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The Geomorphology of the Glacial Valley Cross Section

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ABSTRACT

Several alpine valley systems in the southeastern Beartooth Mountains, Montana and Wyoming, have been examined using techniques similar to methods of stream system analysis. The general equation $y = a x^b$ is the most adequate mathematical model for the cross valley profile; b values range between 1.5 and 2.0, indicating a parabolic form. As intensity of erosion increases in the glacial valley system, the b value also increases, indicating relatively deeper and narrower valley cross sections. The law of stream numbers, the law of stream lengths, and the bifurcation ratio, derived from fluvial geomorphology, are also applicable in glacial geomorphology.

INTRODUCTION

OBJECTIVES

Studies in fluvial geomorphology have introduced the concept of the drainage basin as a geomorphic unit and have theoretically and empirically deduced many relationships among the parameters of discharge, drainage basin area, channel width, channel depth, slope, and roughness. Studies of such relationships have been attempted only on fluvial systems; this paper applies them to an alpine glacial system. The purposes of this report are (1) to introduce fluvial geomorphic concepts and methods into glacial geomorphic studies, and (2) to use these procedures to determine the nature of the interaction between the process of alpine glaciation and the resulting form of the glacial valley by the use of concepts adapted from fluvial geomorphology.

Textbooks have frequently described the glacial valley as U-shaped in cross section (e.g., Thornbury, 1954, p. 371). Davis (1916) suggested a catenary curve. As Svensson (1959) pointed out, this is rarely true of the bedrock cross section; he presents evidence to indicate that a parabola is the best approximation of cross valley form.

Playfair's Law, which states that valleys are proportionate to the streams flowing in them, was one of the earliest attempts to view the landscape in an organized manner (Playfair, 1802). Although later workers showed that there are exceptions to this "law" (Salisbury *et al.*, 1968), it was almost the only attempt to explore the relationships involved in the fluvial landscape until the mid-1940s when R. E. Horton introduced the study of the stream system as a component of the drainage basin (Horton, 1945). From these studies were derived many expressions, commonly of an exponential nature, which show the relationships among drainage basin parameters. In the 1950s the concepts of hydraulic geometry, which encompass many of the parameters of processes and forms found in the fluvial channel, were developed (e.g., Leopold and Maddock, 1953; Leopold and Miller, 1956). This work is summarized by Leopold *et al.* (1964) and Dury (1969). These

studies are referred to in this paper collectively as "fluvial geomorphic geometry" because they examine the results of fluvial processes operating on a land surface producing certain geometrically describable forms.

GLACIAL GEOMORPHIC GEOMETRY

"Glacial geomorphic geometry" is here defined as the study of the geometric form of glacial landforms and is concerned with the relationship between these forms and the processes which created them. In this paper only alpine glaciers and valleys are considered. These networks are basically similar to those of streams and stream valleys, although there are significant differences. The physical behavior of ice is very different from that of water; fluvial channels are almost always much smaller than glacial channels. Channels eroded by glacial ice occupy large sections of sizable preglacial valleys. Further, it is difficult to determine precisely the discharge for former glacial channels. Despite these differences, the similarities are impressive. Although ice differs physically from water, they both move through systems of connecting channels which are commonly the result of statistically random growth (Shreve, 1966, 1967; Werner, 1969). Glaciers frequently follow valley systems previously occupied by streams, adding to the similarity. Further, though there are great scale differences between fluvial

and glacial channels, many of the laws relating channel parameters are dimensionless and so are not affected by these differences. The lack of accurate discharge data may be circumvented (below).

The similarity between the two systems was recognized in A. Penck's "Law of Adjusted Cross Sections" (Penck, 1905, cited in Cotton, 1958, p. 318 and von Engeln, 1942, p. 457), which is a direct analogue of Playfair's Law. Penck's Law states that the valley cross section is proportional to the amount of ice flowing through it. This law may be subject to revisions or exceptions, as is Playfair's Law.

Networks of glaciers, either with or without ice, are similar to stream networks and are therefore susceptible to fluvial analytic techniques. The random statistical nature of the drainage network which produces the effect of allometric growth in stream channels (Nordbeck, 1965) produces the same effect in a glacier network. Many of the "laws" of morphometric analysis are direct results of this allometric growth, and thus should be applicable to glacial systems. This suggests that ice streams or the valleys once occupied by glaciers can be viewed as part of a system like that of a stream network. The law of stream numbers, the law of stream lengths, the bifurcation ratio, and other relationships may all have their counterparts in a glacial situation.

