugate pair
    - No exotic matter fields
    - No vector-like pairs (apart from the one Higgs pair)
- 3 complex structure, 3 Kähler, and 13 vector bundle moduli

The holomorphic $SU(4)$ vector bundle V leading to this observable sector is slope-stable. A rigorous proof of this will be presented in [13]. Note that, although very similar to the supersymmetric standard model, our observable sector differs in two significant ways. These are, first, the appearance of an additional gauged $B - L$ symmetry and, second, the existence of $6 + 13$ moduli fields, all uncharged under the gauge group.

The structure of the hidden sector depends on the choice of a stable, holomorphic vector bundle V'. The topology of V', that is, its second Chern class, is constrained by two conditions: first, the anomaly cancellation equation

$$c_2(V') = c_2(TX) - c_2(V) - [\mathcal{W}], \tag{1}$$

where $[\mathcal{W}]$ is a possible effective five-brane class and, second, a necessary condition of slope-stability given by

$$\int_X \omega \wedge c_2(V') > 0 \tag{2}$$

for some Kähler class ω. Often, this inequality is the only obstruction to finding stable bundles. We expect that the second condition is sufficiently strong that a subset of the bundles V' satisfying it are slope-stable. Applying these conditions to the specific Calabi-Yau threefold and $SU(4)$ observable sector bundle discussed above, one can conclude the following.

Hidden Sector

- One expects there to exist holomorphic vector bundles V' on the hidden sector which satisfy the anomaly cancellation condition and are slope-stable for Kähler classes ω for which the observable bundle V is also stable.

We have not explicitly constructed such hidden sector bundles. A search for these is underway[1]. We will assume their existence in the remainder of this paper.

The vacua presented above are a small subset of the heterotic standard model vacua presented in [7]. As discussed below, their construction involves subtleties in the analysis of the so-called "ideal sheaf" in the observable sector vector bundle, which were previously overlooked. They appear to be the minimal such vacua, all others containing either additional pairs of Higgs–Higgs conjugate fields and/or vector-like pairs of families in the observable sector. For this reason, we will refer to these vacua as "minimal" heterotic standard models.

We note that, to our knowledge, these are the only vacua[2] whose spectrum in the observable sector has exactly the matter content of the MSSM. Other superstring constructions [9, 17, 18, 19, 20] lead to vacua whose zero mode spectrum contains either exotic multiplets or substantial numbers of vector-like pairs of Higgs and family fields, or

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[1] Although exhibiting explicit $N = 1$ supersymmetric hidden sectors is of interest, it is not clear that it is necessary, or even desirable, from the phenomenological point of view. For example, supersymmetry breaking purely by gaugino condensation in the hidden sector may not lead to moduli stabilization with a small positive cosmological constant [14]. This might require the addition of anti-five-branes in the vacuum, as in [15], corresponding to an antieffective component of the five-brane class $[\mathcal{W}]$ in the anomaly cancellation condition. Allowing anti-five-branes in the hidden sector would greatly simplify the search for stable hidden sector vector bundles.

[2] At least until yesterday [16], when a nice generalization of the construction presented in [9] (which makes stability manifest) appeared. Their model differs from ours in two respects. First, it uses a rank 5 vector bundle instead of a rank 4 one. Second, their one pair of Higgs fields arises in a codimension-two region in the moduli space, whereas our Higgs fields are generically present.

both. Although these might obtain an *intermediate* scale mass through cubic couplings with moduli (assuming these interactions satisfy appropriate selection rules and the expectation values of the moduli are sufficiently large), they can never be entirely removed from the spectrum. To do so would violate the decoupling theorem. For these reasons, we speculate that heterotic standard models and, in particular, the minimal heterotic standard model described in this paper may be of phenomenological significance.

