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Henrik Hasseltoft

Institutions: Stockholm School of Economics

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THE “FED-MODEL” AND THE CHANGING CORRELATION OF STOCK AND BOND RETURNS: AN EQUILIBRIUM APPROACH

Henrik Hasseltoft*

January 2009

Abstract

This paper presents an equilibrium model that provides a rational explanation for two features of data that have been considered puzzling: The positive relation between US dividend yields and nominal interest rates, often called the Fed-model, and the time-varying correlation of US stock and bond returns. Key ingredients are time-varying first and second moments of consumption growth, inflation, and dividend growth in conjunction with Epstein-Zin and Weil recursive preferences. Historically in the US, inflation has signalled low future consumption growth. The representative agent therefore dislikes positive inflation shocks and demands a positive risk premium for holding assets that are poor inflation hedges, such as equity and nominal bonds. As a result, risk premiums on equity and nominal bonds comove positively through their exposure to macroeconomic volatility. This generates a positive correlation between dividend yields and nominal yields and between stock and bond returns. High levels of macro volatility in the late 1970s and early 1980s caused stock and bond returns to comove strongly. The subsequent moderation in aggregate economic risk has brought correlations lower. The model is able to produce correlations that can switch sign by including the covariances between consumption growth, inflation, and dividend growth as state variables.

COMMENTS WELCOME

*Stockholm School of Economics and The Institute for Financial Research. Correspondence to the author, The Institute for Financial Research, Drottninggatan 89, SE-113 60 Stockholm, Sweden. E-mail: henrik.hasseltoft@hhs.se. Tel:+46-8-7285127, Fax:+46-8-7285130. I would like to thank Magnus Dahlquist, Peter Schotman, Romeo Tedongap, and seminar participants at the Stockholm School of Economics and The Institute for Financial Research for helpful comments. Financial support from Bankforskninginstitutet is gratefully acknowledged. All errors are mine.

1 Introduction

The correlation between US stock and bond returns has varied substantially over time, reaching highly positive levels in the late 1970s and early 1980s while turning negative in the late 1990s. Several statistical models have been put forward to model the time variation but little work has been done on explaining the phenomenon within an equilibrium model. A second feature of data that has been considered puzzling is the highly positive correlation between US dividend yields and nominal interest rates, a relation often referred to as the Fed-model.¹ From the Gordon growth formula, dividend yields are given by the real discount rate on equity minus the real dividend growth rate. Since changes in expected inflation and bond risk premiums have been the main source of variation in nominal yields (e.g. Campbell and Ammer, 1993, and Best et al., 1998), the positive correlation observed in data implies that one or both of these components must either be positively associated with real discount rates or negatively associated with real dividend growth rates.² However in the literature, it has been considered implausible for inflation to have rational effects on any of these two real components of dividend yields. Instead, a behavioral explanation in the form of inflation illusion (e.g., Modigliani and Cohn, 1979, and Campbell and Vuolteenaho, 2004) has been put forward.

I propose a representative agent asset pricing model that provides a rational explanation for these two features of data. Key ingredients of the model are exogenous consumption growth, inflation, and dividend growth in conjunction with Epstein and Zin (1989) and Weil (1989) recursive preferences. In post 1952 US data, inflation has signalled low future consumption growth. The representative agent therefore dislikes positive inflation shocks. As a result, the agent demands a positive risk premium for holding assets that are poor hedges against inflation, for example equity and nominal bonds. This mechanism causes risk premiums on equity and nominal bonds to comove positively through their common exposure to macroeconomic volatility. Dividend yields and nominal yields therefore become positively associated, allowing the model to match correlations

¹The Fed-model is not endorsed by the Federal Reserve. The name comes from Ed Yardeni at Prudential Securities in the mid 1990s following research reports at the Federal Reserve describing the relation.

²While the Gordon growth formula also can be written in nominal terms, changes in expected inflation is expected to have offsetting effects on the nominal parts of the discount rate and the dividend growth rate. This leaves the real components to explain the correlation.

observed in data. The comovement of risk premiums is consistent with evidence in for example Fama and French (1989) and Campbell and Ammer (1993). In particular, the model suggests that inflation volatility plays a key role for determining risk premiums on both stocks and bonds. Similarly, stock and bond returns move together through common changes in risk premiums. Changes in macroeconomic risk, measured as time-varying second moments of consumption growth, inflation, and dividend growth, are shown to account for a large part of changes in realized correlations between stock and bond returns. The high correlations seen in the late 1970s and early 1980s are attributed to high volatility of inflation and consumption growth. The subsequent drop in correlations is explained by lower macroeconomic volatility. During the late 1990s, correlations turned sharply negative. The model is able to produce correlations that can switch sign by including the covariances between consumption growth, inflation, and dividend growth as state variables. The negative correlations are partly captured by the model as a result of low economic volatility together with a positive covariance between dividend growth and inflation. A series of small positive shocks to inflation generated negative bond returns while positive cash flow shocks raised stock prices through both higher dividend growth and a lower equity risk premium. The discount rate effect on equity arises since positive cash flow shocks that occur during times of positive inflation shocks (bad times) imply that equity is a hedge against periods of high marginal utility. While the model performs well in capturing low-frequency movements in realized correlations, it is more challenging for the model to predict quarter-to-quarter changes in correlations. Model-implied conditional correlations predict realized quarterly correlations significantly and with an R^2 of 13%. Including the lagged realized correlation as explanatory variable does not drive out the significance of the model forecast.

The dynamics of the model are presented in two specifications. In the first specification, I build on Piazzesi and Schneider (2006) and model consumption growth and inflation as a VARMA(1,1) process written in state-space form with all shocks in the economy being homoscedastic. I extend their setup by also modeling dividend growth and introducing equity in to the model. I also allow for the elasticity of intertemporal substitution (EIS) to be different from one and I provide

approximate analytical solutions to the model.³ The codependence of consumption growth and inflation is important for the results of the paper as it makes real asset prices and the price-dividend ratio functions of expected inflation. This generates a direct channel through which the Fed-model is explained. Both expected and unexpected inflation are estimated to have a negative effect on future consumption growth which therefore leads to positive risk premiums on risky assets such as equity and nominal bonds due to inflation shocks. When the EIS is above one, model-implied price-dividend ratios are negatively related to expected inflation. This is supported by empirical evidence provided in the paper.⁴ The specification also draws on the so called long-run risk model of Bansal and Yaron (2004) in which expected real consumption growth contains a small persistent component. However, their model does not consider inflation and its effect on real consumption growth, the real pricing kernel, and therefore on risk premiums in the economy. I show that the real effects of inflation are important to consider and therefore draws an important distinction between the two models. Dividend growth is also modeled differently than in Bansal and Yaron (2004). Rather than assuming expected dividend growth to be a function of expected consumption growth, I allow expected dividend growth to be a separate state variable for price-dividend ratios. This is shown to be important for explaining the two features in data as it drives a wedge between stock and bond prices. Together with expected dividend growth, the conditional means of consumption growth and inflation serve as state variables in the economy. In the second specification, I extend the first by introducing heteroscedasticity. The volatility of shocks to the three macro variables and their covariances are allowed to change over time, which enables me to compute a time-varying conditional correlation between stock and bond returns. In addition to the conditional first moments, the conditional variance of consumption growth, inflation, and dividend growth, and their covariances serve as state variables in the second specification.

The main contributions of the paper are twofold. First, I show that the so called Fed-model

³Bansal et al. (2007a) and Attanasio and Vissing-Jorgensen (2003) estimate the EIS to be in excess of one while Hall (1988) and Campbell (1999) document that it is close to zero. Lustig et al. (2008) provide evidence that the wealth-consumption ratio varies over time, indicating that the EIS is not equal to one.

⁴It is well known in the literature that the so called long-run risk model of Bansal and Yaron (2004) needs an EIS in excess of one to capture the negative relation between consumption volatility and price-dividend ratios, and the positive relation between expected consumption growth and price-dividend ratios. The model in this paper needs an EIS above one for a different reason, namely to capture the negative relation between price-dividend ratios and expected inflation.

can be explained within a rational consumption-based equilibrium model. This stands in contrast to the hitherto dominant explanation in the form of inflation illusion. Second, I show that the changing correlation between stock and bond returns to a large extent can be explained using the same simple equilibrium model. To the best of my knowledge, this is the first equilibrium model to provide a rational explanation of the Fed-model and the first consumption-based equilibrium model that is able to account for a large part of changes in realized correlations between stock and bond returns.

This paper builds on the literature of pricing stocks and bonds in equilibrium. Early contributions include Cox et al. (1985), Mehra and Prescott (1985), Campbell (1986), and Dunn and Singleton (1986). The recursive preferences of Epstein and Zin (1989) and Weil (1989) have been used extensively in the asset-pricing literature (e.g., Campbell, 1993, 1996, 1999, Duffie et al., 1997, and Restoy and Weil, 1998). Bansal and Yaron (2004) show that recursive preferences in conjunction with a time-varying first and second moment of consumption growth can explain the level of the equity risk premium and its variation over time. Hasseltoft (2008) extends the long-run risk model to the term structure of interest rates and shows that a calibrated version of the model is capable of explaining deviations from the expectations hypothesis, the upward sloping nominal yield curve, and the predictive power of the yield curve. It is also shown that the cyclicity of nominal interest rates and yield spreads depend on the relative values of the EIS and the correlation between real consumption growth and inflation. Bansal and Shaliastovich (2008) explain violations of both the expectation hypothesis in bond markets and the uncovered interest rate parity in currency markets using the long-run risk model. Piazzesi and Schneider (2006) make use of recursive preferences and show that the nominal yield curve slopes up if inflation is bad news for future consumption growth. They also explore the role of learning about macroeconomic fundamentals.

The positive correlation of US dividend yields and nominal interest rates, 0.30 for the period 1952-2007 and 0.74 for the period 1965-2007, is commonly known as the Fed-model. It is widely used among practitioners as a tool for comparing the relative valuations of the stock and bond markets and has received academic interest as rational explanations have been considered implausible. From the Gordon growth model, the dividend yield is given by the real discount rate on equity minus the

real dividend growth rate. Similarly, variations in nominal yields can be decomposed into changes in real interest rates, inflation, and future excess returns. Evidence suggest that the latter two components are the dominant factors (e.g. Campbell and Ammer, 1993, and Best et al., 1998). For the positive correlation in data to arise, changes in inflation and bond risk premiums must be negatively associated with real dividend growth rates and/or positively associated with real discount rates. If the Gordon growth formula is written in nominal terms, changes in expected inflation is expected to have offsetting effects on the nominal parts of dividend growth rates and discount rates which leaves the real components to explain the high correlation. Surprisingly, none of these explanations have found empirical support in the literature until recently. Instead an explanation in the form of inflation illusion, originally put forward by Modigliani and Cohn (1979), has found empirical support (e.g., Ritter and Warr, 2002, Asness, 2003, Campbell and Vuolteenaho, 2004, and Cohen et al., 2005). This explanation suggests that investors are irrational and fail to properly adjust the expected nominal dividend growth rate with changes in expected inflation but fully adjust the nominal discount rate. Alternatively, one can view it as investors are discounting real cash flows with nominal interest rates. This implies that stocks are undervalued in periods of high inflation from the viewpoint of a rational investor. In a recent empirical paper, Bekaert and Engstrom (2008) argue that rational mechanisms are at work and ascribe the high correlation to the large incidence of stagflation in US data. They show that the correlation between equity yields and bond yields is mainly driven by a correlation between expected inflation and the equity risk premium as periods of high expected inflation are associated with periods of high risk aversion and high economic uncertainty. Hence, it seems that any rational equilibrium model that would like to explain the Fed-model must contain a link between inflation and the equity risk premium. The model in this paper contains exactly that.

The relation between stock and bond returns has received great academic interest and is of central importance for asset allocation decisions. Early contributions focus on the unconditional correlation. Shiller and Beltratti (1992) fail to match the observed comovement using a present-value model. Campbell and Ammer (1993) decompose the variance of stock and bond returns and find offsetting effects from changes in real interest rates, excess returns, and expected inflation.

Barsky (1989) explore the role of changes in risk and productivity growth for the behavior of stock and bond returns within a general equilibrium framework. Recently, the focus has shifted towards understanding the conditional correlation of stock and bond returns which displays large time variation. Several statistical models have been put forward to shed light on the comovement. Scruggs and Glabadanidis (2003) find that models that impose a constant correlation restriction on the covariance matrix between stock and bond returns are strongly rejected. Connolly et al. (2005) document a negative relation between stock market uncertainty and the future correlation of stock and bond returns. Baele et al. (2007) attempt to explain the time-varying correlation using macro factors but conclude that their factors fit the reality poorly and argue that liquidity factors help explain the time variation. Campbell et al. (2008) specify an exogenous process for the real stochastic discount factor and estimate a term structure model using inflation and asset price data. One of their state variables is the covariance between inflation and the real economy which they link to the changing covariance of stock and bond returns and changes in bond risk premia. Nominal bonds are expected to be a poor hedge against economic fluctuations when inflation is countercyclical since it implies pro-cyclical bond returns. Investors therefore demand a positive risk premium to hold them. David and Veronesi (2008) explore the role of learning about inflation and real earnings for the second moments of stock and bond returns. They estimate their model using asset price data as well and forecast the covariance of stock and bond returns quite accurately. In contrast to these papers, I build on the traditional literature on consumption-based asset pricing and address the issue of changing correlations using a consumption-based equilibrium model. This implies a real stochastic discount factor that is based on consumption growth. Furthermore, only macro data is used to estimate the model which provides a test of how much fundamental macro variables can account for features of asset prices.

This paper is also related to the literature on whether stocks provide a hedge or not against inflation. Several articles have established a negative relation between inflation and stock returns. Fama and Schwert (1977) document that common stock returns are inversely related to expected inflation. Fama (1981) argues that inflation proxies for real activity. He argues that higher expected output increases both stock prices and the demand for real money. If the latter is not accommodated

with a similar change in money supply, the quantity theory of money suggests a drop in the price level. Hence, a negative relation between stock returns and inflation. Kaul (1987) stresses the combination of money demand and counter-cyclical money supply effects for the stock return-inflation relation. Boudoukh et al. (1994) explore the cross-sectional relation between stock returns and inflation and find that non-cyclical industries tend to covary positively with inflation while the reverse holds for cyclical industries. Pilotte (2003) documents that dividend yields and capital gains are differently related to expected inflation.

2 The Model

This section describes the dynamics of consumption growth, inflation, and dividend growth, which are presented in two specifications. Specification I models the first moments, assuming all macro shocks have constant second moments. Specification II allows for heteroscedasticity and models the second moments. Section 2.2 describes the preferences of the representative agent. Section 2.3 solves the model and provides solutions for equity and bond prices.

2.1 Dynamics

2.1.1 Specification I: Homoscedasticity

Consumption growth, inflation, and dividend growth are modeled in state-space form. Let $\mathbf{z}_{t+1} = [\Delta c_{t+1}, \pi_{t+1}, \Delta d_{t+1}]'$ denote the logarithmic consumption growth, inflation, and dividend growth and let $\mathbf{x}_t = [x_c, x_\pi, x_d]'$ represent the time-varying part of the conditional means. The components

of \mathbf{x}_t serve as state variables in the economy. The following dynamics are assumed:

$$\mathbf{z}_{t+1} = \boldsymbol{\mu} + \mathbf{x}_t + \boldsymbol{\eta}_{t+1}, \quad (1)$$

$$\mathbf{x}_{t+1} = \boldsymbol{\beta}\mathbf{x}_t + \boldsymbol{\delta}\boldsymbol{\eta}_{t+1}, \quad (2)$$

$$\boldsymbol{\eta}_{t+1} \sim N.i.i.d. (0, \boldsymbol{\Omega}),$$

$$\boldsymbol{\mu} = [\mu_c, \mu_\pi, \mu_d]',$$

$$\boldsymbol{\eta} = [\eta_c, \eta_\pi, \eta_d]',$$

$$\boldsymbol{\Omega} = \begin{pmatrix} \sigma_c^2 & \sigma_{c\pi} & \sigma_{cd} \\ \sigma_{c\pi} & \sigma_\pi^2 & \sigma_{\pi d} \\ \sigma_{cd} & \sigma_{\pi d} & \sigma_d^2 \end{pmatrix}.$$

All shocks in the economy are assumed to be homoscedastic, subject to the variance-covariance matrix $\boldsymbol{\Omega}$. Shocks to inflation and consumption growth are correlated through $\sigma_{c\pi}$, shocks to consumption and dividend growth through σ_{cd} , and shocks to dividend growth and inflation through $\sigma_{\pi d}$. The persistence of shocks and their effect on the conditional means are governed by $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_1 & \beta_2 & 0 \\ \beta_3 & \beta_4 & 0 \\ \beta_5 & 0 & \beta_6 \end{pmatrix},$$

$$\boldsymbol{\delta} = \begin{pmatrix} \delta_1 & \delta_2 & 0 \\ \delta_3 & \delta_4 & 0 \\ \delta_5 & 0 & \delta_6 \end{pmatrix}.$$

The state-space system can be written as a VARMA(1,1):

$$\mathbf{z}_{t+1} - \boldsymbol{\mu} = \boldsymbol{\beta}(\mathbf{z}_t - \boldsymbol{\mu}) + \boldsymbol{\eta}_{t+1} + \boldsymbol{\alpha}\boldsymbol{\eta}_t, \quad (3)$$

where $\boldsymbol{\alpha}$ is a 3-by-3 matrix. Taking the conditional mean of (3) and letting $\mathbf{x}_t = \boldsymbol{\beta}(\mathbf{z}_t - \boldsymbol{\mu}) + \boldsymbol{\alpha}\boldsymbol{\eta}_t$ denote the conditional mean yields the state-space system above where $\boldsymbol{\delta} = \boldsymbol{\beta} + \boldsymbol{\alpha}$.

Consider first the relation between expected consumption growth and inflation, which follows Piazzesi and Schneider (2006). The two conditional means are interdependent through β_2 and β_3 . This allows real asset prices and valuation ratios, for example the price-dividend ratio, to be a function of expected inflation since the conditional mean of $x_{c,t+1}$ depends on $x_{\pi,t}$. So if one conjectures that asset prices are a function of expected consumption growth, it implies they also are a function of expected inflation. Hence, $x_{c,t}$ and $x_{\pi,t}$ both serve as state variables. This implies a direct link between expected inflation and the real pricing kernel which therefore have implications for risk premiums in the economy. This way of capturing the real effects of inflation is important for explaining the Fed-model and the changing correlation of stock and bond returns. Specification I nests dynamics used in other consumption-based equilibrium models. For example, Wachter (2006) models consumption growth and inflation as two separate ARMA(1,1) processes with correlated shocks. This translates into setting $\beta_2 = \beta_3 = \delta_2 = \delta_3 = 0$. Bansal and Yaron (2004) assume that shocks to realized and expected consumption growth are different and uncorrelated. However, the specification above nests their dynamics if one set the two shocks equal, leave out realized inflation completely, and then set $\beta_2 = \beta_3 = \beta_4 = \delta_2 = \delta_3 = \delta_4 = 0$. Hasseltoft (2008) models expected inflation as an AR(1) process with lagged consumption shocks but keeping expected consumption growth only a function of its own lag and shocks. This implies setting $\beta_2 = \beta_3 = \delta_2 = 0$.

The realized dividend growth rate is modeled analogously to consumption growth and inflation, with an unconditional mean, μ_d , and a time-varying part, x_d .⁵ This specification is different from the common approach in consumption-based models of modeling dividend growth rates as a function of expected consumption growth times a leverage parameter (e.g., Abel, 1999, Bansal and Yaron, 2004). Modeling dividend growth in such a way has the less desirable property of producing a correlation equal to one between expected consumption growth and expected dividend growth. The specification used in this paper breaks that link which makes the expected dividend growth a state variable for price-dividend ratios. This drives a wedge between price-dividend ratios and nominal interest rates in the model as it captures cash flow shocks affecting equity. This allows the model to produce a realistic correlation between price-dividend ratios and nominal interest rates.

⁵Consumption and dividends are not cointegrated. Considering the long-run relation between consumption and dividends is potentially important, as for example argued in Bansal et al. (2007b).

It also improves the ability of the model to match the observed correlation between stock and bond returns as it contributes to a low or even negative correlation between asset returns as discussed in Section 6.

Expected dividend growth rates are allowed to depend on lagged expected consumption growth through β_5 . The effect of expected dividend growth on future consumption growth and inflation, entries (1,3) and (2,3) of β , are both set to zero. Relaxing the restriction would make expected dividend growth rates a state variable for bond prices. The economic rationale for why changes in cash flow growth rates should affect prices of Treasury bonds is not clear, and I therefore choose to restrict the dynamics. The effect of expected inflation on future expected dividend growth rates, entry (3,2) of β , is set to zero. This is done for two reasons. First, it emphasizes the effect inflation has on risk premiums through its relation with future consumption growth. That is, the emphasis is on one of the two rational channels that can generate a positive correlation between dividend yields and nominal interest rates. Second, the unconditional correlation between dividend growth and inflation is close to zero in data, indicating that the second possible rational channel of explaining the Fed-model is less important. Empirical evidence provided in Bekaert and Engstrom (2008) indicate that the relation between expected inflation and the cash flow component of dividend yields plays a minor role.⁶ Similarly, the matrix δ is subject to the same restrictions.

2.1.2 Specification II: Heteroscedasticity

Empirical evidence suggests that inflation (e.g., Engle, 1982, and Bollerslev, 1986), consumption growth (e.g., Kandel and Stambaugh 1990, and Bansal et al., 2005), and dividend growth (e.g., Bansal and Yaron, 2000) are all subject to heteroscedasticity. The volatilities of shocks to the economy are therefore modeled as time varying, incorporating a notion of changing economic uncertainty. The conditional second moments of the shocks are later used to compute the conditional correlation between stock and bond returns.