MORPHOLOGY AND PROCESS

CROSS SECTION MODELS

Several subjective studies have previously been made of the valley cross sections of alpine glacial valleys (Harker, 1899; Davis, 1900, 1906, 1916; Hershey, 1900; Coleman, 1913; Crosby, 1928; Lewis, 1947). From the first statement by McGee (1883) to the more recent works on the subject (e.g., Flint, 1957, p. 94), the term U-shaped has been commonly used. Mathematical analysis was first applied by Svensson (1959) who concluded that the cross section was parabolic. Studies of the rock profile under existing glaciers by means of a gravity survey have also indicated a parabolic form (Ostenso and Holmes, 1962; Kanasewich, 1963; Corbato, 1965).

Because a semicircular channel cross section is hydraulically the most efficient form (Giles, 1962, p. 172), glacial valley cross sections might be expected to approach this form. This does not occur because of the velocity distribution within the glacier; ice moves most rapidly in the valley cen-

ter (Meier, 1960), thus concentrating erosion in the bottom of the cross section. This may also happen in fluvial channels and may produce a parabola in that situation (Koechlin, 1924; Lane, 1955). Many preglacial alpine valleys were probably somewhat V-shaped, which would facilitate the velocity effect in increasing erosion at the valley bottom relative to the sides. Thus glacial valleys retain some influence of the preglacial configuration.

Studies of fluvial hydraulic geometry have conclusively shown that the width and depth of the channel are related by power functions to the discharge moving through the cross section; this may also be true in the glacial situation. (See Table 1 for explanation of symbols used throughout the text.)

From hydraulic geometry we derive:

$$d = c \, Q^{f} \qquad (1)$$

$$\frac{d^{\left(\frac{1}{f}\right)}}{\left(\frac{1}{f}\right)_c} = Q \qquad (2)$$

Also from hydraulic geometry:

$$w = a\, Q^b \qquad (3)$$

Substituting for Q from equation (2):

$$w = a\left[\frac{d^{\left(\frac{1}{f}\right)}}{\left(\frac{1}{f}\right)_c}\right]^b \qquad (4)$$

which is identical to:

$$w = \frac{a\, d^{\left(\frac{b}{f}\right)}}{\left(\frac{b}{f}\right)_c} \qquad (5)$$

finally, solving for d (channel depth):

$$\left[\frac{c}{\left(\frac{f}{b}\right)_a}\right] w^{\left(\frac{f}{b}\right)} = d \qquad (6)$$

Equation (6) indicates that the regression model relating valley width to depth is a power function in which the exponent determines the type of curve described. If the exponent has a value of $+2$, the curve is a normal parabola; a value of $+1.5$ indicates a semicubic parabola; and -1 indicates an equilateral hyperbola.

The general form of the power function model:

$$y = a\, x^b \qquad (7)$$

describes a curve where y is the vertical and x the horizontal distance from the origin to a point on the curve. Figure 1 shows differences resulting from changes in the exponent and coefficient.

Svensson (1959) found that a cross section of Lapporten Valley, Sweden, gave b values of 2.045 and 2.177, which suggested that the cross section was close to that of a normal parabola. The

general usefulness of his results is, however, impaired by the small number of observations on which they are based. His work is valuable because it is the only previous effort to make a quantitative statement about the morphology of the glacial valley.

A mathematical model that describes the curve approximating the cross section of the glacial valley is not a complete description, as shown in Figure 2: two valleys might have the same regression model, but have very different form. This is because the equation describes an endless curve, such that as $x \to \infty$, $y \to \infty$. When used in addition to the regression model, the form ratio, used in fluvial geomorphology to describe channel geometry (Morisawa, 1968, p. 111), gives a complete, quantitative, and dimensionless representation of the geometry of the cross section. The form ratio (F_R) is the ratio of valley depth (D) to valley top width (W_T):

$$F_R = D/W_T \qquad (8)$$

As shown in Figure 2, though two valleys have similar regression models, their form ratios might be different. Conversely, the form ratio is not usable alone, but must be supplemented by the regression model to produce an accurate representation (Figure 3).

