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The fate of dense stellar systems

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Summary. We investigate the fates of dense star clusters containing ~ 10^8 stars. If such systems have evolved to the stage when disruptive stellar collisions are important, the collisionally liberated gaseous debris may settle towards the centre. Spitzer and collaborators suggest that this gas condenses into a population of 'new' main sequence stars. Such a stellar subsystem commences an autonomous contraction, at a rate controlled by collisions among the 'new' stars themselves, when it attains a fraction ϵ = (ellipticity of original cluster)$^{1/2}$ of the total cluster mass. The gas liberated by stellar collisions can no longer recondense into stars when the concomitant luminosity rises to the Eddington limit for the subsystem: at this stage the subsystem must dissolve into an amorphous gas cloud. We discuss possible evolutionary tracks for the massive object thus formed. For a cluster of $10^8 N_8$ solar-type stars, the maximum fraction of rest mass energy released before a massive object forms is only ~ $3 \times 10^{-4}\, \epsilon^{13/9}\, N_8^{4/7}$.

Before a cluster reaches the stage where stellar collisions are disruptive, runaway coalescence may already have led to the build-up of a central massive object. Alternatively, multiple supernovae could leave a post-coalescence cluster composed of compact stellar-mass remnants. Such a cluster sheds most of its members by evaporation before evolving to the stage where it collapses relativistically, unless sufficient gas is present to catalyse the contraction by interacting magnetically or gravitationally with the remnants.

Dense star clusters may be responsible for some of the low-level manifestations of activity in galactic nuclei; but they are probably merely precursor stages of the more spectacular quasar-type phenomena, which develop after a massive object has formed.

1 Introduction

Star densities in the nuclei of normal galaxies are known to be $\gtrsim 10^6$ pc^{-3}, and it has often been suggested that the power supply in active nuclei and quasars derives from a very dense stellar system that is undergoing some kind of violent evolution. Various authors have discussed different processes whereby continuing contraction may lead to expulsion of matter, or the conversion of stellar material into some compact form.

General discussions have been given by Peebles (1972) and Saslaw (1973). The first discussion of stellar collisions (coalescence and disruption) was given by Gold, Axford & Ray (1965), but the most detailed attempt to construct evolutionary sequences has been made by Spitzer and his collaborators (see especially Spitzer & Saslaw 1966; Spitzer & Stone 1967; Spitzer 1971). These authors suppose that the cluster is originally composed of main sequence stars, and make the reasonable assumption that any gas liberated by collisions quickly cools and condenses into new stars of the same kind. They find that stellar collisions eventually become very efficient at dissipating cluster energy, but that the small amount of angular momentum initially present plays a critical role in determining the final state: according to Spitzer and collaborators, the evolution leads to a cold tightly-bound disc of stars whose relative velocities are insufficient to cause further disruptive collisions.

We review this work in Section 2. In Section 3, we suggest that the evolution would actually continue in a more dramatic fashion than envisaged by Spitzer: the dense self-gravitating disc of stars would be unstable to the Ostriker & Peebles (1973) global instability, which would regenerate high random velocities and permit stellar collisions and destruction to proceed at an ever-accelerating pace. Eventually the luminosity generated by the collisions provides so much radiation pressure that the debris cannot recondense into new stars: at this stage the material merges into an amorphous supermassive cloud.

It is far from certain how a supermassive star ends its life. It will eventually collapse or undergo a nuclear explosion. The fraction of its rest-mass energy that can be released is less than ~ 0.01 unless magnetic fields or differential rotation can stave off the post-Newtonian instability. Although its luminosity may be very high, the bright phase is brief compared to the earlier evolutionary time-scale of the cluster. On the other hand, a cluster whose constituent stars can transform themselves into neutron stars or black holes could subsequently evolve on the (slower) dynamical time-scale, but would yield substantially lower luminosity.

It appears, then, that the hypothesis that material is continually reprocessed into new stars for as long as possible is the one which postpones collapse longest, and yields the

