

The Geometry of Kerr Black Holes

Barrett O'Neill



A K Peters
Wellesley, Massachusetts

Contents

Preface	xi
Introduction	xv
Chapter 1 Background	1
1.1 Manifolds	2
1.2 Tensors	8
1.3 Differential Geometry	12
1.4 Extending Manifolds	20
1.5 Lorentz Vector Spaces	25
1.6 Introduction to General Relativity	31
1.7 Submanifolds	41
1.8 Cartan Computations	49
1.9 Overview of a Kerr Black Hole	55
Chapter 2 Beginning Kerr Spacetime	57
2.1 The Kerr Metric	58
2.2 Boyer–Lindquist Blocks	61
2.3 Special Submanifolds	67
2.4 Ergosphere and Time Machine	70

2.5	Kerr-star Spacetime	79
2.6	Connection Forms	90
2.7	Kerr Curvature à la Cartan	96
Chapter 3 Maximal Extensions		105
3.1	Star-Kerr Spacetime	106
3.2	Maximal Extreme Kerr Spacetime	111
3.3	Extending Slow Kerr Spacetime	116
3.4	Building the Crossing Spheres	121
3.5	Maximal Slow Kerr Spacetime	131
3.6	Bundle Structure of Kerr Spacetime	140
3.7	Isometries of Boyer–Lindquist Blocks	149
3.8	Isometries of M_e and M_s	155
3.9	Topology of Kerr Spacetime	163
3.10	Kerr Chronology	171
Chapter 4 Kerr Geodesics		177
4.1	First-Integrals	178
4.2	Carter Constant	182
4.3	Equations and Extensions	189
4.4	Crossing Horizons	196
4.5	Control of the ϑ Coordinate	201
4.6	Control of the r Coordinate	207
4.7	r - L Plots	214
4.8	First-Integrals and Orbits	222
4.9	Vortical Timelike Geodesics	236
4.10	Timelike Global Trajectories	243
4.11	Axial Geodesics	250
4.12	Geodesics in Horizons	255
4.13	Polar Orbits	262
4.14	Equatorial Geodesics	272
4.15	Approaching the Center	288
Chapter 5 Petrov Types		297
5.1	Weyl Tensor	298
5.2	Hodge Star	303
5.3	Commutativity	308
5.4	Petrov Classification	312

5.5	Principal Null Directions	317
5.6	Type D Curvature	322
5.7	The Optical Scalars	327
5.8	Newman–Penrose Formalism	332
5.9	Bianchi Identities and Type D	341
5.10	Goldberg–Sachs Theorem	345
Appendix A	Units	351
Appendix B	Differential Forms	355
Appendix C	Carter Constant	357
Appendix D	Exterior Products	361
	Index of Notations	365
	Bibliography	367
	Index	371