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THE FORMATION OF BINARIES CONTAINING BLACK HOLES BY THE EXCHANGE OF COMPANIONS AND THE X-RAY SOURCES IN GLOBULAR CLUSTERS

J. G. Hills

Department of Astronomy, University of Michigan, Ann Arbor, Michigan 48104, USA

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SUMMARY

Clark's suggestion that the X-ray sources in globular clusters are binaries containing black holes or neutron stars is viable if we consider their formation by an exchange of companions with primordial binaries rather than by three-star encounters as originally suggested. In this process, which is similar to charge exchange among atomic particles, the black hole or neutron star makes a close encounter with a primordial binary, ejects one of its components, and becomes itself bound to the binary. The model is capable of producing the observed number of X-ray sources in globular clusters if a fraction $f_b = 10^{-3}$–10^{-2} of the ordinary stars in the clusters are binaries with semimajor axes less than 1 AU. However, a consideration of the relative reaction rates reveals that X-ray sources powered by the tidal break-up and capture of ordinary stars by the black holes and neutron stars are likely to radiate several orders of magnitude more energy than that emitted by the X-ray binaries.

Clark (1975) proposes that binaries containing black holes are the X-ray sources in globular clusters and that these binaries form by three-body encounters. The binary companion of the black hole is assumed to evolve off the main sequence, fill its Roche lobe, and dump material on to the collapsed object as in conventional X-ray binaries. Unfortunately the rate of binary formation by this mechanism is almost certainly too low to be significant.

However, the model is highly viable if we consider their formation by an exchange process in which the black hole suffers a close encounter with a primordial binary composed of conventional stars, ejects one of its components, and becomes itself bound in the binary, replacing the star which it ejects. This process, which is somewhat akin to charge exchange among atomic particles, is vastly more efficient than three-star encounters. This is due to the rate of formation of binaries containing black holes being proportional to $f_b n_f n_B$ by the exchange of companions and to $n_f^2 n_B$ by three-star encounters. Here n_f is the space density of ordinary stars, f_b is the fraction of these which are binaries, and n_B is the space density of black holes. The extra space density term, n_f, results in the rate of formation of binaries containing black holes being roughly a factor $F \sim (s/a)^3 f_b$ times larger by the exchange of companions than by three-star encounters. Here a is the semimajor axis of the binary formed and s is the mean spacing between stars. In the core of a typical globular cluster, $s \simeq 0{\cdot}1$ pc $= 2 \times 10^4$ AU. If the companion of the black hole is to overflow its Roche lobe, $a \lesssim 1$ AU so $F \sim 10^{13} f_b$. It is evident that if f_b is almost anything finite, the rate of formation of the binaries by exchange vastly exceeds that by three-star encounters.

As shown in Paper I (Hills 1975a), the cross-section specifying the probability that an encounter between a star of mass M_3 and a binary having components of mass M_1 and M_2 will result in an exchange of companions is given by

$$\sigma_E = \pi a^2 \gamma A(1+\beta a_c/a), \tag{1}$$

where the characteristic semimajor axis,

$$a_c = G(M_1+M_2+M_3)/V_f^2. \tag{2}$$

Numerical experiments show that the dimensionless parameters $\gamma = 0{\cdot}17$ and $\beta = 7{\cdot}3$. Here V_f is the mean velocity of the binary relative to the black hole prior to the encounter. Since the typical black hole is much more massive than the average star in the cluster, equipartition of kinetic energy results in its having a relatively low velocity. Thus we may expect V_f to be nearly the rms velocity of the normal stars in the cluster and the black holes to be concentrated near the centre of the cluster. The factor A is the probability of an exchange taking place when the star encounters the binary at zero impact parameter. The numerical experiments discussed in Paper I show that $A = 1$ if M_3 exceeds M_1 and M_2 and if $a < a_c$. Both of these conditions are met since $a_c \sim 10^2$ AU in a typical globular cluster and we are interested in binaries with $a < 1$ AU. Since $a \ll a_c$, equation (1) reduces to

$$\sigma_E = \gamma\beta(\pi a a_c) \simeq 1{\cdot}25\ \pi a a_c. \tag{3}$$

The mean time that elapses before a given black hole of mass M_3 is captured by a binary through the exchange of companions is given by

$$\tau_E = (n_f f_b \sigma_E v_f)^{-1} = [n_f f_b(1{\cdot}24\ \pi a a_c)\ V_f]^{-1}. \tag{4}$$

We note that τ_E is relatively weakly dependent on a, $\tau_E \propto a^{-1}$.

