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The Fractional Sallen-Key Filter Described by Local Fractional Derivative

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ABSTRACT The local fractional derivative (LFD) has attracted wide attention in the field of engineering application. In this paper, the LFD is used to model the fractional Sallen-Key filter for the first time. The non-differentiable(ND) transfer function is obtained by using the local fractional Laplace transform(LFLT). And the amplitude frequency response is analyzed in detail for different fractional order ς. It is found that the fractional Sallen-Key filter becomes the ordinary one in the special case $\varsigma = 1$. The obtained results of this paper show the powerful ability of local fractional calculus in the analysis of complex problems arising in engineering fields.

INDEX TERMS Local fractional derivative, Sallen-Key filter, fractional circuit systems, local fractional Laplace transform.

I. INTRODUCTION

As a frequency selection circuit, the filter attenuates the signal in a certain frequency range very little, making it pass smoothly; the signal outside this frequency range attenuates very much, making it difficult to pass, that has been widely used in various circuits. The Sallen-Key filter is one of the most widely used filters in engineering. Its circuit prototype is composed of VFA (voltage feedback operational amplifier) and RC elements. Its advantages are simple circuit structure, simple expression of passband gain, pole angle frequency and quality factors, convenient adjustment of quality factors and large adjustable range.

The ordinary Sallen-Key filter can be well described by the integer order derivative, but when the current flows in a fractal media (such as porous media), the integer order derivative becomes ineffective. As is known to all, due to the nonlocal in nature, the fractional calculus is more suitable to model natural and engineering processes, and has become an important mathematical tool to describe many complex problems appearing in science and engineering technology, such as physics [1]–[3], vibration [4], seismic signals [5], porous functions [6], [7], batteries [8], control [9], [10], electronic circuit and filter [11]–[15], heat equation [16], viscoelastic wave equation [17], [18], and so on [16]–[21]. Recently, as a new fractional order theory, the local fractional calculus has attracted much attention in various fields and is successfully applied to describe many ND phenomena ranging from theory to applications [22]–[28]. Inspired by recent works in the fractional filters [12]–[14], we propose a new fractional Sallen-Key filter model by using the LFD in this paper for the first time, where the fractional Sallen-Key filter can be not only used in the complex case of the current flowing in fractal media, such as porous media, but also be used in the general case. The influence of different derivative order $0 < \varsigma \leq 1$ on the filter circuit is analyzed in detail. Also, the concept of ND transfer function is introduced. With the approach presented in this paper, it will be possible to have a better study of the filtering effects in the electrical systems for the current flowing in fractal media, which is expected to open some new perspectives towards the characterization of ND filters via LFDs.

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II. THE LFD AND LFLT

Definition 2.1: The LFD of function $\vartheta(\sigma)$ with order ς $(0 < \varsigma \leq 1)$ is defined as follows [29]:

$$\vartheta^{(\varsigma)}(\sigma_0) = \frac{d^{\varsigma}\vartheta(\sigma)}{d\sigma^{\varsigma}}|_{\sigma=\sigma_0} = \lim_{\sigma\to\sigma_0} \frac{\triangle^{\varsigma}(\vartheta(\sigma) - \vartheta(\sigma_0))}{(\sigma - \sigma_0)^{\varsigma}}, \tag{2.1}$$

TABLE 1. The LFDs of several functions on C-S.

$\vartheta(\sigma)$	$\vartheta^{(\varsigma)}(\sigma)$
k	0
$E_\varsigma(k\sigma^\varsigma)$	$kE_\varsigma(k\sigma^\varsigma)$
$\frac{\sigma^{k\varsigma}}{\Gamma(1+k\varsigma)}$	$\frac{\sigma^{(k-1)\varsigma}}{\Gamma(1+(k-1)\varsigma)}$
$cos_\varsigma(k\sigma^\varsigma)$	$-ksin_\varsigma(k\sigma^\varsigma)$
$sin_\varsigma(k\sigma^\varsigma)$	$kcos_\varsigma(k\sigma^\varsigma)$

and there is [30], [31]

$$\triangle^\varsigma\left[\vartheta(\sigma)-\vartheta(\sigma_0)\right]\cong\Gamma(1+\varsigma)\left[\vartheta(\sigma)-\vartheta(\sigma_0)\right]. \quad (2.2)$$

with the Euler's Gamma function as $\Gamma(1+\varsigma) =: \int_0^\infty \vartheta^{\varsigma-1} exp(-\vartheta)\, d\vartheta$.

