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THE FACTOR CONTENT OF FOREIGN TRADE

by

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1. Introduction

In the factor proportions trade theory (the Heckscher-Ohlin theory) the factor content of the pattern of trade plays a major role both in theoretical and in empirical investigations. It is therefore valuable to identify restrictions on permissible vectors of the factor content of trade which necessarily hold in equilibrium. If certain restrictions are empirically testable, they can be used for preliminary testing of the theory before a more thorough examination is undertaken, or they can be used to reject the theory. It is my intention to develop in this paper a set of restrictions on the vectors of the factor content of trade which can be used for these purposes.

It is well known that autarky commodity prices impose restrictions on possible patterns of trade. These restricitons provide a precise prediction of the pattern of trade for a two-country two-commodity world, in the sense that the exporter of a good is readily identifiable from autarky price data (as the country with the lower relative price of the good). I will start by showing that in the presence of factor price equalization autarky factor rewards impose restrictions on the factor content of trade patterns which are completely analogous to the restrictions imposed by autarky commodity prices on the commodity patterns of trade. This result is in line with the view that in the factor proportions model trade in commodities represents indirect trade in factor services. Unfortunately, these restrictions rely on autarky information, and they are, therefore, not very useful for empirical work. In order to overcome this difficulty, I explore an alternative approach.

I argue that common predictions of the factor content of trade flows which are based on post-trade observable data -- the data relevant for empirical studies -- rely on unnecessarily restrictive assumptions; i.e., the assumption that preferences are homothetic and identical across countries and the assumption of factor price equalization. The assumption of identical homothetic preferences seems to be necessary when factor price equalization is assumed, because in this case there is an indeterminancy in the pattern of production. Nevertheless, the two assumptions taken together imply the same factor content of a country's net import vector from the rest of the world for every equilibrium production pattern. However, in a many country world the factor content of <u>bilateral</u> trade flows depends on the realized production patterns, and they cannot, therefore, be predicted. I will show that in the <u>absence</u> of factor price equalization (which many consider to be the more realistic case), there exists an interesting set of restrictions on the factor content of <u>bilateral</u> trade flows which does not rely on specific assumptions about preferences and which is based only on post-trade information. These restrictions enable, therefore, preliminary testing of the factor proportions trade theory without the straightjacket of identical homothetic preferences and factor price equalization.

The general restrictions imposed by autarky commodity prices on net trade vectors are described in Section 2. Analogous restrictions on the net factor content of trade flows for the factor proportions theory are developed in Section 3. The restrictions on the factor content of bilateral trade flows in the absence of factor price equalization and in the absence of specific assumptions about preferences are derived in Section 4. Some concluding comments are provided in Section 5.

2. Autarky Commodity Prices and Comparative Advantage in Commodity Trade

General laws of comparative advantage which relate the pattern of trade to differences in autarky commodity prices have recently been derived by Deardorff (1980) and Dixit and Norman (1980, chp. 4). These laws are applicable to economies with convex technologies and minor restrictions on other aspects. For present purposes I restrict the discussion to free trade equilibria in which all goods are traded and no factors are traded. It is also assumed that the free trade consumption vector of country k, c^k, is not affordable to the economy's consumers in the autarky equilibrium:

$$\text{A.1} \qquad p^{Ak} \cdot c^k \geq p^{Ak} \cdot c^{Ak} \quad \text{for all } k$$

where p^{Ak} stands for country k's autarky equilibrium price vector and c^{Ak} is its autarky consumption vector. Condition A.1 is necessarily satisfied in single consumer economies with balanced trade (it implies the existence of gains from trade). It is however clear that A.1 can also be satisfied in an economy with an unbalanced trade account if its surplus in the trade account is not too large. The existence of a deficit in the trade account is most favorable to A.1. For these reasons I list balanced trade as a separate assumption; i.e.,

$$\text{A.2} \qquad p \cdot t^k = 0 \quad \text{for all } k$$

where p is the free trade price vector and t^k is the net import vector.

