1976

The Fluid Motion Due to a Rotating Disk

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Publication Info

Published in *Journal of the Electrochemical Society*, Volume 123, Issue 3, 1976, pages 383-385.

White, R., Mohr, Jr., C.M., & Newman, J. The Fluid Motion Due to a Rotating Disk. *Journal of the Electrochemical Society*, *123*(3): 383-385.
http://dx.doi.org/10.1149/1.2132830

The coated molybdenum strip was then removed, and a silicon wafer with a (111) orientation and a mirror-polished face was introduced into the melt. An anodic current pulse of 50 mA/cm^2 was then applied for 1 sec. This served to achieve an *in situ* etching of the wafer (approximately 150Å). The polarity was then reversed, the Si wafer becoming the cathode, and constant current electrodeposition established for a predetermined period of time. The sample was then removed, cleaned, and examined. A potentiostat (PAR 173) was used for current control.

Results

Epitaxial deposition of silicon readily occurred upon the single crystal substrate. The current density was found to have an important influence upon the morphology of the deposit. Figure 2(a) shows a scanning electron micrograph of the epitaxial growth obtained at 1 mA/cm^2. Poor coverage of the substrate surface, and a hillock and layer structure can be seen. Such a structure is believed to be due to the adsorption of foreign species, probably oxygen or oxides, which hinder the propagation of kinks and steps.

If the adsorption rate is constant for a given impurity concentration and temperature, increased current density (or rate of deposition) should favor step propagation and more uniform growth. This effect can easily be seen in Fig. 2(b-d), which show deposits obtained at 2, 4, and 6 mA/cm^2, respectively. It should be noted that the epitaxial layer in Fig. 2(c) is only 2μ thick, whereas the others are 10μ thick.

Deposit uniformity across the substrate surface was found to be greatly affected by the current distribution. Since Si is a poor electronic conductor, the current density is greater in areas with a shorter current path within the silicon wafer. This results in an enhanced and more uniform growth close to the melt surface, with poorer growth in remote areas.

To produce a more uniform current distribution, a metallic layer with greater conductivity may be attached to the back side of the wafer, either by a mechanical attachment, or as a coating or film. The metal in contact with the back side of the wafer should be compatible with silicon. It must not have a low melting point alloy or eutectic with silicon, and its solid solubility in Si at the operating temperature must be as small as possible. It may also be important that this layer can be easily removed without damaging the wafer.

It was found that a layer of Ag metal 0.5μ thick, obtained by sputtering, resulted in a substantial improvement in the uniformity of the expitaxial layer. Thicker coatings should produce even more uniform layers. Silver may be removed by dipping in dilute HF, which will not attack silicon. Its solid solubility in silicon at 750°C is extremely low, and the Si-Ag system has one eutectic at 840°C.

Back-reflection Laue x-ray patterns have demonstrated that these layers are epitaxially related to the (111) substrate, even when the growth morphology appears rather rough. Results obtained by Auger spectroscopy and electron microprobe analysis have shown the deposit to be of high purity. Some carbon and oxygen were both found upon the surface after growth by Auger spectroscopy. They disappeared from the signal after a layer about 500Å thick was removed by sputtering. Electron microprobe analysis, with a sensitivity of about 100 ppm, detected no impurities beneath the surface.

Acknowledgments

This work was supported by the Advanced Research Projects Agency and the Office of Naval Research. The authors wish to thank Mr. L. J. Anderson for his capable SEM photography, and Mr. W. H. Holmes for his invaluable assistance in sample preparation.

Manuscript submitted Oct. 20, 1975; revised manuscript received Nov. 10, 1975. This was Paper 259 RNP presented at the Dallas, Texas, Meeting of the Society, Oct. 5-9, 1975.

Any discussion of this paper will appear in a Discussion Section to be published in the December 1976 JOURNAL. All discussions for the December 1976 Discussion Section should be submitted by Aug. 1, 1976.

Publication costs of this article were partially assisted by Stanford University.

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Brief Communication

The Fluid Motion Due to a Rotating Disk

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The solution of the Navier-Stokes equation for fluid motion due to a rotating disk includes characteristic parameters as presented below. We report here the most accurate values available for three of these parameters and compare to them values obtained by a numerical integration technique developed by Newman (3, 5).

In 1921, von Kármán (2) presented a separation of variables solution technique for the motion of an incompressible, Newtonian fluid which transformed the Navier-Stokes equation into a set of coupled, nonlinear, ordinary differential equations. By defining the following dimensionless variables $\zeta = z\sqrt{\Omega/\nu}$, $P = p/\mu\Omega$, $G = v_\theta/r\Omega$, $F = v_r/r\Omega$, and $H = v_z/\sqrt{\nu\Omega}$, the transformed equations may be written as

$$2F + H' = 0 \quad [1]$$

$$F^2 - G^2 + HF' = F'' \quad [2]$$

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Key words: laminar flow.

$$2FG + HG' = G'' \quad [3]$$

$$HH' + P' = H'' \quad [4]$$

where the prime designates differentiation with respect to ζ. The boundary conditions are

$$H = F = 0,\ G = 1 \text{ at } \zeta = 0 \quad [5]$$

and

$$F = G = 0 \text{ at } \zeta = \infty \quad [6]$$

Cochran (1) solved Eq. [1-3] (subject to boundary conditions [5] and [6]) by expanding the components of the velocity field first in power series in the dimensionless distance from the disk, which were assumed to be valid near the disk

$$F = a\zeta - \frac{1}{2}\zeta^2 - \frac{b}{3}\zeta^3 + \dots \quad [7]$$

$$G = 1 + b\zeta + \frac{1}{3}a\zeta^3 + \dots \quad [8]$$

$$H = -a\zeta^2 + \frac{1}{3}\zeta^3 + \frac{b}{6}\zeta^4 + \dots \quad [9]$$

and second in exponential series which were assumed to be valid far from the disk