If there is $\vartheta^{(\varsigma)}(\sigma) = D^{(\varsigma)}\vartheta(\sigma)$, the LFD of higher order can be expressed as:

$$\vartheta^{(k\varsigma)}(\sigma) = \underbrace{D^{(\varsigma)}\ldots D^{(\varsigma)}}_{k\ times}\vartheta(\sigma), \quad (2.3)$$

That is

$$\frac{\partial^{(k\varsigma)}\vartheta(\sigma)}{\partial\sigma^{k\varsigma}} = \underbrace{\frac{\partial^\varsigma}{\partial\sigma^\varsigma}\ldots\frac{\partial^\varsigma}{\partial\sigma^\varsigma}}_{k\ times}\vartheta(\sigma). \quad (2.4)$$

The properties of the LFD are:

(a) $D^{(\varsigma)}\left[\vartheta(\sigma)\pm\nu(\sigma)\right] = D^{(\varsigma)}\vartheta(\sigma)\pm D^{(\varsigma)}\nu(\sigma)$,

(b) $D^{(\varsigma)}\left[\vartheta(\sigma)\nu(\sigma)\right] = \nu(\sigma)\left[D^{(\varsigma)}\vartheta(\sigma)\right]+\vartheta(\sigma)\left[D^{(\varsigma)}\nu(\sigma)\right]$,

(c) $D^{(\varsigma)}\left[\vartheta(\sigma)/\nu(\sigma)\right] = \left\{\left[D^{(\varsigma)}\vartheta(\sigma)\right]\nu(\sigma)-\vartheta(\sigma)\left[D^{(\varsigma)}\nu(\sigma)\right]\right\}/\nu(\sigma)^2,\ \nu(\sigma)\neq 0$.

Definition 2.2: The definitions of several common functions that on Cantor sets(C-S) with a fractional dimension ς, such as Mittag-Leffler function-$E_\varsigma(k\sigma^\varsigma)$, Sine function-$sin_\varsigma(k\sigma^\varsigma)$ and Cosine function-$cos_\varsigma(k\sigma^\varsigma)$ are given as follows [32]:

$$E_\varsigma\left(k\sigma^\varsigma\right) = \sum_{m=0}^{\infty}\frac{k^\varsigma\sigma^{m\varsigma}}{\Gamma(1+m\varsigma)}, \quad (2.5)$$

$$sin_\varsigma(k\sigma^\varsigma) = \sum_{m=0}^{\infty}\frac{(-k)^\varsigma\sigma^{(2m+1)\varsigma}}{\Gamma\left[1+(2m+1)\varsigma\right]}, \quad (2.6)$$

$$cos_\varsigma(k\sigma^\varsigma) = \sum_{m=0}^{\infty}\frac{(-k)^\varsigma\sigma^{2m\varsigma}}{\Gamma(1+2m\varsigma)}. \quad (2.7)$$

where $m, k \in N$. And the LFDs of several functions are given in Table 1.

Definition 2.3: Here we note the LFLT of function $\vartheta(\sigma)$ as $\mathbb{L}_\varsigma\left[\vartheta(\sigma)\right] = \mathbb{S}_\varsigma^\vartheta(\epsilon)$, then LFLT is defined as [32]:

$$\mathbb{L}_\varsigma\left[\vartheta(\sigma)\right] = \mathbb{S}_\varsigma^\vartheta(\epsilon) = \frac{1}{\Gamma(1+\varsigma)}\int_0^\infty\vartheta(\sigma)E_\varsigma(-\sigma^\varsigma\epsilon^\varsigma)(d\sigma)^\varsigma. \quad (2.8)$$

where $\mathbb{L}_\varsigma$ is the LFLT operator.

Theorem 1: There is the following theorem for the LFLT:

$$\mathbb{L}_\varsigma\left[\vartheta^{(i\varsigma)}(\sigma)\right] = \epsilon_\varsigma^{i\varsigma}\left[\vartheta(\sigma)\right]-\sum_{j=0}^{i-1}\epsilon^{(i-1-j)}\vartheta^{(i\varsigma)}(0). \quad (2.9)$$

TABLE 2. The LFLTs of several functions on C-S.

$\vartheta(\sigma)$	$\mathbb{L}_\varsigma\left[\vartheta(\sigma)\right]$
1	$\frac{1}{\epsilon^\varsigma}$
$E_\varsigma(k\sigma^\varsigma)$	$\frac{1}{\epsilon^\varsigma-k}$
$\frac{\sigma^{k\varsigma}}{\Gamma(1+k\varsigma)}$	$\frac{1}{\epsilon^{\varsigma(k+1)}}$
$cos_\varsigma(k\sigma^\varsigma)$	$\frac{\epsilon^\varsigma}{\epsilon^{2\varsigma}+k^2}$
$sin_\varsigma(k\sigma^\varsigma)$	$\frac{k^\varsigma}{\epsilon^{2\varsigma}+k^2}$

The LFLTs of several functions on C-S are presented in Table 2.