PROCESS INTENSITY

Since the glacial process distorts the hydraulically perfect circular cross section, it is reasonable to suppose that the more intense the glaciation, the more distortion of the semicircular cross section becomes apparent, i.e., deeper and relatively more narrow. Intensity of erosion process increases within a glacial system in response to several factors: (1) the order of the segment across
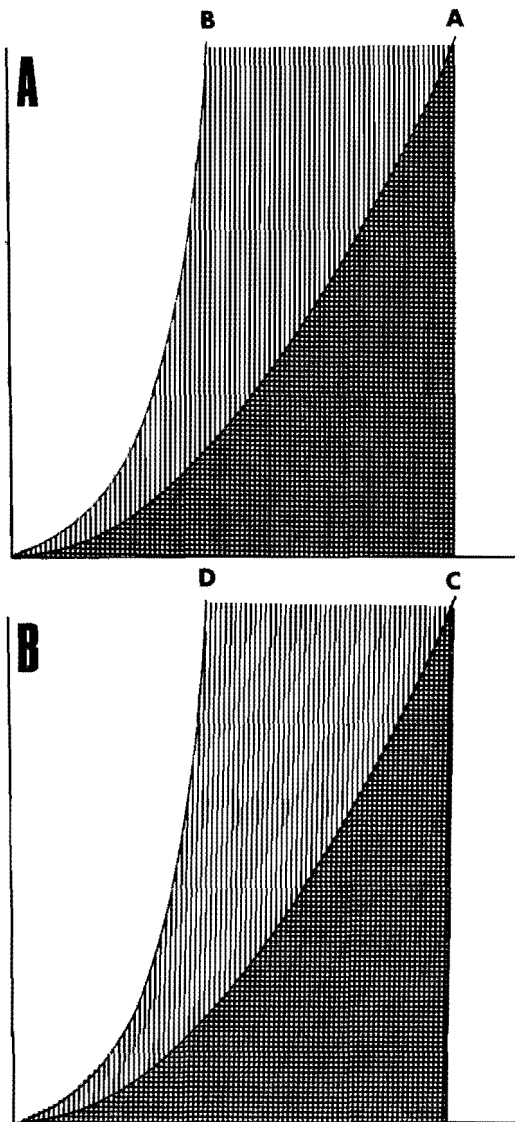
FIGURE 1. A. Form differences resulting from changes in the exponent of the equation $y = a\,x^b$: curve A shows a model with an exponent of value b; curve B shows a model with an exponent of value $b + 1$ (when b is small). Both models have the same a value. B. Form differences resulting from changes in the coefficient; curve C is the result of a model with a coefficient of value a; curve D is one with a coefficient of value $a + 1$ (when a is small). Both models have the same b value.

FIGURE 2. Though both curves have the same model, curve A has a much smaller form ratio than curve B.

which the cross section is sampled; (2) the energy gradient from the segment measured to the segment it joins, evaluated by order difference; (3) the orientation and location of the cirque a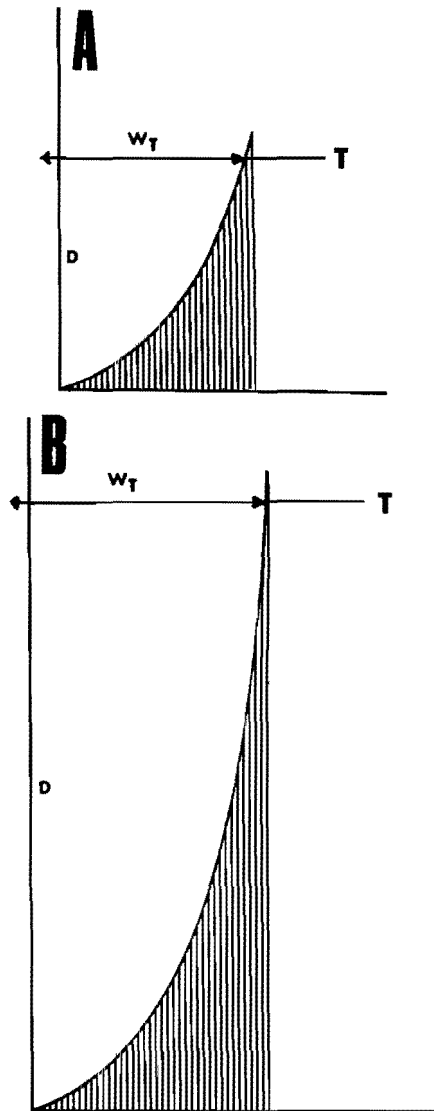rea feeding ice to the valley of the measured cross section; (4) the effect of cornering, or ice moving around corners as it joins a main stream.

