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The formation of HD 149026 b

C. Broeg[1,2,3]★ and G. Wuchterl[3]

[1]*Astrophysical Institute and Obervatory, Friedrich-Schiller University, Schillergäßchen 2-3, Jena, 07745, Germany*
[2]*Max-Planck Institute for extraterrestrial Physics, Giessenbachstrasse, Garching, 85741, Germany*
[3]*Thüringer Landessternwarte, Sternwarte 5, Tautenburg, 07778, Germany*

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ABSTRACT

Today, many extrasolar planets have been detected. Some of them exhibit properties quite different from the planets in our Solar system and they have eluded attempts to explain their formation. One such case is HD 149026 b. It was discovered by Sato et al. A transit-determined orbital inclination results in a total mass of 114 $M_\oplus$. The unusually small radius can be explained by a condensible element core with an inferred mass of 67 $M_\oplus$ for the best-fitting theoretical model.

In the core accretion model, giant planets are assumed to form around a growing core of condensible materials. With increasing core mass, the amount of gravitationally bound envelope mass increases. This continues up to the so-called critical core mass – the largest core allowing a hydrostatic envelope. For larger cores, the lack of static solutions forces a dynamic evolution of the protoplanet, accreting large amounts of gas or ejecting the envelope in the process. This would prevent the formation of HD 149026 b.

By studying all possible hydrostatic equilibria we could show that HD 149026 b can remain hydrostatic up to the inferred heavy core. This is possible if it is formed in situ in a relatively low-pressure nebula. This formation process is confirmed by fluid-dynamic calculations using the environmental conditions as determined by the hydrostatic models.

We present a quantitative in situ formation scenario for the massive core planet HD 149026 b. Furthermore, we predict a wide range of possible core masses for close-in planets like HD 149026 b. This is different from migration, where typical critical core masses should be expected.

Key words: planets and satellites: formation – planetary systems: formation.

1 INTRODUCTION

At the moment, there are two competing theories for giant planet formation. In one theory, the solar nebula fragments as a direct result of gravitational instability to form a giant planet (Cameron 1978; Decampli & Cameron 1979). In the second, the core accretion scenario, a core or protoplanet forms through accretion of planetesimals and when its mass reaches some critical value the surrounding gaseous envelope is supposed to become unstable and collapse on to the core, forming the giant planet in the process (Perri & Cameron 1974; Mizuno, Nakazawa & Hayashi 1978). For an in-depth discussion see Wuchterl, Guillot & Lissauer (2000). In this Letter, we will assume that the nebula is gravitationally stable and follow the second idea.

Soon after the discovery of the first exo-planet, 51 Peg b, by Mayor & Queloz (1995), theorists have come up with several migration theories predicting that the planets form at large orbital distances (around 4–5 au) and migrate inwards. Both continuous (Lin, Bodenheimer & Richardson 1996) and sudden migration (the jumping Jupiters of Weidenschilling 1977) have been proposed. On the other hand, Wuchterl (1996, 1997) showed that in situ formation could occur if sufficent amounts of solids and gases are available in the planets' feeding-zones.

The planet HD 149026 b was discovered by Sato et al. (2005) at a distance a of only 0.042 au. Because it was discovered by both the radial velocity method and the transit method, its mass and density are known. It has a total mass of 114 $M_\oplus$ and an unusually large density. Calculations by the discovery team give the most likely core mass as 67 $M_\oplus$.

We will show that the large core of HD 149026 b cannot be explained by migration. It has to form in situ to allow such a large subcritical core-mass.

2 MODELLING THE EQUILIBRIUM ENVELOPE STRUCTURES

Every protoplanet in our model consists of a solid core of constant density ($\rho_{\rm core} = 5500\,{\rm kg\,m^{-3}}$)[1] and an envelope of hydrogen and

★E-mail: broeg@astro.uni-jena.de

[1] A core density of 10 500 gives similar results.

helium with a helium mass fraction of 0.24. The composition of the envelope is assumed to be protosolar. Following the recommended procedure for the SCVH equations of state, that contain now high-Z elements, their effects are accounted for by a somewhat enhanced He-mass fraction. Heavy elements and their condensates are fully taken into account in the opacities in order to calculate radiative transfer efficiency in detail.

