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The Full Costs of Urban Transport. Part I: Economic Efficiency in Bus Operations; Preliminary Intermodal Cost Comparison and Policy Implications

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THE FULL COSTS OF URBAN TRANSPORT

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PART I: Economic Efficiency in Bus Operations;  
Preliminary Intermodal Cost Comparisons and  
Policy Implications

BY Peter Fisher  
and  
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## ABSTRACT

The downward spiral frequently experienced by transit companies when they increase fares to make up for declining demand is well known. It is characteristic of an increasing-returns-to-scale industry where consumer inputs are essential to the production process. This paper formally incorporates both agency and consumer inputs in a cost framework and explores the short- and long-run costs of bus service. In so doing it is possible to trade off both consumer and agency costs, to arrive at optimal service levels. It is these service levels, with the mode "performing at its best," that form the basis of cost comparisons between transportation alternatives.

We find that off-peak frequencies should be considerably shorter than those typically prevailing on bus transit systems, and that marginal cost pricing would require a zero fare in the off-peak and a fare of up to 20¢ in the peak. This suggests that current rates might be continued in the peak and abolished in the off-peak. A subsidy to maintain this scheme would be about 10% of the average cost of service when operation is in long-run equilibrium (optimized). In the realm of cost comparisons between alternative modes, options based upon the car are found to be among the cheapest for trips to the CBD except in the peak. In terms of public modes, dial-a-ride systems seem to have some economic justification for low density neighborhoods as a feeder to linehaul buses operating on freeways, but at high densities integrated bus service appears to be far more viable. The latter is markedly cheaper than a feeder bus + rapid rail alternative. Finally, there seems to be little economic justification for the current interest in non-standard buses for transit service.

#### ACKNOWLEDGEMENTS

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In two decades the national level of transit ridership has fallen in almost direct proportion to the rise in automobile usage. Twice as many cars are in use today as in 1954. But, despite a surge in ridership during the energy crisis, half as many transit fares will be paid this year as were paid twenty years ago.

Russell E. Train  
Administrator, EPA  
Pittsburgh 1974

## 1. Introduction

It is by now a commonplace that public transit is an important component of an efficient allocation of transport resources. And yet it is equally clear, as Train points out, that when faced with a choice, the travel preferences of the urban public are otherwise -- there is no rush to join the buses.

There appear to be at least two reasons for this situation: first that public transit often fails to optimize simultaneously over agency and consumer (time and effort) inputs, both of which are essential to its production. This is more true of off-peak service than peak, when scheduling tends to be on a "leave-when-full" basis.<sup>1</sup> The second reason is that private transport is beset with a "social cost" problem, in that the externalities generated by trip-making are not perceived by the individual as part of the costs of his or her travel, pollution and highway congestion being prime examples. In the case of pollution the spillover costs are inflicted on the public at large; with congestion they are inflicted only on other users of the road system. Congestion is a peak phenomenon producing driving modes conducive to high vehicular emissions and thus exacerbating pollution; therefore both "social costs" are associated with rush-hour conditions. As the individual only considers the private costs

when deciding between modes, it is likely that the resulting mix will be sub-optimal, frequently emphasizing the automobile at the expense of public transit.

However in a city where there are significant disparities in income, pricing the various modes to reflect these dimensions may have serious repercussions for social welfare. On the other hand, the costs of reaching the optimum may make it impractical. Under such conditions "second best" pricing, where the transit fare is lowered from the marginal toll to compensate for underpricing of the automobile, is often advocated (Sherman 1972; Abe 1973).

An assessment of the full costs of urban transit is therefore crucial if there is to be rational planning of transportation facilities. It is the purpose of this paper to report on these costs, primarily in relation to the efficient provision of a bus-based transit system -- the major source of public transportation in North American cities. To do this we construct a supply-oriented framework, incorporating both the consumer and agency inputs into the production process. This allows an exploration of the various factors that affect the short- and long-run costs of scheduled service. Further, least-cost levels of operation can be established, which when contrasted with the comparable costs of other modes performing at their best (Keeler, Small and Cluff, 1975; Pozdena 1974) make it possible to begin to define an optimal mix.

It should be noted that the method of analysis is that of partial equilibrium: each mode is optimized separately without taking account of interdependencies. Hence the results represent a first pass at the problem. Construction of a general equilibrium model is clearly the next step, although the difficulties of doing this should not be under-estimated. But

Small (1974) has made a beginning here with an economic analysis of the equilibrium established between buses and cars on freeways.

The research presented in this paper is a component part of a wide-ranging study of the Comparative Costs of Bay Area Transportation Modes, and accordingly the bulk of the unit cost estimates and applications of the methodology relate to the San Francisco Bay Area. Two points need to be made in this regard: first, there is a large measure of empirical uncertainty about the values to be assigned to the consumers' shadow prices of time and effort. Our estimates are drawn from the logit analysis of McFadden (1972, 1974a, 1974b) and Chan (1973) on the Bay Area commuters. Nevertheless, our theoretical framework is sufficiently general to permit exploration of the sensitivity of outcomes to different assumptions. Secondly, our major data base for bus operations is the Alameda-Contra Costa Transit Company (AC Transit), reputed to be one of the more efficiently run transit companies in the nation. Use of AC Transit data conforms to a general strategy of ensuring that the most efficient provision of bus service be the basis of comparison with other modes, subject to similar efficiency provisions. The use of this company's unit costs in our abstract framework allows us to draw broad policy conclusions for bus transit in general.

Finally, the term "demand" which occurs frequently throughout this paper does not refer to the conventional demand function of economic theory. The question of the empirical determination of the demand function for urban public transportation is the subject of other studies, notably the current work of McFadden. We, on the other hand, assume various distributions of "requests for service," which are generally independent of price charged, and derive the corresponding average cost and optimality conditions. This can be done for the demand corresponding



to any given price: thus the possibility of substituting empirically estimated demand functions remains open, contributing to the level of generality of the analysis.

## 2 Data-Derivation of Unit Costs

The costs of running a bus system fall into three categories: variable costs, which depend on the level of service; fixed costs, which are essentially invariant over a wide range of service levels; and consumer costs, which are incurred directly by the bus passenger. Since fixed costs determine only the "shutdown level" of the system, that is to say the level of revenues at which the company will decide to cease production, and not the marginal costs on which optimality calculations depend, this paper will not consider fixed costs further. In this category are administrative costs, and the value of the system's land and structures. We assume throughout that fixed costs can be covered: this certainly holds true for our service archetype, AC Transit.

All cost values in this report are in terms of 1972 dollars. Capital values assume a 6% interest rate.

### 2.1 Agency Costs

Variable costs relate to demand dependent expenses of operation, varying either with the vehicle-miles run by the system (costs of fuel, maintenance, and the capital costs of the fleet, etc.) or with the number of hours of operation (wages and fringe benefits, etc.).<sup>2</sup>

#### Operating Costs with Standard Size Buses

Drawing upon annual 1972 AC Transit data relating to operation of its standard size (50 seat) fleet, these hourly and mileage costs are disaggregated on a unit basis in Table 1 (Viton, 1974a).

Drivers' wages account for 89% of the payroll of the system. Fringe costs such as social security, state employment insurance, etc., are not broken down by department in the data. These are small and to a first approximation can be assumed directly proportional to wage costs.

Table 1

## BUS OPERATING COSTS

<u>Category</u>	<u>25-seat</u>	<u>50-seat (standard)</u>	<u>75-seat</u>
HOURLY COSTS	\$8.10/hour	\$9.28/hour	\$10.98/hour
Drivers' wages	5.68	7.94	
Fringes	1.98	1.21	
Advertising	.44	.13	
MILEAGE COSTS	\$.3093/mile	\$.3007/mile	\$.4732/mile
Capital Cost (6%)	.1000	.1172	.2358
Maintenance	.0983	.1073	.1163
Fuel & Oil	.0308	.0251	.0559
Tires	.0108	.0116	.0156
Timetables & Tickets	.0024	.0047	.0071
Claims	.0475	.0153	.0230
Right-of-Way (6%)	.0195	.0195	.0195

"Advertising" includes those costs that directly affect demand (materials, phones, etc.) and excludes wages payed to advertising employees.

Bus capital costs are based upon the 1972 replacement value of AC's 721 vehicle fleet. Since the buses vary in age and condition, conceptual problems arise as to whether the assessed value should be on the basis of replacement with new buses, or a matched replacement via new and secondhand markets, according to age and/or condition. The latter is a complex issue and warrants further research: it includes problems centering upon the scarcity and the representativeness of existing second-hand market data. To permit comparisons with smaller and larger buses recently introduced to the market, our capital cost is based upon replacement by new standard size buses at 1972 prices.

As a pointer to AC's efficiency, maintenance is "preventive," hence the figure given is lower than the neighboring Muni system which repairs buses only on breakdown (Lee, 1974). Right-of-Way costs are attributed to the system using general marginal cost figures of Bay Area arterials, as reported by Finke and Cluff (1974). This includes the cost of supporting services such as police, sheriff, coroner and those provided by elected and appointed officials.

The "total direct cost" (TDC), that is, the cost borne directly by the company in providing service over a route of L miles at a service speed of V miles per hour, is given by aggregating the items in Table 1 viz:  $TDC_{50} = 9.28 \frac{L}{V} + .300_7 L$ .

#### Operating Costs with Non-Standard Size Buses

Alameda-Contra Costa Transit, in common with most North American bus companies, runs service with standard size coaches, seating between 45 and 51 passengers. Recently, considerable interest has been generated

in vehicles of other sizes. For example, transit companies on the West Coast, including AC, have been service testing a demonstration articulated bus seating 75 passengers, while MBS Transit in New York has purchased 10 double decker buses from Britain. At the other end of the scale, Santa Clara Transit is using larger than usual buses for their dial-a-bus system to keep open an option of a possible later useage in arterial service.<sup>3</sup>

With a view to contributing to the burgeoning discussion of these "alternatives," sections 3 and 4 make comparative cost analyses according to size. For this it is necessary to estimate the total direct cost of these two non-standard buses, as if in service with AC Transit.

In the case of the smaller vehicle we have some estimates of the cost of running a 17-seat TC-25 Twin Coach bus in Haddonfield, N.J. (Clemons, 1974). However, the Haddonfield (Dial-a-Ride) buses are designed for luxury travel with sparse seating. To provide seating accommodation comparable with AC standard size buses, there would need to be 25 seats.<sup>4</sup>

Cost estimates disaggregated by hours or mileage are given in Table 1. Wages of drivers and supervisors are assumed to vary directly with the passenger capacity of the bus. This is the case with airline pilots, and it seems reasonable to assume that driving a 50 seat bus, which also involves fare collection (as well as overseeing operation) is somehow more arduous than with the smaller capacity vehicle, and is remunerated accordingly. To the extent that this assumption is violated, then the gap between the costs of running the two vehicles will be narrowed.<sup>5</sup> Note that advertising costs are higher and may reflect intense publicity for a demand activated service or alternatively smaller scale economies of the Haddonfield operation. There is simply not enough information to reject the figure as it stands.

It is assumed that Haddonfield is maintaining its fleet efficiently and on a comparable basis to AC's maintenance procedures. However, if their procedures are closer to Muni's than to AC's scheme, then the true costs of maintenance will be higher than reported here. As with wages, claims and ticket costs are taken to vary directly with passenger capacity. Finally, the average age of the Haddonfield fleet is much less than that of AC Transit. This means that while costs reported here may be attainable in the short run, as the 25-seat buses age, costs will increase, narrowing the gap between these two sizes of vehicle.

The total direct cost is given (as before) by aggregating the items in Table 1.

$$\text{viz: } TDC_{25} \approx 8.10 \frac{L}{V} + 0.309_3 L.$$

Comparable estimates of the cost of running the 75-seat bus are much more difficult, since there are currently no vehicles of this size in regular full-scale service in North America and certainly no formal operating statistics. The actual bus on which our cost estimates are based is a Volvo on loan from the Stockholm Transit Authority for demonstration in the U.S., under the sponsorship of Seattle's Metro.

Because there is no direct information on the way hourly costs vary with this larger bus, we extrapolate from the corresponding smaller bus values, viz:

$$\text{hourly costs } (\$) = 5.58 + 0.068 \times \text{Capacity}$$

Hence hourly costs amount to \$10.98 per hour.

The Metro estimates the price of the Volvo "between \$75,000 and \$90,000." Suppose that it costs \$82,500, the mid point of the range, and has a life of 15 years. Assuming a utilization equal to the AC average of 3,000 miles per month, this is equivalent to a capital cost of

\$0.235<sub>8</sub> per mile. As for other costs varying with miles, Metro reports paying about \$0.0540 per mile in fuel and \$0.0019 per mile in oil (a total of \$0.0559 per mile) during demonstration service. There are two additional wheels over the standard size bus, so tire expenditures will be 1-1/3 times greater. No information exists as to maintenance cost, so we again extrapolate from the corresponding smaller bus values:

$$\text{Maintenance Costs (\$)} = 0.0893 + .0036 \times \text{Capacity}$$

Hence maintenance costs are estimated at \$0.1163 per mile. The total direct cost is therefore

$$\text{TDC}_{75} = 10.98 \frac{L}{V} + 0.473_2 L.$$

## 2.2 Consumer Costs

The second major component of travel cost is the value of the consumer's "non-money" input into the production process, that is, the price of his travel in terms (a) time (opportunity cost) and (b) effort (disutility) and comfort (utility).

This contribution can be divided into two types: an in-vehicle travel input, which in the absence of highway congestion will be near constant for a given trip; and an out-of-vehicle travel input from the consumer's origin point to the vehicle. The latter is generally decomposed into walking to, and waiting at, the bus stop, and sometimes, transfer.

The opportunity cost component is purely time dependent, whereas the disutility/utility component depends on the time spent in some type of input activity: in-vehicle or out-of-vehicle effort/comfort. However, measurement problems generally prevent discrimination between these two components.

The question is, what dollar amount should we assign to each type of input to measure its (combined) opportunity cost and disutility?

There has been a great deal of work of late in this area: McFadden (1972), (1974a), (1974b), and Chan (1973) have as part of the BART Impact Study used sophisticated econometric techniques to measure values for the Bay Area; a good summary of other work is given by Hensher (1974). The estimates used in most of the numerical analysis to follow, derive from Chan's work on commutation, and may be summarized as follows:

Table 2

Type of Non-Money Travel Input	Unit Shadow Price
Walk	$v_w$ : \$9.00 per hour
Wait	$v_w$ : \$9.00 per hour
Transfer	$v_w$ : \$9.00 per hour
In-vehicle	$v_v$ : \$3.00 per hour

The data is drawn from trips of both bus and automobile travelers and corresponds to a median income level. No distinction is drawn between the in-vehicle shadow price for car and transit. However, it would seem that the additional comfort (utility) of the car is offset by driving under rush-hour conditions (disutility).<sup>6</sup>

These shadow prices are socio-economically dependent and hence likely to vary with geographic region, although there is a good agreement between the various (logit) studies as to the magnitude of scaling between the in-vehicle and out-of-vehicle components (Hensher, *op. cit.*)

The values of Table 2 are used in the numerical cost calculations because they are rigorously derived and relate to the same region as the agency costs. The contention that AC Transit is an "efficiency archetype" carries the implication that the (imputed) costs of its operation are capable of replication in other cities. Hence, while agency costs would



not vary with city, some (local) adjustment to  $v_v$  and  $v_w$  could be required, slightly changing the numerical cost calculations.

### 2.3 The Peak - Off-Peak Cost Problem

In presenting our operating cost estimates, we have not distinguished between the costs of providing peak and off-peak service. The estimates given lie between these two bounds. Goldstein (1974) has made a preliminary estimate of the "peak penalty" (that is, the increased hourly cost of providing peak service brought on by union agreements regarding the proportion of working days that may be split up). He estimates that the peak penalty is about 20%; that is, hourly costs for the peak are 20% above those incurred in the non-peak period. These are order-of-magnitude estimates: to refine them further a more detailed institutional study of union contracts would need to be undertaken. To round out the estimation of peak costs, all capital costs of the fleet would need to be attributed to the peak.

Finally, to capture all the differences between the two periods there is a need to distinguish between the unit shadow prices of time and effort at different periods. Off-peak valuations will in general tend to be lower than the peak valuations; but how much lower remains something of an open question. For example, Chan (1973) reports a slightly lower value for the off-peak trip, but is unable to reject the statistical hypothesis that the time valuations in the two periods are equal,<sup>7</sup> which highlights the difficulties involved in accurately estimating the costs in question.

### 3. Methodology - A Structure for Assessing the Short and Long Run Costs of Bus Service

Our aim is to establish a generalized average cost function (Mohring [1972], Byrne and Vuchic [1971], Hurdle [1973a,b], Newell [1971], Boyd et al [1973], Douglas & Miller [1974]) and to explore thoroughly the inter-relationships between the consumer and agency components to increase our understanding of the economics of providing service. This will be at first under the most simple assumptions, (viz: random arrival at bus stop, no capacity constraints, and point-to-point linehaul service), after which these restrictions will be progressively relaxed.

#### 3.1 A Generalized Average Cost Function

Consider a line-haul system transporting  $Q$  passengers in a service interval<sup>8</sup>  $t_s$  between two stops distant  $L$  apart, at a speed of  $V$ . All passengers board at one point and alight at the other. To address the optimality question we ask: at what time should the bus be dispatched to achieve the most favorable tradeoff between agency and user costs?

Suppose that it costs the bus company  $TDC [L, V]$  per trip and that passengers have a wait shadow price of  $\$v_w$  per unit time and an in-vehicle shadow price of  $\$v_v$  per unit time.<sup>9</sup> The  $i$ th passenger's probability of arrival at the stop can be described by a probability (density) function  $p_i[t]$ , such that  $\int_0^{t_s} p_i[t]dt = 1$ . If we assume that these individual probabilities are independent and that the  $Q$  passengers show the same arrival behavior ( $p_i[t] = p[t]$ ) which is independent of dispatch (schedule), then by time  $t = \psi$ , the number of arrivals will have risen to

$$q[\psi] = Q \int_0^{\psi} p[t]dt \quad (1)$$

and the average wait time or "schedule delay" will be:

$$\bar{d}[\psi] = \frac{\int_0^{\psi} (\psi-t)p[t]dt}{\int_0^{\psi} p[t]dt} = \frac{\int_0^{\psi} q[t]dt}{q[\psi]} \quad (2)$$

and in this case identical to the "frequency delay" component: the average duration between arrival and the next scheduled departure. The frequency delay is dependent on the cumulative distribution of arrivals (cumulative demand)  $q[t]$ , rather than the individual probability of arrival  $p[t]$ .

The total cost per patron (i.e., the average cost) is

$$AC[\psi] = \frac{TDC[L, V]}{q[\psi]} + v_v \frac{L}{V} + v_w d[\psi] \quad (3)$$

Hence to find the best tradeoff between Agency and Consumer Costs, we differentiate (3) with respect to  $\psi$  and set the result equal to zero:<sup>10</sup>

$$\text{i.e., } \frac{TDC[L, V]}{[q(\psi)]^2} q'[\psi] - v_w d'[\psi] = 0 \quad (4)$$

which may be solved for  $\psi$  to give the optimal dispatch time.

### 3.2 Single Frequency Bus Service

#### Uniformly Distributed Demand

If we assume that the arrival of  $Q$  patrons (within a service interval of unit time [ $t_s = 1$ ]) is indiscriminant with respect to time (random) then from (1), by time  $t$  cumulative demand will be  $q[t] = Qt$ ,<sup>11</sup> and by substituting in equations (1), (2), and (3), the average cost corresponding to the first bus is

$$AC[\psi] = \frac{TDC}{Q\psi} + v_v \frac{L}{V} + v_w \frac{\psi}{2} \quad (5)$$

where the average wait time is half the dispatch time.

## Sensitivity Analysis:

Dependence of Average Cost on the Dispersal of Demand

In general, demand is not uniformly distributed in time, but characterized by peaks of commuting, shopping trips, etc. The assumption of uniform demand is therefore limited to short service intervals, usually of one hour or less.

Our first consideration is to see how average cost (3) depends upon the distribution of demand. For example, consider two hypothetical (but algebraically simple) arrival probabilities

$$p[t] = 2t \quad \text{and} \quad p[t] = 6t[1-t],$$

with a unit time service interval. The first function is linear whilst the second is a quadratic approximation to a normal distribution, with a peak at  $t = 1/2$ . Assuming that all  $Q$  patrons have the same arrival behavior, substitution of the above functions in the expressions for cumulative demand  $q[\psi]$  and the average wait time  $\bar{d}[\psi]$ , lead to average costs of<sup>12</sup>

$$AC[\psi] = \frac{TDC}{Q\psi^2} + v_v \frac{L}{V} + v_w \frac{\psi}{3} \quad (6)$$

and

$$AC[\psi] = \frac{TDC}{Q\psi(3\psi-2\psi^2)} + v_v \frac{L}{V} + v_w \frac{\psi}{2} \cdot \frac{2-\psi}{3-2\psi} \quad (7)$$

respectively.

Note that in the case of the demand distribution corresponding to  $p[t] = 2t$ , the number of riders is lower than with the uniform demand distribution, so average agency costs are higher. However since there are relatively fewer initial arrivals, the frequency delay is only 2/3 of the uniform case, so the higher agency costs tend to be offset. Hence the best tradeoff of agency and user costs is struck at a longer dispatch time but a higher cost than in (5).

With the demand distribution corresponding to  $p[t] = 6t[1-t]$ , cumulative demand is relatively low at short dispatch times but begins to approach the uniform distribution case after half the service interval, when arrivals are declining. Hence at short dispatch times the average agency cost is higher than in the uniform case; however the average wait time is lower (by approximately 7/10's for  $t < t_{s/2}$ ) because initial arrivals are relatively fewer until the dispatch time approaches the duration of the service interval. Accordingly the best tradeoff is reached at a longer dispatch time, but at a comparable dollar value.

These results are illustrated in figure (1) for a set of parameters representative of line haul conditions on East Bay AC Transit service, with a 50 seat bus and a service interval of one hour.

### Conclusions

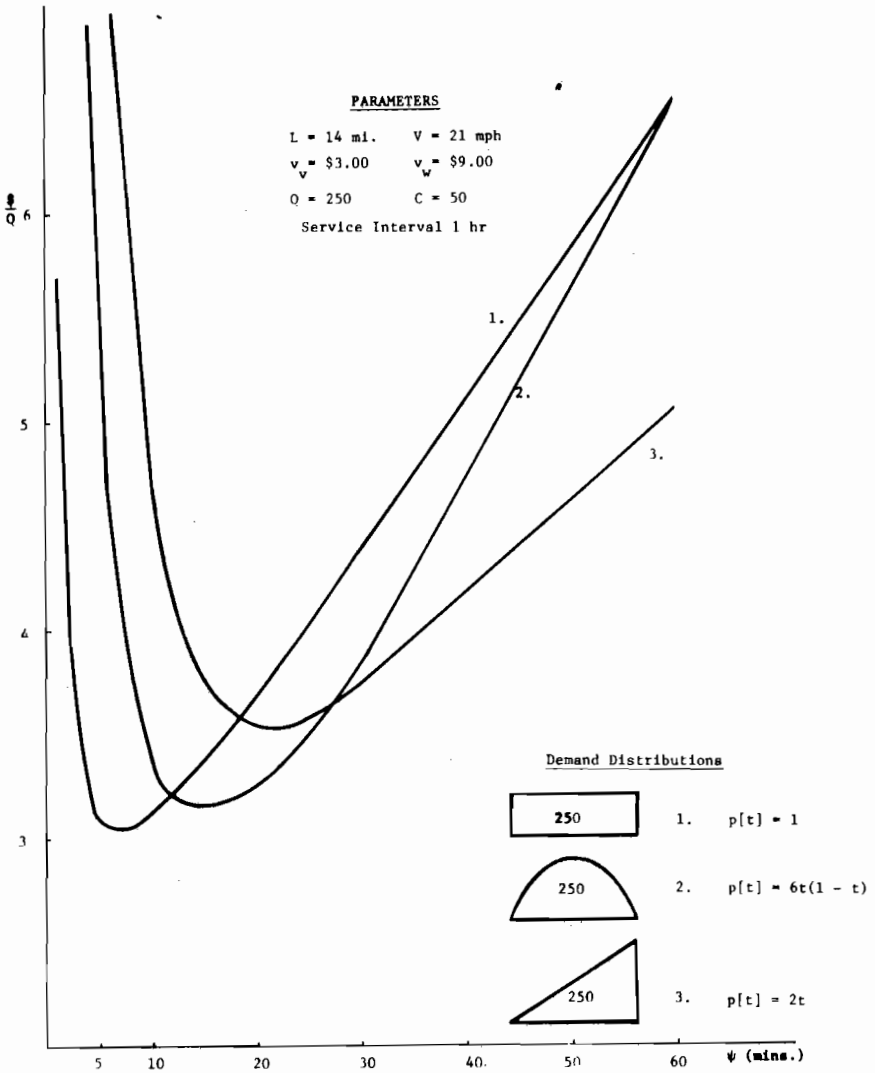
Some limited general conclusions are possible: First, demand distributions that are characterized by few initial arrivals, tend to have less favorable agency-user cost tradeoffs. Secondly, the frequency delay for the first bus is quite dependent on the distribution of demand.

### 3.3 Multiple Frequency Bus Service

However the foregoing relates only to the first bus, and not to multiple frequency service, for which we need to establish the corresponding average agency and user cost components. As linehaul conditions are assumed, with no intermediate stops, the in-vehicle cost component remains constant, and attention can be focussed on the average (over the service interval) of the agency and user schedule delay cost components, based on a regular headway (dispatch).

FIGURE 1

RELATIONSHIP BETWEEN AVERAGE COST, DISPATCH TIME,  
AND THE DISTRIBUTION OF DEMAND.



### Average Agency and User Wait Costs

The average agency cost is the Total Direct Cost per trip divided by the average number of passengers per bus viz: the total demand  $Q$  divided by the number of buses. Therefore, the average number of passengers per bus,  $\bar{q}[\psi]$ , is independent of the demand so that

$$\bar{q}[\psi] = Q\psi \quad (8)$$

However the question of wait time is more complex: Generalizing (2), the schedule or frequency delay within the  $n$ th interbus interval is given as

$$d_n[\psi] = \frac{\int_{(n-1)\psi}^{n\psi} (n\psi-t) p[t]dt}{\int_{(n-1)\psi}^{n\psi} p[t]dt} = \frac{\psi q[(n-1)\psi] + \int_{(n-1)\psi}^{n\psi} q[t]dt}{q[n\psi] - q[(n-1)\psi]} \quad (9)$$

The average frequency delay is therefore

$$\begin{aligned} \bar{d}[\psi] &= \frac{1}{Q} \sum_{n=1}^{1/\psi} \int_{(n-1)\psi}^{n\psi} p[t]dt \cdot \int_{(n-1)\psi}^{n\psi} (n\psi-t)p[t]dt \quad (10) \\ &= \frac{1}{Q} \sum_{n=1}^{1/\psi} \{q[n\psi] - q[(n-1)\psi]\} \cdot \{\psi q[(n-1)\psi] + \int_{(n-1)\psi}^{n\psi} q[t]dt\} \end{aligned}$$

This represents the schedule (frequency) delays of each interval, weighted according to demand in each interval (riders per bus).

#### Sensitivity Analysis: Dependence of the Average Frequency Delay on the Distribution of Demand (A) Simple Distributions

Assuming that demand is uniformly distributed so that  $q[t]=Qt$ , then the frequency delay within the  $n$ th interbus interval reduces to  $d_n[\psi] = \frac{\psi}{2}$ .

But demand within all interbus intervals is identical, so that the average frequency delay is  $\bar{d}[\psi] = \frac{\psi}{2}$ , or half the headway.

For demand corresponding to  $p[t] = 2t$ , the frequency delay within the  $n$ th interval is

$$d_n[\psi] = \frac{3n-2}{3(2n-1)} \psi = \frac{1}{3} \left\{ \frac{3}{2 - \frac{1}{n}} - \frac{2}{2n-1} \right\} \psi \quad (11)$$

Hence for the first interval  $d_1[\psi] = \psi/3$  (as previously derived) for the second interval it is  $(4/9)\psi$ , etc. Note that as the headway gets very small,  $d_n[\psi]$  approaches half the headway, since the dispersal of demand during interbus intervals becomes virtually uniform.

For the demand distribution corresponding to  $p[t] = 6t[1-t]$ , the frequency delay within the  $n$ th interval is given by

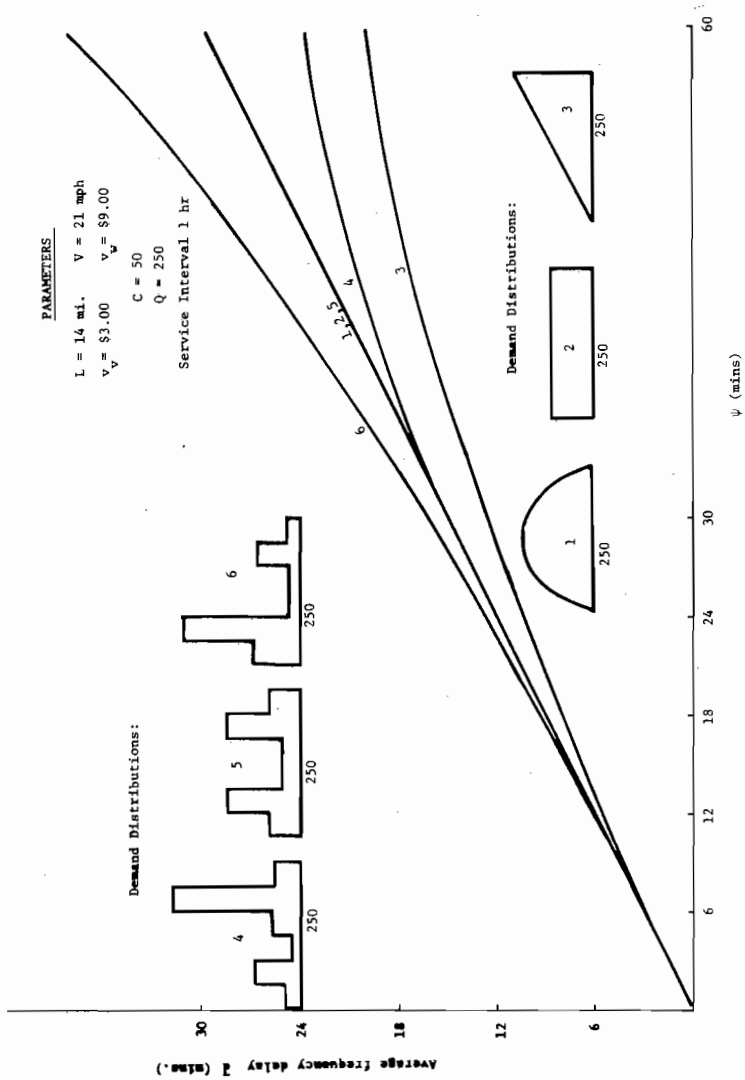
$$d_n[\psi] = \frac{3n-2 + (4n - 3n^2 - 1.5)\psi}{6n-2 + 2(3n - 3n^2 - 1)\psi} \cdot \psi \quad (12)$$

Evaluation of the average frequency delay for the three cases above, with respect to the example of Section 3.2, results in the average cost curves of figure (2). Notice that the average frequency delay is consistently half the headway for both the uniform distribution and the distribution corresponding to  $p[t] = 6t[1-t]$ , because of their very similar cumulative demand viz:  $q[t] = Qt$  and  $q[t] = Qt[3t - 2t^2]$ . However for the case of  $p[t] = 2t$ , where initial cumulative demand is relatively low, there is a deviation from this rule beyond headways of one quarter of the service interval. Such headways are rarely optimal; and user schedule knowledge would no doubt upset the assumption of arrivals independent of headway.



FIGURE 7

RELATIONSHIP BETWEEN AVERAGE FREQUENCY DELAY, HEADWAY,  
AND THE DISTRIBUTION OF DEMAND.



(B) General Distributions

Proceeding further, we simulate a more realistic demand behavior: that patronage is made up of  $g$  patron types or groups, characterized by  $g$  distinct arrival behaviors. The  $i$ th group has  $Q_i$  members, such that

$$Q = \sum_{i=1}^g Q_i, \text{ with a likelihood of arrival concentrated within a narrow}$$

period.<sup>13</sup> For simplicity we assume that the probability of arrival is described by the same functional form for all groups, but the latter is set within different periods for each, i.e., maximum centered at  $g$  points over the service interval,  $\frac{t_s}{g}$  apart.

Assuming a quadratic approximation to normally distributed probability of arrival, the  $i$ th group, with a maximum arrival likelihood at time  $t_i = \frac{t_s}{g} \cdot i$ , has a behavior

$$p_i[t] = \{1 - (t - \tau)\rho\} \cdot \frac{6}{\tau^3} \{t - t_i\} \{\tau - t + t_i\}$$

$$\text{for } t_i - \tau/2 \leq t \leq t_i + \tau/2$$

and

$$p_i[t] = \{t_s - \tau\}\rho$$

$$\text{for } 0 < t < t_i - \tau/2 \quad \text{and} \quad t_s > t > t_i + \tau/2 \quad (13)$$

where  $\int_0^{t_s} p_i[t] dt = 1$  for all  $i$ ,  $\tau$  is the base width of the quadratic function, and  $\rho$  is the value of  $p_i[t]$  outside the range  $(t_i - \tau/2, t_i + \tau/2)$ , and can be set to a very small number. Accordingly any pattern of demand can be simulated through adjustment of relative membership levels  $Q_i$ .

To evaluate average frequency delay corresponding to (13) a discrete summation is now necessary. It is shown in Appendix A-1 that the equivalent of the continuous expression (10) is

$$\bar{d}[\psi] = \frac{1}{Q} \sum_{n=1}^{1/\psi} \sum_{i=1}^g q_{i,n} d_{i,n} \quad (14)$$

where  $d_{i,n}$  is the frequency delay of the  $i$ th patron group in the  $n$ th interbus interval, and  $q_{i,n}$  is the corresponding demand. This relationship assumes a regular headway,<sup>14</sup> and represents the frequency delays for each bus, weighted according to the number of passengers (drawn from the demand distribution) in each interbus interval.

Similarly the discrete equivalent to (8), the average number of passengers per bus is

$$\bar{q}[\psi] = \psi \sum_{n=1}^{1/\psi} \sum_{i=1}^g q_{i,n} \quad (15)$$

where  $\sum_{i=1}^g q_{i,n}$  is the number of riders on the  $n$ th bus.

#### Estimation of Average Cost (-- Algorithm SRAC1)

The above equations embody the main computational aspects of algorithm SRAC1 ([Short Run] Average Cost 1), and are the basis of more general algorithms SRAC2, SRAC3 and SRAC4, to be introduced later. SRAC1 takes as input: the service interval, shadow prices  $v_v$  and  $v_w$ , speed  $V$ , distance  $L$ , and dependent functional relationships for TDC and  $p[t]$ . To evaluate average cost, the service interval is partitioned into equal sub-intervals, the end of these sub-intervals being marked by the arrival of a bus to collect patrons who have accumulated in that interval. Because of the finite integration, average cost estimates are confined to headways that coincide with integration intervals, and are computed according to equations (3), (14) and (15) for varying levels of demand  $Q$ . Output is

average cost as a function of headway for each level of demand. The minimum value of average cost can therefore be derived by interpolation.

