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The Fate of a Five-Dimensional Rotating Black Hole via Hawking Radiation

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We study the evolution of a five-dimensional rotating black hole emitting scalar field radiation via the Hawking process for arbitrary initial values of the two rotation parameters a and b. It is found that any such black hole whose initial rotation parameters are both nonzero evolves toward an asymptotic state $a/M^{1/2} = b/M^{1/2} = \text{const}(\neq 0)$, where this constant is independent of the initial values of a and b.

The conventional view of black hole evaporation is that, regardless of its initial state, Hawking radiation will cause a black hole to approach an uncharged, zero angular momentum state long before all its mass has been lost. For this reason, in some works, it is assumed that as a black hole evaporates close to the Planck scale, where quantum gravity is required to determine its evolution, the final asymptotic state is described by Schwarzschild solution.

However, Chambers, Hiscock and Taylor[1)] investigated, in some detail, the evolution of a Kerr black hole emitting scalar field radiation via the Hawking process, and showed that the ratio of the black hole's specific angular momentum to its mass, $\tilde{a} = a/M$, evolves toward a stable nonzero value ($\tilde{a} \to 0.555$). This means that a rotating black hole will evolve toward a final state with non-zero angular momentum if there is a scalar field. In this Letter, we extend the analysis of Chambers, Hiscock and Taylor to a higher-dimensional case for the reasons described below. Considering the five-dimensional case specifically, we investigate the evolution of a five-dimensional rotating Myers-Perry (MP) black hole[2)] with two rotation parameters through scalar field radiation.

Recently, black holes in N (≥ 4) dimensions have attracted much attention. This is due to interest in the brane world scenario.[3),4)] From a phenomenological point of view, the most exciting possibility for the brane world scenario is that it might be possible to produce higher-dimensional mini-black holes in particle colliders, such as the CERN Large Hadron Collider (LHC), or to find them in cosmic ray events.[5)] A black hole produced in this manner would evaporate rapidly and emit many stan-

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dard particles. Hence it would lose most of its mass and angular momenta through Hawking radiation[6] or superradiance, which is intrinsic to a rotating or charged black hole.[7),8)] A few hot quanta emitted in the final Planck phase, which cannot be treated semiclassically, would not consist of the main part of the decay products.[9)] In most of the literature, the "spin-down" phase of black hole evolution, in which a black hole loses its angular momenta, is simply ignored, and a Schwarzschild black hole is assumed.

In generic particle collisions, however, the impact parameter will be non-zero. Therefore, most black holes produced in a collider would be rotating and could be described by a higher-dimensional MP solution,[2)] or other rotating objects, such as a black ring.[10)] For this reason, we focus on a "spin-down" phase through scalar field radiation. A five-dimensional rotating black hole possesses three Killing vectors: ∂t, $\partial\phi$, and $\partial\psi$. Therefore a five-dimensional black hole has two rotation parameters. For a five-dimensional MP black hole with one rotation parameter, Ida, Oda and Park[11)] found the formulae for the black body factor in a low-frequency expansion and the power spectra of the Hawking radiation. However, if a brane is not infinitely thin but, rather, has a thickness in the order of a fundamental scale ($\sim$ TeV), we expect there to exist a second component of angular momentum. We therefore study the case of two rotation parameters. Frolov and Stojković first derived expressions for the energy and angular momentum fluxes from a five-dimensional rotating black hole with two rotation parameters.[12)] In this work, we numerically evaluated the quantum radiation from a five-dimensional rotating black hole with two rotation parameters, a and b, which we assume to be positive, without loss of generality. We found that such a black hole evolves toward an asymptotic state characterized by a stable values $a = b \sim 0.1975\,(8M/3\pi)^{1/2}$, where M is the mass of the black hole. We also show that the asymptotic state can be described by $a \sim 0.1183\,(8M/3\pi)^{1/2}$ and $b = 0$ if one of the initial rotation parameters is exactly zero.

We start with the quantum radiation of a massless scalar field Φ, which is minimally coupled, for a five-dimensional MP black hole with two rotation parameters,[12)] (see also Ref. 13) for details). To quantize the scalar field, we expand it as $\Phi = R(r)\Theta(\theta)e^{im\phi}e^{in\psi}e^{-i\omega t}$. For the vacuum state, we adopt the (past) Unruh vacuum state $|U^-\rangle$, which mimics the state of collapse of a star to a black hole.[7)] Calculating the vacuum expectation value of the energy-momentum tensor of the scalar field, we can evaluate the emission rates of the total energy and angular momenta, which give the changes of the black hole mass M and angular momenta J_ϕ and J_ψ as