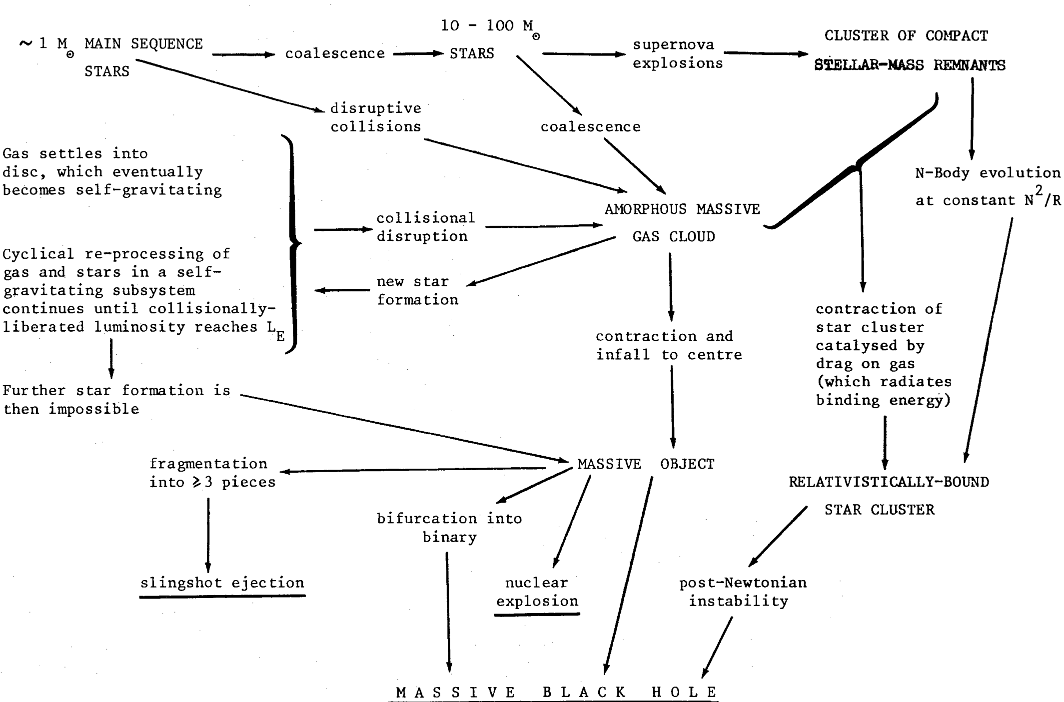

Figure 1

longest sustained level of high luminosity for an isolated star cluster. However, it is uncertain whether a star cluster can generally evolve to the state where disruptive collisions occur without first having passed through a 'coalescence phase', during which massive stars could be built up. These stars, exploding as supernovae, could leave compact stellar-mass remnants. We explore in Section 4 how these remnants affect the subsequent evolution of a cluster.

In Fig. 1 we sketch some of the evolutionary possibilities in a 'flow diagram', which illustrates the general tendency for gravitational binding energy to increase, and the likelihood that the final state is a massive black hole.

2 Summary of Spitzer–Saslaw–Stone scenario

A nearly spherical aggregate of stars develops a 'core–halo' structure. We are interested in the core, which contains N stars of mean mass M_*, distributed roughly uniformly within a radius R. When direct stellar collisions are unimportant, core stars 'evaporate' into the halo, carrying very little positive energy with respect to the core. Hence the core evolves at constant total energy, with

$$R \propto N^2.$$

Since N decreases in this process, the binding energy per core star increases, and the velocity dispersion varies as

$$v_c^2 \simeq GM_* N/2R \propto N^{-1} \propto R^{-1/2}. \tag{2.1}$$

Two-body relaxation effects occur on a time-scale 10–100 times the 'reference time'

$$t_R = 4.8 \times 10^7 N_8^{1/2} R_{17}^{3/2} m_*^{-1/2} [\log(N/2)]^{-1} \text{ yr} \tag{2.2}$$

(where R_{17} is in units of 10^{17} cm, N_8 is in units of 10^8 stars, and m_* is the mass of a typical star in solar units). For any given cluster parameters, the time-scale t_c for each star to suffer a physical collision depends on both geometrical and gravitational cross-sections. Collisions can affect cluster evolution only when $t_c \lesssim t_R$, which requires $v_{rel} \simeq \sqrt{2} v_c \gtrsim v_{esc}$, where v_{esc} is the escape speed from a star of mass M_*.

When t_R is relatively close to t_c, the inelasticity of the collision process may result in a substantial fraction of the colliding stars coalescing. The probable importance of coalescence is still under debate (see Section 4), and seems to depend sensitively on the IMF of the original stars (Colgate 1967; Sanders 1970). Coalescence, if effective, will result in a large number of compact bodies, neutron stars and black holes, being formed before the cluster has a chance to evolve much beyond the $v_{rel} = v_{esc}$ stage. The effects of these bodies, as well as of the explosive mass loss which may have accompanied their formation, on the subsequent evolution of the cluster, are discussed in Section 4. We assume here, with Spitzer, Saslaw and Stone, that neither the effects of coalescence, nor the certain presence of stellar-mass compact objects resulting from the normal evolution of higher than average mass stars, will have a substantial effect. Indeed, we neglect all effects due to the distribution of stellar masses, although Spitzer (1969) and Lightman & Fall (1978) have argued that a cluster whose stars obey a Salpeter mass function will display considerable core structure brought about by mass segregation.

When $v_{rel} = v_{esc}$, a typical collision liberates $\zeta = 0.05$ of the mass of the colliding stars, though ζ rises steeply as the relative velocity v_{rel} increases further (Seidl & Cameron 1972). Comparing the collisional mass-loss time-scale with the stellar-dynamical evolution time-scale (10–100 t_R) we conclude that collisions start to dominate the overall core contraction as soon as $v_{\rm rel}$ exceeds $v_{\rm esc}$, i.e.

$$R_{17} \lesssim 35 N_8 R_*/R_\odot \approx 35 N_8 m_*^\alpha. \tag{2.3}$$

The latter approximate equality comes from the numerically determined mass–radius relation for main sequence stars (*cf.* Iben 1967), which holds with $\alpha \approx 0.56$ for $1\,M_\odot < M_* < 120\,M_\odot$.