#### Application to More General Dispersions of Demand

Accompanying figure (2) are three peaked demand distributions, labeled as: "right-skew," "left-skew," and "symmetric." These result from an assumption of thirteen patron types, over a service interval of one hour. Likelihood of arrival is concentrated in a 12 minute period for each group, and is described by equation (13), with a base demand  $Q = 72$ , and group populations  $Q_i$  of:

6,14,26,6,2,0,0,0,2,4,9,2,1 (left-skew)

1,2,4,9,2,0,0,0,1,6,14,26,6 (right-skew)

and 4,8,12,8,4,0,0,0,4,8,12,8,4 (symmetric), have been used.

In Figure (2) the average frequency delays resulting from these distributions have been evaluated with SRAC1 for the example of Section 3.2. We see that the frequency delay of the symmetric distribution has the same relationship to headway as do the previously worked distributions: the half-headway rule holds because all have similar cumulative demand functions  $q[t]$ . The right and left skew distributions also conform to the half-headway rule until about a half an hour headway.

#### Findings Regarding Average Frequency Delay

If demand is symmetrically distributed over the service interval the average frequency delay is just a function of headway. Furthermore, this independence holds for arbitrary distributions at headways of up to one quarter of the service interval (15 minutes).

Different patron types could well have different incomes (different disutilities and opportunity costs of travel), with different starting times. It was found however

that there was no difference in the minimum cost value and its corresponding headway, using either the mean shadow price of time and effort inputs of all patron groups or the individual values. Agency and user in-vehicle costs are clearly the same in each case, so the problem is to show that the wait costs are the same:

This can be seen by rearrangement of (14): if  $v_{w_i} d_{i,n}$  is the cost of the frequency delay incurred by the  $i$ th patron group in the  $n$ th interbus interval, then the average frequency delay or wait cost is

$$\overline{CS}(\psi) = \frac{1}{Q} \sum_{n=1}^{1/\psi} \sum_{i=1}^g q_{i,n} d_{i,n} v_{w_i} = \sum_{i=1}^g v_{w_i} \left\{ \frac{1}{Q} \sum_{n=1}^{1/\psi} q_{i,n} d_{i,n} \right\}.$$

Anticipating (17) we can reduce this to:

$$\overline{CS}(\psi) = \sum_{i=1}^g v_{w_i} \left\{ \frac{\psi_i}{2} \cdot \frac{1}{g} \right\} = \frac{\psi}{2} \frac{1}{g} \sum_{i=1}^g v_{w_i} = \bar{v}_w \cdot \frac{\psi}{2}, \text{ where } \bar{v}_w = \frac{1}{g} \sum_{i=1}^g v_{w_i}.$$

That is, the averaging can be implicit or explicit.

### General Results and Conclusions

Some important conclusions follow as to the component cost inter-relationships: First for a given demand  $Q$ , the average number of passengers per bus is independent of the distribution of demand and therefore independent of the underlying arrival probabilities. We have the general result

$$\bar{q}(\psi) = Q\psi \quad (16)$$

where  $\psi$  is the headway.

Secondly the average frequency delay  $\bar{d}(\psi)$  for service frequencies of at least four buses per hour is simply a function of headway, independent of the distribution of demand, and is given as

$$\bar{d}(\psi) = \frac{\psi}{2} \quad (17)$$

(i.e., half the headway). This result is familiar to the literature, but appears to have a wider validity than generally appreciated.

Its origin lies in the averaging process wherein as the headway gets smaller and smaller, the demand within each interbus interval becomes more and more uniform. Moreover in deriving the average frequency delay, interbus intervals in which this is more than half the headway tend to be offset by interbus intervals where it is less than half the headway, i.e., arrivals tend to take on a random character. It is only when the headway is quite large that the pattern of demand is influential.

Therefore if we substitute (16) and (17) in the general average cost expression (3), we have a similar independence for AC:

$$AC = \frac{TDC}{Q\psi} + v_v \frac{L}{V} + v_w \frac{\psi}{2} \quad (18)$$

wherein

$$AFC = \frac{TDC}{Q\psi} \quad (19)$$

(agency input)

and

$$AVC = v_v \frac{L}{V} + v_w \frac{\psi}{2} \quad (20)$$

(user's time and effort inputs).

The SRMC is given as

$$SRMC = \frac{\partial}{\partial Q} (Q \cdot AC) = v_v \frac{L}{V} + v_w \frac{\psi}{2} \quad (21)$$

Hence  $SRMC = AVC$ , since an additional rider inflicts no delay on others under point to point linehaul conditions. In other words, there are no implicit constraints to produce congestion effects.

From equation (4), the minimum average cost headway is

$$\psi = \left\{ \frac{2 \text{TDC}}{v_w} \right\}^{1/2} Q^{-1/2} \quad (22)$$

whence substitution in (18) yields<sup>15</sup>

$$\begin{aligned} AC_{\min} &= \frac{\text{TDC}}{Q \left\{ \frac{2\text{TDC}}{v_w} \right\}^{1/2}} Q^{-1/2} + v_v \frac{L}{V} + \frac{v_w}{2} \left\{ \frac{2\text{TDC}}{v_w} \right\}^{1/2} Q^{-1/2} \\ &= \{2 \text{TDC } v_w\}^{1/2} Q^{-1/2} + v_v \frac{L}{V} \end{aligned} \quad (23)$$

In practice these expressions are supplanted by numerical computation with SRACL.

Note the familiar result that the minimum cost frequency is proportional to the square root of demand for service and is independent of the way demand is dispersed over the service interval. Significant scale economies are possible with increasing ridership; for example, if demand doubles, we only need  $\sqrt{2}$  as many buses (per service interval) and average cost declines by  $(1 - \frac{1}{\sqrt{2}})$  of Agency Cost. Figures 3(a) and 3(b) show the relationship between the unit shadow price of wait ( $v_w$  \$ per hour) and minimum cost and associated headway.

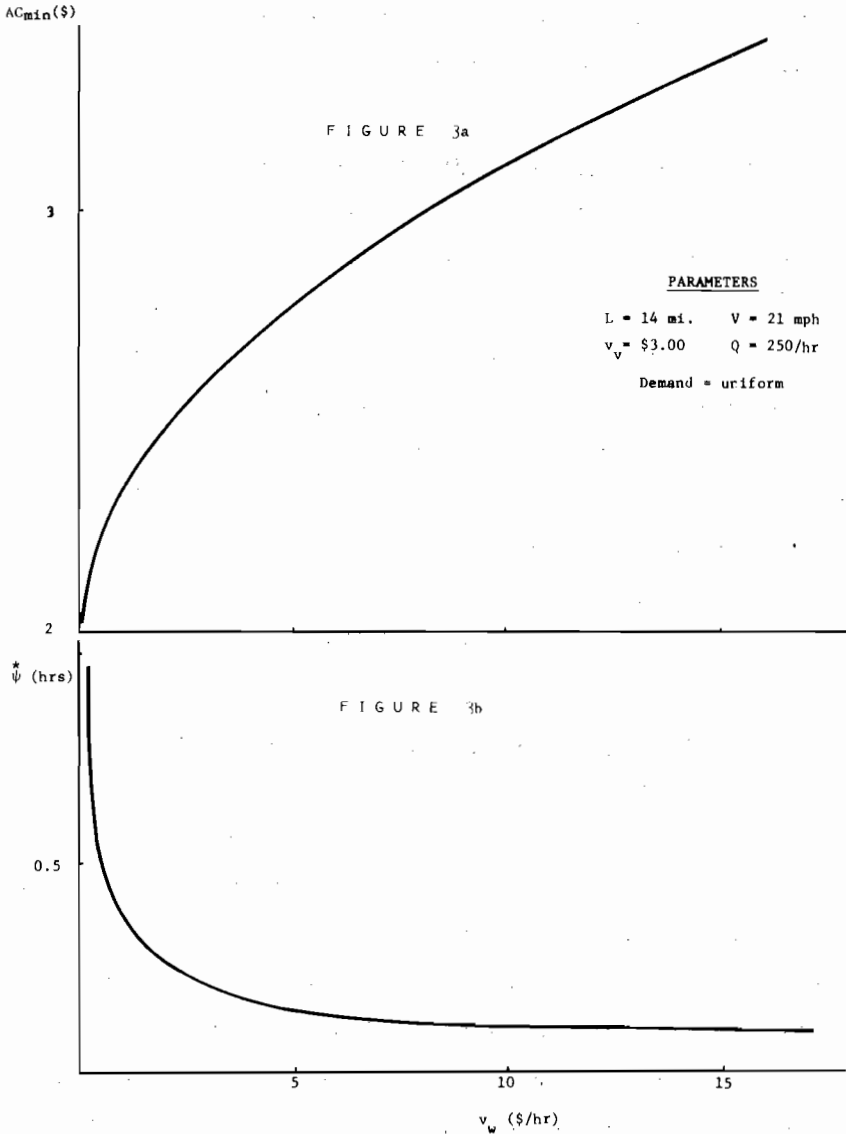
Costs are mirrored about the minimum average cost headway, since for any  $m$ , then  $AC(m\psi)$  equated to  $AC(\psi/m)$ , implies

$$\psi = \left\{ \frac{2\text{TDC}}{v_w Q} \right\}^{1/2} \quad \text{which is just the optimal headway. Hence reducing the}$$

(minimum cost) headway by a given proportion, or increasing that headway by the same proportion, increases the average cost by the same amount.

FIGURE 3

RELATIONSHIP OF THE SHADOW PRICE OF WAIT TO MINIMUM  
AVERAGE COST AND ASSOCIATED HEADWAY.





In conclusion, for patronages sufficiently low that a capacity constraint is not encountered, bus service provided at a rate proportional to that to which people travel is not optimal. Although the frequency delay is proportional to half the headway, the net frequency delay or wait under minimum cost conditions, derived from by (22) and (17), is not independent of demand. This is a consequence of bus service showing increasing returns to scale.

### 3.4 Multiple Frequency Bus Service with Capacity Constraints

Hitherto we have been considering an idealized case of a bus with unlimited capacity in linehaul operation. But our TDC estimate has been based on the costs of a standard 50 seat coach. If we impose a capacity constraint we will find that running buses too infrequently for a given net demand will lead to more people arriving in the interbus interval than the bus can accommodate, and therefore some will have their departure displaced to later buses. This displacement of the patron's actual departure from the nearest departure is called "stochastic delay." Under capacity constraint conditions, we can also introduce the concept of a load factor  $K$ , the ratio of the number of riders to the capacity of the bus, i.e.,  $K = \frac{Q\psi}{C}$ .

#### Estimation of Average Costs with Capacity Constraints (Algorithm SRAC2)

Algorithm, SRAC2 incorporates a size constraint, and is built on SRAC1. The service interval is partitioned in the same way, but loading on the bus arriving at the end of each interbus interval is subject to the capacity constraint. An accounting routine is incorporated that records not only the wait time of those who are successful in catching the bus at the end of the interbus interval in which they arrive, but also keeps track of those who forego buses, and are later successful. This is equivalent to the Markov process described by Douglas & Miller (1974).

Accommodation on each bus is on a first-come-first-served basis. This can be visualized as a queue of patrons forming at the bus stop, conforming to the arrival sequence determined by the demand distribution. At the end of each interbus interval, SRAC2 examines the queue and removes from its head the number corresponding to the number of seats available on the bus (bus capacity). It then computes their schedule delay by noting when they arrived and in what interval. The process is repeated for each bus.

The average schedule delay is the individual schedule delays (frequency and stochastic) of the boarders of each bus, weighted by their respective numbers, and in analogy to equation (14) is given by (Appendix A-2):

$$\bar{d}(\psi) = \frac{1}{Q} \sum_{n=1}^{1/\psi} \sum_{m=1}^{n'} q_{n,m} d_{n,m} \quad (24)$$

such that  $\sum_{m=1}^{n'} q_{n,m} \leq C$ , where  $C$  is the bus capacity (seats), with

$n' \leq n$ , and  $q_{n,m}$  is the number of boarders of the  $n$ th bus who arrived in the  $m$ th interval. Afterwards  $q_{n,m} = 0$  for all  $m$ , for which total absorption takes place, and if a residue for  $q_{n,n'}$  results, then it is reset to

$q_{n,n'} = \sum_{m=1}^{n'} q_{n,m} - C$ . In this expression  $d_{n,m}$  is the schedule delay of

those who arrive in the  $m$ th interval and succeed in boarding the bus arriving at the end of the  $n$ th interval. The corresponding average number of passengers per bus is <sup>16</sup>

$$\bar{q}(\psi) = \psi \sum_{n=1}^{1/\psi} \sum_{m=1}^{n'} q_{n,m} = Q\psi \quad (25)$$

Constrained Minimum Average Cost

Figure (4) shows the effect of size constraint  $C$  on the variation of average cost, where the unconstrained curve corresponding to a bus of unlimited capacity is the envelope of the family of curves for different size vehicles.<sup>17</sup> Note that if the vehicle size constraint is encountered at headways prior to that of unconstrained minimum, the minimum average cost is increased. This corresponds to the familiar "leave-when-full" dispatch rule, and is encountered when  $C \leq Q \psi^*$  or from (22)

$$Q \geq C^2 v_w / 2 \text{ TDC.} \quad (26)$$

This rule is clearly optimal since there is no point in delaying the dispatch of a fully laden bus. Hence in figure (4), the minimum corresponding to a 25 seat bus is "constrained" (unity load factor), whilst the other minima are "unconstrained." This constrained headway is

$$\psi^* = C/Q \quad (27)$$

and the associated minimum average cost is given by substitution in equation (18), viz:

$$AC_{\min} = \frac{\text{TDC}}{C} + v_v \frac{L}{V} + v_w \frac{C}{Q} \quad (28)$$

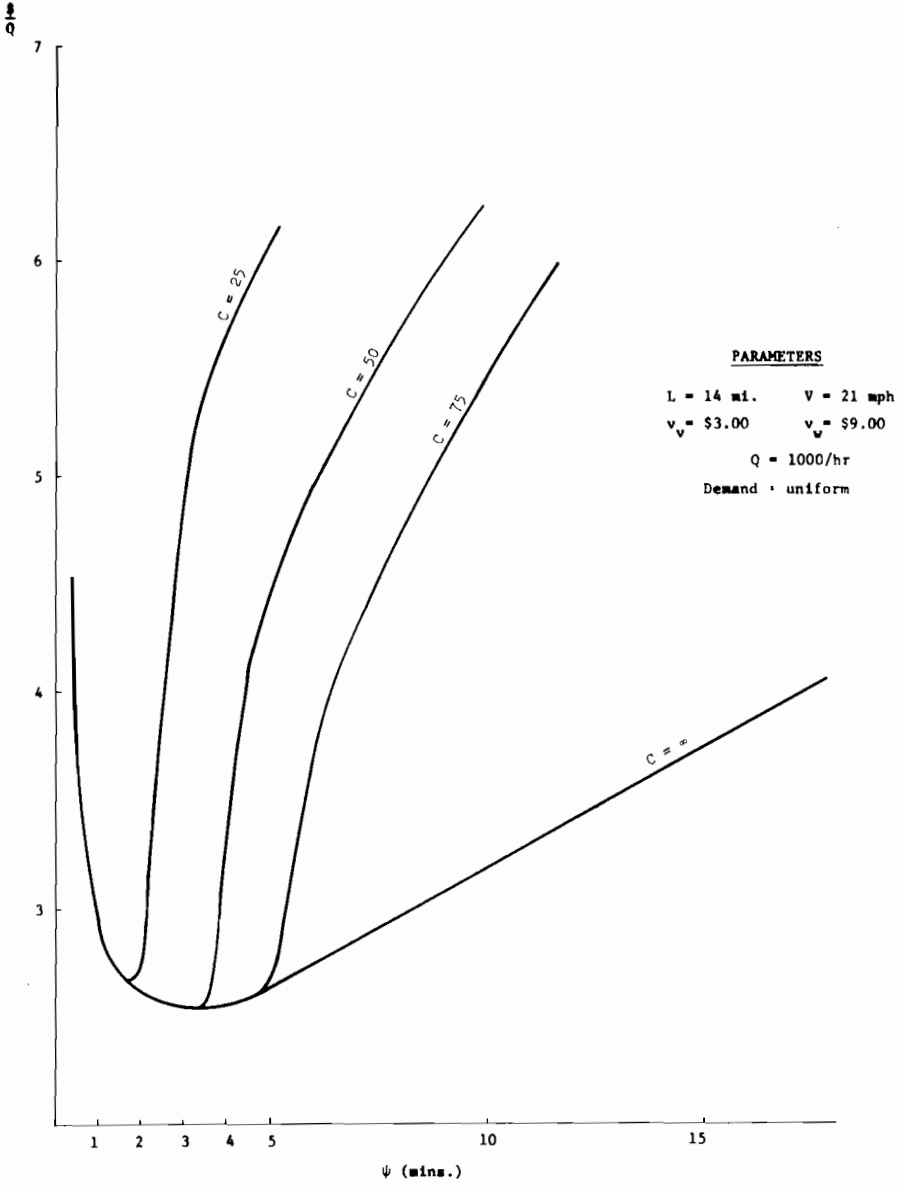
The scale economies observed with (22) and (23) are now enhanced.

Estimation of Minimum Average Cost With Capacity Constraints  
(Algorithm LRAC1)

LRAC1 is an algorithm that computes the minimum average cost by a direct iterative procedure. Beginning at zero headway ( $\psi=0$ ), each iteration advances the headway. At the  $l$ th iteration  $\psi_l Q$  and  $AC_l$  are computed

FIGURE 4

EFFECT OF BUS CAPACITY ON THE VARIATION  
OF AVERAGE COST.



(according to [3]). If  $\psi_{\ell}Q > C$ , then  $AC_{\ell-1}$  is a constrained minimum, otherwise it tests for  $AC_{\ell} \geq AC_{\ell-1}$ . If this is true, then  $AC_{\ell-1}$  is an unconstrained minimum, failing this it proceeds to the next iteration. The algorithm is rapid and well suited to derivation of long run average cost envelopes, and is used for this purpose in Section 4.

#### Comparative Average Costs as a Function of Capacity

There is another important feature of Figure 4. Notice that as vehicle size decreases there is markedly lower cost tolerance in scheduling about the optimal headway. If the headway with a small vehicle is a little bit too long, then stochastic delay congestion costs occur because all the people who arrive within a given interbus interval may not be cleared by the first arriving bus, and costs rise steeply as wait accumulates. On the other hand if the headway is a bit too short then the load factor per bus is too low and costs rise steeply again. However with larger buses we have a wider latitude of headway around the optimum without departing significantly from the minimum average cost value, because the likelihood of stochastic delay is reduced.

#### Sensitivity Analyses

##### A. An Exploration of the Form of Stochastic Delay

The average cost curves in Figure 4 correspond to a uniform distribution of demand. It will be noticed that their right hand upper reaches are turning over. This appears to be the result of a failure to clear the given catchment at longer headways, and therefore not totally accounting for the schedule delay of the patrons who are unable to board within the defined service interval. To check this explanation, components of average cost corresponding to a bus capacity of 50 (and an unlimited capacity, for reference) were computed with SRAC2, according to:

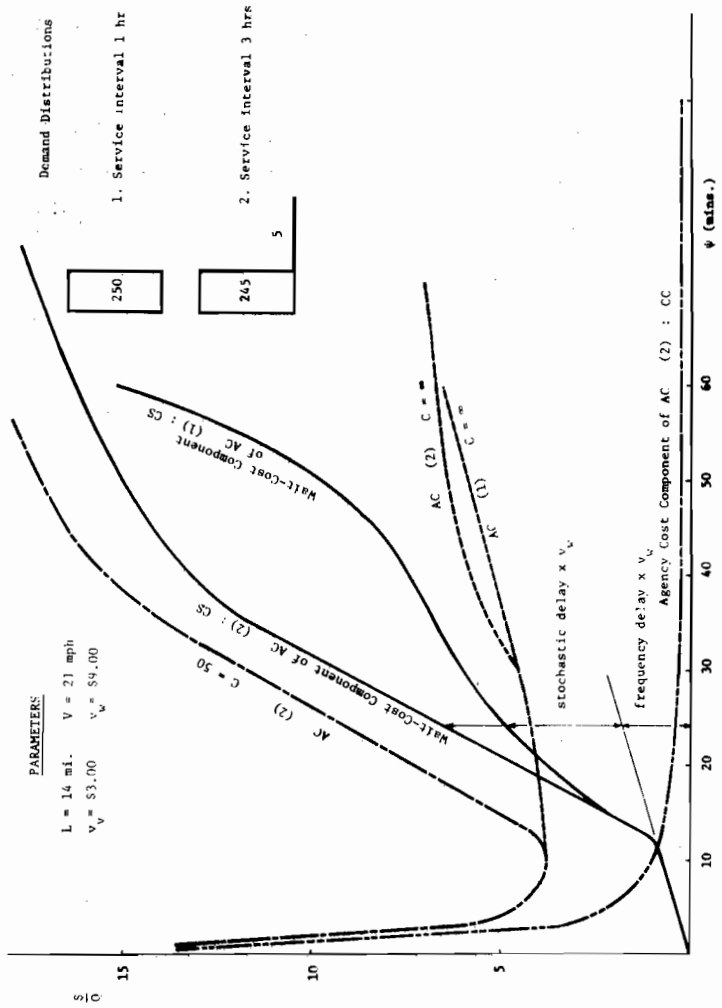
- (a) a service interval of one hour, and a uniform distribution of demand with 250 patrons, and
- (b) a service interval of three hours, and a uniform distribution of demand within each hour, but with virtually all 250 patrons confined to the first hour.<sup>18</sup> This is to ensure that non-boarders from the first hour are accommodated in the remaining two hours, when there are effectively no new patrons added to the queue.

Figure (5) shows the resulting consumer schedule delay cost curves. Notice that the "wait equals half headway" rule breaks down after 12 minutes headway, with the onset of stochastic delay (missed buses). Under the assumptions of (a), the wait of non-boarders beyond the hour is not tracked, and costs are deflated artificially, whereas under (b), they are tracked and those not accommodated at the end of the first hour progressively fill up later scheduled buses. This occurs up to a headway of 36 minutes, beyond which patrons are increasingly unsuccessful. Up to the 36 minute point costs rise linearly, as a result of uniform arrival and orderly queueing. Note that under (a), failure to clear the catchment does not result in significant cost deflation until 16 minutes, or more generally, twice the optimal headway. A simultaneous examination of agency and average costs (also graphed) show that the consumer wait process described above, is the underlying mechanism of the turning-over effect observed with the average cost.

#### B. The Inter-relationship between the Distribution of Demand and Minimum Cost

Tests made in Section 3.3 to determine the inter-relationship between demand distribution and the minimum average cost were repeated incorporating the capacity restraint. Using the uniform and peaked ("symmetric"

FIGURE 1  
 THE INFLUENCE OF THE DISPERSAL OF DEMAND  
 ON STOCHASTIC DELAY.



and "right-skew") distributions given in Figure 6a, average costs were computed by SRAC2. The results show that at low levels of demand (Figure 6a) when the best tradeoff is unconstrained, the minimum average cost (and headway) is, as before, insensitive to the distribution, but has a marked dependence at higher levels when the capacity constraint is encountered (Figure 6b). (The average cost corresponding to the skew distribution, after diverging from the minimum, goes through an inflection: the result of stochastic delay being momentarily curtailed by a lull in arrivals. The latter could arise from staggered starting times.)

### C. Limitations of the Regular Headway Assumption

An underlying assumption contained in the general average cost relationship (3), is the independence of arrival behavior with respect to headway.<sup>19</sup> To make for an independence and simplify analysis, evaluation of average cost has centered on the notion of regular headway.

Minimum Average Cost: The assumption of regular headway to derive minimum average cost is clearly justifiable when demand approximates a uniform distribution. However when it shows a pronounced peaking, minimum cost scheduling theoretically requires adjustment of dispatch times (headway) to meet cumulative demand. For example, when the level of demand is too high to allow a free tradeoff between agency and user costs, dispatch would be on a leave-when-full basis. The derivation of minimum average cost on the basis of regular headway, is equivalent to smoothing the dispatch times over the service interval. When the demand is high, and wait expensive, this technique focusses upon the minimization of stochastic delay, rather than agency cost (when cumulative demand is slack). The higher the concentration of demand, the closer the optimal (regular) headway



FIGURE 6a

THE EFFECT OF THE DISTRIBUTION OF DEMAND  
ON AVERAGE COST.

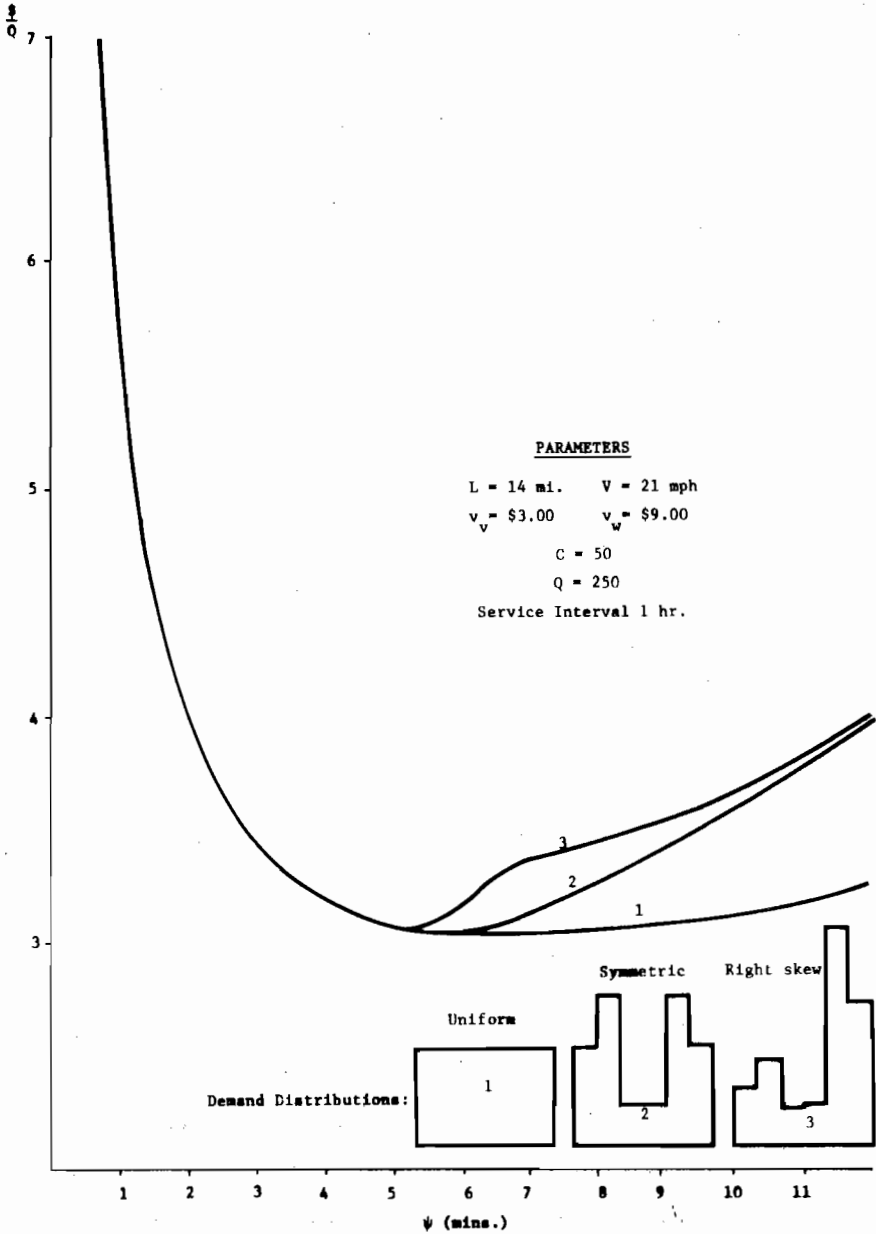
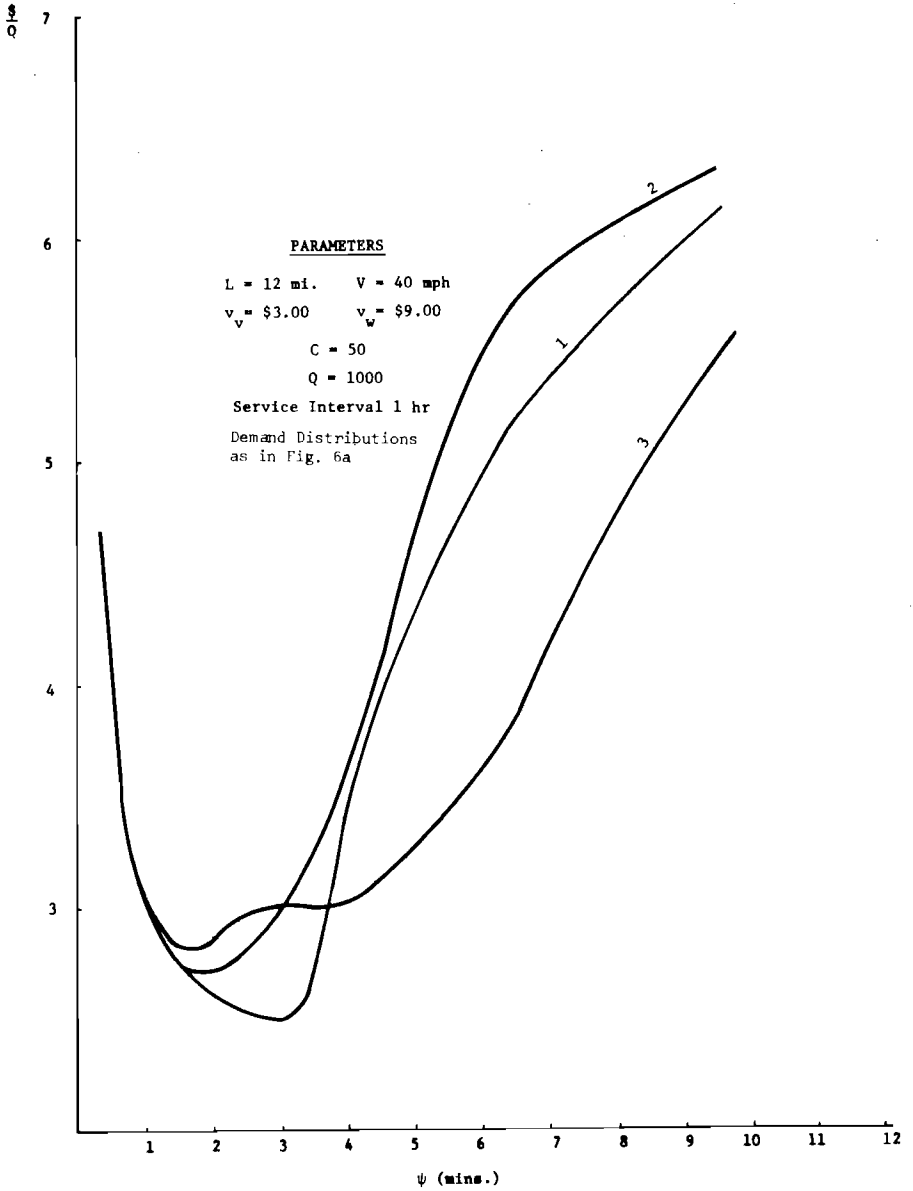


FIGURE 6b

THE EFFECT OF THE DISTRIBUTION OF DEMAND  
ON AVERAGE COST.



is to the typical dispatch-when-full interval in the peak. This accounts for the reduced minimum cost headways shown by the peaked distributions in Figures 6a and 6b. It is important to note that some stochastic delay exists at the optimal frequency in this case but it is small.<sup>20</sup> Regularization of headway on an hourly basis is not an uncommon practice of bus companies and leads to some stochastic delay during the rush hour.

Since the smoothed (regular) headway tends to be an over-estimate of the typical optimal dispatch interval during the peak, and a corresponding under-estimate during the off-peak, the averaging process incorporated in SRAC2 results in a minimum average cost very close to that found by free adjustment of dispatch with LRAC1.

General Average Cost: The evaluation of average costs in general is analytically complex, particularly if a capacity constraint is incorporated. When the demand is not uniformly distributed, some systematic basis for varying dispatch (headway) needs to be assumed to represent a simple agency input. The most elementary basis for dispatch is the regular headway.

However a more adequate treatment may call for division of the service interval into intervals with more or less uniformly distributed demand, and independent regularized headways. But a single agency input for analysis of average cost over the whole service interval is difficult, and it is not clear what would be the basis of a simultaneous minimization of average costs over all intervals.

Consequently, regular headway will continue to be the main basis of analysis of average costs in this paper. One hour service intervals with uniformly distributed demand will also be used, except for a detailed analysis of costs associated with stochastic delay, as this process implies some slackening in demand outside the immediate service interval being considered.

### Average Costs as a Function of Frequency, Capacity and Demand

Figure 7a shows the relationship of average cost to headway for various levels of uniformly distributed demand; Figure 7b shows their disaggregation into agency and user components. Note the scale economies with increasing demand.

