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P. Madsen
Danfoss A/S

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THE FLEXIBILITY MATRIX FOR A ONE LINE STRUCTURE AND ITS

APPLICATION FOR STRESS AND VIBRATION ANALYSIS IN HERMETIC COMPRESSORS

P. Madsen
Danfoss, Nordborg, Denmark

INTRODUCTION

Dealing with design of pressure tubes for hermetic compressors, it is very important at an early stage to be able to evaluate the stresses in the tube, when it is subjected to given, static displacements.

Mostly the pressure tube has a complicated, spatial form, which makes it difficult to apply the usual elementary methods of calculation. An extra difficulty arises because the tube is statically indeterminate to the sixth degree.

In the following a systematic method applicable to a one line structure is described. The principle of the method does not represent anything new. It is thoroughly described in ref.(1) and the basic principles have been used for many years in applied mechanics. However the method is so applicable that it ought to be used in other areas.

TRANSPORT MATRICES

Consider two points A and B of a body D, which may be an element of a larger, continous structure
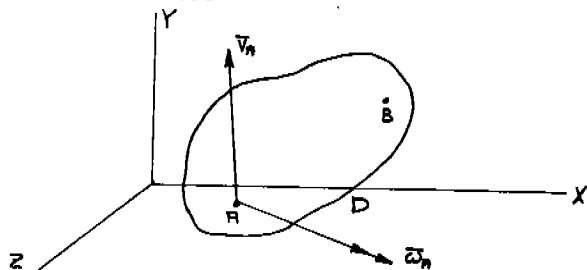

Figure 1

Assume that D moves as a rigid body. The displacement of point A can be described by translation $\bar{V}_A$ and rotation $\bar{\omega}_A$. It is desired to know the corresponding displacement of point B expressed as a function of $\bar{V}_A$ and $\bar{\omega}_A$.

We have for the translation

$$(1) \qquad \bar{V}_B = \bar{V}_A + \bar{\omega}_A \times \overline{AB}$$

and for the rotation

$$(2) \qquad \bar{\omega}_B = \bar{\omega}_A$$

where $\overline{AB} = (a, b, c)$ is the vector from A to B.

Translations and rotations are collected in the column vector

$$\{V\} = \left\{ \frac{\bar{V}}{\bar{\omega}} \right\}$$

and the displacement vector at point B can be written

$$(3) \qquad \{V\}_B = [T]_{AB} \{V\}_A$$

where the transport matrix is defined as

$$[T]_{AB} = \begin{bmatrix} 1 & 0 & 0 & 0 & c & -b \\ 0 & 1 & 0 & -c & 0 & a \\ 0 & 0 & 1 & b & -a & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The transport matrix has the property of transferring a displacement from one point to another in the rigid body.

By a corresponding method forces can be transferred from one point to another. Assume that the element is subjected to a system of external forces and that equilibrium exist when the force vector $\bar{q}_B$ and the moment vector $\bar{m}_B$ are added at point B. We wish to find the static equivalent force vector of point A expressed by $\bar{q}_B$ and $\bar{m}_B$. We have

$$\bar{q}_A = \bar{q}_B$$

$$\bar{m}_A = \bar{m}_B + \overline{AB} \times \bar{q}_B$$

and collecting the forces and the moments in a column vector

$$\{Q\} = \left\{ \frac{\bar{q}}{\bar{m}} \right\}$$

the static equivalent force system at A

can be written

(4)
$$\{Q\}_A = [T]^*_{AB}\{Q\}_B$$

where * indicates a transposed matrix.
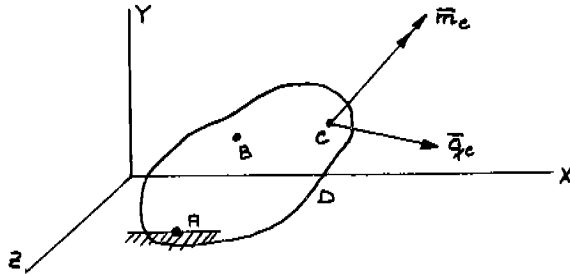
FLEXIBILITY MATRICES

Figure 2

We now assume that the element is fixed at point A and is subjected to a force system $\{Q\}_c$ at C. Assuming that HOOKE's law is valid for the material and stability problems do not exist, the deformations at C is proportional to the force system, and

(5)
$$\{V\}_c = [F]_{AC}\{Q\}_c$$

where $[F]_{AC}$ is the flexibility matrix of the section AC in point C. Each element of the flexibility matrix represent a displacement caused by a unit value of one of the actions while the other actions are zero. In general the element f_{ij} is the i-th displacement due to a unit value of the j-th action.

In the same way we introduce the flexibility matrix $[F]_{AB}$ and $[F]_{BC}$. We wish to find the relationship between the matrices $[F]_{AB}$, $[F]_{AC}$ and $[F]_{BC}$.