III. THE ND LUMPED ELEMENTS AND KIRCHHOFF's CURRENT LAW

A. THE ND RESISTOR

Definition 3.1: The Ohm's Law of the ND resistor in the fractional circuit systems is defined as [33], [34]:

$$i_{\varsigma,R}(\sigma) = \frac{u_{\varsigma,R}(\sigma)}{R_\varsigma}. \quad (3.1)$$

where $u_{\varsigma,R}(\sigma)$ and $i_{\varsigma,R}(\sigma)$ are the ND voltage and ND current of the ND resistor R_ς respectively.

B. THE ND CAPACITOR

The capacitors play an important role in tuning, bypass, coupling, filtering and other circuits. According to the definition of current, we define the ND current by LFD as follow [33], [34]:

$$i_\varsigma(\sigma) = \frac{d^\varsigma\varnothing_\varsigma(\sigma)}{d\sigma^\varsigma}, \quad (3.2)$$

where $\varnothing_\varsigma(\sigma)$ is the ND charge.

Definition 3.2: The ND capacitance of the ND capacitor can be defined as [33], [34]:

$$C_\varsigma = \frac{\varnothing_{\varsigma,C}(\sigma)}{u_{\varsigma,C}(\sigma)}, \quad (3.3)$$

where $u_{\varsigma,C}(\sigma)$ indicates the ND voltage.

Using Eq.(3.2) and Eq.(3.3), yields:

$$i_{\varsigma,C}(\sigma) = C_\varsigma\frac{d^\varsigma u_{\varsigma,C}(\sigma)}{d\sigma^\varsigma}. \quad (3.4)$$

C. THE KIRCHHOFF's CURRENT LAW

Kirchhoff's current law is also known as the node current law. It states that at any time on any node in the circuit, the sum of the current flowing into the node is equal to the sum of the current flowing out of the node (see Fig.1), which can be expressed as:

$$\sum_{m=1}^{n} i_{\varsigma,m}(\sigma) = 0. \quad (3.5)$$

where $i_{\varsigma,m}(\sigma)$ is the m^{th} current entering or leaving the node, and it is the current flowing through the m^{th} branch connected with the node.

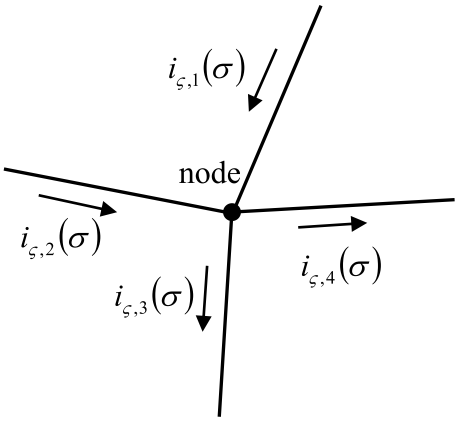

FIGURE 1. The KCL within LFD.

IV. THE SALLEN-KEY FILTER WITHIN LFD

The ND Sallen-Key filter described by the LFD is shown in Fig.2, where all the voltage and current directions we used are reference directions. Applying the concept of virtual open, we have

$$i_{\varsigma,+}(\sigma) = i_{\varsigma,-}(\sigma) = 0, \tag{4.1}$$

there is

$$i_{\varsigma,R_1}(\sigma) = i_{\varsigma,R_F}(\sigma), \tag{4.2}$$

which leads to

$$u_{\varsigma,o}(\sigma) = \left(1 + \frac{R_{\varsigma,1}}{R_{\varsigma,F}}\right) u_{\varsigma,R_1}(\sigma). \tag{4.3}$$

Using the virtual short theory, it gives

$$u_{\varsigma,-}(\sigma) = u_{\varsigma,+}(\sigma), \tag{4.4}$$

Based on the above expression, we get

$$u_{\varsigma,R_1}(\sigma) = u_{\varsigma,C_2}(\sigma), \tag{4.5}$$

The Eq.(4.3) takes the following form by using the above expression

$$u_{\varsigma,o}(\sigma) = \left(1 + \frac{R_{\varsigma,1}}{R_{\varsigma,F}}\right) u_{\varsigma,C_2}(\sigma). \tag{4.6}$$