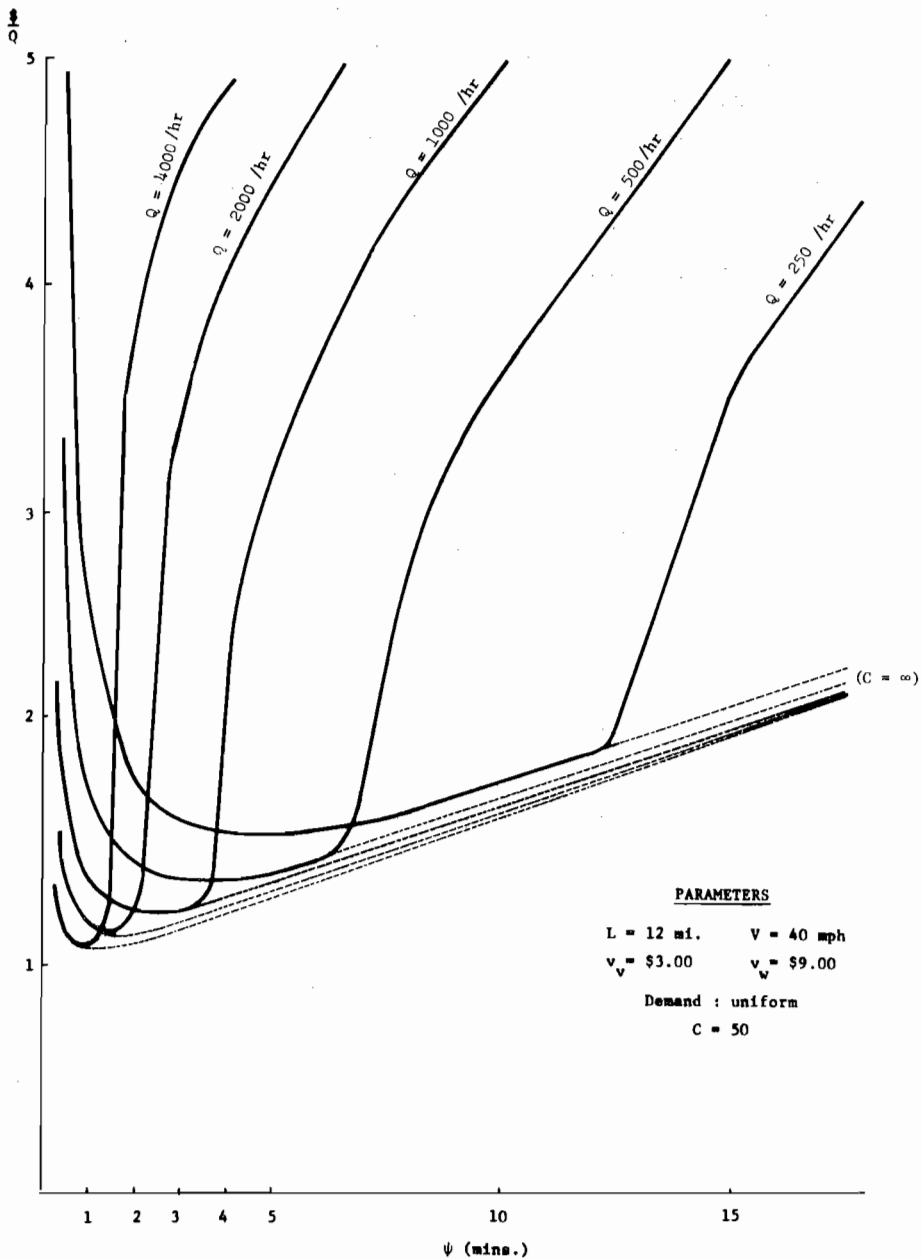
The average costs shown in Figure 4, contain no adjustment of TDC for varying bus size  $C$ . For example a 25 seat bus is cheaper in operating and capital costs than the 50 seat bus. Making these adjustments reference to Section 2.1 gives the improved average cost estimates of Figures 8(a), (b), and (c). Notice the marked cost tolerance with lower levels of demand and larger vehicles. Figures 9(a), (b), and (c) (alternatively) show average cost as a function of demand, for fixed frequencies  $\frac{1}{\psi}$ . Patronage  $Q$  reaches the system capacity constraint at  $Q_0 = C/\psi$ , after which stochastic delay occurs, with rising average cost. Headway variations are significantly cheaper with the 75 seat vehicle over a wide volume of demand.

### Stochastic Delay Congestion Costs

To examine costs associated with the stochastic delay process, the service interval is widened and demand distributed on a peak-off-peak basis. Whereupon the various fixed frequencies can be interpreted as, in turn, the best the company can provide (subject to labor and equipment constraints) to move passengers over the peak period. These limitations are reflected in increased delay costs to the consumers of the service. Figures 10(a), (b), and (c) show the AVC and AFC components as a function of demand corresponding to a three hour service interval (1 hour peak) with peak to off-peak patronage in the ratio of 5:1.

FIGURE 7a

RELATIONSHIP OF AVERAGE COST TO HEADWAY FOR VARIOUS  
LEVELS OF DEMAND.



**PARAMETERS**

$L = 12$  mi.     $V = 40$  mph  
 $v_v = \$3.00$      $v_w = \$9.00$   
 Demand : uniform  
 $C = 50$

FIGURE 7b

COSTS IN FIGURE 7a DISAGGREGATED INTO USER WAIT COSTS (CS) AND AGENCY COSTS (CC).

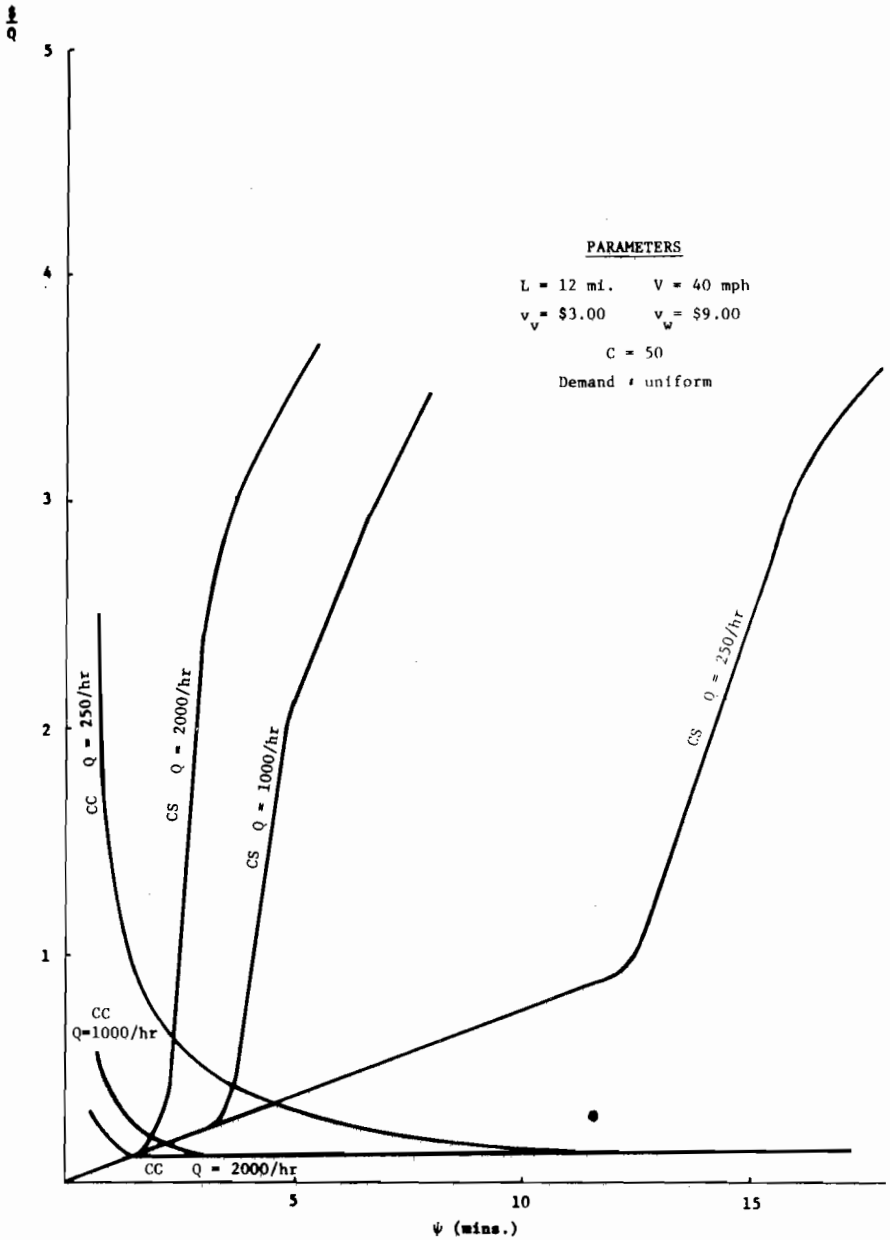


FIGURE 9a

AVERAGE COST AS A FUNCTION OF BUS CAPACITY  
AND HEADWAY.

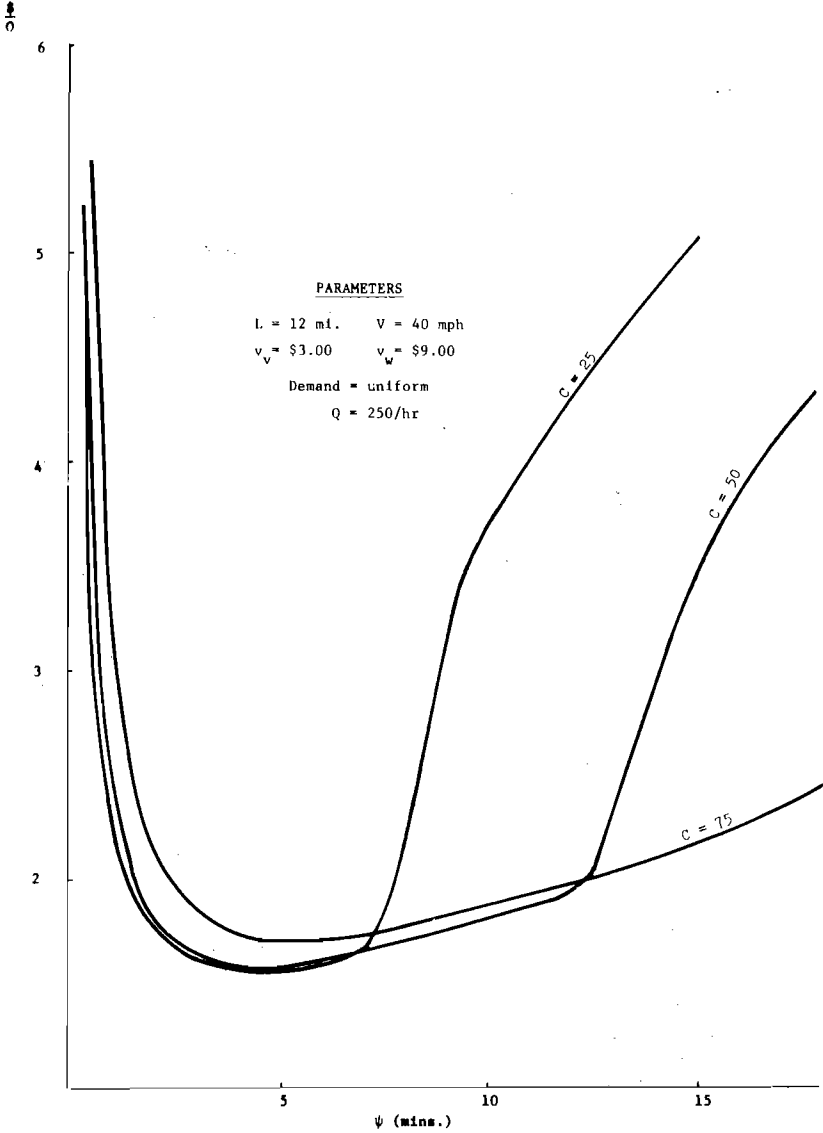


FIGURE 8b

AVERAGE COST AS A FUNCTION OF BUS CAPACITY  
AND HEADWAY.

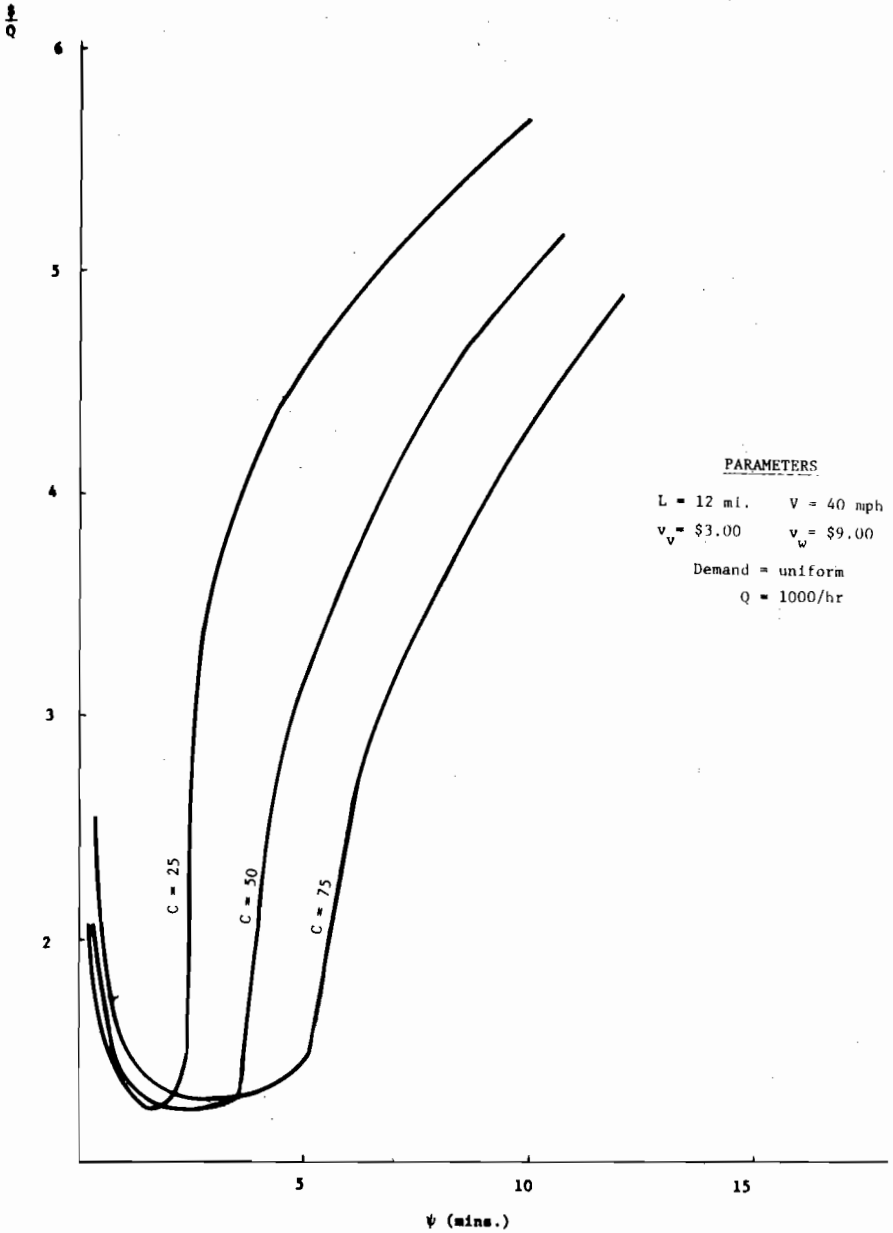




FIGURE 8c

AVERAGE COST AS A FUNCTION OF BUS CAPACITY  
AND HEADWAY.

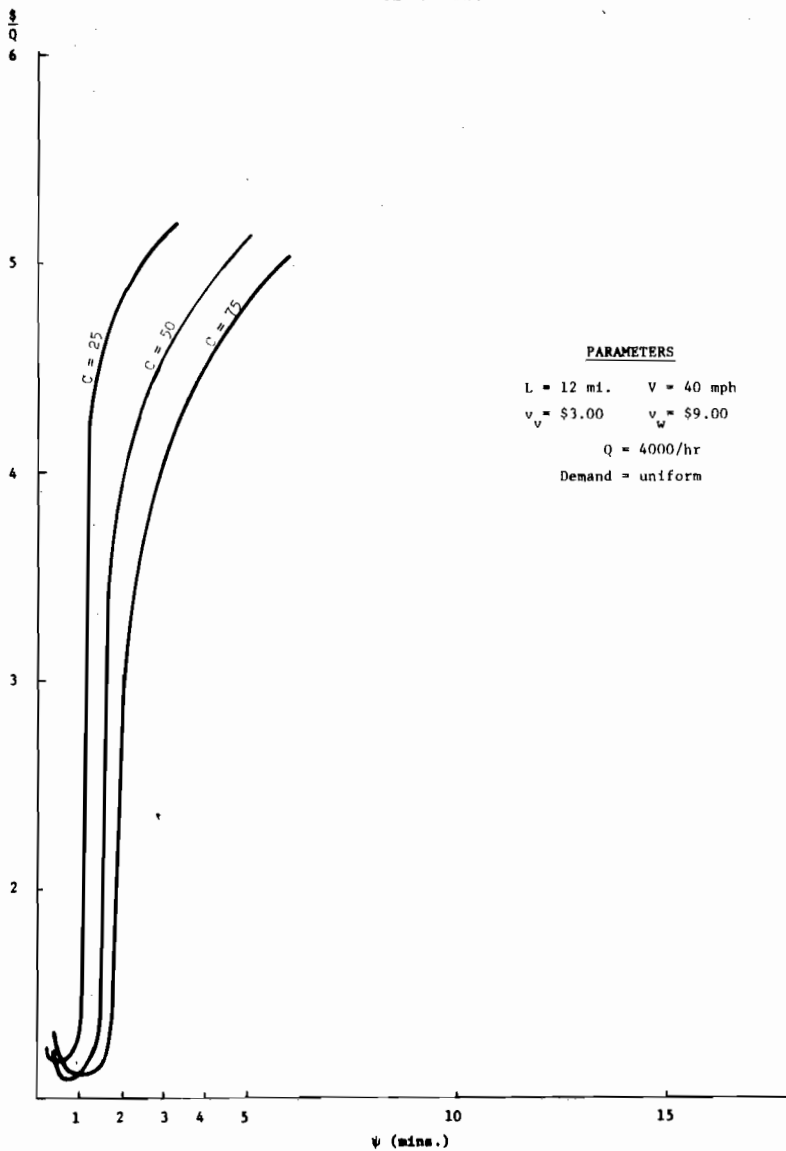


FIGURE 9a

AVERAGE COST AS A FUNCTION OF FREQUENCY AND LEVEL OF DEMAND.

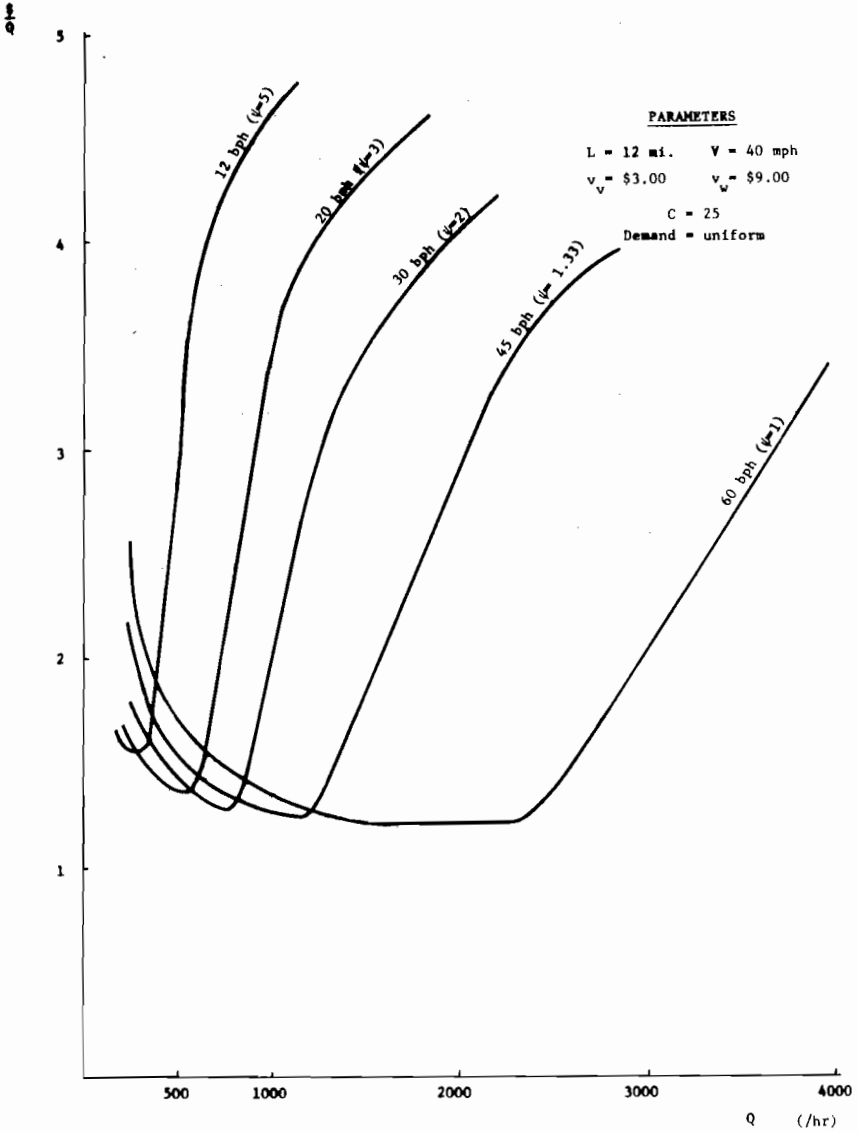


FIGURE 9b

AVERAGE COST AS A FUNCTION OF FREQUENCY AND  
LEVEL OF DEMAND.

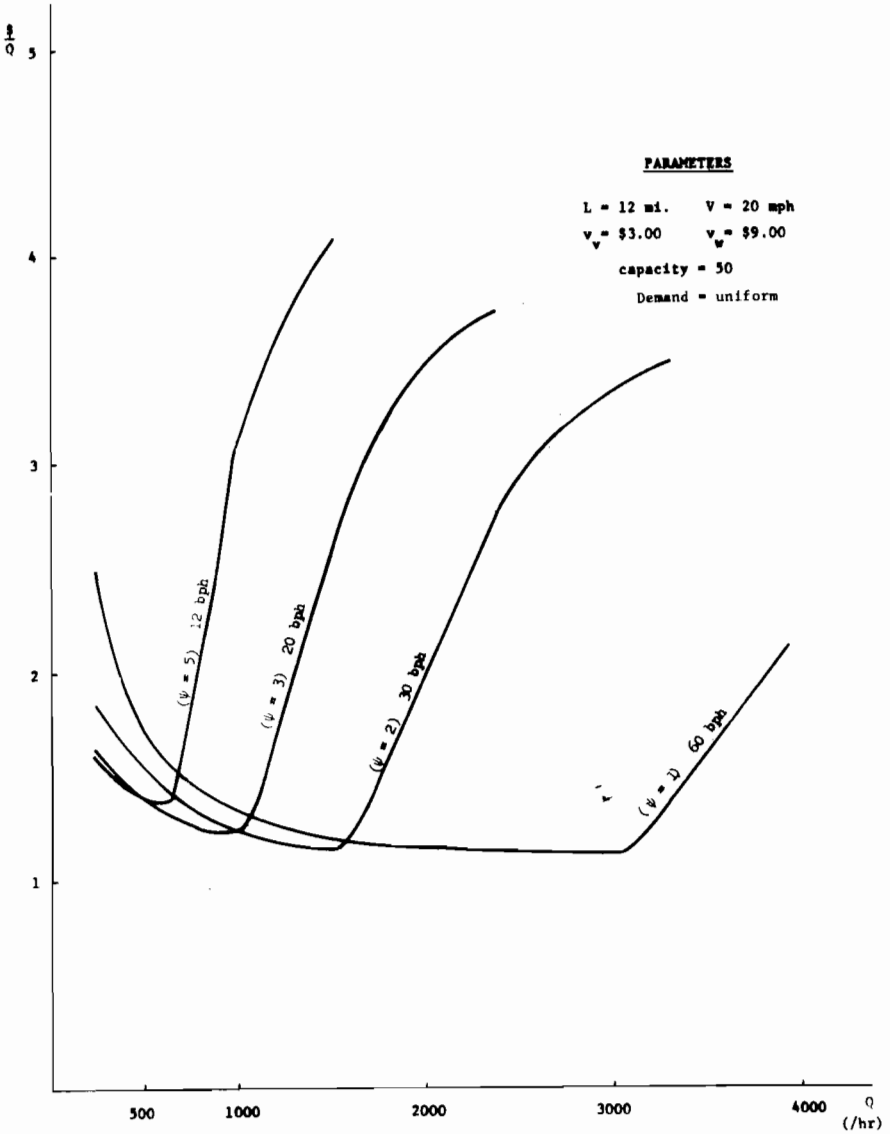


FIGURE 9c

AVERAGE COST AS A FUNCTION OF FREQUENCY AND  
LEVEL OF DEMAND.

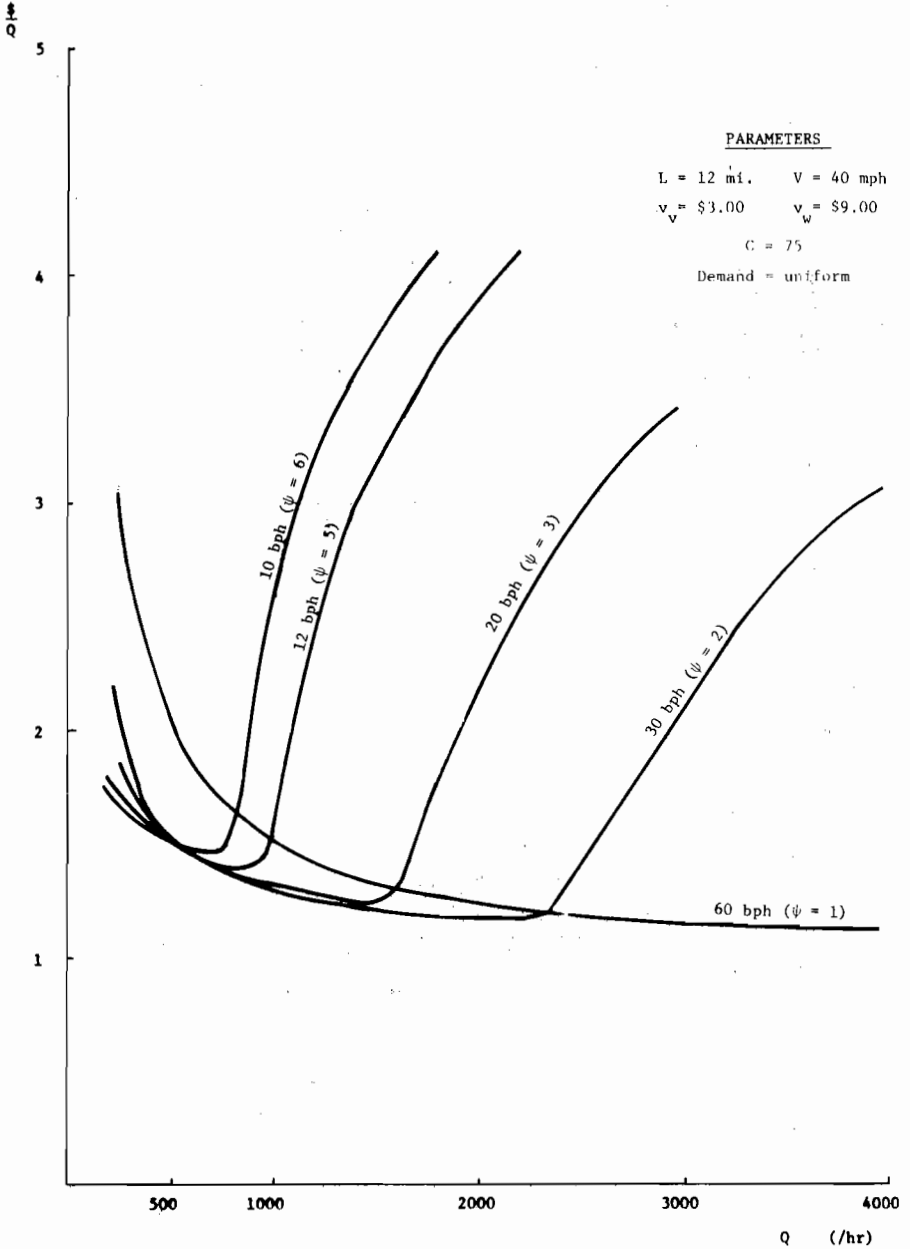


FIGURE 10a

AVERAGE VARIABLE AND FIXED COSTS AS A FUNCTION  
OF FREQUENCY (SYSTEM CAPACITY).  
25-SEAT BUS.

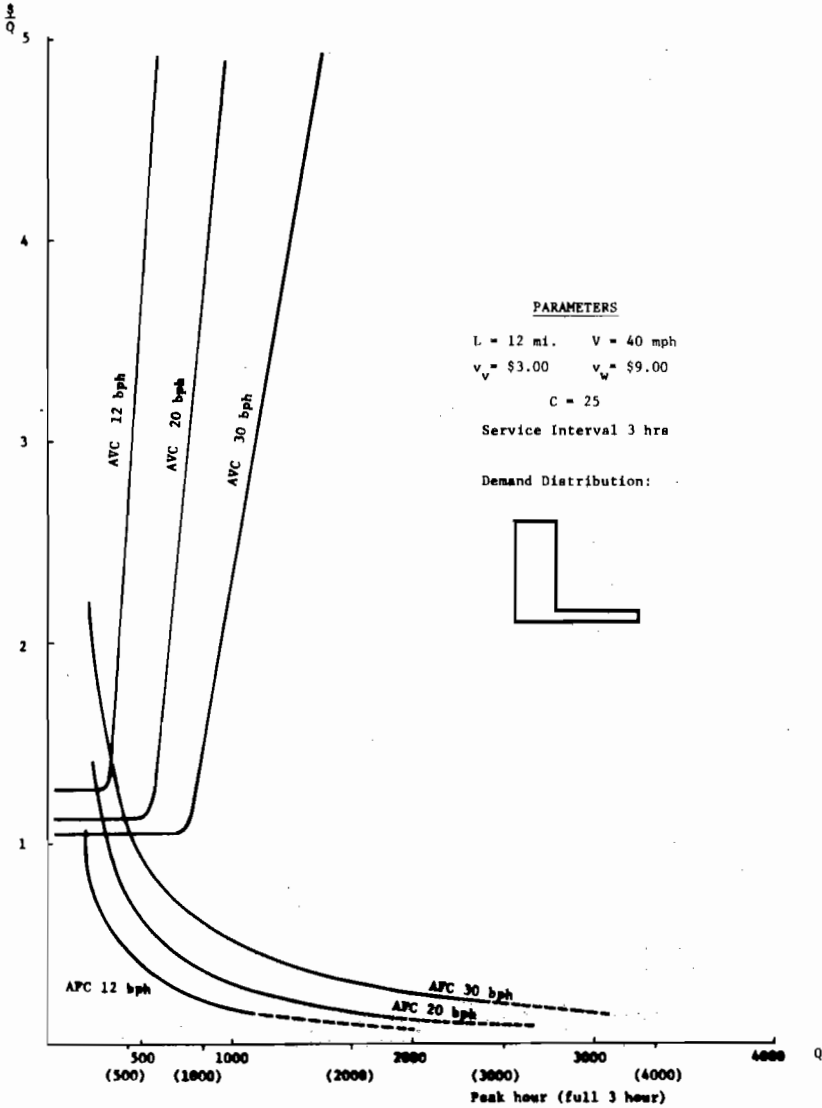


FIGURE 10b

AVERAGE VARIABLE AND FIXED COSTS AS A FUNCTION  
OF FREQUENCY (SYSTEM CAPACITY).

50-SEAT BUS.

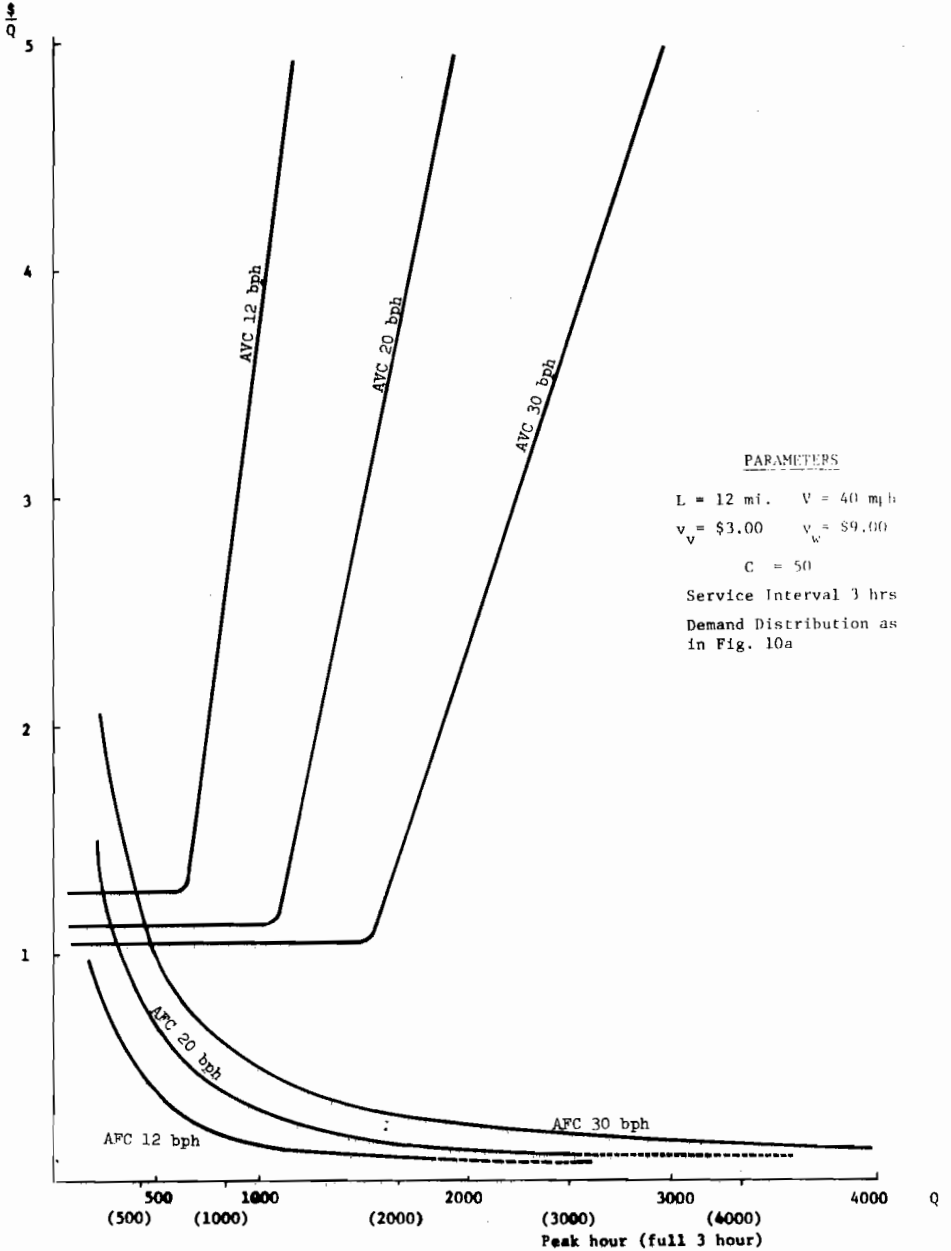
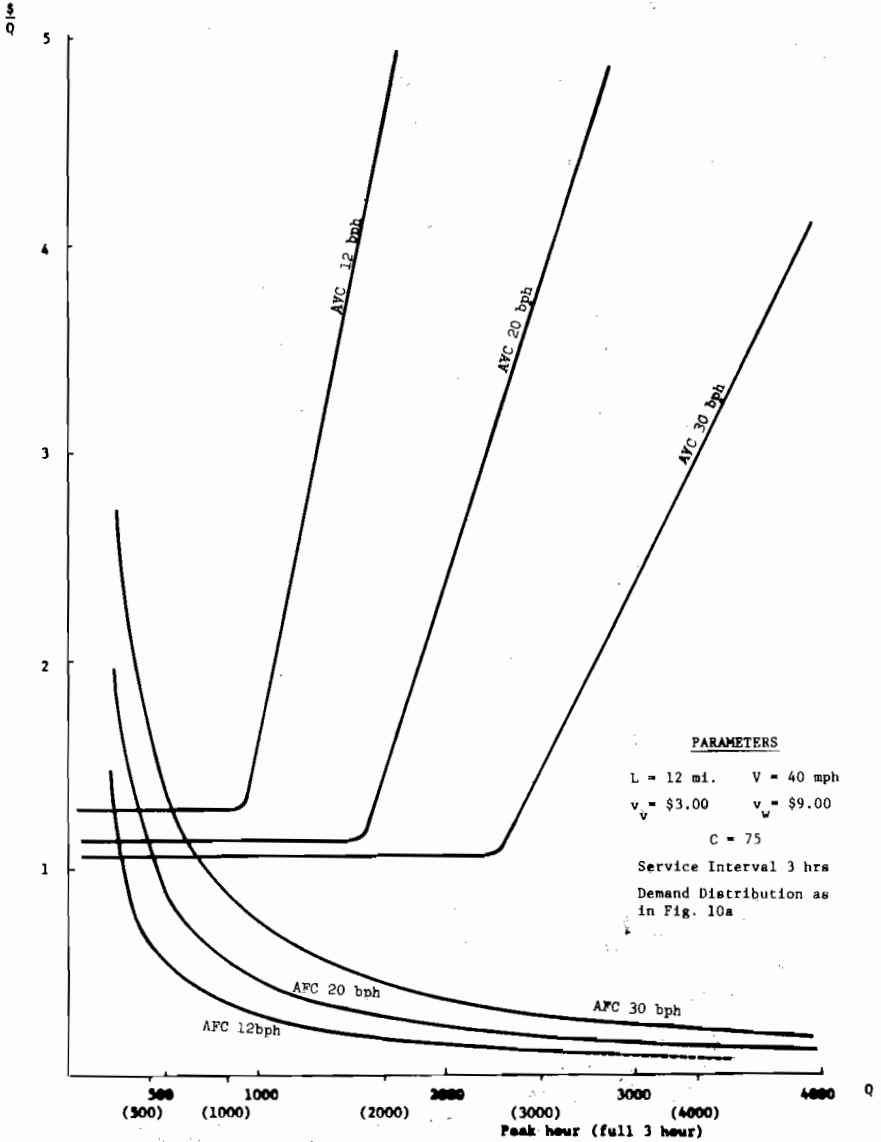


FIGURE 10c

AVERAGE VARIABLE AND FIXED COSTS AS A FUNCTION  
OF FREQUENCY (SYSTEM CAPACITY).  
75-SEAT BUS.