A glacier system can be ordered in a manner similar to a stream network. Several ordering schemes have been used for streams (e.g., Horton, 1945; Strahler, 1957; and Scheidegger, 1965). In most ordering systems the segments are ordered on a rank scale that attempts to reflect discharge. Thus the higher the segment order, the greater the discharge, the more intense the process of glacial erosion, and the deeper and relatively more narrow the valley cross section becomes. The
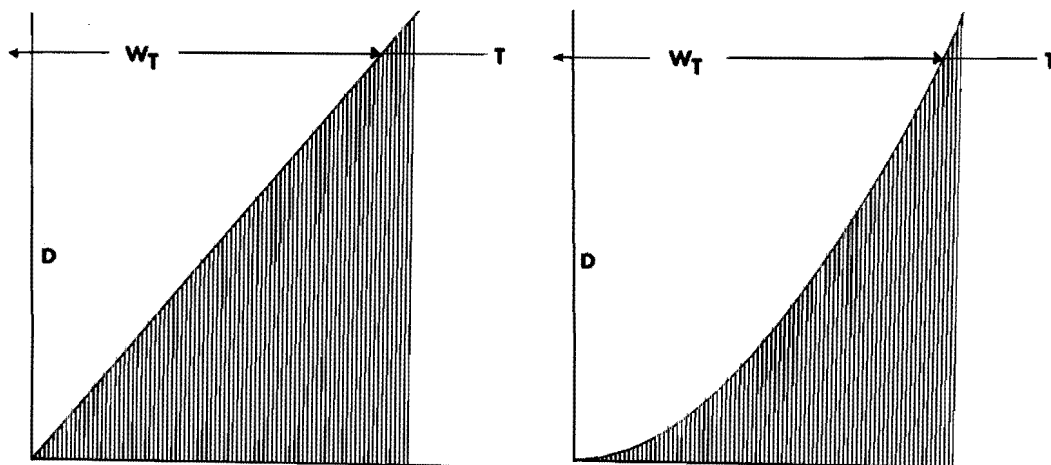
FIGURE 3. Both curves have the same form ratio but different models.

valley also becomes deeper with an increase in energy gradient from tributary to main stream. The latter effect is that of order difference, that is, the difference between the order of the tributary and the order of the stream it joins.

Orientation and location of the cirques feeding the valley sampled influence the cross section through the activity of the glacial regimen. Location is particularly important if it is downwind of a sizable plateau which can contribute large amounts of drifted snow.

Within the stream of ice, there are variations in erosional intensity, particularly where cornering is involved. The ice on the outside of the curve or corner erodes more rapidly, making the valley cross section asymmetrical. Intense plucking may take place on the outside wall, and intense abrasion on the inside; this may steepen the outside

wall much more rapidly than the inside one since plucking is able to remove much more material than abrasion (Flint, 1957, p. 78; Davies, 1969, p. 105). These effects produce asymmetrical channels regardless of the discharge (Charlesworth, 1924, cited in Dury, 1964).

From the above discussion and the model presented in equation (7) it can be seen that the values of a and b in the mathematical model increase as order increases, as energy gradient increases, as cirque location and orientation increase the regimen activity, and as erosion increases on the outside of a corner. Equation (8) shows that the value of the form ratio also increases. These numerical changes reflect the geometry changes as the valley becomes relatively deeper and narrower with the above-mentioned increasing variables.

METHODS OF INVESTIGATION

STUDY AREA

Selection of a suitable study area can eliminate many problems by attempting to hold several variables constant. A study of the type presented here is most accurate and most useful if the area selected meets as many of the following requirements as possible. (1) Topographic maps of the area must be of a scale no smaller than 1:63,360 and be photogrammetrically compiled. This insures that the maps, a primary data source, will be accurate and present the data in a form that can be analyzed. (2) Good quality air photographs are needed to identify surface materials and to supplement the maps. (3) Access to the area on the ground is desirable in order to check the maps

and the interpretation of the photographs. (4) Relatively uniform, homogeneous lithology holds the effects of geology constant. (5) A glacier system developed on a regional slope of uniform direction, as opposed to a dome or basin area, holds constant the variable of aspect. (6) A completely developed branching system with at least 100 clearly defined first order tributaries requires an extensive valley glacier system. (7) Finally, the system to be examined should be devoid of large existing glaciers and the valleys should be free from large amounts of surficial deposits so that the true bedrock profile is visible in the photographs and is represented on the topographic maps.