Spitzer & Saslaw showed that gas released by collisions is able to cool by free–free emission in time-scales much shorter than any of the evolutionary time-scales in the problem, and will therefore fall towards the centre of the core. However, even if the ellipticity of the core ϵ^2 is very small indeed, the gas will not be able to complete its collapse quasi-spherically, but will rather settle into a thin disc of characteristic radius

$$R_{\rm d} \approx \epsilon^{1/2} R \tag{2.4}$$

and thickness given by its temperature (probably $\lesssim 10^4$ K). (2.4) is valid for a disk which is not self-gravitating in the radial direction. At first, the disk will be dominated by the mean cluster field in the vertical direction as well, but when its density exceeds the local mean density of stars, the disk becomes vertically self-gravitating and Jeans unstable. At this point, Spitzer & Saslaw hypothesize that ordinary main sequence stars form on a time-scale much shorter than any of the core evolution times. The new stars do not remain in the disk but diffuse out of it due to two-body encounters with the more energetic core stars, to form a roughly spherical swarm of new stars amidst the core. If this swarm does not become strongly self-gravitating, a steady-state distribution of new stars may be set up on a time-scale much shorter than the core evolution time. The flux of collisionally liberated gas is $\dot{M}_{\rm coll} = \zeta N M_* t_{\rm c}^{-1}$. The collision time-scale $t_{\rm c}$ is proportional to $R^3/NR_*^2 v_{\rm c} \propto N^{5/2} M_*^{3/2} R^{-7/2} R_*^{-2}$. Calculating the constant of proportionality one finds

$$\dot{M}_{\rm coll} = \zeta N M_* t_{\rm c}^{-1} \simeq 1676 \zeta N_8^{5/2} R_{17}^{-7/2} m_*^{3/2+2\alpha} M_\odot/{\rm yr}. \tag{2.5}$$

The condition that self-gravity does not affect the new stars is equivalent to $M_{\rm new} < NM_* (R_{\rm d}/R)^3$, or

$$\zeta \epsilon^{-1/2} (t_{\rm R}/t_{\rm c}) < 1. \tag{2.6}$$

Condition (2.6) fails even for the initial collisional evolution at $t_{\rm R} \approx t_{\rm c}$ if $\epsilon < \zeta^2 \approx 2.5 \times 10^{-3}$ i.e. for initial core ellipticities smaller than $\sim 6.3 \times 10^{-6}$. If (2.6) *is* satisfied initially, then the core stars may be reprocessed several times before a relatively small component of new stars is able to form a self-gravitating system. Denoting conditions at the outset of the collision-dominated regime by subscript zero, we find that before self-gravitation sets in,

$$\epsilon = \epsilon_0 (R_0/R)^{1/2}$$

while

$$t_{\rm R}/t_{\rm c} \propto v_{\rm c}^4 \propto R^{-2}$$

at constant N. Hence the new stars become a self-gravitating system when

$$(R_0/R)^2 \simeq t_{\rm R}/t_{\rm c} \simeq \epsilon_0^{4/7}/\zeta^{8/7}. \tag{2.7}$$

Note that at this stage the new stars comprise only a small fraction ($\sim \epsilon^{3/2}$) of the total core mass.

Domination of the new stars by their own gravitation does not mean that they break away dynamically from the cluster core, however. The velocity dispersion of the self-bound system is, at first, smaller than $v_{\rm c}$ by a factor $\epsilon^{1/2}$, and the internal relaxation time shorter than $t_{\rm R}$ by a factor $\epsilon^{3/2}$. If mass continues to be pumped into the bound system at a rate

$\dot{M}_{\text{coll}}$, the ratio of the mass-doubling time to the internal relaxation time is roughly $t_c/\zeta t_R$, which exceeds unity at first.

As long as the velocity dispersion within the system of new stars remains smaller than v_c, it is conceivable that two-body encounters between old and new stars could efficiently lift new stars out of the self-bound region, and cause the contraction of the core as a whole. Assuming that the angular momentum per unit mass of the infalling material remains fixed at $J = \epsilon_0 R_0 v_{c0}$, the mass M_b and radius R_b of the self-bound system are related by:

$$R_b = 2J^2/GM_b \approx \epsilon_0^2 (M_* N/M_b) R_0 = \epsilon^2 (M_* N/M_b) R. \tag{2.8}$$

One can show that a steady balance between infall of liberated gas and outflow of new stars cannot be maintained once the new stars become a self-gravitating system. Two-body effects cannot succeed in dispersing the increased binding energy throughout the core, and the self-bound region begins to gain mass as $\dot{M}_{\text{coll}}$, while decreasing its radius. The analysis of Spitzer & Saslaw stops short of this contraction, while Spitzer & Stone analyse it only in so far as the increasingly deep potential well at the centre of the core can yield greater luminosities per gram of liberated gas falling into the disk. Once the new stars do become self-bound, relaxation effects between the core stars and the bound sub-system become unimportant. According to Spitzer & Stone, the evolution thereafter becomes rather dull. The luminosity peaks as a result of the earlier relaxation effects (their figure 1 and table 2) and then declines as core stars are depleted, while a cold, increasingly flat disk shrinks towards its final radius, consistent with angular momentum conservation, of $\epsilon_0^2 R_0$. This process occurs on a time-scale comparable with the core depletion time, which in the end is rather long, and although the total amount of energy released is fixed by the final disk parameters, the accompanying luminosity is relatively low.

We envisage a rather more spectacular final scene for the bound sub-system of recycled stars. As the system contracts, the random motions of its stellar population will be kept stirred up to its virial value by collective effects. The result is that stellar collisions will occur within the subsystem at a pace which rapidly overtakes that in the main core. In the next section we follow the evolution of this subsystem, and argue that it naturally proceeds to the formation of a massive central object while the binding energy is still a small fraction of c^2 per unit mass.

