THE FORMATION OF WHITE DWARFS IN CLOSE BINARY SYSTEMS*†

Ronald F. Webbink
Department of Astronomy
University of Illinois
Urbana, Illinois 61801

ABSTRACT

Three major modes of interaction in binary systems are identified, corresponding to (I) physical contact, (II) tidal mass loss from a star with a radiative envelope, and (III) tidal mass loss from a star with a convective envelope. The qualitative evolution of binary systems in each of these cases is outlined, leading to the formation of single white dwarfs, long-period remnants of quasi-conservative mass transfer, or short-period cataclysmic binaries. The origin of short-period double white dwarfs in a second phase of mass transfer is outlined. Implications for the frequency of different systems of these types, and of planetary nebulae with close binary nuclei, are discussed.

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1. INTRODUCTION

The evolution of isolated stars is broadly characterized by gradual expansion, interrupted occasionally as the star adjusts to a new thermal equilibrium following core exhaustion or ignition of some fuel cycle (see, for example, the reviews by Iben [1967b, 1974]). Of course, for single stars this process concludes with either exhaustion or ejection of the non-degenerate envelope. In a close binary system, however, the expansion may be arrested by tidal mass loss to a companion star (see, for instance, the reviews by Plavec [1968, 1973], Paczynski [1971b], or Thomas [1977]). The remnant left by such a star in the binary case depends very strongly on the evolutionary state of the star at the instant mass loss begins, as we shall see below.

As a starting point, it is useful to diagram the radii of stars of various masses at significant transitions during their evolutionary trek. Such a diagram is illustrated in Figure 1 for a particular choice of composition ($X = 0.70$, $Z = 0.02$) and mixing length ($\ell/H_p = 1.5$). This particular diagram was constructed from a uniform set of models by Webbink (1975b and unpublished) and Eggleton (unpublished), supplemented by those of Iben (1965, 1966abc, 1967a) and Lamb, Iben, and Howard (1976). The radii at planetary nebula ejection correspond to the envelope mass-core mass relation deduced by Wood and Cahn (1976), but no account has been taken of physical or thermal pulsations at any stage in the evolution of these stars.

The critical radii plotted here depend weakly on the choice of composition and treatment of convection, but the general morphology of this diagram is preserved. Not shown in Figure 1 are the radii of intermediate and low-mass stars during core helium burning. Such stars have smaller radii than their red giant predecessors, and so cannot initiate tidal mass loss in their state. In addition, no evolutionary states beyond carbon ignition are shown. This owes principally to the lack of suitable calculations, but the final evolution of stars reaching this point is so rapid that significant further growth in radius is not anticipated.

From this radius-mass diagram, a period-mass diagram for close binaries similar to that introduced by Plavec (1968, 1973) can be constructed (Figure 2). By combining circular Newtonian orbits with the Roche geometry, one obtains a relationship between binary period, P, and

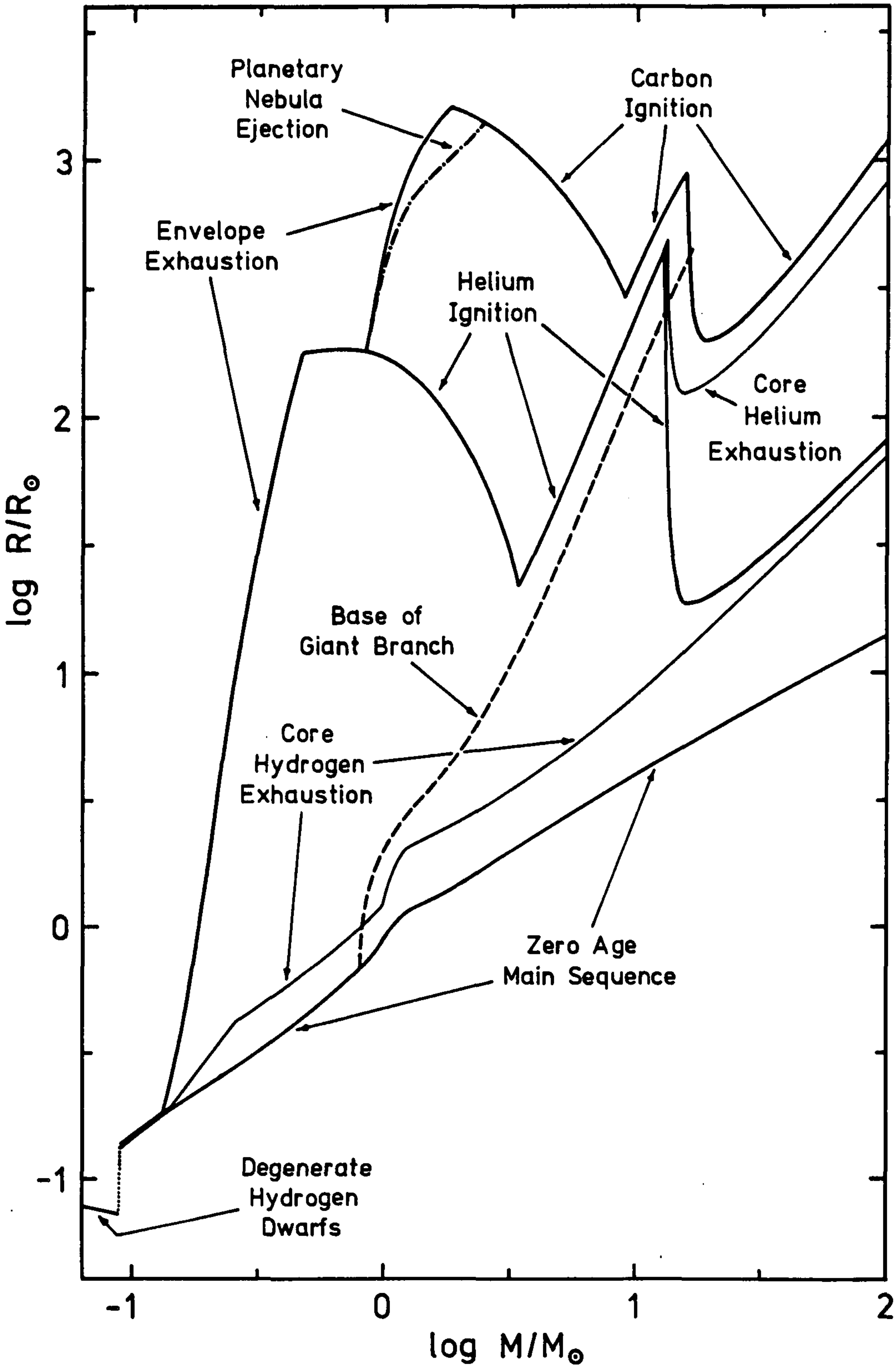

Figure 1. Radii of stars of solar composition at various stages of their evolution.