The agency cost AFC is defined according to (19)

$$\text{viz} \quad \text{AFC} = \frac{\text{TDC}}{Q\psi}$$

and continues to decline after missed buses at  $Q_0$ , since peak loading is progressively displaced to off-peak buses with (otherwise) small load factors.

This displacement is reflected by a basically linearly increasing consumer cost AVC, defined by<sup>21</sup>

$$\text{AVC} = v_v \frac{L}{V} + v_w \frac{\psi}{2} + v_w f(Q-Q_0) \quad (29)$$

where  $v_w \frac{\psi}{2}$  is the frequency delay and

$v_w f(Q-Q_0)$  is the stochastic delay, such that

$$f(Q-Q_0) = 0 \text{ for } Q \leq Q_0 = \frac{C}{\psi}$$

The Average Cost is therefore:

$$\text{AC} = \text{AFC} + \text{AVC} = \frac{\text{TDC}}{Q\psi} + v_v \frac{L}{V} + v_w \frac{\psi}{2} + v_w f(Q-Q_0) \quad (30)$$

and the Short Run Marginal Cost is

$$\text{SRMC} = \frac{\partial}{\partial Q} (Q \cdot \text{AC}) = v_v \frac{L}{V} + v_w \frac{\psi}{2} + v_w f(Q-Q_0) + Qv_w f'(Q-Q_0) \quad (31)$$

$$\text{Hence} \quad \text{SRMC} - \text{AVC} = Qv_w f'(Q-Q_0) \quad (32)$$

Thus a demand dependent gap is introduced between Short Run Marginal and Average Variable Costs, because the marginal passenger increases the level of congestion.



The relationship between AVC and SRMC for various frequencies with a standard coach is shown in Figure 11. The curves labelled LRAC and LRMC approximate a "restrained" long run equilibrium derived from the lower bound envelope of average costs for the three hour period. Notice how steeply the SRMC rises after the constraint is encountered.<sup>22</sup> This reflects the cascade nature of the congestion process (for the marginal rider produces substantial spill-over costs) and the high price of waiting ( $v_w = \$9/\text{hour}$ ). Each frequency is optimal at a level of demand where  $\text{SRMC} = \text{LRMC}$ , that is where long run equilibrium as well as short run equilibrium has been achieved. If this occurs after the demand  $Q_0$ , the best tradeoff is constrained. For example, the frequencies of 20 and 30 buses per hour, are optimal and constrained at demand levels of 1000 and 1500 patrons (per peak hour) respectively. The relatively low frequency of 12 buses per hour is optimal and unconstrained at 600 patrons per peak hour.

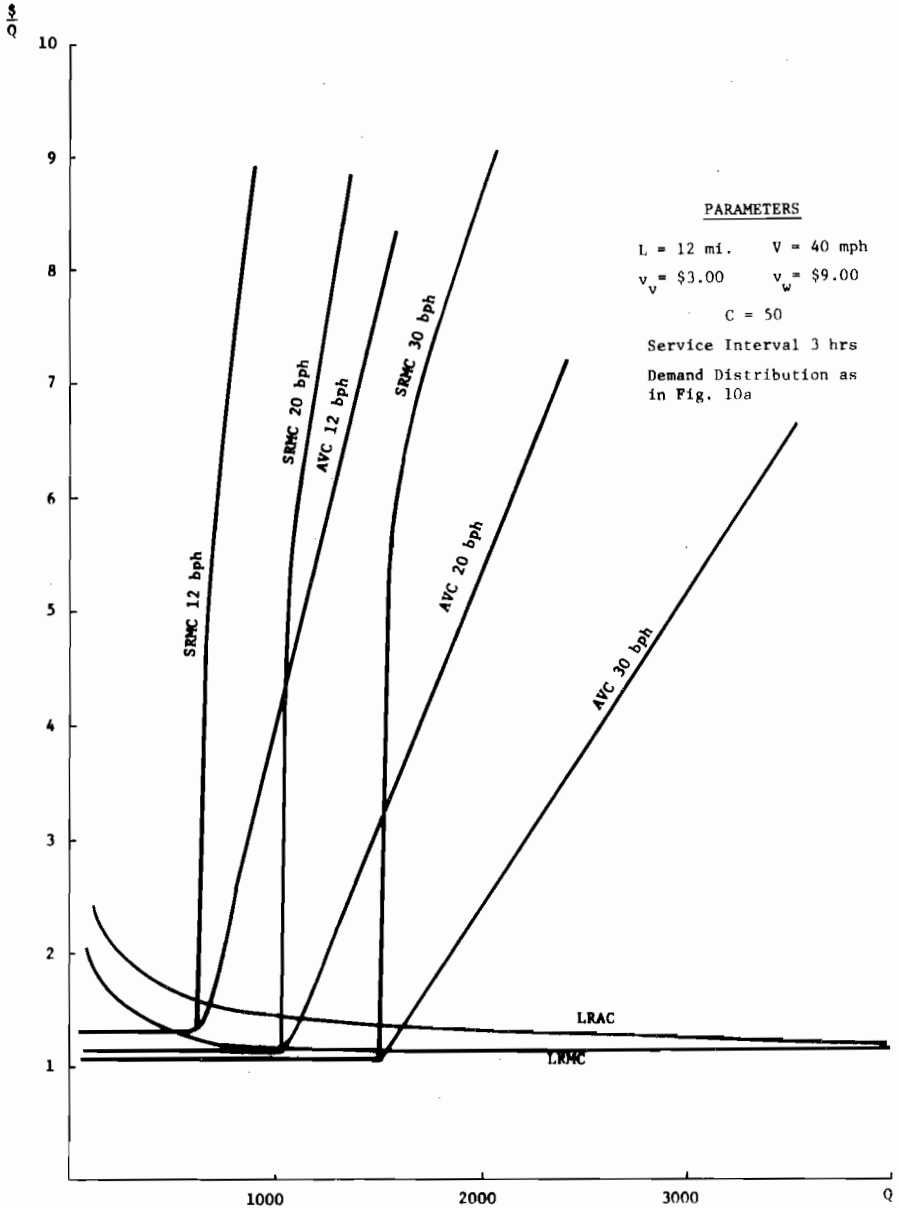
#### Marginal Cost Pricing - Congestion Tolls

Three different pricing systems (Wohl 1970) are applicable to bus transit operations:

- (a) Uniform Daily Fare: This is a commonly used money charge added to the user's time and effort inputs. The user cost is then roughly comparable to the Average Variable Cost
- (b) Zero fare: no money charge
- (c) Marginal Cost Fare: This money charge, when added to user's time and effort inputs, makes the price of a trip equal to the Short Run Marginal Cost. Since Short Run Marginal Cost varies with the level of demand, and hence with congestion, this is also referred to as "Congestion Toll Pricing."

FIGURE 11

SHORT AND LONG RUN COSTS OF FIGURE 10b AS A  
FUNCTION OF LEVEL OF DEMAND.



In equation (29) and Figure 11 the Average Variable Cost is the user's time and effort input, hence the Marginal Cost Fare or congestion toll, is simply the gap between SRMC and AVC as given by (32). This is similar to Mohring's (1972) example, where the congestion toll corresponds to load and discharge delay. This affects schedule delay by reducing frequency; a process Mohring describes as a "system effect." Load and discharge delay is incorporated in Section 3.4; in the present case the congestion toll corresponds only to stochastic delay.

This toll is designed to make the user pay for the costs he inflicts on the system, that is, by increasing the general level of stochastic delay.<sup>23</sup> Marginal Cost Pricing is therefore an efficiency measure that formally incorporates the externalities ("unperceived costs")<sup>24</sup> that result from an individual's travel, into his cost reckoning process. Therefore it tends to produce a more efficient (even) distribution of demand, e.g., smooths out the load factors.

If equipment and/or labor (Goldstein, 1974) restraints<sup>25</sup> do not permit a company to provide a level of service during periods of peak demand comparable to the optimum frequency, then, as is clear from Figure 11, considerable stochastic delay tolls need to be levied to induce the marginal passenger to forego his travel or displace it to another period. In this case the tolls are generally well in excess of the average fixed and variable costs. They can be put to use in many ways, for example, used positively as a "bonus" to induce some passengers to another period (Vickrey, 1967) (Hedges, 1969) or, within the framework of this analysis where demand is a fixed input, used to expand the fleet size and/or labor pool and implement optimal scheduling.

Under peak conditions a small degree of stochastic delay is favored at the optimum because the spill-over increases the otherwise low load factors of the adjoining slack period. However the toll is now very small (of the order of a uniform daily fare) and could go toward covering operating and maintenance costs (AFC). When demand is slack, so that the capacity constraint of the system is not encountered (there is no congestion) the SRMC and AVC are equivalent, being simply the user's time and effort inputs. Hence (32)  $SRMC - AVC = 0$ , and no fare should be charged. (Kraft, 1973; Kemp, 1974). For example, in Figure 11, with a slack demand ( $Q = 600$ ) the toll is zero. Whereas at demand levels of  $Q = 1000$  and  $1500$  corresponding to rush hour conditions, fares of the order of  $10\text{¢}$  and  $18\text{¢}$  (per peak hour patron) should be charged.

Congestion tolls corresponding to (32) as a function of capacity and frequency are shown in Figures 12(a), (b), and (c). Note how sharply these increase with excess demand  $Q - Q_0$ , particularly for the smaller vehicle at low frequencies. Thus suppose a bus company has a comfortable surplus of labor but equipment just adequate to provide optimal scheduling to meet an existing peak hour demand of 1000 patrons, that is, it runs 20 buses per hour. If demand should suddenly increase to 1500 patrons, then stochastic delay will occur. From Figure 12(b), the company would need to levy a toll of \$7 per head, or, charging only rush hour passengers, a toll of \$8.40 per head, which would raise \$12,600 per peak hour. To restore optimal scheduling an additional 10 buses per hour would be required, at a capital cost of  $\$10 \times 40,740 = \$407,740$ . Hence the number of levied peak hours to raise this capital would be  $407,740 \div 12,600 \approx 32$ . In practice, fleet additions could be progressive, and, with each addition, the toll lowered. These monies could alternatively be used to alleviate labor restraints if these were limiting service level.

FIGURE 12a

SHORT RUN CONGESTION TOLLS AS A FUNCTION OF  
FREQUENCY (SYSTEM CAPACITY).

25-SEAT BUS

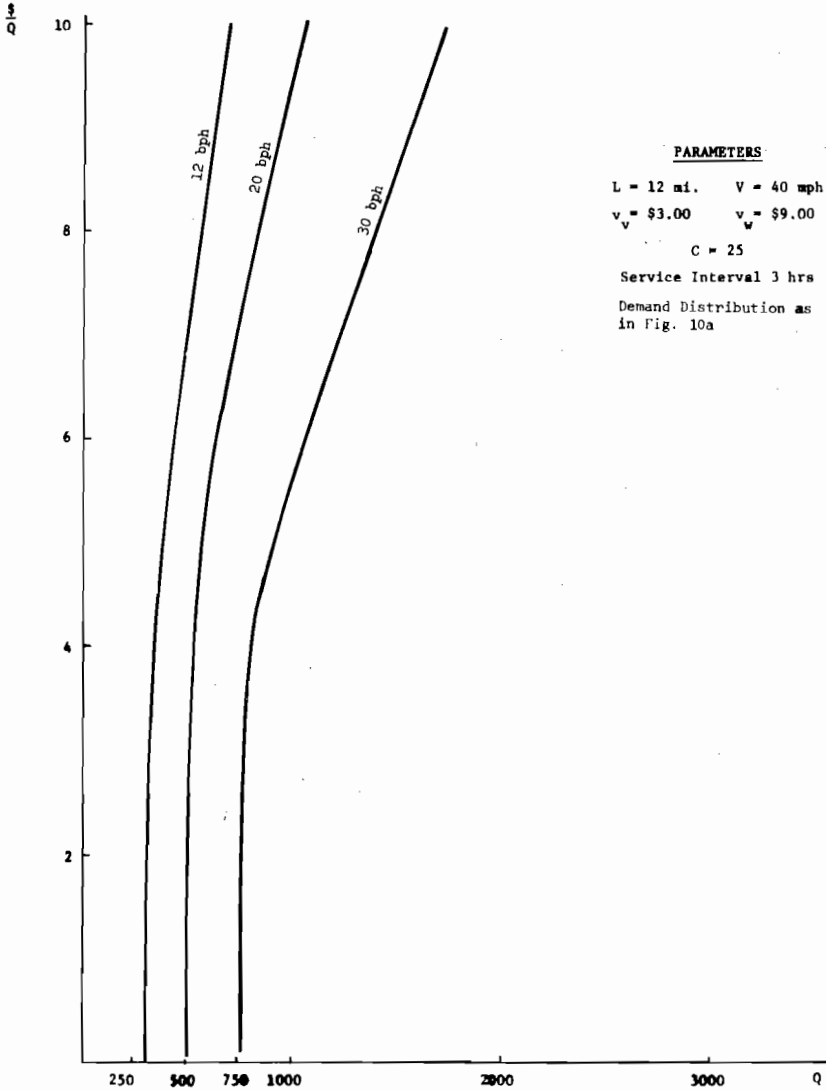
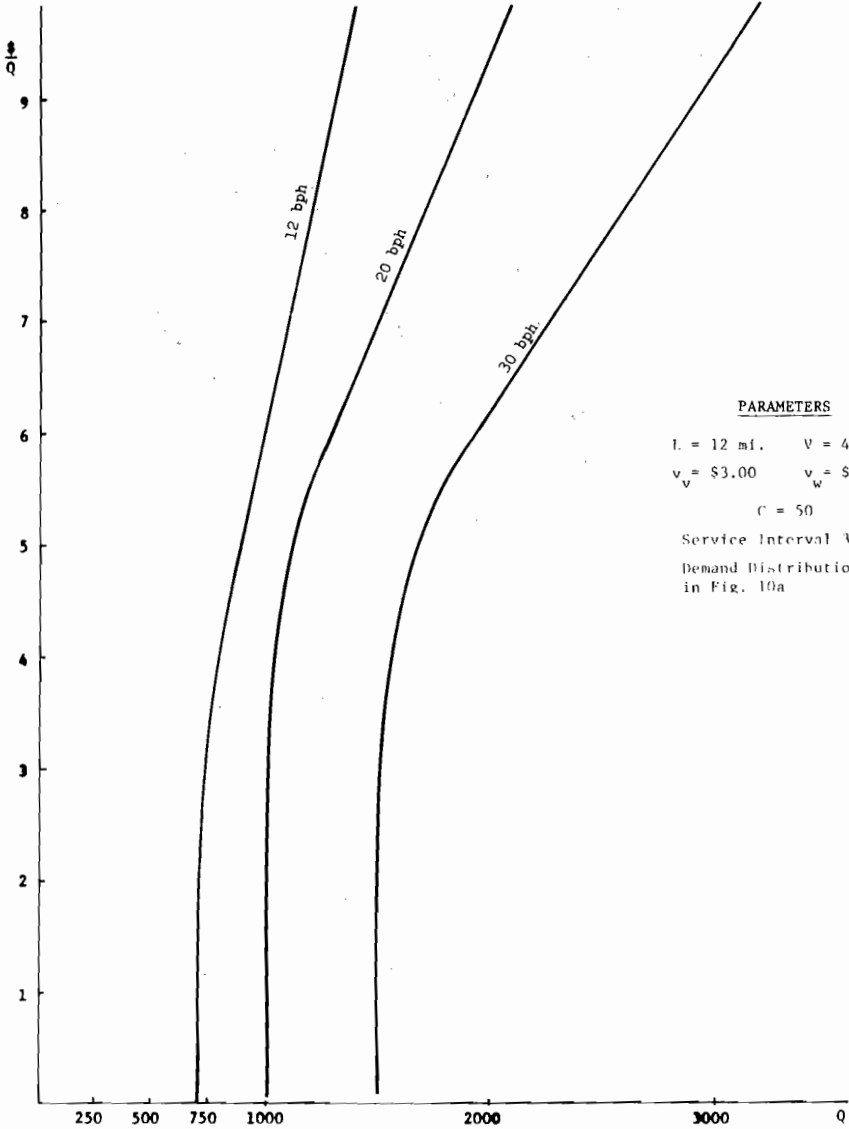


FIGURE 12b

SHORT RUN CONGESTION TOLLS AS A FUNCTION OF  
FREQUENCY (SYSTEM CAPACITY).

50-SEAT BUS.



PARAMETERS

$l = 12$  mi.       $v = 40$  mph

$v_v = \$3.00$        $v_w = \$9.00$

$c = 50$

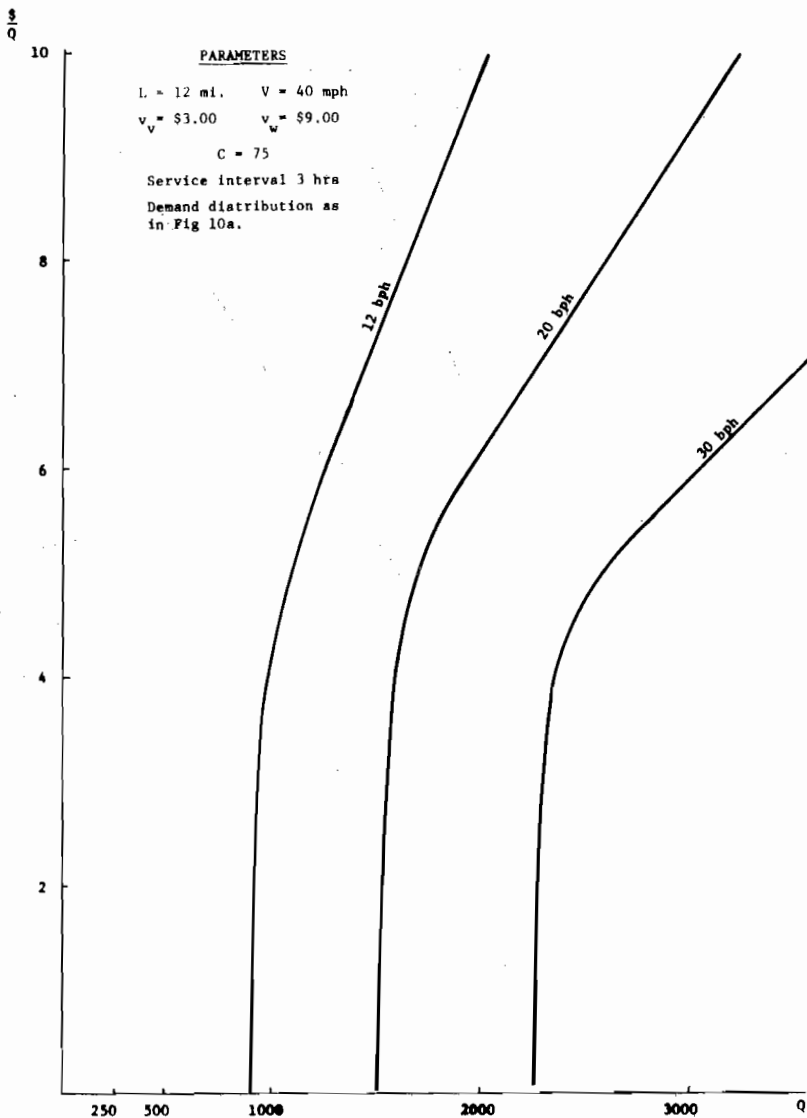
Service Interval 3 hrs

Demand Distribution as  
in Fig. 10a

FIGURE 12c

SHORT RUN CONGESTION TOLLS AS A FUNCTION OF  
FREQUENCY (SYSTEM CAPACITY).

75-SEAT BUS.



A further example of fleet supplementation by congestion tolling is given in Table 3. It may seem unreasonable to charge a congestion toll of \$8 per person. This is because we are unaccustomed to thinking in terms of an explicit costing of time and effort. The dollar value given to  $v_w$  (\$9 per hour) represents in large part the ardour of waiting.

For a given service frequency, increasing demand (in the presence of stochastic delay) means that the average patron tends to wait longer and longer as fully loaded departing buses make less and less impact on the queue he faces. The fares levied in Table 3, mean that the (SRMC) price paid (time and effort plus congestion toll) is well above the price of the cheapest cost service for that level of demand when the toll would be quite small. The high toll deters patrons (an alternative activity valued at up to  $\$v_v$  per hour can be found) or in the context of this analysis, permits a rapid expansion of the fleet.

The tolls of Table 3, apart from being technically difficult to levy, would be politically untenable and inequitable (Vickrey, 1955, Abe, 1973, etc.). Further, without marginal cost pricing for all modes they would most likely lead to a worse than second best outcome (Lipsey and Lancaster, 1956).

If users of a substitute mode, like car drivers on a congested highway, are not levied a toll to also equate the price of their travel to the marginal cost, then bus passengers will be "tolled off" the system to automobiles, leading to an inefficient allocation of transport resources. The situation is complicated by the fact that buses and automobiles usually share a common right-of-way so that there is an interdependence of the congestion experienced by both modes. But this



TABLE 3

## EXPANSION OF FLEET THROUGH CONGESTION TOLLING

Peak Hour Demand Level	2,000	2,000	2,000	2,000
Capacity of Single Vehicle of Fleet	50	50	75	75
Max (Peak Hour) Frequency with Existing Fleet	30	20	20	12
Short Run Toll/Patron	\$ 6.16	\$ 9.40	\$ 6.16	\$ 9.80
Short Run Toll/Patron (levying peak hour patrons only)	\$ 7.39	\$ 11.28	\$ 7.39	\$ 11.76
Net Toll Monies per Peak Hour	\$14,780	\$22,560	\$14,780	\$23,520
Additional Buses Required to minimize stochastic delay	10	20	7	15
Capital Cost	\$407,740	\$815,480	\$577,500	\$1,237,500
Number of Peak Hours SR Toll levied, to Purchase Equipment	28	36	39	52

is generated **disproportionately**, as the marginal automobile user makes a far greater contribution to highway congestion than the marginal bus user.

Automobile users are generally charged a price equivalent to average variable cost, and in these circumstances, Sherman (1971) finds that with perfect (and perhaps imperfect) substitutability between modes (car and bus) the bus fare should be such that the price is below average variable cost (which in terms of our analysis implies zero fare) requiring a subsidy. However if a marginal pricing regime is implemented, and the bulk of highway congestion attributed to the automobile user, then the congestion toll for a 12 mile trip with optimum flow conditions, is about \$2 per car (Keeler, 1974) which is an order of magnitude larger than the comparable rush hour fare indicated in Figure 11 for the (competing) bus mode operating optimally.

### Long Run Equilibrium

It is contended in Section 4, that the envelopes formed by the average cost curves Figures 9(a), (b), and (c), comprise estimates of a "restrained" long run average cost: each envelope corresponding to the long run equilibrium of a (single route) "firm" whose operation is restricted to one size vehicle. These envelopes are shown in Figure 13. Notice that the long run average costs do not vary significantly between firms (vehicle sizes): there is virtually no difference between the 25 and 50 seat vehicles in small passenger markets. This stems from a disproportionately low capital cost of the standard coach as a result of mass production scale economies.

### Derivation of Long Run Marginal Cost and Operating Subsidies

Given a firm operating buses of capacity  $C$ , then for demand  $Q \leq \frac{C^2 v_w}{2TDC}$ , its LRAC is described by (23); otherwise the minimum average cost is constrained and its LRAC is given by (27).

The Long Run Marginal Cost corresponding to (23) is

$$LRMC = \frac{\partial}{\partial Q} (Q \cdot LRAC) = \frac{1}{2} [2TDC v_w]^{1/2} Q^{-1/2} + v_v \frac{L}{V} \quad (33)$$

Inspection of (23) reveals that the agency and user components are equal, hence (33) is the sum of the agency and in-vehicle user costs.

Thus from (23) and (33) we have

$$LRAC - LRMC = \left[ \frac{TDC \cdot v_w}{2} \right]^{1/2} Q^{-1/2} \quad (34)$$

which is the user wait cost.

Similarly when LRAC is a constrained minimum given by (27) then

$$\text{LRMC} = \frac{\partial}{\partial Q} (Q \cdot \text{LRAC}) = \frac{\text{TDC}}{C} + v_v \frac{L}{V} \quad (35)$$

Hence from (27) and (35) we have

$$\text{LRAC} - \text{LRMC} = \frac{v_w}{2} \frac{C}{Q} \quad (36)$$

which is again the user wait cost.

Thus the gap between LRAC and LRMC is equivalent to the cost of the average user's wait (average schedule delay).

A given frequency  $1/\psi$  is optimal at some demand level  $Q$  where  $\text{SRMC} = \text{LRMC}$ . Equating (21) and (33) or (21) and (35), this is seen to correspond to matching the agency cost with the average user's wait cost. Therefore the gap between LRAC and LRMC, (34) and (36) is also equivalent to the agency cost.

The extent to which the money component of LRMC (equal to the toll portion of SRMC [32]), falls short of the money costs of providing service (LRAC) is made up by a subsidy which is equal to the Agency Cost (AFC). This subsidy frees the company from charging to match operating costs, and allows implementation of efficient pricing. Alternatively, if the transit system is going to be operated anyway, there is no need to charge the user any more than the costs (externalities) he or she creates, and the deficit can then be made up by a subsidy.

In Figure 11, these subsidies are:

<u>Q</u>	<u>Subsidy/head</u>
600	\$0.40
1000	\$0.28
1500	\$0.16

Note the presence of significant scale economies.

The relationship between subsidy and capacity is shown by Figure 13. As is clear from (34) and (36) the larger the bus, the higher the subsidy (agency cost).

For example, at a demand of 4000 patrons per hour, these are:

<u>Capacity</u>	<u>Subsidy/head</u>	<u>LRAC</u>	<u>Proportion of LRAC</u>
25	\$0.02	\$1.17	2%
50	\$0.04	\$1.08	4%
75	\$0.08	\$1.12	8%

Although the subsidy per head for a 25 seat vehicle is only half that for a 50 seat vehicle the LRAC is about 10% more expensive and the congestion toll (fare) would be higher. Note that the 75 seat bus remains more expensive than a 50 seat vehicle, even at a demand level of 4000. These and other issues are discussed in Section 4.

### 3.5 Multiple Frequency Bus Service with Capacity Constraints and Multiple Stops

The foregoing analyses have been restricted to point-to-point linehaul conditions, where load and discharge delays do not affect in-vehicle time. Extending the analysis to include collection at more than one stop permits a fuller examination of real-world congestion effects, and provides a foundation for the integrated service considerations in Section 5.

#### Estimation of Average Cost with Multiple Stops and Capacity Constraint (Algorithm SRAC3)

To derive average cost when there are intermediate stops, we need to estimate cost components associated with the average journey. Both the agency cost (CC) and in-vehicle time and effort cost (CV) are affected by the distribution and number of collection points  $S$  along the route. It is shown in Appendix A-3 that these are given by:



$$CC = \frac{1}{Q} \sum_{n=1}^S \frac{1/\psi}{\sum_{s=1}^S} TDC(n,s) \quad (37)$$

and

$$CV = \frac{v_v}{Q} \sum_{n=1}^S \frac{1/\psi}{\sum_{s=1}^S} q_{n,s'} \frac{\Delta L_s}{v_{n,s}} \quad (38)$$

where:

TDC (n,s) is the Total Direct Cost for the nth bus between the sth and s-lth stops

$v_{n,s}$  is the mean traverse speed (stationary to stationary) between the sth and s-lth stops,

and  $\Delta L_s$  is the distance between these stops.

The average schedule delay is:

$$\bar{d} = \frac{1}{Q} \sum_{n=1}^S \frac{1/\psi}{\sum_{s'=1}^S} q_{n,s'} d_{n,s'} \quad (39)$$

where  $d_{n,s'}$  is the schedule delay incurred for the nth bus at the s'th stop.

Algorithm SRAC3 is an extension of SRAC2, based on (37), (38), and (39), and includes the capacity constraint.<sup>26</sup> Corrections to the service speed through loading, acceleration and braking have also been incorporated.<sup>27</sup>

Loading (and discharge) delays affect the in-vehicle cost (38) (Mohring's "own bus effect") and to a lesser extent agency cost (37), and are present at all levels of demand, so that SRMC is in excess of AVC even before encountering the capacity constraint.<sup>28</sup> However calculations with SRAC3 confirm that this congestion is very small in the presence of slack demand, so that the toll is minimal with respect to the peak, when stochastic

congestion is superimposed. Loading congestion also imposes a constraint on the frequency of service with a fixed fleet (Mohring's "system effect") affecting the level of stochastic delay congestion when fleet size is insufficient for optimal scheduling. Further research into the magnitude of these processes is proposed with SRAC3.

#### Minimum Average Cost as a Function of Stopping Frequency

Figure (14a) shows the minimum average cost and its underlying components, for a standard bus in shuttle service with a demand level of 250 patrons per hour. It is assumed that the latter are evenly distributed over  $S$  equi-distant stops, and ride to the termination point.

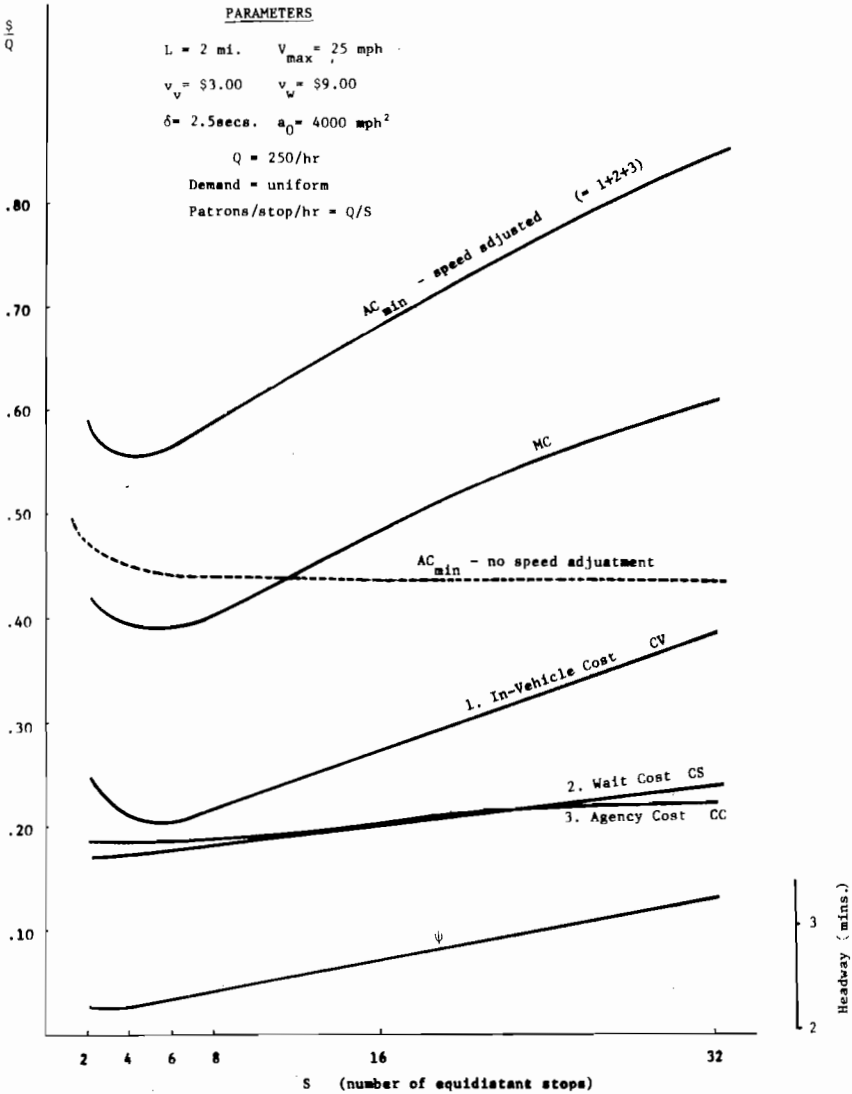
The minimum average cost with no speed adjustment is also included. Notice that this declines rapidly at first, because average in-vehicle time is lower if passengers can board closer to their destination. However when delays through braking, loading, and acceleration, are included, this rises again because with increased stopping frequency, service speed is reduced. For a given headway, increasing the number of stops increases the agency cost because the total direct cost for the journey of the bus is borne by a smaller complement of passengers, also the time taken to cover the route is longer. Therefore the best cost trade-off is at a longer headway when the agency component is lower. That is, increasing the schedule delay effects a higher load factor and a higher subsidy is required.