With the help of Eq.(3.4), there is the following expression

$$i_{\varsigma,C_2}(\sigma) = C_\varsigma \frac{d^\varsigma u_{\varsigma,C_2}(\sigma)}{d\sigma^\varsigma}, \tag{4.7}$$

According to Eq.(4.1), we obtain

$$i_{\varsigma,2}(\sigma) = i_{\varsigma,C_2}(\sigma). \tag{4.8}$$

Then the expression of $u_{\varsigma,2}(\sigma)$ is given as

$$u_{\varsigma,2}(\sigma) = R_\varsigma C_\varsigma \frac{d^\varsigma u_{\varsigma,C_2}(\sigma)}{d\sigma^\varsigma}, \tag{4.9}$$

By inspection, there is

$$u_{\varsigma,C_1}(\sigma) = u_{\varsigma,2}(\sigma) + u_{\varsigma,C_2}(\sigma) - u_{\varsigma,o}(\sigma), \tag{4.10}$$

Substituting Eqs.(4.3), (4.5) and (4.9) into Eq.(4.10), and taking a simplify yields

$$u_{\varsigma,C_1}(\sigma) = R_\varsigma C_\varsigma \frac{d^\varsigma u_{\varsigma,C_2}(\sigma)}{d\sigma^\varsigma} - \frac{R_{\varsigma,F}}{R_{\varsigma,1}} u_{\varsigma,c_2}(\sigma), \tag{4.11}$$

Similarly, we get the following expression

$$i_{\varsigma,C_1}(\sigma) = C_\varsigma \frac{d^\varsigma u_{\varsigma,C_1}(\sigma)}{d\sigma^\varsigma}, \tag{4.12}$$

By taking Eq.(4.11) into Eq.(4.12), there is the following result

$$i_{\varsigma,C_1}(\sigma) = R_\varsigma C_\varsigma^2 \frac{d^{2\varsigma} u_{\varsigma,C_2}(\sigma)}{d\sigma^{2\varsigma}} - C_\varsigma \frac{R_{\varsigma,F}}{R_{\varsigma,1}} \frac{d^\varsigma u_{\varsigma,C_2}(\sigma)}{d\sigma^\varsigma}, \tag{4.13}$$

We now use the KCL, then there is

$$i_{\varsigma,1}(\sigma) = i_{\varsigma,C_1}(\sigma) + i_{\varsigma,2}(\sigma), \tag{4.14}$$

Substitution of Eqs.(4.7), (4.8) and (4.13) into the above equation, which results in

$$i_{\varsigma,1}(\sigma) = R_\varsigma C_\varsigma^2 \frac{d^{2\varsigma} u_{\varsigma,C_2}(\sigma)}{d\sigma^{2\varsigma}} - C_\varsigma \frac{R_{\varsigma,F}}{R_{\varsigma,1}} \frac{d^\varsigma u_{\varsigma,C_2}(\sigma)}{d\sigma^\varsigma} + C_\varsigma \frac{d^\varsigma u_{\varsigma,C_2}(\sigma)}{d\sigma^\varsigma}, \tag{4.15}$$

The expression for the input signal $u_{\varsigma,i}(\sigma)$ is given as

$$u_{\varsigma,i}(\sigma) = u_{\varsigma,1}(\sigma) + u_{\varsigma,2}(\sigma) + u_{\varsigma,C_2}(\sigma), \tag{4.16}$$

where

$$u_{\varsigma,1}(\sigma) = i_{\varsigma,1}(\sigma) R_\varsigma, \tag{4.17}$$

We now plug Eq.(4.15) into the Eq.(4.17), giving that

$$u_{\varsigma,1}(\sigma) = R_\varsigma^2 C_\varsigma^2 \frac{d^{2\varsigma} u_{\varsigma,C_2}(\sigma)}{d\sigma^{2\varsigma}} - R_\varsigma C_\varsigma \frac{R_{\varsigma,F}}{R_{\varsigma,1}} \frac{d^\varsigma u_{\varsigma,C_2}(\sigma)}{d\sigma^\varsigma} + R_\varsigma C_\varsigma \frac{d^\varsigma u_{\varsigma,C_2}(\sigma)}{d\sigma^\varsigma}, \tag{4.18}$$

By using Eq.(4.9) and Eq.(4.18), the Eq.(4.16) becomes

$$u_{\varsigma,i}(\sigma) = R_\varsigma^2 C_\varsigma^2 \frac{d^{2\varsigma} u_{\varsigma,C_2}(\sigma)}{d\sigma^{2\varsigma}} - R_\varsigma C_\varsigma \frac{R_{\varsigma,F}}{R_{\varsigma,1}} \frac{d^\varsigma u_{\varsigma,C_2}(\sigma)}{d\sigma^\varsigma} + 2R_\varsigma C_\varsigma \frac{d^\varsigma u_{\varsigma,C_2}(\sigma)}{d\sigma^\varsigma} + u_{\varsigma,C_2}(\sigma), \tag{4.19}$$