mean density, $\bar{\rho}$,

$$\log P = -0.433 - \frac{1}{2}\log\bar{\rho} - \frac{3}{2}\log f(\mu), \qquad (1)$$

which must be satisfied by any star just filling its Roche surface. (Throughout this review, we will deal only with circular orbits, for which the Roche potentials are well-defined. The eccentric-orbit problem, while certainly relevant to long-period systems [cf. Bouigue 1974], has largely defied resolution [see, e.g., Nduka 1971].) In this expression, P is in days and $\bar{\rho}$ in solar units. The term in $f(\mu)$ is a small correction for the mass fraction, μ, of the lobe-filling star, $\mu = M_1/(M_1 + M_2) = 1/(1 + q)$, where $q = M_2/M_1$ is the mass ratio of the binary, and $f(\mu)$ is given by

$$f(\mu) = \frac{3}{2}\left(\frac{3}{\mu}\right)^{1/3}\frac{R_{crit}}{A}, \qquad (2)$$

where R_{crit} is the tidal radius and A the binary separation. The correction due to this term is at most -0.37 in log P (for $\mu = 1$), and amounts to less than 0.10 in absolute value for $\mu < 0.8$ (cf. Kopal 1959).

An evolving primary (the more massive component) in a close binary decreases in mean density (increases in radius) as it evolves, until the critical density dictated by the binary period is reached, at which point tidal mass loss commences. Thus Figure 2 provides a ready reference to the evolutionary state of a mass-losing component in a system of given orbital period at the onset of tidal mass transfer.

In the following discussion, an attempt is made to outline comprehensively the general phenomenon of binary evolution, showing what kinds of primordial binaries produce what sorts of remnants. As such, it is perhaps not so much a review as a synthesis of current thought on the subject. It is worth bearing in mind, in considering white dwarf production, that unless a white dwarf announces its presence by accreting material from its companion, or by undergoing a thermonuclear outburst, it is likely to be detected in a close binary only if it is young and luminous (e.g., Feige 24: Thorstensen, et al., 1978), or if it retains a very close, cold companion (e.g., V471 Tau = BD +16° 516: Young and Nelson 1972; GK Vir = PG 1413+01: Green, Richstone and Schmidt 1978).

2. MODES OF MASS EXCHANGE

In the traditional notation, stages of tidal mass exchange in models of close binary systems have been identified by the phase of nuclear evolution occupied by the unstable component:

Case A --- core hydrogen burning
Case B --- shell hydrogen burning
Case C --- core or shell helium burning.

This convention arose out of the original modeling calculations by Kippenhahn and Weigert (1967) and by Lauterborn (1970). Hybrid cases, involving a transition from one nuclear phase to the next during mass exchange may occur, and are frequently denoted by a combination, AB (e.g., Ziolkowski 1970; Horn 1971) or BC (Tutukov, Yungel'son, and Klayman 1973). There is no common notation identifying contact binaries, in which both

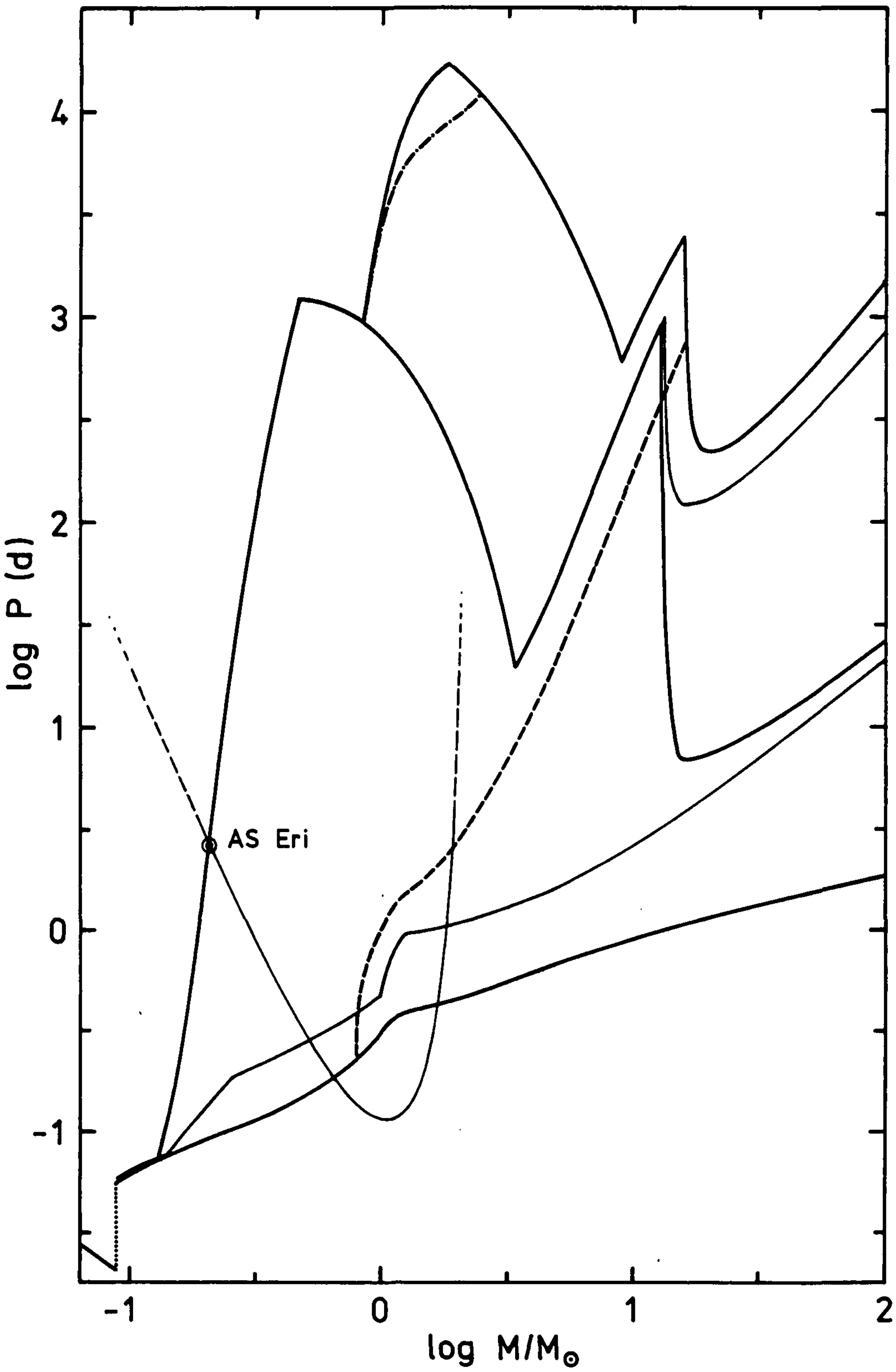

Figure 2. Limiting binary periods for the onset of tidal instability at various stages of evolution of the primary star. The individual curves correspond to the limiting radii of Figure 1, and are exact for a binary of unit mass ratio. The curve passing through the point representing the secondary in AS Eri is the trajectory followed by a system conserving total mass and orbital angular momentum.

stars fill their Roche lobes.