$$\dot{M} = -\pi \sum_{lmn} \int_0^\infty d\omega\, \frac{\omega^2}{\omega_+} \frac{\Gamma_{lmn}}{e^{2\pi\omega_+/\kappa} - 1}, \tag{1}$$

$$\dot{J}_\phi = -\pi \sum_{lmn} \int_0^\infty d\omega\, \frac{m\omega}{\omega_+} \frac{\Gamma_{lmn}}{e^{2\pi\omega_+/\kappa} - 1}, \tag{2}$$

$$\dot{J}_\psi = -\pi \sum_{lmn} \int_0^\infty d\omega\, \frac{n\omega}{\omega_+} \frac{\Gamma_{lmn}}{e^{2\pi\omega_+/\kappa} - 1}, \tag{3}$$

where $\omega_+ = \omega - m\Omega_\phi - n\Omega_\psi$, $\kappa = (r_+^2 - r_-^2)/2Mr_+$, l is the eigenvalue of the angular function $\Theta(\theta)$, and Γ_{lmn} is the greybody factor, which is identical to the absorption probability of the incoming wave of the corresponding mode. The values r_+ and r_- represent the event horizon and the inner horizon of the black hole, respectively. The quantities $\Omega_\phi = a/(r_+^2 + a^2)$ and $\Omega_\psi = b/(r_+^2 + b^2)$ are the two angular velocities at the horizon r_+. The superradiance modes are given by the condition $0 < \omega < m\Omega_\phi + n\Omega_\psi$. From this condition, we find the interesting feature that a counter-rotating particle can be created by superradiance (i.e. if $\Omega_\phi \gg \Omega_\psi$ and $m \geq 1$) because the superradiance condition is satisfied for a counter-rotating particle ($n < 0$) (see Ref. 13) for details).

Using the above formula for the quantum creation of a scalar field, we investigate the evolution of a five-dimensional MP black hole with two rotation parameters. From the condition for the existence of horizon(s), we obtain the condition $a+b \leq r_s$ constraining the angular momenta, where r_s is a typical scale length which is related to the gravitational mass M of the black hole as $r_s^2 = 8M/3\pi$.

As shown by Page,[14] it is convenient to introduce scale invariant rates of change for the mass and angular momenta of an evaporating black hole as

$$f \equiv -r_s^2 \dot{M}, \quad g_a \equiv -\frac{r_s}{a_*}\dot{J}_\phi, \quad \text{and} \quad g_b \equiv -\frac{r_s}{b_*}\dot{J}_\psi, \tag{4}$$

where $a_* = a/r_s$ and $b_* = b/r_s$. In terms of the scale invariant functions f, g_a, and g_b, the time evolution equations for a_* and b_* are given by

$$\frac{\dot{a_*}}{a_*} = -\frac{8}{3\pi}\frac{fh_a}{r_s^4} \quad \text{and} \quad \frac{\dot{b_*}}{b_*} = -\frac{8}{3\pi}\frac{fh_b}{r_s^4}, \tag{5}$$

where the dimensionless functions h_a and h_b are defined as

$$h_a \equiv \frac{d\ln a_*}{d\ln M} = \frac{3}{2}\left(\frac{g_a}{f} - 1\right) \quad \text{and} \quad h_b \equiv \frac{d\ln b_*}{d\ln M} = \frac{3}{2}\left(\frac{g_b}{f} - 1\right). \tag{6}$$

We now discuss the evolution of a_* and b_*, as determined through the numerical evaluation of f, g_a and g_b. Henceforth, we use units such that $r_s = 1$. In the dynamical system (5), a fixed point plays an important role. It is defined by $h_a = 0$ and $h_b = 0$. Note that f is positive definite. If h_a (h_b) is positive, then a_* (b_*) decreases, while if h_a (h_b) negative, then a_* (b_*) increases. Because h_a (h_b) depends not only on a_* (b_*) but also on b_* (a_*), $h_a = 0$ ($h_b = 0$) gives a curve in the a_*-b_* plane. Since there is symmetry between a_* and b_*, the fixed point should be symmetrical, too.

We first discuss the behavior of the mass and angular momentum loss rates in the case $a = b$ (and hence $a_* = b_*$). Figure 1 displays the mass loss rate $f(a_*)$ in terms of a_* ($= b_*$). The mass loss rate through the scalar radiation is more effective at smaller values of a_*. We depict the angular momentum loss rate $g_a(a_*)$ ($= g_b(a_*)$) in Fig. 2. The function $g_a(a_*)$ has a maximum at $a_* = a_*^{(\max)} \approx 0.3844$. We plot the function $h_a(a_*)$ ($= h_a(a_*)$) in Fig. 3. We find $h_a(a_*) = 0$ at $a_* = a_*^{(\mathrm{cr})} \approx 0.1975$, which is a fixed point in the present dynamical system. An important property of the function $h_a(a_*)$ is that $h_a(a_*) < 0$ [$h_a(a_*) > 0$] for $a_* < a_*^{(\mathrm{cr})}$ [$a_* > a_*^{(\mathrm{cr})}$].