Now, since $c^k = x^k + t^k$ and $c^{Ak} = x^{Ak}$ where x^k is the free trade production vector and x^{Ak} is the autarky production vector, A.1 implies:

$$p^{Ak} \cdot t^k \geq p^{Ak} \cdot x^{Ak} - p^{Ak} \cdot x^k \geq 0$$

where the last inequality stems from the fact that in a competitive equilibrium the economy's value added is maximal. Using this result, the balanced trade assumption A.2, and the market clearing conditions, we obtain the following set of restrictions on the net import vectors:

(1) $\quad p^{Ak} \cdot t^k \geq 0 \quad$ for all k

(2) $\quad p \cdot t^k = 0 \quad$ for all k

(3) $\quad \sum_k t^k = 0$

Conditions (1) - (3) describe in a sense the weakest set of restrictions that can be imposed on trade vectors without having more detailed information about the economies. They do, however, provide some insight into the determinants of trade patterns. The combination of (1) and (2) implies the following general law of comparative advantage:

$$(\lambda^{Ak} p^{Ak} - \lambda^k p) \cdot t^k \geq 0 \quad \text{for all } k$$

for all nonnegative scalars λ^{Ak} and λ^k. Since the lambdas can be arbitrarily chosen, this law says that every country imports on average commodities whose relative price is lower in the trading equilibrium than in autarky and it exports on average commodities whose relative price is higher in the trading equilibrium than in autarky. These averages can be calculated using any desirable combination of lambdas.

For a world that consists of two countries one can derive a law of comparative advantage which does not require balanced trade. For a two country world (1) and (3) imply:

$$(\lambda^{Ak}p^{Ak} - \lambda^{A\ell}p^{A\ell}) \cdot t^k \geq 0$$

for $\ell \neq k$ and for all nonnegative scalars λ^{Ak} and $\lambda^{A\ell}$. This law says that within the limits of unbalanced trade permitted by A.1 for a two country world, every country imports on average commodities which are relatively expensive in its autarky equilibrium as compared to its trading partner and it exports on average commodities which are relatively cheap in its autarky equilibrium as compared to its trading partner. For the two commodity case this law provides a precise prediction of the pattern of trade in the sense that every country's export good is readily identifiable as the relatively cheaper good in autarky.

3. Autarky Factor Rewards and Comparative Advantage in the Factor Content of Trade

In this section I develop a characterization of the factor content of trade flows for the factor propositions trade theory which is analogous to the characterization of the commodity trade pattern that was described in the previous section. For this purpose the following assumption is made:

A.3 There is no joint production and every commodity is produced by means of primary inputs only with quasi-concave, positively linear homogeneous production functions, which are identical across countries.

Observe that technological differences across countries as well as joint production and diminishing returns to scale were not excluded in the previous characterization of commodity trade vectors. Here they are. Condition A.3 also excludes the existence of intermediate inputs. This is done, however, only in order to simplify the exposition. I also assume:

A.4 No country imports commodities that it exports.

Now let $T^{k\ell}$ stand for gross imports of commodities by country k from country ℓ. Clearly, $T^{k\ell} \geq 0$, and due to the last assumption $T_i^{k\ell} = 0$ whenever commodity i is not traded between countries k and ℓ or it is exported by k to ℓ. Using the gorss commodity import vectors, we define the gross import vectors of factor content by:

(4) $$T_V^{k\ell} = A^{\ell} T^{k\ell}$$

where $A^{\ell} \equiv A(w^{\ell})$ is the matrix of cost minimizing techniques of production for the free trade equilibrium factor reward vector in country ℓ, w^{ℓ}. This measures the actual factor content of trade using the *input-output coefficients* of the country of *origin. Net import vectors by country* k *from* country ℓ for commodities and factor *content, denoted by lower case letters*, *are therefore*:

(5) $$t^{k\ell} = T^{k\ell} - T^{\ell k}$$

(6) $$t_V^{k\ell} = T_V^{k\ell} - T_V^{\ell k}$$

And *finally, net imports of commodities* and factor content by country k are:

(7) $$t^k = \sum_\ell t^{k\ell}$$

(8) $$t_V^k = \sum_\ell t_V^{k\ell}$$

Let $\Pi(p, V)$ be the function that describes the maximal value of GDP derivable from the factor endowment vector V by means of the common technology (it is common to all countries). Let V^k stand for country k's vector of factor endowments. From the definition of $\Pi(\cdot)$ and the fact that x^k could be produced in autarky, $p^{Ak} \cdot x^k \leq \Pi(p^{Ak}, V^k)$. Moreover, if country k were made a gift of the factors actually embodied in its net imports, t_V^k, then (since technology is common to all countries) it could produce with them at least the value of these net imports, and probably more by an efficient rearrangement of production (see Deardorff (1982)). Hence:

$$p^{Ak} \cdot (x^k + t^k) \leq \Pi(p^{Ak}, V^k + t_V^k)$$

Now, since $c^k = x^k + t^k$, then using A.1 and the fact that $\Pi(\cdot)$ is concave in V, the above inequality implies:

$$\begin{aligned} p^{Ak} \cdot c^{Ak} &\leq \Pi(p^{Ak}, V^k) + \Pi_V(p^{Ak}, V^k) \cdot t_V^k \\ &= p^{Ak} \cdot x^{Ak} + w^{Ak} \cdot t_V^k \end{aligned}$$

where $\Pi_V(\cdot)$ is the gradient of $\Pi(\cdot)$ with respect to V and it equals the competitive factor reward vector, x^{Ak} is the autarky output vector, and w^{Ak} is the autarky factor reward vector. Since $c^{Ak} = x^{Ak}$, this inequality reduces to:

$$(9') \qquad w^{Ak} \cdot t_V^k \geq 0$$

This is analogous to A.1, and it applies to the unbalanced trade patterns to which A.1 applies (see also Deardorff (1982, Section III)).

Now assume:

A.4 Free trade brings about factor price equalization; i.e., $w^k = w$ for all k.

Then, using (9') and the market clearing conditions, we may state that assumptions A.1 - A.4 imply the following set of restrictions on the net import vectors of factor content:

$$(9) \qquad w^{Ak} \cdot t_V^k \geq 0 \qquad \text{for all } k$$

$$(10) \qquad w \cdot t_V^k = 0 \qquad \text{for all } k$$

$$(11) \qquad \sum_k t_V^k = 0$$

These conditions are precisely analogous to conditions (1) - (3) that were derived for commodity trade vectors. It is, therefore, possible to state generalizations of the price version of the factor-content Heckscher-Ohlin theorem analogous to the generalized laws of comparative advantage derived in the previous section. In particular, (9) - (10) imply

$$(\lambda^{Ak} w^{Ak} - \lambda^k w) \cdot t_V^k \geq 0 \qquad \text{for all } k$$

i.e., every country imports on average the content of those factors of production whose relative price is higher in autarky than in free trade and it exports on average the content of those factors of production which are relatively cheaper in autarky than in free trade. This result depends on the assumptions of factor price equalization and balanced trade. These assumptions are not required for the law of comparative advantage for a two country world, which is derivable from (9) and (11); i.e.,

$$(\lambda^{Ak}w^{Ak} - \lambda^{A\ell}w^{A\ell}) \cdot t_V^k \geq 0$$

and which states that every country imports on average the content of those factors of production which are relatively expensive in its autarky equilibrium as compared to its trading partner, and it exports on average the content of those factors of production which are relatively cheaper in its autarky equilibrium as compared to its trading partner (see also Deardorff (1982) and Ethier (1982) for this result). For the two-factor case, say labor and capital -- and irrespective of the number of commodities -- this law provides a precise prediction of the net exporter of every factor content on the basis of the autarky wage rental ratio. The country with the higher autarky wage rental ratio is a net importer of labor content and a net exporter of capital content.

To summarize, conditions (9) and (11) describe the general restrictions on the factor content of trade vectors, with the additional restriction (10) applying whenever there is factor price equalization and balanced trade. However, since these restrictions rely on autarky data, and these data are not available, they are not very useful for empirical investigations. It is, therefore, desirable to develop restrictions on trade vectors which rely on data that can be obtained from observations of trading equilibria.

4. The Factor Content of Trade Without Factor Price Equalization

The factor proportions trade theory has traditionally predicted trade patterns by means of the value definition of relative factor abundance (i.e., by means of autarky relative factor rewards) without restrictions on preferences, or by means of the quantity definition of relative factor abundance (i.e., the relative endowments of factors of production) assuming that preferences are homothetic and identical across countries. In the second approach the assumption of factor price equalization has played a dominant role in most discussions of the many-factor case.