While this convention enjoys wide usage, providing a rough guide to the nuclear structure of the primary, the characteristics of the mass transfer process itself define broadly a different classification scheme, based on three different modes of mass exchange. These are the following:

2.1 Mode I: Contact Interaction

The most tightly bound, lowest-angular momentum zero-age binaries are those which are physically in contact, the W Ursae Majoris systems. The properties of these systems have been reviewed extensively elsewhere (Kraft 1967; Rucinski 1973, 1974; Binnendijk 1977). Their significance in the context of binary interaction is that, despite the fact that they must be nearly unevolved systems (e.g., TX Cnc in Praesepe: Whelan, Worden, and Mochnacki 1973), the two components have essentially identical surface temperatures, departing radically from the main sequence mass-luminosity relationship. Furthermore, from their high space density (Shapley 1948) and presence in old open clusters (e.g., AH Cnc in M67: Whelan, et al., 1979), it can be inferred that this contact state is secularly stable, apart from nuclear evolution (see Van't Veer [1975] for a different point of view). These systems lie near the zero-age main sequence in Figure 2.

A substantial transfer of luminosity from primary to secondary is required in these binaries (Lucy 1968), quite apart from mass exchange. The details of this interaction are at present far from clear, but a number of discussions of the transport problem have been published, concerning large-scale circulations (Hazlehurst and Meyer-Hofmeister 1973; Moses 1976; Nariai 1976; Webbink 1977c), convective interchange (Ivanov 1976), or temperature discontinuities in the envelope of one or the other component (Shu, Lubow and Anderson 1979a). Ivanov found convective interchange ineffectual, and the contact discontinuity model of Shu, Lubow, and Anderson has been widely criticized as unphysical (Hazlehurst and Refsdal 1978; Lucy and Wilson 1979; Papaloizou and Pringle 1979; Smith, Robertson, and Smith 1979; but see Shu, Lubow, and Anderson 1979ab). Quite apart from the details of energy exchange, however, the tendency of the more rapidly evolving primary to expand as hydrogen is depleted in its core can apparently be accommodated only if there is a net mass flow from secondary to primary on this nuclear time scale (Moss 1971; Biermann and Thomas 1973; Webbink 1976a). The binary may also suffer a cyclic thermal instability during this phase (Lucy 1976; Flannery 1976; Williams and Roxburgh 1976; Robertson and Eggleton 1977).

In addition to binaries in contact at zero age, intermediate and high-mass binaries may evolve to this state in consequence of the rapid (thermal time scale) phase of mass transfer which occurs when initially detached systems first interact tidally. The secondary expands rapidly out of thermal equilibrium, typically reaching contact after only a few percent of the primary's mass has been transferred. This phenomenon was first demonstrated by Benson (1970) and Yungel'son (1973), and has since been confirmed by many investigators. The physical reasons for this anomalous growth of the secondary are closely related to those for the contraction of a similar star undergoing thermal time scale mass loss (see Benson 1970, Webbink 1976b for a more detailed discussion). It must be stressed that expansion arises not because of the accretion energy of material falling to the stellar surface, but from the internal readjustment of the star to pressure equilibrium as its mass increases. This tendency to evolve into contact increases markedly for binaries with disparate masses, both because of the greater orbital shrinkage during mass reversal, and because of the great difference in thermal time scales between components. In any case, notwithstanding the different paths followed by these stars in reaching the contact state, their evolution from this point onwards probably resembles that of the W Ursae Majoris stars (Webbink 1979b). Observationally, many examples of massive contact binaries are now known (e.g., Leung and Wilson 1976; Leung and Schneider 1978; Eggen 1978).

2.2 Mode II: Mass Loss from Radiative Envelopes

This was the first mode of tidal mass loss to be modeled in detail (Paczynski 1967; Kippenhahn, Kohl, and Weigert 1967; Plavec, et al., 1968), and is now relatively well-understood. The onset of tidal mass loss brings an exponential growth in the mass transfer rate, until a value characteristic of the thermal time scale of the mass-losing star is achieved:

$$\dot{M}_1 \text{ (max)} \approx - \frac{M_1}{\tau_{KH}} \quad . \tag{3}$$

Mass loss continues at roughly this rate until the shrinkage of the tidal lobe about the unstable star abates, as a consequence of reversal of the mass ratio. The star can then return to thermal equilibrium, and the mass transfer rate drops to one dictated by its expansion due to nuclear evolution.

A star with a radiative envelope is able to pass through the mass reversal phase without greatly exceeding its tidal radius at any time because of its ability to contract sufficiently rapidly in response to mass loss to accommodate its shrinking tidal limit. Such a star has, by virtue of its stability against convection, a specific entropy profile in its envelope which increases outwards. Thus, as material is removed from the surface, energy must be supplied to subsurface layers to restore thermal equilibrium. If mass loss is sufficiently rapid, this energy deficit cannot be supplied immediately, with the result that the envelope is cooler and denser than normal for a star of its mass: the mass-losing star becomes undersized and underluminous for its mass.

2.3 Mode III: Mass Loss from Convective Envelopes and Degenerate Stars

Since the ability of a radiative star to shrink with its Roche lobe arises from its very stability against convection, it is not surprising that the circumstances attending mass loss from convective envelopes lead to a very different response. The behavior of stars of this type was first noted by Paczynski (1965), and early discussions of the phenomenon offered by Paczynski, Ziolkowski, and Zytkow (1970) and by Lauterborn and Weigert (1972).

Since in a convective envelope the specific entropy of matter decreases outwards, no energy must be supplied such a star as it loses mass to restore thermal equilibrium: the envelope must in fact cool to achieve this state. Consequently, rather than withering in response to mass loss, these stars tend to expand (notwithstanding the shrinkage of the tidal limit), depending on the depth of this surface convection zone. (The behavior of these stars is in fact well-approximated by mixed polytropes with envelopes of index $n = {}^3/_2$.) The mass loss rate increases to values limited essentially only by the sonic velocity in the unstable envelope.