The dynamics for the first moments are kept the same as above, but the conditional variance-

⁶I have estimated the system allowing for an interaction between expected inflation and future expected dividend growth rates. The coefficient turns out to be negative but is not statistically different from zero at usual significance levels.

covariance matrix of the shocks are allowed to change over time. The variances and the covariances are modeled as separate autoregressive processes subject to random shocks that are assumed to be normally distributed and uncorrelated. Assume the macro shocks $\boldsymbol{\eta}_{t+1}$ to be normally distributed with mean zero and subject to the conditional variance-covariance matrix $\boldsymbol{\Omega}_t$, which takes the form:

$$\boldsymbol{\Omega}_t = \begin{pmatrix} \sigma_{c,t}^2 & \sigma_{c\pi,t} & \sigma_{cd,t} \\ \sigma_{c\pi,t} & \sigma_{\pi,t}^2 & \sigma_{\pi d,t} \\ \sigma_{cd,t} & \sigma_{\pi d,t} & \sigma_{d,t}^2 \end{pmatrix},$$

and let the variances and covariances follow:

$$\sigma_{c,t+1}^2 = \alpha_c + \phi_c(\sigma_{c,t}^2 - \alpha_c) + \tau_c \epsilon_{c,t+1}, \quad (4)$$

$$\sigma_{\pi,t+1}^2 = \alpha_\pi + \phi_\pi(\sigma_{\pi,t}^2 - \alpha_\pi) + \tau_\pi \epsilon_{\pi,t+1}, \quad (5)$$

$$\sigma_{d,t+1}^2 = \alpha_d + \phi_d(\sigma_{d,t}^2 - \alpha_d) + \tau_d \epsilon_{d,t+1}, \quad (6)$$

$$\sigma_{c\pi,t+1} = \alpha_{c\pi} + \phi_{c\pi}(\sigma_{c\pi,t} - \alpha_{c\pi}) + \tau_{c\pi} \epsilon_{c\pi,t+1}, \quad (7)$$

$$\sigma_{cd,t+1} = \alpha_{cd} + \phi_{cd}(\sigma_{cd,t} - \alpha_{cd}) + \tau_{cd} \epsilon_{cd,t+1}, \quad (8)$$

$$\sigma_{\pi d,t+1} = \alpha_{\pi d} + \phi_{\pi d}(\sigma_{\pi d,t} - \alpha_{\pi d}) + \tau_{\pi d} \epsilon_{\pi d,t+1}, \quad (9)$$

where α denotes the unconditional mean of each process, the ϕ parameters govern the persistence of shocks to the second moments, and the τ parameters determine the volatility of the second moments. These dynamics are extensions of Bansal and Yaron (2004) who model consumption volatility as in (4). The assumption of conditionally normal second moments is made out of convenience since it allows for closed-form affine solutions of asset prices. These dynamics are therefore assumed for modeling purposes. When estimating the second moments in Section 3, I use the diagonal VEC-model of Bollerslev et al. (1988). The estimated parameters are then mapped into the dynamics above.

2.2 Investor Preferences

The representative agent in the economy has Epstein and Zin (1989) and Weil (1989) recursive preferences:

$$U_t = \left\{ (1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \quad (10)$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, $\gamma \geq 0$ denotes the risk aversion coefficient and $\psi \geq 0$ the elasticity of intertemporal substitution (EIS). The discount factor is represented by δ . This preference specification allows time preferences to be separated from risk preferences. This stands in contrast to time-separable expected utility in which the desire to smooth consumption over states and over time are interlinked. The agent prefers early (late) resolution of risk when the risk aversion is larger (smaller) than the reciprocal of the EIS. A preference for early resolution and an EIS above one imply that $\theta < 1$. This specification nests the time-separable power utility model for $\gamma = \frac{1}{\psi}$ (i.e., $\theta = 1$).

The agent is subject to the following budget constraint:

$$W_{t+1} = R_{c,t+1} (W_t - C_t), \quad (11)$$

where the agent's total wealth is denoted W_t , $W_t - C_t$ is the amount of wealth invested in asset markets and $R_{c,t+1}$ denotes the gross return on the agents total wealth portfolio. This asset delivers aggregate consumption as its dividends each period. Epstein and Zin (1989) show that this economy implies an Euler equation for asset return $R_{i,t+1}$ in the form of:

$$E_t \left[\underbrace{\delta^\theta G_{t+1}^{\frac{-\theta}{\psi}} R_{c,t+1}^{-(1-\theta)}}_{M_{t+1}} R_{i,t+1} \right] = 1, \quad (12)$$

where G_{t+1} denotes the aggregate gross growth rate of consumption and M_{t+1} denotes the intertemporal marginal rate of substitution (IMRS). The logarithm of the IMRS can be written as:

$$m_{t+1} = \theta \ln(\delta) - \frac{\theta}{\psi} \Delta c_{t+1} - (1 - \theta) r_{c,t+1}, \quad (13)$$

where $\ln R_{c,t+1} = r_{c,t+1}$ and $\ln G_{t+1} = \Delta c_{t+1}$. Note that the IMRS depends on both consumption

growth and on the return from the total wealth portfolio. Recall that $\theta = 1$ under power utility, which brings us back to the standard time-separable IMRS.

2.3 Solving the model

This section solves the model for the case of constant and time-varying second moments of the macro shocks. The solutions are presented in Sections 2.3.1 and 2.3.2. Common to the two cases are the returns on the aggregate wealth portfolio and the market portfolio, which are approximated using the analytical solutions found in Campbell and Shiller (1988):

$$r_{c,t+1} = k_{c,0} + k_{c,1}pc_{t+1} - pc_t + \Delta c_{t+1}, \quad (14)$$

$$r_{m,t+1} = k_{d,0} + k_{d,1}pd_{t+1} - pd_t + \Delta d_{t+1}, \quad (15)$$

where pc_t and pd_t denote the log price-consumption ratio and the log price-dividend ratio, and the constants k_c and k_d are functions of the average level of pc_t and pd_t , denoted \bar{pc} and \bar{pd} .⁷ Specifically, the constants are:

$$k_{c,1} = \frac{\exp(\bar{pc})}{1 + \exp(\bar{pc})}, \quad (16)$$

$$k_{c,0} = \ln(1 + \exp(\bar{pc})) - k_{c,1}\bar{pc}, \quad (17)$$

and similarly for the k_d coefficients.

2.3.1 Specification I: Homoscedasticity

All asset prices and valuation ratios are conjectured to be functions of the time-varying conditional means of the three macro variables. Starting with the log price-consumption ratio, it is conjectured to be a linear function of expected consumption growth, x_c , expected inflation, x_π , and expected dividend growth, x_d :

$$pc_t = A_{c,0} + A_{c,1}x_{c,t} + A_{c,2}x_{\pi,t} + A_{c,3}x_{d,t}. \quad (18)$$

⁷Bansal et al. (2007a) show that the approximate analytical solutions for the returns are close to the numerical solutions and deliver similar model implications.

Using the standard Euler equation together with the dynamics of consumption growth and inflation one can solve for the coefficients. Appendix A.1 explains how to solve for the A-coefficients and reports the expression for $A_{c,0}$. The remaining coefficients are given by:

$$\begin{aligned} A_{c,1} &= \frac{1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - \frac{1}{\psi}}{1 - k_{c,1}\beta_1}, \\ A_{c,2} &= \frac{k_{c,1}\beta_2(1 - \frac{1}{\psi})}{(1 - k_{c,1}\beta_4)(1 - k_{c,1}\beta_1) - k_{c,1}^2\beta_2\beta_3}, \\ A_{c,3} &= 0. \end{aligned}$$

First note that $A_{c,3}$ equals zero as a result of the restrictions imposed on the dynamics, which were discussed in Section 2.1.1. Coefficient, $A_{c,1}$, measures the response of the price-consumption ratio to changes in expected consumption growth. The denominator is positive given that $\beta_1 < 1$ so its sign depends on whether expected consumption growth implies good or bad news for future inflation, β_3 , on whether the price-consumption ratio responds positively or negatively to inflation expectations, $A_{c,2}$, and on whether the EIS, ψ , is above or below one. Setting β_3 and/or $A_{c,2}$ equal to zero gives the expression for $A_{c,1}$ found in Bansal and Yaron (2004). However, the influence of inflation on real variables is key to the paper and represents an important distinction from the long-run risk model. The denominator of $A_{c,2}$ is positive for plausible parameter values so its sign depends on β_2 and the EIS. For example, high inflation expectations will depress the price-consumption ratio ($A_{c,2} < 0$) when high inflation signals low future consumption growth, $\beta_2 < 0$, and the EIS is above one. An EIS in excess of one implies that the intertemporal substitution effect dominates the wealth effect. As high inflation signals low future returns, agents sell risky assets which leads to lower valuation ratios. In the case of expected utility ($\frac{1}{\psi} = \gamma$), a risk aversion coefficient above one instead implies that the wealth effect dominates which results in a positive value of $A_{c,2}$ given that $\beta_2 < 0$. Section 4 provides empirical evidence that expected inflation is negatively related to the price-consumption ratio and accounts for a large part of its variation.⁸

The following expression represents innovations to the real pricing kernel, where vector λ rep-

⁸The unobservable price-consumption ratio is proxied by the wealth-consumption ratio measured in Lustig et al. (2008).

represents the market prices of risk:

$$m_{t+1} - E_t(m_{t+1}) = \lambda_{\eta_c} \eta_{c,t+1} + \lambda_{\eta_\pi} \eta_{\pi,t+1}, \quad (19)$$

$$\lambda_{\eta_c} = -\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\delta_1 + k_{c,1}A_{c,2}\delta_3 + k_{c,1}A_{c,3}\delta_5 + 1), \quad (20)$$

$$\lambda_{\eta_\pi} = (\theta - 1)(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4). \quad (21)$$

The model allows both shocks to consumption growth and inflation to be priced. For example, $\lambda_{\eta_\pi} > 0$ implies that the representative agent dislikes positive inflation shocks and therefore requires a higher risk premium for assets that perform badly in periods of high inflation. Hence, this represents an additional part of risk premiums in the economy compared to models in which only consumption shocks are priced, e.g., Bansal and Yaron (2004). Recall that $\theta = 1$ under power utility, which means that inflation risk is not priced and the price of consumption risk collapses to $-\frac{1}{\psi} = -\gamma$.

The log price-dividend ratio is conjectured to be a linear function of the same three state variables as above:

$$pd_t = A_{d,0} + A_{d,1}x_{c,t} + A_{d,2}x_{\pi,t} + A_{d,3}x_{d,t}. \quad (22)$$

Again, the coefficients are solved for using the standard Euler equation. Appendix A.2. describes the derivations and reports the expression for $A_{d,0}$. The remaining coefficients are given by:

$$\begin{aligned} A_{d,1} &= \frac{-\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}(A_{c,1}\beta_1 + A_{c,2}\beta_3 + A_{c,3}\beta_5) - A_{c,1} + 1) + k_{d,1}(A_{d,2}\beta_3 + A_{d,3}\beta_5)}{1 - k_{d,1}\beta_1}, \\ A_{d,2} &= \frac{(1 - k_{d,1}\beta_1)X + Y}{(1 - k_{d,1}\beta_4)(1 - k_{d,1}\beta_1) - k_{d,1}^2\beta_2\beta_3}, \\ A_{d,3} &= \frac{(\theta - 1)A_{c,3}(k_{c,1}\beta_6 - 1) + 1}{1 - k_{d,1}\beta_6}, \\ X &= (\theta - 1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}), \\ Y &= k_{d,1}\beta_2 \left[-\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 - A_{c,1} + 1) + k_{d,1}A_{d,3}\beta_5 \right], \end{aligned}$$

Since $A_{c,3}$ equals zero, $A_{d,3}$ equals $\frac{1}{1 - k_{d,1}\beta_6}$ which is positive given that $\beta_6 < 1$. This implies

that higher expected dividend growth naturally raises price-dividend ratios. Coefficient $A_{d,2}$ is in general negative when the EIS is above one and when high expected inflation is a signal of low future consumption growth, $\beta_2 < 0$. Expected consumption growth and price-dividend ratios are in general positively associated ($A_{d,1} > 0$) for high values of the EIS and negative values of β_3 , provided that $A_{d,2}$ is negative. As for the price-consumption ratio, Section 4 provides empirical evidence that expected inflation is negatively related to price-dividend ratios.

Real and nominal log bond prices are conjectured to be functions of the same state variables:

$$q_{t,n} = D_{0,n} + D_{1,n}x_{c,t} + D_{2,n}x_{\pi,t} + D_{3,n}x_{d,t}, \quad (23)$$

$$q_{t,n}^{\$} = D_{0,n}^{\$} + D_{1,n}^{\$}x_{c,t} + D_{2,n}^{\$}x_{\pi,t} + D_{3,n}^{\$}x_{d,t}. \quad (24)$$

Let $y_{t,n} = \frac{1}{n}q_{t,n}$ and $y_{t,n}^{\$} = \frac{1}{n}q_{t,n}^{\$}$ denote the n -period continuously compounded real and nominal yield. Then:

$$y_{t,n} = -\frac{1}{n}(D_{0,n} + D_{1,n}x_{c,t} + D_{2,n}x_{\pi,t} + D_{3,n}x_{d,t}), \quad (25)$$

$$y_{t,n}^{\$} = -\frac{1}{n}(D_{0,n}^{\$} + D_{1,n}^{\$}x_{c,t} + D_{2,n}^{\$}x_{\pi,t} + D_{3,n}^{\$}x_{d,t}), \quad (26)$$

where the D-coefficients determine how yields respond to changes in expected consumption growth, inflation, and dividend growth. Solving for nominal log bond prices requires the use of the nominal log pricing kernel which is determined by the difference between the real log pricing kernel and the inflation rate:

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}. \quad (27)$$

Appendix A.3 and A.4 show how to solve for the coefficients and report the expressions for $D_{0,n}$

and $D_{0,n}^{\$}$. The remaining coefficients are given by:

$$\begin{aligned}
D_{1,n} &= -\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 - A_{c,1} + 1) + D_{1,n-1}\beta_1 + D_{2,n-1}\beta_3, \\
D_{2,n} &= (\theta - 1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}) + D_{1,n-1}\beta_2 + D_{2,n-1}\beta_4, \\
D_{3,n} &= 0, \\
D_{1,n}^{\$} &= -\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 - A_{c,1} + 1) + D_{1,n-1}^{\$}\beta_1 + D_{2,n-1}^{\$}\beta_3, \\
D_{2,n}^{\$} &= (\theta - 1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}) + D_{1,n-1}^{\$}\beta_2 + D_{2,n-1}^{\$}\beta_4 - 1, \\
D_{3,n}^{\$} &= 0,
\end{aligned}$$

where $D_{i,0}$ and $D_{i,0}^{\$}$ equal zero for $i = 1, 2, 3$. For plausible parameter values, real yields in the model increase in response to positive shocks to consumption growth. This suggests that $D_{1,n}$ is negative and means that real bonds act as a hedge against bad times. They are therefore subject to negative risk premiums in the economy which makes the real yield curve slope downwards. This is supported by empirical evidence from UK-index linked bonds which have been trading since the mid 1980s (e.g., Evans, 1998, and Piazzesi and Schneider, 2006) Unfortunately, data for US index-linked bonds only date back to 1997 but indicate a positively sloped yield curve. However, the rather short sample period and the fact that the market was illiquid at the inception of trading warrants some caution in interpreting the data. The pro-cyclical nature of real yields is also consistent with the empirical findings of Chapman (1997) and Ang et al. (2008). Real yields decrease in response to higher expected inflation if high inflation is bad news for future consumption growth, i.e., $D_{2,n}$ is then positive. This is consistent with earlier studies such as Fama and Gibbons (1982), Pennacchi (1991), and Boudoukh (1993). Ang et al. (2008) also document a negative relation between real rates and expected inflation but find the correlation to be positive for longer horizons.

As expected, high expected inflation depresses nominal bond prices leading $D_{2,n}^{\$}$ to be negative. The reaction of nominal bonds to changes in expected consumption growth depends on the relation between expected consumption growth and future inflation captured by β_3 . A positive β_3 implies a negative $D_{1,n}^{\$}$ since high consumption growth then signals high future inflation which is bad news for nominal bond returns. The economic intuition for why changes in dividend growth rates should

affect bond prices is not clear, so both real and nominal bonds have a zero loading on x_d . The zero loadings arise due to the restrictions imposed on the dynamics in Section 2.1.1.

2.3.2 Specification II: Heteroscedasticity

Having solved for asset prices using the conditional means as state variables, this section conjectures that asset prices also are functions of the conditional variances and covariances of consumption growth, inflation, and dividend growth. The coefficients in front of the conditional means remain the same as above, wherefore only the solutions for the second moments are reported.

The log price-consumption ratio is conjectured to be a linear function of the following state variables:

$$pc_t = A_{c,0} + A_{c,1}x_{c,t} + A_{c,2}x_{\pi,t} + A_{c,3}x_{d,t} + A_{c,4}\sigma_{c,t}^2 + A_{c,5}\sigma_{\pi,t}^2 + A_{c,6}\sigma_{d,t}^2 + A_{c,7}\sigma_{c\pi,t} + A_{c,8}\sigma_{cd,t} + A_{c,9}\sigma_{\pi d,t}. \quad (28)$$

Appendix A.1 shows that the solutions are given by:

$$\begin{aligned} A_{c,4} &= \frac{-0.5X^2}{\theta(\phi_c k_{c,1} - 1)}, \\ A_{c,5} &= \frac{-0.5Y^2}{\theta(\phi_\pi k_{c,1} - 1)}, \\ A_{c,6} &= 0, \\ A_{c,7} &= \frac{-XY}{\theta(\phi_{c\pi} k_{c,1} - 1)}, \\ A_{c,8} &= 0, \\ A_{c,9} &= 0, \\ X &= [\lambda_{\eta_c} + k_{c,1}(A_{c,1}\delta_1 + A_{c,2}\delta_3 + A_{c,3}\delta_5 + 1)], \\ Y &= [\lambda_{\eta_\pi} + k_{c,1}(A_{c,1}\delta_2 + A_{c,2}\delta_4)], \end{aligned}$$

where λ_{η_c} and λ_{η_π} are the market prices of risk, found in (20)-(21). For negative values of θ , that is when the risk aversion and the IES both are above one, $A_{c,4}$ and $A_{c,5}$ are both negative since ϕ_c , ϕ_π , and $k_{c,1}$ are all positive and less than one. Increased volatility of consumption growth and inflation therefore depresses the log price-consumption ratio. The need for an IES above one to

capture the negative relations between the log price-consumption ratio and macro volatility is the same as in the long-run risk model. The response of the price-consumption ratio to the covariance of consumption growth and inflation, $\sigma_{c\pi,t}$, is determined by X and Y which are closely related to the market prices of risk of consumption and inflation shocks. For example, $A_{c,7}$ tends to be positive when positive consumption shocks lower the marginal utility of the agent while positive inflation shocks increase the marginal utility. A pro-cyclical inflation process is then associated with a higher price-consumption ratio. The remaining coefficients are zero as a result of the restrictions imposed on the dynamics.

The log price-dividend ratio is conjectured to be a function of same state variables:

$$pd_t = A_{d,0} + A_{d,1}x_{c,t} + A_{d,2}x_{\pi,t} + A_{d,3}x_{d,t} + A_{d,4}\sigma_{c,t}^2 + A_{d,5}\sigma_{\pi,t}^2 + A_{d,6}\sigma_{d,t}^2 + A_{d,7}\sigma_{c\pi,t} + A_{d,8}\sigma_{cd,t} + A_{d,9}\sigma_{\pi d,t}. \quad (29)$$

Appendix A.2 shows that the solutions are given by:

$$\begin{aligned} A_{d,4} &= \frac{0.5X^2 + (\theta - 1)A_{c,4}(k_{c,1}\phi_c - 1)}{(1 - k_{d,1}\phi_c)}, \\ A_{d,5} &= \frac{0.5Y^2 + (\theta - 1)A_{c,5}(k_{c,1}\phi_{\pi} - 1)}{(1 - k_{d,1}\phi_{\pi})}, \\ A_{d,6} &= \frac{0.5Z^2 + (\theta - 1)A_{c,6}(k_{c,1}\phi_d - 1)}{(1 - k_{d,1}\phi_d)}, \\ A_{d,7} &= \frac{XY + (\theta - 1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1)}{(1 - k_{d,1}\phi_{c\pi})}, \\ A_{d,8} &= \frac{XZ + (\theta - 1)A_{c,8}(k_{c,1}\phi_{cd} - 1)}{(1 - k_{d,1}\phi_{cd})}, \\ A_{d,9} &= \frac{YZ + (\theta - 1)A_{c,9}(k_{c,1}\phi_{\pi d} - 1)}{(1 - k_{d,1}\phi_{\pi d})}, \\ X &= [\lambda_{\eta_c} + k_{d,1}(A_{d,1}\delta_1 + A_{d,2}\delta_3 + A_{d,3}\delta_5)], \\ Y &= [\lambda_{\eta_{\pi}} + k_{d,1}(A_{d,1}\delta_2 + A_{d,2}\delta_4)], \\ Z &= [k_{d,1}A_{d,3}\delta_6 + 1]. \end{aligned}$$

As for the price-consumption ratio above, a high value of the EIS is needed to produce a negative relation between price-dividend ratios and macroeconomic volatility. The last three coefficients

determine how the price-dividend ratio responds to changes in the covariances between the three macro variables.