We now specify, in more detail, the properties of the these minimal vacua and indicate how they are determined. The requisite Calabi-Yau threefold, X, is constructed as follows [17]. Let $\widetilde{X}$ be a simply connected Calabi-Yau threefold which is an elliptic fibration over a rational elliptic surface, dP_9. It was shown in [6] that $\widetilde{X}$ factors into the fiber product $\widetilde{X} = B_1 \times_{\mathbb{P}^1} B_2$, where B_1 and B_2 are both dP_9 surfaces. Furthermore, $\widetilde{X}$ is elliptically fibered with respect to each projection map $\pi_i : \widetilde{X} \to B_i$, $i = 1, 2$. In a restricted region of their moduli space, such manifolds can be shown to admit a $\mathbb{Z}_3 \times \mathbb{Z}_3$ group action which is fixed-point free. It follows that

$$X = \frac{\widetilde{X}}{\mathbb{Z}_3 \times \mathbb{Z}_3} \tag{3}$$

is a smooth Calabi-Yau threefold that is torus-fibered over a singular $d\mathbb{P}_9$ and has non-trivial fundamental group

$$\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3 \,, \tag{4}$$

as desired. It was shown in [6] that X has

$$h^{1,1}(X) = 3 \,, \quad h^{2,1}(X) = 3 \tag{5}$$

Kähler and complex structure moduli respectively; that is, a total of 6 geometric moduli.

We now construct a holomorphic vector bundle, V, on X with structure group

$$G = SU(4) \tag{6}$$

contained in the E_8 of the observable sector. For this bundle to admit a gauge connection satisfying the hermitian Yang-Mills equations, it must be slope-stable. The connection spontaneously breaks the observable sector E_8 gauge symmetry to

$$E_8 \longrightarrow Spin(10) \,, \tag{7}$$

as desired. We produce V by building stable, holomorphic vector bundles $\widetilde{V}$ with structure group $SU(4)$ over $\widetilde{X}$ that are equivariant under the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$. This is accomplished by generalizing the method of "bundle extensions" introduced in [8]. The bundle V is then given as

$$V = \frac{\widetilde{V}}{\mathbb{Z}_3 \times \mathbb{Z}_3} \,. \tag{8}$$

Realistic particle physics phenomenology imposes additional constraints on $\widetilde{V}$. Recall that with respect to $SU(4) \times Spin(10)$ the adjoint representation of E_8 decomposes as

$$\mathbf{248} = \big(\mathbf{1}, \mathbf{45}\big) \oplus \big(\mathbf{4}, \mathbf{16}\big) \oplus \big(\overline{\mathbf{4}}, \overline{\mathbf{16}}\big) \oplus \big(\mathbf{6}, \mathbf{10}\big) \oplus \big(\mathbf{15}, \mathbf{1}\big) \,. \tag{9}$$

The number of $\mathbf{45}$ multiplets is given by

$$h^0\left(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\right) = 1. \tag{10}$$

Hence, there are $Spin(10)$ gauge fields in the low energy theory, but no adjoint Higgs multiplets. The chiral families of quarks/leptons will descend from the excess of $\mathbf{16}$ over $\overline{\mathbf{16}}$ representations. To ensure that there are three generations of quarks and leptons after quotienting out $\mathbb{Z}_3 \times \mathbb{Z}_3$, one must require that

$$n_{\overline{\mathbf{16}}} - n_{\mathbf{16}} = \frac{1}{2} c_3\big(\widetilde{V}\big) = -3 \cdot \big|\mathbb{Z}_3 \times \mathbb{Z}_3\big| = -27 \,, \tag{11}$$

where $n_{\overline{\mathbf{16}}}$, $n_{\mathbf{16}}$ are the numbers of $\overline{\mathbf{16}}$ and $\mathbf{16}$ multiplets, respectively, and $c_3(\widetilde{V})$ is the third Chern class of $\widetilde{V}$.

The number of $\overline{\mathbf{16}}$ zero modes [9] is given by $h^1\big(\widetilde{X}, \widetilde{V}^*\big)$. Therefore, if we demand that there be no vector-like matter fields arising from $\overline{\mathbf{16}}$-$\mathbf{16}$ pairs, $\widetilde{V}$ must be constrained so that

$$h^1\left(\widetilde{X}, \widetilde{V}^*\right) = 0 \,. \tag{12}$$

Similarly, the number of $\mathbf{10}$ zero modes is $h^1\big(\widetilde{X}, \wedge^2 \widetilde{V}\big)$. However, since the Higgs fields arise from the decomposition of the $\mathbf{10}$, one must not set the associated cohomology to zero. Rather, we restrict $\widetilde{V}$ so that $h^1\big(\widetilde{X}, \wedge^2 \widetilde{V}\big)$ is minimal, but non-vanishing. Subject to all the constraints that $\widetilde{V}$ must satisfy, we find that the minimal number of $\mathbf{10}$ representations is

$$h^1\left(\widetilde{X}, \wedge^2 \widetilde{V}\right) = 4 \,. \tag{13}$$

In [7], the smallest dimension of this cohomology group that we could find in the heterotic standard model context was $h^1(\widetilde{X}, \wedge^2 \widetilde{V}) = 14$. However, as discussed below, a more detailed analysis of the ideal sheaf involved in the construction of the vector bundle allows one to reduce this from 14 to 4.