Figure (14a) indicates an optimum at about 4 stops over 2 miles, or about one every half mile. A more complete consideration should make a tradeoff with link (walk) costs: for example by assuming a demand catchment of one quarter of a mile width on each side of the route we



FIGURE 14a

RELATIONSHIP OF MINIMUM AVERAGE COST AND ITS COMPONENTS  
TO STOPPING FREQUENCY FOR A SHUTTLE SERVICE.



have estimated the mean link cost (in terms of walk at \$9/hour) as a function of the number of stops. The cost tradeoff is made in Figure (14b), and shows an optimal spacing of about one stop every  $1/4$  mile.

Repeating the analysis for conditions approximating linehaul on a freeway, Figure (15), a stop spacing anywhere between  $5/8$  and 1 mile is sufficient. However collection points (ingress/egress ramps) are unlikely to be closer than a mile, and in any case it is better to stop the bus as little as possible for reasons of safety and rider comfort, so the one mile spacing is best. This result would also apply to kiss-and-ride and park-and-ride if the same catchment were assumed.

### 3.6 Backhauling

There has been an implicit assumption throughout this section that the demand per route is the same irrespective of the direction of the trip. While this is generally true of the off-peak, in the peak the demand is usually much greater in one direction than the other, so that there is an imbalance in bus requirements.

There are three possible responses to this problem, any of which may be exercised in concert: first, those buses not needed again may be stored in the company yards: and, assuming a perfectly amenable union, the drivers temporarily discharged. In practice, labor contracts often have drivers following private pursuits on company time. There is clearly a cost here, and one which should be borne by the peak travelers. The second response is to resort to "deadhauling," that is return some of the buses additional to those in regular (revenue earning) backhaul by running empty over the fastest return path. The brevity of the peak imposes a restraint on the effectiveness of this recycling, which some

FIGURE 14b

TRADE-OFF BETWEEN MINIMUM AVERAGE COST OF FIGURE 14a  
AND SPATIALLY RELATED USER COST (WALK).

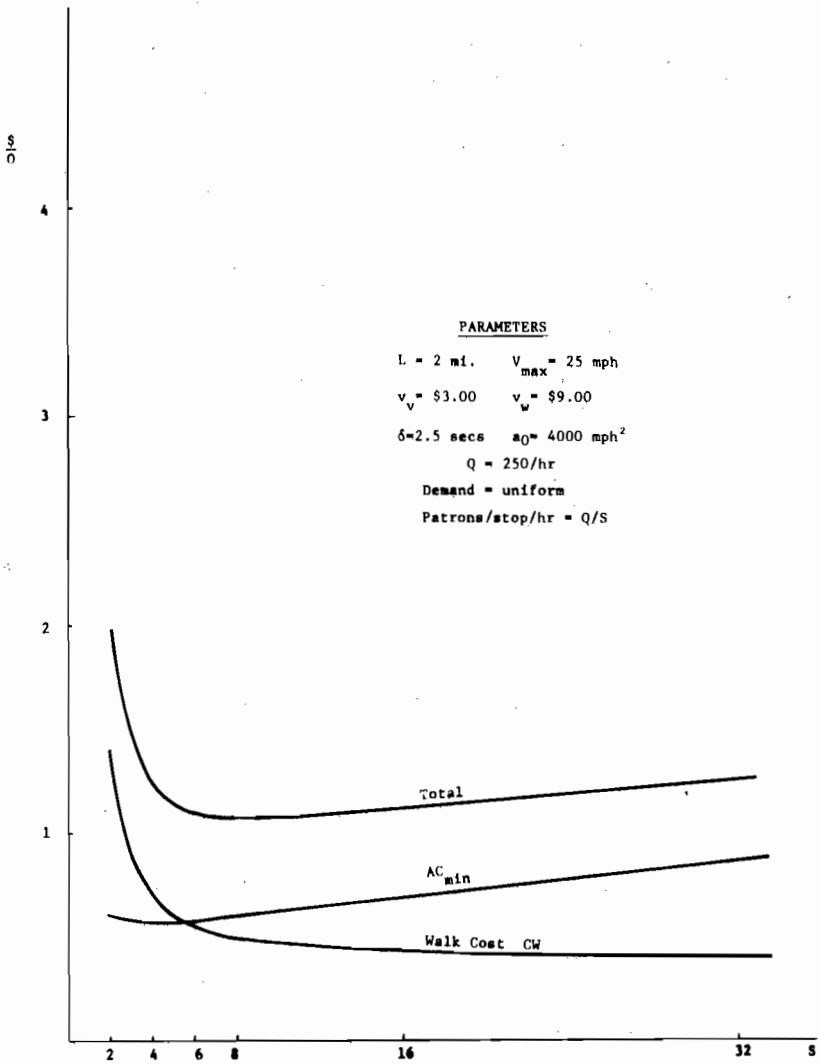
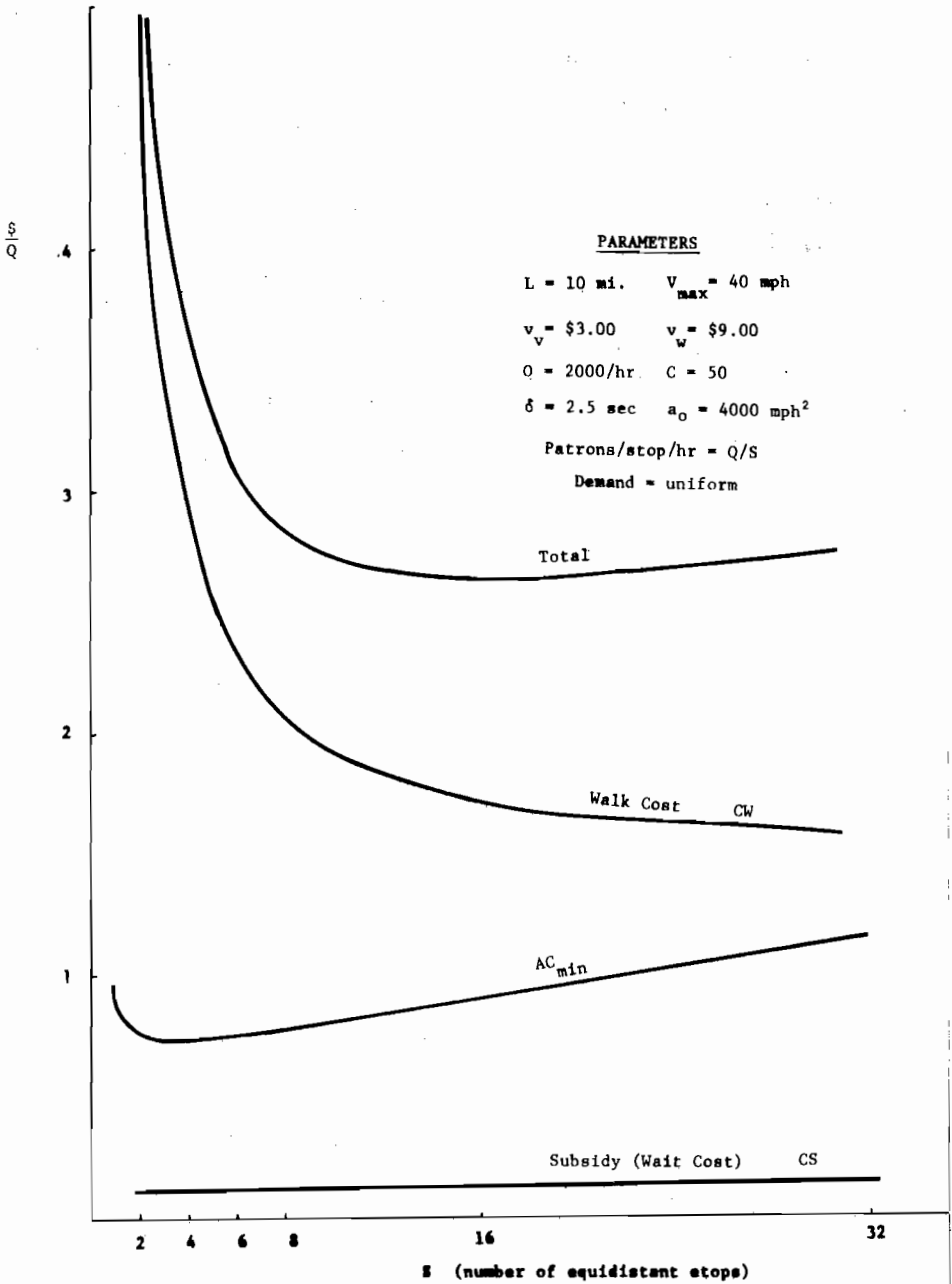


FIGURE 15

TRADE-OFF BETWEEN MINIMUM AVERAGE COST AND SPATIALLY RELATED USER COST FOR LINEHAUL SERVICE.



companies partly overcome by combining deadhauling with substitution of buses between routes. Again, the costs should be allocated to the peak users. The third response is to use the buses in charter service, although the opportunities here are limited. The work of this section suggests another strategy: converting this excess capacity into an improved off-peak service, hence providing optimal scheduling.

Since we have not formally analysed the peak-off peak cost problem, patronage has been assumed comparable in both directions. However, the inclusion of different inward and outward bound levels of demand is theoretically simple, but would considerably complicate our answers by adding another dimension to them: for each inward demand level  $Q$ , there would be an associated outward demand level  $Q^*$ , and optimal scheduling would be effected on the basis of both.

#### 4.0 Long Run Equilibria and System Capacity

In economic theory the long run is distinguished from the short run by the variability of all factors of production. While in the short run some factors are fixed, in the long run all factor inputs can be varied. In the case of bus operations both frequency  $1/\psi$  and vehicle size  $C$  can be varied to adjust system capacity  $C/\psi$  to the long term demand  $Q$ . In reality, increases to the frequency or number of buses available to run service are principally made through purchasing buses (and labor), rather than the minor economies that result from adjusting service speed  $V$  (which apart from manipulating stop frequency is virtually endogenous) and route length  $L$  to increase turn-around.

In section 3.4 we considered a special version of the distinction between long and short run operation: the firm could, in the long run, expand its capital stock (system capacity) by variation of frequency through the acquisition of more buses of the same size. However it clearly makes sense to also allow for the expansion of system capacity by buying buses of different sizes. In section 2 we developed agency cost functions for three size buses: the "standard" 50-seat bus; the 25-seat version of the Haddonfield 17-seat bus; and the 75-seat articulated bus.

#### 4.1 A Parametric Long Run Cost Relationship

Consider an industry composed of efficient bus companies, each of which runs point-to-point line haul service over a single route. The  $j^{\text{th}}$  company operates a route of length  $L_j$  at a service speed of  $V_j$ ; the demand which must be met is  $Q_j$ , and its patrons have time and effort shadow prices of  $v_{vj}$  and  $v_{wj}$ . Suppose that it operates buses of size  $C_j$ , thus determining the agency cost TDC  $[L_j, V_j]$ . If passengers arrive at the starting point

randomly over the service interval, then the optimal headway is given by (22) or, if the capacity constraint is encountered, by (26). The corresponding minimum average costs are given by (23) or (27). Each firm can increase its system capacity only by acquiring buses of the same size.

As the various parameters for each company vary, we have an industry operating in the long run in the sense that all factors, including bus size, are variable. Then if we assume that the cost differences among firms varying only in system capacity are attributable to a random variable, "entrepreneurship," then we might model the industry long-run cost function by an equation of the Cobb-Douglas type:

$$\text{LRAC} = a Q_w^\alpha V_w^\beta V_v^\gamma L^\delta V_v^\epsilon.$$

Since we do not actually observe the industry just sketched, we proceed by simulation of the data required. Each of the parameters is allowed to vary over the three values noted in table 4. With six parameters each varied three ways, this is equivalent to observations upon  $3^6$  (=792) mono-route companies. The minimum average cost is calculated from equations (23) or (26) using LRAC1, and the Cobb-Douglas-type cost function fitted by ordinary least squares, since it is linear in the logarithms of the variables.

viz:

$$\text{LRAC} = 2.2155 Q_w^{-.1087} V_w^{.6940} V_v^{.1187} L^{.8776} V_v^{-.7963} \quad (40)$$

The coefficients of this equation minimize the sum of the squared differences between the actual and calculated values. Given that the  $R^2$  value is .9893, it is apparent that the fit is excellent. Further, if the assumption is made that the random variables of "entrepreneurship" are normally distributed with zero mean and variance  $\sigma^2$ , and that the covariance of "entrepreneurship" among different firms is zero, then it

TABLE 4

<u>Parameter</u>	<u>Values</u>		
Q (persons)	500	1000	2500
$v_w$ (\$)	5.00	9.00	12.00
$v_v$ (\$)	1.00	2.00	4.00
C (seats)	25	50	75
L (miles)	5.0	10.0	20.0
V (mph)	10.0	20.0	40.0



can be shown that the coefficients are all estimated very accurately. That is, the ratio of the coefficients to their estimated standard errors exceeds the value of a t-distributed variate at the 99% level of statistical significance.

We can now estimate the Long Run Marginal Cost:

$$\begin{aligned} \text{LRMC} &= \frac{\partial}{\partial Q} (\text{LRAC} \cdot Q) = \text{LRAC} + Q \frac{\partial}{\partial Q} \text{LRAC} \\ &= (1 - .1087) \text{LRAC} \\ &= .8913 \text{LRAC}. \end{aligned} \tag{41}$$

This represents a value comparable to averaging the LRMC drawn from (33) and (35) and accords well with Mohring's (1972) observation of long-run increasing returns to scale: for as LRMC is less than LRAC, then LRAC is falling.

Similarly, the subsidy required to provide efficient allocation of resources, by allowing the price of a trip to be set equal to LRMC, is just the difference between LRAC and LRMC:

$$\begin{aligned} \text{Subsidy} &= \text{LRAC} - \text{LRMC} \\ &= \text{LRAC} - .8913 \text{LRAC} \\ &= .1087 \text{LRAC}. \end{aligned} \tag{42}$$

That is, on average, a subsidy of about 10% of the average cost is required.

The various coefficients of the estimated long-run average cost function can be interpreted as elasticities with respect to changes in the inputs. Thus for example, the coefficient of  $-.1087$  for  $Q$  (the number of passengers) indicates that, holding all other factors constant, a 10% increase in passengers will lead to a 1.1% decrease in average costs.

By the same token, a 10% increase in distance travelled, at unchanged speed, will increase average costs by 8.8%.

Note that we are not arguing that the cost function (40) represents the long-run behavior of any existing firm, but simply that if firms were to allocate their fleet according to the optimality rules derived in section 3, and the passengers had a uniform arrival rate with time and effort shadow prices of  $v_v$  and  $v_w$ , then the long-run average cost would be substantially as derived. In this context our approach has the advantage of not pre-judging the question of the values of  $v_v$  and  $v_w$ , which as pointed out in section 2 vary both with socio-economic group and geographic region.

Finally, it should be re-emphasized that we have not differentiated between peak and off-peak service. This is an important deficiency, which centers on difficulties in the measurement rather than methodological arena.

#### 4.2 Linear Efficiency Regions

In sections 3.4 and 4.1 we modelled the individual firm as being, in a sense, "locked in" to the way it could expand its fleet: it could alter the number but not the size of its buses. In practice, there are a number of reasons why this may apply: In the "bus market," manufacturers tend to specialize in, and capture the market for, different size vehicles (e.g., Volvo and M.A.N. supply articulated buses, Twin-Coach and Flexible supply small buses, and G.M. has a virtual monopoly of the "standard" coach). Running a fleet of different makes could lead to problems in the maintenance division: more spare parts would have to be stocked; and preventive maintenance schedules would be more complicated. And since familiarity is an important component of safe driving, there might well be a cost in this area as well.

Nevertheless, many companies do regard a fleet of different-sized vehicles as feasible. The question then arises: suppose we know the magnitudes of the consumer shadow prices  $v_v$  and  $v_w$  and consider a given route of length  $L$  and speed  $V$ . Then for what ranges of (uniformly distributed) demand are the various sizes most efficient -- that is, constitute a least minimum average cost operation?

Given a demand level  $Q$ , the exact answers can be obtained by simply solving equation (23) or (27) for each size. But we are asking a slightly different question: at what  $Q$  will, say, the 25-seat vehicle be as costly to operate as the 50-seat vehicle? To answer this we approximate points of equal cost by an iso-cost function that varies linearly with  $v_w$  and  $Q$ . This is a good approximation, except at very low levels of demand.

Figures 16a, b, c and 17 a, b, c show the regions of cost efficiency. As can be seen, given the level of  $v_w$ , the area in which the 25-seat bus is less costly than the standard bus diminishes steadily as the trip time  $\frac{L}{V}$  (the ratio of distance to speed) rises, whilst the 75-passenger articulated bus becomes more competitive; but only significantly it seems, at ceiling levels of demand.

The impression that the standard coach is most economical over a wide range of trip characteristics, demand, and values of  $v_w$  is reinforced by the thin lines on the figures which distinguish the (linearized) regions where the cost savings with non-standard buses are less than 5¢ per passenger.<sup>29</sup> It is apparent that in most cases the region of efficiency for the 50-seat bus and those where it has less than a 5¢ disadvantage comprehends much of the  $v_w - Q$  plane. This fact might be of interest to transit companies seeking to minimize cost while retaining a maximum of flexibility in operations. That is, to justify purchasing an articulated

FIGURE 16a

RELATIVE COST EFFICIENCY OF BUSES ACCORDING TO SIZE  
HALF MILE POINT-TO-POINT SHUTTLE SERVICE.

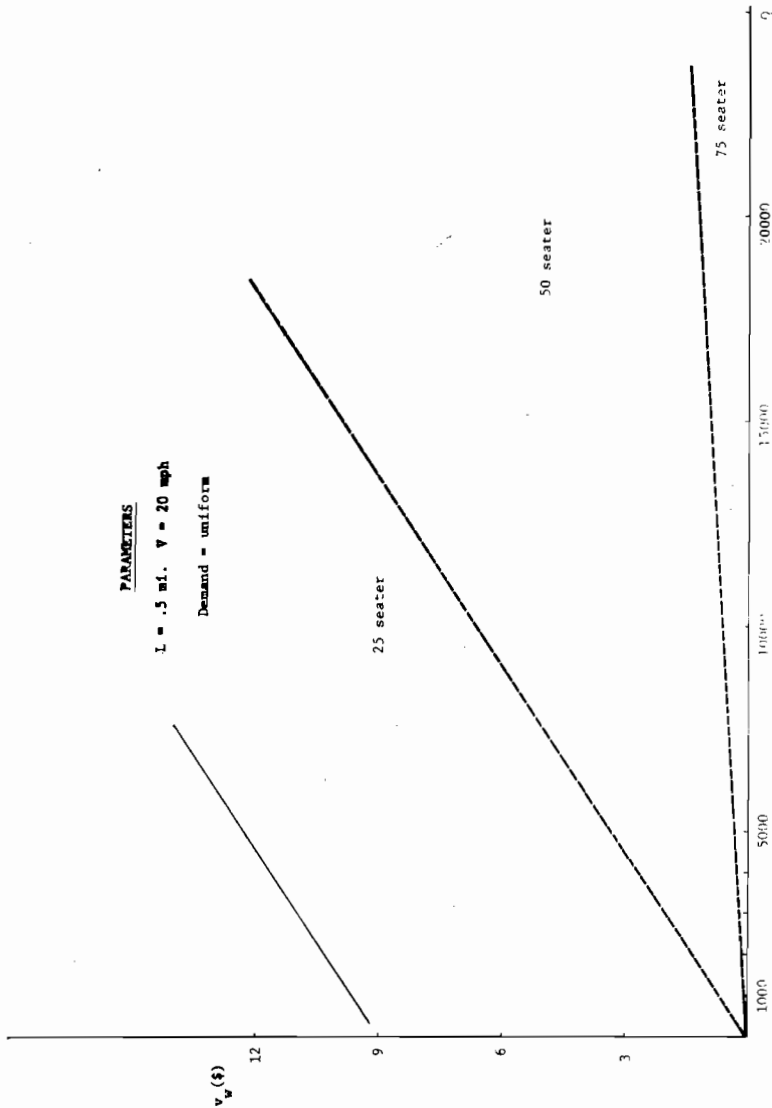


FIGURE 16b

RELATIVE COST EFFICIENCY OF BUSES ACCORDING TO SIZE  
ONE MILE POINT-TO-POINT SHUTTLE SERVICE.

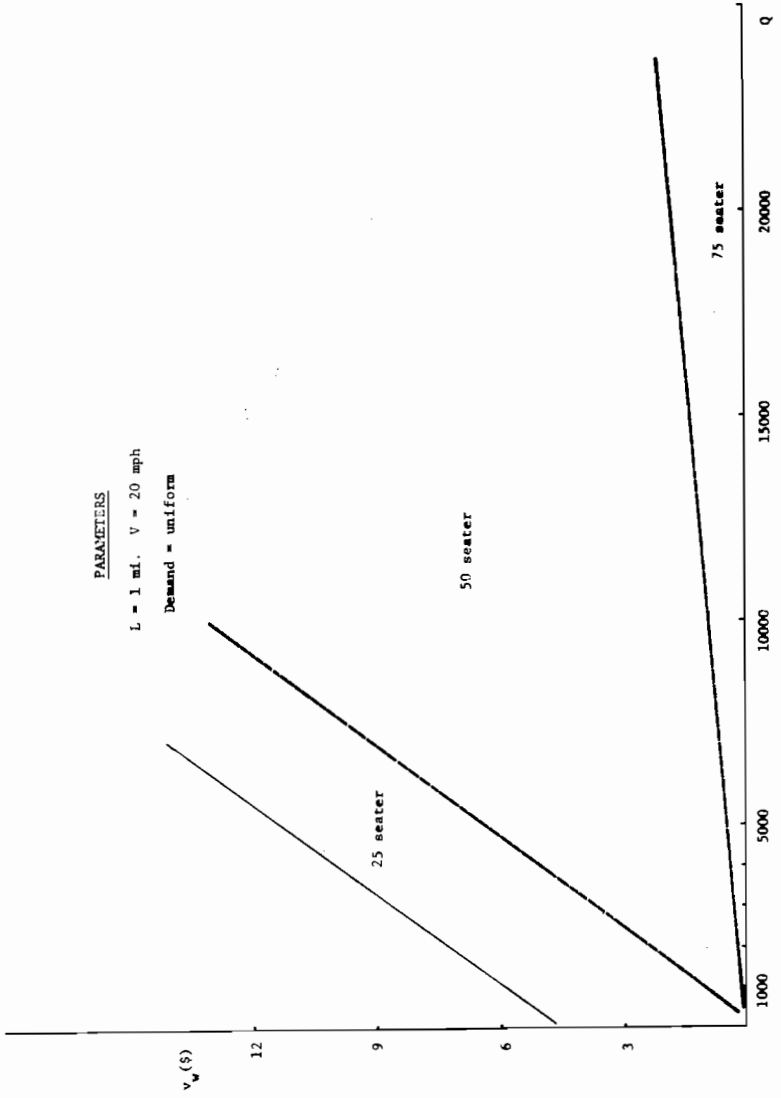


FIGURE 16c

RELATIVE COST EFFICIENCY OF BUSES ACCORDING TO SIZE  
TWO MILE POINT-TO-POINT SHUTTLE SERVICE.

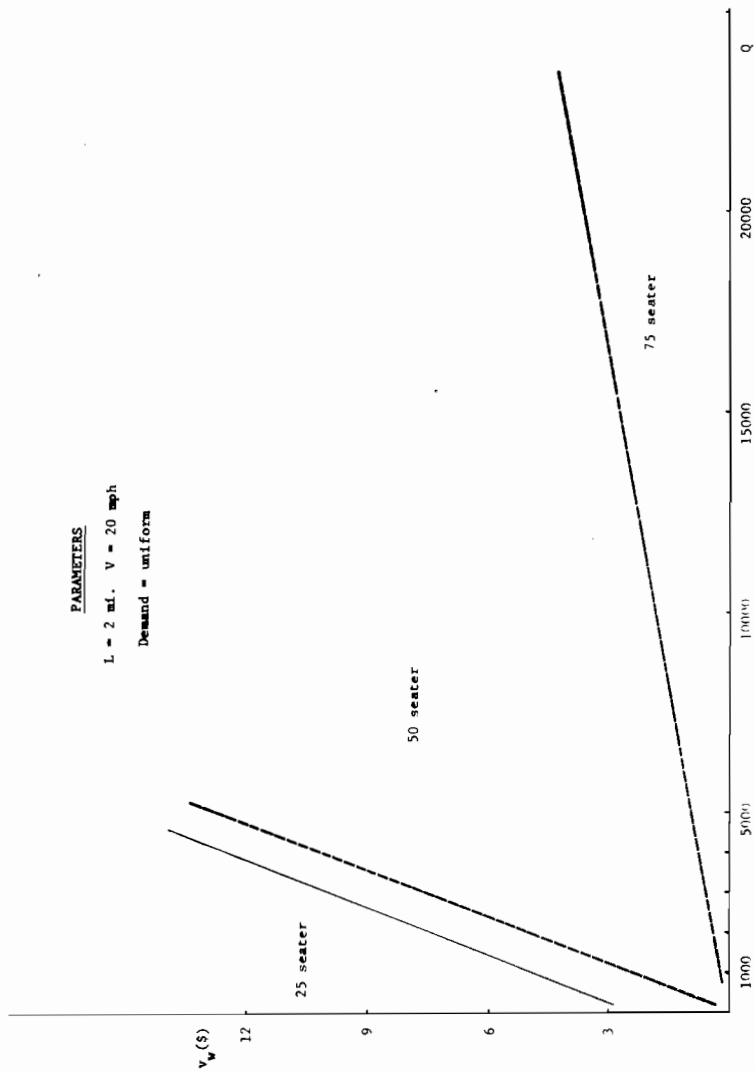


FIGURE 17a

RELATIVE COST EFFICIENCY OF BUSES ACCORDING TO SIZE  
SIX MILE POINT-TO-POINT LINEHAUL SERVICE.

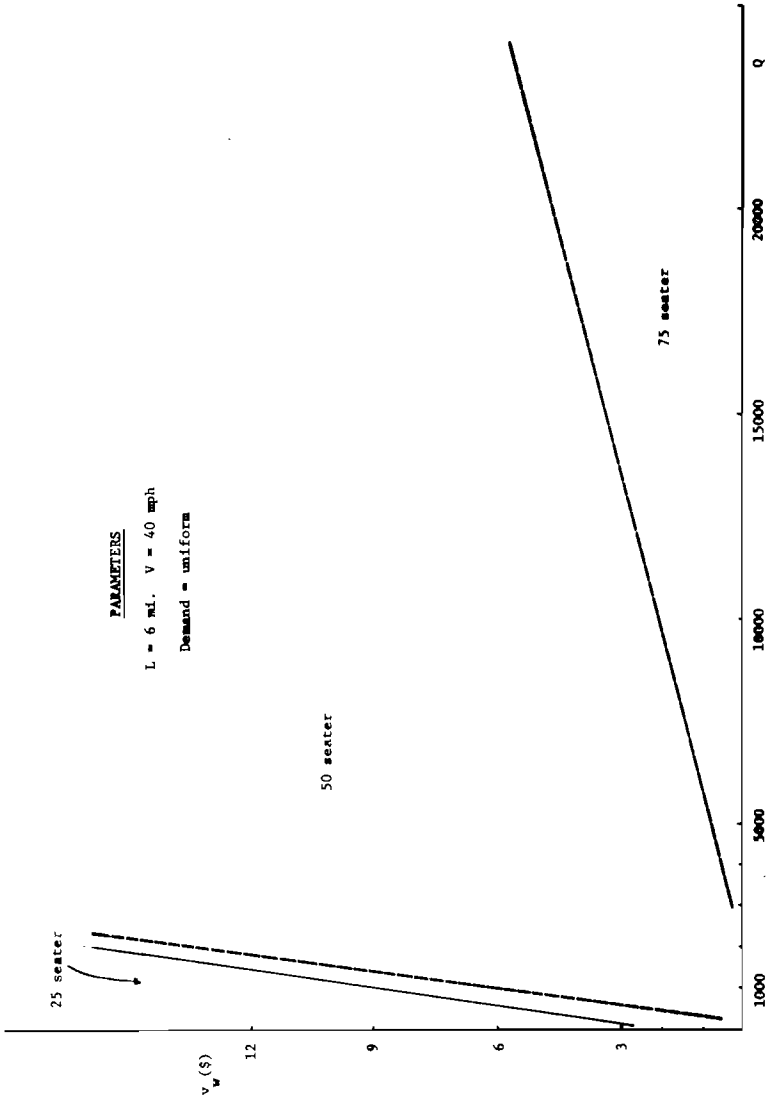


FIGURE 17b

RELATIVE COST EFFICIENCY OF BUSES ACCORDING TO SIZE  
TWELVE MILE POINT-TO-POINT LINEHAUL SERVICE.

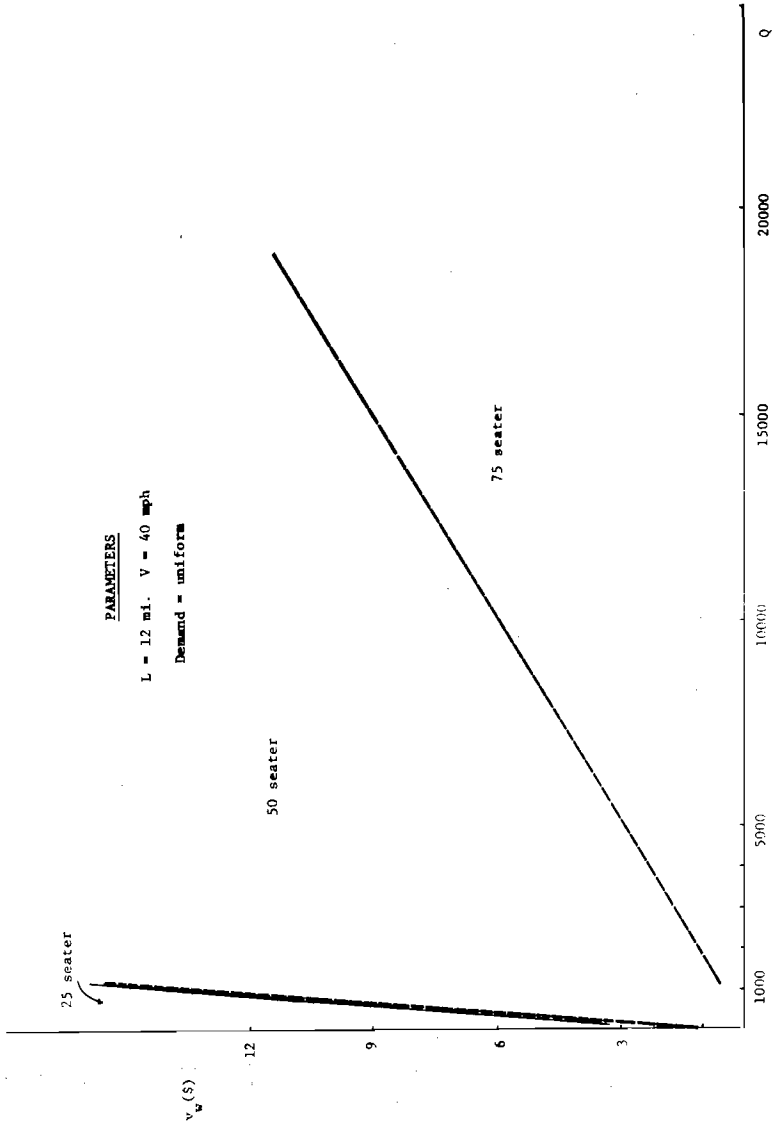
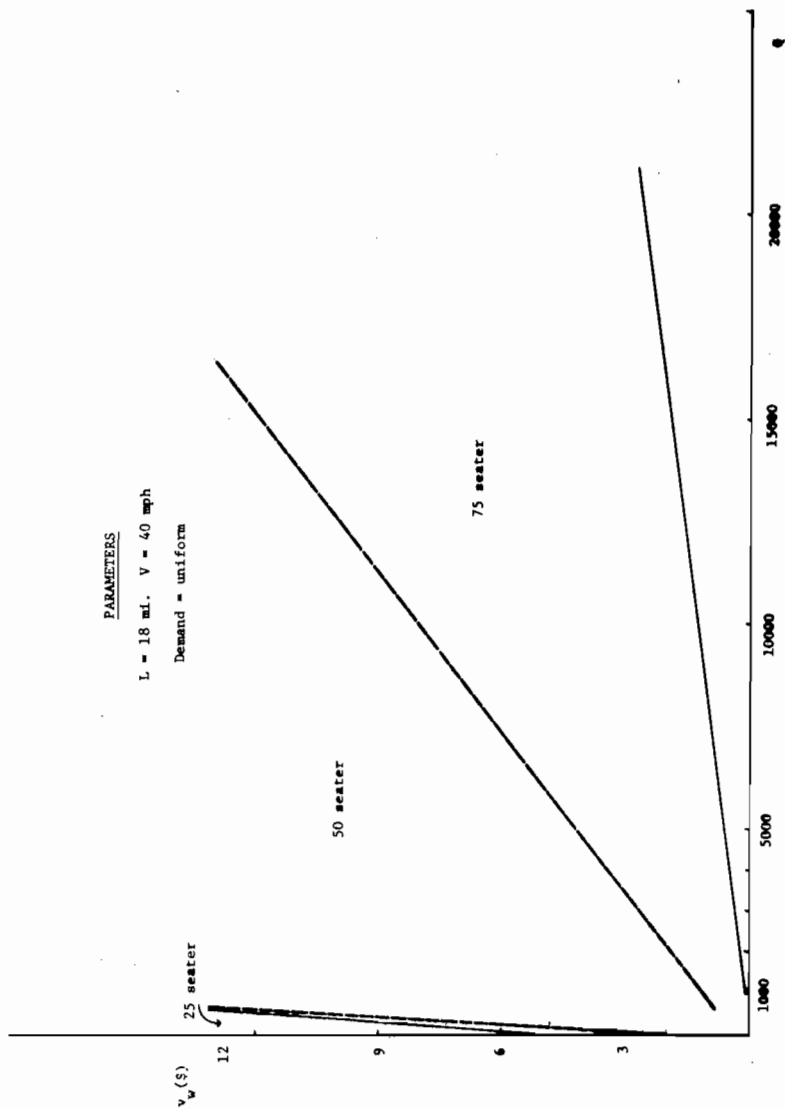




FIGURE 17c

RELATIVE COST EFFICIENCY OF BUSES ACCORDING TO SIZE  
EIGHTEEN MILE POINT-TO-POINT LINEHAUL SERVICE.



bus it has to be assumed that the route on which it is to be operated is capable of generating a very large demand, or that its patrons will not appreciably mind waiting for the bus to come along (i.e., have a low shadow price  $v_w$ ). If these conditions cannot be envisaged over the route in question, then it would seem better to invest in the standard bus.



## 5. Integrated Bus Service and Inter-Modal Cost Comparisons

In section 3.5 we modeled the costs associated with multiple-stop bus service, and widened the analysis to include the spatially related user costs. We noted that increasing the stopping frequency of a bus while reducing the mean distance walked to a stop also increases the time required to traverse the route.