The general results that were developed in the previous section are in the spirit of the first tradition. They can, however, also be fitted into the second tradition. For under the assumption of identical homothetic tastes it has been shown by Dixit and Norman (1980, chp. 4) that differences in factor endowments are negatively correlated with appropriately normalized differences in autarky factor rewards. It can, in fact, be shown, using their method, that:

$$(\bar{w}^{Ak} - \bar{w}^{A\ell}) \cdot (\lambda^k V^k - \lambda^\ell V^\ell) \leq 0$$

for every nonnegative λ^k and λ^ℓ, where bars over the factor reward vectors indicate that they are normalized (i.e., every w^{Ak} is divided by a particularly chosen scalar such that $(\bar{w}^{Ak} - \bar{w}^{A\ell})$ has positive as well as negative components). Hence, on average countries have in autarky relatively higher rewards of factors of production with which they are relatively poorly endowed and they have relatively lower rewards of factors of production with which they are relatively well endowed. This relationship between relative factor endowments and autarky relative factor rewards can be used together with conditions (9) - (11) to predict the factor content of the pattern of trade on the basis of cross country differences in relative factor endowments. However, once identical homothetic tastes are assumed, one can use other methods to provide a more precise prediction of the factor content of trade in the presence of factor price equalization (see Vanek (1968)).

The above described approaches to the prediction of trade patterns leave the trade specialist in an uncomfortable position; in order to test the factor proportions theory without restrictions on preferences he needs autarky data (i.e., autarky factor rewards) which is not available, or he has to test the theory jointly with the hypothesis that preferences are homothetic, identical across countries, and that in the trading equilibrium factor prices are equalized. It is argued in what follows that the assumption of factor price equlization, which many believe to be unrealistic, is not a virtue for the purpose of trade pattern predictions but rather a vice. It will be shown that in the absence of factor price equalization one can predict the factor content of trade from post trade data without restricting preferences, and that this can be done not only for every country's net import vector but also for bilateral trade patterns. This I believe to be the most important contribution of this paper.

The basic insight for the general result derived below can be obtained from the two factor many goods many countries case. Figure 1 depicts a Lerner diagram in which the isoquants numbered from one to six describe output levels of goods one to six, respectively, each one worth one dollar at the free trade prices. There exist only six goods, which are produced with capital and labor. There are three countries with their capital labor ratios represented

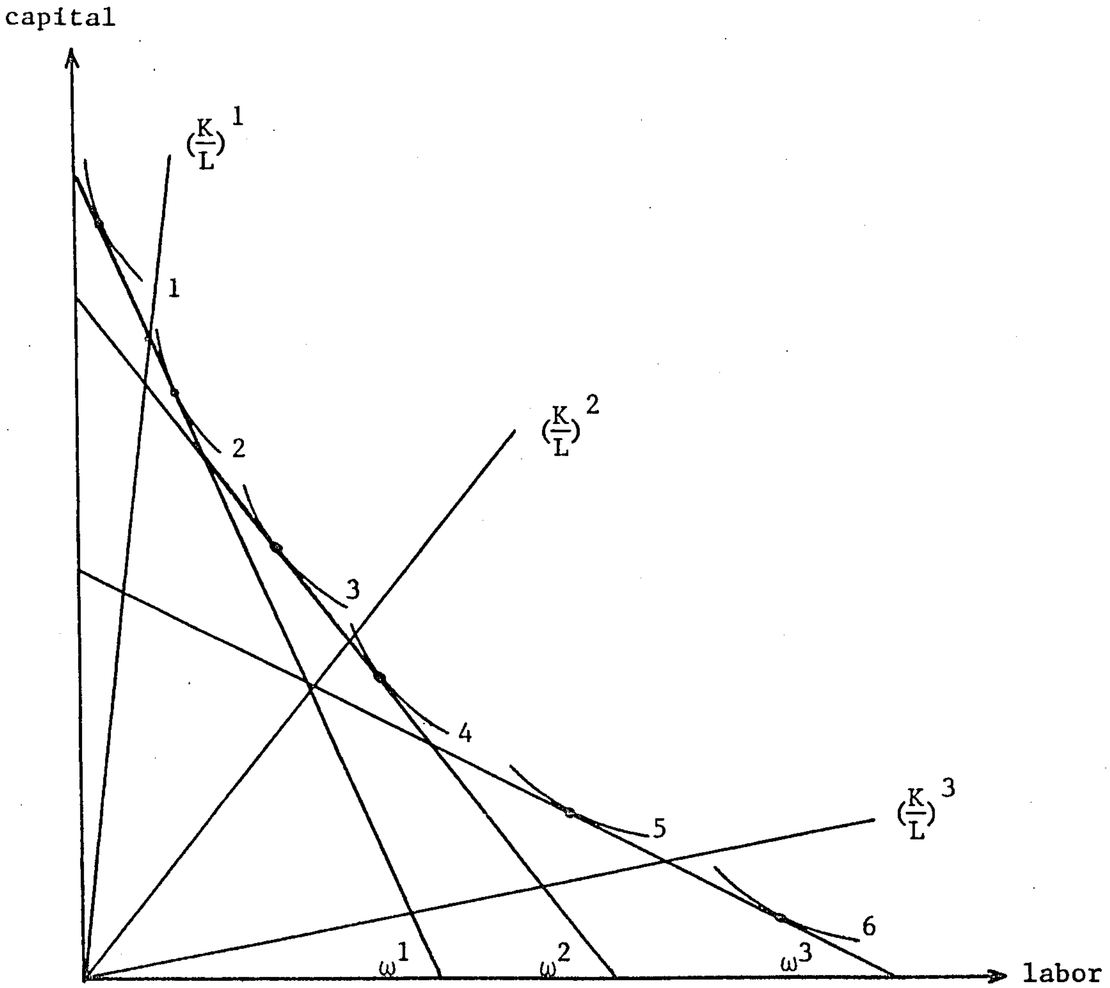
capital
$(\frac{K}{L})^1$
1
2
$(\frac{K}{L})^2$
3
4
5
$(\frac{K}{L})^3$
6
ω^1
ω^2
ω^3
labor

