THE GEOMETRICAL LANGUAGE OF CONTINUUM MECHANICS

This book presents the fundamental concepts of modern Differential Geometry within the framework of Continuum Mechanics. It is divided into three parts of roughly equal length. The book opens with a motivational chapter to impress upon the reader that Differential Geometry is indeed the natural language of Continuum Mechanics or, better still, that the latter is a prime example of the application and materialization of the former. In the second part, the fundamental notions of Differential Geometry are presented with rigor using a writing style that is as informal as possible. Differentiable manifolds, tangent bundles, exterior derivatives, Lie derivatives, and Lie groups are illustrated in terms of their mechanical interpretations. The third part includes the theory of fibre bundles, G-structures, and groupoids, which are applicable to bodies with internal structure and to the description of material inhomogeneity. The abstract notions of Differential Geometry are thus illuminated by practical and intuitively meaningful engineering applications.

Marcelo Epstein is currently a Professor of Mechanical Engineering at the University of Calgary, Canada. His main research revolves around the various aspects of modern Continuum Mechanics and its applications. A secondary related area of interest is biomechanics. He is a Fellow of the American Academy of Mechanics, recipient of the Cancam prize, and University Professor of Rational Mechanics. He is also adjunct Professor in the Faculties of Humanities and Kinesiology at the University of Calgary.

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To Silvio's memory

"Heu ... frater adempte mihi!"

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Preface

Il naufragar m'è dolce in questo mare.
GIACOMO LEOPARDI

In mathematics and the sciences, just as in the arts, there is no substitute for the masterpieces. Fortunately, both in Differential Geometry and in Continuum Mechanics, we possess a veritable treasure trove of fundamental masterpieces, classical as well as modern. Reading them may elicit a pleasure, even an emotion, comparable to that aroused by playing a Mozart piano sonata, the same sensation of perfection and beauty, the sweetness of drowning in such an ocean. This comparison is fair also in a different sense, namely, that for common mortals to achieve these spiritual heights requires a considerable amount of study and work, without which we must resign ourselves to the intuitive feelings evoked by the senses and to the assurances of the critics or the arbiters of taste. The aim of this book is to provide some familiarity with the basic ideas of Differential Geometry as they become actualized in the context of Continuum Mechanics so that the reader can feel more at home with the masters.

Differential Geometry is a rather sophisticated blend of Algebra, Topology, and Analysis. My selection of topics as a nonmathematician is rather haphazard and the depth and rigor of the treatment vary from topic to topic. As befits human nature, I am understandably forgiving of myself for this lack of consistency, but the reader may not be so kind, in which case I will not take it as a personal offence. One way or the other, there is no escape from the fact that the subject matter is extremely difficult and requires dedication and patience, particularly for the various interconnected levels of generalization and abstraction. In this respect, the presence of Continuum Mechanics as a materialization of many of the important geometric notions is of invaluable help. Learning can be regarded as a helical process, whereby each turn elevates us to the next storey of a building with an infinite number of floors. On each floor the same ideas dwell as on all others, but the perspective is wider. The speed of ascent is strongly dependent on the natural abilities and background of the visitor. Ideally, a good book should help to accelerate this process, regardless of the particular floor of departure. In some cases, a good book may help the visitor to abandon the building altogether in justifiable alarm and despair.

The plan and structure of the book emerge quite clearly from a cursory glance at the Contents, which probably reflects my own predilections and priorities. Having been trained initially in Civil Engineering, my own ascent to modern Differential Geometry was rather tortuous and slow: from beams and shells to classical geometry of curves and surfaces, from Linear Elasticity to Continuum Mechanics, General Relativity, continuous distributions of dislocations, and beyond. Along the way, I was privileged to have the help of Jędrzej Śniatycki (Calgary), Reuven Segev (Beer Sheva), Marek Elżanowski (Portland), and Manuel de León (Madrid) – to each of whom I owe more than I can express. These notes are in part the result of two graduate courses in engineering, the first at the invitation of Carlos Corona at the Universidad Politécnica de Madrid in the spring of 2006 and the second at the invitation of Ben Nadler at the University of Alberta in the spring of 2009. Both courses were helpful in providing a sounding board for the presentation and, in the second case, getting assistance from the students in spotting errors in the manuscript and providing valuable suggestions.

Calgary, July 2009