The relatively few detailed calculations of mass loss in this mode have all used variants of a model of the mass loss process due to Jedrzejec (1969). This treats the problem as one of the free expansion of the unstable envelope through a sonic surface at the constriction of equipotential surfaces near the inner Lagrangian point. Even in calculations conservative of mass and angular momentum (Paczynski and Sienkiewicz 1972; Plavec, Ulrich and Polidan 1973; Webbink 1977ab), peak mass mass loss rates of order

$$\dot{M}_1 \text{ (max)} \stackrel{\sim}{\sim} - \frac{M_1}{(\tau_{hyd}\ \tau_{KH})^{1/2}} \tag{4}$$

are achieved, where τ_{hyd} is the sound travel time through the star, and τ_{KH} is the Kelvin time, as before. Even among low-mass stars, this rate exceeds 10^{-3} $M_\odot$ yr^{-1}. Rates several orders of magnitude greater than this may be encountered if substantial angular momentum loss occurs (Drobyshevski and Reznikov 1974ab; Webbink 1979a).

Mass loss of a very similar character could also be anticipated from lobe-filling degenerate stars. In this case, expansion in response to mass loss is fed by energy drawn from the Fermi sea of electrons, rather than from the thermal energy of the non-degenerate gas discussed above (cf. Pringle and Webbink 1975).

The possibility of lobe-filling degenerate stars is not so far removed from reality as might be supposed. Many models of WZ Sge, for example, invoke such stars (e.g., Krzeminski and Kraft 1964; Ritter 1978), and a similar model has been proposed for AM CVn = HZ 29 (Faulkner, Flannery, and Warner 1972). In general, however, a degenerate star can only fill its tidal lobe if its companion is also degenerate, in which case the lobe-filling star is always the less massive one. In these circumstances, runaway mass transfer occurs only if the mass ratio is near unity, or if severe angular momentum losses accompany the exchange process.

3. ENDPOINTS OF MASS TRANSFER

It is apparent that the behavior of a binary undergoing mass exchange is at least qualitatively associated with the evolutionary state of its more massive component at the onset of instability. There is no simple correspondence with the traditional Cases A, B, and C, however. The following discussion is an attempt to recast current theoretical work in terms of the various modes of mass transfer outlined above, and to identify the descendant binary types which result from variants of these modes.

It is appropriate to list first a few simple and obvious constraints on the types of remnants which may be produced by a given binary:

(1) The remnant object cannot possess more mass or angular momentum than its progenitor.

(2) Nuclear evolution is essentially irreversible. Indeed, only in exceptional circumstances is significant mixing through a composition gradient, or across a boundary, likely to occur in response to mass exchange (cf. Webbink 1977a). Thus a remnant cannot in general have a smaller core mass than its progenitor.

(3) If the total duration of mass exchange is short compared with the nuclear time scale of the primary (lobe-filling) star, the core of the remnant (i.e., that portion completely exhausted of one or more fuels) will differ insignificantly in mass from that of its progenitor. As a practical matter, this situation normally demands thermal or dynamical (convective) time scale mass transfer, which does not normally cease until all fuel is removed from the outermost nuclear shell source. That is, the remnant tends to be the stripped core of its progenitor.

(4) If the terminal phases of mass exchange proceed at rates slow compared with the thermal time scale of the mass-losing star, the core mass of that star continues to grow as it loses mass up to the point of detachment, provided that the progenitor star at the onset of mass loss had not already developed so massive a core that violations of the first and second constraints above are implied for a thermal equilibrium remnant at detachment (cf. Refsdal and Weigert, 1971).

Before surveying different types of mass transfer in this context, however, one other point is noteworthy. If we fix the total mass and orbital angular momentum, J, of a binary, where

$$J^2 = G\,(M_1 + M_2)^3\, A\, \mu^2\, (1 - \mu)^2 \quad , \qquad (5)$$

a unique relationship is defined by the Roche geometry between the mass of a lobe-filling component and the binary period. This relationship is illustrated in Figure 2 with a curve appropriate to the well-studied Algol-type binary AS Eridani (see below). This fiducial curve may in fact be translated vertically or horizontally in the figure, without change of scale, to explore the variations in orbital parameters of any other binary (under the assumption of conservation of total mass and orbital angular momentum), by fitting the curve through the point corresponding to the orbital period and mass of the lobe-filling component, in such a way that the minimum of the curve falls at one-half the total mass of the binary. This is a very valuable expedient in exploring possible progenitors or descendants of observed systems, provided that the conservative assumption can reasonably be justified.

In Figure 3, the various evolutionary stages delineated in Figures 1 and 2 are identified with the modes of mass exchange appropriate to stars first reaching tidal instability in those stages. We can now discuss how binaries evolve in each of these regimes. In order to simplify this discussion, however, attention will be concentrated on systems of essentially unit initial mass ratio, but in which only one component is evolving. This is a reasonable restriction, given the primordial distribution in mass ratios (cf. Trimble 1974, 1978; Abt and Levy 1976, 1978;

