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THE EXTENT OF MEASUREMENT ERROR IN LONGITUDINAL EARNINGS DATA: DO TWO WRONGS MAKE A RIGHT?

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THE EXTENT OF MEASUREMENT ERROR IN LONGITUDINAL EARNINGS DATA: DO TWO WRONGS MAKE A RIGHT?

<u>ABSTRACT</u>

This paper examines the properties and prevalence of measurement error in longitudinal earnings data. The analysis compares Current Population Survey data to administrative Social Security payroll tax records for a sample of heads of households over two years. In contrast to the typically assumed properties of measurement error, the results indicate that errors are serially correlated over two years and negatively correlated with true earnings (i.e., mean reverting). Moreover, reported earnings are more reliable for females than males. Overall, the ratio of the variance of the signal to the total variance is .82 for men and .92 for women. These ratios fall to .65 and .81 when the data are specified in first-differences. The estimates suggest that longitudinal earnings data may be more reliable than previously believed.

John Bound Department of Economics University of Michigan Ann Arbor, MI 48109 Alan B. Krueger Industrial Relations Section Firestone Library Princeton University Princeton, NJ 08544 Without knowing the extent of the falsifications that actually occur in economic statistics, it is impossible to estimate their influence upon economic theory.

Oskar Morgenstern

Economists have long been concerned that mismeasurement of data leads to spurious results or obscures true economic relationships.¹ This concern has been heightened by the increased use of longitudinal data sets. If measurement errors are uncorrelated over time, then statistical problems caused by the mismeasurement of economic data may be greatly exacerbated when longitudinal data are used to estimate fixed-effects or first-differenced regressions (Griliches and Hausman, 1986). On the other hand, if economic data are consistently misreported over time, then first-differencing could conceivably increase the reliability of longitudinal data. This paper examines the nature and extent of response errors in longitudinal data on individual-reported labor earnings.

The analysis uses a unique data set that links two consecutive March Current Population Surveys (CPS) to employer-reported Social Security earnings records. Data are thus available on each employee's self-reported earnings and employer-reported earnings at two points in time. Our maintained assumption is that Social Security payroll records are a relatively accurate measure of true earnings. Although the results are specific to annual earnings data reported in the CPS, they have potentially important implications for a variety of empirical studies. It is common for annual earnings to appear as a left-hand-side variable

¹ References containing discussions of the prevalence of measurement error and its effects include Morgenstern (1963), Griliches (1974, 1985), Ashenfelter and Solon (1982), Mellow and Sider (1983), Duncan and Hill (1985), and Solon (1987).

in human capital earnings regressions, and as a right-hand-side variable in unemployment duration models, labor supply studies, consumption functions, and other empirical analyses. Particularly in the area of labor supply, many researchers have come to conclude that measurement error in earnings has a dominant impact on empirical findings (e.g., Altonji, 1986, Abowd and Card, 1987, and Ashenfelter, 1984) A better understanding of the nature and magnitude of measurement error is necessary to properly account for its influence on empirical analyses.

The paper is organized as follows. Section one describes the data set that we use. Section two reviews the effect that measurement errors have on econometric analyses under a variety of assumptions, and presents a convenient summary statistic to measure the reliability of a variable. Section three estimates the properties of measurement error in longitudinal earnings data. The analysis uses Social Security payroll records as a measure of true earnings, and estimates limited dependent variable models to account for the censoring of Social Security earnings at the taxable maximum. In section four we address several potential problems that might affect the robustness and representativeness of our results.

The main findings of the paper can be simply summarized: Differencing CPS earnings exacerbates measurement error problems, but even in the first-differenced data there is more news than noise. Our estimates suggest that for men greater than 80% of the measured cross-sectional variation in yearly earnings corresponds to true variation, while nearly 65% of the observed variation in first-differenced earnings is true variation. For the sample of women these figures rise to 90% in the cross-section and 80% in

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first-differences. Moreover, measurement error appears to be positively auto-correlated over two years, and negatively correlated with true earnings. These last two findings are inconsistent with the assumptions of classical measurement error which are typically employed in empirical studies. I. Data

The CPS is a widely used source of labor market data. A stratified, cluster sample containing about 55,000 households is surveyed every month, and half of the households are resurveyed the following year.² Each March the Annual Demographic Supplement to the CPS asks individuals a question on their annual labor earnings in the <u>previous calendar year</u>. If an individual is not home at the time of the survey, another household member is asked to give a "proxy response" for that individual. The annual earnings question is deliberately asked in the month of March because it is hoped that individuals will be especially knowledgeable of their income at this time of year since income tax returns are due in April. The annual earnings concept in the CPS is meant to reflect all wages, salary, tips and bonuses before deductions received from employment in the preceding calendar year.

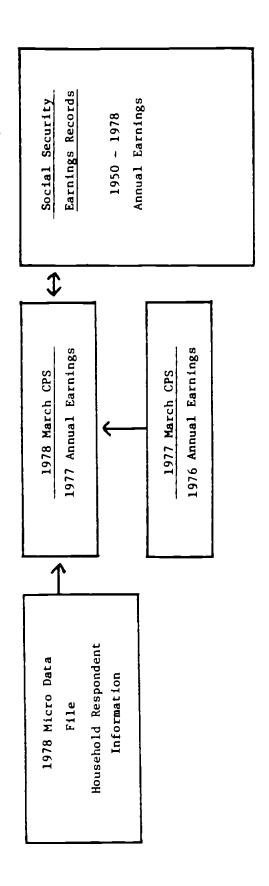
The diagram in Figure 1 outlines the steps we took to generate a longitudinal employer-employee data set with CPS data. The starting point was the 1978 CPS-SER Exact Match File. This file was created by a joint project of the Census Bureau and Social Security Administration (SSA), and contains survey responses for persons in the March 1978 CPS Annual Demographic File linked to their respective earnings information in SSA administrative records. Individuals in the March 1978 CPS were asked a supplemental question on their social security number. If the social security number of the head of the household was not obtained in

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² Households in the selected addresses are in the survey for four consecutive months, not interviewed for the following eight months, and then back in the survey for a final four months. Thus, in any given month half of the households will be re-interviewed the following year, and the other half were interviewed in the previous year.