The above expression can be rewritten as

$$u_{\varsigma,i}(\sigma) = R_\varsigma^2 C_\varsigma^2 \frac{d^{2\varsigma} u_{\varsigma,C_2}(\sigma)}{d\sigma^{2\varsigma}} + \left(3 - A_\varsigma\right) R_\varsigma C_\varsigma \frac{d^\varsigma u_{\varsigma,C_2}(\sigma)}{d\sigma^\varsigma} + u_{\varsigma,C_2}(\sigma), \tag{4.20}$$

where

$$A_\varsigma = 1 + \frac{R_{\varsigma,F}}{R_{\varsigma,1}}. \tag{4.21}$$

Applying the LFLT to Eq.(4.6) and Eq.(4.20) respectively, yields the results as [35]

$$\mathbb{S}_\varsigma^{u_{\varsigma,o}}(\epsilon) = A_\varsigma \mathbb{S}_\varsigma^{u_{\varsigma,C_2}}(\epsilon), \tag{4.22}$$

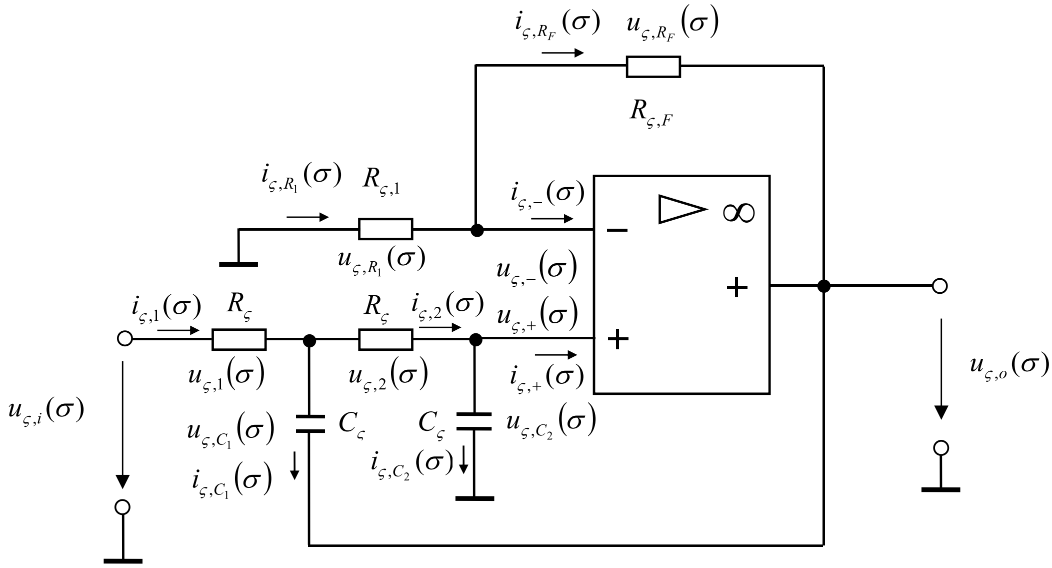

FIGURE 2. The Salen-Key filter model within LFD.