The inclusion of all the time and effort costs of a trip in the analysis means that the full price of door-to-door travel can now be evaluated and cost comparisons made with other modes.

With the exception of a rail rapid transit, urban transport technologies have a local collection capability. In the case of a bus, this can take the form of a local or feeder service operating over a fixed route with frequent stops, or the hybrid dial-a-bus, a recent innovation, which has no fixed route, simply collecting patrons at their door. A good feeder or local service would be comfortably accessible on foot.

### 5.1 A Framework for Assessing Inter-Modal Travel Costs

The comparisons are to be made for corridor trips, permitting the inclusion of rail rapid transit. These are considered focussed on a high density node (or activity center) such as a CBD, in anticipation that the more general urban trip can be analyzed by relaxing these restrictions once the framework is operative.

A trip consists of a feeder or local journey over semi-arterial/arterial streets and a line haul journey over a freeway or fixed track facility.<sup>30</sup> The average walk on termination of the journey is assumed comparable for all modes and is not included.

When the linehaul mode is rail, there is clearly a transfer and change of mode at the beginning of linehaul, as is also the case with dial-a-bus. However a transfer and a possible change of mode need not occur with bus or car because the same vehicle can continue its journey by entering the freeway. In the case of bus, this mode of operation is termed "integrated" or "throat" service, the Seattle Blue Streak and the Washington Shirley Highway services being the most notable examples.

To formalize the distinction, suppose the net demand for transport to destination D via corridor TD is contained within a circular catchment of radius  $r_0$  (Figure 18a). The typical trip therefore comprises a local journey over a distance  $r$ , followed by a fixed linehaul journey over distance L. To estimate the cost associated with the average journey we need to make an assumption about the variation of demand as a function of radius  $r$  from the feeder "terminus" T: suppose that demand density falls off uniformly with distance such that it is zero at the ring boundary  $r = r_0$ , that is:

$$P(r) = P_0 \left(1 - \frac{r}{r_0}\right)$$

where  $P_0$  is the demand density at T. This function is shown in Figure 18b. Therefore the net demand within radius  $r$  is

$$\begin{aligned} Q(r) &= \int P(r) dA = 2\pi \int_0^r P(r) r dr = 2\pi P_0 \int_0^r \left(1 - \frac{r}{r_0}\right) r dr \\ &= \pi P_0 r^2 \left[1 - \frac{2r}{3r_0}\right] \\ &\approx \frac{2}{3} \pi P_0 r^2 \end{aligned}$$

This linear approximation derives from an increasing area being offset by a declining demand density.

From the differential calculus relationship  $\Delta Q = \frac{dQ}{dr} \Delta r$ , we have:

$$\Delta Q \approx \frac{2\pi}{3} P_o \Delta r \quad (44)$$

which shows that increments in distance  $r$  are accompanied by proportionate increases in demand.

The total demand is therefore  $Q = \frac{2\pi}{3} P_o r_o$ , which is equivalent to a terminus density of  $P_o = \frac{3}{2\pi} \frac{Q}{r_o}$ .

## 5.2 Integrated Bus Service - Average Cost

### (Algorithm SRAC4)

Consider the above catchment to be served by  $R$  equally spaced radial feeder routes of length  $r_o$ ,<sup>31</sup> having  $S$  equally spaced stops. Since demand is isotropic, each route has a catchment of size  $Q_{\mathcal{R}} = \frac{Q}{R}$  which according to (44) is shared equally amongst the  $S$  stops, the demand per stop being  $Q_{\mathcal{R}S} = \frac{Q_{\mathcal{R}}}{S} = \frac{Q}{RS}$ . The mean walk distance to each stop can be obtained by integrating over its catchment area,<sup>32</sup> Figure 18c. If minimization of walk were the sole objective then there would be no theoretical upper limit on either the number of routes or stops. But too many routes would lead to very low load factors significantly increasing agency cost (37), while too many stops would slow service speed, markedly increasing both the in-vehicle cost (38) and the agency cost. Therefore a simultaneous tradeoff<sup>33</sup> is needed between the number of stops and feeder routes on the one hand, and the mean walk distance on the other.

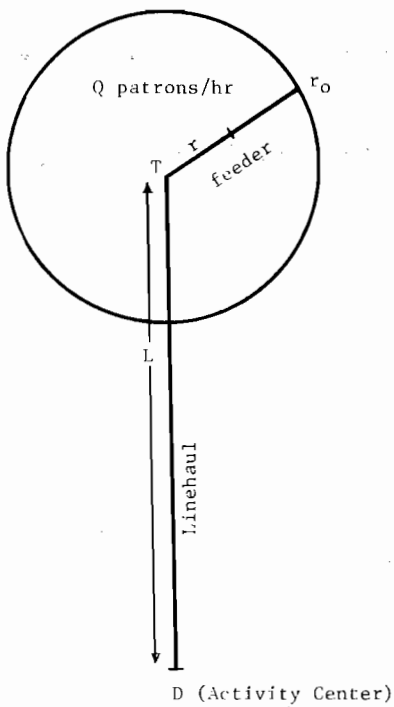


FIGURE 18a

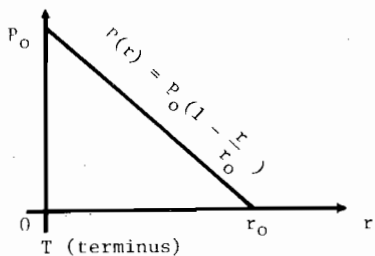


FIGURE 18b

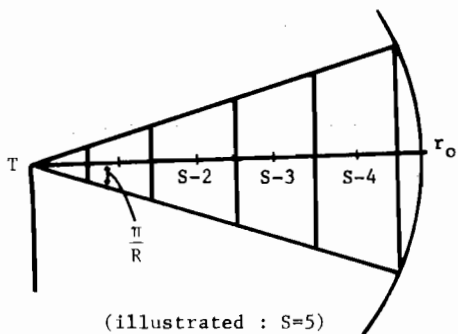


FIGURE 18c

(illustrated :  $S=5$ )

A journey by a small vehicle on the feeder leg and a larger vehicle on the linehaul leg has clearly a higher minimum average cost than utilization of the same vehicle throughout, if only because the latter eliminates transfer costs. The problem is further compounded by the "mismatch" in vehicle sizes at T, requiring more than one feeder bus load to fill a linehaul bus. If the linehaul buses were dispatched on a single feeder-load basis, any cost advantages of using the smaller vehicle would be eliminated. Yet there are quite significant problems of synchronization of feeder services to eliminate wait at the feeder terminus T, or at least to make it as small as possible.<sup>34</sup> Even then, it is not clear that this would ensure a minimization of overall costs.

Because this process is hard to treat analytically and its costs are decidedly suboptimal with respect to throat or integrated service, we shall not evaluate it further. (The poor cost showings of the small feeder type bus over the standard bus, as elaborated in sections 3.4 and 4.2, coupled with the need to provide interchange facilities at T, lend further credence to the strategy of concentrating on integrated service.)

Algorithm SRAC4 (Appendix B-1) is an adaption of SRAC3 that computes the average cost of the typical journey by integrated bus: Given a point-to-point linehaul distance L and a demand catchment of radius  $r_0$  (Figure 18b), it evaluates this cost as a function of demand level Q ( $Q_{\lambda}$ ), headway  $\psi$ , number of stops S and feeder routes R, via (37), (38), (39), (24), and (25). Corrections to the service speed on account of stopping and loading are also included. By systematically varying headway, number of stops and routes, the minimum average cost can be found by interpolation. Note that the frequency on the linehaul leg, resulting from the convergence of R (identical) feeder routes, is  $R/\psi$ .



### Sensitivity Analysis

In Figure 19 we have simultaneously varied the number of stops and routes, corresponding to feeder and linehaul lengths of 2 and 12 miles, and to an hourly demand of 1000 spread over a catchment of radius 2 miles. In this and all following cases, the shadow price of walk is equated to the shadow price of wait  $v_w$  (section 2.2). Figure 19 shows that the optimal "stop-route" combination is about nine stops (or one every one quarter of a mile) and 12 feeder routes, or in our notation,  $S = 9, R = 12$ . Beyond nine stops the minimum cost again increases, because the bus becomes appreciably slowed.

At this stopping frequency twelve feeder routes are close to optimal over much of the range of contemplated demand: see Figure 20. Further, with increasing demand the cost minima became flatter, enabling a wide latitude in the number of feeder routes without appreciably departing from optimality.

Figure 21 again illustrates the effect of density, but this time in terms of breadth of catchment. Stop spacing is held constant and the number of feeder routes varied as a function of catchment radius  $r_0$ , for a fixed demand  $Q = 1000$ . Not unexpectedly, the higher the density the lower the cost of meeting the given demand, and the fewer feeder routes needed.

### Average Cost as a Function of Linehaul Distance and Demand

Using SRAC<sup>4</sup>, the average cost of integrated service with a standard bus has been evaluated as a function of demand for three different linehaul distances, Figures 22a, b, and c. A demand catchment of  $r_0 = 2$  miles has been assumed with nine stops, and the number of

FIGURE 19

INTEGRATED BUS SERVICE: RELATIONSHIP OF AVERAGE COST TO STOP FREQUENCY (S) AND NUMBER OF FEEDER ROUTES (R).

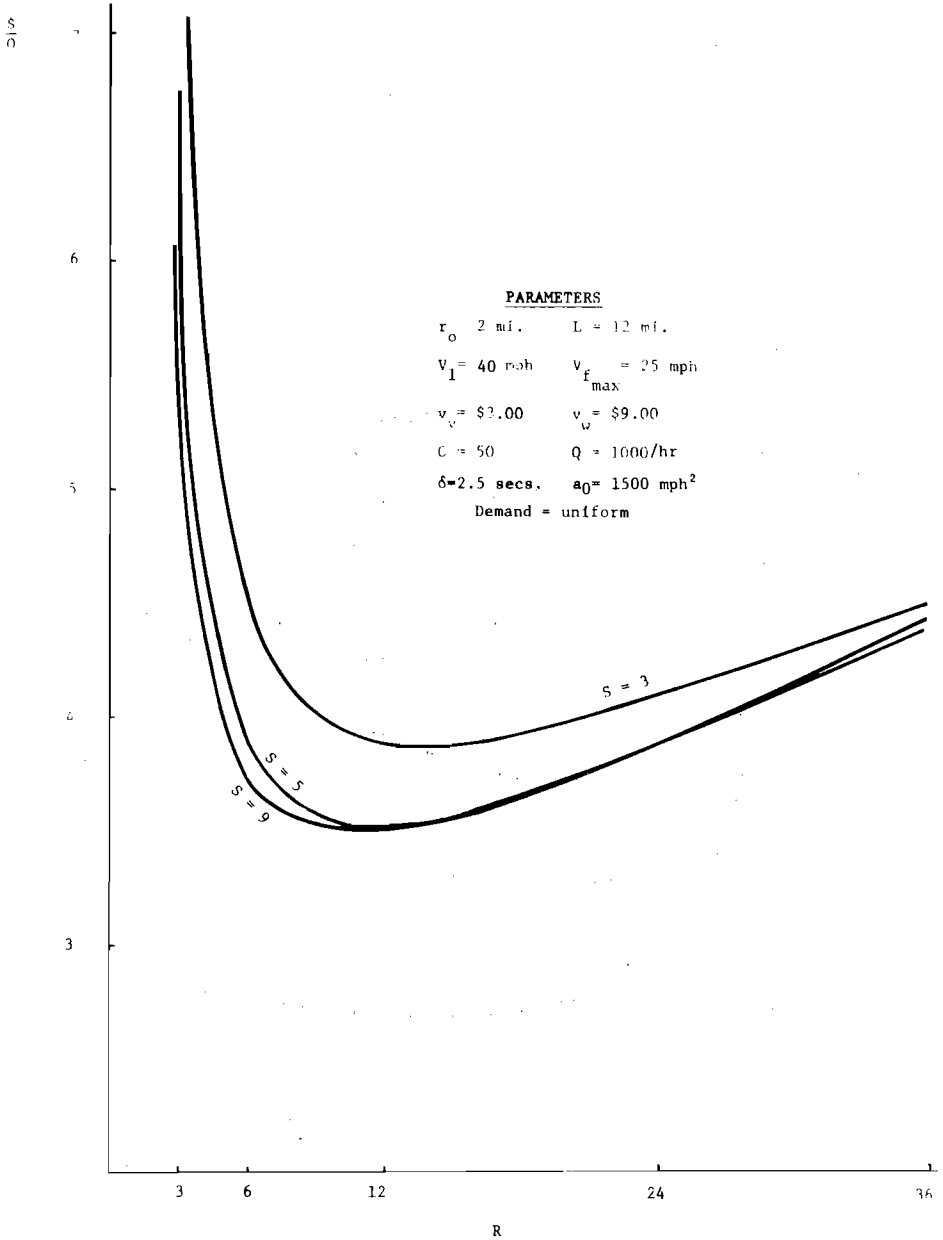


FIGURE 20

INTEGRATED BUS SERVICE: RELATIONSHIP OF AVERAGE COST TO  
LEVEL OF DEMAND AND NUMBER OF FEEDER ROUTES (R).

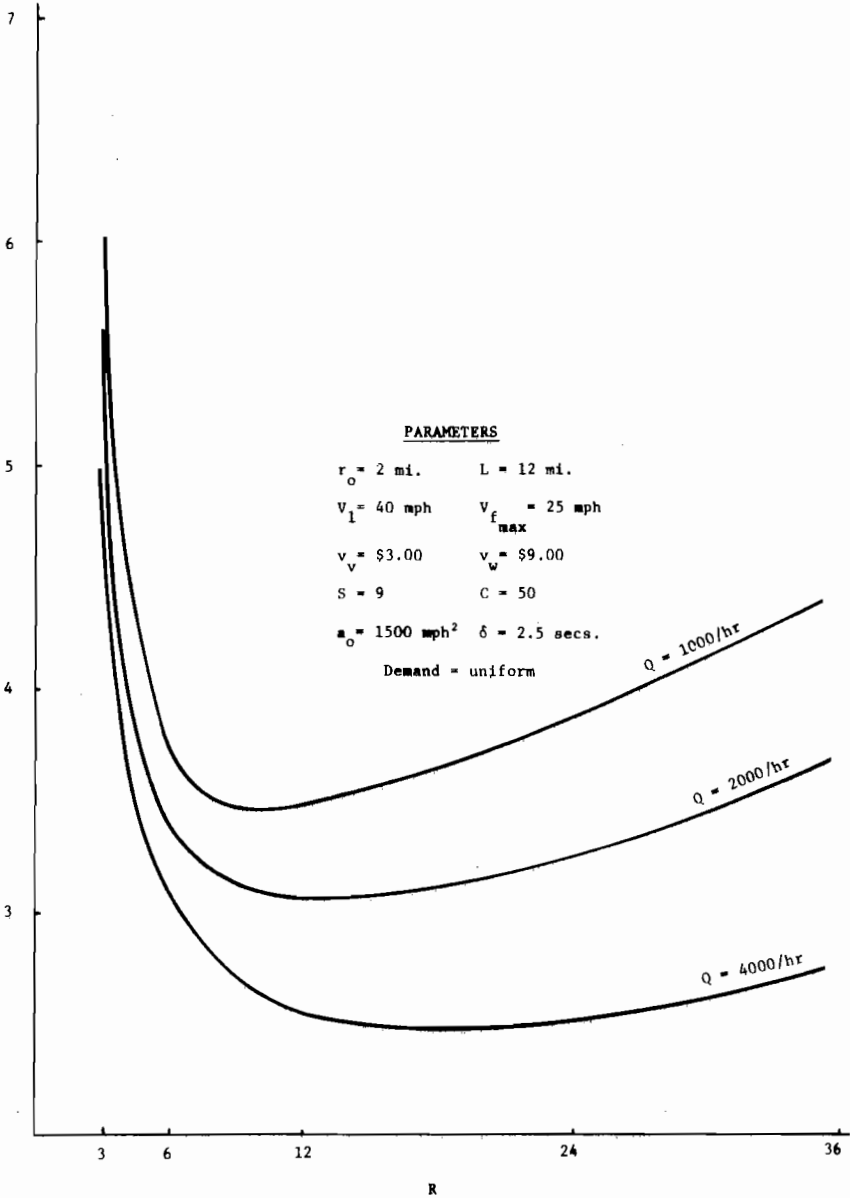


FIGURE 21

INTEGRATED BUS SERVICE: RELATIONSHIP OF AVERAGE COST TO SPATIAL DISPERSAL OF DEMAND ( $r_0$ ) AND NUMBER OF FEEDER ROUTES ( $R$ ).

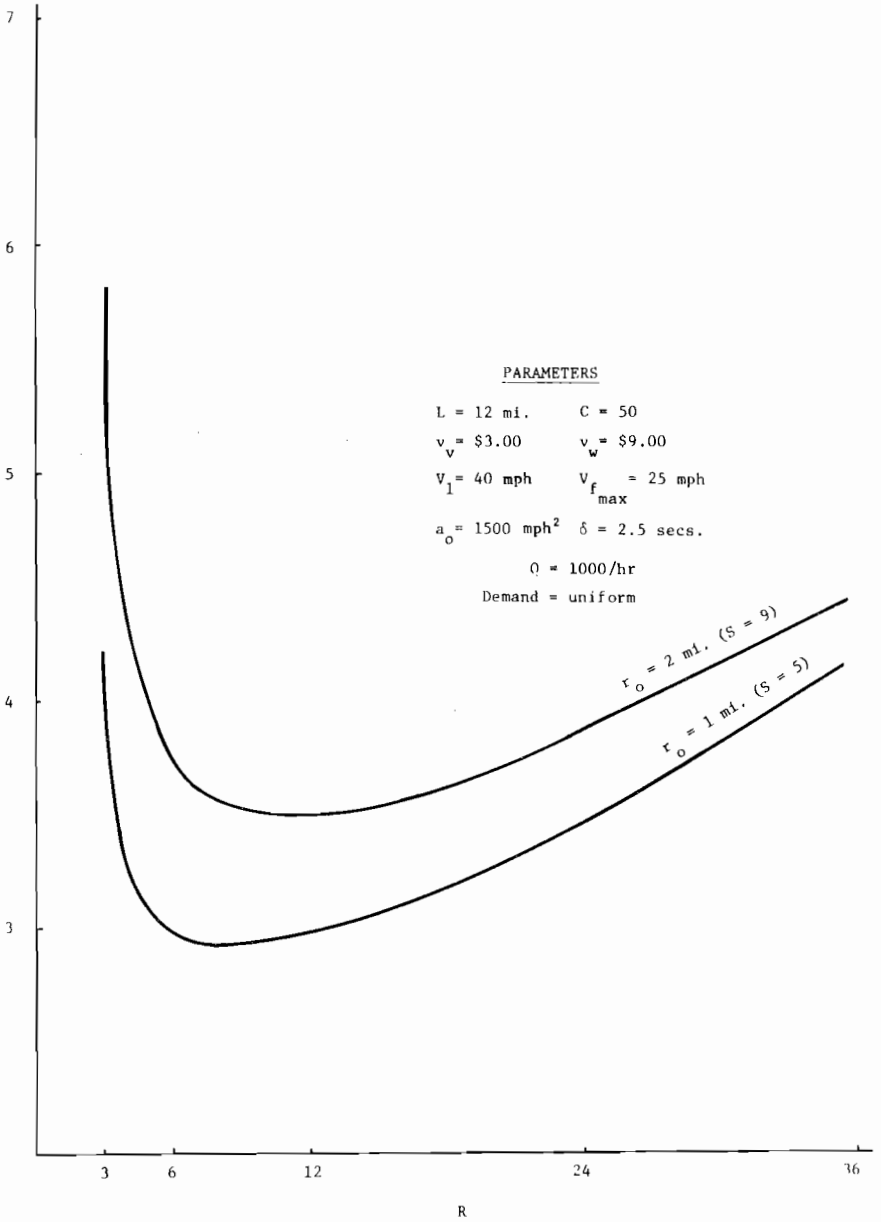


FIGURE 10a

MINIMUM AVERAGE COSTS OF ALTERNATIVE MODAL COMBINATIONS AS A FUNCTION OF LEVEL OF DEMAND - 6 MI., LINEHAUL JOURNEY.

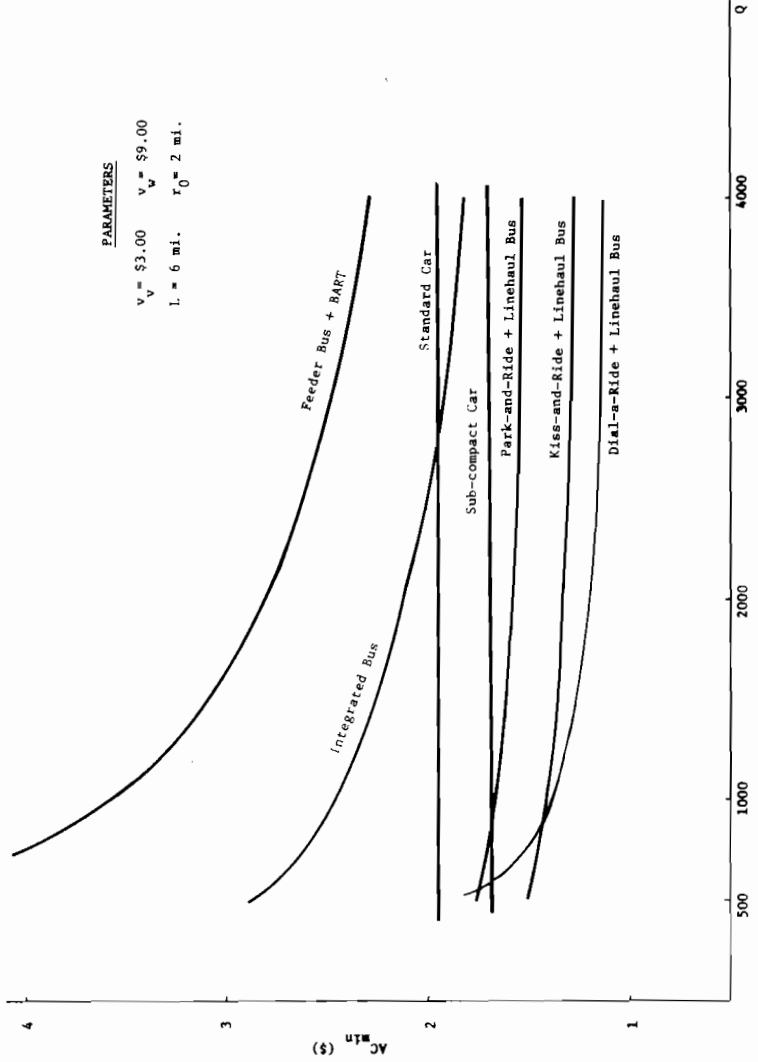


FIGURE 10b

MINIMUM AVERAGE COSTS OF ALTERNATIVE MODAL COMBINATIONS AS A FUNCTION OF LEVEL OF DEMAND - 10 MI., LINEHAUL JOURNEY.

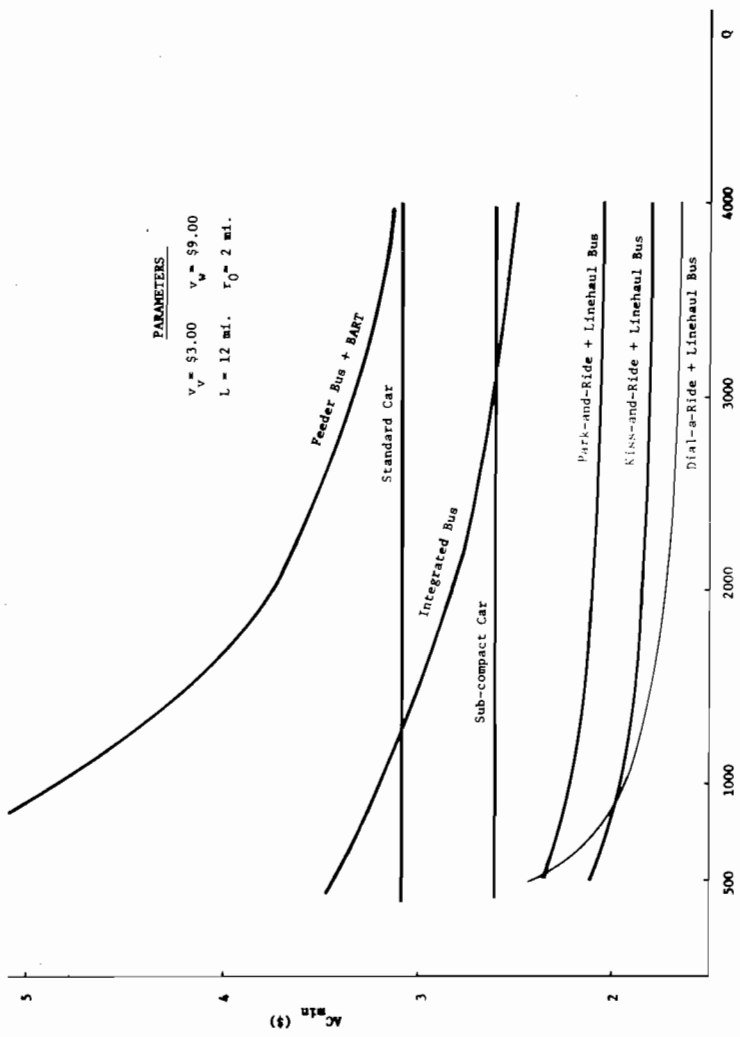
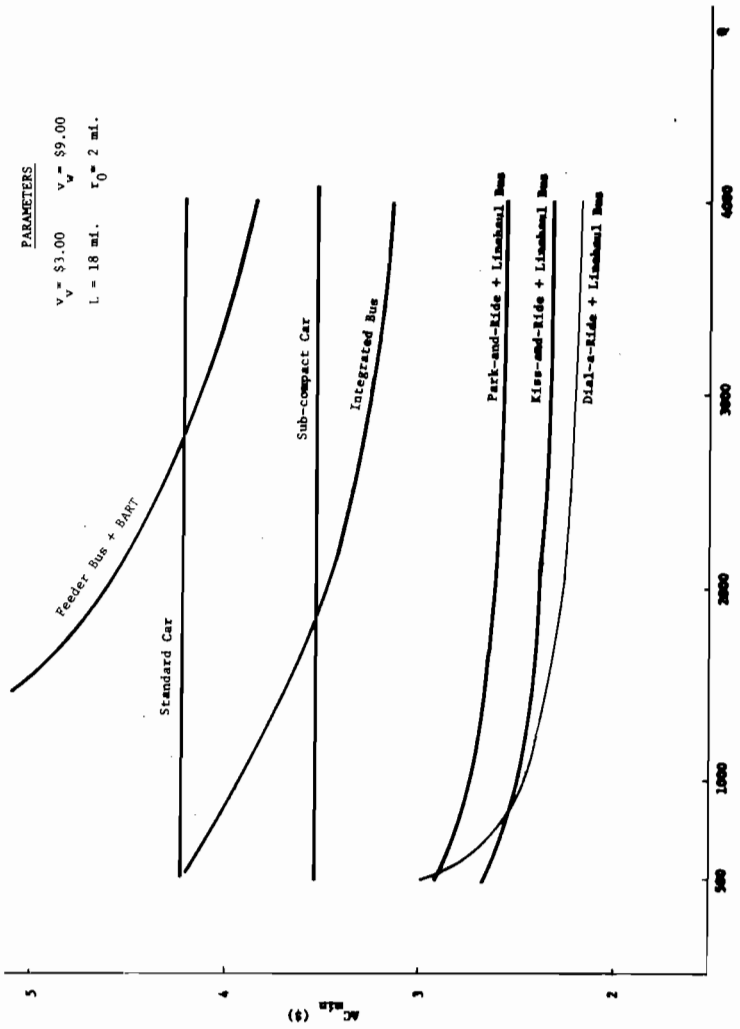


FIGURE 27c

MINIMUM AVERAGE COSTS OF ALTERNATIVE MODAL COMBINATIONS AS  
A FUNCTION OF LEVEL OF DEMAND - 18 MI. LINEHAUL JOURNEY.



feeder routes optimized, which often corresponds to  $R = 12$ . On the local leg a maximum speed of  $V_{f \text{ max}} = 25$  mph and an acceleration/deceleration of  $a_o = 4000$  mph<sup>2</sup> has been assumed with a loading time of  $\delta = 2.5$  seconds per passenger. On the linehaul leg, a speed of  $V_1 = 40$  mph has been used which corresponds to rush-hour conditions.<sup>35</sup>

Notice that integrated service is relatively expensive at low demand levels, for to avoid small load factors the catchment per stop is large, making for a higher walk cost. The scale economies in Figure 22b are about double those observed over a comparable distance for point-to-point linehaul in Figure 13.

Figures 23a, b, and c on the other hand show minimum average cost as a function of the linehaul length. Since on this portion of the journey costs vary in direct proportion to distance and time, average costs increase linearly as linehaul becomes the major component of the journey. As expected, the subsidy payable increases with the length of the route, but the magnitude of the scale economies observed above reduce its value substantially when demand reaches the 4000 patrons per hour level.

### 5.3 Alternative Modes - Average Costs

We come now to the identification of other modes, or modal combinations, that provide alternative ways of making the above trip, and the evaluation of the average cost associated with a typical trip by each mode within the framework established in section 5.1.

The following "alternatives" have been selected:

- |                                     |   |            |
|-------------------------------------|---|------------|
| (a) automobile                      | } | mixed mode |
| (b) park-and-ride + linehaul bus    |   |            |
| (c) kiss-and-ride + linehaul bus    |   |            |
| (d) dial-a-ride + linehaul bus      |   |            |
| (e) feeder bus + rail rapid transit |   |            |



FIGURE 23a

MINIMUM AVERAGE COSTS OF ALTERNATIVE MODAL COMBINATIONS AS  
A FUNCTION OF LINEHAUL DISTANCE - LOW LEVEL OF DEMAND.

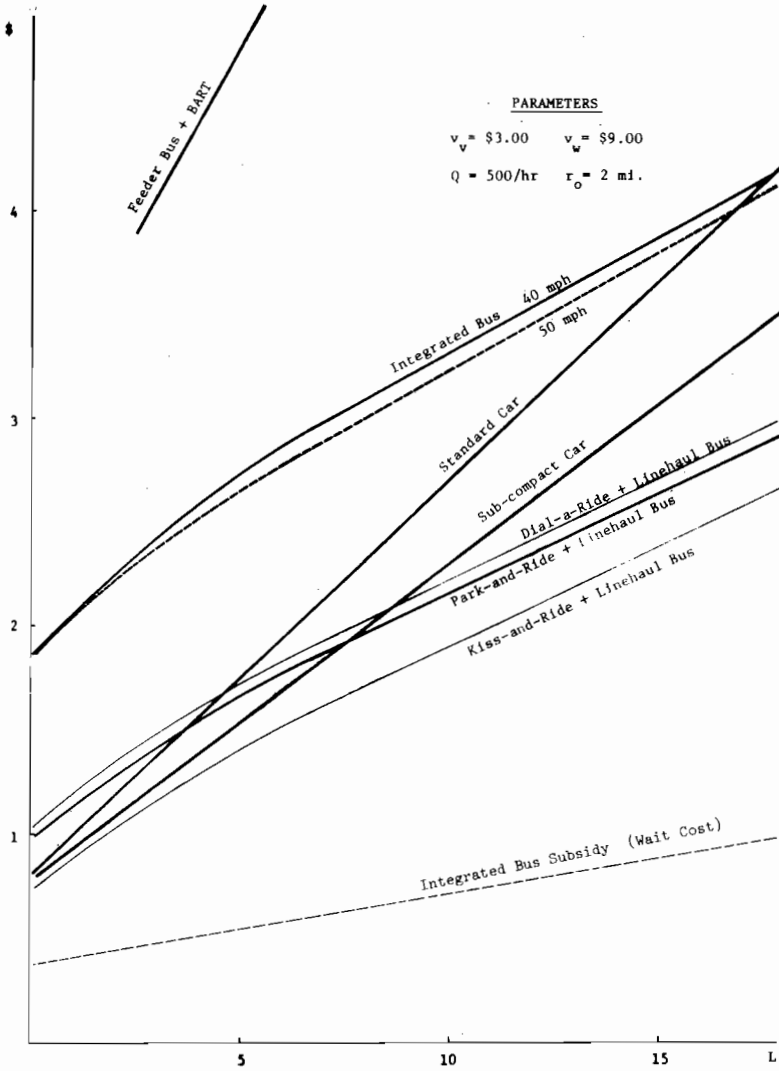


FIGURE 29b

MINIMUM AVERAGE COSTS OF ALTERNATIVE MODAL COMBINATIONS AS A FUNCTION OF LINEHAUL DISTANCE - MODERATE LEVEL OF DEMAND.