Detailed considerations show that the companion star is not able to overflow its Roche radius in the red giant stage unless the initial semimajor axis, a_0, of the primordial (pre-encounter) binary is less than 1 AU. The smallest semimajor axis of any primordial binary is $a_{min} \simeq 10^{-2}$ AU $= 2\ R_\odot$. Thus the primordial binaries of interest in the formation of the X-ray binaries have semimajor axes in the range $a_0 = (0{\cdot}01\text{–}1)$ AU. In calculating τ_E by equation (4) we let $a = \langle a_0 \rangle = 0{\cdot}1$ AU, the logarithmic average semimajor axis of the binaries of interest. We also let f_b be the fraction of all ordinary stars in the system which are binaries with semimajor axes less than 1 AU. The system M15, which we take to be representative of the globular clusters with X-ray sources, has a central density of $3 \times 10^4\ M_\odot/\mathrm{pc}^3$ and an rms stellar velocity dispersion of $V_f = 9{\cdot}3\ \mathrm{km\ s^{-1}}$ (Peterson & King 1975). If the average stellar mass in the core is $0{\cdot}5\ M_\odot$, $a_c = 113$ AU for a black hole with a mass $M_3 = 10\ M_\odot$ and $a_c = 26$ AU for a neutron star with $M_3 = 1{\cdot}5\ M_\odot$. From equation (4) we find that in M15 the mean time that elapses before a given black hole is captured by a binary with $a_0 < 1$ AU is $\tau_E = 1{\cdot}7 \times 10^9\ \mathrm{yr}/f_b$. For a neutron star this is $\tau_E = 7{\cdot}5 \times 10^9\ \mathrm{yr/f_b}$. The total number of X-ray binaries in a globular cluster at a particular time is

$$N_x = N_B \zeta(\tau_c/\tau_E) \tag{5}$$

where $\tau_c = 1{\cdot}2 \times 10^{10}$ yr is the age of the cluster, ζ is the fraction of the binaries containing black holes in which the companion has evolved off the sequence and is presently overflowing its Roche lobe, and N_B is the number of black holes or neutron stars in the system. We adopt Clark's estimate that $\zeta = 0{\cdot}03$. If the main sequence stars in the system initially followed a Salpeter (1955) mass-distribution-function, there were several thousand of them more massive than 10 $M_\odot$. Thus we assume that $N_B \sim 10^3$ for either neutron stars or black holes. If $N_X = 1$ for the core of M15, equation (5) requires that $f_b = 5 \times 10^{-3}$ for black holes and $f_b = 2 \times 10^{-2}$ for neutron stars. Considering that not all globular clusters with dense stellar cores are X-ray sources, it is possible that the required f_b may be reduced by another order of magnitude to 10^{-3}. The value of f_b for globular clusters is unknown, although the high densities in globular cluster cores indicate that $f_b < 0.15$ for binaries with $a_0 \lesssim 30$ AU (Hills 1975b). Despite the uncertainty in f_b, the required $f_b = 10^{-3}$–10^{-2} is modest enough to suggest that the model is plausible.