2.1 Model equations and assumptions

The hydrostatic equilibrium configuration of the envelopes is given by the well known equations of stellar structure:

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \rho(P, T), \tag{1}$$

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM}{r^2}\rho(P, T), \tag{2}$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = \frac{T}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\nabla(P, T), \tag{3}$$

$$\frac{\mathrm{d}L}{\mathrm{d}r} = 0, \tag{4}$$

$$\frac{\mathrm{d}U}{\mathrm{d}r} = \frac{GM}{r^2}. \tag{5}$$

This, together with the boundary conditions (see Section 2.2), determines the values for the mass M, the pressure P, the temperature T, the luminosity L, and the gravitational potential U as a function of the radius r. G is the gravitational constant and ∇ is calculated as

$$\nabla = \min_{\mathrm{smooth}}(\nabla_{\mathrm{rad}}, \nabla_{\mathrm{s}}), \tag{6}$$

i.e. the adiabatic temperature gradient ∇_{s} or the temperature gradient as caused by radiative energy transport in the diffusion approximation, ∇_{rad} – whichever is smaller. This corresponds to the use of zero entropy gradient convection and the application of the Schwarzschild criterion, but is continuously differentiable across the transition region. ∇_{s} is directly given by the equation of state, and ∇_{rad} is calculated as

$$\nabla_{\mathrm{rad}} = \frac{3}{64\pi\sigma G}\frac{\kappa L P}{T^4 M}$$

(see Mihalas & Weibel-Mihalas 1999, chapter 6 for a derivation of this formula and explanation of all parameters). κ is the Rosseland-mean opacity and is determined by using tabulated values (see Section 2.3.2).

2.2 Boundary conditions

To solve the differential equation system we specify the following boundary conditions.

(i) The core radius is defined using a fixed core density ρ_{c}:

$$r_{\mathrm{core}} = \sqrt[3]{\frac{3}{4\pi}\frac{M}{\rho_{\mathrm{c}}}}.$$

(ii) The outer radius is given by the Hill radius:

$$r_{\mathrm{Hill}} = a\sqrt[3]{\frac{M}{3M_*}}, \tag{7}$$

where a is the planet's semimajor axis, M its mass, and M_* the mass of the host star.

(iii) The luminosity L is defined as the energy libration rate obtained for a constant planetesimal accretion rate and the dissipation of the planetesimal kinetic energy at the core surface:

$$U(r_{\mathrm{core}}) = -\int_{r_c}^{r_{\mathrm{Hill}}} \frac{GM_r}{r^2}\,\mathrm{d}r, \quad \text{and}$$
$$L = -(U - U_0)\dot{M}, \tag{8}$$

where U_0 is arbitrary. U_0 is the gravitational potential at r_{Hill}.

2.3 Constituent relations

To fully specify the differential equation system, ∇_{S}, ρ, and κ need to be specified. We use the following equation of state and opacity tables.

2.3.1 Equation of state

$\nabla_{\mathrm{s}}(P, T)$, and ρ (P, T) are interpolated from Saumon, Chabrier & van Horn (1995). First hydrogen and helium are interpolated independently, then the mixed quantity is determined. ∇_{s} is calculated via spline derivatives from the mixed entropy including the mixing entropy term, as suggested in Saumon et al. (1995). The helium mass fraction is $Y = 0.24$.

2.3.2 Opacity

Rosseland-mean opacities $\kappa(\rho, T)$ are interpolated from a combined table: opacities include Rosseland-mean dust opacities from Pollack, McKay & Christofferson (1985, $\lg T \leqslant 2.3$), Alexander & Ferguson (1994) values in the molecular range, and Weiss, Keady & Magee (1990) Los Alamos high-temperature opacities.

3 UTILIZING ALL EQUILIBRIA TO DETERMINE THE ENVIRONMENTAL PROPERTIES FOR HD 149026 B

In order to solve the system of differential equations, a range of parameters must be provided, namely the

(i) core mass M_{core},
(ii) pressure at the core, P_{core},
(iii) mass of the host star, M_*,
(iv) semimajor axis of the planet, a,
(v) nebula temperature, T_{neb}, and
(vi) planetesimal accretion rate, $\dot{M}$.