3 Later evolution

3.1 EVOLUTION OF THE SELF-GRAVITATING STELLAR SUBSYSTEM

It is fairly well established that an isolated axisymmetric self-gravitating system, whether of stars or fluid, is violently unstable when the ratio of rotational kinetic energy to gravitational binding energy, t, exceeds $\simeq 0.14$ (Ostriker & Peebles 1973, and references cited therein). This instability leads, within a dynamical time-scale, to a quasi-steady state which marginally satisfies the stability criterion. For a stellar system this may either be an axisymmetric distribution with a 'hot' core of diminished radius surrounded by a few distant stars carrying a substantial fraction of the angular momentum; or a 'hot' bar, whose limiting form is a 'binary' of compact bound subsystems orbiting each other. In what follows, we suppose that most (or, to be more quantitative, that $\gtrsim 1/2$) of the mass is able to contract in a roughly axisymmetric fashion, as it loses energy in the course of its evolution. There are various mechanisms (e.g. the maintainance of a weak bar or gaseous viscosity) which could expel angular momentum to the core or to the remaining $\lesssim 1/2$ of the subsystem mass at the required rate. A gaseous subsystem may either develop a bar or eject rings, depending on the amount of differential rotation.

The subsystem of newly-formed stars is unstable to the bar instability as soon as it becomes self-gravitating (equation (2.7)), and is continually forced into instability as new, high angular momentum mass is supplied by the cluster core. Whether or not new stars have time to form, the overall radius of the self-gravitating subsystem will be determined by conservation of angular momentum according to (2.8). Since the subsystem remains heated to its virial temperature, it evolves not to the flat, cold disk of stars envisaged by Spitzer & Stone, but rather to a 'hot' system, in which, if star formation remains efficient, violent stellar collisions eventually dominate the evolution.

On a time-scale which is short compared to the liberation time of the remaining core mass, the velocity dispersion within the subsystem rises from $\sim \epsilon^{-1/2} v_c$ to $\gtrsim v_c$, and it passes into the regime of disruptive collisions. At first, the evolution of the subsystem then proceeds on the time-scale for doubling its mass,

$$t_d \sim M_b/\dot{M}_{coll} \sim (M_b/\zeta N M_*) t_c, \tag{3.1}$$

but eventually the internal collision time-scale,

$$t_{c,b} \sim (R_b/R)^{7/2} (NM_*/M_b)^{3/2} t_c, \tag{3.2}$$

shrinks to the point where subsequent evolution of the subsystem, if new stars are still being formed efficiently, occurs at constant mass and is dominated by a fully internal cycle of star formation and destruction. This phase begins when $t_d \zeta_b \sim t_{c,b}$, where ζ_b is the efficiency of gas liberation when two subsystem stars collide. In terms of core and subsystem parameters, this condition may be written as

$$M_b/NM_* \sim (\zeta/\zeta_b)^{1/6} \epsilon^{7/6}. \tag{3.3}$$

Equations (3.2) and (3.3) are valid only when the velocity dispersion in the bound system, v_b, exceeds v_{esc}. The strongest constraint we can place on v_b relies on the fact that the core velocity dispersion, v_c, satisfies $v_c \gtrsim v_{esc}$, while $v_b \gtrsim v_c$ when $M_b/NM_* \gtrsim \epsilon$. In this regime, $(\zeta/\zeta_b)^{1/6} \sim 1$, while $\epsilon^{7/6}$ differs little from ϵ. Therefore we take

$$M_b/NM_* \sim \epsilon \tag{3.4}$$

as the condition for collisions to begin to dominate the evolution of the bound subsystem.

This latest phase is characterized by constant total angular momentum as well as constant mass. Hence, the characteristic radius within which most of the angular momentum is carried must remain at $\gtrsim \epsilon R$. However, to satisfy the Ostriker–Peebles criterion, the system must become more centrally condensed as it loses random kinetic energy in collisions, with most of the newly-formed stars populating a tightly bound central region of ever-increasing collision frequency. If the subsystem remained almost spherical, collisions would occur at a rate given by (3.2). If it evolved into a disk with thickness $h \ll r_b$, where r_b is the characteristic radius containing half the mass (the configuration satisfying the Ostriker–Peebles criterion by containing stellar orbits with a large dispersion in eccentricity), the collision rate would exceed (3.2) by a factor $\sim r_b/h$. Regardless of the value of h, we now show that the system must evolve in a time $\sim (\epsilon/\zeta_b) t_c$ to a point where new stars can no longer form.

3.2 WHEN STAR FORMATION MUST STOP

In the absence of a detailed theory of star formation, we can nonetheless determine the extreme condition under which efficient star formation can occur. As the hot part of the subsystem contracts due to collisional dissipation, its luminosity is given by an expression similar to (2.5) except for an extra factor (r_b/h) which allows for possible flattening:

$$L_b \sim \zeta_b (r_b/h) v_b^2 M_b t_{c,b}^{-1} \sim 7 \times 10^{45} \zeta_b (r_b/h) \epsilon^{7/2} \cdot N_8^{7/2} r_{b17}^{9/2} m_*^{2\alpha + 5/2} \text{ erg/s} \tag{3.5}$$

(Spitzer & Saslaw 1966). When r_b equals R_b, at the point where collisions rather than mass gain begin to drive the evolution, L_b is roughly equal to L_c, the luminosity due to infalling debris from collisions in the core. However, once the collisional runaway begins within the subsystem, $L_c \propto (v_b/v_c)^2 \propto R_b/r_b$ while $L_b \propto (r_b/h)(R_b/r_b)^{9/2}$. Therefore during this phase, the luminosity due to collisional evolution of the subsystem exceeds L_c.