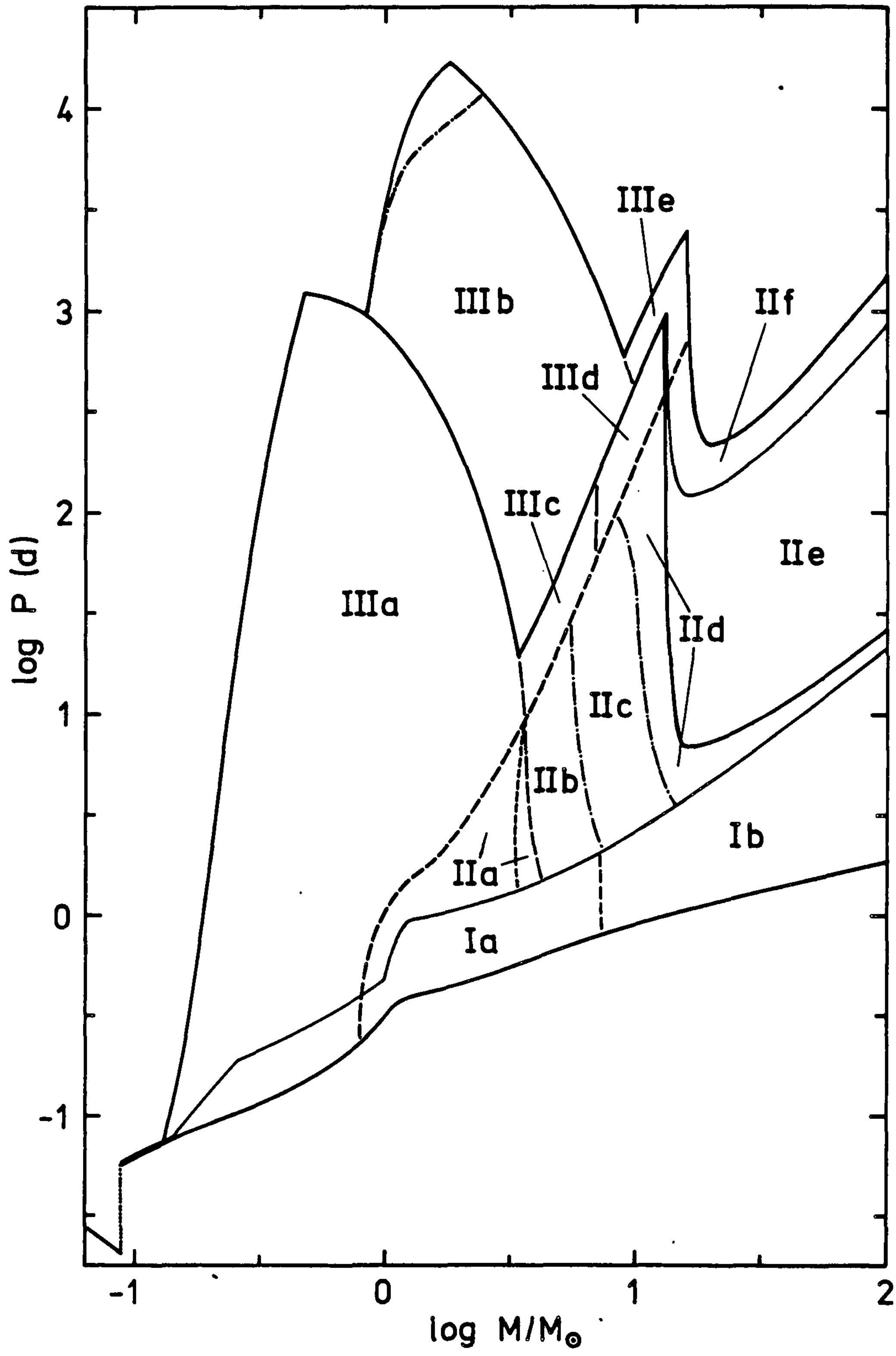

Figure 3. Identification of the major modes of mass mass exchange, and variants within them, in the period-mass diagram. The qualitative evolution of systems in these various regions is outlined in the text.

Lucy and Ricco 1979) and the strong mass-dependence of stellar lifetimes. In addition, a formal definition of the stellar core mass has been adopted from Webbink (1975b) in delineating the subdivisions in Figure 3. For stars with thick shells, this function places the boundary approximately at the center of the shell. In this case, decreases in core mass are possible if the surface convection zone encroaches on a composition gradient, but this effect is pronounced only among higher-mass asymptotic giants where the hydrogen-burning shell may be weak or extinct. For thin burning shells, the core masses are well-defined.

3.1 Mode I

The structure and evolution of contact binaries have been the object of numerous studies and much controversy, as mentioned above (§ 2.1). As intimated there, however, the long term survival of these systems demands the accumulation of mass by the more massive component. In this case, the very limited angular momentum of the progenitor system must eventually be absorbed by this star, resulting in stellar coalescence, which probably occurs very rapidly following core hydrogen exhaustion in the primary (Webbink 1976a). The most promising candidates for binaries undergoing coalescence are probably a small number of objects (e.g., UZ Lib, HD 199178) resembling FK Comae (Bopp 1977), a peculiar, low-amplitude ellipsoidal variable containing a rapidly rotating, emission-line G giant apparently embedded in an extended disk. Evolution of the completely coalesced star is not expected to differ significantly from that of primordial single stars of similar mass.

Attention was also called above to many studies showing formation of contact binaries due to the rapid swelling of accreting stars. As a matter of expedience, it has been assumed in Figure 3 (regions Ia and Ib) that this course of evolution dominates systems evolving in the traditional Case A. In fact, this phenomenon is in all likelihood extremely sensitive to the initial mass ratio, as indicated above, and has been found to occur in models of more widely separated systems (Flannery and Ulrich 1977). Indeed, among evolutionary studies including detailed models of the accreting star, the only instance in which mass ratio reversal has been successfully negotiated is one involving a very low-mass accreting star with a deep convective envelope (Webbink 1977a). The remaining studies all involve rather close systems with significantly different component masses, however. A number of them have carried calculations beyond the onset of contact by assuming the common surface now coincides with a single equipotential surface, but neglecting the energy exchange which is so pronounced in W Ursae Majoris systems. This omission is a serious one, as a strong case can be made that the onset of contact leads to reversal of the mass flow (Webbink 1979b), rather than the overflow of the outer Lagrangian surface which otherwise occurs. Avoidance of deep contact demands a subsequent course of evolution not unlike that outlined for the W Ursae Majoris stars (Webbink 1976b).

For massive contact binaries, total mass $M \gtrsim 14\ M_\odot$ (region Ib), an alternative possibility to coalescence may occur. This is because core helium ignition occurs immediately after the primary has left the main sequence, and may permit the original primary to contract to the helium main sequence, rather than forcing the gross expansion which otherwise leads to coalescence. Such a course of evolution may be required to explain the short binary periods of many Wolf-Rayet binaries (Packer and De Greve 1979).

In summary, then, with the possible exception of massive systems producing Wolf-Rayet components too massive to become white dwarfs anyway, mass exchange in Mode I probably leads to single stellar remnants not much different from those left by primordial single stars of similar total mass.

3.2 Mode II

These are binaries which encounter mass exchange while the primary is crossing the Hertzsprung gap (regions IIa-f in Figure 3). The anticipated mass transfer rates are sufficiently moderate that, provided creation of a secularly stable contact state can be avoided, mass ratio reversal may be expected to proceed quasi-conservatively, i.e., without catastrophic losses of systemic mass and angular momentum. This expectation is indeed borne out by the high observed frequency of Algol-type eclipsing binaries.

Systems with primary masses $M_1 \lesssim 3.5\ M_\odot$ (IIa in Figure 3) follow the initial thermal time scale mass ratio reversal with a prolonged phase during which mass is lost to the secondary on an approximately nuclear time scale as the degenerate core grows, up to the point where the primary's envelope has essentially been exhausted. Evolution of this type has been computed by many authors (see, for example, the reviews by Paczynski [1971] and Thomas [1977]; Webbink [1975a] gives a bibliography of all calculations up to 1975), and the post-mass-reversal systems are unmistakably of the Algol type. This association is supported by studies both of individual binaries (e.g., Plavec 1973) and of the statistics of Algol systems (e.g., Ziolkowski 1976), but despite the qualitative accord between theory and observation, substantial angular momentum losses, exceeding 50 percent in many cases, are required to produce quantitative agreement (Popov 1968, 1970; Svechnikov 1973; Plavec 1973; Ziolkowski 1976). The detailed study of AS Eri by Refsdal, Roth and Weigert (1974) can be particularly recommended in this regard. As can be seen in Figure 3, this system must now be near completion of mass transfer, as less massive secondaries can no longer fill their Roche surfaces. Significant angular momentum losses must have occurred since mass reversal, in view of the unreasonably small separation otherwise deduced at mass equality. The precise mechanisms by which these losses occur have yet to be identified, although some non-conservative effects have been discussed by Wilson and Stothers (1975), Lin and Pringle (1976), and Savonije (1978), for example.