We now present a stable vector bundle $\widetilde{V}$ satisfying constraints eqns. (11), (12) and (13). This is constructed as an extension

$$0 \longrightarrow V_1 \longrightarrow \widetilde{V} \longrightarrow V_2 \longrightarrow 0 \tag{14}$$

of two rank 2 bundles, V_1 and V_2. Each of these is the tensor product of a line bundle with a rank 2 bundle pulled back from a dP_9 factor of $\widetilde{X}$. Using the two projection maps, we define[3]

$$V_1 = \mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2) \otimes {\pi_1}^*(W_1)\,, \quad V_2 = \mathcal{O}_{\widetilde{X}}(\tau_1 - \tau_2) \otimes {\pi_2}^*(W_2)\,, \tag{15}$$

where

$$span\{\tau_1, \tau_2, \phi\} = H^2(\widetilde{X}, \mathbb{C})^{\mathbb{Z}_3 \times \mathbb{Z}_3} \tag{16}$$

is the $\mathbb{Z}_3 \times \mathbb{Z}_3$ invariant part of the Kähler moduli space. The two bundles, W_1 on B_1 and W_2 on B_2, are constructed via an equivariant version of the Serre construction as

$$0 \longrightarrow \chi_1 \mathcal{O}_{B_1}(-f_1) \longrightarrow W_1 \longrightarrow \chi_1^2 \mathcal{O}_{B_1}(f_1) \otimes I_3^{B_1} \longrightarrow 0 \tag{17}$$

and

$$0 \longrightarrow \chi_2^2 \mathcal{O}_{B_2}(-f_2) \longrightarrow W_2 \longrightarrow \chi_2 \mathcal{O}_{B_2}(f_2) \otimes I_6^{B_2} \longrightarrow 0\,, \tag{18}$$

where $I_3^{B_1}$ and $I_6^{B_2}$ denote the ideal sheaf[4] of 3 and 6 points in B_1 and B_2 respectively. Characters χ_1 and χ_2 are third roots of unity which generate the first and second factors of $\mathbb{Z}_3 \times \mathbb{Z}_3$.

The crucial new observation occurs in the definitions of W_1 and W_2. Satisfying condition eq. (11) requires that one use ideal sheaves of 9 points in total. In our previous papers [7], we chose W_1 to be the trivial bundle and defined W_2 as an extension of two rank 1 bundles, one of which contained a single ideal sheaf, I_9. This comprises 9 points, as it must. However, it is possible to use several such sheaves in the definitions of W_1 and W_2, as long as the total number of points is 9. Note that while the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on $\widetilde{X}$ only has orbits consisting of 9 points, the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on the base surfaces B_1 and B_2 is not free and, in fact, has orbits of 9 and of 3 points. This allows one to define the ideal sheaf $I_3^{B_1}$ using the fixed points of the second $\mathbb{Z}_3$ on B_1 and the ideal sheaf $I_6^{B_2}$ using the fixed points of the second $\mathbb{Z}_3$ on B_2 taken with multiplicity 2. That is,

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[3] See [7] for our notation of line bundles $\mathcal{O}_{\widetilde{X}}(\cdots)$, etc.

[4] The analytic functions vanishing at the respective points.

previously we only considered the case where the total of 9 points were distributed as[5] $0+9$. In this paper, we distribute the points into two different ideal sheaves as $3+6$. This allows us to obtain the precise MSSM matter content.

We now extend the observable sector bundle V by adding a Wilson line, W, with holonomy

$$\mathrm{Hol}(W) = \mathbb{Z}_3 \times \mathbb{Z}_3 \subset Spin(10)\,. \tag{19}$$

The associated gauge connection spontaneously breaks $Spin(10)$ as

$$Spin(10) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}\,, \tag{20}$$

where $SU(3)_C \times SU(2)_L \times U(1)_Y$ is the standard model gauge group. Since $\mathbb{Z}_3 \times \mathbb{Z}_3$ is Abelian and $\mathrm{rank}\big(Spin(10)\big) = 5$, an additional rank one factor must appear. For the chosen embedding of $\mathbb{Z}_3 \times \mathbb{Z}_3$, this is precisely the gauged $B-L$ symmetry.