Star formation must cease at or before the point where L_b equals the Eddington luminosity for the subsystem, $L_{E_b} \sim 10^{38} m_b$ erg/s, m_b being the mass in solar units. Were L_b to exceed L_{E_b}, gas released in collisions would be blown out of the subsystem altogether before it could collect centrally to form new stars. Clearly it is energetically impossible for the tightly bound subsystem to unbind itself in this manner, utilizing no energy other than the internal kinetic energy given by the virial theorem. Most of its stellar content must therefore have been transformed into a hot amorphous self-gravitating cloud by the time the characteristic radius reaches

$$r_E \sim 8 \times 10^{16} \zeta_b^{2/9} \epsilon^{5/9} N_8^{5/9} m_*^{4\alpha/9+1/3} \text{ cm.} \tag{3.6}$$

It should be stressed that r_E is a *lower limit* on the radius at which the subsystem must cease star formation. There are a number of processes which may inhibit or eliminate star formation at an earlier stage. For example, hierarchical fragmentation may be hindered by rotation in the disk, or a tendency towards coalescence with neighbouring fragments; and protostars may be prevented from reaching the main sequence by encounters with ordinary stars. Crude estimates of the importance of such effects suggest that they will probably not shut down star formation very much before r_E is reached, if at all.

The principal conclusion of this section is that runaway processes in the centre of a stellar system must give rise to a central supermassive gas cloud by the time a fraction ϵ of its mass has been released in collisions. We cannot predict subsequent events in detail, because we do not know – for instance – how the cloud's angular momentum will be redistributed during its evolution; but we outline below some plausible evolutionary tracks.

3.3 FURTHER EVOLUTION AFTER A CENTRAL MASSIVE OBJECT HAS FORMED

Although the central cloud will be radiation-pressure supported in the vertical direction, its binding energy will be comparable with its rotational energy, a fraction $x \simeq 0(1)$ of its gravitational potential energy. Since it radiates its internal energy at a rate L_E, the timescale for radiating away from its initial binding energy is

$$t_{rad} \lesssim 5 \times 10^8 x (r_s/r_E) \text{ yr} \sim 2 \times 10^5 x \zeta_b^{-2/9} \epsilon^{4/9} N_8^{4/9} m_*^{2/3-4\alpha/9} \text{ yr} \tag{3.7}$$

where $r_s = 2GM_b/c^2 = 3 \times 10^5 m_b$ cm is the Schwarzschild radius. Assuming that it is in the mass range when radiation pressure is dominant, its temperature is

$$T_i \lesssim 8.2 \times 10^{12} m_b^{-1/2} (r_s/r_E) \sim 4 \times 10^5 \zeta_b^{-2/7} \epsilon^{-1/18} N_8^{-1/18} m_*^{1/6-4\alpha/9} \text{K.} \tag{3.8}$$

Nuclear burning cannot yield luminosities comparable with L_E unless T can become considerably higher than T_i, which would happen only if redistribution of angular momentum permitted a fraction of the cloud mass to contract radially.

If angular momentum is not lost (*cf.* Ozernoi & Usov 1973) or redistributed, the gaseous body will increase its ellipticity, within a time t_{rad}, until it becomes strongly bar-like, and prone to fission. If it bifurcates, the two supermassive stars thus formed may each possess considerable rotational energy; multiple fissions (*cf.* Salpeter 1971) may then occur. A system of three or more bodies will evolve via 'slingshot' ejection in a few dynamical timescales (Saslaw, Aarseth & Valtonen 1974). The presence of angular momentum in the remaining bodies will have the effect discussed by Fowler (1966), Fricke (1974), and others, of stabilizing the individual self-gravitating units against post-Newtonian instability. Although this means that the final collapse to a black hole may take as long as 5×10^8 yr, it also removes the possibility that the onset of nuclear burning will unbind the system entirely (Fowler 1966). Instead, the nuclear burning stage, if it occurs, will manifest itself as a hiatus in the collapse, lasting no more than $\sim 10^6$ yr. In the unlikely event that fission results in individual supermassive stars with virtually no rotation, onset of the post-Newtonian instability will occur early, with collapse to a black hole for units more massive than $3 \times 10^6 \, M_\odot$, and explosion of units with $M \lesssim 3 \times 10^6 \, M_\odot$ (Fricke 1973; Von Hoerner & Saslaw 1976). Explosions would eject the central compact mass from the core altogether, returning the star cluster nearly to its state prior to the collisional runaway in its central subsystem. The process could then repeat, until eventually the whole mass of the core was dispersed.

A central binary that does not explode exhibits a large quadrupole moment during its early evolution, regardless of how rapidly its constituents contract to form black holes. A surviving binary of total mass M_b, in a circular orbit of semimajor axis $\gtrsim r_\mathrm{E}$, is too extended for gravitational waves to carry off most of the angular momentum before it is lost via gravitational interactions with the core stars, with the collisionally liberated gas which is still falling in to form a disk of radius $\sim \epsilon R$, and with new stars which are being formed from this gas. The product of the first collisional runaway, whether a binary or a single differentially rotating object, will be able to lose much of its angular momentum on a time-scale $\lesssim \epsilon t_\mathrm{R}$, (i.e. before a second collisional runaway occurs) and presumably evolves into a black hole.

