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The forward and inverse problem in tissue optics based on the radiative transfer equation: a brief review

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Abstract

This note serves as an introduction to two papers by Klose et al. [Optical tomography using the time-independent equation of radiative transfer, Parts 1 (JQSRT 2002;72:691–713) and 2 (JQSRT 2002;72:715–732)] and provides a brief review of the latest developments in optical tomography of scattering tissue. We discuss advancements made in solving the forward model for light propagation based on the radiative transfer equation, in reconstructing scattering and absorption cross sections of tissue, and in molecular imaging of luminescent sources.

Keywords

Radiative transfer; Discrete ordinates; Simplified spherical harmonics; Optical tomography; Molecular imaging; Fluorescence; Bioluminescence

Optical tomography (OT) is a medical imaging modality that calculates three-dimensional (3D) maps of absorption and scattering coefficients in biological tissue by using a radiative transfer model for visible or near-infrared light. The reconstructed optical property maps aid the diagnosis of pathological dysfunction in clinical and pre-clinical imaging. For example, changes in the absorption coefficient, caused by changes in blood-oxygenation levels, facilitate the study of tumor development or brain function. OT provides an experimental means for monitoring these changes [1].

The mathematical framework of OT is given, in general, by the combination of a forward model for light propagation and an inverse model for reconstructing the optical property maps from predicted and measured partial boundary currents. The most commonly applied forward model in OT has been the diffusion equation due to its mathematical simplicity and the availability of a vast amount of fast and efficient numerical solvers [1]. The diffusion equation is a low-order approximation to the more generally applicable radiative transfer equation (RTE) and, as such, is only valid in the diffusion limit wherein scattering dominates absorption. A two-part contribution by Klose et al. [2,3] offered a dramatic improvement by providing the first experimentally validated image reconstruction algorithm beyond the diffusion limit based on the time-independent RTE.

Since the publication of these two papers in 2002, major advances have been achieved in OT with respect to solving the forward model based on the RTE [4]. The discrete-ordinates (S_N)

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method, for example, has been further developed for 3D [5] or curved geometries [6], spherical harmonics (P_N) methods have been implemented for use in tissue optics [7,8], and a simplified spherical harmonics (SP_N) approximation to the RTE has been derived [9]. Furthermore, the RTE-based OT has also been extended to the time domain (TD) [10,11,12] and frequency domain (FD) [13,14,15,16,17,18]. Finally, different methods for solving the inverse source problem based on the RTE have been developed and play a major role in molecular imaging of fluorescent and bioluminescent sources in scattering tissue. A comprehensive review of RTE-based molecular imaging and tomography is given in [4].

A major improvement in tissue optics with the RTE has been achieved by significantly accelerating the solution process of the forward model while maintaining the same level of physical and numerical accuracy. Here, the RTE is approximated by the SP_N equations which form a small set of coupled diffusion equations [9]. For example, the SP_3 approximation (N=3) consists of only two equations. Their solution can be obtained ~100 times faster than solving a set of 288 equations of a S_{16} discrete-ordinates method [9]. Most importantly, the ray effect, which is typical of the S_N method, is not caused by the SP_N approximation. Finally, solutions to the SP_N equations also outperform the diffusion solution in terms of physical accuracy, especially when strongly absorbing tissues, light sources in the vicinity of a boundary, or small tissue geometries are considered [9].

The RTE has also been solved in the FD and TD using the S_N [11,13,14,15] or SP_N [17,18] methods. By increasing the amount of independent measurement data, solving the RTE in FD or TD promises to improve the image quality of the reconstructed optical property maps. FD and TD solutions with respect to the inverse problem partially overcome limitations caused by a cross-talk between physically independent absorption and scattering cross sections that can be observed with time-independent solutions.

In recent years, molecular imaging has emerged as an investigative tool for pre-clinical imaging of optical reporter probes in small animals. Molecular imaging uses fluorescent or bioluminescent sources inside tissue as imaging contrast. They can be used for studying gene expression, protein–protein interaction, or other effects caused by pharmacological intervention on a molecular or cellular level. In optical molecular tomography, an inverse model based on the RTE recovers the 3D distribution of light-emitting sources while fluorescence or bioluminescence light intensities at the tissue surface are measured [4].

For example, the inverse fluorescent source problem based on the RTE was solved with an S_N approximation [5,19], with a S_N approximation in the FD [16], and with a SP_N approximation [20]. The latter uses only the spectral dependence of light absorption in tissue for reconstructing the unknown 3D fluorescent source distribution.

Bioluminescence tomography also deals with solving an inverse source problem like fluorescence tomography, taken into account that no excitation source for light-emission stimulation is present. Here, bioluminescent sources have been reconstructed with methods based on the S_N [6] and SP_N [21,22,23] approximations. Overall, the majority of solution techniques use multi-spectral light intensity measurements. They exploit the spectrally dependent light absorption of tissue between 580 nm and 660 nm [21,22,23]. In fact, highly absorbing tissues at wavelengths <650 nm of the bioluminescence spectrum necessitate RTE solutions in order to obtain sufficiently accurate forward model predictions.

In summary, the development and application of different solution methods based on the RTE and its high-order approximations have proven to be a valuable tool in tissue optics and OT. They will continue to facilitate and accelerate discoveries in biomedical research.

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