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The area selected for study, the Beartooth Mountains, meets these requirements. Located in south-central Montana and northwestern Wyoming, the Beartooth Mountains, referred to on some maps as the Snowy Mountains, form the northwest boundary of the Big Horn Basin and are the front range of the Rocky Mountains in this region. The range, about 70 km wide and 130 km long, trends northwest and southeast and is bordered on the north and west sides by the Yellowstone River. The Yellowstone Plateau and Clark's Fork of the Yellowstone River are to the south and west, and the Big Horn Basin and the High Plains lie to the east.

The range itself is a relatively flat-topped, uplifted metamorphic block with upturned beds of sedimentary rocks on the north and east and faults on the west and south (Bevan, 1923; Hughes, 1933; Foose *et al.*, 1961). The uplifted plateau is 1,200 to 1,500 m above the plains to the east and the Yellowstone River to the west, with portions of the upland surface having an elevation of up to 3,400 m. The highest point of the range, Granite Peak (3,817 m), is the highest point in Montana. The plateau is dissected by numerous glacial troughs, the result of successive glaciations of the range during the Pleistocene epoch (Bevan, 1946).

U.S. Geological Survey maps of the study area are of scale 1:62,500 with a contour interval of 80 feet (24 m) and are photogrammetrically compiled. Excellent air photographs (scale 1:15,840) are available from the U.S. Forest Service. The area is reasonably accessible by means of U.S. Route 212 and several unimproved dirt roads.

The study area is composed of the drainage basins of six valleys in the far southeastern corner of the range: Littlerock Creek, Rock Creek, Lake Fork, West Fork, East Rosebud Creek, and West Rosebud Creek. The combined area of these basins is about 780 km², and there are 127 first order, cirque-headed segments.

Measurement and Analysis

The network of glacial valleys studied was delineated by the extent of ice during the Pinedale I glacial maximum, about 23,000 BP (Richmond, 1965). The extent of the ice of this stade is easily identified on photographs because of the sizable moraines deposited. The system of glaciers present at that time was mapped using topographic maps and air photographs, and the resulting networks were topologically transformed to straight segment networks to make the process of ordering simpler. The resulting networks were ordered according to the Strahler ordering system (Strah-

ler, 1957), where each fingertip segment is first order; two first order segments join to form a second order segment, and so forth.

After ordering, the segments were assigned identification numbers and those to be measured randomly selected. Twenty first order valleys with order difference of zero were selected along with 20 first order valleys with order difference of two, 10 second order valleys, and 10 third order valleys.

The cross valley measurements were made approximately 25% of the segment length upstream from the order changing junction. Use of air photographs insured that the measurements taken from the map were of a bedrock profile and allowed identification of the glacial trimlines which were indicators of the upper limit of the curve that modeled the cross section.

The measurements of the points of the cross section were made along a line from the thalweg to the trimline orthogonal to the contour lines. This was done for both sides of the valley. The x value for the computation of the model constants was measured from the thalweg horizontally to a point on the valley side, and the y value was measured vertically from the thalweg to that point. Each point thus had unique coordinates. Five points were arbitrarily selected between the thalweg and trimline on each side of the valley and all 10 were used in the calculation of constants for the valley. All were considered positive so the resulting model in effect "folded" the valley cross section halves over, one on top of the other. The least squares solution of y on x calculated by computer approximates the curves of the two half cross sections and represents the whole cross section. The equation of this curve is the mathematical descriptive model of the valley cross section; residuals indicate asymmetry and other irregularities.

There are two principal sources of error: the maps from which the measurements are taken and the method of measurement. The researcher has no control over errors in the map and it is assumed that the photogrammetric methods used in compilation have minimized these errors. Error might also be manifested in the measurement procedure, as the profile along which the points are measured is projected vertically down onto the map, while the true cross profile is tilted from the vertical slightly because of the gradient of the valley. From trigonometry, it can be seen that this would affect the y measurement according to the value of the cosine of the gradient of the valley. In a valley with a gradient of 8° the error of measurement would be slightly less than 1%, which is considered acceptable in this study.

MODELS

Figure 4 shows that the Law of Stream Numbers appears to hold true for the glacial situation; the number of segments of a given order is exponentially related to the order number. The bifurcation ratio between first and second order streams is 3.91; between second and third 4.56; between third and fourth 3.50. The mean bifurcation ratio for all orders is 3.99, or about 4.00, a value which has also been found in many stream networks (Leopold *et al.*, 1964; Morisawa, 1968). This is apparently because of the random statistical nature of the networks.