$$F = Ae^{-\alpha\zeta} - \frac{(A^2 + B^2)e^{-2\alpha\zeta}}{2\alpha^2} + \frac{A(A^2 + B^2)e^{-3\alpha\zeta}}{4\alpha^4} + \dots \quad [10]$$

$$G = Be^{-\alpha\zeta} - \frac{B(A^2 + B^2)}{12\alpha^4}e^{-3\alpha\zeta} + \dots \quad [11]$$

$$H = -\alpha + \frac{2A}{\alpha}e^{-\alpha\zeta} - \frac{(A^2 + B^2)}{2\alpha^3}e^{-2\alpha\zeta} + \frac{A(A^2 + B^2)}{6\alpha^5}e^{-3\alpha\zeta} + \dots \quad [12]$$

We (5) followed Cochran's suggestion and required the two sets of expansions to yield, at $\zeta = 1$, the same values of the functions as well as the derivatives of F and G. In this manner we obtained the following values for the characteristic parameters[2]

$$a = 0.51023262,\ b = -0.61592201,\ \alpha = 0.88447411,$$
$$A = 0.92486353,\ B = 1.20221175 \quad [13]$$

Benton (7) solved this problem by utilizing a technique suggested by Fettis (8) and has tabulated to four significant figures the velocity field, its derivatives, and the pressure field.[3]

To demonstrate the utility of Newman's (3-5) solution technique, estimates of the parameters a, b, α, A, and B were obtained by solving this boundary value problem numerically. The governing Eq. [1-3] were first linearized (3-5) about trial values and then cast in finite-difference form accurate to order h^2. The boundary conditions given by Eq. [5] were applied directly, whereas it was necessary to approximate those given by Eq. [6] at some finite value of ζ, ζ_{max}. The following expressions, derived from Eq. [10] and [11], were used for that purpose

$$F' = H_x F - \frac{(F^2 + G^2)}{2H_x} + \dots \quad [14]$$

and

$$G' = H_x G + \dots \quad [15]$$

where H_x was our estimate of $-\alpha$ according to

$$H_x = H + \frac{2F}{H_x} + \frac{(F^2 + G^2)}{2H_x^3}\left(1 + \frac{F}{3H_x^2}\right) + \dots \quad [16]$$

which was developed from Eq. [12]. Equations [14], [15], and [16] were also linearized about trial values and expressed in finite-difference form accurate to order h^2. The resulting system of equations was solved by technique developed (3, 4) and extended (5) by Newman. Estimates of the five parameters obtained this way are

$$a = 0.51023262,\ b = -0.61592201,\ \alpha = 0.88447410,$$
$$A = 0.92486322,\ B = 1.20221104 \quad [17]$$

Clearly, these are very accurate estimates of the parameters given by Eq. [13]. The poorest estimate is for B, which is in error by only seven digits in the eighth significant figure.

The very attractive feature of Newman's solution technique, in addition to its accuracy, is its suitability for solving complicated boundary value problems directly without the development of specialized techniques, such as Cochran's for the present problem.

Acknowledgments

This work was supported by the U.S. Energy Research and Development Administration.

Manuscript submitted Sept. 18, 1975; revised manuscript received Nov. 10, 1975.

Any discussion of this paper will appear in a Discussion Section to be published in the December 1976 JOURNAL. All discussions for the December 1976 Discussion Section should be submitted by Aug. 1, 1976.

Publication costs of this article were partially assisted by the University of California, Berkeley.

LIST OF SYMBOLS

English characters:

- a characteristic parameter equal to $F'(0)$
- b characteristic parameter equal to $G'(0)$
- A characteristic parameter
- B characteristic parameter
- F dimensionless radial velocity
- G dimensionless velocity component in the tangential direction
- H dimensionless velocity component in the normal direction (from the disk)
- h dimensionless step size
- P dimensionless dynamic pressure
- p dynamic pressure, dyne/cm^2
- r radial distance from the axis of the disk, cm
- v_r velocity component in the radial direction, cm/sec
- v_θ velocity component in the tangential direction, cm/sec
- v_z velocity component in the normal direction, cm/sec
- z normal distance from the disk, cm

Greek characters:

- α characteristic parameter equal to $-H(\infty)$
- ζ dimensionless normal distance from the disk
- μ viscosity of fluid, g/cm-sec
- ν kinematic viscosity of fluid, cm^2/sec
- Ω rotation speed of the disk, sec^{-1}

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[2] Values for a and b accurate to 12 significant figures are available in Ref. (6).

[3] Unfortunately, a minus sign for the pressure derivative in his equation (10) is missing; consequently, the sign of the entries in his Table 2 for $P - P_0$ should be changed.

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Erratum

In the paper, "Lead-Acid Batteries: Use of Carbon Fiber-Lead Wire Grids at the Positive Electrode," by J. L. Weininger and C. R. Morelock, which appeared on pp. 1161-1167 in the September 1975 JOURNAL, Vol. 122, No. 9, the following resistivity for carbon fibers, lead, and lead dioxide should replace the incorrectly stated conductivities of the same materials in column 1, p. 1161:

pyrolyzed polyacrylonitrile fibers

$$\rho C_{fiber} = 0.86 \times 10^{-3} \text{ to } 2.5 \times 10^{-3} \text{ ohm-cm}$$

lead: $\rho_{Pb} = 2.1 \times 10^{-5}$ ohm-cm

lead dioxide: $\rho_{\alpha\text{-}PbO_2} = 1 \times 10^{-3}$ ohm-cm

$\rho_{\beta\text{-}PbO_2} = 4 \times 10^{-3}$ ohm-cm