Figure 1

Ceneration of the Longitudinal CPS-Social Security Data Set



The 1978 BLS Micro Data File containing information on the household respondent and type of interview formation can not be matched for 1977 because of changes in the identification codes in the BLS Micro was merged to the data set by the authors on the basis of unique household identifiers. Similar in-March 1978 CPS for household heads on the basis of household ID numbers and personal characteristics. The match between social security earnings histories and the March 1978 CPS was carried out by the Census Bureau and Social Security Administration. The authors matched the March 1977 CPS to the Data File. Notes:

the initial interview, CPS surveyors were instructed to call the household again. Exact matches between CPS and SSA records were then identified on the basis of the respondent's social security number, name, age, sex, race and line number. Table 1 gives the sample size at each step of the way for the sample of household heads. In only about half of the cases could respondents to the CPS survey be positively linked to their corresponding Social Security records.

The resulting matched CPS-SER file contains a single cross-section of responses to the March CPS questions for 1978, and a time-series of annual covered SSA earnings records for each individual from 1950 to 1978. The Social Security earnings records are taken directly from employer-reported Form 941 quarterly payroll tax records, which are used by the Social Security Administration to calculate OASDHI benefits and determine insured status. Nearly 99 percent of the population we examine is in covered employment (Alvey and Cobleigh, 1980). One important limitation of the SSA earnings data is that earnings are censored at the taxable maximum. The maximum was \$16,500 in 1977 and \$15,300 in 1976. Although in these years relatively few women (5%) have earnings above the Social Security earnings ceiling, nearly half of the males' earnings are censored at the earnings maximum.

The CPS earnings question and Social Security earnings records closely approximate the same concept. In fact, CPS surveyors are allowed to examine respondents' tax forms and W2 statements if they are offered during the interview. And surveyors are explicitly instructed as to which line of the tax form to use in filling-out the questionnaire.³ It

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³ See "March 1988 CPS Interviewers Instructions: Section I," Bureau of the Census, 1988.

	Sample	Number Remaining	Percent of Previous Row
1.	Heads of households in matchable rotation groups.	27,485	
2.	Successful matches between 1978 CPS and 1977 CPS (based on household ID number, age, education, race and sex).	18,048	65.7
3.	Successful matches between 1978 CPS and Social Security data (based on soc. sec. number, name, age, sex, race and line number).	9,137	50.6
4a.	Males	7,303	79.9
5a.	Private, employed workers with positive annual earnings in covered employment.	3,463	47.4
6a.	Non-imputed CPS earnings in 1978 and 1977.	2,924	84.4
7a.	Non-truncated social security earnings in 1977 and 1976.	1,575	53.9

Continued

Table 1 - Continued

Sample	Number Remaining	Percent of Previous Row
b. Females	1,834	20.1
b. Private, employed workers with positive annual earnings in covered employment.	556	30.3
b. Non-imputed CPS earnings in 1978 and 1977.	465	83.6
'b. Non-truncated social security earnings in 1977 and 1976.	444	95.5

Notes:

- a. CPS earnings refer to the previous year.
- b. Social Security taxable earnings limit was \$15,300 in 1976 and \$16,500 in 1977.

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is not known, however, how often tax forms are actually referred to in the course of CPS interviews.⁴

To create a longitudinal data set containing employee and employer earnings reports at two points in time, we have matched the 1977 March CPS Annual Demographic File to the 1978 CPS-SER data set. This match is based on each respondent's unique CPS household identification number, age, education, sex and race. Half of the observations in the March 1977 CPS are in rotation groups that are re-interviewed in the March 1978 CPS. Because the match between CPS and SSA records is most accurate and complete for heads of households, we focus solely on this group. About two-thirds of the observations in the appropriate rotation groups in the 1978 CPS could be successfully matched to the 1977 CPS.

The 1978 and 1977 March CPS Public Use Demographic Files do not contain information on whether earnings data are reported by individuals directly through a self-response or by another household member in a proxy response. Since whether an individual answered a question directly by him or herself may influence the accuracy of the response, and since the likelihood of self-response may differ systematically across groups of the population (e.g. males and females), this is a potentially important omission. However, the BLS Basic Micro-Data File, which is used internally by the BLS, contains information on the identification of the individual who answered the household questions. The person who answered the household questions most likely self-responded to the earnings question as well. We obtained the 1978 BLS Basic Micro-Data File from the Census Bureau and merged it to the CPS-SER. Unfortunately,

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⁴ Two other widely used labor market surveyors, PSID and NLS, similarly allow surveyors to examine tax records.

because of inconsistencies in the BLS Basic Micro-Data File, comparable information cannot be merged to the Demographic File for 1977. Nonetheless, the additional information on the household respondent for 1978 gives some indication of the effect of self-response on the accuracy of earnings data.

We narrow the sample to private, nonagricultural workers with positive CPS and Social Security earnings. Individuals who did not report their earnings to CPS and were then assigned earnings via the "hot deck" procedure are also eliminated from the sample.⁵ We further eliminate from the sample workers who earned any self-employment income during the year or were in occupations that are likely to receive tips. Most importantly, to improve the accuracy of the Social Security earnings data we restrict the sample to workers whose longest job during the year was in a covered industry and occupation.⁶ Despite these final restrictions, individuals who move between covered and uncovered employment during the course of the year will have lower Social Security earnings than true earnings. As a result, in specifications estimated in section three we further limit the sample to employees who have only one employer during the year. Earnings for these workers should consist entirely of covered earnings.

⁵ See Lillard, Smith and Welch (1986) for a description and analysis of the hot deck procedure used by the Census Bureau to impute incomes of nonrespondents.

Employees in the following occupations are eliminated from the sample because they are not covered by the Social Security Act or because they are likely to receive unreported tips: bartenders, busboys, waiters, baggage porters, barbers, bootblacks, hairdressers, housekeepers, laundresses, maids, taxicab drivers, newsboys, clergymen, religious workers N.E.C., farmers, and agriculture and fisheries workers.

The three-way link between the 1978 CPS, Social Security, and 1977 CPS, and the subsequent restrictions placed on the sample resulted in a great deal of sample attrition (see Table 1). Since in the process of matching observations over time and between data sources we have eliminated from the sample individuals with inconsistent responses to the age, sex, education or race questions, there is reason to suspect that the final sample is biased in the direction of giving more accurate responses than the typical employee.