Parameters (i) and (ii) are our independent parameters; they are varied in a scale-free way, i.e. equidistant in the logarithm.

Parameters (iii), (iv) and (v) are determined by the host star and the position of the planet. The nebula temperature can be calculated in thermal equilibrium with the star:

$$T_{\mathrm{neb}} = 280\left(\frac{a}{1\,\mathrm{au}}\right)^{-1/2}\left(\frac{L_*}{\mathrm{L}_\odot}\right)^{1/4}\,\mathrm{K} \tag{9}$$

(see Hayashi 1981; Hayashi, Nakazawa & Nakagawa 1985).

The only remaining free parameter is the planetesimal accretion rate $\dot{M}$. In the case of HD 149026 b we choose an unusually large value of $\dot{M} = 10^{-2}\,\mathrm{M}_\oplus\,\mathrm{yr}^{-1}$. This value corresponds to particle-in-a-box planetesimal accretion for the density at the position of HD 149026 b in a minimum mass solar nebula (Hayashi 1981; Hayashi et al. 1985) with a gravitational enhancement factor $F_{\mathrm{g}} = 21$.

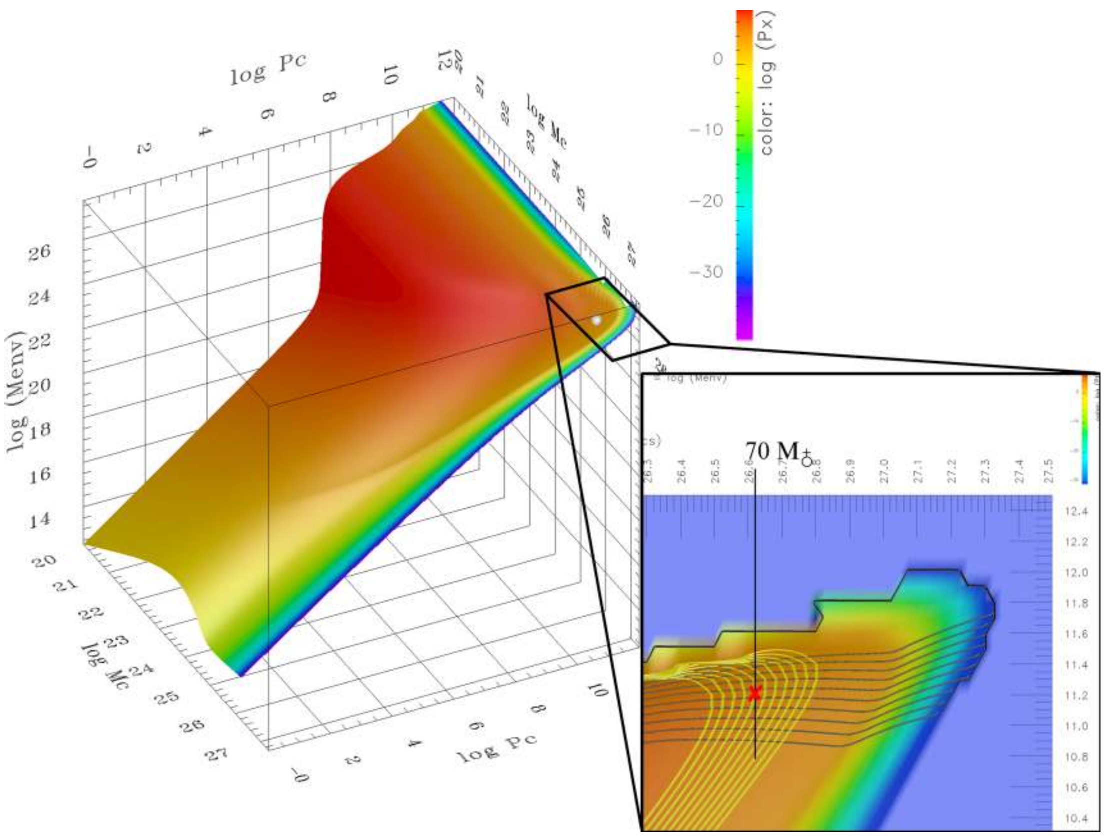

Figure 1. This figure shows the envelope mass ($\log M_{\rm env}$) as a function of core mass and pressure at the core surface (log Mc and log Pc in the figure, respectively). The logarithms are taken from the corresponding values in SI units (kg/Pa). The results are given as a surface in three dimensions and the surface colour is mapped from the outside pressure (log Px). The blow-up on the right-hand side shows the region of interest around a core mass of 70 $\mathrm{M}_\oplus$. To determine the nebula conditions for a critical core mass of 70 $\mathrm{M}_\oplus$, we have drawn lines of constant nebula pressure (isobars, yellow, in the range $\lg P\,\mathrm{Pa}^{-1} = 3...4$, step 0.1) and lines of constant envelope mass (grey, in the range $\lg M_{\rm env}\,\mathrm{kg}^{-1} = 26...27$, step 0.1). The critical core mass for a given nebula pressure is given as the largest possible core mass for a given isobar. Thus the isobar that is tangentially touched by the line of 70 $\mathrm{M}_\oplus$ determines the nebula pressure for which the critical core mass is 70 $\mathrm{M}_\oplus$. It is $\lg P\,\mathrm{Pa}^{-1} = 3.6$. Using the grey lines we can immediately find the envelope mass corresponding to this critical core mass; it is 47 $\mathrm{M}_\oplus$, giving a total mass of roughly 117 $\mathrm{M}_\oplus$. This is almost exactly the total mass of HD 149026 b.