One complication we have not considered is the recoil velocity of the binary. On the average, the semimajor axis of the binary after an exchange is about the same or slightly smaller than its original value (Hills 1975a). Thus there is a net increase in the binding energy of the binary when the black hole of mass M_3 replaces the star of mass M_1. Energy conservation causes the binary and ejected star to recoil away from each other at relatively high velocity. If we ignore the velocity of the black hole with respect to the binary before the encounter, the recoil velocity of the binary is

$$V_B = \left[\frac{(M_3 - M_1)}{(M_2 + M_3)}\left(\frac{M_2}{M_1}\right)\frac{G(M_1 + M_2 + M_3)}{a_0}\right]^{1/2}\left[\frac{M_1}{(M_1 + M_2 + M_3)}\right]. \tag{6}$$

The recoil velocity of star M_1 is found by replacing M_1 by $(M_2 + M_3)$ in the numerator of the second term in square brackets. The escape velocity from the centre of M15 is $V_{esc} = 36$ km s^{-1} (Peterson & King 1975). If we let $M_1 = M_2 = 0.5\ M_\odot$, we find that $V_B > V_{esc}$ if $a_0 \lesssim 0.014$ AU for $M_3 = 10\ M_\odot$ and $a_0 \lesssim 0.034$ AU for $M_3 = 1.5\ M_\odot$. Thus exchange will cause only the most tightly bound primordial binaries to be ejected from the cluster. While the effect of the recoil velocity is not very significant in M15, it will be so in clusters having lower escape velocities. This may explain, in part, the preference of the X-ray sources for the more centrally condensed clusters with high central escape velocities. The ejected binaries may be responsible for the X-ray sources in the galactic bulge.

The black holes and neutron stars in a cluster may also emit X-rays from an accretion disk acquired by the tidal break-up and capture of ordinary stars passing within their Roche limits (Hills 1976; this is an obvious extension of the QSO model discussed in Hills 1975c). Fabian, Pringle & Rees (1975) also propose that a black hole may capture a close binary companion from an initially hyperbolic orbit by the tidal dissipation produced when this star passes just outside the Roche radius of the black hole. The major uncertainty here is whether or not the tidal distortion is sufficiently irreversible to allow capture. If such captures occur, the process is competitive with tidal break-up. The main uncertainty in the tidal break-up theory is the fraction, f_a, of the mass of the relatively massive accretion disk which is ultimately accreted by the compact object. In a given system the time-averaged ratio of the power radiated by these accretion disks to the power radiated by the X-ray binaries formed by exchange is given roughly by

$$\frac{L_a}{L_b} = \frac{f_a \sigma_a}{f_a'(\zeta f_b)\,\sigma_E} = \frac{f_a R_T}{f_a'(\zeta f_b)\,a_0} \tag{7}$$

(see equations (3) and (4)). Here R_T is the tidal radius of the collapsed object and f_a' is the fraction of the mass of the binary companion ultimately accreted by the black hole. We have assumed that the average mass of the binary companions of the black holes is the same as that of the stars they capture by tidal break-up. Here $R_T = 4{\cdot}3\,R_\odot$ for a black hole with $M_3 = 10\,M_\odot$ and $R_T = 2{\cdot}3\,R_\odot$ for a neutron star with $M_3 = 1{\cdot}5\,M_\odot$. Since $\langle a_0 \rangle \sim 0{\cdot}1$ AU $= 20\,R_\odot$, $R_T/a \sim 10^{-1}$. Equation (7) reduces to $L_a/L_b \simeq 3 \times 10^{+|2-3|}(f_a/f_a')$ if $\zeta = 0{\cdot}03$ and $f_b = 10^{-3}$–10^{-2}. Thus $L_a/L_b \gg 1$ unless no more than a minute fraction, $f_a \sim 10^{-3} f_a'$, of the accretion disk is ultimately accreted by the compact object.

It is entirely possible that not all the radiation from an accretion disk reaches the observer as X-rays. The large mass of the disk may produce such a thick envelope of gas around the source that only a small fraction of its X-rays escapes absorption. This may account for the infrared sources found in certain globular clusters (MacGregor, Phillips & Selby 1973; Cohen & Fawley 1974). These have luminosities comparable to those of the X-ray sources and they show a decided preference for clusters with dense stellar cores.

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