Figure 1

by the rays $(K/L)^{\ell}$ and their free trade wage rental ratios represented by the slopes ω^{ℓ} of the one dollar cost lines. In the equilibrium described by Figure 1, country 1 -- which has the highest capital labor ratio -- produces goods 1 and 2; country 2 -- with the intermediate capital labor ratio -- produces goods 3 and 4; while country 3 -- the relatively most labor abundant country -- produces goods 5 and 6. It is now a simple matter to observe that the more capital rich a country is, the <u>more</u> capital and <u>less</u> labor it uses per dollar output in <u>all</u> lines of production (more generally, it never uses less capital and more labor). Hence, whatever trade there may exist between <u>two</u> countries, exports of the relatively capital rich country will embody a higher capital labor ratio than the exports of the relatively labor rich country. This describes a clear bilateral factor content pattern of trade (see Brecher and Choudhri (1982)).

In order to generalize this result, observe that due to the fact that technologies are identical across countries and $\Pi(\cdot)$ is concave in V:

$$\begin{aligned} p \cdot (x^k + T^{k\ell}) &\leq \Pi(p, V^k + T_V^{k\ell}) \\ &\leq \Pi(p, V^k) + \Pi_V(p, V^k) \cdot T_V^{k\ell} \\ &= p \cdot x^k + w^k \cdot T_V^{k\ell} \end{aligned}$$

implying:

$$p \cdot T^{k\ell} \leq w^k \cdot T_V^{k\ell} \quad \text{for all } \ell \text{ and } k \tag{12}$$

Due to constant returns to scale every line of production brakes even in equilibrium, implying:

$$(13) \qquad p \cdot T^{k\ell} = w^{\ell} \cdot T_V^{k\ell} \quad \text{for all } \ell \text{ and } k$$

Combining (12) with (13) yields:

$$(14) \qquad (w^k - w^{\ell}) \cdot T_V^{k\ell} \geq 0 \quad \text{for all } \ell \text{ and } k$$

and by symmetry:

$$(15) \qquad (w^k - w^{\ell}) \cdot T_V^{\ell k} \leq 0 \quad \text{for all } \ell \text{ and } k$$

One way to interpret conditions (14) and (15) is by combining them in order to obtain:

$$(16) \qquad (w^k - w^{\ell}) \cdot t_V^{k\ell} \geq 0$$

which says that on average country k is a net importer from country ℓ of the content of those factors of production that are cheaper in ℓ than in k and it is a net exporter to ℓ of the content of those factors of production that are cheaper in k than in ℓ. Also, since (see Appendix for a proof):

$$(17) \qquad (w^k - w^{\ell}) \cdot (\lambda^k V^k - \lambda^{\ell} V^{\ell}) \leq 0 \quad \text{for all } \ell \text{ and } k$$