Similarly, the real and nominal log bond prices are conjectured to be functions of the same state variables:

$$q_{t,n} = D_{0,n} + D_{1,n}x_{c,t} + D_{2,n}x_{\pi,t} + D_{3,n}x_{d,t} + D_{4,n}\sigma_{c,t}^2 + D_{5,n}\sigma_{\pi,t}^2 + D_{6,n}\sigma_{d,t}^2 + D_{7,n}\sigma_{c\pi,t} + D_{8,n}\sigma_{cd,t} + D_{9,n}\sigma_{\pi d,t}, \quad (30)$$

$$q_{t,n}^{\$} = D_{0,n}^{\$} + D_{1,n}^{\$}x_{c,t} + D_{2,n}^{\$}x_{\pi,t} + D_{3,n}^{\$}x_{d,t} + D_{4,n}^{\$}\sigma_{c,t}^2 + D_{5,n}^{\$}\sigma_{\pi,t}^2 + D_{6,n}^{\$}\sigma_{d,t}^2 + D_{7,n}^{\$}\sigma_{c\pi,t} + D_{8,n}^{\$}\sigma_{cd,t} + D_{9,n}^{\$}\sigma_{\pi d,t}. \quad (31)$$

Appendix A.3 and A.4 show how to solve for the coefficients. The coefficients for real bonds are for brevity reported in Appendix A.3 while the coefficients for nominal bonds are given by:

$$\begin{aligned} D_{4,n}^{\$} &= (\theta - 1)A_{c,4}(k_{c,1}\phi_c - 1) + D_{4,n-1}^{\$}\phi_c + 0.5X^2, \\ D_{5,n}^{\$} &= (\theta - 1)A_{c,5}(k_{c,1}\phi_{\pi} - 1) + D_{5,n-1}^{\$}\phi_{\pi} + 0.5Y^2, \\ D_{6,n}^{\$} &= 0, \\ D_{7,n}^{\$} &= (\theta - 1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1) + D_{7,n-1}^{\$}\phi_{c\pi} + XY, \\ D_{8,n}^{\$} &= 0, \\ D_{9,n}^{\$} &= 0, \\ X &= \left[\lambda_{\eta_c} + D_{1,n-1}^{\$}\delta_1 + D_{2,n-1}^{\$}\delta_3 + D_{3,n-1}^{\$}\delta_5 \right], \\ Y &= \left[\lambda_{\eta_{\pi}} - 1 + D_{1,n-1}^{\$}\delta_2 + D_{2,n-1}^{\$}\delta_4 \right], \end{aligned}$$

where $D_{i,0}$ and $D_{i,0}^{\$}$ equal zero for $i = 4, 5, \dots, 9$. An increase in the volatility of consumption growth and inflation has a negative effect on bond prices, provided a high EIS and high values of the persistence parameters. In that case, nominal bonds do not provide a hedge against periods of high economic turbulence and higher macroeconomic volatility therefore raises nominal yields. Coefficient $D_{7,n}^{\$}$ determines how nominal yields response to changes in the covariance of consumption growth and inflation and depends on the market prices of risk of consumption and inflation shocks, the first terms in X and Y, and on whether bond prices increase or decrease in response

to the shocks, the remaining terms in X and Y . A positive $D_{7,n}^{\$}$ implies higher bond prices and lower yields when inflation is procyclical. The volatility of dividend growth and the covariance terms involving dividend growth have no effect on real and nominal bond prices. This is due to the restrictions imposed on the δ matrix in Section 2.1.1..

3 Data and Estimation

This section explains the data used in the paper and the estimation of the homoscedastic and heteroscedastic dynamics specified in Section 2.1. Preference parameters are calibrated to match unconditional moments of asset prices and are therefore discussed in Section 4.

3.1 Data

Quarterly aggregate US consumption data on nondurables and services is collected from Bureau of Economic Analysis for the period 1952-2007. Inflation is computed as in Piazzesi and Schneider (2006) using the price index corresponding to the consumption data. Value-weighted market returns (NYSE/AMEX/NASDAQ) are retrieved from CRSP. Nominal interest rates are collected from the Fama-Bliss file in CRSP and from the website of J. Huston McCulloch. The former set of yields are used to match unconditional moments and the latter are used for computing quarterly bond returns since they make it possible to compute quarterly returns on a five year bond. Daily stock returns and daily 5-year nominal interest rates for the period January 1962 - December 2007 are collected from CRSP and the Federal Reserve Bank in St. Louis, respectively. The daily data is used for computing the correlation between stock and bond returns within each quarter. Dividend growth is computed using monthly CRSP returns including and excluding dividends. The procedure follows Bansal et al. (2005) among others. Quarterly dividends are formed by summing dividends for each quarter. To mitigate seasonalities, a moving four-quarter average is used. Real dividend growth rates are found by taking the log first difference and deflating using the constructed inflation series.

Table 1 provides summary statistics of the data. Dividend growth displays the highest volatility of the three series, followed by inflation and consumption growth. Inflation displays positive autocorrelations over both one and two years while the autocorrelations for consumption growth

and inflation are not statistically different from zero. The unconditional correlation between inflation and consumption growth is negative, -0.35 , and statistically significant at the 10% level. The correlation between consumption growth and dividend growth and between dividend growth and inflation have a positive and negative sign, respectively.

3.2 Estimation

3.2.1 Specification I

The state space system for consumption growth, inflation, and dividend growth is estimated using maximum likelihood (e.g., Hamilton, 1994) and quarterly US data for the period 1952:2 - 2007:4. The three series are demeaned. The assumption of Gaussian error terms, $\boldsymbol{\eta}$, gives a log-likelihood function that is the sum of Gaussian densities. The following objective function is maximized:

$$\sum_{t=1}^T \left[-\frac{1}{2}n \ln(2\pi) - \frac{1}{2} \ln |\boldsymbol{\Omega}| - \frac{1}{2} \boldsymbol{\eta}'_t \boldsymbol{\Omega}^{-1} \boldsymbol{\eta}_t \right], \quad (32)$$

where $\boldsymbol{\eta}_t = \mathbf{z}_t - \boldsymbol{\mu} - \mathbf{x}_{t-1}$, T equals the number of observations and n equals the dimension of vector \mathbf{z}_t , 223 and 3 respectively. The likelihood function is evaluated by finding the state vector recursively from $\mathbf{x}_{t+1} = \boldsymbol{\beta} \mathbf{x}_t + \boldsymbol{\delta} \boldsymbol{\eta}_{t+1}$, where \mathbf{x}_0 is set equal to its unconditional mean of zero.

Table 2 presents the estimated parameter values. Negative values of β_2 and δ_2 imply that both expected and unexpected inflation lead to lower future consumption growth. As a result, the agent demands a positive risk premium on assets that are poor hedges against inflation, e.g. equity and nominal bonds. It also makes price-dividend ratios negatively related to expected inflation. The covariance of shocks to consumption growth and inflation is negative and statistically significant. Using the estimated parameter values, the model is simulated 2,000 times in which each simulation contains 223 quarters. The model-implied macro moments are displayed in Table 1 together with the sample moments. The model provides an overall good fit to data.

Figure 1 depicts time series of realized and expected consumption growth, inflation, and dividend growth using the fitted values from the estimation. The time-varying part of the conditional means are extracted from data. The solid lines represent the sum of μ and x for each macro variable

and the dashed lines are the realized values. The stagflation period in the 1970s is evident from the spike in inflation and the sharp drop in consumption growth. Noteworthy is also the gradual decline in inflation from 1980 up the beginning of the 2000s. The spike in dividend growth at the end of the sample period is the result of a large difference between cum and ex-dividend market returns obtained from CRSP for November 2004.⁹

Using the estimated parameters, it is straightforward to back out the implied macro shocks. Figure 2 displays the squared shocks to the three macro variables, which provides an understanding of how the macroeconomic volatility has changed over time. Consumption growth was subject to several large shocks during the early part of the sample period while the magnitude of the shocks have decreased over time, with the exception of spikes in the early 1980s and early 1990s. Inflation experienced a period of large shocks during the 1970s and 1980s after which the size of the shocks has decreased.

Figure 3 depicts the cross products of shocks to consumption growth and inflation, shocks to consumption growth and dividend growth, and shocks to inflation and dividend growth. This provides an understanding of how the covariances have changed over time. The period of stagflation in the 1970s is evident from the first graph in which shocks to consumption growth and inflation had opposite signs. After that period, the cross products have stayed close to zero. One can also note that the product of shocks to consumption growth and inflation has stayed negative throughout most of the sample period. The relation between shocks to consumption growth and dividend growth has on the other hand stayed mostly positive, with the exception of the late 1980s. Shocks to inflation and dividend growth were negatively correlated in the mid 1970s and positively correlated in the late 1990s. As will be discussed later, the positive shocks to both dividend growth and inflation in the late 1990s lead to negative correlations in the model between stock and bond returns since higher dividend growth increases stock returns while bond returns suffer from positive inflation shocks.

⁹The large shock to dividend growth stems from a very large special payment by Microsoft.

3.2.2 Specification II

The objective of this section is to estimate processes for the time-varying conditional second moments. I choose to model the conditional variances and covariances as in the diagonal VEC-model of Bollerslev et al. (1988). Let the macro shocks, $\boldsymbol{\eta}_{t+1}$, be normally distributed with mean zero and subject to the conditional variance-covariance matrix \mathbf{H}_t . Let $\text{vech}(\mathbf{H}_t)$ be an operator that stacks the columns of the lower triangular of the variance-covariance matrix. Then, every element of $\text{vech}(\mathbf{H}_t)$ follows a univariate process:

$$h_{c,t} = c_c + a_c \eta_{c,t}^2 + b_c h_{c,t-1}, \quad (33)$$

$$h_{c\pi,t} = c_{c\pi} + a_{c\pi} \eta_{c\pi,t}^2 + b_{c\pi} h_{c\pi,t-1}, \quad (34)$$

$$h_{cd,t} = c_{cd} + a_{cd} \eta_{cd,t}^2 + b_{cd} h_{cd,t-1}, \quad (35)$$

$$h_{\pi,t} = c_{\pi} + a_{\pi} \eta_{\pi,t}^2 + b_{\pi} h_{\pi,t-1}, \quad (36)$$

$$h_{\pi d,t} = c_{\pi d} + a_{\pi d} \eta_{\pi d,t}^2 + b_{\pi d} h_{\pi d,t-1}, \quad (37)$$

$$h_{d,t} = c_d + a_d \eta_{d,t}^2 + b_d h_{d,t-1}. \quad (38)$$

The system is estimated using maximum likelihood and the shocks extracted from data in Section 3.2.1 are used as inputs. Table 3 presents the estimation results. The b parameters are all estimated to be in excess of 0.80, indicating that the second moments are persistent processes. Using the estimated parameters and the shocks, I compute time series of the implied conditional second moments. Figure 4 plots the conditional volatilities and Figure 5 plots the conditional covariances. The conditional volatility of consumption growth has been lower in the second half of the sample with notable spikes in the early 1980s and 1990s. Inflation displayed high volatility in the late 1970s and early 1980s after which the volatility declined. The current decade has seen an increase in the volatility of inflation, up to levels last seen in the 1980s. Dividend growth experienced a period of increased volatility from the mid 1980s to the mid 1990s and in the early 2000s. The conditional covariance between inflation and dividend growth has been mostly negative with the exception of the late 1950s and the late 1990s. The covariance between inflation and consumption growth is estimated to have been negative throughout the entire sample period with a sharp drop in the mid

1970s. Finally, the covariance between consumption growth and dividend growth has been positive throughout the sample period.

The estimated parameter values are used when simulating the model. The mapping between the VEC-model and the model's dynamics for the second moments is done in the following way. A GARCH model of the form $h_t = c + bh_{t-1} + a\eta_t^2$ can be rewritten as $h_t = c + (a+b)h_{t-1} + a(\eta_t^2 - h_{t-1})$ where the last term can be viewed as a shock to volatility with mean zero, conditional on information at time $t - 1$. Assuming for simplicity that the shock is normally distributed, the expression maps into the model's volatility dynamics specified in Section 2.1.2.. I therefore use the sum of the estimated a and b for each second moment as persistence parameters in the model. The unconditional mean of the second moments in the model, that is the α parameters in Section 2.1.2., are set as $\alpha = \frac{c}{1-a-b}$ for each process. The remaining parameter which governs the volatility of volatility, τ , is set as to match the unconditional variance of the model's second moments with the ones estimated from data. The unconditional variance of the second moments within the model are given by $\frac{\tau^2}{1-\phi^2}$. The τ parameters are therefore set as $\tau = \sqrt{Var(h)(1-\phi^2)}$ for each of the second moments. Table 4 presents the parameter values used for the model's second moments.

4 Implications for Asset prices

Having estimated the dynamics, the three preference parameters remain to be determined. It is well known from Bansal and Yaron (2004) that models using Epstein-Zin recursive preferences in conjunction with persistent shocks to expected consumption growth can match the observed equity premium for plausible values of risk aversion. However, as is discussed in for example Bansal and Yaron (2000) and Hasseltoft (2008) these models are sensitive to the persistence of macro shocks. A high persistence allows the model to lower the required risk aversion in order to match risk premiums in the economy. The well-known downward bias of estimated persistence parameters in finite samples (Kendall, 1954) also leads to a higher risk aversion needed for matching the level of risk premiums (Bansal et al., 2007a).

I choose to use a risk aversion coefficient of 10 which seems to be a plausible level of risk aversion in the literature. The discount factor, δ , is set to 0.997 and the EIS, ψ , is set to 1.5.

The magnitude of the EIS is subject to controversy as mentioned earlier in the paper. It is known in the literature that the so called long-run risk model needs an EIS above one to be able to explain features of asset price data. The model presented in this paper needs an EIS in excess of one for a different reason. Given that high inflation expectations are estimated to signal low future consumption growth ($\beta_2 < 0$), an EIS above one is needed to produce a negative relation between expected inflation and the price-consumption ratio and the price-dividend ratio. This implies that the intertemporal substitution effect dominates the wealth effect and agents therefore sell risky assets in anticipation of lower future returns, yielding lower valuation ratios. The negative signs are supported by empirical evidence by running a contemporaneous regression of a proxy for the unobservable price-consumption ratio, stemming from Lustig et al. (2008), and the observed price-dividend ratio onto the extracted state variables. Table 5 reports the results. Starting with the price-consumption ratio, the results report a positive and statistically significant coefficient when only x_c is used as explanatory variable. This is also the case in the model if one set $A_{c,2}$ equal to zero and keeping the EIS above one. However, adding x_π as explanatory variable drives out the significance of x_c and changes the sign of the coefficient to negative while the coefficient for x_π is negative and strongly significant. The R^2 also increases from 13% to 32%. Results for the price-dividend ratio are similar. Using only x_c as explanatory variable yields a positive and statistically significant slope coefficient. However, adding expected inflation drives out the significance of expected consumption growth and changes its sign to negative. Expected inflation enters negative and highly significant while the R^2 increases from 3% to 13%. Expected dividend growth enters positive, as it is in the model, but is not statistically significant.

Table 6 reports the implied unconditional moments of asset prices. The average equity excess return implied by the homoscedastic model when using the estimated parameter values to simulate the model, is 0.24% which is much lower than the 5.52% observed in data. The average yield spread between a 5-year bond and a 3-month bond implied by the model is 0.15% compared to 0.96% in data. However, Table 6 shows that by increasing the persistence parameter of inflation, β_4 , to 1.055 which is less than one standard error away from the point estimate, the model generates an

average equity excess return of 4.16% and a slope of the yield curve of 1.50%.¹⁰ The risk aversion is kept at 10 for the calibrated model. This highlights the sensitivity of the model to the persistence in the macro variables and also shows that the model is able to give a reasonable match to data using parameter values that are close to the point estimates. Model-implied dividend yields have a correlation of 0.34 with observed dividend yields and the correlation of model yields with actual yields are 0.72, 0.65, and 0.30 for the short rate, the 5-year rate, and the yield spread respectively.

Turning to the heteroscedastic model (Specification II), I set the preference parameters as above. Again, I evaluate the model implications using both the estimated parameter values and the calibrated value of β_4 . The last two columns of Table 6 report the asset pricing implications. Using the point estimates, the model generates an equity premium of 0.14% and a yield curve slope of 0.23%. Both values are smaller than those observed in data. However, the sensitivity of the model to a small increase in inflation persistence is highlighted in the last column. Setting β_4 equal to 1.055 results in an average equity premium of 5.03% and a yield curve slope of 1.66%. The unconditional moments of the model are sensitive to small alterations in other parameters as well, suggesting that a more extensive calibration exercise is likely to yield an even better match to data. For example, Hasseltoft (2008) shows that these type of models also are sensitive to changes in the persistence of volatility shocks.

5 Explaining the Fed-model

The so called Fed-model refers to the positive correlation between US dividend yields and US nominal interest rates. Figure 6 plots the US dividend yield and the 5-year nominal Treasury yield for the period 1952-2007. The two series display an unconditional correlation of 0.30 for the entire sample period, but 0.74 for the period 1965-2007. This phenomenon has been considered puzzling since it implies that changes in expected inflation and bond risk premiums, which have been the main drivers of nominal interest rates (e.g. Campbell and Ammer, 1993, and Best et al., 1998), should be associated with movements in dividend yields. The puzzle can be illustrated by

¹⁰The model-implied autocorrelations of inflation when setting β_4 equal to 1.055 are 0.77 and 0.66 for a one-year and two-year horizon, respectively.

considering the Gordon growth model that expresses the dividend-price ratio in steady state as:

$$\frac{D}{P} = R - G, \quad (39)$$

where R is the real discount rate on equities and G is the real dividend growth rate. The real discount, R , can be decomposed into the real risk free rate and the equity risk premium. For the positive correlations in data to arise, expected inflation and bond risk premiums must be either positively associated with real interest rates and equity risk premiums or negatively correlated with dividend growth rates. Surprisingly, the literature has considered any of these explanations to be unlikely. Instead, an explanation in the form of inflation illusion has found wide support (e.g., Ritter and Warr, 2002, Asness, 2003, Campbell and Vuolteenaho, 2004, and Cohen et al., 2005). This entails irrational investors who fail to adjust the expected nominal dividend growth for changes in inflation but they adjust the nominal discount rate. In effect, investors discount real cash flows using nominal interest rates, leading equities to be undervalued from the viewpoint of a rational investor in times of high inflation. However in a recent empirical paper, Bekaert and Engstrom (2008) argue that rational mechanisms are at work. Using a vector autoregressive framework, they ascribe the high correlation of dividend yields and bond yields to mainly a positive relation between expected inflation and the equity risk premium. They proxy the equity risk premium with a measure of economic uncertainty and a consumption-based measure of risk aversion and find that they are both positively correlated with expected inflation. Using cross-country data, they further argue that the correlation between dividend yields and bond yields are higher in countries with a higher average incidence of stagflation. Hence, any equilibrium model that tries to explain the Fed-model seems to need a link between expected inflation and the equity risk premium. The model presented in this paper contains such a link.

As the objective of this section is explain the unconditional correlation between dividend yields and nominal yields, it suffices to analyze the homoscedastic case of the model. Consider the unconditional correlation between the log dividend yield, dp_t and the nominal yield on a bond with

a maturity of n periods, $y_{n,t}^{\$}$:

$$\begin{aligned}
Cov(dp_t, y_{t,n}^{\$}) &= Cov(-pd_t, y_{t,n}^{\$}), \\
&= Cov(-A_{d,1}x_{c,t} - A_{d,2}x_{\pi,t} - A_{d,3}x_{d,t}, -\frac{1}{n}(D_{1,n}^{\$}x_{c,t} + D_{2,n}^{\$}x_{\pi,t} + D_{3,n}^{\$}x_{d,t})), \\
&= A\sigma_{x_c}^2 + B\sigma_{x_{\pi}}^2 + C\sigma_{x_d}^2 + D\sigma_{x_c x_{\pi}} + E\sigma_{x_c x_d} + F\sigma_{x_{\pi} x_d}, \\
A &= (-A_{d,1})\left(-\frac{D_{1,n}^{\$}}{n}\right), \\
B &= (-A_{d,2})\left(-\frac{D_{2,n}^{\$}}{n}\right), \\
C &= (-A_{d,3})\left(-\frac{D_{3,n}^{\$}}{n}\right), \\
D &= \left((-A_{d,1})\left(-\frac{D_{2,n}^{\$}}{n}\right) + (-A_{d,2})\left(-\frac{D_{1,n}^{\$}}{n}\right) \right), \\
E &= \left((-A_{d,1})\left(-\frac{D_{3,n}^{\$}}{n}\right) + (-A_{d,3})\left(-\frac{D_{1,n}^{\$}}{n}\right) \right), \\
F &= \left((-A_{d,2})\left(-\frac{D_{3,n}^{\$}}{n}\right) + (-A_{d,3})\left(-\frac{D_{2,n}^{\$}}{n}\right) \right).
\end{aligned} \tag{40}$$

Given the estimated and calibrated parameters, the second term is dominating which means that shocks to inflation serve as the main determinant for the covariance between dividend yields and nominal yields. Specifically, B is positive indicating that the covariance is increasing in the volatility of inflation. Coefficient A is also positive meaning that macroeconomic volatility in general is suggested to play a key role for explaining the Fed-model. The loading on dividend volatility is zero since expected dividend growth only affects dividend yields and not bond prices, i.e. C equals zero. Coefficient D is positive while both E and F are negative. For example, periods in which both dividend growth and inflation increase imply lower dividend yields through higher cash flows and higher nominal yields through higher inflation. This generates a negative covariance between the two variables. As mentioned above, variations in dividend yields can be decomposed into changes in real interest rates, risk premiums, and dividend growth while changes in nominal yields can be decomposed into changes in real interest rates, inflation, and risk premiums. While variations in real interest rates is likely to induce a positive correlation between the two, it has been shown that changes in real rates contributes little to changes in asset prices (e.g. Campbell and Ammer, 1993).

Instead a likely channel for the positive correlation to arise is through common changes in risk premiums.