Those systems evolving within a Hubble time leave binary remnants containing low-mass helium white dwarfs, $0.21\ M_\odot \lesssim M_{wd} \lesssim 0.46\ M_\odot$, with main sequence companions of a few (1.5 to 6) solar masses. Because detachment occurs in these systems as the secondary crosses the line appropriate to envelope exhaustion in Figure 3, the relation between white dwarf mass and remnant period is in fact the one given by this line (Refsdal and Weigert 1971), i.e., these two quantities must be very highly correlated for Algol remnants.

Binaries with primary masses $M_1 \gtrsim 3.5\ M_\odot$ enter the Hertzsprung gap with sufficiently massive cores that little or no core growth occurs during the mass transfer phase, although the hydrogen burning shell continues to narrow. The very long terminal phase of core growth and nuclear time scale mass loss following mass reversal, characteristic of the less massive systems, is thus much curtailed.

For a very small range of masses, $3.3\ M_\odot \lesssim M_1 \lesssim 4.3\ M_\odot$ (the portion of IIa to the right of the dashed line in Figure 3), the primary star may yet be stripped below the threshold for helium ignition, ending life as a $0.34\ M_\odot$ - $0.46\ M_\odot$ helium white dwarf. Somewhat more massive primaries, $3.6\ M_\odot \lesssim M_1 \lesssim 7.6\ M_\odot$ (IIb), develop sufficiently large helium cores on the main sequence that ignition is assured in any case, at which point mass transfer is terminated (Ziolkowski 1970). These stars pass through a presumably detached helium star phase on their way to becoming $0.46\ M_\odot$ - $1.0\ M_\odot$ carbon-oxygen white dwarfs. No evolutionary calculations of this type have yet been carried beyond helium ignition, however.

For somewhat more massive primaries yet, $5.6\ M_\odot \lesssim M_1 \lesssim 14\ M_\odot$ (IIc), a second phase of tidal mass loss by the primary may follow core helium exhaustion by the helium star remnant (De Greve, De Loore, and van Dessel 1978, and references therein). This is because, even completely denuded of their hydrogen envelopes, the helium stars ($1.0\ M_\odot \lesssim M_{He} \lesssim 2.6\ M_\odot$) left by the first phase of mass transfer expand considerably during their shell helium burning phases (e.g., Paczynski 1971a). The primary is thus again stripped down, this time to its helium burning shell, eventually leaving a $1.0\ M_\odot$ - $1.4\ M_\odot$ carbon-oxygen white dwarf. During this final phase of mass loss, some growth of the carbon-oxygen core may be possible among the lower-mass helium stars in this mass range, leading to a pronounced peak in the white dwarf mass function near $1.0\ M_\odot$ where the helium star limiting radius is a very strong function of mass.

Binaries containing primary components falling in regions IId-f leave helium star remnants of sufficient mass that they do not develop the deeply degenerate carbon cores those of lower mass do prior to carbon ignition (compare, e.g., Kippenhahn and Weigert 1967; Tutukov, Yungel'son and Klayman 1973; De Loore and De Greve 1976). Their growth in radius following core helium exhaustion is then much smaller than the lower mass helium stars, and a second phase of mass loss does not occur. As the remnants in the more massive systems have masses upwards of $1.9\ M_\odot$ (IId) or $4\ M_\odot$ (IIe-f), they may be expected to leave collapsed (neutron star or black hole) remnants. The further evolution of these systems, as progenitors of X-ray binaries, has been reviewed extensively elsewhere (van den Heuvel 1976; Massevitch, Tutukov, and Yungel'son 1976).

3.3 Mode III

Clearly, a binary is extremely unstable in which the more massive component fills its tidal surface, and yet is unable to contract as this surface does in response to mass loss. Although some studies have modeled mass transfer in this case as conservative of mass and angular momentum, it has since become clear that the enormous mass transfer rates which are encountered probably activate a number of powerful mechanisms for extensive mass and angular momentum loss from these systems. These include supercritical accretion onto the main sequence companion (Plavec, Ulrich, and Polidan 1973; Webbink 1975a, 1979a; Bath 1977), loss of material through an outer Lagrangian point or similar rotational instability (Ritter 1975, 1976; Webbink 1975a, 1979a; Nariai and Sugimoto 1976; Lin 1977; Livio, Salzman, and Shaviv 1979), and dissipation within an embedding envelope (Sparks and Stecher 1974; Alexander, Chau and Henriksen 1976; Paczynski 1976; Taam, Bodenheimer and Ostriker 1978; Meyer and Meyer-Hofmeister 1979).

The association of classical novae and dwarf novae with the descendants of binaries interacting in this mode of mass loss (Ritter 1975, 1976; Webbink 1975a; Paczynski 1976) is now well-supported by several lines of evidence (cf. Webbink 1979a), most notably the existence of planetary nuclei with close binary nuclei (Mendez and Niemela 1977; Bond, Liller and Mannery 1978; Acker 1978). Probably all of the nonconservative mechanisms cited above contribute in some measure to the extensive mass and angular momentum losses implied, but the details of the transformation remain largely matters of speculation (see Webbink [1979a] for a discussion of possible intermediate objects). More to the point, rotational shedding and common envelope evolution constitute such a severe drain on systemic angular momentum that it is not clear what arrests the spiral toward coalescence (cf. Meyer and Meyer-Hofmeister 1979). Nevertheless, the high apparent space density of remnant systems (Kraft 1965; Bath and Shaviv 1978) would seem to imply that a substantial fraction of binaries undergoing this transformation decouple in a state still not far removed from tidal interaction.

Whatever the details of this mass transfer process, it is possible to identify several end products (Figure 3), corresponding to different initial states of the primary core, as the entire duration of mass loss is in any case far too short to permit it to evolve significantly. Shorter-period low-mass systems (IIIa) interact before the degenerate helium core has grown to the critical mass for helium ignition, thus leaving low-mass (0.18 $M_\odot$ - 0.46 $M_\odot$, for systems of age less than a Hubble time) helium white dwarf remnants. Because of the very slow evolution of such low-mass cores toward the white dwarf track in the Hertzsprung-Russell diagram (Webbink 1975b), few of them are likely to be capable of exciting the binary ejecta as planetary nebulae (Livio, Salzman, and Shaviv 1979). This limitation is less severe among longer-period systems (IIIb), which interact after the primary has reached the asymptotic giant branch. These systems leave more massive (0.56 $M_\odot$ - 1.4 $M_\odot$) carbon-oxygen white dwarfs. Webbink (1979a) associates the striking absence of cataclysmic binaries with orbital periods between 2 and 3 hours with this discontinuity in core masses at the division between regions IIIa and IIIb; the helium white dwarfs arising out of IIIa form the ultra-short period systems with orbital periods below 2 hours, while the carbon-oxygen white dwarfs from IIIb form the longer-period group above 3 hours. The discontinuity in core masses reflects advance of the hydrogen burning shell while the primary has contracted into core helium burning.