for all nonnegative λ^k and λ^{ℓ}, and since the vector $(w^k - w^{\ell})$ has both positive and negative components (since in a free trade equilibrium a country cannot produce cheaper every good as compared to another country), the factors with which country k is relatively well endowed are on average cheaper in k than in ℓ and factors with which country k is relatively poorly endowed are on average more expensive in k than in ℓ. Loosely interpreted, (16) and (17) imply that in the absence of factor price equalization (and without restricting preferences) country k is in some sense on average a net imported from ℓ of the content of those factors with which country ℓ is relatively well endowed as compared to k and it is on average a net exporter to ℓ of the content of those factors with which country k is relatively well endowed. The last interpretation of (16) is appealing, but it is rather loose due to the necessity to rely on an indirect link via (17), while the covariance relationship is not transitive. Moreover, there might be cases in which $t_V^{k\ell}$ will have all positive or all negative components, because in a many country world bilateral trade accounts need not balance (even when every country's trade account is balanced). However, independently of interpretation, (16) represents a valid and interesting restriction on bilateral trade patterns.

Another insight into (16) is obtained by observing that (4) - (6), together with A.4, imply $t_V^{k\ell} = A^{k\ell}t^{k\ell}$, where $A^{k\ell}$ is a matrix consisting of columns from A^k and A^ℓ such that column i is taken from A^k if k exports good i to ℓ and it is taken from A^ℓ if ℓ exports good i to k. If good i is not traded between k and ℓ it does not matter from which matrix this column is taken, because the i'th component of $t^{k\ell}$ is in this case zero. Combining this observation with (16), we obtain:

(18) $$(w^k - w^\ell)^T A^{k\ell} t^{k\ell} \geq 0$$

which says that on average country k imports from ℓ goods which are relatively intensive in factors of production which are more expensive in k than in ℓ and it exports to ℓ goods which are relatively intensive in factors of production that are cheaper in k than in ℓ. When combined with (17), this provides a bilateral prediction of the Heckscher-Ohlin type, except that relative factor intensities are measured by means of the coefficients of the matrix $A^{k\ell}$ which consists of the activities of the bilateral exporters. If $t^{k\ell}$ happens to have all components of the same sign, (18) represents a restriction on the bilateral vector of trade in goods despite the trade being unidirectional.

An alternative to the last two approaches is to interpret conditions (14) and (15) directly. Observe that they imply that the imports of k from ℓ contain on average relatively more content of factors which are cheaper in ℓ than in k as compared to the exports of k to ℓ. By means of (17) this also implies that the factor content of k's imports from ℓ has on average a higher ratio of factors of production with which k is well endowed relative to ℓ than the comparable ratio in k's exports to ℓ.

The last approach can perhaps be better understood by the following transformation of conditions (14) - (15). Let:

$$M^{k\ell} = \{i \mid w_i^k \geq w_i^\ell\}$$

$$N^{k\ell} = \{i \mid w_i^k < w_i^\ell\}$$

The set $M^{k\ell}$ is the set of all factors of production that are not cheaper in k than in ℓ while $N^{k\ell}$ is the set of factors of production that are cheaper in k. Define also:

$$\alpha_i^{k\ell} = (w_i^k - w_i^\ell)/\alpha^{k\ell} \quad \text{for } i \in M^{k\ell}$$

$$\beta_i^{k\ell} = (w_i^\ell - w_i^k)/\beta^{k\ell} \quad \text{for } i \in N^{k\ell}$$

where

$$\alpha^{k\ell} = \sum_{i \in M^{k\ell}} (w_i^k - w_i^\ell)$$

$$\beta^{k\ell} = \sum_{i \in N^{k\ell}} (w_i^\ell - w_i^k)$$

The numbers $\alpha_i^{k\ell}$ are nonnegative weights which add up to one and the numbers $\beta_i^{k\ell}$ are also nonnegative weights which add up to one, while $\alpha^{k\ell}$ and $\beta^{k\ell}$ are positive numbers. Using these definitions, (14) and (15) imply:

$$(19) \qquad \frac{\sum_{i\varepsilon N^{k\ell}} \beta_i^{k\ell} T_{Vi}^{\ell k}}{\sum_{i\varepsilon M^{k\ell}} \alpha_i^{k\ell} T_{Vi}^{\ell k}} \geq \frac{\alpha^{k\ell}}{\beta^{k\ell}} \geq \frac{\sum_{i\varepsilon N^{k\ell}} \beta_i^{k\ell} T_{Vi}^{k\ell}}{\sum_{i\varepsilon M^{k\ell}} \alpha_i^{k\ell} T_{Vi}^{k\ell}}$$