For HD 149026 b the correct values are therefore:

(i) $M_* = 1.3\,\mathrm{M}_\odot$, $L_* = 2.72\,\mathrm{L}_\odot$,
(ii) $a = 0.042$ au,
(iii) $T_{\rm neb} = 1754$ K, and
(iv) $\dot{M} = 10^{-2}\,\mathrm{M}_\oplus\,a^{-1}$.

Using these values we have calculated all hydrostatic envelope structures by solving the equation system described in Section 2 for a wide range of core masses and pressures. Using this solution manifold it is easy to determine the correct environment that allows such a large critical core mass of roughly 70 $\mathrm{M}_\oplus$ (see Fig. 1).

The results show that a hydrostatic equilibrium of gaseous envelope and solid core is indeed possible at the specified position, if the nebula pressure is $P = 10^{3.6}$ Pa = 3980 Pa. In that case the envelope mass is $\lg M_{\rm env}\,\mathrm{kg}^{-1} = 26.45$ or roughly 47 $\mathrm{M}_\oplus$, leading to a total of 117 $\mathrm{M}_\oplus$ for the protoplanet.[2] This is very close to the observed mass of 114 $\mathrm{M}_\oplus$.

It should be noted that this is only the case very close to the host star. With increasing distance, the accretion rate $\dot{M}$ decreases, and even more importantly the hill radius increases for constant planet mass. Both effects lead to a reduced critical core mass. Therefore, such a large core is only allowed for very close-in planets, such as HD 149026 b.

4 FLUID-DYNAMIC FORMATION OF HD 149026 B

Knowing the environmental conditions, especially the values of the nebula pressure $P_{\rm neb}$ and accretion rate $\dot{M}$, we tried to reproduce the in situ formation of HD 149026 b with a thorough fluid-dynamic calculation.

The algorithm for the dynamical calculations is described in Wuchterl (1990, 1991a,b). For this Letter we use a modified convection theory as in Wuchterl & Tscharnuter (2003). These calculations use a different equation of state (not SCVH, see Wuchterl 1989) and slightly different dust opacities, namely for interstellar dust instead of the Pollack et al. protosolar dust, see Wuchterl & Tscharnuter (2003).

Because the dynamical calculation uses a particle-in-a-box accretion scheme instead of a constant accretion rate, we use a gravitational enhancement factor $F_{\rm g} = 21$ (or a Safronov number $\theta = 10$) which leads to a peak accretion rate of $\dot{M} = 10^{-2}\,\mathrm{M}_\oplus\,\mathrm{yr}^{-1}$ as required.