In addition to these systems, mass loss in this mode can also lead to cataclysmic-like binaries containing helium stars. This may occur when an intermediate-mass star develops a sufficiently large helium core to insure ignition, but encounters tidal instability on the giant branch before helium ignition. If its mass lies between 3.5 $M_\odot$ and 7 $M_\odot$ (IIIc), a helium star of 0.46 $M_\odot$ to 1.4 $M_\odot$ should be produced, whereas beyond 7 $M_\odot$ (IIId) the helium star remnants range upwards from 1.4 $M_\odot$ to 4 $M_\odot$. The novalike variable V Sge has been suggested as an example of such a helium star cataclysmic binary (Whyte and Eggleton 1979). In the absence of a further phase of mass exchange during shell helium burning, these should leave carbon-oxygen white dwarfs (IIIc) or neutron stars (IIId). However, above 0.8 $M_\odot$, sufficient expansion of the helium star may be expected, to force a second phase of mass transfer from the primary, leaving the ultimate fate of these systems in doubt. If stripped to their carbon-oxygen cores, they would leave remnants in the range from 0.8 $M_\odot$ to 2.0 $M_\odot$, with those of Chandrasekhar mass arising from 2.5 $M_\odot$ helium stars, corresponding to $\sim$10 $M_\odot$ progenitors. A final possibility occurs for relatively massive stars, 9 $M_\odot \lesssim M_1 \lesssim 16$ $M_\odot$ (IIIe), for which transfer in this mode could leave massive helium stars (1.4 $M_\odot$ to 6 $M_\odot$) in shell-burning configurations, although the possibility remains that some of these stars could be stripped even further (down to the helium-burning shell). Unfortunately, the evolution of helium star cataclysmic systems has not previously been explored, and these deductions must be based solely on the general principles outlined in the introduction to this section.

4. THE SECOND PHASE OF MASS TRANSFER

Some time after the primary has completed its evolution, a second phase of binary interaction may occur, in which the role of the tidally unstable star is filled by the primordially less massive star, the secondary. Of course, such a phase does not exist if the first phase of mass transfer led to coalescence (Ia).

Space does not permit a detailed discussion of the secular evolution of the cataclysmic-like binaries produced from mass transfer in Mode III. This is itself an extremely complex subject, made more difficult by the uncertain extent to which systemic mass and angular momentum loss is provoked by the outbursting characteristics. General discussions have been published by Faulkner (1976), Warner (1978), and Whyte and Eggleton (1979). It is worth noting, however, that the true space density of binaries of this general type (degenerate star plus main sequence companion) may well be utterly dominated by detached systems of the V471 Tau or GK Vir type. Intrinsically, these systems are extremely faint, so they are far less conspicuous than the violently eruptive dwarf novae or classical novae. Given the relatively long life times already anticipated for cataclysmic variables, of order 10^9 years, it is still possible that they are preceded by an even longer detached state (similar to the inactive state hypothesized for X-ray binaries), with characteristic lifetimes of the order of a Hubble time or even longer. In this case, we would only sample a fraction of these binaries as cataclysmic variables within the age of the Galaxy. Leaving aside such speculations, however, the ultimate evolution of these binaries is probably characterized by gradual and complete erosion of the secondary (references above; also Chau and Lauterborn 1977), only in exceptional circumstances (e.g., GK Per) leaving a double white dwarf.

Figure 4 illustrates the evolutionary state of the original secondaries of Mode II mass transfer systems at the onset of the second phase