Hence, by (19), the $\beta_i^{k\ell}$ weighted average of factors which are cheaper in k than in ℓ compared to the $\alpha_i^{k\ell}$ weighted average of factors which are cheaper in ℓ than in k is relatively higher in k's exports to ℓ than in its imports from ℓ. In the two-factor case condition (19) identifies precisely the relative intensity of k's exports to ℓ as compared to its imports from ℓ, in terms of the differences in factor rewards. When combined with (17), the identification can be made in terms of relative factor abundance. To see this, observe that by first choosing $(\lambda^k = 0, \lambda^\ell > 0)$ and afterwards choosing $(\lambda^k > 0, \lambda^\ell = 0)$ equation (17) implies:

$$(20) \qquad \frac{\sum_{i\varepsilon N^{k\ell}} \beta_i^{k\ell} V_i^k}{\sum_{i\varepsilon M^{k\ell}} \alpha_i^{k\ell} V_i^k} \geq \frac{\alpha^{k\ell}}{\beta^{k\ell}} \geq \frac{\sum_{i\varepsilon N^k} \beta_i^{k\ell} V_i^\ell}{\sum_{i\varepsilon M^{k\ell}} \alpha_i^{k\ell} V_i^\ell}$$

Hence, the weighted average of factors of production which are relatively abundant in k's export vector to ℓ as compared to its import vector from ℓ (as seen from (19)) is also the weighted average of factors of production with which country k is well endowed relative to country ℓ (as seen from (20)). This makes precise the sense in which country k exports to ℓ relatively more of the factor content of those factors of production with which it is relatively well endowed and it imports from ℓ relatively more of the factor content of those factors of production with which it is relatively poorly endowed.

To summarize, conditions (14) - (15) provide restrictions on the factor content of bilateral trade patterns as functions of differences in factor rewards. These represent a generalization of the insight derived from the two factor, many goods and many countries case. The importance of these restrictions stems from the fact that they do not depend on the structure of preferences and that they can be tested directly by means of post-trade data.

5. Concluding Comments

I have shown that in the presence of factor price equalization the predictions of the factor proportions trade theory concerning the factor content of the pattern of trade can be formulated in terms of differences in autarky relative factor rewards in complete analogy to the predictions of commodity trade patterns based on differences in relative autarky commodity prices. Moreover, I have shown that in the absence of factor price equalization the factor content of bilateral trade patterns can be predicted from post-trade data without imposing restrictions on preferences. These predictions should prove useful in tests of the factor proportions trade theory. These tests are directly applicable to economies in which there are external effects, including interindustry spillover effects, for the case in which the definition of identical production functions is extended to include the external effects (as discussed in Helpman (1983)), as well as to economies with differentiated products for which the scale of operation of firms within a given industry does not vary across countries. As with all other conditions which are expressed in the form of correlations, the tightness of my conditions depends on diamensionality. Hence, the smaller the number of factors of production, the more useful are these conditions in predicting the factor content of trade. They are particularly useful in cases in which the number of goods is large relative to the number of factors of production and the economies are at least partially specialized in production.

Appendix

The purpose of this appendix is to prove the validity of (17). From concavity of $\Pi(\cdot)$ in V, we obtain:

$$\Pi(p, \lambda^k V^k) \leq \Pi(p, \lambda^\ell V^\ell) + \Pi_V(p, \lambda^\ell V^\ell) \cdot (\lambda^k V^k - \lambda^\ell V^\ell)$$

$$= \Pi(p, \lambda^\ell V^k) + w^\ell \cdot (\lambda^k V^k - \lambda^\ell V^\ell)$$

because $\Pi_V(\cdot)$ is homogeneous of degree zero in V. Reversing indices, we obtain:

$$\Pi(p, \lambda^\ell V^\ell) \leq \Pi(p, \lambda^k V^k) + w^k \cdot (\lambda^\ell V^\ell - \lambda^k V^k)$$

Combining the two inequalities yields:

$$(w^k - w^\ell) \cdot (\lambda^k V^k - \lambda^\ell V^\ell) \leq 0$$

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