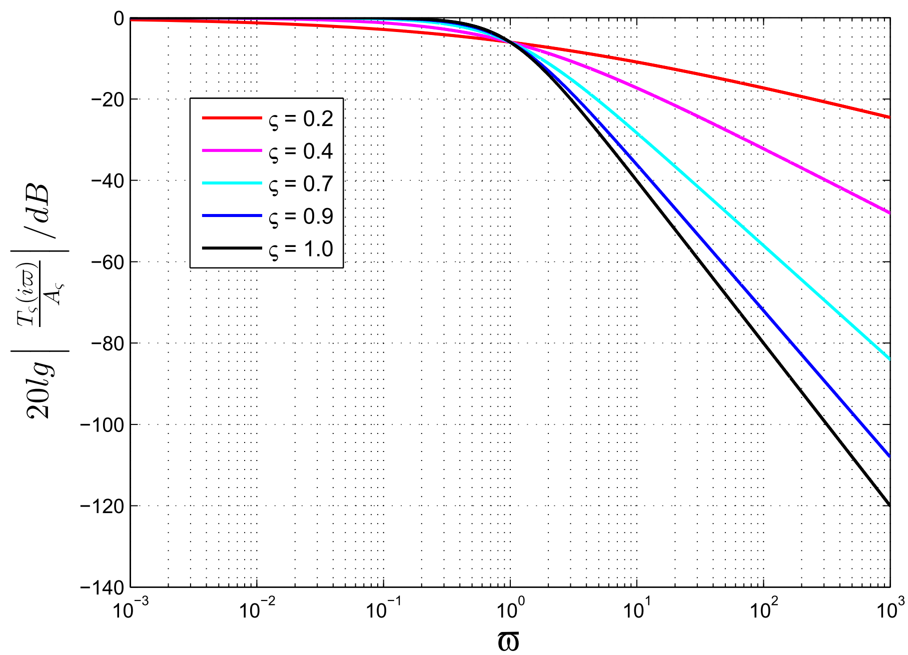

FIGURE 3. The curve of the ND logarithmic AFC with $\varsigma = 0.2, 0.4, 0.7, 0.9, 1.0$ at $\mathbb{Z}_\varsigma = 1$ and $A_\varsigma = 1$.

And

$$\begin{aligned}&\mathbb{S}_\varsigma^{u_{\varsigma,i}}(\epsilon)\\&=R_\varsigma^2C_\varsigma^2\left[\epsilon^{2\varsigma}\mathbb{S}_\varsigma^{u_{\varsigma,C_2}}(\epsilon)-\epsilon^\varsigma u_{\varsigma,C_2}^{(\varsigma)}(0)-u_{\varsigma,C_2}(0)\right]\\&\quad+\left(3-A_\varsigma\right)R_\varsigma C_\varsigma\left[\epsilon^\varsigma\mathbb{S}_\varsigma^{u_{\varsigma,C_2}}(\epsilon)-u_{\varsigma,C_2}(0)\right]\\&\quad+\mathbb{S}_\varsigma^{u_{\varsigma,C_2}}(\epsilon).\end{aligned}\tag{4.23}$$

The Eq.(4.23) can be rewritten as the following form by using the zero-state of $u_{\varsigma,C_2}(0)=0$.

$$\begin{aligned}\mathbb{S}_\varsigma^{u_{\varsigma,i}}(\epsilon)&=R_\varsigma^2C_\varsigma^2\epsilon^{2\varsigma}\mathbb{S}_\varsigma^{u_{\varsigma,C_2}}(\epsilon)\\&\quad+\left(3-A_\varsigma\right)R_\varsigma C_\varsigma\epsilon^\varsigma\mathbb{S}_\varsigma^{u_{\varsigma,C_2}}(\epsilon)+\mathbb{S}_\varsigma^{u_{\varsigma,C_2}}(\epsilon),\end{aligned}\tag{4.24}$$

Then there is

$$\begin{aligned}T_\varsigma(\epsilon)&=\frac{\mathbb{S}_\varsigma^{u_{\varsigma,o}}(\epsilon)}{\mathbb{S}_\varsigma^{u_{\varsigma,i}}(\epsilon)}\\&=\frac{A_\varsigma}{R_\varsigma^2C_\varsigma^2\epsilon^{2\varsigma}+\left(3-A_\varsigma\right)R_\varsigma C_\varsigma\epsilon^\varsigma+1},\end{aligned}\tag{4.25}$$

We now apply the $\epsilon=i\varpi$ and $\mathbb{Z}_\varsigma=R_\varsigma C_\varsigma$ to Eq.(4.25), which gives ND transfer function as

$$T_\varsigma(i\varpi)=\frac{A_\varsigma}{\mathbb{Z}_\varsigma^2(i\varpi)^{2\varsigma}+\left(3-A_\varsigma\right)\mathbb{Z}_\varsigma(i\varpi)^\varsigma+1},\tag{4.26}$$