As discussed in [9], the zero mode spectrum of $V \oplus W$ on X is determined as follows. Let R be a representation of $Spin(10)$, and denote the associated $\widetilde{V}$ bundle by $U_R(\widetilde{V})$. Find the representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on $H^1\big(\widetilde{X}, U_R(\widetilde{V})\big)$ and tensor this with the representation of the Wilson line on R. The zero mode spectrum is then the invariant subspace under this joint group action. Let us apply this to the case at hand. To begin with, the single **45** decomposes into the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ gauge fields. Now consider the $\overline{\mathbf{16}}$ representation. It follows from eq. (12) that no such representations occur. Hence, no $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ fields arising from vector-like $\overline{\mathbf{16}}$-**16** pairs appear in the spectrum, as desired. Next examine the **16** representation. The constraints (11) and (12) imply that

$$h^1\left(\widetilde{X}, \widetilde{V}\right) = 27\,. \tag{21}$$

One can calculate the $\mathbb{Z}_3 \times \mathbb{Z}_3$ representation on $H^1\big(\widetilde{X}, \widetilde{V}\big)$, as well as the Wilson line action on **16**. We find that

$$H^1\big(\widetilde{X}, \widetilde{V}\big) = RG^{\oplus 3}, \tag{22}$$

where RG is the regular representation of $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ given by

$$RG = 1 \oplus \chi_1 \oplus \chi_2 \oplus \chi_1^2 \oplus \chi_2^2 \oplus \chi_1\chi_2 \oplus \chi_1^2\chi_2 \oplus \chi_1\chi_2^2 \oplus \chi_1^2\chi_2^2. \tag{23}$$

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[5] The ideal sheaf of 0 points is just the trivial line bundle.

Furthermore, the Wilson line action can be chosen so that

$$\mathbf{16} = \Big[\chi_1\chi_2^2\big(\mathbf{3},\mathbf{2},1,1\big) \oplus \chi_2^2\big(\mathbf{1},\mathbf{1},6,3\big) \oplus \chi_1^2\chi_2^2\big(\overline{\mathbf{3}},\mathbf{1},-4,-1\big)\Big] \oplus \\ \oplus \Big[\big(\mathbf{1},\mathbf{2},-3,-3\big) \oplus \chi_1^2\big(\overline{\mathbf{3}},\mathbf{1},2,-1\big)\Big] \oplus \chi_2\big(\mathbf{1},\mathbf{1},0,3\big). \quad (24)$$

Tensoring these together, we find that the invariant subspace consists of three families of quarks and leptons, each family transforming as

$$\big(\mathbf{3},\mathbf{2},1,1\big)\,, \quad \big(\overline{\mathbf{3}},\mathbf{1},-4,-1\big)\,, \quad \big(\overline{\mathbf{3}},\mathbf{1},2,-1\big) \qquad (25)$$

and

$$\big(\mathbf{1},\mathbf{2},-3,-3\big)\,, \quad \big(\mathbf{1},\mathbf{1},6,3\big)\,, \quad \big(\mathbf{1},\mathbf{1},0,3\big) \qquad (26)$$

under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. We have displayed the quantum numbers $3Y$ and $3(B-L)$ for convenience. Note from eq. (26) that each family contains a right-handed neutrino, as desired.

Next, consider the **10** representation. Recall from eq. (13) that $h^1\big(\widetilde{X}, \wedge^2\widetilde{V}\big) = 4$. We find that the representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$ in $H^1\big(\widetilde{X}, \wedge^2\widetilde{V}\big)$ is given by

$$H^1\big(\widetilde{X}, \wedge^2\widetilde{V}\big) = \chi_2 \oplus \chi_2^2 \oplus \chi_1\chi_2^2 \oplus \chi_1^2\chi_2\,. \qquad (27)$$

Furthermore, the Wilson line W action is

$$\mathbf{10} = \Big[\chi_2^2\big(\mathbf{1},\mathbf{2},3,0\big) \oplus \chi_1^2\chi_2^2\big(\mathbf{3},\mathbf{1},-2,-2\big)\Big] \oplus \Big[\chi_2\big(\mathbf{1},\overline{\mathbf{2}},-3,0\big) \oplus \chi_1\chi_2\big(\overline{\mathbf{3}},\mathbf{1},2,2\big)\Big]\,. \quad (28)$$

Tensoring these actions together, one finds that the invariant subspace consists of a single copy of

$$\big(\mathbf{1},\mathbf{2},3,0\big)\,, \quad \big(\mathbf{1},\overline{\mathbf{2}},-3,0\big)\,. \qquad (29)$$

That is, there is precisely one pair of Higgs–Higgs conjugate fields occurring as zero modes of our vacuum.