From (4) the force system at B can be written

(6)
$$\{Q\}_B = [T]^*_{BC}\{Q\}_c$$

and according to (5) the displacement at B is

$$\{V\}_B = [F]_{AB}\{Q\}_B$$

and using (6), we have

$$\{V\}_B = [F]_{AB}[T]^*_{BC}\{Q\}_c$$

From (3) this displacement can be transported to point C giving

(7)
$$\{V\}^1_c = [T]_{BC}[F]_{AB}[T]^*_{BC}\{Q\}_c$$

The displacement vector $\{V\}^1_c$ is due to the deformations of the element AB. To this is added the displacements due to the deformations in the element BC

(8)
$$\{V\}^2_c = [F]_{BC}\{Q\}_c$$

and

$$\{V\}_c = \{V\}^1_c + \{V\}^2_c$$

from (7) and (8)

(9)
$$\{V\}_c = [T]_{BC}[F]_{AB}[T]^*_{BC}\{Q\}_c + [F]_{BC}\{Q\}_c$$

Substituting (5) in (9), we have

$$[F]_{AC}\{Q\}_c = [T]_{BC}[F]_{AB}[T]^*_{BC}\{Q\}_c + [F]_{BC}\{Q\}_c$$

which may be written

(10)
$$[F]_{AC} = [T]_{BC}[F]_{AB}[T]^*_{BC} + [F]_{BC}$$

The equation shows how the resulting flexibility matrix of the two sections AB and BC coupled in series is calculated.
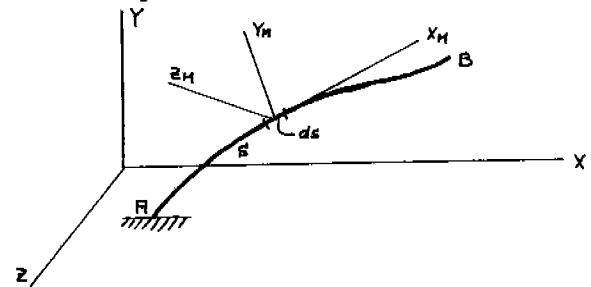

Figure 3

Let AB in figure 3 be a beam that is fixed at A and free at B. Consider an element ds situated at a distance s from the support measured along the centroidal axis of the beam. Imposing a coordinate system x_M, y_M, z_M upon the element, where x_M is the tangent to the centroidal axis and y_M and z_M coincide with the principal axes of the cross-section.

In this coordinate system the flexibility matrix of the element is

$$[F(s)]\,ds = \begin{bmatrix} \frac{1}{EA} & & & & & \text{SYMMETRIC} \\ 0 & \frac{\mu}{GA} & & & & \\ 0 & 0 & \frac{\mu}{GA} & & & \\ 0 & 0 & 0 & \frac{1}{GK} & & \\ 0 & 0 & 0 & 0 & \frac{1}{EI_z} & \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{EI_y} \end{bmatrix} ds$$

where E is the modulus of elasticity, G is the modulus of rigidity, A is the cross-sectional area, I_z and I_y are the principal moments of inertia, K is the polar moment of inertia, and μ is a factor that depends on the form of the cross-section.

The element system is rotated so that the axes coincide with the principal coordinate system. Denoting the rotating matrix $[R]$ we have

$$[F(s)]'ds = [R(s)][F(s)][R(s)]^* ds$$

where $'$ indicates a rotated matrix.

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Using equation (10) summing all infinite-simal elements from the support to the free end, we obtain

$$(11) \quad [F]_{\text{RB}} = \int_0^{s_B} [T(s)]_{s_B} [R(s)] [F(s)] [R(s)]^* [T(s)]^*_{s_B} ds$$

where $[T(s)]_{s_B}$ is the transport matrix from s to B.

Using equation (11) the flexibility matrix of an arbitrary one line structure can be calculated. In order to reduce the computer time it is an advantage to calculate the flexibility matrices for the standard elements of the system, first of all the straight and the curved elements. The total flexibility matrix can now be determined by successive rotations and transports using equation (10).
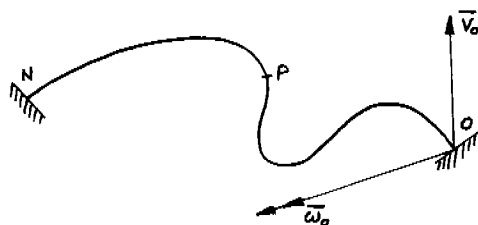
APPLICATIONS TO HERMETIC COMPRESSORS

Figure 4

Consider figure 4 and assume that the diagram represent a pressure tube for a hermetic compressor. It is further assumed that we have calculated the flexibility matrix $[F]_{ON}$ of the tube at point O.

In a compressor the tube is fixed at both ends, and if the support at O is subjected to a displacement $\{V\}_0$ the reaction forces $\{Q\}_0$ can be determined applying equation (5)

$$\{Q\}_o = [F]_{ON}^{-1} \{V\}_o$$

Now the static equivalent force vector at an arbitrary point P can be calculated by transferring the reaction forces to the afore-mentioned point

$$\{Q\}_P = [T]_{OP}^* \{Q\}_o$$

where $[T]_{OP}$ is the transport matrix from O to P.

The force vector $\{Q\}_P$ is obtained in the directions of the principal coordinate system, but a rotation of the vector gives us the forces in the directions of the principal axes of the cross-section

$$\{Q\}_P' = [R]_P \{Q\}_P$$

where $[R]_P$ is a rotation matrix. By application of the theory of elasticity the normal and the shear stresses of the cross-section are found, and these can be used calculating

a reference stress by using an acceptable theory of strength.

When investigating theoretically the vibrations of a hermetic compressor it is very important to know the stiffness matrix of the suspension springs and the pressure tube. The stiffness matrix can be determined by measuring the flexibility in different directions, setting in the values in the flexibility matrix and then inverting the matrix.
The diagonal elements in the flexibility matrix are simple to determine, whereas the off-diagonal elements can be extremely difficult to obtain. In this case the present method will give the result.

We have used the method at DANFOSS for some time for investigations of pressure tubes. Several tubes have been analysed and on the basis of this alternative designs have been worked out. In a single instance it was shown that a good geometrical form significantly reduced the maximal stresses and consequently reduced the risk for failure.

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