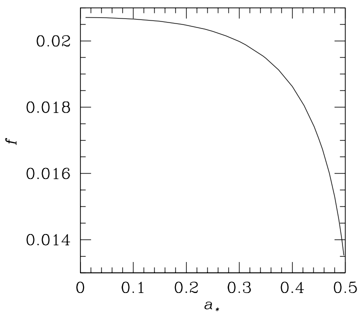

Fig. 1. The scale invariant quantity f, which represents the mass loss rate, as a function of a_* for the case $a_* = b_*$. The function f is positive definite by definition.

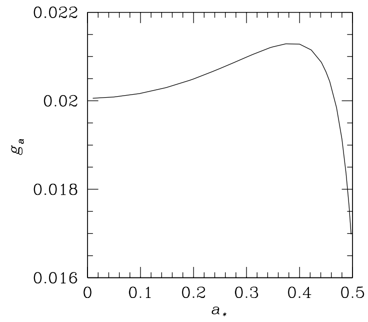

Fig. 2. The scale invariant quantity g_a, which represents the loss rate of the angular momentum J_ϕ, as a function of a_* for the case $a_* = b_*$, for which $g_a = g_b$. The function g_a is positive definite by definition.

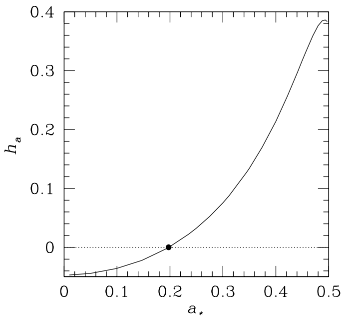

Fig. 3. The scale invariant quantity h_a, which represents the change rate of a_*, as a function of a_* for the casef $a_* = b_*$, for which $h_a = h_b$. The function $h_a(a_*)$ has a zero at $a_* = a_*^{(\mathrm{cr})} \simeq 0.1975$ (a black spot).

As a result, the fixed point $(a_*, b_*) = (a_*^{(\mathrm{cr})}, a_*^{(\mathrm{cr})})$ is stable along the line $a_* = b_*$. Hence, a black hole formed with equal rotation parameters, $a_* = b_* \neq 0$, will eventually reach an asymptotic state characterized by a_* $(= b_*) = a_*^{(\mathrm{cr})}$, through scalar field radiation.

In order to investigate the more generic case ($a \neq b$), we have to analyze Eq. (5). For this purpose, we depict the contour plots of f and g_a in Figs. 4 and 5, respectively. (g_b is obtained by exchanging the axes for a_* and b_* in Fig. 5.)

In the a_*-b_* plane, the region in which $a_* + b_* > 1$ is forbidden, because there is no horizon (the black region in Figs. 4 and 5). In Fig. 4, there are two bright regions (one for large a_* and small b_*, and one for small a_* and large b_*), where f becomes large. This means that the creation rate is high in these regions. In Fig. 5, there is only one bright region (for large a_* and small b_*). Therefore, the angular momentum J_ϕ is emitted effectively only in this region. This is the superradiance effect. For the angular momentum J_ψ, if b_* is large, we find effective emission. This means that the superradiance modes give a dominant contribution to the particle creation.

There is one interesting observation here: If the two rotation parameters are equal (i.e. $a_* = b_*$), the emission rates are suppressed even if the black hole is in

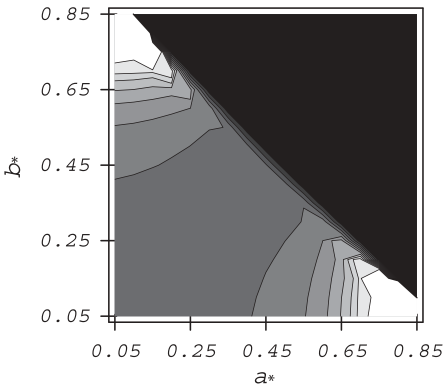

Fig. 4. The contours of f in the a_*-b_* plane. The darkest and brightest regions correspond to zero and $f_{\max}$ (the maximum of f), which is given by $f_{\max} \simeq f(0.85, 0.05) = 4.349$, respectively. The difference between two contours is $f_{\max}/10$. The black region is forbidden, because there is no horizon in this region.