Finally, consider the **1** representation of the $Spin(10)$ gauge group. It follows from (9), the above discussion, and the fact that the Wilson line action on **1** is trivial that the number of **1** zero modes is given by the $\mathbb{Z}_3 \times \mathbb{Z}_3$ invariant subspace of $H^1\big(\widetilde{X}, \widetilde{V}\otimes\widetilde{V}^*\big)$, which is denoted by $H^1\big(\widetilde{X}, \widetilde{V}\otimes\widetilde{V}^*\big)^{\mathbb{Z}_3\times\mathbb{Z}_3}$. Using the formalism developed in [11], we find that

$$h^1\left(\widetilde{X}, \widetilde{V}\otimes\widetilde{V}^*\right)^{\mathbb{Z}_3\times\mathbb{Z}_3} = 13. \qquad (30)$$

That is, there are 13 vector bundle moduli.

Putting these results together, we conclude that the zero mode spectrum of the observable sector has gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, contains three families of quarks and leptons each with a right-handed neutrino, has one Higgs–Higgs conjugate pair, and contains no exotic fields or additional vector-like pairs of multiplets of any kind. Additionally, there are 13 vector bundle moduli.

As a final step, one must demonstrate that $\widetilde{V}$ is slope-stable. This has been proven, in detail, and will be presented in [13]. Here, suffice it to say that $\widetilde{V}$ will be stable with respect to any Kähler class in a finite three-dimensional region of Kähler moduli space containing the point

$$\omega = 3\big(2\tau_1 + 3\tau_2 + \phi\big). \tag{31}$$

Henceforth, we restrict our discussion to this region of moduli space, which we denote by $\mathcal{K}^s$.

Another important constraint for realistic compactifications is the existence of Yukawa couplings. Recall that (via the Kaluza-Klein reduction) the massless fields are associated with a number of vector-bundle valued harmonic one-forms Ψ_i on the Calabi-Yau threefold. Their Yukawa coupling is then given by the integral

$$\lambda_{ijk} = \frac{1}{9}\int_{\widetilde{X}} \Omega \wedge \mathrm{Tr}\Big(\Psi_i \wedge \Psi_j \wedge \Psi_k\Big)\,, \tag{32}$$

where the Tr denotes a suitable contraction of the vector bundle indices. The integral is only non-zero if the legs of the three one-forms Ψ_i span the π_1-fiber direction, the π_2 fiber direction, and the base $\mathbb{P}^1$ direction. This is the case here. A detailed analysis reveals that we do, indeed, have non-vanishing Yukawa couplings [21].

Thus far, we have discussed the vector bundle of the observable sector. However, the vacuum can contain a stable, holomorphic vector bundle, $\widetilde{V}'$, on X whose structure group is in the E_8' of the hidden sector. The requirement of anomaly cancellation relates the observable and hidden sector bundles, imposing the constraint that

$$c_2\big(\widetilde{V}'\big) = c_2\big(T\widetilde{X}\big) - c_2\big(\widetilde{V}\big) - [\mathcal{W}], \tag{33}$$

where $[\mathcal{W}]$ must be an effective class and c_2 is the second Chern class. In the strongly coupled heterotic string, $[\mathcal{W}]$ is the class of the holomorphic curve around which a bulk space five-brane is wrapped. In the weakly coupled case $[\mathcal{W}]$ must vanish. We have previously constructed $\widetilde{X}$ and $\widetilde{V}$ and, hence, can compute $c_2\big(T\widetilde{X}\big)$ and $c_2\big(\widetilde{V}\big)$. They are found to be

$$c_2\big(T\widetilde{X}\big) = 12\big(\tau_1^2 + \tau_2^2\big), \quad c_2\big(\widetilde{V}\big) = \tau_1^2 + 4\tau_2^2 + 4\tau_1\tau_2 \tag{34}$$

respectively. Inserting these results, eq. (33) becomes a constraint on the hidden sector bundle $\widetilde{V}'$. Henceforth, we assume that $\widetilde{V}'$ satisfies (33). The easiest possibility is that $\widetilde{V}'$ is the trivial bundle. However, in this case, we find that $[\mathcal{W}]$ is not effective. Hence, we must choose the hidden sector bundle $\widetilde{V}'$ to be non-trivial.