Despite the difficulty in matching the various data sets and the imposed sample restrictions, the comparison between the means of several key variables in the final sample and the population of privately employed heads of households shown in Table 2 suggests that the sample may be reasonably representative of its population. In particular, the mean age, education, weeks worked, and self-response rate in the final sample and in the full March 1978 CPS are similar. On the other hand, the average reported CPS earnings for calendar year 1977 is 11% greater for men and 21% greater for women in the final sample than in the full CPS. This difference in part reflects higher average earnings of covered jobs than uncovered jobs, the deletion of workers with self-employment income, and the restriction that workers in the CPS-SER sample have positive earnings in two consecutive years.

II. Models of Measurement Error

We assume an individual's reported earnings in natural logarithms in year t (CPS_t) equals his true log earnings (η_t) plus an error (v_t) :

$$CPS_t = \eta_t + v_t.$$

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Table 2: Comparison of Final Sample and Population

	l	Men	W	Women	
Variable	(1)	(2)	(3)	(4)	
	Sample	Population	Sample	Population	
Education	11.93	11.96	11.60	11.69	
	(2.93)	(3.20)	(2.65)	(2.84)	
Age	41.99	40.38	45.56	41.67	
	(12.31)	(13.35)	(14.27)	(15.57)	
Weeks of	48.83	47.08	46.31	43.12	
Employment	(8.51)	(10.68)	(11.91)	(14.45)	
White	.93	.91	.86	.82	
	(.26)	(.28)	(.35)	(.38)	
Never Married	.04	.08	.23	.29	
	(.20)	(.28)	(.42)	(.46)	
SMSA	.69	.60	.73	.67	
	(.46)	(.49)	(.44)	(.47)	
North Central	.30	.26	.35	.26	
	(.46)	(.44)	(.48)	(.44)	
North East	.23	.21	.18	.21	
	(.42)	(.41)	(.38)	(.41)	
South	.33	.28	.30	. 29	
	(.47)	(.45)	(.46)	(.45)	
West	.14	.24	.17	.24	
	(.35)	(.43)	(.38)	(.43)	
Annual Earnings	15,586	14,000	7,906		
in 1977	(7,928)	(8,531)	(4,544)		
Self-Respondent	.32	.36	.91	.82	
	(.46)	(.48)	(.28)	(.38)	

Means with Standard Deviations in Parentheses^a

Notes:

a. Population is the entire March 1978 CPS sample of male or female heads of households with non-zero earnings in the previous year. Sample sizes for columns (1) through (4) are 2,924, 23,096, 465, and 5,222, respectively.

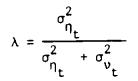
In the case of "classical" measurement error it is assumed that v_t is identically, independently distributed with $E(v_t) = cov(\eta_t, v_t) = 0$. In what shall be called mean reverting measurement error, we assume $E(v_t) =$ 0 and $cov(\eta_t, v_t) \neq 0$.⁷

There are several plausible alternative measures of the accuracy of economic data.⁸ First, the variance of $v_t \left(\sigma_{v_t}^2\right)$ is directly related to the noise added to the residual variance of a cross-sectional regression when the dependent variable contains an additive white noise error component. If the dependent variable is specified in first-differences and we assume that error variances are constant over time, then the variability added to an OLS regression because of classical measurement error is $\sigma_{\Delta v}^2 = 2\sigma_v^2(1-\rho)$ where ρ is the serial correlation coefficient of v. If the measurement error in the dependent variable is correlated with any of the independent variables, however, the measurement error problem is more severe since the estimated coefficients will be biased and inconsistent.

Perhaps a more useful summary measure relates to the magnitude of the attenuation bias imparted on an OLS regression coefficient because an <u>independent variable</u> is measured with error. In the simplest case of a cross-sectional bivariate regression where the independent variable (earnings) is reported with an additive white noise error, the proportional attenuation bias is given asymptotically by $1-\lambda$, where

 $^{^7}$ We refer to this case as "mean reverting" measurement error because as shown below we find a negative correlation between $v_{\rm t}$ and $\eta_{\rm t}$.

^o See Fuller (1987) for a review of statistical models of the effect of measurement error.



The quantity λ is often referred to as the reliability of the data.

If the independent variable is measured in first-differences and if we assume that both true earnings and measurement error are stationary series, then the reliability of ΔCPS_{t} is:

$$\lambda = \frac{\sigma_{\eta}^2}{\sigma_{\eta}^2 + \sigma_{\nu}^2 \frac{(1-\rho)}{(1-r)}}$$

where r is the first order serial correlation in true earnings, and ρ is the first order serial correlation in measurement error. If the (positive) serial correlation in earnings exceeds the (positive) serial correlation in measurement error, the attenuation bias will be greater in longitudinal data than in cross-sectional data (Griliches and Hausman, 1986).

In the more general case when $cov(\eta_t, v_t) \neq 0$, the simple signal-to-total-variance formulas are no longer appropriate estimates of the reliability of the data. It can easily be shown that a nonzero covariance between η_t and v_t leads to the addition of covariance terms to the numerator and denominator of the least squares attenuation bias formula (1- λ). However, the generalization of the reliability formulas can be compactly written as

$$\lambda = \frac{\operatorname{cov}(\operatorname{CPS}_{t}, \eta_{t})}{\operatorname{var}(\operatorname{CPS}_{t})}$$

for cross-sectional data, and as

$$\lambda = \frac{\operatorname{cov}(\Delta CPS, \Delta \eta)}{\operatorname{var}(\Delta CPS)}$$

for first-differenced data.

The reliability of a variable in the general case, it should be noted, is just the slope coefficient from an OLS regression of the correctly measured variable on the mismeasured variable.⁹ As a result, λ can also be interpreted as the proportion of a change in the observed variable that translates into a change in the true, latent variable.

III. Results

To implement the above formulas, we initially assume that Social Security payroll records are an accurate measure of true earnings (or that any error in Social Security earnings records is sufficiently small that it can safely be ignored), and measure η_t by the log of employer-reported Social Security earnings data denoted SSE_t. We calculate v_t by the formula $v_t = CPS_t - SSE_t$. The subscript t refers to wages in period 1 (1976) or period 2 (1977).

If the measurement error is "classical" in the sense that $cov(\eta_t, v_t) = 0$, the sample of workers whose earnings fall below the Social Security taxable maximum (K_t) yields a random sample of v_t . This convenient result follows because selecting workers on the condition that $\eta_t < K_t$ is equivalent to sampling on an orthogonal variable. More generally, however, if η_t and v_t are not independent, sampling on η_t will

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It should also be noted that these estimates of λ are robust to the presence of additive white noise measurement error in the variable used to measure η_+ .

leave us with a nonrandom sample of v_t . We subsequently handle this truncation problem by making the standard assumption of joint normality.