Consider the equity risk premium for the homoscedastic case:

$$\begin{aligned}
E_t[r_{m,t+1} - r_{f,t}] + \frac{1}{2}Var_t[r_{m,t+1}] &= -Cov_t[m_{t+1}, r_{m,t+1}], & (41) \\
&= -Cov_t[A\eta_{c,t+1} + B\eta_{\pi,t+1}, C\eta_{c,t+1} + D\eta_{\pi,t+1} + E\eta_{d,t+1}], \\
&= -[AC\sigma_c^2 + BD\sigma_\pi^2 + (AD + BC)\sigma_{c\pi} + AE\sigma_{cd} + BE\sigma_{\pi d}], \\
A &= \lambda_{\eta_c}, \\
B &= \lambda_{\eta_\pi}, \\
C &= k_{d,1}A_{d,1}\delta_1 + k_{d,1}A_{d,2}\delta_3 + k_{d,1}A_{d,3}\delta_5, \\
D &= k_{d,1}A_{d,1}\delta_2 + k_{d,1}A_{d,2}\delta_4, \\
E &= k_{d,1}A_{d,3}\delta_6 + 1,
\end{aligned}$$

which is determined by the conditional covariance between the real pricing kernel and the real market return. As positive inflation shocks signal worse economic conditions, the marginal utility is increasing in shocks to inflation. The representative agent therefore dislikes higher inflation and the market price of inflation risk, λ_{η_π} , has therefore a positive sign. The market return on the other hand decreases in response to inflation shocks, $D < 0$, implying that stocks are a poor hedge against inflation. Inflation shocks therefore cause a negative conditional covariance between m_{t+1} and $r_{m,t+1}$ and contributes to a positive equity risk premium. This part of the equity risk premium is absent in models that do not consider the real effects of inflation, for example the long-run risk model.

The yield on a nominal bond can be written as the sum of the corresponding real yield, the expected inflation over the bond's maturity, the inflation risk premium, and a Jensen's inequality term. The following holds for the nominal short rate:

$$y_{t,3m}^{\$} = y_{t,3m} + E_t(\pi_{t+1}) + Cov_t(m_{t+1}, \pi_{t+1}) - \frac{1}{2}Var_t(\pi_{t+1}),$$

where the covariance term represents the inflation risk premium. Positive inflation shocks that occur during periods of high marginal utility imply a positive inflation risk premium as nominal bonds then perform badly in bad times. Nominal yields therefore increase as a result. Solving for the inflation risk premium in the model yields:

$$\begin{aligned}
Cov_t(m_{t+1}, \pi_{t+1}) &= Cov_t(A\eta_{c,t+1} + B\eta_{\pi,t+1}, \eta_{\pi,t+1}), \\
&= B\sigma_{\pi}^2 + A\sigma_{c\pi}, \\
A &= \lambda_{\eta^c}, \\
B &= \lambda_{\eta^{\pi}}.
\end{aligned} \tag{42}$$

Recall that the price of inflation risk, $\lambda_{\eta^{\pi}}$, is positive, so higher inflation volatility raises risk premiums on nominal bonds. The price of consumption risk, λ_{η^c} , is negative so a counter-cyclical inflation process contributes to a higher inflation risk premium. This arises since bonds then perform badly in periods of low consumption growth. That is, bond returns are procyclical. Analyzing risk premiums on equity and bonds jointly suggests that risk premiums on both assets load positively on the unconditional volatility of inflation. In fact, dividend yields and nominal yields become positively correlated in the model mainly through the positive correlation of risk premiums on equity and bonds.

Table 7 reports observed and model-implied correlations for both the homoscedastic and heteroscedastic case. The model produces a high comovement between dividend yields and 5-year nominal interest rates. The correlation coefficients for the homoscedastic case are 0.82 for the whole sample period and 0.81 for the period 1965-2007 compared to 0.30 and 0.74 in data. The heteroscedastic model generates similar coefficients, 0.73 and 0.72. The model reproduces the high correlations observed in data solely through rational channels, mainly the common effect of inflation on equity and bond risk premiums. Turning off the effect of inflation being bad news for consumption growth, i.e., setting $\beta_2 = 0$, reduces the model-implied correlation drastically to 0.17.

Recall the expressions for the log price-dividend ratio and for the 5-year log nominal bond price:

$$\begin{aligned}
 pd_t &= A_{d,0} + A_{d,1}x_{c,t} + A_{d,2}x_{\pi,t} + A_{d,3}x_{d,t}, \\
 q_{5y,t}^{\$} &= D_{0,5y}^{\$} + D_{1,5y}^{\$}x_{c,t} + D_{2,5y}^{\$}x_{\pi,t} + D_{3,5y}^{\$}x_{d,t}.
 \end{aligned}$$

The fact that an increase in both expected and unexpected inflation imply bad news for future consumption growth leads to positive risk premiums on equity and a negative relation between inflation and price-dividend ratios. That is, coefficient $A_{d,2}$ is negative. As expected, nominal bond prices decline in response to higher inflation which implies that $D_{2,5y}^{\$}$ is negative. Hence, dividend yields and nominal yields become positively associated.

6 Explaining the correlation of stock and bond returns

The unconditional correlation of US stock and bond returns for the period 1952:2-2007:4 is slightly positive, 0.10. However, it has varied substantially through time. Figure 7 displays a 20-quarter rolling correlation between nominal US stock returns and nominal returns on the 5-year US Treasury bond. The 1950s and early 1960s experienced negative correlations which turned positive during the 1970s and early 1980s. The comovement of stock and bond returns declined sharply in 1987 at the time of the stock market crash. The correlation subsequently turned positive in the early 1990s before the stock market boom in the late 1990s evolved with associated negative correlations. The correlations remained negative in the early 2000, at the time stock prices were falling and the Federal Reserve were lowering short rates in response to lower economic activity. This section makes use of the heteroscedastic case of the model since the objective is to explain the changing conditional correlations. Risk premiums on equity and bonds are therefore time varying in this section. Before exploring the implications of the model for the correlation of stock and bond returns, consider the

equity risk premium for the heteroscedastic case:

$$\begin{aligned}
E_t[r_{m,t+1} - r_{f,t}] + \frac{1}{2}Var_t[r_{m,t+1}] &= -Cov_t[m_{t+1}, r_{m,t+1}], \\
&= -[AC\sigma_{c,t}^2 + BD\sigma_{\pi,t}^2 + (AD + BC)\sigma_{c\pi,t} + AE\sigma_{cd,t} + BE\sigma_{\pi d,t} + F], \\
F &= (\theta - 1)k_{c,1}k_{d,1}(A_{c,4}A_{d,4}\tau_c^2 + A_{c,5}A_{d,5}\tau_\pi^2 + A_{c,6}A_{d,6}\tau_d^2 + A_{c,7}A_{d,7}\tau_{c\pi}^2 \\
&\quad + A_{c,8}A_{d,8}\tau_{cd}^2 + A_{c,9}A_{d,9}\tau_{\pi d}^2),
\end{aligned} \tag{43}$$

where coefficients A-E are the same as in (41). Risk premiums on equity vary over time in response to changes in the second moments of consumption growth, inflation, and dividend growth. The dominant factors for determining risk premiums on equity is inflation volatility, $\sigma_{\pi,t}^2$, followed by the covariance between dividend growth and inflation, $\sigma_{\pi d,t}$. Expected excess returns increase as the volatility of inflation increases. That is, stocks are risky assets as they perform badly in periods of high macroeconomic volatility. This relates to the so called long-run risk model in which consumption volatility plays an important role. However in contrast to that model, inflation volatility turns out to be the major driver of risk premiums when the real effects of inflation are taken into account. A positive conditional covariance between dividend growth and inflation contributes to a lower equity risk premium since high dividend growth in bad inflationary times implies that stocks are a good hedge. As will be discussed below, a positive $\sigma_{\pi d,t}$ is suggested to have played an important role for making stock and bond returns negatively correlated in the late 1990s.

Solving for the inflation risk premium in the heteroscedastic case yields:

$$\begin{aligned}
Cov_t(m_{t+1}, \pi_{t+1}) &= Cov_t(A\eta_{c,t+1} + B\eta_{\pi,t+1}, \eta_{\pi,t+1}), \\
&= B\sigma_{\pi,t}^2 + A\sigma_{c\pi,t},
\end{aligned} \tag{44}$$

where A and B are the same as in (42). Again, inflation volatility is the dominant factor. As discussed in Section 5, inflation volatility moves risk premiums on equity and bonds in the same direction suggesting that their returns should be positively correlated through their common expo-

sure to macroeconomic risk. A highly volatile inflation rate implies that macro risk becomes the main determinant for changes in risk premiums. When inflation is less volatile, its effect on risk premiums is less dominant which leaves room for other factors to affect the comovement between asset returns. One such factor is changes in dividend growth that only affects equity in the model. I choose to analyze the correlation between stock and bond returns in two different ways. First, I analyze the model-implied conditional correlations. Second, I compute rolling correlations of model-implied realized returns and compare to rolling correlations of actual returns.

The heteroscedastic dynamics in Specification II allow me to compute time-varying conditional correlations implied by the model. First consider the model-implied quarterly conditional covariance between nominal stock returns and returns on a 5-year nominal bond:

$$Cov_t(r_{m,t+1} + \pi_{t+1}, h_{t+1,60m}^{\$}) = A + B\sigma_{c,t}^2 + C\sigma_{\pi,t}^2 + D\sigma_{c\pi,t} + E\sigma_{cd,t} + F\sigma_{\pi d,t}, \quad (45)$$

where $h_{t+1,60m}^{\$} = q_{t+1,57m}^{\$} - q_{t,60m}^{\$}$ and where the expressions for the coefficients A to F are different from the ones used above. The two most important factors to consider are again the volatility of inflation and the covariance between inflation and dividend growth. Coefficient C is by far the largest indicating that changes in inflation volatility has the strongest effect on the covariance between asset returns. This stems from its common effect on equity and bond risk premiums. The covariance of returns react negatively to periods in which dividend growth and inflation are positively associated since higher dividend growth raises stock returns while bond returns suffer from an increase in inflation, i.e., coefficient F is negative. A period of rising dividend growth together with an increase in inflation does not only lead to an outperformance of stocks versus bonds due to higher cash flows. A positive covariance between the two also lowers the equity risk premium since high dividend growth in bad inflationary times imply that stocks are a good hedge against less favorable times. Given that inflation volatility is low, a positive covariance between dividend growth and inflation can therefore generate a negative correlation between stock and bond returns. This is what the model suggests happened during the late 1990s. Note that the variance of the macroeconomic variables only contributes to a positive covariance in the model. The covariance terms are therefore important as they allow the model-implied correlations to switch sign.

Dividing the conditional covariance by the product of the conditional volatility of stock and bond returns allows me to compute conditional correlations. I use the extracted second moments from Section 3.2.2. to compute a time series of model-implied correlations. Figure 8 plots the model-implied quarterly conditional correlations. The model generates low conditional correlations in the beginning of the sample, highly positive correlations during the late 1970s and early 1980s and subsequently lower correlations. The model predicts correlations close to zero in the late 1990s, falling short of the sharply negative realized correlations observed for the same period. Similarly, the model did not foresee the negative correlations that occurred during the late 1950s. To evaluate the predictive ability of the model, I regress observed correlations within each quarter, that is between time t to $t + 1$, computed using daily data onto the model-implied conditional correlation at time t . Table 8 reports the results. The regression yields a statistically significant coefficient and an R_{adj}^2 of 13%. For comparison, regressing the observed quarterly correlation onto its own lag yields a statistically significant slope coefficient and an R_{adj}^2 of 38%. The last regression in the table includes the model forecast as an independent variable together with the lagged observed correlation. Including the model's prediction increases the R^2 from 38% to 41% while the model coefficient remains statistically significant. The model therefore seems to contain information beyond what is included in lagged correlations. Restricting the time period to 1970-2007 improves the model's ability to explain variations in future correlations, raising the R^2 from 13% to 20%.¹¹

Next I consider the implications of the model for realized stock and bond returns. Nominal quarterly stock returns are formed as: $r_{m,t+1}^{\$} = k_{0,m} + k_{1,m}pd_{t+1} - pd_t + \Delta d_{t+1} + \pi_{t+1}$ and nominal quarterly returns for the 5-year bond are computed as the difference between the 57-month log bond price and the 60-month log bond price, $h_{\$t+1,60m} = q_{t+1,57m}^{\$} - q_{t,60m}^{\$}$. I then form 20-quarter rolling correlations of the returns. Figure 9 displays model correlations vs. correlations of actual returns. The model correlations fit data quite well with a correlation between the two series of 0.71. The model therefore seems to capture low-frequency movements in realized correlations quite accurately. The late 1970s and early 1980s experienced high levels of volatility in both

¹¹Regression results for the subperiod are not reported in order to conserve space, but are available upon request.

consumption growth and inflation which made both stocks and bonds risky, generating a high positive correlation of returns. Volatility levels gradually decreased during the 1980s with the result of a gradual decline in the correlation. The downturn in model-implied correlations in the late 1980s are mainly due to an increase in the volatility of stock returns as the volatility of dividend growth increased sharply during that period. The early 1990s saw a spike in the correlations as consumption growth volatility increased sharply together with lower stock return volatility. Subsequently, up to year 2000, macroeconomic volatility decreased to historically low levels with the effect of lower correlations. The negative model correlations in the late 1990s arise as a result of low economic volatility together with a positive covariance between dividend growth and inflation. This positive covariance has two effects on equity returns in the model. For example, consider a period in which both dividend growth and inflation increase. First, stock returns respond positively through higher cash flows. Second, it lowers the equity risk premium since positive cash flow shocks that occur in bad times (rising inflation) imply that stocks are a hedge against bad inflationary times. At the same time, a slowly increasing inflation rate in the late 1990s made bonds perform badly, yielding negative bond returns. The model therefore attributes the negative correlations to an outperformance of stocks vs. bonds through higher cash flows and a lower equity risk premium together with a series of small positive inflation shocks. The fact that the model does not predict negative correlations in the late 1990s but capture some of the negative realized correlations is due to a higher covariance of dividend growth and inflation than expected. The model-implied correlations increased sharply in the early 2000s as the volatility of inflation started to pick up. However, in data the correlations remained negative as stock prices fell and the Federal Reserve started cutting interest rates to stimulate the economy, yielding positive bond returns. This is not the first paper to have difficulties explaining and matching the extent of the negative correlations observed throughout the current decade. The unconditional correlation of stock and bond returns is 0.25 in the model, which is somewhat higher than the 0.10 observed in data.

7 Conclusion

This paper proposes a consumption-based equilibrium model that to a large extent can explain two features of data that have been considered puzzling. First, the paper shows that the strikingly high correlation between US dividend yields and nominal interest can be explained within a rational model through a risk-premium channel. This stands in contrast to the hitherto dominant explanation in the form of inflation illusion. Second, the model attributes a large part of changes in realized correlations between stock and bond returns to changes in macroeconomic risk. High volatility of consumption growth and inflation caused stock and bond returns to comove strongly in the late 1970s and early 1980s. Risk premiums on both equity and bonds in the model react similarly to changes in macroeconomic volatility, making their returns positively correlated. The subsequent decline in aggregate economic risk from the early 1980s until 2000 is suggested to have brought correlations lower. The negative correlations observed in the late 1990s are partly attributed to low levels of macroeconomic volatility in conjunction with a positive covariance between dividend growth and inflation.

However, there are still some unresolved issues from the perspective of the model. First, the estimated negative relation between consumption growth and inflation is a statistical relation. The model is silent on what the actual underlying mechanisms are. A possible explanation would be monetary policy through which a central bank, keen on bringing down inflation expectations, raise short rates such that consumption growth contracts in the following periods. An interesting area for future research would be to examine the role of monetary policy for explaining the so called Fed-model and correlations between asset returns, but also its role for determining risk premiums in general.

Second, starting in year 2000, inflation volatility started to increase and has today reached levels last seen in the early 1980s. This implies a positive correlation between stock and bond returns in the model. However, correlations in data have remained negative throughout the current decade which suggests that other forces are at work. The extent of the negative correlations have puzzled many others in the literature as well and warrants a further investigation. A particularly interesting period to analyze is the recent financial crisis in which stock and bond returns have

tended to be negatively correlated. Expecting consumption-based models to fully explain asset correlations during such extreme periods is perhaps too much to hope for. Instead, modeling so called liquidity factors jointly with macro factors is likely to yield more insights into so called “flight-to-safety” periods.

8 Appendix

Sections A.1 - A.4 solve the model using approximate analytical solutions for the case of heteroscedasticity (Specification II). Coefficients for the conditional first moments, x_c, x_π, x_d , are the same for the homoscedastic Specification I and for the heteroscedastic Specification II.

A.1 The price-consumption ratio

The coefficients governing the price-consumption ratio are derived using the logarithm of the intertemporal marginal rate of substitution, $m_{t+1} = \theta \ln(\delta) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)r_{c,t+1}$, together with the dynamics of consumption growth and inflation in Section 2.1.1, the volatility dynamics in Section 2.1.2., and the approximation of the return on the consumption paying asset, $r_{c,t+1} = k_{c,0} + k_{c,1}pc_{t+1} - pc_t + \Delta c_{t+1}$, where $pc_t = A_{c,0} + A_{c,1}x_{c,t} + A_{c,2}x_{\pi,t} + A_{c,3}x_{d,t} + A_{c,4}\sigma_{c,t}^2 + A_{c,5}\sigma_{\pi,t}^2 + A_{c,6}\sigma_{d,t}^2 + A_{c,7}\sigma_{c\pi,t} + A_{c,8}\sigma_{cd,t} + A_{c,9}\sigma_{\pi d,t}$. Consider the Euler equation for the consumption claim:

$$E_t \left[\exp\left(\theta \ln(\delta) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)r_{c,t+1} + r_{c,t+1}\right) \right] = 1.$$

Due to the conditional normality of Δc and the state variables, and therefore also r_c , the log Euler condition can be written as:

$$E_t [m_{t+1} + r_{c,t+1}] + \frac{1}{2} Var_t [m_{t+1} + r_{c,t+1}] = 0.$$

The conditional mean is given by:

$$\begin{aligned}
E_t [m_{t+1} + r_{c,t+1}] &= \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + \theta(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,4}\alpha_c(1 - \phi_c) + A_{c,5}\alpha_\pi(1 - \phi_\pi) + \\
&A_{c,6}\alpha_d(1 - \phi_d) + A_{c,7}\alpha_{c\pi}(1 - \phi_{c\pi}) + A_{c,8}\alpha_{cd}(1 - \phi_{cd}) + A_{c,9}\alpha_{\pi d}(1 - \phi_{\pi d})) - A_{c,0} + \mu_c) + \\
&x_{c,t} \left[-\frac{\theta}{\psi} + \theta(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - A_{c,1} + 1) \right] + \\
&x_{\pi,t} [\theta(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2})] + x_{d,t} [\theta A_{c,3}(k_{c,1}\beta_6 - 1)] + \\
&\sigma_{c,t}^2 [\theta A_{c,4}(k_{c,1}\phi_c - 1)] + \sigma_{\pi,t}^2 [\theta A_{c,5}(k_{c,1}\phi_\pi - 1)] + \\
&\sigma_{d,t}^2 [\theta A_{c,6}(k_{c,1}\phi_d - 1)] + \sigma_{c\pi,t} [\theta A_{c,7}(k_{c,1}\phi_{c\pi} - 1)] + \\
&\sigma_{cd,t} [\theta A_{c,8}(k_{c,1}\phi_{cd} - 1)] + \sigma_{\pi d,t} [\theta A_{c,9}(k_{c,1}\phi_{\pi d} - 1)].
\end{aligned}$$

and the conditional variance is given by:

$$\begin{aligned}
Var_t [m_{t+1} + r_{c,t+1}] &= \sigma_{c,t}^2 X^2 + \sigma_{\pi,t}^2 Y^2 + \sigma_{d,t}^2 Z^2 + 2\sigma_{c\pi,t} XY + 2\sigma_{cd,t} XZ + 2\sigma_{\pi d,t} YZ + \\
&(\theta k_{c,1} A_{c,4} \tau_c)^2 + (\theta k_{c,1} A_{c,5} \tau_\pi)^2 + (\theta k_{c,1} A_{c,6} \tau_d)^2 + (\theta k_{c,1} A_{c,7} \tau_{c\pi})^2 + \\
&(\theta k_{c,1} A_{c,8} \tau_{cd})^2 + (\theta k_{c,1} A_{c,9} \tau_{\pi d})^2, \\
X &= \left[-\frac{\theta}{\psi} + \theta(k_{c,1}A_{c,1}\delta_1 + k_{c,1}A_{c,2}\delta_3 + k_{c,1}A_{c,3}\delta_5 + 1) \right], \\
Y &= [\theta(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4)], \\
Z &= [\theta(k_{c,1}A_{c,3}\delta_6)].
\end{aligned}$$

Setting the conditional moments equal to zero and solving for the A_c -coefficients yield the following expressions:

$$\begin{aligned}
A_{c,0} &= (\theta(1 - k_{c,1}))^{-1} \left[\theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + \theta(k_{c,0} + k_{c,1}(A_{c,4}\alpha_c(1 - \phi_c) + A_{c,5}\alpha_\pi(1 - \phi_\pi) + \right. \\
&\quad A_{c,6}\alpha_d(1 - \phi_d) + A_{c,7}\alpha_{c\pi}(1 - \phi_{c\pi}) + A_{c,8}\alpha_{cd}(1 - \phi_{cd}) + A_{c,9}\alpha_{\pi d}(1 - \phi_{\pi d})) + \mu_c) + \\
&\quad 0.5((\theta k_{c,1}A_{c,4}\tau_c)^2 + (\theta k_{c,1}A_{c,5}\tau_\pi)^2 + (\theta k_{c,1}A_{c,6}\tau_d)^2 + (\theta k_{c,1}A_{c,7}\tau_{c\pi})^2 + \\
&\quad \left. (\theta k_{c,1}A_{c,8}\tau_{cd})^2 + (\theta k_{c,1}A_{c,9}\tau_{\pi d})^2) \right], \\
A_{c,1} &= \frac{1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - \frac{1}{\psi}}{1 - k_{c,1}\beta_1}, \\
A_{c,2} &= \frac{k_{c,1}A_{c,1}\beta_2}{1 - k_{c,1}\beta_4}, \\
A_{c,3} &= 0, \\
A_{c,4} &= \frac{-0.5X^2}{\theta(k_{c,1}\phi_c - 1)}, \\
A_{c,5} &= \frac{-0.5Y^2}{\theta(k_{c,1}\phi_\pi - 1)}, \\
A_{c,6} &= \frac{-0.5Z^2}{\theta(k_{c,1}\phi_d - 1)}, \\
A_{c,7} &= \frac{-XY}{\theta(k_{c,1}\phi_{c\pi} - 1)}, \\
A_{c,8} &= \frac{-XZ}{\theta(k_{c,1}\phi_{cd} - 1)}, \\
A_{c,9} &= \frac{-YZ}{\theta(k_{c,1}\phi_{\pi d} - 1)},
\end{aligned}$$

where X,Y, and Z are determined as above. Coefficients $A_{c,6}$, $A_{c,8}$, and $A_{c,9}$ are zero since $A_{c,3}$ equals zero. Note that $A_{c,1}$ and $A_{c,2}$ are determined jointly. Solving the simultaneous equation system by substituting $A_{c,1}$ into $A_{c,2}$ returns:

$$A_{c,2} = \frac{k_{c,1}\beta_2(1 - \frac{1}{\psi})}{(1 - k_{c,1}\beta_4)(1 - k_{c,1}\beta_1) - k_{c,1}^2\beta_2\beta_3}.$$

The homoscedastic dynamics (Specification I) has a different $A_{c,0}$ term, namely:

$$A_{c,0} = \frac{\left[\theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + \theta(k_{c,0} + \mu_c) + 0.5(\sigma_c^2 X^2 + \sigma_\pi^2 Y^2 + \sigma_d^2 Z^2 + 2\sigma_{c\pi} XY + 2\sigma_{cd} XZ + 2\sigma_{\pi d} YZ) \right]}{\theta(1 - k_{c,1})},$$

where X,Y, and Z are determined as above.