[2] Repeating these calculations with a core density of $\rho_{\rm core} = 10500\,\mathrm{kg\,m}^{-3}$ and a critical core mass of 67 $\mathrm{M}_\oplus$ leads to the same nebula pressure ($P = 10^{3.6}$ Pa), a slightly larger envelope mass ($\lg M_{\rm env}\,\mathrm{kg}^{-1} = 26.5$ or 53 $\mathrm{M}_\oplus$) and a total mass of 120 $\mathrm{M}_\oplus$.

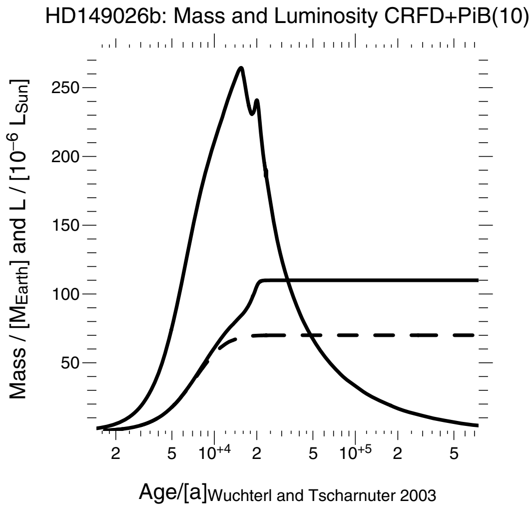

Figure 2. Possible evolution of HD 149026 b. The figure shows the time-evolution of three quantities: the core mass (dashed line), the total mass (solid), and the luminosity (solid). The age is defined as in Wuchterl & Tscharnuter (2003), and CRFD+PiB(10) stands for fully hydrodynamic simulation with Safronov number 10. The smooth luminosity curve is a strong hint for quasi-static evolution which is confirmed by looking at the data. The dynamic terms are negligible.

In spite of the slight differences regarding the constituent relations, the dynamic calculations confirm the hydrostatic model. The evolution of HD 149026 b is plotted in Fig. 2. The dynamic calculation confirms quasi-static evolution for the entire formation process of HD 149026 b, no instabilities occur. This formation scenario explains the high core mass and shows no dynamic accretion phase – the planet grows hydrostatically all the way to its final mass.

5 SOLVING THE FEEDING-ZONE PROBLEM

So far we have shown how HD 149026 b could have formed provided that there is enough material available to form the planet at its current position. The lack of building material is usually considered the strongest argument against in situ formation of close-in planets.

It is true that in a classical feeding zone, i.e. 3–4 hill radii (equation 7) on both sides of the orbit of the planet, there is not enough material available in a gravitationally stable disc. This problem can be circumvented by assuming a continuous flow of material on to the star as is the case for accretion discs. According to Hartmann et al. (1998), typical T Tauri discs with an age of 1 Myr have relatively low accretion rates; they say

'The median accretion rate for T Tauri stars of age $\sim$1 Myr is $10^{-8}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$; the intrinsic scatter at a given age may be as large as 1 order of magnitude.'

For now, we will assume a very fast formation of the planet of 10^5 yr and calculate the amount of material that passes the orbit of the hypothetical in situ planet. Using Hartmann's estimate, the material passing a close-in planet during its formation is theerfore $M = 10^5 \times 10^{-8}\,\mathrm{M}_\odot = 333\,\mathrm{M}_\oplus$. For the envelope this is quite enough mass, but what about the heavy element core? For solar composition gas we have a mass fraction of condensible material of $\approx$1/56 outside of the ice-line and $\approx$1/240 inside (Hayashi et al. 1985). Typical disc-lifetimes are $\sim$1 Myr. We can now calculate the amount of condensible material that passes the protoplanet's orbit during the lifetime of the disc:

$$M_{\mathrm{cond}} = 10^6 \times 10^{-8}\,\mathrm{M}_\odot/240 \approx 14\,\mathrm{M}_\oplus. \quad (10)$$

Keeping in mind that HD 149026 b is an extreme case, it is quite appropriate to use the upper limit given by Hartman: a disc accretion rate of $10^{-7}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$. This provides $140\,\mathrm{M}_\oplus$ of condensible material at the orbit of the planet.