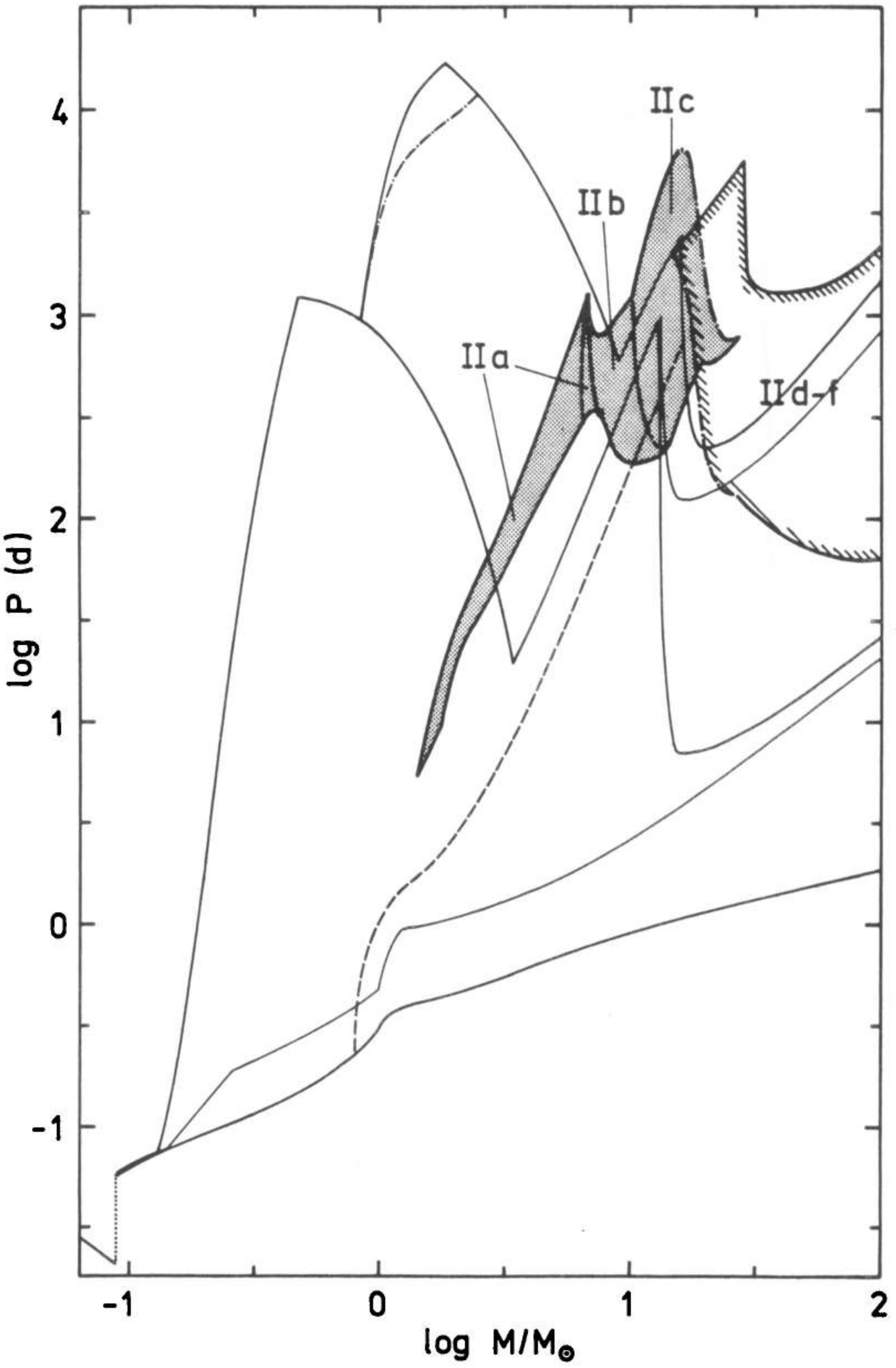

Figure 4. Remnants of the first phase of mass transfer in Mode II, for systems of unit initial mass ratio conserving total mass and orbital angular momentum. The mass is that of the original secondary, which itself loses mass during the second phase of mass transfer. Shaded regions (IIa-c) denote stars with white dwarf companions, the hatched region (IId-f) those with collapsed companions.

of transfer. It has been assumed here that the primordial systems had unit mass ratios, and strictly conserved mass and orbital angular momentum. In fact, as we have already pointed out, significant losses occur in these systems, which would reduce the remnant periods significantly, as well as giving a modest reduction in the final mass of the secondary (accreting star). On the other hand, mass losses in a stellar wind approaching the second phase of transfer would increase the binary period, while again decreasing the secondary remnant mass. Finally, primordial mass ratios less than unity yield secondary remnant masses correspondingly smaller, and may also yield shorter orbital periods when the remnant primary mass is dictated by the core mass at the onset of tidal instability (IIb-e). The indicated regions are thus highly idealized, and real systems may be found at both smaller masses and shorter orbital periods than illustrated in Figure 4.

The noteworthy feature of the region occupied by Mode II remnants is that, as a rule, the second phase of mass transfer occurs only after the original secondary has reached the base of the giant branch, i.e., in Mode III. This phase of mass transfer has been very little studied. We merely note that the deduced properties of systems containing massive white dwarfs (IIb-c) approaching this phase are in qualitative accord, aside from a tendency toward somewhat more massive giant components, with the wind accretion, nuclear outburst model of symbiotic stars proposed by Tutukov and Yungel'son (1976). Nevertheless, accretion models onto either main sequence stars or massive white dwarfs (Bath 1977; Webbink 1979a) are also permitted within this evolutionary framework. What do not occur are the very long period, low-mass giant plus white dwarf systems discussed by Whelan and Iben (1973) as Type I supernovae progenitors.

As in first phase transfer in Mode III, extensive mass and angular momentum loss may again be anticipated during tidal interaction in this case, but the remnant binary will now consist of a close double white dwarf (Webbink 1975a). The prospect for such a system surviving this intense interaction as a binary is actually much improved over that for the classical nova progenitors outlined above, as in this case the very compact cores permit extraction of up to two orders of magnitude more orbital energy for envelope ejection before either core can itself be disrupted. Double white dwarfs formed in this way may have masses ranging from 0.26 $M_\odot$ + 0.21 $M_\odot$ to 1.4 $M_\odot$ + 0.9 $M_\odot$. As a rule, the more massive component is the younger white dwarf of the pair. A few candidates for double white dwarfs are known (AM CVn: Faulkner, Flannery and Warner 1972; GP Com: Warner 1972; LB 3459: Kilkenny, Penfold and Hilditch 1979), but none of these instances are certain. From an evolutionary standpoint, yet a third phase of tidal interaction might be expected, but only when the orbital period had been eroded to the order of one minute by gravitational radiation (cf. Pringle and Webbink 1975), at which point the gravitational radiation time scale is scarcely a few thousand years. Mass transfer on this sort of time scale between white dwarfs implies such high accretion luminosities that extensive mass and angular momentum losses should be expected.

Two points are worth noting in closing this discussion. The first is that any residual hydrogen envelope on either white dwarf can scarcely survive the onion-skin-like tidal mass transfer between them. Mass transfer or coalescence in this case may thus be relevant to the formation of

DB white dwarfs. The second is that, given the extremely long inactive lifetimes ($>10^{10}$ years) inferred for such white dwarf-main sequence binaries as V471 Tau and GK Vir, a similar situation among double white dwarf systems would open the possibility that one component could finally be pushed over the Chandrasekhar limit after an almost arbitrarily long hiatus in mass transfer. This situation is similar to that discussed by Warner (1974) for the production of Type I supernovae from cataclysmic variables, except that the material accreted by the more massive white dwarf is in this case far less explosive.

5. CONCLUSION

A few words are appropriate in closing, on the subject of the frequencies with which real stars may actually follow one or another of the evolutionary itineraries outlined above. Unfortunately, space does not permit a detailed discussion here of the statistics of primordial (unevolved) binaries. The observational study of this problem probably least subject to uncontrolled selection effects is that of Abt and Levy (1976, 1978; see also Heintz 1969). Their results regarding the frequency of close binaries among both B stars and solar-type stars are consistent with a uniform distribution in the logarithm of the orbital period, with a probability of approximately 0.14 per decade in orbital period that a given main sequence star has a less massive stellar companion. A substantial fraction, possibly even a majority, of stellar systems (single, double or multiple) must therefore evolve not as isolated stars, but as close binary systems.

One point to be made is that the discussion of Mode II transfer above (§ 3.2) showed that, quite apart from questions of mass loss from isolated stars, white dwarf production is possible in binaries with primaries as massive as $\sim 14\ M_\odot$. Caution is therefore warranted in the interpretation of white dwarfs in young open clusters as indicative of a high threshold for planetary nebula ejection in isolated stars (cf. Tuchman, Sack, and Barkat 1978).

Another point is that the origin discussed above for cataclysmic-like short period binaries implies that they are extremely common objects. Assuming a uniform birth rate and a Salpeter (1955) mass function, and allowing for mass loss from isolated stars (after Wood and Cahn 1976), an estimated 38 percent of all stellar deaths (counting binary stars as single objects) should produce binaries of this type (regions IIIa and IIIb in Figure 3). Another ~ 6 percent ultimately produce double white dwarfs (IIa-c).

A corollary of the above statement is that planetary nebulae with close binary nuclei should be extremely common. According to Wood and Cahn (1976), isolated stars below $\sim 4.4\ M_\odot$ become planetary nebulae. If we suppose binaries evolving initially in modes IIIb-c and IIb-c ultimately produce visible planetaries, we should expect them to contribute ~ 20 percent (white dwarf plus main sequence star) and ~ 4 percent (double white dwarf), respectively, of the total planetary birth rate (allowing that not all stars below the planetary threshold mass are single). Substantially higher contributions are possible if the lower mass binary remnants also illuminate their nebulae, or if not all single star remnants do so.

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