A.2 The price-dividend ratio

The coefficients governing the price-dividend ratio are found in an analogous manner. The Euler condition for the market return, $r_{m,t+1}$, is written as:

$$E_t [m_{t+1} + r_{m,t+1}] + \frac{1}{2} Var_t [m_{t+1} + r_{m,t+1}],$$

where $r_{m,t+1} = k_{d,0} + k_{d,1}pd_{t+1} - pd_t + \Delta d_{t+1}$ and $pd_t = A_{d,0} + A_{d,1}x_{c,t} + A_{d,2}x_{\pi,t} + A_{d,3}x_{d,t} + A_{d,4}\sigma_{c,t}^2 + A_{d,5}\sigma_{\pi,t}^2 + A_{d,6}\sigma_{d,t}^2 + A_{d,7}\sigma_{c\pi,t} + A_{d,8}\sigma_{cd,t} + A_{d,9}\sigma_{\pi d,t}$. Coefficients $k_{d,0}$ and $k_{d,1}$ are defined as:

$$\begin{aligned} k_{d,0} &= \ln(1 + \exp(\bar{pd})) - k_{d,1}\bar{pd}, \\ k_{d,1} &= \frac{\exp(\bar{pd})}{1 + \exp(\bar{pd})}. \end{aligned}$$

Using the dynamics of consumption growth, inflation, dividend growth, and the second moments, the conditional mean is given by:

$$\begin{aligned}
E_t[m_{t+1} + r_{m,t+1}] &= \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,4}\alpha_c(1 - \phi_c) + A_{c,5}\alpha_\pi(1 - \phi_\pi) + \\
&A_{c,6}\alpha_d(1 - \phi_d) + A_{c,7}\alpha_{c\pi}(1 - \phi_{c\pi}) + A_{c,8}\alpha_{cd}(1 - \phi_{cd}) + A_{c,9}\alpha_{\pi d}(1 - \phi_{\pi d})) - A_{c,0} + \mu_c) + \\
&+ k_{d,0} + k_{d,1}(A_{d,0} + A_{d,4}\alpha_c(1 - \phi_c) + A_{d,5}\alpha_\pi(1 - \phi_\pi) + A_{d,6}\alpha_d(1 - \phi_d) + \\
&A_{d,7}\alpha_{c\pi}(1 - \phi_{c\pi}) + A_{d,8}\alpha_{cd}(1 - \phi_{cd}) + A_{d,9}\alpha_{\pi d}(1 - \phi_{\pi d})) - A_{d,0} + \mu_d + \\
&x_{c,t} \left[-\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - A_{c,1} + 1) + k_{d,1}A_{d,1}\beta_1 + \right. \\
&k_{d,1}A_{d,2}\beta_3 + k_{d,1}A_{d,3}\beta_5 - A_{d,1} \left. \right] + \\
&x_{\pi,t} [(\theta - 1)(k_{d,1}A_{d,1}\beta_2 + k_{d,1}A_{d,2}\beta_4 - A_{d,2}) + k_{d,1}A_{d,1}\beta_2 + k_{d,1}A_{d,2}\beta_4 - A_{d,2}] + \\
&x_{d,t} [(\theta - 1)A_{c,3}(k_{c,1}\beta_6 - 1) + k_{d,1}A_{d,3}\beta_6 - A_{d,3} + 1], \\
&\sigma_{c,t}^2 [(\theta - 1)A_{c,4}(k_{c,1}\phi_c - 1) + A_{d,4}(k_{d,1}\phi_c - 1)], \\
&\sigma_{\pi,t}^2 [(\theta - 1)A_{c,5}(k_{c,1}\phi_\pi - 1) + A_{d,5}(k_{d,1}\phi_\pi - 1)], \\
&\sigma_{d,t}^2 [(\theta - 1)A_{c,6}(k_{c,1}\phi_d - 1) + A_{d,6}(k_{d,1}\phi_d - 1)], \\
&\sigma_{c\pi,t}^2 [(\theta - 1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1) + A_{d,7}(k_{d,1}\phi_{c\pi} - 1)], \\
&\sigma_{cd,t}^2 [(\theta - 1)A_{c,8}(k_{c,1}\phi_{cd} - 1) + A_{d,8}(k_{d,1}\phi_{cd} - 1)], \\
&\sigma_{\pi d,t}^2 [(\theta - 1)A_{c,9}(k_{c,1}\phi_{\pi d} - 1) + A_{d,9}(k_{d,1}\phi_{\pi d} - 1)].
\end{aligned}$$

The conditional variance is given by:

$$\begin{aligned}
Var_t [m_{t+1} + r_{m,t+1}] &= \sigma_{c,t}^2 X^2 + \sigma_{\pi,t}^2 Y^2 + \sigma_{d,t}^2 Z^2 + 2\sigma_{c\pi,t} XY + 2\sigma_{cd,t} XZ + 2\sigma_{\pi d,t} YZ + \\
& ((\theta - 1)k_{c,1}A_{c,4}\tau_c + k_{d,1}A_{d,4}\tau_c)^2 + ((\theta - 1)k_{c,1}A_{c,5}\tau_\pi + k_{d,1}A_{d,5}\tau_\pi)^2 + \\
& ((\theta - 1)k_{c,1}A_{c,6}\tau_d + k_{d,1}A_{d,6}\tau_d)^2 + ((\theta - 1)k_{c,1}A_{c,7}\tau_{c\pi} + k_{d,1}A_{d,7}\tau_{c\pi})^2 + \\
& ((\theta - 1)k_{c,1}A_{c,8}\tau_{cd} + k_{d,1}A_{d,8}\tau_{cd})^2 + ((\theta - 1)k_{c,1}A_{c,9}\tau_{\pi d} + k_{d,1}A_{d,9}\tau_{\pi d})^2, \\
X &= \left[-\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\delta_1 + k_{c,1}A_{c,2}\delta_3 + k_{c,1}A_{c,3}\delta_5 + 1) + \right. \\
& \left. k_{d,1}A_{d,1}\delta_1 + k_{d,1}A_{d,2}\delta_3 + k_{d,1}A_{d,3}\delta_5 \right], \\
Y &= [(\theta - 1)(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4) + k_{d,1}A_{d,1}\delta_2 + k_{d,1}A_{d,2}\delta_4], \\
Z &= [(\theta - 1)(k_{c,1}A_{c,3}\delta_6) + k_{d,1}A_{d,3}\delta_6 + 1].
\end{aligned}$$

Setting the conditional moments equal to zero and solving for the A_d -coefficients yield the following expressions:

$$\begin{aligned}
A_{d,0} &= (1 - k_{d,1})^{-1} \left[\theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,4}\alpha_c(1 - \phi_c) + A_{c,5}\alpha_\pi(1 - \phi_\pi) + \right. \\
&\quad A_{c,6}\alpha_d(1 - \phi_d) + A_{c,7}\alpha_{c\pi}(1 - \phi_{c\pi}) + A_{c,8}\alpha_{cd}(1 - \phi_{cd}) + A_{c,9}\alpha_{\pi d}(1 - \phi_{\pi d})) - A_{c,0} + \mu_c) + k_{d,0} + \\
&\quad k_{d,1}(A_{d,4}\alpha_c(1 - \phi_c) + A_{d,5}\alpha_\pi(1 - \phi_\pi) + A_{d,6}\alpha_d(1 - \phi_d) + \\
&\quad A_{d,7}\alpha_{c\pi}(1 - \phi_{c\pi}) + A_{d,8}\alpha_{cd}(1 - \phi_{cd}) + A_{d,9}\alpha_{\pi d}(1 - \phi_{\pi d})) + \mu_d + \\
&\quad 0.5(((\theta - 1)k_{c,1}A_{c,4}\tau_c + k_{d,1}A_{d,4}\tau_c)^2 + ((\theta - 1)k_{c,1}A_{c,5}\tau_\pi + k_{d,1}A_{d,5}\tau_\pi)^2 + \\
&\quad ((\theta - 1)k_{c,1}A_{c,6}\tau_d + k_{d,1}A_{d,6}\tau_d)^2 + ((\theta - 1)k_{c,1}A_{c,7}\tau_{c\pi} + k_{d,1}A_{d,7}\tau_{c\pi})^2 + \\
&\quad \left. ((\theta - 1)k_{c,1}A_{c,8}\tau_{cd} + k_{d,1}A_{d,8}\tau_{cd})^2 + ((\theta - 1)k_{c,1}A_{c,9}\tau_{\pi d} + k_{d,1}A_{d,9}\tau_{\pi d})^2) \right], \\
A_{d,1} &= \frac{-\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - A_{c,1} + 1) + k_{d,1}A_{d,2}\beta_3 + k_{d,1}A_{d,3}\beta_5}{1 - k_{d,1}\beta_1}, \\
A_{d,2} &= \frac{(\theta - 1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}) + k_{d,1}A_{d,1}\beta_2}{1 - k_{d,1}\beta_4}, \\
A_{d,3} &= \frac{(\theta - 1)A_{c,3}(k_{c,1}\beta_6 - 1) + 1}{1 - k_{d,1}\beta_6}, \\
A_{d,4} &= \frac{(\theta - 1)A_{c,4}(k_{c,1}\phi_c - 1) + 0.5X^2}{1 - k_{d,1}\phi_c}, \\
A_{d,5} &= \frac{(\theta - 1)A_{c,5}(k_{c,1}\phi_\pi - 1) + 0.5Y^2}{1 - k_{d,1}\phi_\pi}, \\
A_{d,6} &= \frac{(\theta - 1)A_{c,6}(k_{c,1}\phi_d - 1) + 0.5Z^2}{1 - k_{d,1}\phi_d}, \\
A_{d,7} &= \frac{(\theta - 1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1) + XY}{1 - k_{d,1}\phi_{c\pi}}, \\
A_{d,8} &= \frac{(\theta - 1)A_{c,8}(k_{c,1}\phi_{cd} - 1) + XZ}{1 - k_{d,1}\phi_{cd}}, \\
A_{d,9} &= \frac{(\theta - 1)A_{c,9}(k_{c,1}\phi_{\pi d} - 1) + ZY}{1 - k_{d,1}\phi_{\pi d}},
\end{aligned}$$

where X,Y, and Z are determined as above. Similar to the consumption paying asset, $A_{d,1}$ and $A_{d,2}$ are determined jointly. Substituting $A_{d,1}$ into $A_{d,2}$ yields:

$$\begin{aligned} A_{d,2} &= \frac{(1 - k_{d,1}\beta_1)X + Y}{(1 - k_{d,1}\beta_4)(1 - k_{d,1}\beta_1) - k_{d,1}^2\beta_2\beta_3}, \\ X &= (\theta - 1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}), \\ Y &= k_{d,1}\beta_2 \left[-\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - A_{c,1} + 1) + k_{d,1}A_{d,3}\beta_5 \right]. \end{aligned}$$

The homoscedastic dynamics (Specification I) has a different $A_{d,0}$ term, namely:

$$\begin{aligned} A_{d,0} &= ((1 - k_{d,1}))^{-1} \left[\theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}A_{c,0} - A_{c,0} + \mu_c) + k_{d,0} + \mu_d + \right. \\ &\quad \left. 0.5(\sigma_c^2 X^2 + \sigma_\pi^2 Y^2 + \sigma_d^2 Z^2 + 2\sigma_{c\pi}XY + 2\sigma_{cd}XZ + 2\sigma_{\pi d}YZ) \right] \end{aligned}$$

where X,Y, and Z are determined as for conditional variance above.

A.3 Real bonds

The Euler condition for a real bond takes the form:

$$Q_{t,n} = E_t [M_{t+1}Q_{t+1,n-1}],$$

where $Q_{t,n} = \exp(D_{0,n} + D_{1,n}x_{c,t} + D_{2,n}x_{\pi,t} + D_{3,n}x_{d,t} + D_{4,n}\sigma_{c,t}^{\$} + D_{5,n}\sigma_{\pi,t}^{\$} + D_{6,n}\sigma_{d,t}^{\$} + D_{7,n}\sigma_{c\pi,t} + D_{8,n}\sigma_{cd,t} + D_{9,n}\sigma_{\pi d,t})$. Again, using the conditional lognormality of the state variables:

$$q_{t,n} = E_t [m_{t+1} + q_{t+1,n-1}] + \frac{1}{2} Var_t [m_{t+1} + q_{t+1,n-1}].$$

The conditional mean is given by:

$$\begin{aligned}
E_t [m_{t+1} + q_{t+1,n-1}] &= \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,4}\alpha_c(1 - \phi_c) + A_{c,5}\alpha_\pi(1 - \phi_\pi) + \\
&A_{c,6}\alpha_d(1 - \phi_d) + A_{c,7}\alpha_{c\pi}(1 - \phi_{c\pi}) + A_{c,8}\alpha_{cd}(1 - \phi_{cd}) + A_{c,9}\alpha_{\pi d}(1 - \phi_{\pi d})) - A_{c,0} + \mu_c) + D_{0,n-1} + \\
&D_{4,n-1}\alpha_c(1 - \phi_c) + D_{5,n-1}\alpha_\pi(1 - \phi_\pi) + D_{6,n-1}\alpha_d(1 - \phi_d) + \\
&D_{7,n-1}\alpha_{c\pi}(1 - \phi_{c\pi}) + D_{8,n-1}\alpha_{cd}(1 - \phi_{cd}) + D_{9,n-1}\alpha_{\pi d}(1 - \phi_{\pi d}) + \\
&x_{c,t} \left[-\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - A_{c,1} + 1) + D_{1,n-1}\beta_1 + D_{2,n-1}\beta_3 + D_{3,n-1}\beta_5 \right] + \\
&x_{\pi,t} [(\theta - 1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}) + D_{1,n-1}\beta_2 + D_{2,n-1}\beta_4] + \\
&x_{d,t} [(\theta - 1)A_{c,3}(k_{c,1}\beta_6 - 1) + D_{3,n-1}\beta_6] + \\
&\sigma_{c,t}^2 [(\theta - 1)A_{c,4}(k_{c,1}\phi_c - 1) + D_{4,n-1}\phi_c] + \\
&\sigma_{\pi,t}^2 [(\theta - 1)A_{c,5}(k_{c,1}\phi_\pi - 1) + D_{5,n-1}\phi_\pi] + \\
&\sigma_{d,t}^2 [(\theta - 1)A_{c,6}(k_{c,1}\phi_d - 1) + D_{6,n-1}\phi_d] + \\
&\sigma_{c\pi,t} [(\theta - 1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1) + D_{7,n-1}\phi_{c\pi}] + \\
&\sigma_{cd,t} [(\theta - 1)A_{c,8}(k_{c,1}\phi_{cd} - 1) + D_{8,n-1}\phi_{cd}] + \\
&\sigma_{\pi d,t} [(\theta - 1)A_{c,9}(k_{c,1}\phi_{\pi d} - 1) + D_{9,n-1}\phi_{\pi d}],
\end{aligned}$$

and the conditional variance by:

$$\begin{aligned}
Var_t [m_{t+1} + q_{t+1,n-1}] &= \sigma_{c,t}^2 X^2 + \sigma_{\pi,t}^2 Y^2 + \sigma_{d,t}^2 Z^2 + 2\sigma_{c\pi,t} XY + 2\sigma_{cd,t} XZ + 2\sigma_{\pi d,t} YZ + \\
&((\theta - 1)k_{c,1}A_{c,4}\tau_c + D_{4,n-1}\tau_c)^2 + ((\theta - 1)k_{c,1}A_{c,5}\tau_\pi + D_{5,n-1}\tau_\pi)^2 + ((\theta - 1)k_{c,1}A_{c,6}\tau_d + D_{6,n-1}\tau_d)^2 + \\
&((\theta - 1)k_{c,1}A_{c,7}\tau_{c\pi} + D_{7,n-1}\tau_{c\pi})^2 + ((\theta - 1)k_{c,1}A_{c,8}\tau_{cd} + D_{8,n-1}\tau_{cd})^2 + ((\theta - 1)k_{c,1}A_{c,9}\tau_{\pi d} + D_{9,n-1}\tau_{\pi d})^2, \\
X &= \left[-\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\delta_1 + k_{c,1}A_{c,2}\delta_3 + k_{c,1}A_{c,3}\delta_5 + 1) + D_{1,n-1}\delta_1 + D_{2,n-1}\delta_3 + D_{3,n-1}\delta_5 \right], \\
Y &= [(\theta - 1)(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4) + D_{1,n-1}\delta_2 + D_{2,n-1}\delta_4], \\
Z &= [(\theta - 1)(k_{c,1}A_{c,3}\delta_6) + D_{3,n-1}\delta_6].
\end{aligned}$$

Matching the coefficients gives:

$$\begin{aligned}
D_{0,n} &= \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,4}\alpha_c(1 - \phi_c) + A_{c,5}\alpha_\pi(1 - \phi_\pi) + \\
&\quad A_{c,6}\alpha_d(1 - \phi_d) + A_{c,7}\alpha_{c\pi}(1 - \phi_{c\pi}) + A_{c,8}\alpha_{cd}(1 - \phi_{cd}) + A_{c,9}\alpha_{\pi d}(1 - \phi_{\pi d})) - A_{c,0} + \mu_c) + D_{0,n-1} + \\
&\quad D_{4,n-1}\alpha_c(1 - \phi_c) + D_{5,n-1}\alpha_\pi(1 - \phi_\pi) + D_{6,n-1}\alpha_d(1 - \phi_d) + \\
&\quad D_{7,n-1}\alpha_{c\pi}(1 - \phi_{c\pi}) + D_{8,n-1}\alpha_{cd}(1 - \phi_{cd}) + D_{9,n-1}\alpha_{\pi d}(1 - \phi_{\pi d}) + \\
&\quad 0.5(((\theta - 1)k_{c,1}A_{c,4}\tau_c + D_{4,n-1}\tau_c)^2 + ((\theta - 1)k_{c,1}A_{c,5}\tau_\pi + D_{5,n-1}\tau_\pi)^2 + \\
&\quad ((\theta - 1)k_{c,1}A_{c,6}\tau_d + D_{6,n-1}\tau_d)^2 + ((\theta - 1)k_{c,1}A_{c,7}\tau_{c\pi} + D_{7,n-1}\tau_{c\pi})^2 + ((\theta - 1)k_{c,1}A_{c,8}\tau_{cd} + D_{8,n-1}\tau_{cd})^2 + \\
&\quad ((\theta - 1)k_{c,1}A_{c,9}\tau_{\pi d} + D_{9,n-1}\tau_{\pi d})^2), \\
D_{1,n} &= -\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - A_{c,1} + 1) + D_{1,n-1}\beta_1 + D_{2,n-1}\beta_3, \\
D_{2,n} &= (\theta - 1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}) + D_{1,n-1}\beta_2 + D_{2,n-1}\beta_4, \\
D_{3,n} &= (\theta - 1)A_{c,3}(k_{c,1}\beta_6 - 1) + D_{3,n-1}\beta_6, \\
D_{4,n} &= (\theta - 1)A_{c,4}(k_{c,1}\phi_c - 1) + D_{4,n-1}\phi_c + 0.5X^2, \\
D_{5,n} &= (\theta - 1)A_{c,5}(k_{c,1}\phi_\pi - 1) + D_{5,n-1}\phi_\pi + 0.5Y^2, \\
D_{6,n} &= (\theta - 1)A_{c,6}(k_{c,1}\phi_d - 1) + D_{6,n-1}\phi_d + 0.5Z^2, \\
D_{7,n} &= (\theta - 1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1) + D_{7,n-1}\phi_{c\pi} + XY, \\
D_{8,n} &= (\theta - 1)A_{c,8}(k_{c,1}\phi_{cd} - 1) + D_{8,n-1}\phi_{cd} + XZ, \\
D_{9,n} &= (\theta - 1)A_{c,9}(k_{c,1}\phi_{\pi d} - 1) + D_{9,n-1}\phi_{\pi d} + YZ.
\end{aligned}$$

The coefficients are computed recursively using the fact that $D_{i,0} = 0$ for $i = 0, 1, 2$. Note that coefficients $D_{3,n}$, $D_{6,n}$, $D_{8,n}$, and $D_{9,n}$ are zero since $A_{c,3}$, $A_{c,6}$, $A_{c,8}$, and $A_{c,9}$ equal zero. The homoscedastic dynamics (Specification I) has a different $D_{0,n}$ term, namely:

$$\begin{aligned}
D_{0,n} &= \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}A_{c,0} - A_{c,0} + \mu_c) + D_{0,n-1} + \\
&\quad 0.5(\sigma_c^2 X^2 + \sigma_\pi^2 Y^2 + \sigma_d^2 Z^2 + 2\sigma_{c\pi}XY + 2\sigma_{cd}XZ + 2\sigma_{\pi d}YZ)
\end{aligned}$$

where X, Y, and Z are determined as for conditional variance above.