However, simply satisfying (33) is not sufficient. As discussed previously, $\widetilde{V}'$ must also be slope-stable. As a guide to constructing stable, holomorphic vector bundles $\widetilde{V}'$ in the hidden sector, we note the following condition. It can be shown that if $\widetilde{V}'$ is slope-stable with respect to a Kähler class ω, it must satisfy the "Bogomolov inequality"

$$\int_{\widetilde{X}} c_2\big(\widetilde{V}'\big) \wedge \omega > 0. \tag{35}$$

Note that if $c_2\big(\widetilde{V}'\big)$ is Poincare dual to an effective (antieffective) curve, then (35) is satisfied (never satisfied) for any choice of Kähler class. Most vector bundles $\widetilde{V}'$ have a second Chern class whose Poincare dual is neither effective nor antieffective. In this case, constraint (35) is satisfied for ω's contained in a non-vanishing subspace of the Kähler cone. One can explicitly analyze this subspace using the second Chern class derived from anomaly condition (33). It is simplest to limit our discussion to $\widetilde{V}'$ for which $[\mathcal{W}] = 0$. The generalization to the case where $[\mathcal{W}]$ is non-vanishing is straightforward. In this case, eqns. (33) and (34) imply that

$$c_2\big(\widetilde{V}'\big) = 11\tau_1^2 + 8\tau_2^2 - 4\tau_1\tau_2. \tag{36}$$

Recalling from (16) that τ_1,τ_2 and ϕ are a basis for the $\mathbb{Z}_3 \times \mathbb{Z}_3$ invariant Kähler moduli space, we can parameterize an arbitrary such Kähler class by

$$\omega = x_1\tau_1 + x_2\tau_2 + y\phi. \tag{37}$$

Then, using the relations $\tau_1^3 = \tau_2^3 = \phi^2 = 0$, $\tau_1\phi = 3\tau_1^2$ and $\tau_2\phi = 3\tau_2^2$ we see using (36) and (37) that

$$c_2\big(\widetilde{V}'\big) \wedge \omega = 4x_1 + 7x_2 - 12y. \tag{38}$$

It follows that constraint (35) will be satisfied if

$$4x_1 + 7x_2 - 12y > 0. \tag{39}$$

This defines a three-dimensional region of moduli space which we denote by $\mathcal{K}^B$. Note that the Kähler class (31) for which the observable sector bundle $\widetilde{V}$ was proven to be stable also satisfies (39). Hence,

$$\mathcal{K}^s \cap \mathcal{K}^B \neq \emptyset. \tag{40}$$

In fact, one can show that $\mathcal{K}^s \cap \mathcal{K}^B$ is a finite three-dimensional subcone of the Kähler cone. It follows that both $\widetilde{V}$ and $\widetilde{V}'$ can, in principle, be slope-stable with respect to any Kähler class $\omega \in \mathcal{K}^s \cap \mathcal{K}^B$.

Fix $\omega \in \mathcal{K}^s \cap \mathcal{K}^B$. There are numerous vector bundles $\widetilde{V}'$ with second Chern class (36) which satisfy condition (35) for this choice of ω. Since (35) is only a necessary condition for stability, we expect that many such $\widetilde{V}'$ are not stable. Indeed, one can construct explicit examples for which this is the case. However, (35) is a very strong condition and it is believed that at least some $\widetilde{V}'$ are slope-stable with respect to ω. Furthermore, since one may choose any ω in the three-dimensional space $\mathcal{K}^s \cap \mathcal{K}^B$, it becomes even more probable that there exist slope-stable vector bundles $\widetilde{V}'$ with respect to at least one such ω.

We conclude that one expects that there should exist hidden sector holomorphic vector bundles $\widetilde{V}'$ that satisfy the anomaly cancellation condition and are slope-stable. Explicit examples of such bundles will be presented elsewhere.

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