Figure 2 presents a plot of the density of v_t for men who were below the Social Security earnings limit in 1977, and Figure 3 contains a similar plot for women. Both plots show a symmetric, unimodal distribution of errors. Although the distribution is bell shaped the tails are thicker than one would expect with a normal distribution (the standard deviation of v_t in each case is more than three times its interquartile range). In addition, the plots display a large spike near zero -- 14% of the women and 12% of the men report earnings that match their Social Security Form 941 earnings to the exact dollar amount, and more than 40% of male and female respondents are within 2.5 percent.

The summary statistics given at the bottom of the figures indicate that in these truncated samples, measurement error has approximately a zero mean, although women have a slight tendency to under-report their earnings. Furthermore, the magnitude of the error is substantial. The error variance is slightly greater than .10 for men and .05 for women. The error variance represents 27.6% of the total variance in CPS earnings for men and 8.9% for women. Below we explore whether the seemingly greater accuracy in earnings data for women can be explained by their higher propensity to self-respond to the survey.

We next turn our attention to the question of the potential biases these errors might have on the estimation of economic relationships. Using an error-ridden measure of earnings as a left-hand-side variable will produce biased results only if v_t (or Δv_t) is correlated with a right-hand-side regressor. Table 3 presents results of regressing v_t on a variety of typical human capital and demographic variables. Tobit

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procedures are used to account for truncation of Social Security earnings data. The results indicate, for example, that earnings of younger men and married men tend to be under-reported.

The magnitude of the coefficients reported in the table equals the magnitude of the bias expected for these variables when earnings is the dependent variable of a regression. Although the coefficients in each Tobit are jointly statistically significant, the estimated bias for each variable is small compared to its typical effect in an earnings equation. The R^2 's implied by these equations are between .01 and .02 for men, and between .05 and .09 for women.¹⁰ These results suggest that the mismeasurement of earnings leads to little bias when CPS earnings are on the left-hand-side of a regression.

Variance-Covariance Matrix of True Earnings and Measurement Error

Next we estimate the variance-covariance matrix of η_t and v_t . This matrix is of interest because it gives an indication of the magnitude and covariance structure of measurement error over two years. Since Social Security earnings reports in our data set are censored at the taxable maximum, we must impute the latent variance-covariance matrix. Below we give a brief description of the limited dependent variable techniques that are used to account for the censoring problem. A rigorous derivation of our estimator is presented in the Appendix.

¹⁰ These R²'s are calculated as $1 - \sigma_e^2 / \sigma_0^2$ where σ_e^2 represents the estimated residual variance and σ_0^2 represents the estimated residual variance when no explanatory variables are included in the equation.

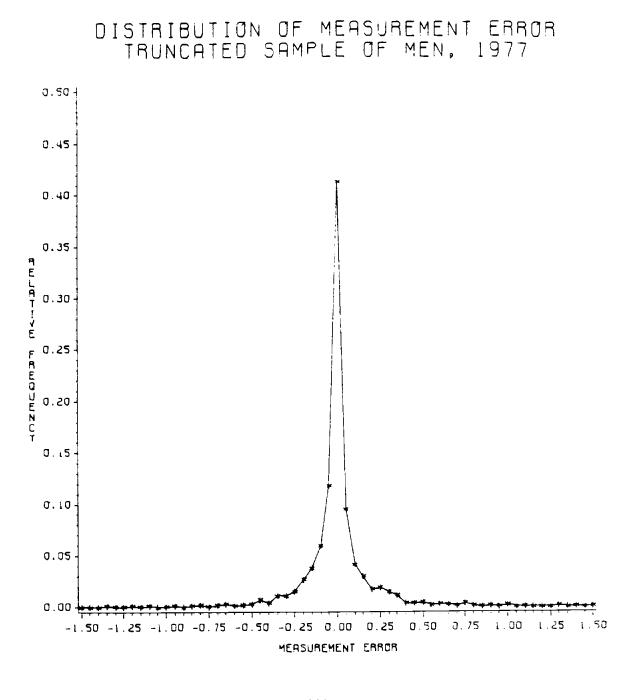
	M	Men		men
	\overline{v}_1	v ₂	v ₁	v ₂
Intercept	. 139	201	386	. 152
	(.078)	(.079)	(.078)	(.102)
Education	.000	002	.012	001
	(.002)	(.003)	(.005)	(.005)
Age	002	002	001	002
	(.001)	(.001)	(.001)	(.001)
Log Weeks Worked	024	.063	.072	001
	(.016)	(.014)	(.015)	(.020)
White	043	.009	.060	.009
	(.019)	(.025)	(.031)	(.035)
Never Married	.052	.063	021	.025
	(.024)	(.039)	(.026)	(.036)
SMSA	.002	020	041	022
	(.015)	(.018)	(.026)	(.036)
North East	.007	018	.014	050
	(.024)	(.031)	(.033)	(.039)
North Central	.008	010	023	128
	(.023)	(.029)	(.030)	(.042)
South	.017	003	021	068
	(.022)	(.028)	(.035)	(.039)
ô	. 278	. 335	. 200	. 220
x ²	26.8	26.7	39.40	23.04
Log-likelihood	870.81	559.88	450.79	403.47
Implied R ²	.022	. 009	. 089	.0498

Table 3: Factors Associated with Measurement Error^{a, b}

Notes: a. Sample size for men is 2,709 and for women is 425.

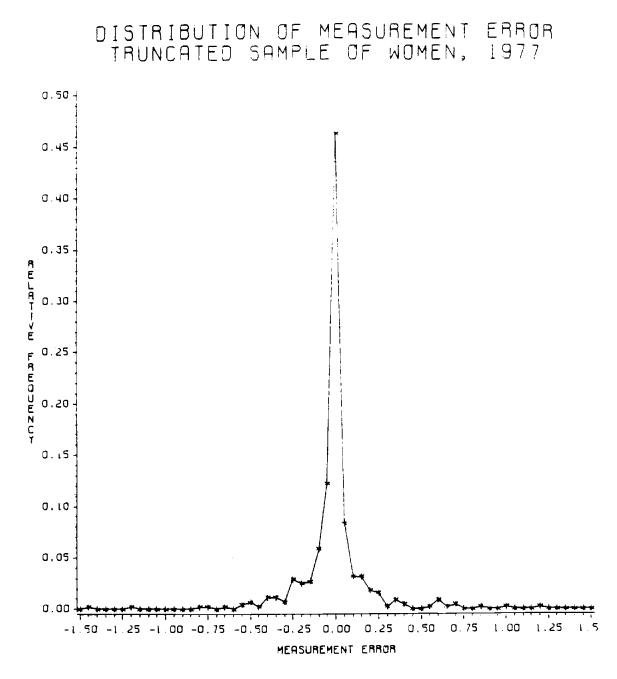
b. v_1 is difference between CPS and Social Security earnings in 1976, and v_2 is difference between CPS and Social Security earnings for 1977.