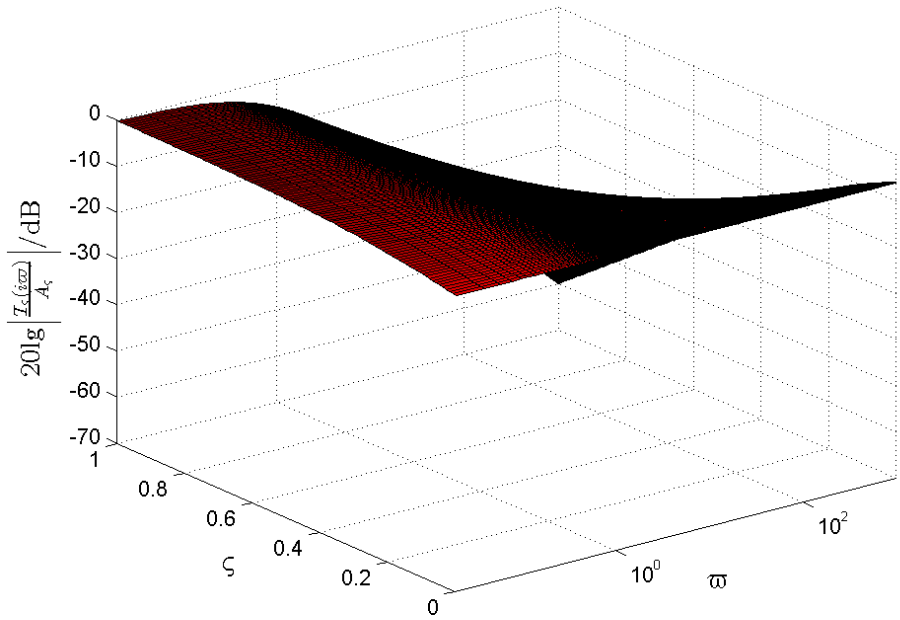

FIGURE 4. The 3-D graph of ND logarithmic AFC with different fractional orders ς at $\mathbb{Z}_\varsigma = 1$ and $A_\varsigma = 1$.

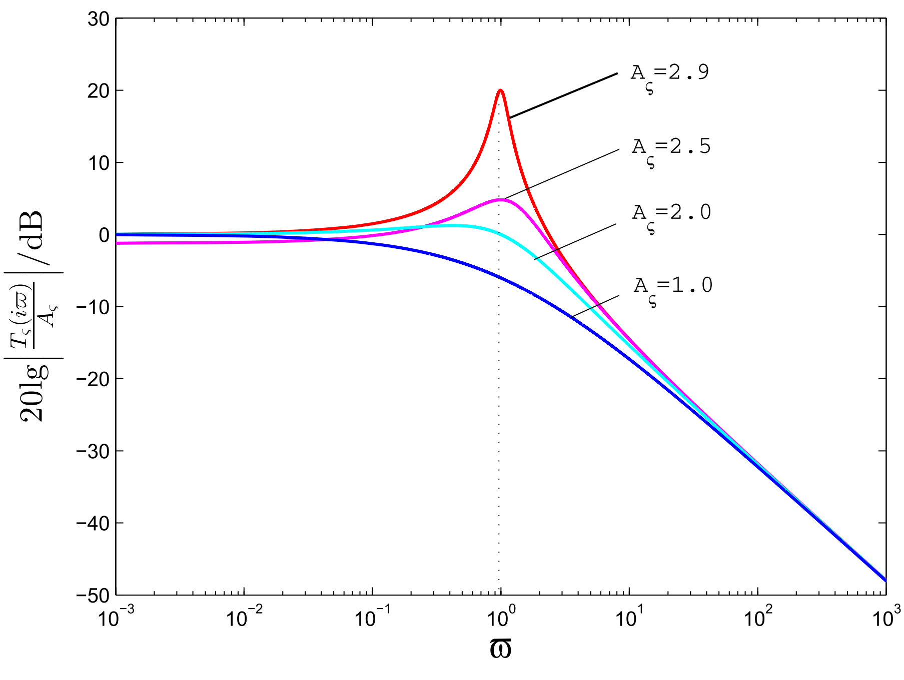

FIGURE 5. Curve of the ND logarithmic AFC with different $A_\varsigma = 2.9, 2.5, 2.0, 1.0$ at $\mathbb{Z}_\varsigma = 1$ and $\varsigma = 0.4$.

From which, we get the expressions of ND amplitude-frequency characteristic(AFC) and ND phase-frequency characteristics(PFC) as:

$$\left|T_\varsigma(i\varpi)\right| = \frac{A_\varsigma}{\sqrt{\left(1-\mathbb{Z}_\varsigma^2\varpi^{2\varsigma}\right)^2 + \left(3-A_\varsigma\right)^2\mathbb{Z}_\varsigma^2\varpi^{2\varsigma}}}, \quad (4.27)$$

and

$$\Phi_\varsigma(\varpi) = -arctan\left[\frac{\left(3-A_\varsigma\right)\mathbb{Z}_\varsigma\varpi^\varsigma}{1-\mathbb{Z}_\varsigma^2\varpi^{2\varsigma}}\right]. \quad (4.28)$$

It should be noted that the fractional Sallen-Key filter becomes to the ordinary one for the special case $\varsigma = 1$.