After a central black hole forms, gas will drain into it via an accretion disc. However, if the central mass has collapsed to form a black hole then the total rate at which energy is released in infall will exceed the Eddington luminosity of the central mass by a factor

$$\sim 0.1 \, \dot{M}_\mathrm{coll} c^2 / L_\mathrm{E} \sim 4 \times 10^7 m_\mathrm{c}^{-1} (\dot{M}_\mathrm{coll} / M_\odot \, \mathrm{yr}^{-1}).$$

It is conceivable (Shakura & Sunyaev 1973) that radiation pressure could blow a large fraction of the disk mass out along the vertical direction. However, such a wind would not escape to infinity, but would be trapped within the potential well of the core and quickly returned to the vicinity of the black hole. Therefore, acceptance of the Shakura & Sunyaev picture would not mean that the central mass grows more slowly, but merely that the infalling mass cannot fall directly into the central black hole. The black hole would then become 'clothed' in a supermassive star, and constitute a progressively smaller fraction of the central mass. An alternative view suggests that the radiative efficiency falls as $\dot{M}$ rises, so that the black hole can accrete the gas at an arbitrarily high rate (Maraschi, Reina & Treves 1976; Begelman 1978a, b).

If the accretion disc becomes self-gravitating, Jeans-style instabilities would quickly generate a large viscosity (Paczynski 1977), or else permit the disc to fragment into stars, whose Coulomb-type interactions provide, in effect, a high viscosity. Thus the mass resident in a thin accretion disc can never become more than a small fraction of the central mass (Begelman, in preparation).

The luminosities and time-scales characterizing the core's collisional self-destruction and infall into a central black hole are reminiscent of those supposed for active galactic nuclei and Seyferts, but it is difficult to stretch the parameters to include the more luminous QSO's (up to 10^{48} erg/s). Frank (1978) has found that tidal disruption of stars by a massive black hole and an otherwise non-collisional core is also insufficient, and suggests collisions within the core as the principal source of the gas. In view of the upper limits on $\dot{M}_\mathrm{coll}$, this appears plausible for $M_* \sim 1 \, M_\odot$, only if $\epsilon \sim 0(1)$ and $10^8 < N < 10^{10}$. Time-scales and mass loss rates are more favourable as N approaches 10^8, and $\dot{M}_\mathrm{coll}$ may attain $\sim 100 \, M_\odot$/yr for $\sim 10^6$ yr. However, it is unclear whether luminosities exceeding L_E ($\sim 10^{46}$ erg/s for a $10^8 M_\odot$ central mass) could escape from such a system; certainly, some highly non-axisymmetric form of convective radiation transfer, such as photon bubbles (Prendergast & Spiegel 1973) would have to occur with very high efficiency. The L_E problem is overcome by allowing N to tend towards 10^{10}, but in this limit $\dot{M}_{\text{coll}}$ will only barely attain $\gtrsim 1 M_\odot$/yr when $\epsilon \approx 1$; and such a mass flux cannot yield steady luminosities in excess of $\sim 10^{46}$ erg/s. We note, in passing, that one way to relieve the low $\dot{M}_{\text{coll}}$ problem at the $N \sim 10^{10}$ end is to postulate $M_* \gtrsim 5 M_\odot$.

If the collisional self-destruction of the core is not in itself the quasar (or Seyfert, etc.), it may very well be the precursor phase, in which a massive black hole could be built up over a short time. This model has two advantages over the growth by tidal capture model proposed by Hills (1975) – there is no need to postulate a seed black hole and the central massive object can, in principle, grow to $10^8 M_\odot$ in substantially less than 10^9 yr.

4 Coalescence, clusters of compact objects, etc.

4.1 COALESCENCE

Coalescence of main sequence stars can play a dominant role only when v_c is comparable with the escape velocity v_{esc} from the surface of a typical star (i.e. 500–1000 km/s): when v_c is smaller than 0.6–0.7 v_{esc} (Colgate 1967) or 0.3 v_{esc} (Lightman & Shapiro 1978), geometrical encounters are unimportant on the time-scale $\sim 50\, t_R$ over which the cluster evolves by evaporation; and when v_c is larger than $\sim$ 1000 km/s (Sanders 1970), any collisions would lead to disruption. These limiting velocities define a 'coalescence strip' in the M–R plane. The effects of coalescence have been considered by Colgate (1967) and by Sanders (1970), whose conclusions are rather different. Colgate argues that the coalescence process saturates when stars of ~ 50–$100\, M_\odot$ have been produced, because these are so diffuse that ordinary $1 M_\odot$ stars pass straight through them without being captured. Sanders (1970), however, finds that in a cluster of $10^7 M_\odot$ the coalescence runs away, leading to the build-up of a few supermassive stars. For an initial cluster mass of $\gtrsim 10^8 M_\odot$, this runaway is prevented because the collision time-scale (proportional to the square of the cluster mass for a given v_c) is longer than the main sequence lifetime for a star of $10 M_\odot$. Unfortunately, uncertain details of the physics affect the coalescence phase sensitively. If there were no mass loss from the system, coalescence, being a dissipative process, would double the cluster binding energy, and with it v_c^2, by the time a typical star had undergone one collision. Explosive mass loss in which the ejecta leave the cluster altogether would by itself *reduce* v_c. However, coalescent build-up of massive stars, followed by their explosive disintegration, can never decrease the binding energy. This means that, as mass is lost, v_c rises at least as steeply as (cluster mass)$^{-1/2}$. Thus, even a system in which maximal mass loss occurs would have evolved from the coalescence strip into the disruptive regime before its mass would have been reduced by a factor ~ 10. Sanders did not allow for the tendency of the massive stars to sink towards the centre of the cluster on a time-scale $\sim t_R(m/\langle m_* \rangle)^{-1}$. We estimate that this effect permits runaway coalescence in clusters of initial mass up to $\sim 10^8 M_\odot$. This is one of the quickest routes to the formation of a massive object in a dense stellar system.