FIGURE 2



Mean	.004
Variance	.114
Interquartile Range	049 to .033

FIGURE 3



Mean	017
Variance	.051
Interquartile Range	049 to .020

<u>A.</u>	Maximum Like	lihood Esti	imates; Full	Sample
	η ₁	η ₂	v ₁	v ₂
η ₁	.458 (.012)			
n ₂	.382 (.012)	.529 (.015)		
ν ₁	089 (.004)	054 (.005)	.083 (.003)	
v ₂	060 (.006)	104 (.007)	.039 (.003)	.116 (.004)

Variance-Covariance Matri	x for	Male	Workers
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	B. Resul	ts for Trun	icated Sampl	. <u>e</u>
	n ₁	n ₂	۰ ₁	v ₂
η ₁	.327 (.012)			
η ₂	.230 (.011)	.401 (.014)		
۰ ₁	051 (.004)	014 (.005)	.087 (.003)	
ν ₂	022 (.005)	051 (.006)	.037 (.003)	.114 (.004)

Notes: Asymptotic standard errors in parentheses. η_t is the true annual earnings as measured by social security payroll records and v_t is the difference between reported earnings in the CPS and social security payroll records.

Table 4

Variance-Covariance Matrix for Female Workers

<u>A.</u>	. Maximum Likelihood Estimates; Full Sample					
	η ₁	٩ ₂	۰ 1	v ₂		
n ₁	.739 (.049)					
η ₂	.485 (.041)	.625 (.043)				
v ₁	016 (.008)	009 (.002)	.048 (.003)			
v ₂	.011 (.008)	005 (.008)	.005 (.002)	.051 (.003)		

	B. Resul	lts for Tru	ncated Sampl	. <u>e</u>
	η ₁	n ₂	v ₁	v ₂
η ₁	.706 (.047)			
η ₂	.444 (.037)	.588 (.040)		
۰ 1	015 (.009)	008 (.008)	.049 (.003)	
v ₂	.014 (.009)	.000 (.008)	.005 (.002)	.051 (.003)

Notes: Asymptotic standard errors in parentheses. η_t is the true annual earnings as measured by social security payroll records and v_t is the difference between reported earnings in the CPS and social security payroll records.

Table 5

The easiest way to describe our estimation procedure is as a three step process. In the first step we estimate the parameters $(\alpha_t, \beta_t, \gamma_t)$ of the following two equation system:

$$SSE_{1}^{*} = \alpha_{1} + \beta_{1}CPS_{1} + \gamma_{1}CPS_{2} + \varepsilon_{1}$$
$$SSE_{2}^{*} = \alpha_{2} + \beta_{2}CPS_{1} + \gamma_{2}CPS_{2} + \varepsilon_{2}$$

where SSE^{*}_t is the latent, unobserved Social Security earnings record. The equations are jointly estimated by maximum likelihood assuming a bivariate Tobit likelihood function which allows for a covariance in the error terms $(\sigma_{\varepsilon_1\varepsilon_2})$.

Once the parameters of the system are estimated, in the second step the following consistent formulas are used to derive the latent elements of the CPS-SSE covariance matrix:

$$Var(SSE_{t}^{*}) = \beta_{t}^{2}Var(CPS_{1}) + \gamma_{t}^{2}Var(CPS_{2}) + 2\beta_{t}\gamma_{t}Cov(CPS_{1}, CPS_{2}) + \sigma_{\varepsilon_{t}}^{2}$$

$$Cov(SSE_{1}^{\star}, SSE_{2}^{\star}) = \beta_{1}\beta_{2}Var(CPS_{1}) + \gamma_{1}\gamma_{2}Var(CPS_{2}) + (\beta_{1}\gamma_{2} + \beta_{2}\gamma_{1})Cov(CPS_{1}, CPS_{2}) + \sigma_{\xi_{1}\xi_{2}}$$

 $COV(SSE_{t}^{\star}, CPS_{s}) = \beta_{t}Cov(CPS_{s}, CPS_{1}) + \gamma_{t}Cov(CPS_{s}, CPS_{2}).$

Finally, in the third step the resulting variance-covariance matrix of CPS and SSE is used to derive the variance-covariance matrix of η and v. For example, $\sigma_{\eta_1 v_2} = \text{cov}(\text{SSE}_1^*, \text{CPS}_1) - \text{var}(\text{SSE}_1^*)$. The rest of these transformations are listed in the Appendix.¹¹

Table 4 reports the MLE imputed variance-covariance matrix of η_t and v_t for the full sample of males. The table also reports the observed variance-covariance matrix for the truncated sample of males who earned less than the Social Security maximum. Table 5 contains the corresponding estimates for the sample of female workers.

As expected, the imputed variances of Social Security earnings $(\sigma_{\eta_t}^2)$ exceed the corresponding variances for the noncensored sample. Furthermore, the first order serial correlation in Social Security earnings increases from .635 in the noncensored subsample to .776 in the imputed full sample for men, and from .689 to .914 for women.

The limited dependent variable techniques were employed to allow for a nonzero covariance between η_t and v_t . The results suggest that this estimation technique was necessary since there are large negative correlations between measurement error and true earnings for men, exceeding -.40 in each year. However, for women the negative correlation between measurement error and true earnings is quite small in absolute magnitude. This result implies that measurement errors lead male workers' reported earnings to regress to the mean earnings level.¹² Because of the likelihood of mean reverting error, we will focus our attention on the MLE results, although all of our qualitative conclusions are unchanged in the truncated sample.

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¹¹ In practice, we found it more convenient to reparameterize the likelihood function in terms of the parameters of interest and estimate the variance-covariance matrix in one step. Although conceptually equivalent, this approach has the advantage of yielding the appropriate asymptotic standard errors.