In the case of HD 149026 b we can go even further. HD 149026's metalicity is given as [Fe/H] = 0.36 (from Sato et al. 2005). Assuming that the other heavy elements are similarly enriched, this is an overabundance in heavy elements of $10^{0.36} = 2.3$. In total we end up with $\sim 300\,\mathrm{M}_\oplus$ in heavy elements passing the planet's orbit – this is enough solid material for HD 149026 b.

There is one problem remaining, concerning the condensibles: the high temperature of the nebula. At a temperature of 1754 K and a pressure of 4000 Pa, will there be any condensed specimen left? According to Duschl, Gail & Tscharnuter (1996) the mass fraction of silicon for this $P - T$ regime is 0.99–0.999. Thus the silicates are still available. For other specimen, especially carbon, this is usually not the case. To answer the question of condensible mass fraction precisely, the exact environmental conditions, such as pressure, temperature and chemical composition at the time of HD 149026 b's formation need to be taken into account. Without this information, we can only speculate. As it is very likely that centimetre-sized and larger grains spiral inward to the star, a fraction of the particles will never reach equilibrium before meeting the protoplanet. Once inside the planet, the high-pressure environment prevents evaporation. Therefore we assume that a fraction of the non-silicates can also be accreted on to the core of the protoplanet providing just enough material to form HD 149026 b.

6 NOTES ON MIGRATION

The reader might have pondered the lack of migration in the above discussion – this is partly intended. We wanted to show that while different types of migration can take place for different embryo masses and nebula proberties, they are not strictly necessary for the formation of close-in planets.

In the case of HD 149026 b we can go even further: our calculations show that to obtain such a large core, at least the last phase of core accretion must have occured in close proximity to its present location, i.e. without migration. The 'last phase' in this case refers to the time when the core mass grows beyond $\sim 30\,\mathrm{M}_\oplus$. Before that time, migration could have occured – this is irrelevant for the formation scenario presented here. Once the planet nears its final core mass, the nebula pressure is already very low. Therefore the planet's orbit should be stable against type-I migration.

This scenario has some interesting consequences. The conventional accretion model for planet formation at larger separations predicts a typical core size. Two giant planets forming in the same nebula should be of roughly similar mass, i.e. giant planets orbiting the same star can be expected to have similar core masses. This is not true for the in situ formation scenario presented in this Letter. In the case of HD 149026 b we could show that there is no dynamically triggered gas-accretion phase that sets in beyond a critical core mass. If this is the case for all close-in planets, core masses will only be determined by the amount of available material. Therefore we expect a wide distribution of core masses for close-in planets in the case of in situ formation. This property could be used to distinguish between in situ formation and large-scale migration.

7 CONCLUSIONS

We have demonstrated a plausible formation scenario for HD 149026 b. In situ formation permits the growth of the planet's very large core up to its present size.

Provided that enough material is available, even very low densities (e.g. minimum mass solar nebula values) produce very high planetesimal accretion rates. A Safronov number of 10 is enough to produce accretion rates of $\dot{M} = 10^{-2}\,\mathrm{M}_{\oplus}\,\mathrm{yr}^{-1}$, building a 70-$\mathrm{M}_{\oplus}$ core in less than 10000 yr. By analysis of all possible hydrostatic envelopes around such a large core we could determine the correct nebula pressure and demonstrate a fluid-dynamic model for the formation of HD 149026 b.

In Section 5 we analysed the mass flow in a typical T Tauri star accretion disc at an age of 1 Myr. We could show that for such a case enough material is transported across the orbit of a close-in planet, such as HD 149026 b, to provide sufficient material even when considering grain-evaporation caused by the high temperatures this close to the star.

The planet HD 149026 b could indeed have formed in situ, i.e. at its present position. In that case its entire evolution would have been quasi-static – no dynamic accretion takes place. Forming HD 149026 b at larger distances in this manner – e.g. outside the ice-line – is not possible, as the critical core mass decreases dramatically with increasing distance.

Our model predicts a wide range of core masses for close-in planets while migrating planets should all have similar 'typical' critical cores. This property could be used to distinguish between the two formation scenarios.

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