A.4 Nominal bonds

The Euler condition for the real price of a nominal bond is:

$$\begin{aligned}\frac{Q_{t,n}^{\$}}{\Pi_t} &= E_t \left[M_{t+1} \frac{Q_{t+1,n-1}^{\$}}{\Pi_{t+1}} \right], \\ Q_{t,n}^{\$} &= E_t \left[M_{t+1} \frac{Q_{t+1,n-1}^{\$} \Pi_t}{\Pi_{t+1}} \right],\end{aligned}$$

where the following is conjectured: $Q_{t,n}^{\$} = \exp(D_{0,n}^{\$} + D_{1,n}^{\$}x_{c,t} + D_{2,n}^{\$}x_{\pi,t} + D_{3,n}^{\$}x_{d,t} + D_{4,n}^{\$}\sigma_{c,t}^2 + D_{5,n}^{\$}\sigma_{\pi,t}^2 + D_{6,n}^{\$}\sigma_{d,t}^2 + D_{7,n}^{\$}\sigma_{c\pi,t} + D_{8,n}^{\$}\sigma_{cd,t} + D_{9,n}^{\$}\sigma_{\pi d,t})$. Taking logs and again using the conditional lognormality yields:

$$q_{t,n}^{\$} = E_t \left[m_{t+1} - \pi_{t+1} + q_{t+1,n-1}^{\$} \right] + \frac{1}{2} \text{Var}_t \left[m_{t+1} - \pi_{t+1} + q_{t+1,n-1}^{\$} \right].$$

The conditional mean is given by:

$$\begin{aligned}
E_t \left[m_{t+1} - \pi_{t+1} + q_{t+1,n-1}^{\$} \right] &= \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,4}\alpha_c(1 - \phi_c) + \\
&A_{c,5}\alpha_\pi(1 - \phi_\pi) + A_{c,6}\alpha_d(1 - \phi_d) + A_{c,7}\alpha_{c\pi}(1 - \phi_{c\pi}) + A_{c,8}\alpha_{cd}(1 - \phi_{cd}) + A_{c,9}\alpha_{\pi d}(1 - \phi_{\pi d})) - A_{c,0} + \mu_c) - \\
&\mu_\pi + D_{0,n-1}^{\$} + D_{4,n-1}^{\$}\alpha_c(1 - \phi_c) + D_{5,n-1}^{\$}\alpha_\pi(1 - \phi_\pi) + D_{6,n-1}^{\$}\alpha_d(1 - \phi_d) + \\
&D_{7,n-1}^{\$}\alpha_{c\pi}(1 - \phi_{c\pi}) + D_{8,n-1}^{\$}\alpha_{cd}(1 - \phi_{cd}) + D_{9,n-1}^{\$}\alpha_{\pi d}(1 - \phi_{\pi d}) + \\
&x_{c,t} \left[-\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - A_{c,1} + 1) + D_{1,n-1}^{\$}\beta_1 + D_{2,n-1}^{\$}\beta_3 \right] + \\
&x_{\pi,t} \left[(\theta - 1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}) - 1 + D_{1,n-1}^{\$}\beta_2 + D_{2,n-1}^{\$}\beta_4 \right], \\
&x_{d,t} \left[(\theta - 1)A_{c,3}(k_{c,1}\beta_6 - 1) + D_{3,n-1}^{\$}\beta_6 \right] + \\
&\sigma_{c,t}^2 \left[(\theta - 1)A_{c,4}(k_{c,1}\phi_c - 1) + D_{4,n-1}^{\$}\phi_c \right] + \\
&\sigma_{\pi,t}^2 \left[(\theta - 1)A_{c,5}(k_{c,1}\phi_\pi - 1) + D_{5,n-1}^{\$}\phi_\pi \right] + \\
&\sigma_{d,t}^2 \left[(\theta - 1)A_{c,6}(k_{c,1}\phi_d - 1) + D_{6,n-1}^{\$}\phi_d \right] + \\
&\sigma_{c\pi,t} \left[(\theta - 1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1) + D_{7,n-1}^{\$}\phi_{c\pi} \right] + \\
&\sigma_{cd,t} \left[(\theta - 1)A_{c,8}(k_{c,1}\phi_{cd} - 1) + D_{8,n-1}^{\$}\phi_{cd} \right] + \\
&\sigma_{\pi d,t} \left[(\theta - 1)A_{c,9}(k_{c,1}\phi_{\pi d} - 1) + D_{9,n-1}^{\$}\phi_{\pi d} \right],
\end{aligned}$$

and the conditional variance by:

$$\begin{aligned}
Var_t \left[m_{t+1} - \pi_{t+1} + q_{t+1,n-1}^{\$} \right] &= \sigma_{c,t}^2 X^2 + \sigma_{\pi,t}^2 Y^2 + \sigma_{d,t}^2 Z^2 + 2\sigma_{c\pi,t} XY + 2\sigma_{cd,t} XZ + 2\sigma_{\pi d,t} YZ + \\
&((\theta - 1)k_{c,1}A_{c,4}\tau_c + D_{4,n-1}^{\$}\tau_c)^2 + ((\theta - 1)k_{c,1}A_{c,5}\tau_\pi + D_{5,n-1}^{\$}\tau_\pi)^2 + \\
&((\theta - 1)k_{c,1}A_{c,6}\tau_d + D_{6,n-1}^{\$}\tau_d)^2 + ((\theta - 1)k_{c,1}A_{c,7}\tau_{c\pi} + D_{7,n-1}^{\$}\tau_{c\pi})^2 + \\
&((\theta - 1)k_{c,1}A_{c,8}\tau_{cd} + D_{8,n-1}^{\$}\tau_{cd})^2 + ((\theta - 1)k_{c,1}A_{c,9}\tau_{\pi d} + D_{9,n-1}^{\$}\tau_{\pi d})^2, \\
X &= \left[-\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\delta_1 + k_{c,1}A_{c,2}\delta_3 + k_{c,1}A_{c,3}\delta_5 + 1) + D_{1,n-1}^{\$}\delta_1 + D_{2,n-1}^{\$}\delta_3 + D_{3,n-1}^{\$}\delta_5 \right], \\
Y &= \left[(\theta - 1)(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4) - 1 + D_{1,n-1}^{\$}\delta_2 + D_{2,n-1}^{\$}\delta_4 \right], \\
Z &= \left[(\theta - 1)(k_{c,1}A_{c,3}\delta_6) + D_{3,n-1}^{\$}\delta_6 \right].
\end{aligned}$$

Matching the coefficients gives:

$$\begin{aligned}
D_{0,n}^{\$} &= \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,4}\alpha_c(1 - \phi_c) + \\
&\quad A_{c,5}\alpha_\pi(1 - \phi_\pi) + A_{c,6}\alpha_d(1 - \phi_d) + A_{c,7}\alpha_{c\pi}(1 - \phi_{c\pi}) + A_{c,8}\alpha_{cd}(1 - \phi_{cd}) + A_{c,9}\alpha_{\pi d}(1 - \phi_{\pi d})) - \\
&\quad A_{c,0} + \mu_c) - \mu_\pi + D_{0,n-1}^{\$} + D_{4,n-1}^{\$}\alpha_c(1 - \phi_c) + D_{5,n-1}^{\$}\alpha_\pi(1 - \phi_\pi) + D_{6,n-1}^{\$}\alpha_d(1 - \phi_d) + \\
&\quad D_{7,n-1}^{\$}\alpha_{c\pi}(1 - \phi_{c\pi}) + D_{8,n-1}^{\$}\alpha_{cd}(1 - \phi_{cd}) + D_{9,n-1}^{\$}\alpha_{\pi d}(1 - \phi_{\pi d}) + \\
&\quad 0.5(((\theta - 1)k_{c,1}A_{c,4}\tau_c + D_{4,n-1}^{\$}\tau_c)^2 + ((\theta - 1)k_{c,1}A_{c,5}\tau_\pi + D_{5,n-1}^{\$}\tau_\pi)^2 + \\
&\quad ((\theta - 1)k_{c,1}A_{c,6}\tau_d + D_{6,n-1}^{\$}\tau_d)^2 + ((\theta - 1)k_{c,1}A_{c,7}\tau_{c\pi} + D_{7,n-1}^{\$}\tau_{c\pi})^2 + \\
&\quad ((\theta - 1)k_{c,1}A_{c,8}\tau_{cd} + D_{8,n-1}^{\$}\tau_{cd})^2 + ((\theta - 1)k_{c,1}A_{c,9}\tau_{\pi d} + D_{9,n-1}^{\$}\tau_{\pi d})^2), \\
D_{1,n}^{\$} &= -\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - A_{c,1} + 1) + D_{1,n-1}^{\$}\beta_1 + D_{2,n-1}^{\$}\beta_3, \\
D_{2,n}^{\$} &= (\theta - 1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}) - 1 + D_{1,n-1}^{\$}\beta_2 + D_{2,n-1}^{\$}\beta_4, \\
D_{3,n}^{\$} &= (\theta - 1)A_{c,3}(k_{c,1}\beta_6 - 1) + D_{3,n-1}^{\$}\beta_6, \\
D_{4,n}^{\$} &= (\theta - 1)A_{c,4}(k_{c,1}\phi_c - 1) + D_{4,n-1}^{\$}\phi_c + 0.5X^2, \\
D_{5,n}^{\$} &= (\theta - 1)A_{c,5}(k_{c,1}\phi_\pi - 1) + D_{5,n-1}^{\$}\phi_\pi + 0.5Y^2, \\
D_{6,n}^{\$} &= (\theta - 1)A_{c,6}(k_{c,1}\phi_d - 1) + D_{6,n-1}^{\$}\phi_d + 0.5Z^2, \\
D_{7,n}^{\$} &= (\theta - 1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1) + D_{7,n-1}^{\$}\phi_{c\pi} + XY, \\
D_{8,n}^{\$} &= (\theta - 1)A_{c,8}(k_{c,1}\phi_{cd} - 1) + D_{8,n-1}^{\$}\phi_{cd} + XZ, \\
D_{9,n}^{\$} &= (\theta - 1)A_{c,9}(k_{c,1}\phi_{\pi d} - 1) + D_{9,n-1}^{\$}\phi_{\pi d} + YZ.
\end{aligned}$$

The coefficients are computed recursively using the fact that $D_{i,0}^{\$} = 0$ for $i = 0, 1, 2$. Note that coefficients $D_{3,n}^{\$}, D_{6,n}^{\$}, D_{8,n}^{\$}$, and $D_{9,n}^{\$}$ are zero since $A_{c,3}, A_{c,6}, A_{c,8}$, and $A_{c,9}$ equal zero. The homoscedastic dynamics (Specification I) has a different $D_{0,n}^{\$}$ term, namely:

$$\begin{aligned}
D_{0,n}^{\$} &= \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}A_{c,0} - A_{c,0} + \mu_c) + D_{0,n-1}^{\$} + \\
&\quad 0.5(\sigma_c^2 X^2 + \sigma_\pi^2 Y^2 + \sigma_d^2 Z^2 + 2\sigma_{c\pi}XY + 2\sigma_{cd}XZ + 2\sigma_{\pi d}YZ)
\end{aligned}$$

where X, Y, and Z are determined as for conditional variance above.

A.5 Conditional covariance of stock and bond returns

The conditional covariance between nominal stock and bond returns can be written as:

$$\begin{aligned}
Cov_t[r_{m,t+1} + \pi_{t+1}, h_{t+1,n-1}^{\$}] &= Cov_t[k_{d,1}pd_{t+1} + \Delta d_{t+1} + \pi_{t+1}, q_{t+1,n-1}^{\$}] \\
&= Cov_t[k_{d,1}(A_{d,1}x_{c,t+1} + A_{d,2}x_{\pi,t+1} + A_{d,3}x_{d,t+1} + A_{d,4}\sigma_{c,t}^2 + \\
&\quad A_{d,5}\sigma_{\pi,t}^2 + A_{d,6}\sigma_{d,t}^2 + A_{d,7}\sigma_{c\pi,t} + A_{d,8}\sigma_{cd,t} + A_{d,9}\sigma_{\pi d,t}) + \eta_{d,t+1} + \eta_{\pi,t+1}, \\
&\quad D_{1,n-1}^{\$}x_{c,t} + D_{2,n-1}^{\$}x_{\pi,t} + D_{3,n-1}^{\$}x_{d,t} + D_{4,n-1}^{\$}\sigma_{c,t}^2 + D_{5,n-1}^{\$}\sigma_{\pi,t}^2 + \\
&\quad D_{6,n-1}^{\$}\sigma_{d,t}^2 + D_{7,n-1}^{\$}\sigma_{c\pi,t} + D_{8,n-1}^{\$}\sigma_{cd,t} + D_{9,n-1}^{\$}\sigma_{\pi d,t}] \\
&= Cov_t[\eta_{c,t+1}(k_{d,1}\delta_1 A_{d,1} + k_{d,1}\delta_3 A_{d,2} + k_{d,1}A_{d,3}\delta_5) + \\
&\quad \eta_{\pi,t+1}(k_{d,1}\delta_2 A_{d,1} + k_{d,1}\delta_4 A_{d,2} + 1) + \\
&\quad \eta_{d,t+1}(k_{d,1}\delta_6 A_{d,3} + 1) + k_{d,1}(A_{d,4}\tau_c \epsilon_{c,t+1} + A_{d,5}\tau_{\pi} \epsilon_{\pi,t+1} + A_{d,6}\tau_d \epsilon_{d,t+1} + \\
&\quad A_{d,7}\tau_{c\pi} \epsilon_{c\pi,t+1} + A_{d,8}\tau_{cd} \epsilon_{cd,t+1} + A_{d,9}\tau_{\pi d} \epsilon_{\pi d,t+1}), \\
&\quad \eta_{c,t+1}(D_{1,n-1}^{\$}\delta_1 + D_{2,n-1}^{\$}\delta_3 + D_{3,n-1}^{\$}\delta_5) + \eta_{\pi,t+1}(D_{1,n-1}^{\$}\delta_2 + D_{2,n-1}^{\$}\delta_4) + \\
&\quad \eta_{d,t+1}(D_{3,n-1}^{\$}\delta_6) + D_{4,n-1}^{\$}\tau_c \epsilon_{c,t+1} + D_{5,n-1}^{\$}\tau_{\pi} \epsilon_{\pi,t+1} + D_{6,n-1}^{\$}\tau_d \epsilon_{d,t+1} + \\
&\quad D_{7,n-1}^{\$}\tau_{c\pi} \epsilon_{c\pi,t+1} + D_{8,n-1}^{\$}\tau_{cd} \epsilon_{cd,t+1} + D_{9,n-1}^{\$}\tau_{\pi d} \epsilon_{\pi d,t+1}] \\
&= A + BVar_t[\eta_{c,t+1}] + CVar_t[\eta_{\pi,t+1}] + DCov_t[\eta_{c,t+1}, \eta_{\pi,t+1}] + \\
&\quad ECov_t[\eta_{c,t+1}, \eta_{d,t+1}] + FCov_t[\eta_{\pi,t+1}, \eta_{d,t+1}], \\
A &= k_{d,1}(A_{d,4}D_{4,n-1}^{\$}\tau_c^2 + A_{d,5}D_{5,n-1}^{\$}\tau_{\pi}^2 + A_{d,6}D_{6,n-1}^{\$}\tau_d^2 + A_{d,7}D_{7,n-1}^{\$}\tau_{c\pi}^2 + \\
&\quad A_{d,8}D_{8,n-1}^{\$}\tau_{cd}^2 + A_{d,9}D_{9,n-1}^{\$}\tau_{\pi d}^2), \\
B &= (k_{d,1}\delta_1 A_{d,1} + k_{d,1}\delta_3 A_{d,2} + k_{d,1}A_{d,3}\delta_5)(D_{1,n-1}^{\$}\delta_1 + D_{2,n-1}^{\$}\delta_3 + D_{3,n-1}^{\$}\delta_5), \\
C &= (k_{d,1}\delta_2 A_{d,1} + k_{d,1}\delta_4 A_{d,2} + 1)(D_{1,n-1}^{\$}\delta_2 + D_{2,n-1}^{\$}\delta_4), \\
D &= (k_{d,1}\delta_1 A_{d,1} + k_{d,1}\delta_3 A_{d,2} + k_{d,1}A_{d,3}\delta_5)(D_{1,n-1}^{\$}\delta_2 + D_{2,n-1}^{\$}\delta_4) + \\
&\quad (k_{d,1}\delta_2 A_{d,1} + k_{d,1}\delta_4 A_{d,2} + 1)(D_{1,n-1}^{\$}\delta_1 + D_{2,n-1}^{\$}\delta_3 + D_{3,n-1}^{\$}\delta_5), \\
E &= (k_{d,1}\delta_1 A_{d,1} + k_{d,1}\delta_3 A_{d,2} + k_{d,1}A_{d,3}\delta_5)(D_{3,n-1}^{\$}\delta_6) + \\
&\quad (k_{d,1}\delta_6 A_{d,3} + 1)(D_{1,n-1}^{\$}\delta_1 + D_{2,n-1}^{\$}\delta_3 + D_{3,n-1}^{\$}\delta_5), \\
F &= (k_{d,1}\delta_2 A_{d,1} + k_{d,1}\delta_4 A_{d,2} + 1)(D_{3,n-1}^{\$}\delta_6) + \\
&\quad (k_{d,1}\delta_6 A_{d,3} + 1)(D_{1,n-1}^{\$}\delta_2 + D_{2,n-1}^{\$}\delta_4).
\end{aligned}$$

References

- Abel, Andrew B., 1999, Risk premia and term premia in general equilibrium, *Journal of Monetary Economics* 43, 3–33.
- Ang Andrew, Monika Piazzesi, and Min Wei, 2008, The term structure of real rates and expected inflation, *Journal of Finance* 63, 797–849.
- Attanasio, Orazio P., and Annette Vissing-Jorgensen, 2003, Stock market participation, intertemporal substitution and risk aversion, *American Economic Review* 93, 383–391.
- Baele Lieven, Geert Bekaert, and Koen Inghelbrecht, 2007, The determinants of stock and bond return comovements, Working Paper.
- Bansal, Ravi, and Ivan Shaliastovich, 2008, A long-run risks explanation of predictability puzzles in bond and currency markets, Working Paper.
- Bansal, Ravi, and Amir Yaron, 2000, Risks for the long run: A potential resolution of asset pricing puzzles, NBER Working Paper 8059.
- Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.
- Bansal Ravi, Dana Kiku, and Amir Yaron, 2007, Risks for the long run: Estimation and inference, Working Paper.
- Bansal Ravi, Robert Dittmar, and Dana Kiku, 2007, Cointegration and consumption risks in asset returns, *Review of Financial Studies*, forthcoming.
- Bansal Ravi, Robert F. Dittmar, and Christian T. Lundblad, 2005, Consumption, dividends, and the cross section of equity returns, *Journal of Finance* 60, 1639–1672.
- Bansal Ravi, Varoujan Khatchatrian, and Amir Yaron, 2005, Interpretable asset markets?, *European Economic Review* 49, 531–560.
- Barsky, Robert B., 1989, Why don't the prices of stocks and bonds move together?, *American Economic Review* 79, 1132–1145.
- Bekaert, Geert, and Eric Engstrom, 2008, Inflation and the stock market: Understanding the Fed-model, Working Paper.
- Best Peter, Alistair Byrne, and Antti Ilmanen, 1998, What really happened to U.S. bond yields, *Financial Analysts Journal* 54, 41–49.
- Bollerslev, Tim, 1986, Generalized autoregressive conditional heteroscedasticity, *Journal of Econometrics* 31, 307–327.
- Bollerslev Tim, Robert F. Engle, and Jeffrey M. Wooldridge, 1988, A capital asset pricing model with time-varying covariances, *Journal of Political Economy* 96, 116–131.
- Boudoukh, Jacob, 1993, An equilibrium model of nominal bond prices with inflation-output correlation and stochastic volatility, *Journal of Money, Credit and Banking* 25, 636–665.