V. ANALYSIS OF THE ND AFC

By letting $\mathbb{Z}_\varsigma = 1$ and $A_\varsigma = 1$, we illustrate the curves of the ND logarithmic AFC with different fractional orders ς at $\mathbb{Z}_\varsigma = 1$ and $A_\varsigma = 1$ in Fig.3. It can be seen that the ND Sallen-Key filter has the low-pass filtering characteristics.

By careful observation, it is found that the attenuation of the curves decreases as the angular frequency ϖ increases.

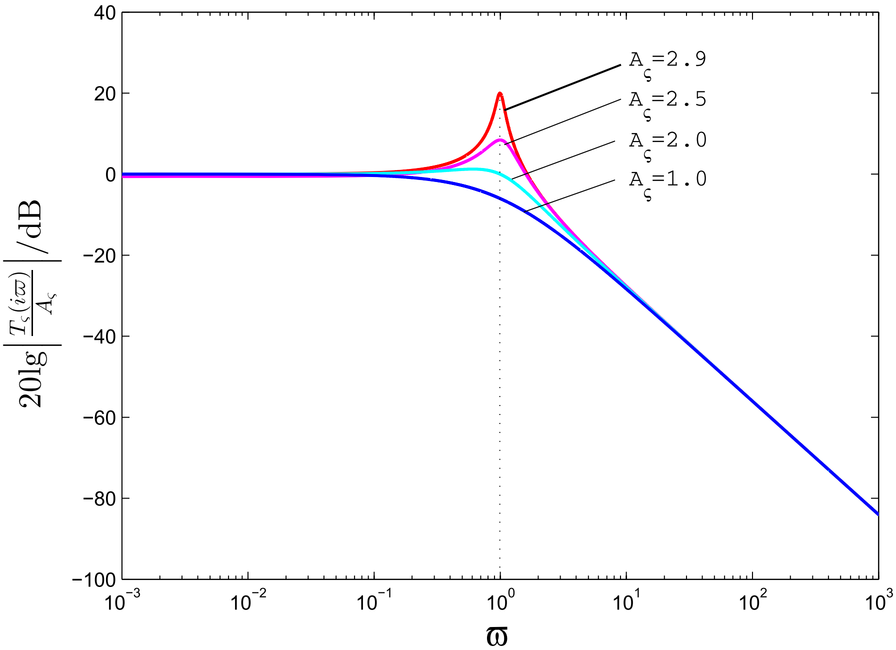

FIGURE 6. Curve of the ND logarithmic AFC with different $A_\varsigma = 2.9, 2.5, 2.0, 1.0$ at $\mathbb{Z}_\varsigma = 1$ and $\varsigma = 0.7$.

Besides that, the larger the value of the fractional order ς is, the faster the curve decays as the angular frequency decreases, That is, the better the frequency selection characteristics is. And the three dimensional(3-D) ND logarithmic AFC graph versus the fractional orders ς when $\mathbb{Z}_\varsigma = 1$ and $A_\varsigma = 1$ is shown in fig 4.

By using $\mathbb{Z}_\varsigma = 1$ and $\varsigma = 0.4$, fig.5 plots the curves of the ND logarithmic AFC with different values of the A_ς. Obviously, there is an obvious protuberance in the falling area of the curves and the larger the value of A_ς is, the more severe the protuberance is, which directly affects the filtering effect of the filter. Therefore, the ND Sallen-Key filter is also called ND voltage controlled voltage source low-pass active filter. When the value of A_ς is small, it can not only maintains the gain of pass-band, but also quickly attenuates the ND AFC of the high frequency band, and avoid the large convex seal of the ND AFC in the falling area of the curves, so the filtering effect is better. When $\varsigma = 0.7$, the curves are illustrated in fig.6, where we get the similar conclusions.

VI. CONCLUSION

In this paper, the fractional Sallen-Key filter modeled by LFD is proposed for the first time, where the ND transfer function is obtained by using the LFLT, and the corresponding ND AFC is also presented and studied in detail. By comparing different derivative order, it found that the larger the value of the fractional order ς is, the better the frequency selection characteristics of the filter is. Also, the ND logarithmic AFC of different A_ς is discussed, which shows the larger the value of A_ς is, the more severe the protuberance is, which directly affects the filtering effect of the filter. It is noteworthy that the fractional Sallen-Key filter becomes the ordinary one for $\varsigma = 1$. The fractional model of the Sallen-Key filter we proposed can be not only used to describe the complex case of the current flowing in fractal media, such as porous media, but also be used to describe the general case. The all results given in this paper are expected to shed a new light of applications of fractional calculus to the study of ND filters via LFDs.

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