 $^{^{12}}$ Duncan and Mathiowetz (1985) obtain similar results for a sample of (primarily male) workers in one production plant.

The results strongly suggest that reporting errors in CPS earnings are positively correlated from one year to the next. The implied auto-correlation coefficient of v_t is .40 for men, and .10 for women. This finding implies that a substantial portion of the measurement error in CPS earnings will "cancel out" when earnings data are differenced. Unfortunately, with only two years of data it is impossible to distinguish an auto-regressive process in the measurement error from a person fixed-effect or from other time-series processes.

Reliability of the Data

We next turn to the magnitude of the attenuation bias that these results imply. Table 6 presents maximum likelihood estimates of the reliability coefficient (λ) with and without taking account of the covariance between η_t and v_t .¹³ The results for male workers are reported for the full sample, and then for subsamples of self-respondents (in period 2) and males who had only one employer over the year. Unfortunately, a comparable disaggregated analysis for women is infeasible because of the small sample size.

First consider the cross-sectional results for the full samples. Under the assumption of classical measurement error, the reliability of CPS earnings data is in the low eighty percent range for men, and the low ninety percent range for women. Allowing for mean reverting measurement error substantially increases these estimates for men, but only slightly for women. This is precisely the direction of the effect of mean reverting measurement error on λ that one would expect if $\sigma_v^2 < \sigma_n^2$.

¹³ The reliability coefficients are just transformations of the covariance matrix of CPS_t and SSE^{*}_t. This is described in the Appendix.

It is interesting to compare our estimate of λ to that found by Duncan and Hill (1985) in their evaluation study of the Panel Study of Income Dynamics. Using self-reported earnings data and employer records for the employees of one manufacturing plant, they estimate that the signal-to-total-variance ratio (assuming classical measurement error) for log annual earnings in 1982 is .76. The signal-to-total variance ratio is somewhat smaller in Duncan and Hill's evaluation study than here, but this difference may at least in part be attributable to the lower variance of the signal inherent in studying one plant.

The results in Table 6 further show that measurement error bias is exacerbated by taking first differences of the CPS earnings data. Nonetheless, the point estimates of the reliability coefficient for ΔCPS_t in the .78 to .85 range should give solace to those who feared that little could be learned from longitudinal studies of earnings because most of the observed changes in these panel data were due to noise. Both the positive serial correlation and mean reversion in the errors increase the reliability of first-differenced data relative to the case of classical measurement error.

To examine the effect of self versus proxy responses on the accuracy of CPS data, we separately examine the subsample of male workers who self-reported their earnings for the calendar year 1977 in the 1978 CPS. Interestingly, there is little evidence that this sample of workers has more reliable data than the entire sample. Mellow and Sider (1983) similarly find that self-responded answers to a variety of CPS questions are not more accurate than proxy responses. This result is particularly relevant for comparisons between men and women because women heads of

Variable	A11 M <u>en</u>	Self- <u>Responding Men</u>	Single Employer Men	All Women
		Classical Meas	urement Error ^b	
1976 Cross-Section	.844	.876	.833	. 939
	(.006)	(.008)	(.008)	(.005)
1977 Cross-Section	.819	.847	.815	.924
1977 01000 500000	(.009)	(.010)	(.006)	(.007)
First Differenced	.648	. 708	.625	. 814
	(.011)	(.008)	(.008)	(.014)
		<u>Mean</u> Reverting M	leasurement Error ^C	
1976 Cross-Section	1.016	.964	1.003	. 958
	(.005)	(.015)	(.038)	(.012)
1977 Cross-Section	.974	.962	.958	. 929
	(.010)	(.013)	(.010)	(.014)
First Differenced	.775	.859	. 719	. 848
	(.017)	(.022)	(.051)	(.018)

Table 6 Maximum Likelihood Estimates of Reliability of Annual Earnings Assuming Classical or Mean Reverting Measurement Error^a

Notes: a. Asymptotic standard errors in parentheses. Sample sizes for columns one through four are 2,924, 922, 2,394 and 465.

b. Classical measurement error reliability statistic assumes η_t and v_t are uncorrelated, but allows for serial correlation in v_t .

For the cross-sections $\lambda_{t} = \frac{\sigma_{\eta_{t}}^{2}}{\sigma_{\eta_{t}}^{2} + \sigma_{v_{t}}^{2}}$, and for the first-differenced data $\lambda = \frac{\sigma_{\Delta \eta}}{\sigma_{\Delta \eta}^{2} + \sigma_{\Delta v}^{2}}$.

c. Mean reverting measurement error reliability statistic allows for a correlation between η_t and v_t as well as for serial correlation in v_t . For the cross-sections, $\lambda_t = \frac{\text{cov}(\text{SSE}_t, \text{CPS}_t)}{\text{var}(\text{CPS}_t)}$ and for the first-differenced data, $\lambda = \frac{\text{cov}(\Delta \text{SSE}, \Delta \text{CPS})}{\text{var}(\Delta \text{CPS})}$. households are far more likely to self-respond to the CPS than men (see Table 2).

The final subsample that we examine is the group of male workers who were employed by a single firm in each of the two consecutive years analyzed. We focus on this sub-sample because Social Security earnings records are likely to be particularly accurate for these workers as they were exclusively employed in covered employment. Interestingly, the results show very similar estimates of the reliability of the data for the single employer sample of workers and the full sample. This finding suggests that possible bias in the estimates of λ due to incomplete coverage of Social Security earnings records is not an important problem.

IV. Potential Problems

Our strategy has been to use Social Security earnings data as a proxy for true earnings to estimate the nature and magnitude of measurement error. It is worth asking how sensitive our conclusions are to a variety of potential problems. We see three major ones. First, Social Security earnings records may not be an accurate measure of individuals' earnings. Second, the sample we use may be unrepresentative in a number of important respects. Third, our estimates may be sensitive to the distributional assumptions we make to account for the truncation in Social Security earnings records. Although we believe that each of these points has some force, we do not believe our conclusions are driven by these potential problems. Below we consider each potential problem in turn.

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a. Errors in Social Security Earnings Records

Social Security earnings records may be inaccurate for our purposes for a variety of reasons: Earnings records may be matched to the wrong person; employer tax reports may be inaccurate; individuals might have some income from uncovered or under-the-table jobs. Five factors lead us to believe that our findings are not a direct result of measurement error in Social Security data.