- Boudoukh Jacob, Matthew Richardson, and Robert F. Whitelaw, 1994, Industry returns and the Fisher effect, *Journal of Finance* 1994, 1595–1615.
- Campbell, John Y., 1986, Bond and stock returns in a simple exchange model, *Quarterly Journal of Economics* 101, 785–804.
- Campbell, John Y., 1993, Intertemporal asset pricing without consumption data, *American Economic Review* 83, 487–512.
- Campbell, John Y., 1996, Understanding risk and return, *Journal of Political Economy* 104, 298–345.
- Campbell, John Y., 1999, Asset prices, consumption and the business cycle, in John B. Taylor, and Michael Woodford, eds.: *Handbook of Macroeconomics* (Elsevier Science, North-Holland, Amsterdam).
- Campbell, John Y., and John Ammer, 1993, What moves the stock and bond markets? A variance decomposition for long-term asset returns, *Journal of Finance* 48, 3–37.
- Campbell, John Y., and Robert J. Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–227.
- Campbell, John Y., and Tuomo Vuolteenaho, 2004, Inflation illusion and stock prices, *American Economic Review* 94, 19–23.
- Campbell John Y., Adi Sunderam, and Luis M. Viceira, 2008, Inflation bets or deflation hedges? The changing risks of nominal bonds, Working Paper.
- Chapman, David A., 1997, The cyclical properties of consumption growth and the real term structure, *Journal of Monetary Economics* 39, 145–172.
- Cohen Randolph B., Christopher Polk, and Tuomo Vuolteenaho, 2005, Money illusion in the stock market: The Modigliani-Cohn hypothesis, NBER Working Paper 8059.
- Connolly Robert, Chris Stivers, and Licheng Sun, 2005, Stock market uncertainty and the stock-bond return relation, *Journal of Financial and Quantitative Analysis* 40, 161–194.
- Cox John C., Jonathan E. Ingersoll, and Stephen A. Ross, 1985, A theory of the term structure of interest rates, *Econometrica* 53, 385–408.
- David, Alexander, and Pietro Veronesi, 2008, Inflation and earnings uncertainty and volatility forecasts: A structural form approach, Working Paper.
- Duffie Darrell, Mark Schroder, and Costis Skiadas, 1997, A term structure model with preferences for the timing of resolution of uncertainty, *Economic Theory* 9, 3–22.
- Dunn, Kenneth, and Kenneth Singleton, 1986, Modeling the term structure of interest rates under non-separable utility and durability of goods, *Journal of Financial Economics* 17, 27–55.
- Engle, Robert F., 1982, Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, *Econometrica* 50, 987–1007.

- Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–968.
- Evans, Martin D., 1998, Real rates, expected inflation, and inflation risk premia, *Journal of Finance* 53, 187–218.
- Fama, Eugene F., 1981, Stock returns, real activity, inflation, and money, *American Economic Review* 71, 545–565.
- Fama, Eugene F., and Kenneth R. French, 1989, Business conditions and expected returns on stocks and bonds, *Journal of Financial Economics* 25, 23–49.
- Fama, Eugene F., and Michael R. Gibbons, 1982, Inflation, real returns and capital investment, *Journal of Monetary Economics* 9, 297–323.
- Fama, Eugene F., and William G. Schwert, 1977, Asset returns and inflation, *Journal of Financial Economics* 5, 115–146.
- Hall, Robert E., 1988, Intertemporal substitution in consumption, *Journal of Political Economy* 96, 339–357.
- Hamilton, James D., 1994, *Time series analysis*, Princeton University Press, Princeton.
- Hasseltoft, Henrik, 2008, Stocks, bonds, and long-run consumption risks, Working Paper.
- Kandel, Shmuel, and Robert F. Stambaugh, 1990, Expectations and volatility of consumption and asset returns, *Review of Financial Studies* 3, 207–232.
- Kaul, Gautam, 1987, Stock returns and inflation, *Journal of Financial Economics* 18, 253–276.
- Kendall, M. G., 1954, Note on bias in the estimation of autocorrelation, *Biometrika* 41, 403–404.
- Lustig Hanno, Stijn van Nieuwerburgh, and Adrien Verdelhan, 2008, The wealth-consumption ratio, Working Paper.
- Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.
- Modigliani, Franco, and Richard A. Cohn, 1979, Inflation, rational valuation and the market, *Financial Analysts Journal* 35, 24–44.
- Newey, Whitney K., and Kenneth D. West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.
- Pennacchi, George G., 1991, Identifying the dynamics of real interest rates and inflation: Evidence using survey data, *Review of Financial Studies* 4, 53–86.
- Piazzesi, Monika, and Martin Schneider, 2006, Equilibrium yield curves, in Michael Woodford Daron Acemoglu, and Kenneth Rogoff, eds.: *NBER Macroeconomics Annual: 2006* (MIT Press, Cambridge).

- Pilotte, Eugene F., 2003, Capital gains, dividend yields, and expected inflation, *Journal of Finance* 2003, 447–466.
- Restoy, Fernando, and Philippe Weil, 1998, Approximate equilibrium asset prices, NBER Working Paper 6611.
- Ritter, Jay R., and Richard S. Warr, 2002, The decline of inflation and the bull market of 1982-1999, *Journal of Financial and Quantitative Analysis* 37, 29–61.
- Scruggs, John T., and Paskalis Glabadanidis, 2003, Risk premia and the dynamic covariance between stock and bond returns, *Journal of Financial and Quantitative Analysis* 38, 295–316.
- Shiller, Robert J., and Andrea E. Beltratti, 1992, Stock prices and bond yields. Can their co-movements be explained in terms of present value models?, *Journal of Monetary Economics* 30, 25–46.
- Wachter, Jessica A., 2006, A consumption-based model of the term structure of interest rates, *Journal of Financial Economics* 79, 365–399.
- Weil, Philippe, 1989, The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics* 24, 401–421.

Table 1: **Macro Moments**

	Sample		Model		
	Moment	SE	Mean	5%	95%
<u>Consumption growth, Δc</u>					
Mean	0.81	(0.05)	0.81	0.72	0.90
Std.Dev.	0.47	(0.04)	0.46	0.42	0.50
AC(4)	0.04	(0.07)	0.07	-0.06	0.20
AC(8)	-0.13	(0.07)	0.00	-0.12	0.13
<u>Dividend growth, Δd</u>					
Mean	0.45	(0.19)	0.45	-0.01	0.91
Std.Dev.	1.91	(0.28)	1.92	1.72	2.13
AC(4)	0.02	(0.17)	0.17	0.04	0.32
AC(8)	0.08	(0.10)	0.07	-0.07	0.23
<u>Inflation, π</u>					
Mean	0.92	(0.09)	0.92	0.54	1.30
Std.Dev.	0.62	(0.08)	0.56	0.43	0.73
AC(4)	0.70	(0.10)	0.60	0.40	0.77
AC(8)	0.50	(0.11)	0.43	0.17	0.67
<u>Correlations</u>					
Δc and Δd	0.15	(0.14)	0.08	-0.05	0.21
Δc and π	-0.35	(0.19)	-0.36	-0.48	-0.23
Δd and π	-0.16	(0.14)	-0.01	-0.21	0.19

This table presents unconditional moments of observed and model-implied data. Means and percentiles for the model are computed over 2,000 simulations each containing 223 quarters. AC(k) denotes the autocorrelation for k lags. Standard errors, denoted SE, are computed as in Newey West (1987), using four lags. The sample period is 1952:2 to 2007:4.

Table 2: **Estimated Parameters: Specification I**

	Estimate	SE
β_1	0.533	(0.157)
β_2	-0.104	(0.052)
β_3	0.281	(0.122)
β_4	1.019	(0.038)
β_5	0.564	(0.413)
β_6	0.799	(0.071)
δ_1	0.245	(0.068)
δ_2	-0.107	(0.092)
δ_3	0.076	(0.050)
δ_4	0.495	(0.064)
δ_5	-0.203	(0.235)
δ_6	0.295	(0.055)
σ_c^2	0.183	(0.017)
σ_π^2	0.100	(0.009)
σ_d^2	2.873	(0.271)
$\sigma_{c,\pi}$	-0.039	(0.009)
$\sigma_{c,d}$	0.089	(0.048)
$\sigma_{\pi,d}$	-0.062	(0.036)

This table presents results from estimating the parameters in Section 2.1.1 using maximum likelihood. The sample period is 1952:2 to 2007:4.

Table 3: **Estimated Parameters: Specification II**

	Estimate	SE
c_c	0.003	(0.004)
$c_{c\pi}$	-0.003	(0.003)
c_{cd}	0.013	(0.162)
c_π	0.003	(0.002)
$c_{\pi d}$	-0.004	(0.009)
c_d	0.086	(0.052)
b_c	0.916	(0.052)
$b_{c\pi}$	0.874	(0.115)
b_{cd}	0.816	(2.286)
b_π	0.881	(0.039)
$b_{\pi d}$	0.841	(0.297)
b_d	0.810	(0.052)
a_c	0.072	(0.041)
$a_{c\pi}$	0.051	(0.042)
a_{cd}	0.012	(0.113)
a_π	0.090	(0.034)
$a_{\pi d}$	0.054	(0.078)
a_d	0.170	(0.042)

This table presents results from estimating the parameters of the diagonal VEC-model in Section 3.2.2. using maximum likelihood. The sample period is 1952:2 to 2007:4.

Table 4: **Parameters governing the model's second moments**

	Value
α_c	0.0000260
$\alpha_{c\pi}$	-0.0000034
α_{cd}	0.0000073
α_π	0.0000114
$\alpha_{\pi d}$	-0.0000037
α_d	0.0004284
ϕ_c	0.988
$\phi_{c\pi}$	0.924
ϕ_{cd}	0.828
ϕ_π	0.970
$\phi_{\pi d}$	0.896
ϕ_d	0.980
τ_c	$1.16 \cdot 10^{-6}$
$\tau_{c\pi}$	$8.35 \cdot 10^{-7}$
τ_{cd}	$4.79 \cdot 10^{-7}$
τ_π	$1.08 \cdot 10^{-6}$
$\tau_{\pi d}$	$1.59 \cdot 10^{-6}$
τ_d	$9.17 \cdot 10^{-5}$

This table presents the parameter values used for the second moments within the model. They are set as to match the first two moments of the model's second moments with the ones estimated from data. Section 3.2.2. describes in detail how the parameters are set.

Table 5: **Regressing observed valuation ratios on estimated state variables**

	pc_t		pc_t		pd_t		pd_t	
	β	t-stat	β	t-stat	β	t-stat	β	t-stat
<u>Explanatory variables</u>								
$x_{c,t}$	0.35	3.73	-0.00	-0.03	0.41	2.17	-0.20	-0.71
$x_{\pi,t}$			-0.18	-5.76			-0.31	-3.26
$x_{d,t}$							0.01	0.15
R^2_{adj}	0.13		0.32		0.03		0.13	

This table presents regression results from regressing the observed log price-consumption ratio (pc) and the log price-dividend ratio (pd) onto the extracted state variables, x_c , x_π , and x_d . The regressions are run contemporaneously. As a proxy for the price-consumption ratio, the wealth-consumption ratio in Lustig et al. (2008) is used. All variables are measured on a quarterly basis. The sample period is 1952:2 to 2006:4. T-stats are based on heteroscedasticity robust standard errors.

Table 6: Unconditional Asset Price Moments

	Sample	Estimated Model Specification I	Calibrated Model Specification I	Estimated Model Specification II	Calibrated Model Specification II
<u>Stock market excess returns</u>					
$E(r_m - r_f)$	5.52	0.24	4.16	0.14	5.03
$\sigma(r_m - r_f)$	16.30	8.96	10.89	11.54	12.91
<u>Price-dividend ratios</u>					
$E(\exp(pd))$	36.38	63.34	20.45	68.16	17.94
$\sigma(pd)$	0.39	0.08	0.14	0.11	0.18
<u>Real and nominal interest rates</u>					
$E(y_{3m})$		3.28	2.74	3.28	2.62
$\sigma(y_{3m})$		0.49	0.49	0.50	0.65
$E(y_{60m} - y_{3m})$		-0.07	-0.24	-0.05	-0.23
$E(y_{3m}^{\$})$	5.13	7.07	6.79	7.07	6.66
$\sigma(y_{3m}^{\$})$	2.87	1.87	2.02	1.88	1.96
$E(y_{60m}^{\$} - y_{3m}^{\$})$	0.96	0.15	1.50	0.23	1.66

This table presents unconditional moments of observed and model-implied asset prices. Specification I and Specification II refer to the homoscedastic and heteroscedastic model, respectively. Moments for the estimated model stem from using the parameter values estimated from data. The calibrated model increases the persistence parameter of inflation, β_4 , with less than one standard error from the point estimate. The log price-dividend ratio is denoted pd . A risk aversion of 10, an elasticity of intertemporal substitution of 1.5, and a discount factor of 0.997 are used. All moments are annualized. The sample period is 1952:2 to 2007:4.

Table 7: **Explaining the Fed Model**

	Sample	Specification I	Specification II
$Corr(DP, y_{60m})$ 1952-2007	0.30	0.82	0.73
$Corr(DP, y_{60m})$ 1965-2007	0.74	0.81	0.72

This table presents correlation coefficients between dividend-price ratios and 60-month nominal yields in data and from the model. Specification I and Specification II refer to the estimated homoscedastic and heteroscedastic model, respectively. The sample periods are 1952:2 to 2007:4 and 1965:1 to 2007:4.

Table 8: **Predicting quarterly correlations**

	$Corr_{Data,t:t+1}$		$Corr_{Data,t:t+1}$		$Corr_{Data,t:t+1}$	
	β	t-stat	β	t-stat	β	t-stat
<u>Explanatory variables</u>						
$Corr_{Model,t}$	0.78	4.84			0.40	3.93
$Corr_{Data,t-1:t}$			0.63	9.00	0.57	7.21
R_{adj}^2	0.13		0.38		0.41	

This table presents regression results from regressing observed quarterly correlations between US stock and bond returns for time t to $t + 1$ ($CORR_{Data,t:t+1}$) onto its own lagged value ($CORR_{Data,t-1:t}$) and the model-implied conditional correlation at time t ($CORR_{Model,t}$). Observed quarterly correlations are computed using daily stock and bond returns within that particular quarter. The sample period is 1962:1 to 2007:4. T-stats are based on heteroscedasticity robust standard errors.

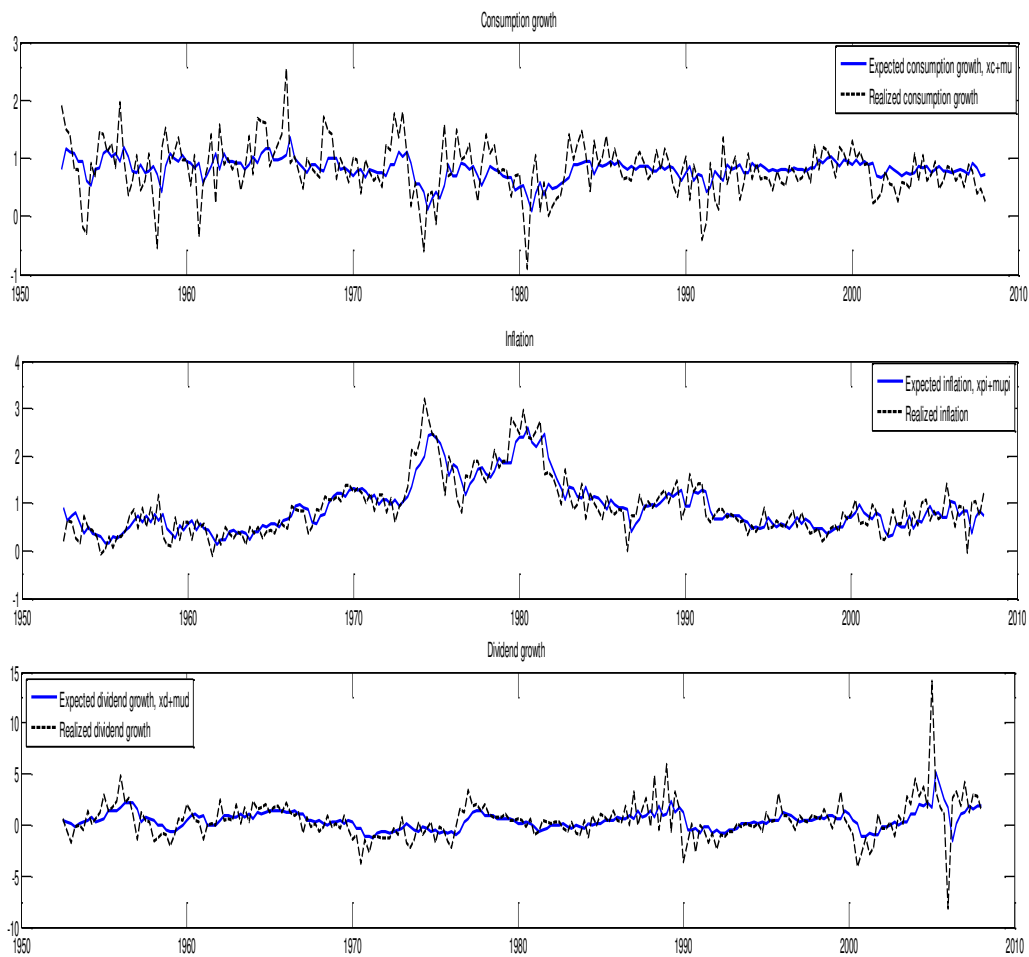


Figure 1: Consumption growth, inflation, and dividend growth. The figure displays realized quarterly consumption growth, inflation, and dividend growth and the extracted conditional means using the estimated parameters from the maximum likelihood estimation. Growth rates are expressed in quarterly units.

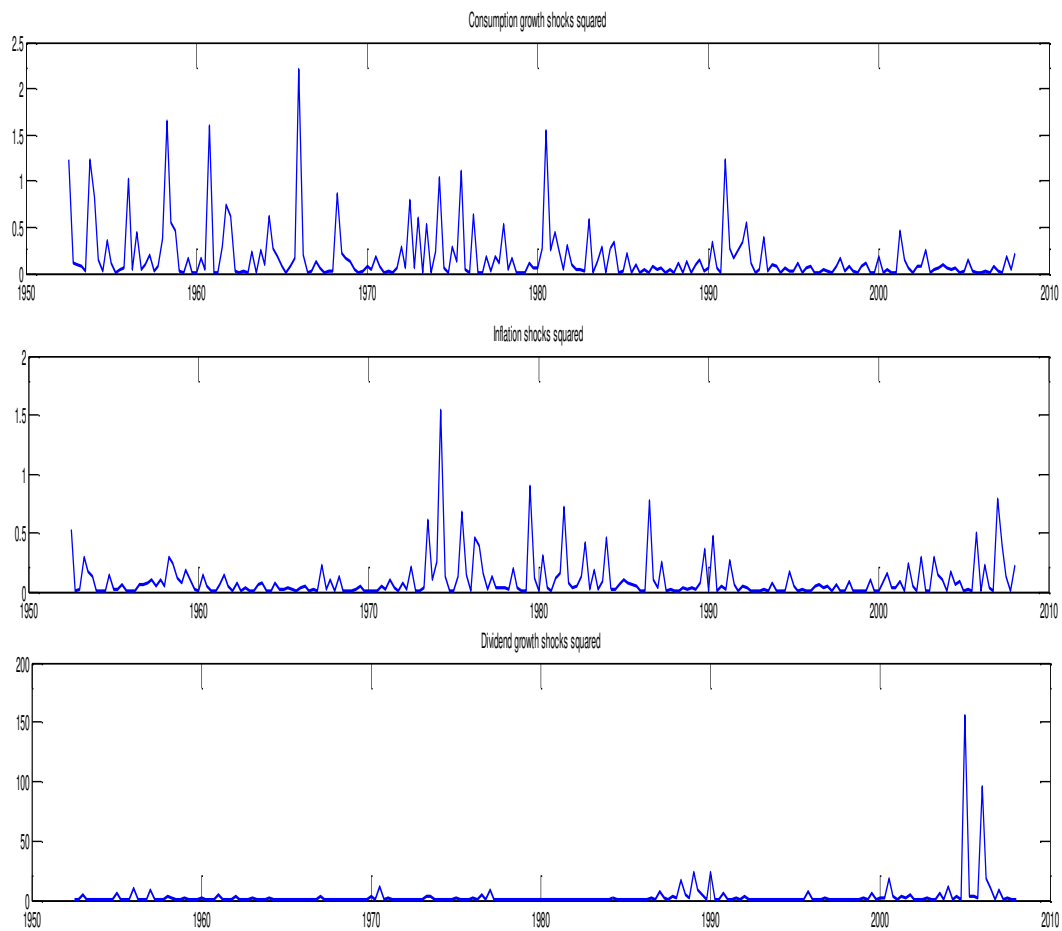


Figure 2: Squared shocks to consumption growth, inflation, and dividend growth. The figure displays squared shocks to quarterly consumption growth, inflation, and dividend growth. All shocks are extracted from the estimated dynamics in Section 3.2.1.