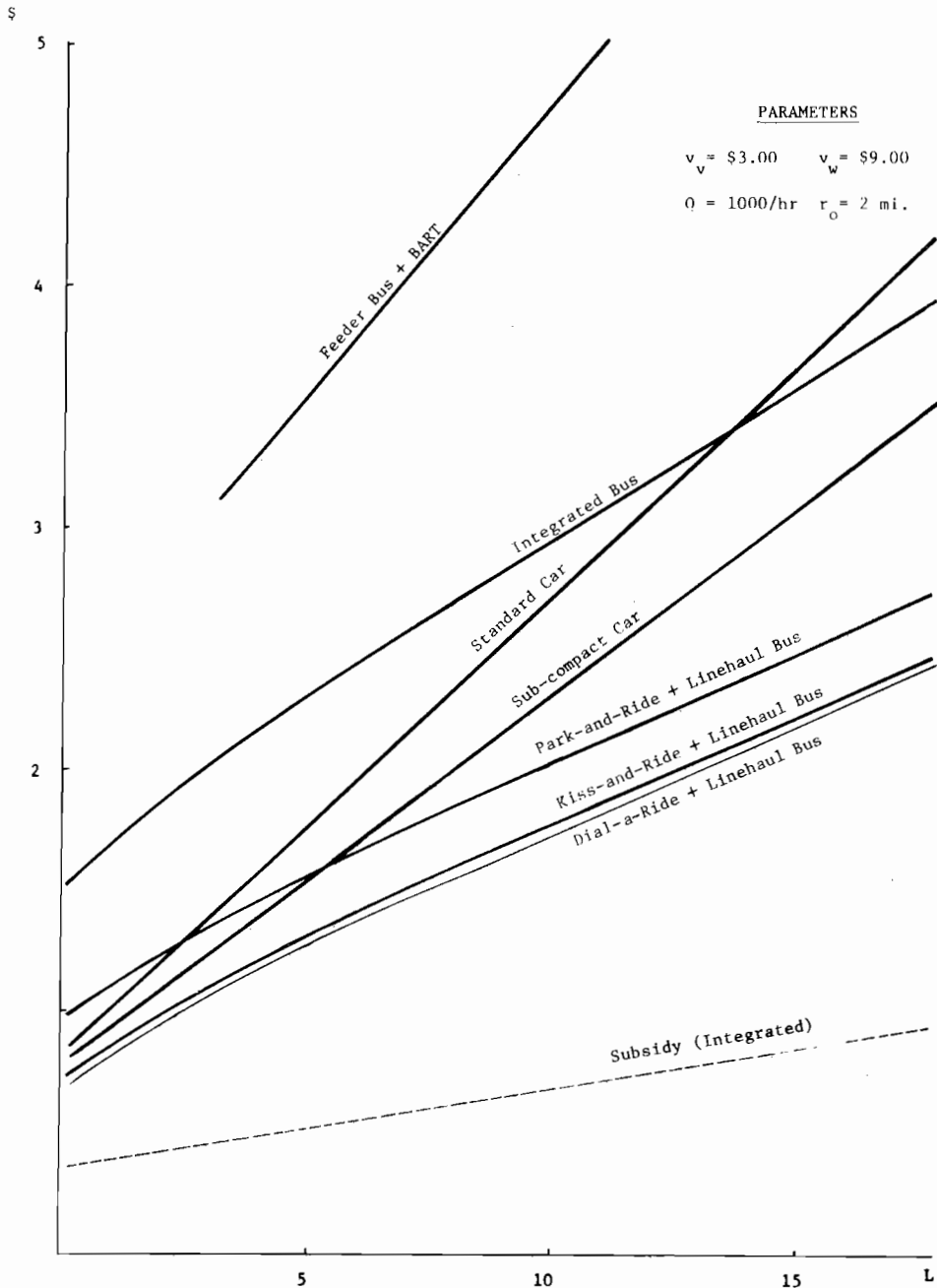
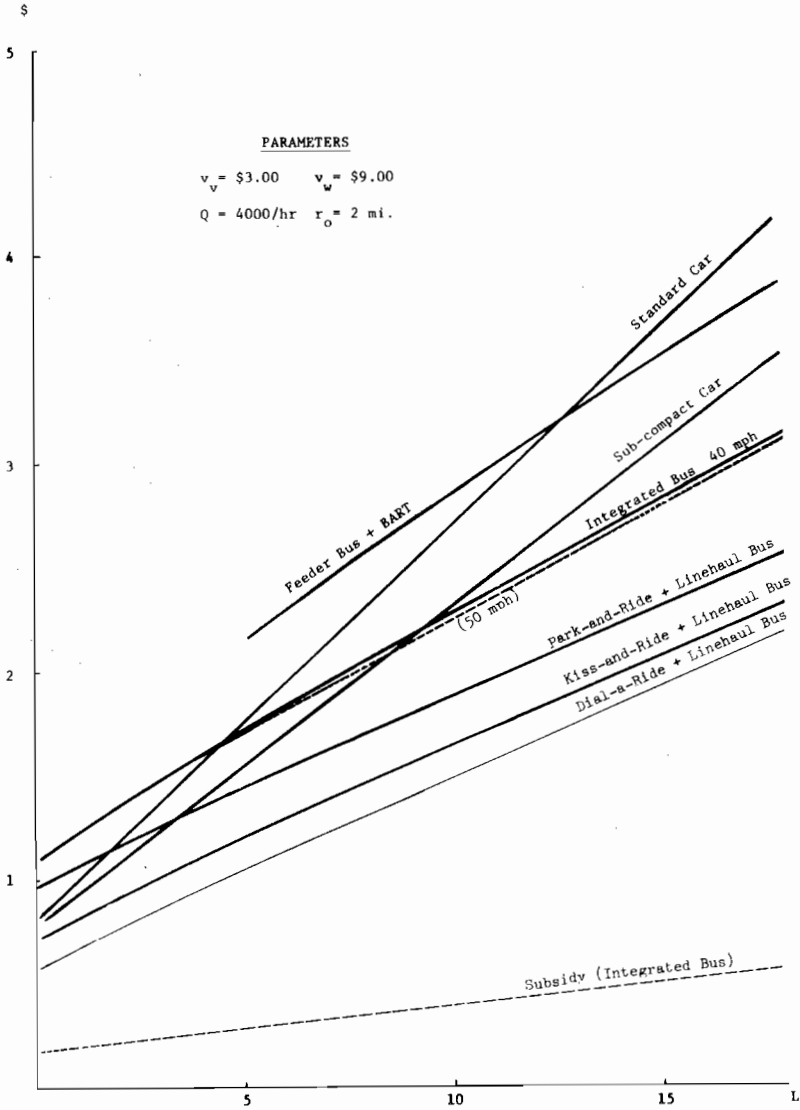


FIGURE 23c

MINIMUM AVERAGE COSTS OF ALTERNATIVE MODAL COMBINATIONS AS  
A FUNCTION OF LINE-HAUL DISTANCE - HIGH LEVEL OF DEMAND.



(a) Costs of an Automobile Trip

In estimating the cost of the average journey by automobile it is important to distinguish between journey characteristics as they apply to bus and to car. The bus, as a mass ridership vehicle, starts at some terminal point ( $r = r_0$ ) and runs into the feeder "terminus" T, collecting passengers en route. Clearly the agency and in-vehicle components of average cost, equations (37) and (38), are larger and smaller respectively than those incurred if all passengers were to load at the starting point. That is, the average trip length is less than the vehicle trip length.

However in the case of car with single occupancy these two lengths are identical, and are given by:

$$\bar{r} = \frac{\int_0^{r_0} r dQ}{\int_0^{r_0} dQ}, \text{ but from (44), } dQ = \frac{2}{3} \pi P_0 dr$$

$$\text{Therefore, } \bar{r} = \frac{\int_0^{r_0} r dr}{\int_0^{r_0} dr} = \frac{r_0}{2} \quad (45)$$

That is, the mean distance driven to the feeder terminus T is half the radius of the demand catchment. Hence the average automobile trip comprises a drive of  $r_0/2$  miles at a mean speed  $V_f$  mph over local streets, followed by a drive of L miles at a mean speed  $V_L$  mph along a freeway. The associated variable costs can now be evaluated.

Drawing on the vehicle-mile cost estimates of Bay Area roads given by Keeler, Small and Cluff (1974), disaggregated figures for arterials and freeways with standard and sub-compact sedans are given in Appendix A-4. A speed of 25 mph ( $V_f$ ) has been assumed on the feeder leg (arterial) and 50 mph ( $V_L$ ) on the linehaul leg (freeway).

Including time and effort inputs, the net variable costs are then:

$$AVC_{std} = 0.161_7 \frac{r_o}{2} + 0.128_4 L + \frac{v_v}{50} [r_o + L] \quad (46)$$

and

$$AVC_{s/cpt} = 0.127_q \frac{r_o}{2} + 0.091_5 L + \frac{v_v}{50} [r_o + L] \quad (47)$$

Externalities of noise and air pollution (\$.0048/vehicle mile) and noise have been omitted, since these are not included with buses.<sup>36</sup> In both cases, the costs are a very small proportion of net vehicle mile costs, and even allowing for disparities between buses and cars, would not affect comparability.

The fixed costs are those of parking:<sup>37</sup> using a 6% interest rate Meyer, Kain and Wohl (1965) estimate an annual cost of \$388 for a low (land value) CBD parking lot. This is equivalent to \$1.06 per day (per round trip) or \$0.53 per one way trip. The equivalent costs for garage (medium land value) and fringe (very low land value) parking are \$0.97 and \$0.27 per one way trip, respectively. Assuming that destination D is a Central Business District and that the average motorist parks in the low CBD lot, then the degree of walk to the final destination is roughly comparable with that for the use of bus and rail. Alternatively, the same cost is incurred by fringe parking and making a local transit trip (27¢ plus 25¢).

Assuming that the parking costs are the same for all size automobiles, and there is no depreciation of the vehicle while it is parked, then the average fixed costs are AFC = 0.53. So the average cost is:

$$AC_{std} = 0.53 + 0.161_7 \frac{r_o}{2} + 0.128_4 L + \frac{v_v}{50} [r_o + L] \quad (48)$$

and

$$AC_{s/cpt} = 0.53 + 0.127_q \frac{r_o}{2} + 0.091_5 L + \frac{v_v}{50} [r_o + L] \quad (49)$$

Note that these are equivalent to minimum average cost estimates, and that the standard and sub-compact cars provide upper and lower bounds respectively for the costs of automobile trips.

(b) Costs of a Park-and-Ride + Linehaul Bus Trip

This park-and-ride alternative entails an automobile trip of length  $\frac{r_o}{2}$ , a transfer of time  $\theta_t$ , and a linehaul bus trip of Length L.

As the transfer is carried out in suburban locations, we can assume parking costs equivalent to fringe CBD parking: \$0.27 per one way trip. Average costs for the automobile trip component are therefore given by adding this figure to (46) or (47) with  $L = 0$ , viz

$$AC_{std} = 0.27 + 0.161_7 \frac{r_o}{2} + \frac{v_v}{50} r_o \quad (50)$$

and  $AC_{s/cpt} = 0.27 + 0.127_9 \frac{r_o}{2} + \frac{v_v}{50} r_o \quad (51)$

The transfer time for the mean trip is somewhat arbitrary: we shall settle for  $\theta_t = 3$  minutes. At a shadow price of  $v_w$  this amounts to a fixed cost of  $3/60 v_w$  dollars.

It is reasonable to suppose that arrival at the terminus is quite random so that the bus component of average variable cost is simply described by (23) or (28). Average cost is then the sum of these various component costs.<sup>38</sup>

(c) Costs of a Kiss-and-Ride + Linehaul Bus Trip

This situation is more complicated. The car makes two round trips to the terminus T per day, or two trips per commuter trip. While its operating costs can be wholly allocated to the commuter, there is the problem of the time and effort costs, etc. incurred by the

ferrying party. One way out is to assume that the benefits derived from having the use of the car during the day equal or exceed these costs in value (willingness to pay) and therefore do not enter into the evaluation of the commutation trip. Average costs for the automobile portion of the journey are therefore similar to (50) and (51) viz:

$$AC_{std} = 0.27 + 2 \cdot 0.161_7 \frac{r_o}{2} + \frac{v_v}{50} r_o \quad (52)$$

or

$$AC_{s/cpt} = 0.27 + 2 \cdot 0.127_9 \frac{r_o}{2} + \frac{v_v}{50} r_o \quad (53)$$

In this case transfer time is assumed halved because the commuter is dropped near the bus or train station entrance at T. Under an assumption of random arrival at this point, average costs are evaluated as for park and ride.

(d) Costs of a Dial-a-Ride + Linehaul Bus Trip

Another link mode is dial-a-ride or demand-activated service. Average costs here are even more difficult to evaluate because of variable route and wait time. Utilizing cost data of Clemons (1974) relating to the operation of the Haddonfield system (Appendix A-5), the total direct costs (TDC) have been estimated as

$$TDC = 15.14 \frac{r_o}{V_f} + 0.451_2 V_f' \quad (54)$$

where  $V_f'$  is the mean speed of the bus.

It must be emphasized that these vehicle-hour and mile costs are no more than first round approximations containing a circuitry factor of 1.5 to account for the meandering nature of the service compared to its fixed route counterpart.

The average cost for the dial-a-ride portion of the journey is therefore <sup>39</sup>

$$AC = \frac{TDC}{\bar{q}} + v_v \frac{r_o}{V_f} \quad (55)$$

where  $\bar{q}$  is the mean number of passengers per bus. If we assume that having once requested dial-a-ride service the commuter can occupy his or her time profitably, then we may exclude wait costs. In the special case of subscription service the patron knows when the bus will arrive, and here the wait is clearly close to zero. There is also a problem of relating  $\bar{q}$  to the demand level  $Q$ ; we make the following assumptions:

<u>Demand density</u>	$\bar{q}$
"light" ( $Q = 500/\text{hr}$ )	3
"medium" ( $Q = 1000/\text{hr}$ )	8 (half-full)
"heavy" ( $Q = 4000/\text{hr}$ )	17 (full)

The remaining portion of the journey has costs identical to those of kiss-and-ride line haul bus.

#### (e) Costs of Feeder Bus Plus Rail Rapid Transit (BART) Trip

Pozdena (1974) has established a similar framework to that out-lined in section 5.1 for evaluating optimal costs associated with an average journey by feeder bus and BART. Drawing upon his work we have a comparable minimum average cost estimate of the form

$$AC_{\min} = a + bQ^{-1/2} + cQ^{-1/3} + dQ^{-1} \quad (56)$$

where the first term corresponds to the transfer and net invehicle time and effort costs; the second to the agency and wait costs on the



linehaul leg; the third to wait, walk and agency costs on the feeder leg; and the fourth to the fixed rail costs. The values for these parameters as a function of linehaul distance are given in Appendix A-6.

Note that in (56) there is no matching of system capacity between the feeder buses and trains. Because of shunting problems, BART does not have the capability of adjusting train lengths to meet changing demand.

#### 5.4 Inter-Modal Cost Comparisons

It is now possible to make some cost comparisons between integrated bus service and these alternative modes. The schematic framework, although abstracting from the cartesian character of local street grids, does not significantly distort relative cost levels, since path lengths of the various modes are comparable in both situations. This is a partial equilibrium analysis: each mode is assumed to function optimally independently of other modes.

In Figures 22(a), (b), and (c) costs of these four alternative modal combinations are shown with integrated bus costs for three different linehaul distances. Note the significant returns to scale of both integrated bus and feeder bus-BART services, especially in the case of the latter where there are high fixed costs of trackage, signalling, stations, etc. By comparison, right of way costs attributable to buses when they share roads with automobiles and trucks are minimal (section 2.1). One factor making the BART combination more expensive is the cost of transfer which is avoided with integrated service.

While feeder-bus+fixed rail is extremely expensive at low levels of demand, it becomes more competitive with integrated bus service as demand climbs, but even at 4000 patrons/hour it is still a long way from

breaching the cost gap. Rough calculations indicate that integrated bus is still cheaper at 20,000 patrons/hour<sup>40</sup> over a route of 18 miles.

Except over the shortest distance (6 miles) where transfer cost is important, park-and-ride + linehaul bus is consistently cheaper than the automobile alone: one gets the best of both worlds since a costly walk is eliminated and the scale economies afforded by bus are reaped over the major portion of the journey. Kiss-and-ride + linehaul bus and dial-a-ride + linehaul bus are cheaper again, but these are "softer" estimates. It is likely that dial-a-ride is costlier at higher levels of demand than indicated, since the assumption of random arrival at T tends to break down as close to full buses produce marked "quanta of arrivals," leading to a capacity mismatch problem. Note that integrated service is cheaper than the least cost auto mode (sub-compact) at higher levels of demand and longer distances, since a finer route coverage is possible, thus minimizing walk. In other words, the virtual elimination of the appreciable disutility of walk at higher demand levels makes integrated service strongly competitive with the car which (still) incurs a significant parking cost at the (CBD) destination.

To gain a clearer understanding of the relationship of costs to linehaul distance we have superimposed the estimates for alternative modes on Figures 23(a), (b), and (c). These costs generally increase linearly with distance: in the case of Feeder bus + BART this is most dramatic at low demand when the large fixed rail costs are divided amongst the few passengers. The rail-based mode is never competitive with any other option except standard car, and only then at more significant demand levels over a long haul (> 12 miles) which resembles an intercity service.

At low demand (500 patrons per hour), Figure 23(a) shows that the automobile (especially the sub-compact sedan) is cheaper than park-and-ride+linehaul bus and dial-a-ride+linehaul bus for distances up to about five miles because it has no transfer cost. This conforms with consumer perceptions of cost, since over shorter distances direct driving to the destination is preferred. For longer journeys, the mixed-mode options based on linehaul bus are the most economical. The integrated bus is not competitive because of the sparse route coverage necessary to maintain economical load factors. Increasing the linehaul speed from 40 to 50 mph does not bridge this gap. With demand set at moderate levels, Figure 23(b), the cost hierarchy is more-or-less maintained, although the range over which the sub-compact automobile is competitive is reduced. Both kiss-and-ride and dial-a-bus options are cheaper than park-and-ride, basically because of a smaller transfer cost.

Finally, at high demand (Figure 23c) all costs have converged quite markedly. The mixed-mode options based on linehaul bus are still generally cheaper whatever the distance. However the higher density now brings integrated service into strong competition with the sub-compact car at distances exceeding 10 miles, and even rail rapid transit overtakes the standard car beyond 12 miles.

### 5.5 Policy Implications

Some policy conclusions can be drawn regarding transportation for metropolitan areas: if we suppose that patronages of 500, 1000 and 4000/hour are typical of the demand for CBD travel from corporate oriented, outer, middle and inner distance suburbs, then in the case of a large city, distances in excess of twelve miles can be interpreted as "outer

suburbs" (Figure 23a), distances between five and 12 miles (Figure 23b) as "middle suburbs," and distances less than five miles (Figure 23c) as "inner suburbs."<sup>41</sup>

The mixed-mode linehaul bus options are the least expensive for journeys from middle and outer suburbs, and (with the exception of kiss-and-ride) are displaced by the sub-compact car for journeys originating in the inner suburbs. However, if congestion costs are included in the automobile estimates then there are indications (Keeler, 1974) that these may be sufficient, even in the case of small cars, to raise costs above the park-and-ride linehaul bus option.

Apart from the preceding consideration, these findings confirm the perceived costs of CBD commuters: over shorter distances the car is utilized; over longer distances they are inclined to park-and-ride.

What comes out very clearly is the importance of the automobile, at least as a link mode, in making a minimum cost journey to a high density activity center. For lower density activity centers (where parking costs are allocatable at the CBD fringe rate) the car is less costly up to moderate distances (10-12 miles). Its superiority in trips linking low density areas to other low density areas is already well known. The role of (subscription) dial-a-ride as an alternative link mode requires a more thorough cost study before similar definitive conclusions can be drawn. But in higher density settlements, it is clear that some investment in integrated bus service is warranted over dial-a-ride service.

Finally in view of the burgeoning controversy of rail rapid transit versus bus systems, it may be useful to examine the highway lane capacity required by a throat bus system serving different districts, as

an alternative to a feeder bus plus BART service. Imagine three settlements: high density (6 miles), moderate density (12 miles) and low density (18 miles) centered on a transportation corridor as shown in Figure 24a. Each settlement is served by an (optimized) integrated bus service which (by application of SRAC<sup>4</sup>) produces the indicated freeway flows, with an aggregate of 408 buses per hour. Such a volume can be achieved with an exclusive lane operation, being well within the theoretical upper limit set by the Highway Capacity Manual (HRB, 1965). This facility would only be a necessity over the inner six miles of the freeway since further out, the flows are relatively light. Note too, that it would be possible to operate far fewer buses without greatly departing from optimality.

Figure 24a also records the average costs for typical journeys from each settlement by both integrated bus service and feeder bus plus BART. Overall, the rail alternative is 50% more expensive than the bus. In Figure 24b the analysis is repeated on a high density settlement transport corridor. Note that bus flows remain within the feasible realm, while the rail alternative is still 30% more expensive.

From the above, it is clear that integrated bus service can match rail in meeting typical demands for corridor transportation at a much cheaper cost, while maintaining flexibility.<sup>42</sup> The performance of the bus rapid transit ways proposed for Atlanta and other U.S. cities, will be watched with keen interest (A.M.A., 1970; MARTA, 1973).<sup>43</sup>

In conclusion, a word of caution is in order regarding interpretation of these results. First, they are dependent upon Bay Area estimates of the shadow prices of wait, in-vehicle time and effort, etc.,

**FIGURE 24**

**OPTIMAL INTEGRATED SERVICE FROM SETTLEMENTS LOCATED  
ON A TRANSPORTATION CORRIDOR.**

PARAMETERS

$V_1 = 40$  mph       $V_{f \max} = 25$  mph  
 $v_v = \$3.00$        $v_w = \$9.00$   
 $C = 50$        $\delta = 2.5$  secs.  
 $S = 9$        $a_0 = 4000$  mph<sup>2</sup>  
 Demand = uniform

TOTAL COST

Bus      \$12,580  
 (BART    \$18,730)

(Feeder Bus + BART Average Cost  
in parentheses)

24 routes	12 routes	12 routes
12 bph/route	6 bph/route	4 bph/route
\$1.82/patron	\$3.20/patron	\$4.20/patron
(\$2.30/patron)	(\$4.85/patron)	(\$9.39/patron)

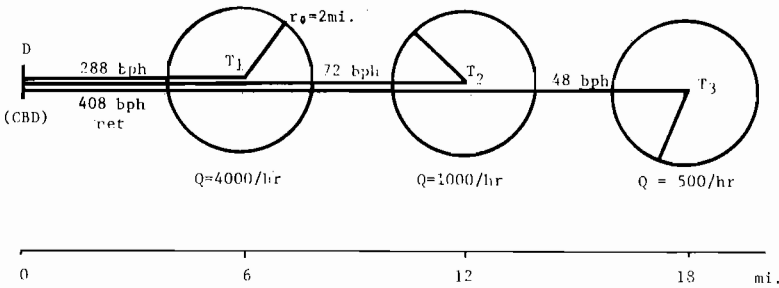


FIGURE 24b

OPTIMAL INTEGRATED SERVICE FROM (HIGH DENSITY) SETTLEMENTS  
LOCATED ON A TRANSPORTATION CORRIDOR.

PARAMETERS

$V_1 = 40$  mph     $V_{f \max} = 25$  mph  
 $v_v = \$3.00$      $v_w = \$9.00$   
 $C = 50$      $\delta = 2.5$  secs.  
 $S = 9$      $a_o = 4000$  mph<sup>2</sup>

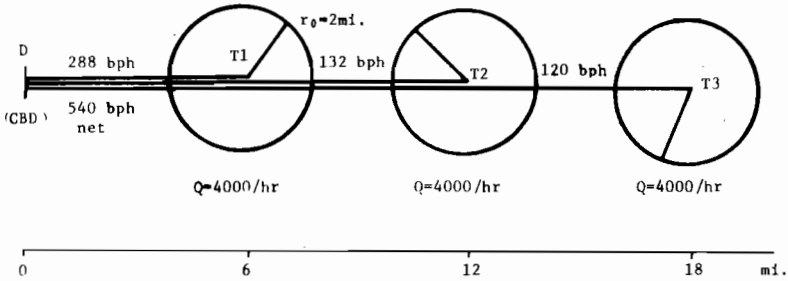
Demand = uniform

TOTAL COST

Bus    \$28,000  
 (BART    \$37,000)

(Feeder Bus + BART Average Cost  
in parentheses)

24 routes	12 routes	12 routes
12 bph / route	11 bph / route	10 bph / route
\$1.82 per patron	\$2.08 per patron	\$3.14 per patron
(\$2.30 per patron)	(\$3.10 per patron)	(\$3.85 per patron)



and these could vary with geographic region, although the relativity of the costs would be unlikely to be upset. Secondly, there is a measure of arbitrariness in the transfer time, and hence transfer cost. Finally, modal interactions are not explicitly accounted for: the approach is that of partial, as opposed to general, equilibrium analysis. The results should therefore be viewed as first round.





## 6. Conclusions

In this report we have constructed a supply-oriented framework of bus operations, and used it to "quantify" the factors that affect the short- and long-run costs of fixed route service. This framework has included the commuters' time and effort inputs into the production process, factors that transit companies do not formally incorporate into their scheduling considerations. Hence section 3 has indicated that headways during the off-peak should be significantly shorter than typically supplied. Such a measure would also reduce dead hauling and under utilization of crews.

Further, an analysis of stochastic delay shows that marginal cost pricing of point-to-point linehaul with standard buses to require a zero fare in the off-peak and a fare between 10 and 20 cents (with optimal scheduling) during the peak. Departure from optimality during this rush hour period raises average cost considerably. When this corresponds to an under-supply of buses, stochastic delay effects are generally very significant, and the short-run marginal cost fare is very steep -- usually more than an order of magnitude greater than the optimal rate above. For a service operating both in short- and long-run equilibrium, the subsidy payable should be about 10% of the average cost of the operation. This is similar to other estimates made for public transit (e.g., Vickrey, 1955).

The peak fares above would be raised somewhat if intermediate stops are permitted, because the marginal user would impose additional delays on his or her fellow travelers by alighting or boarding ("own bus effect"), as well as slowing the return of buses to make other commuter

runs ("system effect"). However if highway users are not similarly congestion-tolled, a second-best solution would require a fare that makes the price less than the marginal cost. In this case the optimal values above provide an estimate of the upper bound.

If transit in its own right is to be an effective competitor with door-to-door modes, it is important that agencies make a straight tradeoff between their own costs and all the consumer's time and effort costs, including wait and transfer. This means that both the spatial density of routes, and the schedule frequency on these routes, need to be considered simultaneously. This is precisely what has been done in section 5, where it was shown that optimized integrated service can be cost-competitive with the automobile. This work suggests that there is a single combination of route coverage, stop spacing and headway, constituting an optimal service for a given area.

Perhaps unexpectedly, the automobile fares well in cost terms, especially as a direct commute vehicle over the short haul or as a link mode to a linehaul bus operating express service on a freeway. This is in large measure due to the fact that it incurs no walk (or transfer) costs. However if priced to take account of its significant spillover costs of highway congestion, it would lose its competitive edge as a direct mode, in the peak, but would remain one of the cheaper means of making off peak trips.

On the other hand, the rapid rail combination does not show up well at all. Even in high density corridors the bus alternative has a sizeable cost advantage. Accordingly, there would seem to be very few cities in North America where transit flows could not be handled by a bus system. In practice the cost disadvantage to rapid rail is further accentuated by its emphasis on high speed linehaul service to reduce

run time, which is at the expense of the time required to join the system. This strategy seems misplaced in view of the fact that riders put a higher price on their out-of-vehicle (link) time than time spent in the vehicle.

But aside from their cost advantages over rail, buses have a far greater operational flexibility. Whereas investment in rapid rail systems builds in a high degree of rigidity, bus systems are adaptable to changes in urban development, travel patterns and new transport technology. In addition optimal service levels can be derived by an interplay of both theoretical considerations and field experimentation, in a way that is clearly not possible with rail. Buses are also able to combine both the feeder and linehaul functions.

But although it may be conceded that bus systems are more economical and flexible, the objection is sometimes raised that they are a "second class transportation" (Wall Street Journal, 1974); with comfort characteristics inferior to those of rapid rail coaches. In the Bay Area the comparison would be between the rather plain buses used by AC Transit and the plushness of the new BART cars. However the recently formed bus system operated by Golden Gate Bridge Highway & Transportation District has demonstrated that this image can be reversed. The district runs a fleet of standard coaches with luxury seating and pleasing decor<sup>44</sup> which appears to have met with success in appealing to the high income market of Marin and Sonoma counties. The costs of upgrading a system to this standard do not appear excessive: Appendix A-7 shows that the additional expense of running Golden Gate buses in lieu of AC buses amounts to, on average, only 5¢ per passenger with optimal scheduling. Hence allowing for this improved comfort in our calculations, would still leave buses with a competitive edge over the rail alternative.

Lately both small and large (articulated) buses have been catching the eye of transit companies. There appears to be a widely held belief that by buying these non-standard size vehicles, operational efficiency could be improved. This is not supported by our results: except for extremes in demand, no economies derive from the operation of small or large buses over fixed routes in lieu of standard size buses. This is in part the result of a disproportionately lower capital cost of the standard bus, arising from mass production scale economies. While it is true that similar price advantages could accrue to other size vehicles, should a very strong market develop for them, this is in the realm of the hypothetical. The addition of different size (brand) vehicles to a fleet may also lead to inefficiencies in maintenance and safety. As small buses cannot effectively be used on arterial routes and large buses on feeder service, buying non-standard buses would therefore seem to be unnecessarily restrictive while offering no cost advantages. But we should add the qualification that these findings are no better than the quality of the available cost data. With more and better data our conclusions would be surer.

In an age of decidedly scarce transportation resources it is important that some basis for deciding among competing modal alternatives be found. We have proposed and explored an economic model of the provision of bus transit services, and have broadened the framework to furnish such intermodal cost comparisons. It is our feeling that an economic analysis can succeed in clarifying many of the issues involved, even if it cannot, without some indication of society's preferences, solve them. It is to this essentially preliminary task that we hope to have made a contribution.

## APPENDIXES



## APPENDIX A-1

Multiple Frequency Bus Service -  
Derivation of Average Frequency Delay and Ridership

(ALGORITHM SRAC1)

(1) The frequency delay in the nth inter-bus interval is the weighted average over g groups. Viz:

$$d_n = \frac{\sum_{i=1}^g q_{i,n} d_{i,n}}{\sum_{i=1}^g i,n}$$

Where  $d_{i,n}$  is the frequency delay of ith patron group in nth interval:<sup>45</sup>

$$d_{i,n} = \frac{\sum_{T=(n-1)\phi+1}^{n\phi} (n\phi-T) p(i,T) \Delta T}{\sum_{T=(n-1)\phi+1}^{n\phi} p(i,T) \Delta T}$$

in which

n is the inter-bus interval,

$\phi$  is the number of integration intervals of duration  $\Delta T$  per inter-bus interval

$p(i,T)$  is the probability of arrival in the (integration) interval T of a patron of the ith type

and  $q_{i,n} = Q_i \sum_{T=(n-1)\phi+1}^{n\phi} p(i,T) \Delta T$  is the number of arrivals in the

nth inter-bus interval of the ith patron type.



The average frequency delay is therefore the frequency delays for each bus, weighted according to the number waiting in each inter-bus interval.

$$\begin{aligned} \bar{d}(\psi) &= \frac{\sum_{n=1}^{1/\psi} \left\{ \sum_{i=1}^g q_{i,n} \right\} \cdot d_n}{\sum_{n=1}^{1/\psi} \left\{ \sum_{i=1}^g q_{i,n} \right\}} \\ &= \frac{1}{Q} \sum_{n=1}^{1/\psi} \sum_{i=1}^g q_{i,n} d_{i,n} \end{aligned}$$

Where  $Q$  is the total number of riders (patronage) of the service.<sup>46</sup>

The average number of passengers per bus is therefore

$$\bar{q}(\psi) = \psi \sum_{n=1}^{1/\psi} \sum_{i=1}^g q_{i,n}$$

## APPENDIX A-2

Multiple Frequency Bus Service with Capacity Constraints -  
Derivation of Average Schedule Delay and Ridership

(ALGORITHM SRAC2)

(1) The schedule delay of the boarders of the  $m$ th bus, who arrive in the  $n$ th inter-bus interval, is:

$$d_{n,m} = d_m + \{n-m\} \cdot \{\phi \cdot \Delta T\},$$

where:

$\{n-m\} \cdot \{\phi \Delta T\}$  is the "stochastic delay,"

and

$$d_m = \frac{\sum_{i=1}^g q_{i,n} d_{i,m}}{\sum_{i=1}^g q_{i,m}} \quad \text{is the "frequency delay."}$$

$d_{i,m}$  is the associated frequency delay of the  $i$ th patron group, and has the form of  $d_{i,n}$ , except that the summation commences from the integration interval beyond which arrivals miss the next scheduled departure.

Therefore the schedule delay of the boarders of the  $n$ th bus is represented by the weighted average

$$d_n = \frac{\sum_{m=1}^{n'} q_{n,m} d_{n,m}}{\sum_{m=1}^{n'} q_{n,m}}$$

where the constraint  $n' \leq n$  applies, such that

$$\sum_{m=1}^{n'} q_{n,m} \leq C$$

in which  $C$  is the bus capacity, and  $q_{n,m}$ , the number of boarders of the  $n$ th bus who were arrived in the  $m$ th interbus interval, afterwards  $q_{n,m} = 0$ , for all  $n'$  for which total absorption takes place, and if a residue for  $q_{n,n'}$  results, then it is reset to

$$q_{n,n'} = \sum_{m=1}^{n'} q_{n,m} - C$$

SRAC2 commences allocation from the first (interbus) interval and forms a patron queue with priority on a "first-come-first-served" basis.

That is in:  $\sum_{m=1}^{n'} q_{n,m} \leq C$  the allocation/addition is from the

first interval ( $m=1$ ) to the  $n'$ th interval.

The average schedule delay is the individual schedule delay of the boarders of each bus, weighted according to their respective numbers.

$$\text{i.e.: } \bar{d}(\psi) = \frac{\sum_{n=1}^{1/\psi} \left\{ \sum_{m=1}^{n'} q_{n,m} \right\} d_n}{\sum_{n=1}^{1/\psi} \sum_{m=1}^{n'} q_{n,m}} = \frac{1}{Q} \sum_{n=1}^{1/\psi} \sum_{m=1}^{n'} q_{n,m} d_{n,m}$$

where  $Q$  is the number of boarders (riders).<sup>47</sup>

Finally the average number of passengers per bus is

$$\bar{q}(\psi) = \psi \sum_{n=1}^{1/\psi} \sum_{m=1}^{n'} q_{n,m} = Q\psi$$

## APPENDIX A-3

Multiple Frequency Bus Service with Multiple Stops  
Derivation of Agency and User Costs

When more than one stop is introduced, the need is to estimate cost components associated with the average journey. Both Agency and Consumer (in-vehicle time) costs are affected by the distribution and number of collection points along the route.

In general, this overall average trip cost is given by

$$\bar{C}(\psi) = \frac{1}{Q} \sum_{n=1}^S \frac{1/\psi}{\sum_{s'=1}^S} q_{n,s'} \sum_{s=s'}^S c_{n,s}$$

Wherein  $c_{n,s}$  = some specific cost incurred with the nth bus from the sth to the s-lth stop,

and  $q_{n,s'}$  = the number of passengers boarding at s'th stop.

$$\text{viz: } q_{n,s'} = \sum_{i=1}^g q_{i,n,s'}$$

(1) The agency cost per trip from s'th stop is experienced by  $q_{n,s'}$  boarders i.e.,

$$C_{n,s} = \text{TDC}_{n,s} / q_{n,s'}$$

Hence substituting in the general equation above we get

$$\begin{aligned} \bar{C}(\psi) &= \frac{1}{Q} \sum_{n=1}^S \frac{1/\psi}{\sum_{s'=1}^S} \sum_{s=s'}^S \text{TDC}_{n,s} \\ &= \frac{1}{Q} \sum_{n=1}^S \frac{1/\psi}{\sum_{s=1}^S} \sum_{s'=1}^s \text{TDC}_{n,s} \end{aligned}$$

(2) The in-vehicle cost component from the  $s$ 'th stop is

$$\sum_{s=s'}^S C_{n,s} = v_v \sum_{s=s'}^S \frac{\Delta L_s}{V_{n,s}} \quad \text{i.e., } C_{n,s} = v_v \frac{\Delta L_s}{V_{n,s}}$$

Therefore the average in-vehicle cost is:

$$\begin{aligned} \overline{CV}(\psi) &= \frac{v_v}{Q} \frac{1/\psi}{\sum_{n=1}^S} \sum_{s'=1}^S q_{n,s'} \sum_{s=s'}^S \frac{\Delta L_s}{V_{n,s}} \\ &= \frac{v_v}{Q} \frac{1/\psi}{\sum_{n=1}^S} \sum_{s=1}^S \left\{ \sum_{s'=1}^s q_{n,s'} \right\} \frac{\Delta L_s}{V_{n,s}} \end{aligned}$$

Where

$$V_{n,s} = V_{n,s'} = \frac{\Delta L_{s'}}{\theta_{s'} + q_{n,s'} \cdot \delta}$$

$$\text{wherein } \theta_{s'} = \frac{v_{\max}}{a_0} + \frac{\Delta L_s}{v_{\max}}, \quad \text{for } \Delta L_{s'} < \frac{v_{\max}^2}{a_0},$$

$$\text{otherwise, } \theta_{s'} = 2 \left[ \frac{\Delta L_{s'}}{a_0} \right]^{1/2},$$

$\delta$  is the loading time per passenger,

$a_0$  is the acceleration/deceleration of the bus,

$v_{\max}$  is its maximum speed, and

$\Delta L_s$  is the distance between  $s$ 'th and  $s'-1$ th stops.