First, we have chosen our samples to minimize errors in Social Security data by requiring that administrative records match CPS records on the basis of age, race, sex and Social Security numbers. Second, our results are not qualitatively changed when we narrow the sample to males whose only job was in covered employment over the year.

Third, the cross-sectional estimates of measurement error implied by our data are similar to those found by other researchers using different data sets (e.g., Duncan and Hill, 1985, and Mellow and Sider, 1983). Fourth, white noise measurement error in Social Security earnings would spuriously induce a negative covariance between v_t and η_t . While a negative covariance is found for men, the corresponding covariance for women is virtually zero, and there is no obvious reason to suspect that Social Security earnings records are more accurate for women than men.

Lastly, we should emphasize that additive white noise measurement error in Social Security earnings would lead the reliability statistics we report in the text to be either downward biased (when λ equals the signal-to-total-variance ratio) or unbiased (when λ is the slope coefficient from the regression of the true variable on the mismeasured variable).

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b. Representativeness of Sample

Of necessity, we restrict our samples to heads of households who remained at the same address over two years, who are successfully matched to their Social Security records, and who received earnings from covered employment in two consecutive years. Imposing these restrictions might generate a sample of individuals that is more reliable than the overall population.¹⁴ While this is true, we doubt that the nature of our sample can explain our results. As mentioned in section one, as far as observable characteristics are concerned the sample is not very different from the full CPS. Furthermore, many of the sample restrictions we impose are similar to the ones imposed by other researchers in analyzing longitudinal earnings data. Thus, the results are reasonably representative of the kinds of samples often analyzed.

c. Normality

We assume joint normality of CPS_t and SSE_t to estimate many of the statistics in the paper. The spike at zero and the thick tails in the distribution of v_t are indicative of non-normality. Nonetheless, there are three reasons to believe that our conclusions are not due to this assumption.

First, when we re-estimate the models by maximum likelihood eliminating the spike at zero and the outliers, the conclusions are qualitatively unchanged. Second, our estimates are similar when we focus solely on the truncated sample. And most importantly, we believe that

¹⁴ On the other hand, we note that movers might experience relatively large true earnings changes. Therefore, excluding these workers may <u>re</u>duce the signal in the data.

under plausible assumptions the truncated sample should give estimates of the reliability of cross-sectional and first-differenced data that <u>understate</u> the true reliability of the data. The intuition behind this result is most easily seen when η_t and v_t are uncorrelated. Since truncation is based on η_t and only indirectly affects v_t , we would expect the truncation to reduce the variance of η_t but not change the variance of v_t . As a result, truncation leads the signal-to-total variance ratio to increase.

To examine the robustness of this result when η and v are correlated, we performed a series of Monte Carlo simulations. In the first case we assumed that the η 's and v's were distributed joint normally, and in the second case we assumed that the v's represented mixture of normals.

Under the first assumption we generated samples of 4 random normal variables, η_1 , η_2 , v_1 and v_2 , with variances of .5, .5, .1 and .1 respectively. The samples were of size 1,000. We assumed that the η 's and the v's were positively auto-correlated across time ($\rho_{\eta_1}, \eta_2^{>0}$ and $\rho_{v_1}, v_2^{>0}$), while the η 's and v's were negatively correlated with each other ($\rho_{v_s}, \eta_t^{<0}$). We varied these correlations between .1 and .9 and between -.1 and -.9 respectively.

Under the second assumption (i.e., v distributed as a mixture of normals), we generated 6 random variables, η_1 , η_2 , ε_1 , ε_1 , μ_1 and μ_2 , with variances .5, .5, .1, .1 and .01 and .01 respectively. Each sample size was 1,000. We assumed that the η 's, the ε 's and the μ 's were all positively correlated over time, but that the ε 's and the μ 's were both negatively correlated with the η 's. The ε 's and the μ 's were also assumed to be positively correlated. Again, we varied the correlations

from between .1 and .9 or from -.1 to -.9. From the ε 's and μ 's we defined two new variables: v_1 and v_2 . With equal probability, the measurement error v_1 was randomly set equal to either ε_1 or μ_1 , while v_2 was randomly set equal to either ε_2 or μ_2 .

In both of the above cases we calculated λ 's for both the full sample and for those with η 's > 0. In all cases that λ 's calculated on the full sample were closer to 1 than they were in the truncated samples. This was true for both cross-sectional and first differenced data. We conclude from these simulations that truncating on the η 's will typically induce a downward bias on reliability measures. Therefore, the λ 's we estimated on the truncated samples likely underestimate the reliability of earnings data.

V. Conclusion

This paper has estimated the properties of measurement error in longitudinal data on earnings. Our estimates contain both good news and bad news as far as longitudinal earnings data are concerned. The good news first: Recent work using panel data has emphasized the fact that differencing data can seriously exacerbate measurement error problems. For earnings data in particular, common wisdom would seem to be that changes are dominated by noise rather than signal. Our estimates suggest that this view is too pessimistic. Fully 75% of the variation in the change in earnings in CPS data represents true earnings variation.

The bad news is that our results suggest that the simple models that have been used to characterize measurement error in past studies are not appropriate. In particular, the standard (classical) assumption has been that measurement error is pure white noise. Our results indicate, however, that measurement error in earnings data is positively auto-correlated and negatively correlated with true earnings. A broader range of measurement error models which allow for non-classical measurement error might be necessary.

The time-series properties of measurement error seem especially important. Unfortunately, with only 2 years of data it is impossible to distinguish a moving average process in the measurement error from an auto-regressive process or from some other time-series process. A valuable extension of this work, therefore, would be an examination of the time-series properties of measurement error in a longer panel of data.

Appendix

This appendix reviews the limited dependent variable methods we used to calculate the variety of estimates reported in the text. First, it should be clear that all the statistics reported in the paper -- λ and the variance-covariance matrix of η_t and v_t -- are straightforward functions of the variance-covariance matrix of Social Security and CPS earnings. Because of censoring problems this matrix is not directly observable, but it can be imputed if we make distributional assumptions.

Specifically, we assume that Social Security and CPS earnings are distributed normally. Consider the following two equation system which relates Social Security earnings to CPS earnings each year:

(1)
$$SSE_1^{\star} = \alpha_1 + \beta_1 CPS_1 + \gamma_1 CPS_2 + \varepsilon_1$$

(2)
$$SSE_2^{\star} = \alpha_2 + \beta_2 CPS_1 + \gamma_2 CPS_2 + \varepsilon_2$$

where SSE_t^* is the latent, unobserved Social Security earnings record. SSE_t^* is only observed if SSE_t is less than the earnings maximum (K_t) . This specification is not meant to be structural. Rather, it is used to estimate the latent variance-covariance matrix of Social Security and CPS earnings.