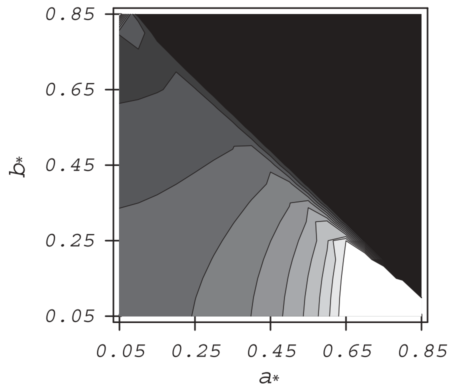

Fig. 5. The contours of g_a in the a_*-b_* plane. The black and white regions correspond to zero and $g_{a,\max}$ (the maximum of g_a), which is given by $g_{a,\max} \simeq f(0.85, 0.05) = 5.92467$, respectively. The difference between two contours is $g_{a,\max}/10$. The black region is forbidden, because there is no horizon in this region.

a maximally rotating state ($a_* = b_* = 0.5$). In the case $a = b$, something strange seems to happen, and the system behaves like a "spherically symmetric" black hole. In fact, the angular equation for $\Theta(\theta)$ in this case is exactly the same as that for the Schwarzschild black hole.[12)] This may suppress the superradiance effect. This is consistent with the result given in Ref. 15), the efficiency of energy extraction for a MP black hole is very small in the case that the rotation parameters are equal.

In order to see the evolution of a black hole in the a_*-b_* plane, we plot the vector field $(\dot{a}_*, \dot{b}_*)$ with arrows in Fig. 6. From this figure, we see how the values of a_* and b_* evolve toward the fixed point. We can also prove that the fixed point is a stable attractor (see Ref. 13) for details).

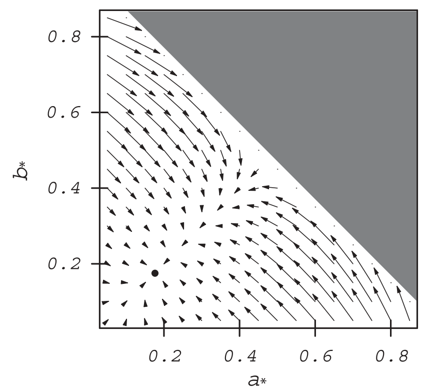

Fig. 6. The vector field describes the direction in which a_* and b_* evolve, i.e. $(\dot{a}_*, \dot{b}_*)$. For any initial values of a_* and b_*, the system evolves toward $a_* = b_* = 0.1975$ (the black spot), which is a stable fixed point. The shaded region is forbidden.

In Fig. 6 the arrows far from the symmetry line of $a_* = b_*$ are very large. Then, if the initial value of a_* (b_*) is large, while that of b_* (a_*) is small, a_* and b_* first approach the same value. Near the fixed point $(a_*^{(\mathrm{cr})}, a_*^{(\mathrm{cr})})$, the arrows are very small, which means that the evolution toward the fixed point is slow. We thus find that after reaching a state with $a_* = b_*$, a_* and b_* eventually evolve together toward the fixed point

$(a_*^{(\mathrm{cr})}, a_*^{(\mathrm{cr})}) \approx (0.1975, 0.1975)$. This means that any rotating black hole with two non-zero rotation parameters will evolve toward a final state with the same specific angular momenta, $a_* = b_* = a_*^{(\mathrm{cr})} \approx 0.1975$. For a black hole with only one non-trivial rotation parameter, i.e. $a \neq 0$ and $b = 0$ exactly, we obtain the stable fixed point from the equation $h_a(a_*, 0) = 0$, which yields $a_* \approx 0.1183$.

Finally, consider the evaporation time of the black hole. In the above analysis, we showed that our dynamical system (5) has one stable attractor, which can be reached through quantum particle production. However, the black hole may evaporate away before this fixed point is reached. Whether this happens depends on the evaporation time and the evolution time in the a_*-b_* plane. We can evaluate the evaporation time scale τ_M using the emission rate of the black hole mass as $\tau_M = -M/\dot{M}$, and we can evaluate the evolution time scale τ_{a_*} using the evolution equation (5) as $\tau_{a_*} = a_*/|\dot{a}_*|$.

We thus find that $\tau_M/\tau_{a_*} = 8|h_a|/(3\pi) \sim O(1)$. However, this does not mean that the black hole will evaporate away before reaching the fixed point. If the integrated evaporation time, which depends on the initial mass of the black hole, is much longer than the evolution time, we have enough time to realize the final state described by the fixed point. Therefore, we conclude that if a black hole has a mass that is larger than the fundamental Planck mass scale, its two specific angular momenta will eventually become equal when it evaporates away.

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