In Colgate's (1967) scenario (see also Arons, Kulsrud & Ostriker 1975), multiple supernova explosions (releasing the gravitational binding energy of individual stellar-mass objects) can generate a supercritical quasar-level luminosity for a few million years. This phase releases a large amount of gas, and leaves behind a cluster of compact stellar-mass remnants – neutron stars or black holes. We outline below the likely eventual fate of such a system.

4.2 CLUSTERS OF COMPACT OBJECTS AND GAS

The endpoint of the coalescence phase, according to Colgate (1967) might be 10^7 compact objects, each of $1-10\,M_\odot$, in a cluster with a velocity dispersion 300–1000 km/s. The value of t_R would have been reduced during the coalescence phase by a factor comparable with the reduction in $N^2 \langle m_* \rangle$. If the cluster subsequently evolved according to (2.1), conserving its total binding energy, only ~ 100 objects would remain in the tightly bound core when this became relativistic. Even though relativistic star clusters have interesting properties (which have been extensively discussed in the literature), this seems an unpromising route for producing them. It is nevertheless interesting to explore whether the contraction of a cluster at constant mass (rather than constant binding energy) would be possible if the compact stars were embedded in a gas cloud capable of dissipating and radiating energy.

Unno (1971), and Bisnovatyi-Kogan & Sunyaev (1972) were the first to study quantitatively the properties of a star cluster embedded in dense gas. The latter authors noted that the presence of stars would help to stabilize a supermassive gas cloud against the post-Newtonian instability, but concluded that a tightly bound cluster of $\lesssim 10^9$ *normal* stars could not coexist with a comparable mass of gas: stellar motions would generate a luminosity exceeding L_E, which would drive the surplus gas away until the situation stabilized. (Their argument is essentially equivalent to our discussion in Section 3 where we show that stellar collisions themselves generate a luminosity exceeding L_E when v_c exceeds a value which increases slowly with cluster mass.)

We now consider whether the presence of gas can catalyse the contraction of a dense cluster of neutron stars and/or black holes. Suppose that a gas mass of $M_g = \xi N_c M_*$ lies within the cluster. Then each object provides an input of mechanical energy of

$$\sim \rho_{gas} v_c^3 r_{ac}^2 \tag{4.1}$$

where $r_{ac} \simeq GM_*/v_c^2$ is the accretion radius for a typical stellar object. The resulting luminosity radiated by the gas is

$$L_{gas} \simeq N_c^{1/2} (GM_*)^{3/2} M_g R_c^{-5/2} \simeq 2 \times 10^{39}\, \xi m_*^{5/2} N_{c8}^{3/2} R_{c18}^{-5/2} \text{ erg/s.} \tag{4.2}$$

This energy L_{gas} is radiated at the expense of the cluster's binding energy. Note that accretion on to the individual compact objects (yielding ηc^2 per unit mass) could yield $\sim \eta (v_c/c)^{-2}$ times as much power. There might also be additional energy from pulsar-type activity (*cf.* Arons *et al.* 1974).

Defining t_{gas} as $(N_c M_* v_c^2 / L_{gas})$ we conclude that $t_{gas} \sim t_R/\xi$, so that the gas cannot dissipate enough energy to modify the ordinary stellar-dynamical evolution of the cluster unless $\xi \gtrsim 1$. In this case (when we essentially already have a supermassive object, rather than a star cluster, as the primary ingredient) the stellar system would be able to contract towards the centre, perhaps providing enough energy to maintain the gas cloud at its Eddington limit, until a relativistic instability ensued. If the individual objects were stellar-mass black holes rather than neutron stars, they would be able to swallow the whole material of the gas cloud on a time-scale similar to that of the overall contraction (Begelman 1978a, b).

(We note parenthetically that the accumulation of a large amount of gas can give rise to new phenomena even in a cluster composed of *normal* stars:

(1) Ablation of material from the surface of normal stars would be important if $\rho_{gas} v_c^3 > \sigma T_*^4$. As pointed out by Unno (1971), evaporation may even occur at lower gas densities, if the gas is hotter than the escape temperature from stellar surfaces, and conduction can occur.

(2) The intense radiation field trapped within the whole gas cloud would modify the

surface boundary condition for each star. The radiation energy density would be

$$E_{\rm rad} \simeq \frac{(L_{\rm gas}+L_*)}{4\pi c R_{\rm c}^2}\,\tau_{\rm es},$$

where

$$\tau_{\rm es} \simeq 10^3\,(M_{\rm g}/10^8\,M_\odot)R_{\rm c18}^{-2}$$

is the optical depth of the cloud to electron scattering. If $E_{\rm rad}$ approaches aT_*^4 the stellar surface boundary condition is modified in a fashion that may trigger mass loss.)

4.3 CLUSTERS OF MAGNETIZED NEUTRON STARS (OR WHITE DWARFS)

If the compact objects are neutron stars with surface magnetic fields of the strengths inferred in pulsars and compact X-ray sources, the cross-section for interaction of the neutron stars with surrounding gas is $\sigma \simeq (r_{\rm mag}^2 + r_{\rm ac}^2)$, where $r_{\rm mag}$ is given by

$$(1 + r_{\rm ac}/r_{\rm mag})^{5/2}\rho_{\rm g}v_{\rm c}^2 \simeq B^2(r_{\rm mag})/4\pi. \tag{4.3}$$

We assume fluid-like behaviour of the gas. For a dipole magnetic field of surface strength B_0, we have

$$B^2(r_{\rm mag}) \simeq B_0^2(r_*/r_{\rm mag})^6 \tag{4.4}$$

where $r_* \simeq 10$ km for a typical neutron star.