Conditional on the CPS earnings, there are four additive components to the log likelihood function describing this system. Each is presented below.

I. If $SSE_1^* < K_1$ and $SSE_2^* < K_2$ log b $(\frac{\varepsilon_1}{\sigma_1}, \frac{\varepsilon_2}{\sigma_2})$ where $b(\cdot)$ is a bivariate normal density and σ_t is the standard deviation of ϵ_t .

II. If $SSE_{1}^{+} > K_{1}$ and $SSE_{2}^{+} < K_{2}$

$$- \log(\sigma_2) - \frac{1}{2\sigma_1^2} \epsilon_1^2 + \log \phi (-Z_1)$$

where ϕ is the cumulative distribution function for a standardized normal variate and

$$z_{1} = [(K_{1} - \alpha_{1} - \beta_{1} CPS_{1} - \gamma_{1} CPS_{2})/\sigma_{1} - \rho_{\overline{\sigma_{2}}}^{\varepsilon_{2}}] \div (1 - \rho^{2})^{1/2}$$

III. If $SSE_1^* < K_1$ and $\overline{SSE_2^*} > K_2$ $-\log (\sigma_1) - \frac{1}{2\sigma_1\sigma_2} \varepsilon_1 \varepsilon_2 + \log \Phi (-Z_2)$ where $Z_2 = [(K_2 - \alpha_2 - \beta_1 CPS_1 - \gamma_1 CPS_2)/\sigma_2 - \rho \frac{\varepsilon_1}{\sigma_1}] \div (1 - \rho^2)^{1/2}$ IV. If $SSE_1^* > K_1$ and $SSE_2^* > K_2$

$$\log B (-\tilde{Z}_{1}, -\tilde{Z}_{2}, \rho)$$

where B is a cumulative bivariate normal distribution function

and
$$\tilde{Z}_t = [K_t - \alpha_t - B_t CPS_t - \gamma_t CPS_t] / \sigma_t$$

Once β_1 , β_2 , γ_1 , γ_2 , $\sigma_{\epsilon_1}^2$, $\sigma_{\epsilon_2}^2$, $\sigma_{\epsilon_1 \epsilon_2}^2$ have been estimated the variance-covariance matrix of Social Security and CPS data can easily be computed.

In particular, the following asymptotic formulas relate the variances and covariances to the estimated parameters:

(3)
$$\operatorname{Var}(\operatorname{SSE}_{t}^{*}) = \beta_{t}^{2} \operatorname{Var}(\operatorname{CPS}_{1}) + \gamma_{t}^{2} \operatorname{Var}(\operatorname{CPS}_{2}) + 2\beta_{t} \gamma_{t} \operatorname{Cov}(\operatorname{CPS}_{1}, \operatorname{CPS}_{2}) + \sigma_{\varepsilon_{t}}^{2}$$

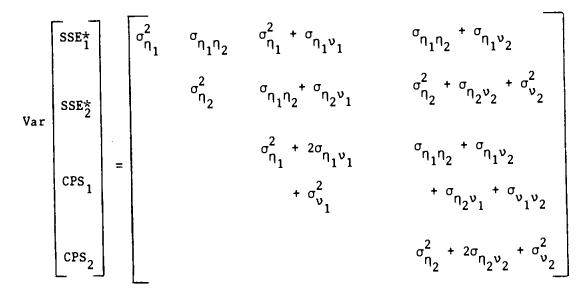
(4)
$$Cov(SSE_1^*, SSE_2^*) = \beta_1 \beta_2 Var(CPS_1) + \gamma_1 \gamma_2 Var(CPS_2)$$

+
$$(\beta_1\gamma_2 + \beta_2\gamma_1)Cov(CPS_1, CPS_2) + \sigma_{\varepsilon_1\varepsilon_2}$$

(5) $Cov(SSE_t^*, CPS_s) = \beta_t Cov(CPS_s, CPS_1) + \gamma_t Cov(CPS_s, CPS_2).$

The variance-covariance elements involving only CPS earnings are estimated directly from the observed data.

Once the variance-covariance matrix of Social Security and CPS earnings has been estimated, it is straightforward to calculate the variance-covariance matrix of η and v, and to calculate the various reliability statistics. The variance-covariance matrix of SSE and CPS may be written as follows:



This system of equations represents a mapping from the variance-covariance matrix of η and v to the variance-covariance matrix of SSE and CPS. Specifically:

$$M \cdot Var \begin{bmatrix} SSE_1^* \\ SSE_2^* \\ CPS_1 \\ CPS_2 \end{bmatrix} = Var \begin{bmatrix} n_1 \\ n_2 \\ v_1 \\ v_2 \end{bmatrix}$$

where M is 16 x 16 matrix of 1's, 0's, and -1's. M can be inverted to solve for the variance-covariance matrix of η and v as an explicit function of the variance-covariance matrix of SSE and CPS.

The signal-to-noise ratios are easily calculated from the variance-covariance matrix of η and v. The reliability statistics which are robust to mean reverting measurement error are calculated as follows:

$$\lambda_{t} = \frac{\text{cov}(\text{CPS}_{t}, \text{SSE}_{t})}{\text{var}(\text{SSE}_{t})} , \text{ and}$$

$$\lambda_{\Delta t} = \frac{\operatorname{cov}(\operatorname{CPS}_2, \operatorname{SSE}_2) + \operatorname{cov}(\operatorname{CPS}_1, \operatorname{SSE}_1) - \operatorname{cov}(\operatorname{CPS}_1, \operatorname{SSE}_2) - \operatorname{cov}(\operatorname{CPS}_2, \operatorname{SSE}_1)}{\operatorname{var}(\operatorname{CPS}_2) + \operatorname{var}(\operatorname{CPS}_1) + 2 \operatorname{cov}(\operatorname{CPS}_1, \operatorname{CPS}_2)}$$

Finally, we note that asymptotic standard errors are computed by re-parameterizing the likelihood function in terms of the parameters of interest using the transformations presented above. These parameters are functions of, among other things, the variances and covariances of log CPS earnings. We account for the sampling variance of CPS earnings by assuming that log CPS earnings follow a joint normal distribution, and add the joint marginal density of these earnings to the likelihood function.

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