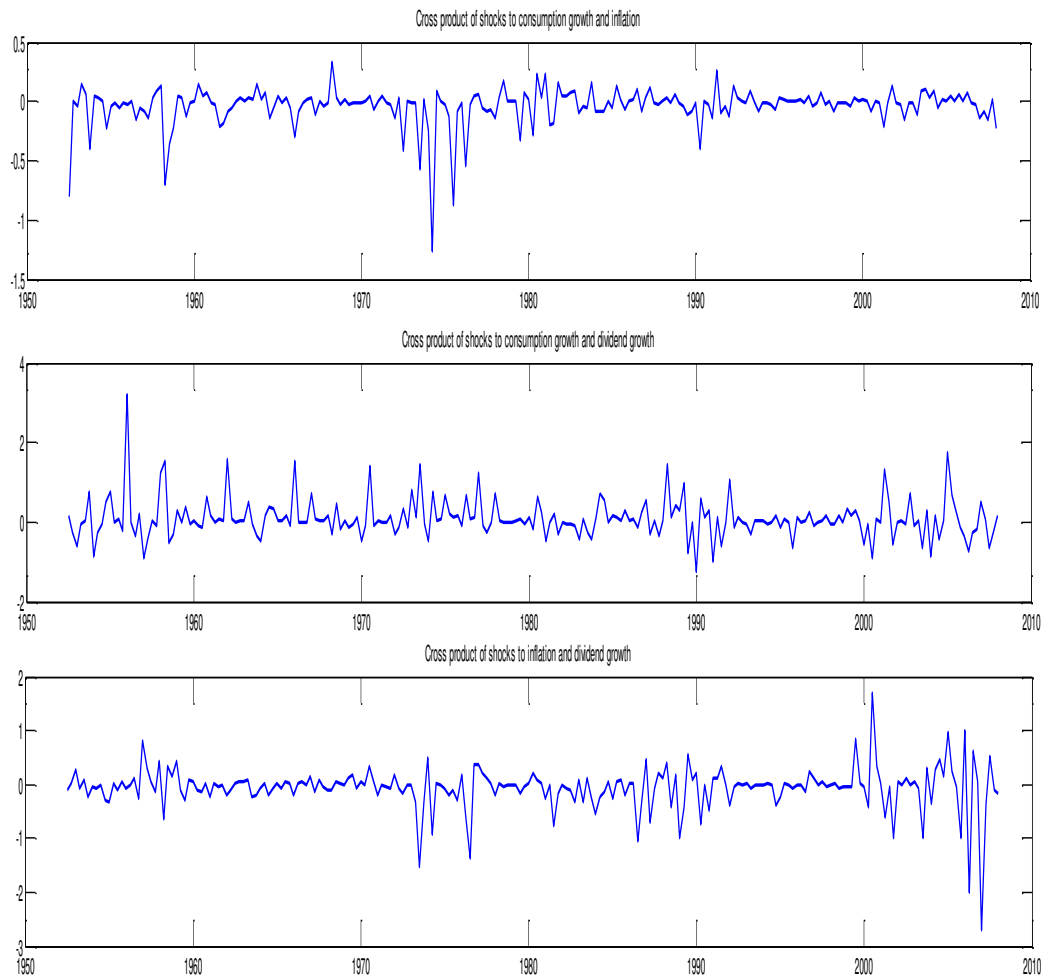


Figure 3: Cross product of shocks to consumption growth, inflation, and dividend growth. The figure displays cross products of shocks to consumption growth and inflation, to consumption growth and dividend growth, and to inflation and dividend growth. All shocks are extracted from the estimated dynamics in Section 3.2.1.

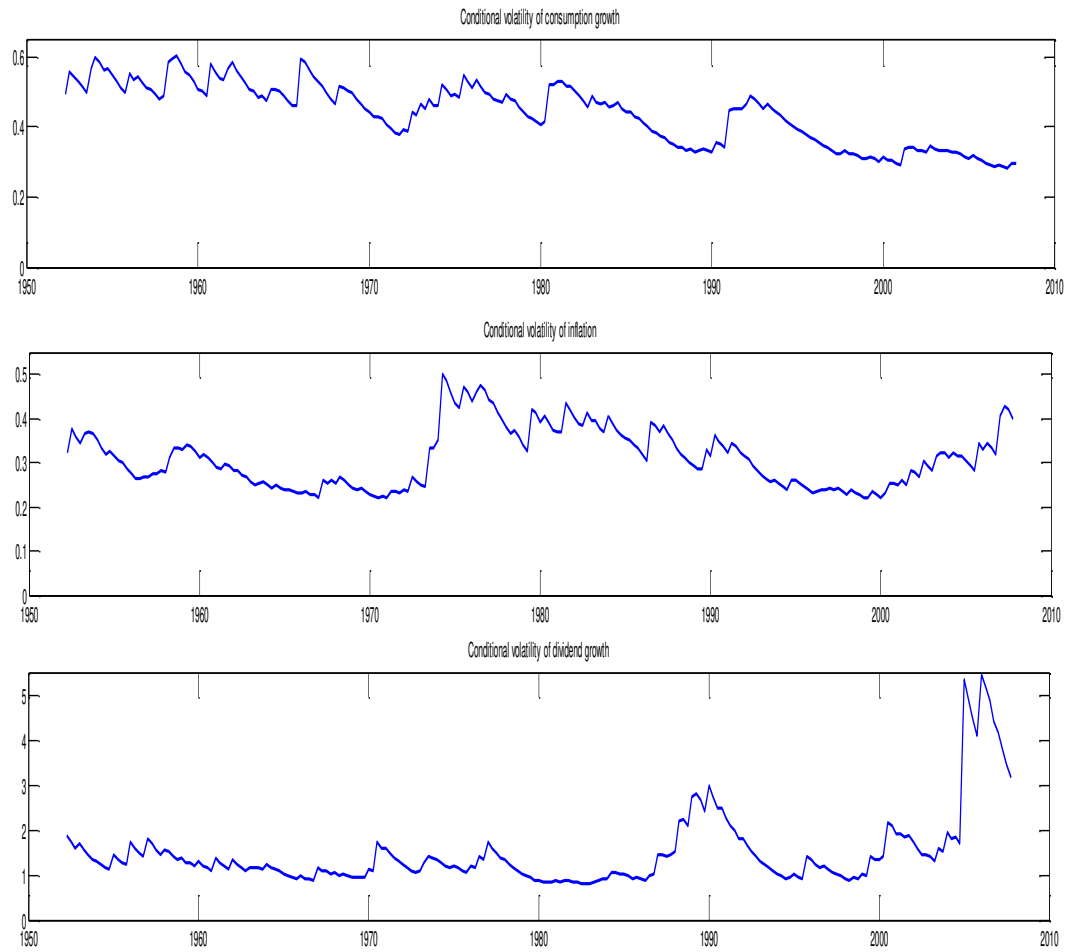


Figure 4: Conditional volatilities of macro variables. The figure displays the conditional standard deviation of consumption growth, inflation, and dividend growth. The volatilities are expressed in quarterly units and stem from the estimated diagonal VEC-model in Section 3.2.2.

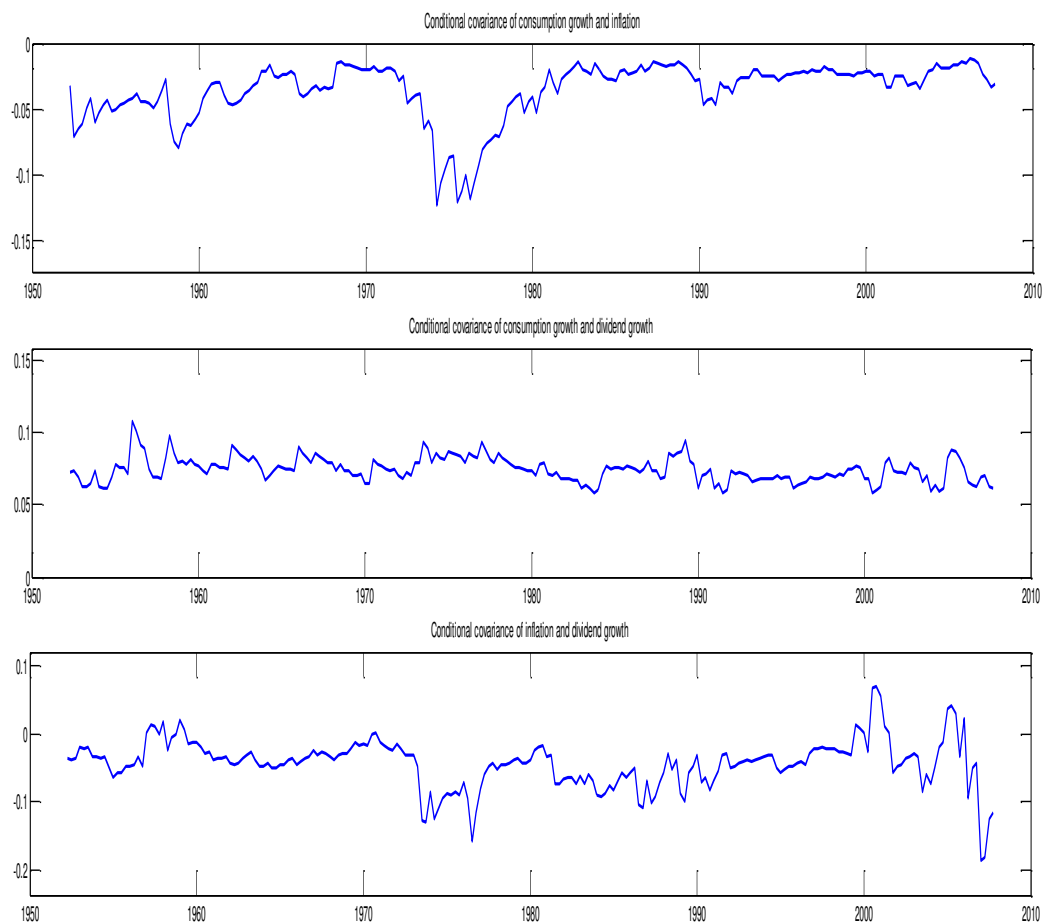


Figure 5: Conditional covariances of macro variables. The figure displays the conditional covariance of consumption growth and inflation, of consumption growth and dividend growth, and of inflation and dividend growth. The covariances are expressed in quarterly units and stem from the estimated diagonal VEC-model in Section 3.2.2.

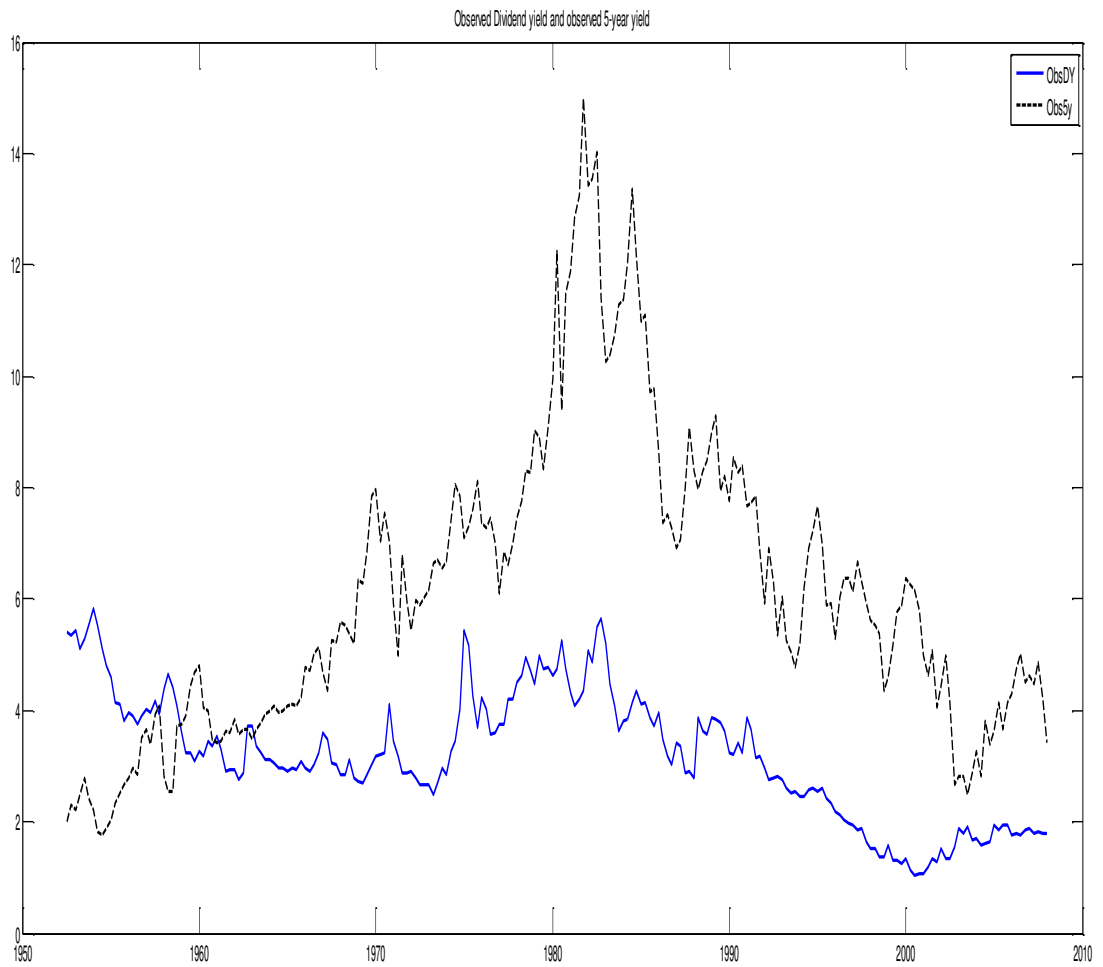


Figure 6: Dividend yields and the 5-year Treasury rate. The figure displays the nominal US 5-year Treasury rate and the US aggregate dividend yield.

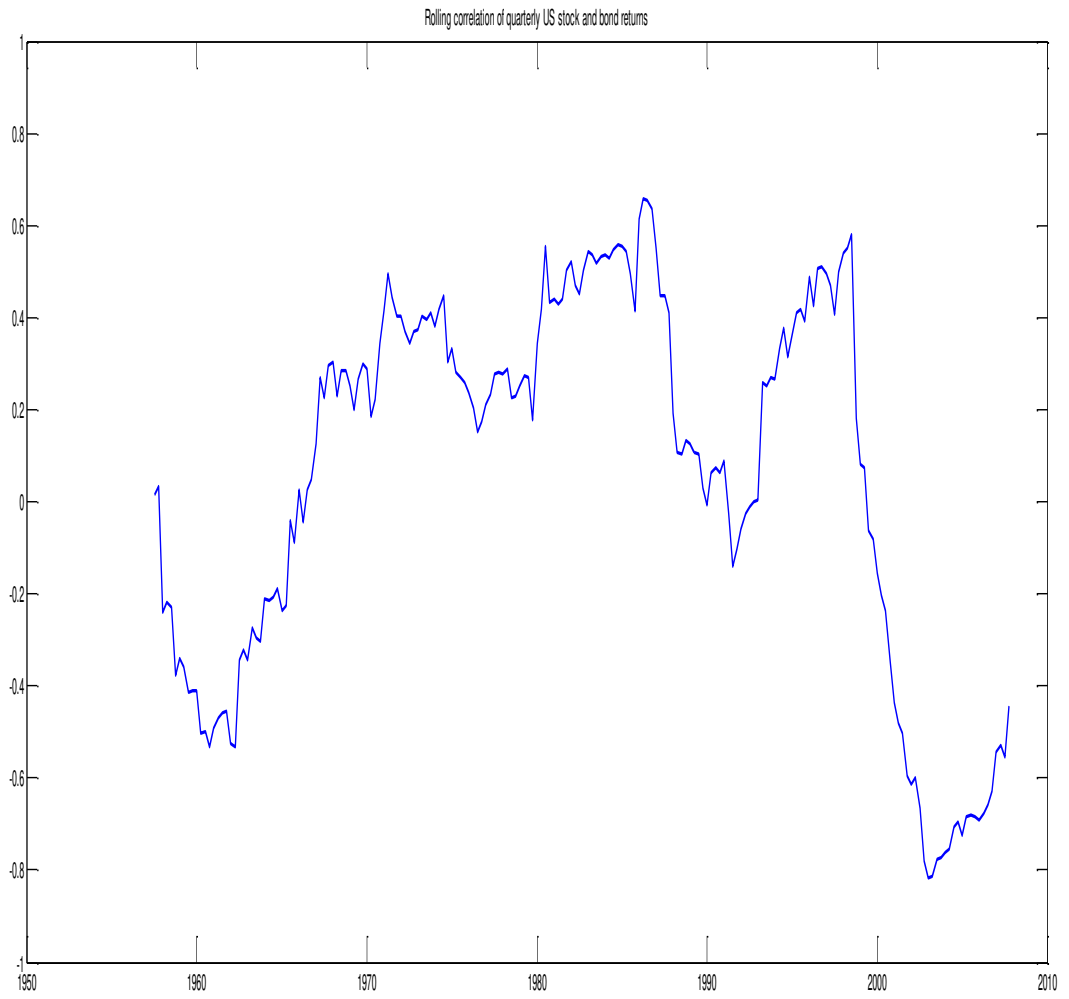


Figure 7: Observed correlation of stock and bond returns. The figure displays a 20-quarter rolling correlation of returns on US stocks and 5-year US Treasury bonds.

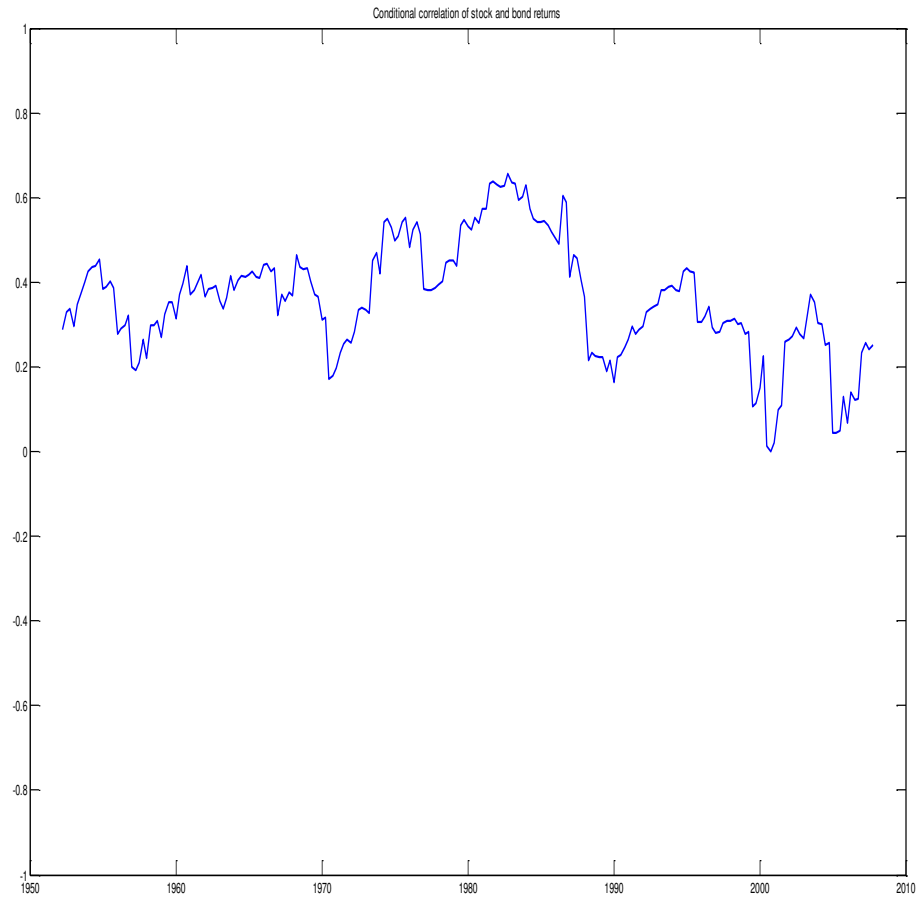


Figure 8: Model-implied conditional correlation of stock and bond returns. The figure displays quarterly model-implied conditional correlations of stock and bond returns.

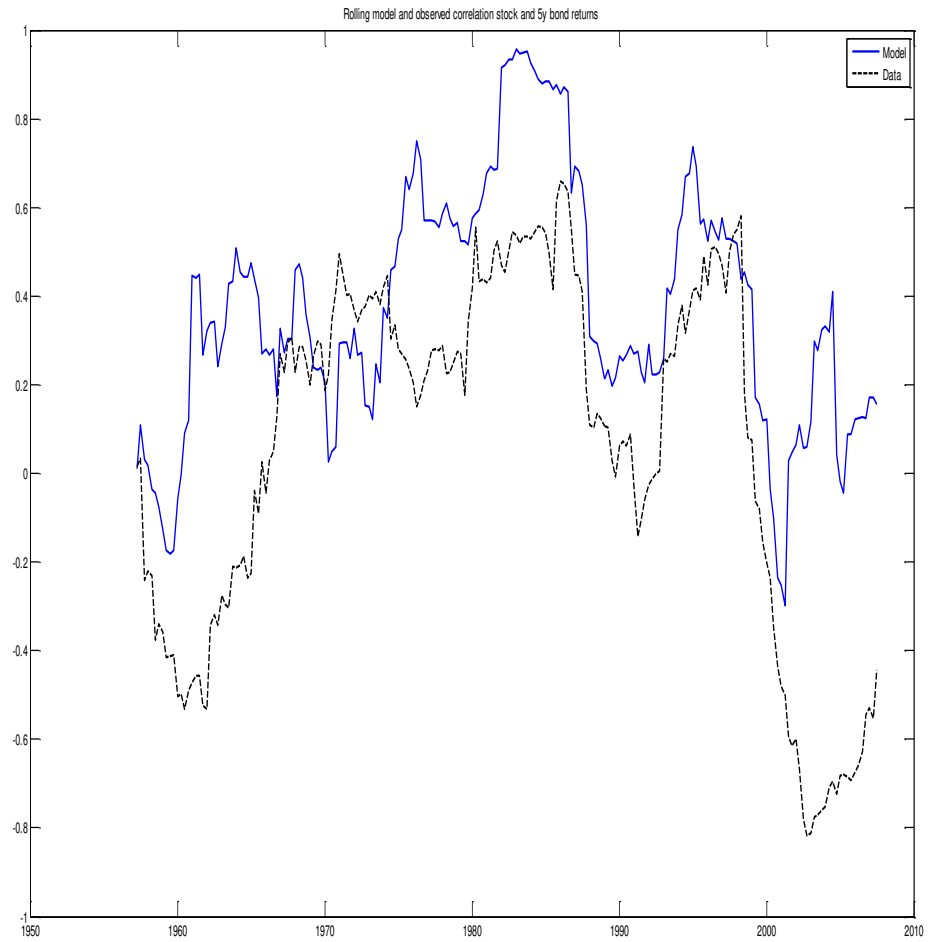


Figure 9: Observed and model-implied correlations of stock and bond returns. The figure displays 20-quarter rolling correlations of stock and bond returns implied from the model and observed in data. The solid line represents model correlations and the dashed line sample correlations.