(3) Average frequency delay cost. This does not involve an averaging process with respect to an average journey.

The frequency delay in the  $n$ th interbus interval over the overall route (viz. over  $S$  stops) is

$$d_n = \sum_{s'=1}^S q_{n,s'} d_{n,s'}$$

Then the weighted average over all interbus intervals (buses) reduces to

$$\bar{d}(\psi) = \frac{1}{Q} \sum_{n=1}^{1/\psi} \sum_{s'=1}^S q_{n,s'} d_{n,s'}$$

Hence the Average Frequency Delay or Wait Cost is

$$\overline{CS}(\psi) = \frac{v_w}{Q} \sum_{n=1}^{1/\psi} \sum_{s'=1}^S q_{n,s'} d_{n,s'}$$

SRAC3 embodies all the above features with the capacity constraint of SRAC2.

## APPENDIX A-4

Automobile (Variable) Costs

Source: Keeler, Cluff and Small (1974). Revised in an unpublished manuscript by Small, September 1974.

Mileage Costs (per Vehicle mile)

<u>Standard Car</u>	<u>Freeway - 50mph</u>	<u>Urban Arterial - 25mph</u>
Maintenance	\$.0444	\$.0567
Accidents	.0150	.0360
Roads	.0195	.0195
Capital	.0495	.0495
TOTAL	.1284	.1617

Subcompact Car

Maintenance	\$.0282	\$.0360
Accidents	.0205	.0491
Roads	.0195	.0195
Capital	.0233	.0233
TOTAL	.0915	.1279

## APPENDIX A-5

Dial-a-Ride Costs (Haddonfield System)

Source: Clemons (1974)

Hourly Costs

Conducting Transportation (included dispatch, drivers wages, etc.)	\$ 9.65
Advertising	.44
	<hr/>
	\$10.09/ hr.

Mileage Costs

Fuel and Oil	\$ .0308
Claims	.0475
Maintenance	.1125
Capital (at 6%)	.1100
	<hr/>
	\$ .3008/ mile

To adjust for the fact that both average trip length and duration are subject to considerable uncertainty with dial-a-ride service (viz: its meandering nature) the above costs are inflated by a circuitry factor of 1.5 (MITRE, 1974) so that hourly costs become \$15.14 per hour and mileage costs, \$0.4512/ mile.



## APPENDIX A-6

Feeder Bus + BART (Minimum) Costs

Pozdena (1974) has set up a framework similar to ours for optimizing the provision of BART services, and like us, trades off agency and consumer (walk, wait and in-vehicle time) components. The result of his optimization procedure is an equation of the form

$$AC_{\min} / \text{hr of operation} = a + b Q^{-1/2} + c Q^{-1/3} + d Q^{-1}$$

where a, b, c and d are determined by the linehaul distance of the trip. The first term in this equation is the transfer and linehaul and feeder in-vehicle time; the second accounts for the linehaul wait and agency cost; the third represents the feeder wait, walk and agency cost; and the fourth term is the fixed rail cost. Since BART routes are fixed, the third term does not vary with linehaul distance. In our comparisons we are interested in trips of 6, 12, and 18 miles: for these cases the coefficients are as follows:

<u>Distance</u>	<u>Coefficient</u>			
	a	b	c	d
6 mi.	.70	31.00	15.40	570.00
12 mi.	1.15	44.30	15.40	1140.00
18 mi.	1.60	54.30	15.40	1710.00

Given these values, the minimum average cost of a BART trip can be calculated.

The question now arises, whether replication of the BART system would incur similar costs. Two suggestions are relevant here: first, that BART has experienced huge (and well-publicized) cost over-runs; these, it might be claimed, need not occur in other such projects. Secondly, many of BART's costs are thought to be developmental in nature, that is to say, costs associated with developing a novel system. Such costs, if any, would not be incurred by potential replicators. With respect to the first suggestion, Merewitz (1972) has compared BART's 45% cost over-run with over-runs on other projects. He finds that "there is no evidence to indicate that [BART's cost-estimating experience] is appreciably different from [other] transit projects in the United States and Europe." Hence, there is reason to suppose that similar cost over-runs would be experienced in replication experiments. On the second point, Merewitz and Pozdena (1974) propose a model for a long-run cost function for rapid transit properties. They estimate this model for a cross-section of such properties, and find (in their Model 5) that the predicted value is "fairly close" to the actual cost figure for BART. Thus in this case too there is the feeling that many of BART's costs would be incurred in replication attempts; though of course all the evidence is not in.

## APPENDIX A-7

Upgrading the Comfort Characteristics Typical of an AC Transit Bus:Golden Gate Cost Comparison

From the operating costs of the Golden Gate Bridge Highway and Transportation District standard size bus system (Viton 1974) we have as an estimate of total direct cost:  $TDC_{GGT} = \$11.9_2 \frac{L}{V} + .327_2 L$ , where L is the route length and V the service speed. Writing  $TDC_{ACT}$  as the total direct cost of the comparable expression derived in section 2 for a 50 seat bus, we can draw the following comparison between agency costs:

<u>Service Characteristics</u>		<u>TDC<sub>GGT</sub></u>	<u>TDC<sub>ACT</sub></u>	<u>Difference</u>
V	L			
40 mph;	6 mi.	\$ 3.75	\$ 3.19	\$ .56
	12 mi.	7.50	6.39	1.11
	18 mi.	11.26	9.59	1.67
20 mph;	.5 mi.	\$ .46	\$ .38	\$ .08
	1.0 mi.	.92	.67	.25
	2.0 mi.	1.84	1.34	.50

Assuming various levels of demand Q, and our usual values of  $v_v = \$3.00$  and  $v_w = \$9.00$ , then the following sample estimates of average cost under optimal scheduling can be derived from (28) or (33):

CASE 1. L = 18 miles, V = 40 mph.

<u>Demand level Q</u>	<u>AC<sub>min</sub> - GGT</u>	<u>AC<sub>min</sub> - ACT</u>	<u>Difference</u>
100	\$2.77	\$2.67	\$ .10
500	1.99	1.93	.06
1000	1.80	1.77	.03
5000	1.64	1.59	.05

CASE 2. L = 6 miles, V = 40 mph.

<u>Demand level Q</u>	<u>AC<sub>min</sub> - GGT</u>	<u>AC<sub>min</sub> - ACT</u>	<u>Difference</u>
100	\$1.27	\$1.21	\$ .06
500	.81	.79	.02
1000	.71	.69	.02
5000	.57	.56	.01

That is, on average, the cost difference between operating with the "plain" (ACT) or "luxury" (GGT) bus amounts to no more than 5¢ per passenger.





```

C      IN-VEHICLE SHADOW PRICE -- VV
      VV=3.00
C
C *****
C
C      DERIVED DATA
C
C      DR IS THE DISTANCE BETWEEN STOPS ON FEEDER LEG
      DR=RO/FLOAT(S-1)
C      CORRECTIONS TO STATIONARY TO STATIONARY TIME DUE TO ACCELERATION
      AND BRAKING.
      TR=VELFF/AO+DR/VELFF
      IF((DR-VELFF**2/AO).LE.0.)TR=2.*SQRT(DR/AO)
      TR1=TR
C      INTEGRATION INTERVAL
      DT=1./FLOAT(N)
C *****
C
C      GENERATION OF ARRIVAL DISTRIBUTION
C
C      UNIFORM DEMAND OPTION
      DO 90 J=1,N
90 P(J,1)=1
      IF(G.EQ.1)GO TO 91
C      GC IS THE CONTROL ON THE NUMBER OF USER TYPES -- G
      GC=4
C      TAU IS THE DURATION IN WHICH THE I TH USER TYPE ARRIVE AT STOP(S)
      ITAU=12
      TAU=FLOAT(ITAU)/FLOAT(N)
      G=(N-ITAU)/GC+1
      M=0
      RHO=.01
      AA=1.-(1.-TAU)*RHO
      DO 9 I=1,G
      M=M+1
      DO 9 J=1,N
      TT=1./FLOAT(N)*(.5+FLOAT(J-1))
C      TRANSLATE (QUADRATIC) PROBABILITY OF ARRIVAL FUNCTION TO G
C      DIFFERENT POSITIONS OVER SERVICE INTERVAL
      T=TT
      IF(I.GT.1)T=TT-(1.-TAU)*FLOAT(I-1)/FLOAT(G-1)
      P(J,M)=(6.*AA/(TAU**3))*T*(TAU-T)
      IF(P(J,M).LT.0.)P(J,M)=RHO
9 CONTINUE
C *****
C
C      VARY NUMBER OF FEEDER ROUTES
91 R1=R1/2
      DO 3 IR=1,RSTEP
      R1=2*R1
      R=R1
      PRINT 101
      PRINT 779,S,RO,L,VELLL,VELFF

```

```

C      PRINT 1779, AO
C
C *****
C      DERIVE FEEDER DEMAND LEVEL FROM NET
C      DO 158 I=1,G
159  QG(I)=FLOAT(QG1(I))/R+.5
C      AQP(I)=0
C      HOURLY DEMAND LEVEL
C      K=1
C      DO 200 KT=1,N
C      IF((TSI*FLOAT(KT)/FLOAT(N)).GT.FLOAT(K))GO TO 201
C      GO TO 202
201  K=K+1
C      AQP(K)=0
202  DO 200 I=1,G
C      AQP(K)=AQP(K)+P(KT,I)*DT*FLOAT(QG(I))
200  QPH(K)=AQP(K)
C      CALCULATE ANGLE OF FEEDER SECTOR
C      PHI=3.14159/R
C      NET DEMAND
C      QP=0
C      DO 110 I=1,G
110  QP=QP+QG(I)
C
C *****
C      SPATIAL USER COST
C      COMPUTE MEAN DISTANCE TO SS TH BUS STOP
C
C      DISCOR IS THE CORRECTION TO WALK FOR NEED TO TAKE CARTESIAN PATHS
C      DISCOR=1.3
C      DO 2000 S=1,SS
C      NX=60
C      IF(SS.EQ.1)GO TO 1223
C      A1=0
C      AA1=0
C      MEAN WALK TO FEEDER BUS STOP
C      DX=DR/FLOAT(NX)
C      DY=DX
C      DO 1221 IX=1,NX
C      X= .5*FLOAT(2*IX-1)*DX-DR/2.
C      NY=((DR*(FLOAT(SS)-.5)+X+DR/2.)*TAN(PHI))/DX
C      DO 1221 IY=1,NY
C      Y= .5*FLOAT(2*IY-1)*DY
C      PD=1
C      A1=A1+PD*DX*DY
1221 AA1=AA1+PD*SQRT(X**2+Y**2)*DX*DY
C      MEAN DISTANCE WALKED TO SS TH STOP
C      OBAR(SS)=AA1/A1*DISCOR
C      GO TO 2000
C
C      1223 A2=0
C      AA2=0
C      DX=.5*DR/FLOAT(NX)
C      DY=DX

```



```

DO 1222 IX=1,NX
X = .5*FLOAT(2*(X-1)*DX
NY=X*TAN(PHI)/DX
DO 1222 IY=1,NY
PD=1
Y=.5*FLOAT(2*(IY-1)*DY
A2=A2+PD*DX*DY
1222 AA2=AA2+PD*SQRT(X**2+Y**2)*CX*DY
DBAR(SS)=AA2/A2*DISCOR
2000 CONTINUE
NR=R
PRINT 2779,NR
PRINT 2779,(DBAR(SS),SS=1,S)

C
C
C *****
C
NBUSCP=BUSCP1
PRINT 777,NBUSCP
PRINT 130
PRINT 778,(QPH(M),M=1,K)
PRINT 17

C
C *****
C
VARY REGULARIZED HEADWAY (1/PSI)

(UNLESS OTHERWISE INDICATED *INTERVAL*=INTERBUS INTERVAL*).
DO 3 PSI=1,N
EXCLUDE HEADWAYS THAT DO NOT COINCIDE WITH INTEGRATION INTERVALS
IF(MOD(N,PSI).NE.0) GO TO 3
NI=N/PSI
HI=FLOAT(PSI)/FLJAT(N)
DO 30 SS=1,S
DO 30 J=1,NI
30 TEST(J,SS)=.TRUE.
CC=0
CV=0
CS=0
CW=0
QI=0
THE TA1=0

C
C *****
C
TREAT BUS ARRIVING AT END OF NBI TH INTERVAL
DO 12 NBI=1,NI
KT1=(NBI-1)*PSI+1
KT2=NBI*PSI
C
ALLOCATE PASSENGERS TO BUS ARRIVING AT END OF NBI TH INTERVAL
C
C *****
C
CCC=0
CVV=0
CSS=0

```

```

CWH=0
Q11=0
C   SFT BUSCAP TO CAPACITY OF (EMPTY) BUS AT START OF RUN
BUSCAP=BUSCP1
C   SCAN OVER STOPS FOR NBI TH BUS
DO 120 SS=1,S
C   DISTRIBUTE QG(I) OVER S STOPS
QGS(1)=FLOAT(2*QG(1))/FLOAT(2*S-1)+.5
IF(SS.EQ.1)QGS(1)=FLOAT(QGS(1))/2+.5
C
C
C *****
C   SCAN FROM FIRST INTERVAL TO (NBI -1)TH INTERVAL TO PICK UP NON-
C   BOARDERS, ALLOCATION PRIORITY INCREASES WITH DECREASING NBI1
C   (FIRST COME-FIRST SERVED)
TRIG=.FALSE.
C1=0
IF(NBI.EQ.1)GO TO 212
N11=NBI-1
DO 210 NBI1=1,N11
IF(TRIG)QNC1(NBI1,SS)=0
IF(TRIG) GO TO 210
TEST(NBI1,SS)=.FALSE.
C1=C1+QNC(NBI1,SS)
C   BUS CAPACITY CCNSTRANT
IF(C1.GT.BUSCAP)GO TO 211
C   EXCESS PASSENGERS ABSORBED
QNC1(NBI1,SS)=QNC(NBI1,SS)
C   RESET PASSENGERS TO RECORD ABSORBTION
QNC(NBI1,SS)=0.
GO TO 210
C   OVERFLOW PASSENGERS ABSORBED
211 QNC1(NBI1,SS)=QNC(NBI1,SS)-(C1-BUSCAP)
C   (OVERFLOW) PASSENGERS NOT ACCOMODATED
QNC(NBI1,SS)=C1-BUSCAP
TRIG=.TRUE.
KT1=KT1
210 CONTINUE
C
C *****
C   FOCUS ATTENTION ON INTERVAL AT HAND (VIZ. NBI TH)
C   BUS FULL .... GO TO 1003 TO RECORD NCN-BOARDERS ORIGINATING IN
C   THIS NBI TH INTERVAL
IF(TRIG)GO TO 1003
C   OVERFLOW ZERO OR LESS THAN BUSCP1.....
C   THEN SEE HOW MANY CAN ACCOMODATE FROM THIS NBI TH INTERVAL
C   FIND TIME INTERVAL CORRESPONDING TO ACHIEVMENT OF FULL BUS LOAD
212 TRIG=.FALSE.
KT22=KT2
B1=C1
DO 1000 KT=KT1,KT2
IF(TRIG) GO TO 1000
DO 1100 I=1,G
1100 B1=B1+P(KT,I)*FLOAT(QGS(I))*DT

```

```

C      BUS CAPACITY CONSTRAINT
C      IF (B1.LT.BUSCAP) GO TO 1000
C      KT2=KT
C      TRIG=,TRUF.
1000 CONTINUE
C
C *****
C
C      COMPUTE SCHEDULE DELAY OF BOARDERS WHO ARRIVE / LEAVE IN THIS
C      NBI TH INTERVAL
C      QC(NBI,SS)=0.
C      DC(NBI,SS)=0.
C      DO 11 I=1,G
C      A=0
C      B=0
C      B1=0
C      DO 10 KT=KT1,KT2
C      B1=B1+P(KT,I)*FLOAT(QGS(I))*DT
C      B=B+P(KT,I)
C      10 A=A+(FLOAT(NBI*PSI-KT)/FLOAT(N)*P(KT,I)
C      DC(NBI,SS)=DC(NBI,SS)+(A/B+DT*FLOAT(KT2-KT22))*B1
C      11 QC(NBI,SS)=QC(NBI,SS)+B1
C      SCHEDULE DELAY FOR BUS ARRIVING AT END OF NBI TH INTERVAL
C      DC(NBI,SS)=DC(NBI,SS)/QC(NBI,SS)
C
C *****
C
C      COMPUTE FREQUENCY DELAY OF NBI TH INTERVAL NON-BOARDERS (VIZ.
C      THOSE WHO ARRIVE TOO LATE, AND MISS OUT ON ARRIVAL OF BUS AT THE
C      END OF INTERVAL)
C      KT11=KT22+1
C      THERE MAY BE ZERO NON-BOARDERS ... CHECK
1003 DNC(NBI,SS)=0.
C      QNC(NBI,SS)=0.
C      IF (KT11.GE.KT2) GO TO 1004
C      DO 810 I=1,G
C      A=0
C      B=0
C      B1=0
C      DO 108 KT=KT11,KT2
C      B1=B1+P(KT,I)*FLOAT(QGS(I))*DT
C      B=B+P(KT,I)
C      108 A=A+FLOAT(NBI*PSI-KT)/FLOAT(N)*P(KT,I)
C      FREQUENCY DELAY WITHIN NBI TH INTERVAL OF NON-BOARDERS
C      DNC(NBI,SS)=DNC(NBI,SS)+A/B*B1
C      810 QNC(NBI,SS)=QNC(NBI,SS)+B1
C      FREQUENCY DELAY OF NON-BOARDERS
C      DNC(NBI,SS)=DNC(NBI,SS)/QNC(NBI,SS)
C
C *****
C
C      COMPUTE SCHEDULE DELAY OF PEOPLE SUCCESSFULLY PICKED UP AT END
C      OF THIS NBI TH INTERVAL
C      SCAN THROUGH INTERVALS TO PICK UP NON-BOARDERS
1034 D=0
C      Q=0

```

```

DO 610 NBI1=1,NBI
C   JUMP OVER RIDERS NOT ACCOMODATED ON BUS AT NBI TH INTERVAL, SS TH
C   STOP
C   IF(NBI1.EQ.NBI)GO TO 611
C   IF(TEST(NBI1,SS)) GO TO 610
C   IF TEST(NBI1,SS)=TRUE THEN RIDERS OF NBI1 TH INTERVAL AT SS TH
C   STOP MISSED OUT
C   DF = FREQUENCY DELAY
C   DF= DNC(NBI1,SS))
C   DS = STOCHASTIC DELAY
C   DS= B1*FLOAT(NBI-NBI1)
C   D=D+(DF+DS)*QNC1(NBI1,SS)
C   GO TO 610
611 IF(TRIG1) GO TO 610
C   ADD ON THIS INTERVALS NBI TH CONTRIBUTION IF APPLICABLE
C   D=D+DC(NBI1,SS)*QC(NBI1,SS)
C   Q = THE NUMBER OF PASSENGERS BOARDING BUS A SS TH STOP AT THE
C   END OF THE NBI TH INTERVAL (L.E. BUSCAP)
C   Q=Q+QC(NBI1,SS)
610 CONTINUE
C
C *****
C   RESET BUS CAPACITY FOR NEXT STOP
C   BUSCP2=BUSCAP
C   BUSCAP=BUSCAP-Q
C   IF(BUSCAP.LT.C.)BUSCAP=0
C
C *****
C   D/Q IS THE AVERAGE SCHEDULE DELAY OF PASSENGER RIDING ON BUS AT END
C   OF THE NBI TH INTERVAL
C   Q1 IS THE PROGRESSIVE TOTAL OF PASSENGERS (BY STOP) BOARDING
C   BUS AT END OF NBI TH INTERVAL
C   Q1=Q1+Q
C   Q11 IS THE PROGRESSIVE TOTAL OF PASSENGERS CARRIED BY BUS AT END
C   OF THE NBI TH INTERVAL
C   Q11=Q11+Q
C
C *****
C   COMPUTE MEAN INTERSTOP SPEED ON LOCAL PORTION OF ROUTE
C   ASSUME PSI REPRESENTS AVERAGE HEADWAY - NO CORRECTION ON HEADWAY
C   FOR LOADING (DELAYS) AND PASSING STOPS (RUN EARLIES)
C   PASS STOPS IF FULLY LOADED
C   TR=TR1
C   IF(BUSCAP.EQ.C.)TR=DR/VELFF+.5*VELFF/AC
C   IF(BUSCAP.EQ.O..AND.BUSCP2.EQ.O.)TR=DR/VELFF
C   VELF=DR/(TR+Q*DELTA)
C   EXCLUDE LOADING TIME CONSIDERATIONS FROM FIRST STOP
C   IF(SS.LT.S.AND.SS.EQ.1)VELF=DR/TR
C   INCLUDE LOADING CONSIDERATIONS INTO LINEHAUL FROM S TH STOP
C   IF(SS.EQ.S)VELL=L/(L/VELL+Q*DELTA)
C   IF(SS.LT.S)THETA1=THETA1+CR/VELF
C   IF(SS.EQ.S)THETA1=THETA1+L/VELL

```

```

C *****
C
C   TRIG2=.TRUE.
C   DO 122 NR[1]=1,NBI
C     TFST FOR OVERFLOW
C     IF(ONC(NR11,SS).NE.0.)TRIG2=.FALSE.
C     TFST FOR NO OVERFLOW FROM EXISTING INTERVAL
C     IF(TRIG) GO TO 122
C     XNRT=(FI(OT(NBI))*8I*TSI+7.
C     IF(TRIG2) PRINT 700,XNBI,PSI
C 122 CONTINUE
C
C *****
C
C   COMPUTE AGENCY COST PER PATRON PER BUS (AT END OF NBI TH
C   INTERVAL)
C   TDCF IS COMPONENT OF TDC PER INTERSTOP DISTANCE
C   IF(SS.FQ.S)GO TO 1445
C   IF(BUSCP1.EQ.25.)TDCF=-.3073*DR+8.10*DR/VELF
C   IF(BUSCP1.EQ.50.)TDCF=-.2987*DR+9.28*DR/VELF
C   IF(BUSCP1.EQ.75.)TDCF=-.4749*DR+10.98*DR/VELF
C   IF(SS.LT.S)GO TO 1444
C 1445 TDCI IS THE COMPONENT OF TDC PER LINEHAUL DISTANCE
C 1445 CONTINUE
C   IF(BUSCP1.EQ.25.)TDCL=-.3073*L+8.10*L/VELL
C   IF(BUSCP1.EQ.50.)TDCL=-.2987*L+9.28*L/VELL
C   IF(BUSCP1.EQ.75.)TDCL=-.4749*L+10.98*L/VELL
C 1444 CONTINUE
C
C   AGENCY COST
C   FEEDER COMPONENTS
C   IF(SS.LT.S)CCC=CCC+TDCF
C   ADD LINEHAUL COMPONENT
C   IF(SS.EQ.S)CCC=CCC+TDCL
C
C   WFIGHTED IN-VEHICLE TIME
C   FEEDER COMPONENTS
C   IF(SS.LT.S)CVV=CVV+DR/VELF*Q11
C   ADD LINEHAUL COMPONENT
C   IF(SS.FQ.S)CVV=CVV+L/VELL*Q11
C
C   WFIGHTED SCHEDULE DELAY
C   CSS=CSS+D
C
C   WFIGHTED WALK TIME
C   CWW=CWW+DBAR(SS)/3.*Q
C
C 120 CONTINUE
C   CC=CC+CCC
C   CV=CV+CVV
C   CS=CS+CSS
C   CW=CW+CWW
C 12 CONTINUE
C *****
C

```

```

TDC=CC/FLOAT(N1)
CC=CC/Q1
CV=CV/Q1*VV
CS=(CS/Q1+DT/2.)*VW*TSI
CW=CW/Q1*VW
AC=CC+CV+CS
TAC=AC+CW
D1=C/(VW*TSI)
THETA=THETA1/FLOAT(N1)
Q=Q1/FLOAT(N1)
QPT=QP*R

```

```

C *****
C

```

```

C RESULT WRITE-OUT
C RPSI IS PSI EXPRESSED IN MINUTES
C RPSI=((FLOAT(PSI)-.5)*DT+.5/FLOAT(N))*TSI*60.
C PRINT 7,N,DT,TDC,THETA,CP,QPT,VV,VW,RPSI,D1,Q,CC,CV,CS,AC,
C XCH,TAC
C 3 CONTINUE

```

```

C *****
C

```

```

101 FORMAT(1H1)
779 FORMAT(1H0,*NUMBER OF STOPS =*,I2,*FEEDER DISTANCE =*,F6.2,* LINE
1HAUL DISTANCE =*,F6.2,* LINE HAUL SPEED =*,F4.0,* MAXIMUM POSSIBLE
1 FEEDER SPEED =*,F4.0)
1779 FORMAT(1H,* ACCELERATION/DECELFRATION =*,F6.0)
2779 FORMAT(1H,*NUMBER OF FEEDER ROUTES =*,I3)
8779 FORMAT(1H,*MEAN WALK DISTANCE TO RESPECTIVE STOPS =*,9F5.2)
777 FORMAT(1H0,*BUS CAPACITY =*I3,/)
778 FORMAT(1H0,*HOURLY LEVEL OF DEMAND =*,10I18)
17 FORMAT(1H0)
100 FORMAT(* N DT TDC THETA DEMND TDEMND VV VW PSI
1 D Q CC CV CS AC CW TAC*)
790 FORMAT(*MISSED BUSES CEASE AT*,F5.2,*-HOURS FOR HEADWAY OF *,I4,*
1MINUTES*)
7 FORMAT(1H0,I4,F6.3,F6.2,F6.3,2I6,F6.0,F6.0,F6.2,8F8.3)

```

```

C *****
C

```

```

STOP
END

```



## FOOTNOTES

<sup>1</sup>For example on AC Transit's transbay route F, buses have an off-peak frequency of twelve minutes or more. A cost tradeoff between agency and consumer (wait and in-vehicle) inputs suggests an optimal frequency of about half of this, or even less when consumer walk is included. As for the peak, we are assuming that the companies have fleets sufficient to minimize stochastic delay (foregoing of buses).

<sup>2</sup>Blachman (1974) has examined returns to scale and factor substitution econometrically in a cross-section of bus transit companies, finding constant returns to scale (vehicle-miles) and fixed factor-proportions in production. Given that this is the case, the "accounting method" of cost allocation we use here will yield correct results.

<sup>3</sup>These buses are Twin Coach TC-31's fitted out with 25 seats. When marketed as commuter buses they have 31 seats.

<sup>4</sup>The Twin-Coach TC-25 bus marketed as a commuter bus has 25 seats.

<sup>5</sup>In the event of a system partially or fully changing over to smaller buses it cannot be envisaged that (existing) drivers would accept lower wages.

<sup>6</sup>For example:

$$\begin{aligned} & \left[ \begin{array}{cc} \text{Utility} & \text{Disutility} \\ \text{(privacy, comfort)} & \text{(congested driving)} \end{array} \right] \text{ car} \\ & = \left[ \begin{array}{cc} \text{Utility} & \text{Disutility} \\ \text{(reading paper)} & \text{(no seat, crowding)} \end{array} \right] \text{ bus} \end{aligned}$$

<sup>7</sup>Chan's sample relates to commutation trips only.

<sup>8</sup>Viz: a representative period of service.

<sup>9</sup>Logit work in this area suggests that  $v_w \approx 3 v_v$ .

<sup>10</sup>This is equivalent to setting LRMC = SRMC.



<sup>11</sup>This is a sufficient, but not a necessary condition of arrival independent of headway. The simplest representation of random arrival is  $p[t]=1$ , which will therefore be used throughout much of this paper.

<sup>12</sup>Hereafter "AC[ $\psi$ ]" shall be denoted by "AC."

<sup>13</sup>It is necessary to assume that the patron has a finite but infinitesimal probability of arrival outside this period (a not unreasonable assumption), to avoid indeterminacy in the solution.

<sup>14</sup>The average frequency delay is not affected by regular scheduling with an irregular distribution of arrivals.

<sup>15</sup>Again, equivalent to setting SRMC = LRMC.

<sup>16</sup>Here Q is the number of boarders, and is identical to the demand if the catchment is cleared.

<sup>17</sup>No adjustment of TDC has been made in this case for vehicle size.

<sup>18</sup>Approximated by a probability of arrival function shared by all riders, such that  $p[t] = .98$  for  $t \leq 1$ , and  $p[t] = .01$  for  $1 < t \leq 3$ ,

with  $\int_0^3 p[t] dt = 1$ .

<sup>19</sup>This assumption will break down if users have a schedule knowledge, since they will tend to time their arrivals just prior to bus departure. Any headway would then seem optimal.

<sup>20</sup>For example, there is a 14% spillover from the peak hour demand of 2000 in Figure 10b. By 4000 the spillover is 8%.

<sup>21</sup>Here  $\psi = m\psi_0$ , where  $\psi_0$  is the headway in hours and m is the number of hours in the service interval.

<sup>22</sup>A similar effect has been recorded by our co-workers (Keeler, Small, and Cluff, 1975) in relation to congestion and lane capacity of roads.

<sup>23</sup>In addition, the marginal user increments the level of "crowding," thus lowering the quality of the service, and making for an elastic capacity limit. Standees would then be expected to cost their in-vehicle time more highly. In the present case the level of demand is presumed fixed, with no standees permitted (as with Golden Gate service).

<sup>24</sup>In addition to costs internal to the system users ("private costs"), externalities can also include costs external to the system users, for example pollution. The latter are extremely small for buses: Small (1974) estimates a value of \$0.001 per vehicle mile for 1970-72 model diesel buses of standard capacity, which is only \$0.0004 per passenger for a fully laden bus on a 20 mile trip.

<sup>25</sup>The extent to which the agency can recirculate its buses (via backhaul etc.) affects the system capacity. The comparability of the average round-trip run time in relation to the (relatively short) peak period limits this utilization.

<sup>26</sup>The introduction of this constraint means that the equality between (20) and (21) ceases.

<sup>27</sup>An acceleration of 4000mph<sup>2</sup> is assumed. This figure is based upon an analysis of technical data relating to Greyhound buses, which for all practical purposes are similar to the standard intra-city bus. The same figure is assumed for deceleration. The loading delay is estimated at 2.5 seconds per passenger (Kennedy, Homburger and Kell, 1973).

<sup>28</sup>Note that road congestion reduces the cruising speed of the bus, which affects the frequency of service possible with a given fleet. If buses are unable to pull out of the traffic stream, a strong inter-dependency develops between loading and road congestion, possibly affecting following buses.

<sup>29</sup>In the absence of a thin line, the entire 75-seat region is up to 5¢ cheaper than the 50-seat region.

<sup>30</sup>There is a "complementary" trip outward in the evening. Some subtle differences prevail between these inward and outward trips: for instance, with buses, stochastic delay is confined to one point in the evening (the downtown terminal) whereas in the morning, it can be distributed over the local stops. Similar asymmetries occur with wait for dial-a-ride and kiss-and-ride + linehaul bus options. None of these differences are sufficient to change the rankings of our intermodal cost comparisons.

<sup>31</sup>In reality this length should be less than the catchment radius, but no great error is introduced by the above approximation.

<sup>32</sup>Assuming that the density of the demand is approximately constant over this area.

<sup>33</sup>Boyd, et al. (1973), DOT (1969) and Meyer, Kain and Wohl (1965) have made some estimates of the cost of integrated bus service and compared these with similar estimates for other modes, but have not made this essential simultaneous optimization. Mohring (1972) and Hurdle (1973a&b) have performed a similar analysis for buses only.

<sup>34</sup> Consider the following example: suppose feeder service is operated by 25-seat buses and linehaul service by 75-seat buses. Assuming all feeder buses have the same load factors, then minimization of wait at terminus T would require a simultaneous arrival of, say, three feeder buses so the linehaul bus could be loaded in one step. Otherwise, riders from the first and second to arrive feeder buses would, in addition to their transfer costs, experience an (in-vehicle) wait to ensure a payload for the larger bus.

<sup>35</sup> A correction of 1.3 has been made to the walk cost to allow for a cartesian path.

<sup>36</sup> The corresponding air pollution cost for buses (1970-72 standard models) is \$.001 per vehicle mile, or on a per passenger basis about one hundredth of the auto pollution cost.

<sup>37</sup> We have not included in any of the auto costs the capital cost of a residential garage or the value of on-street parking.

<sup>38</sup> Note that costs of deprivation of a car in one car households are not included. These would be very difficult to estimate.

<sup>39</sup> "Optimization" in the context of this report: (a tradeoff between user and agency costs) is not meaningful in relation to a service where scheduling is by response to individual demand.

<sup>40</sup> Assume the catchment is a high density settlement generating  $Q = 20,000$  patrons/hour for a linehaul journey of 18 miles (intercity service). If feeder costs are taken to be comparable and both systems have the same linehaul speed (i.e., equal in-vehicle time cost), then agency costs per bus patron are  $\frac{TDC}{50} = \frac{6.37}{50} = \$0.13$ , whilst for BART (from Appendix A-6) they are \$0.25. That is, the bus is cheaper. To move 20,000 people per hour would require 400bph, well within the flow capacity of an exclusive lane facility. (HRB, 1965)

<sup>41</sup> For example, suppose the fringe suburb has a residential density of four households per acre, then within the catchment of two mile radius, there are  $\pi \times 2^2 \times 64 \times 4 \approx 3000$  households. If on average, one in every five of these has a CBD commuter, then this corresponds to about 600 patrons per hour. By this reasoning middle and inner distance suburbs would correspond to about 10 and 30 households per acre, which fits well with reality.

<sup>42</sup> Vickrey (1974) has pointed out that rail rapid transit, unlike throat bus service, combines local collection with a local delivery ability, while not compromising markedly in service speed. Bus stops on freeways no doubt can serve a similar role.

<sup>43</sup> Further research on the economics of exclusive lane bus operation is being conducted by Small (1974).

<sup>44</sup> Golden Gate Bridge & Transit District is planning to fit their buses with bucket seats and personalized sound systems, similar to those used in aircraft. A survey of Shirley Highway express bus riders (Saks et al. 1973) has shown that, apart from air conditioning, comfort ranks rather low; frequency and reliability being the most highly valued. However their sample comprises people who have already made a choice to use the system.

<sup>45</sup> The discrete equivalent of (9).

<sup>46</sup> Assumes buses have an unlimited capacity.

<sup>47</sup> This is identical to the net demand if the catchment is cleared.



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