The effects of the magnetosphere are interesting when

$$r_{\rm mag} > r_{\rm ac} > r_* \tag{4.5}$$

for it is then possible for the stars to transfer energy to the gas on a time-scale $< t_{\rm R}$ even if $\xi < 1$.

The 'ram-pressure luminosity' of each neutron star, resulting from 'sweeping' by its magnetosphere, is

$$L_{\rm mag\ ram} \simeq \rho_{\rm g}\sigma_{\rm mag}v_{\rm c}^3 \simeq 10^{32}(n_{\rm g}/10^{10}\,{\rm cm}^{-3})^{2/3}\left(\frac{v_{\rm c}}{10^9\,{\rm cm/s}}\right)^{7/3}\left(\frac{r_*}{10\,{\rm km}}\right)\left(\frac{B_0}{10^{12}\,{\rm G}}\right)^{2/3}\ {\rm erg/s}. \tag{4.6}$$

Note that, since $\sigma_{\rm ac} \propto v_{\rm c}^4$, the accretion luminosity decreases as $v_{\rm c}$ rises, whereas $L_{\rm mag\ ram}$ *increases.*

The effects of magnetic ram pressure dominate the cluster evolution if $\xi > \sigma_{\rm ac}/\sigma_{\rm mag}$, and a necessary condition for this is

$$v_{\rm c} > 2.5\times 10^8\left(\frac{n_{\rm gas}}{10^{10}\,{\rm cm}^{-3}}\right)^{1/10}\left(\frac{r_*}{10\,{\rm km}}\right)^{-3/5}\left(\frac{B_0}{10^{12}\,{\rm G}}\right)^{-1/5}\ {\rm cm/s}. \tag{4.7}$$

When (4.7) is satisfied, the ram pressure luminosity increases the binding energy of the cluster on a time-scale $t_{\rm ram} \simeq M_* v_{\rm c}^2/L_{\rm mag\ ram}$. The cluster will then become progressively more tightly bound, evolving at constant mass (i.e. bypassing (2.1)) until the post-Newtonian instability causes the collapse of the cluster, or $L_{\rm ram}$ rises to $L_{\rm E}$, at which stage the gas is ejected by radiation pressure. For neutron stars, $r_* \simeq 10^6$ cm and $B_0 \simeq 10^{12}$ G, these effects turn out to be unimportant if $\xi \lesssim 1$ unless the cluster contains $\gtrsim 10^{10}$ stars at the conclusion of the coalescence phase. However, white dwarfs have been observed with surface magnetic fields of up to 10^8 G, and hence magnetic moments up to 10^5 times greater than those of neutron stars. The strongly magnetic white dwarfs in the cluster would spiral to the centre in a time $< t_R$ if

$$\xi > 3.5 \times 10^{-3} (N_c/10^8)^{-1} (r_*/10^9 \,\mathrm{cm})^{-3} (B_0/10^8 \,\mathrm{G})^{-1}, \tag{4.8}$$

and would probably coalesce to form a black hole. If such objects constituted the bulk of the cluster, (4.8) would be the condition for evolution at constant mass. Since white dwarfs are produced by stars undergoing less than explosive mass loss, it is probable that the amount of gas retained in the cluster of white dwarfs would be quite large, with ξ possibly exceeding unity. Alternatively, this gas might be reprocessed into main sequence stars, as suggested by Sanders (1970), in which case the analysis of Sections 2 and 3 applies in the post-coalescence phase.

5 Conclusions

Our aim in this paper has been to investigate the fates of dense clusters containing $\sim 10^8$ stars. In Section 3, we made the same assumptions as Spitzer and his collaborators, and followed the evolution beyond the stage when a self-gravitating subsystem of 'new stars' first forms (comprising a fraction of the total cluster mass of order ϵ, the square root of the cluster ellipticity). The subsystem undergoes a runaway contraction, collisional disruption being balanced by continuing star formation. But the stellar subsystem irreversibly dissolves into an amorphous cloud when it has contracted to a radius given by (3.6): at this stage, the luminosity resulting from disruptive stellar collisions generates so much radiation pressure that further star formation is terminated. The luminosity attains the Eddington limit for the subsystem, but this bright phase is of short duration. For a cluster of $10^8 N_8$ solar-type stars, the maximum fraction of rest-mass energy released before a massive object forms is only

$$\sim 3 \times 10^{-4} \epsilon^{13/9} N_8^{4/7}. \tag{5.1}$$

If star formation were inhibited at a radius larger than (3.6), the energy release would be correspondingly less. Runaway coalescence at the stage when v_c is comparable with the stellar escape velocity v_{esc} could create a massive object at an even earlier stage in the cluster's evolution.

If the cluster contains compact stellar mass objects (perhaps the result of multiple supernovae), then physical collisions among them never become important. A system composed solely of such bodies would evolve according to (2.1), (i.e. at constant binding energy) until a small fraction of the original mass developed into a relativistically bound core. Gravitational or magnetic interactions with gas pervading the system can in principle enable the cluster binding energy ($\propto N^2/R$) to increase (see Fig. 1), but this process is significant only in somewhat contrived conditions.

Even if dense star clusters are the precursors of active galactic nuclei, our discussion suggests that the conspicuous stages of this activity occur *after* a massive object has formed. Once this has happened, a wide variety of consequences may ensue, some of which are depicted in Fig. 1. Different manifestations of nuclear activity may correspond to different evolutionary stages (or even to quite different tracks in the flow diagram). There is no reason why all phenomena should involve the same mechanisms; all possibilities need further theoretical investigation.

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