The Law of Stream Lengths also seems to hold true for the glacial situation, with the cumulative segment lengths being related to segment order by an exponential function (Figure 5). This again is probably a product of the stochastic nature of the networks involved and provides another similarity between fluvial and glacial situations.

The results of the calculation of the constants *a* and *b* for the models and the mean form ratios are given in Table 2. As the order of the segments increases from first to second to third (U_1 through U_3), the *b* values steadily increase and the *a* values generally increase also. This indicates the expected change whereby the cross sections become relatively more narrow and deep. The form ratio reflects this change also, except for the third order segments where the ratio decreased. This is apparently explained by the fact that the higher order segments are nearer to the mountain front where the local relief is less. Similar geometry

changes occur as the intensity of process increases with increasing order different, that is from U_1, $U_d = 0$ to U_1, $U_d = 2$.

Location and orientation has a great effect on the cross sectional geometry. The 40 first order valleys are classified on the basis of an evaluation of their location—favorable or unfavorable—and the constants of their models are averaged. A cirque was judged to be in a favorable location if it had plateau surface to the west, northwest, or southwest. All other cirques were designated unfavorable, but the classification is obviously relative since no cirques would form in truly unfavorable situations. Table 2 shows that those valleys in favorable locations are much deeper and relatively more narrow than those in less favorable locations. The form ratio is also sensitive to changes in intensity of process.

The expected changes are also found when the intensity of process changes because of the effect of cornering. In order to assess the asymmetry of the first order valleys, the measurements of the cross sections are considered in halves, that is, the two valley sides are considered separately, each with its own model. The constants for all the models are averaged in two groups: those on the side of the tributary that is upstream with respect to the main channel and those that are on the downstream side. The mean values are given in Table 2, and show that the upstream sides have been steepened more than the downstream sides; the increase in the mean *b* value is much greater than the slight decrease in the mean *a* value. Not
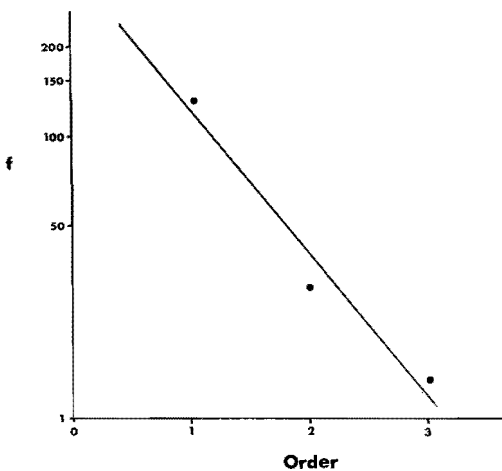

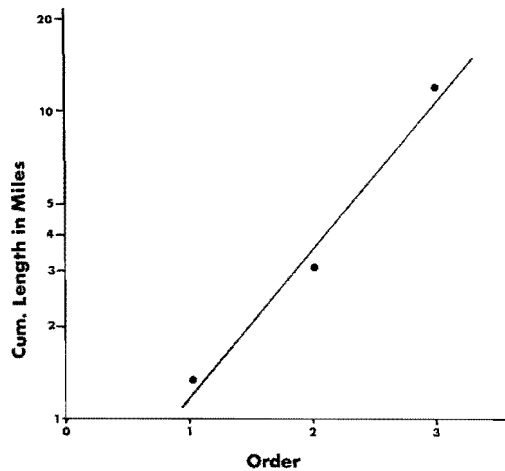FIGURE 4. Glacial stream frequency plotted against order.

FIGURE 5. Glacial stream length (cumulative) plotted against stream order.

Table 2

Results

	a	b	$F_{\underline{R}}$	N[a]
U_1, $U_d=0$	0.0074	1.6261	0.250	20
U_1, $U_d=2$	0.0105	1.7141	0.348	20
U_1, Fav. loc.	0.0106	1.7128	0.369	17
U_1, Unfav. loc.	0.0063	1.4633	0.242	23
U_1, Up side	0.0060	1.7212		40
U_1, Down side	0.0050	1.6780		40
U_1, Mean	0.0090	1.6701	0.304	40
U_2, Mean	0.0024	1.8122	0.445	10
U_3, Mean	0.0134	1.8390	0.393	10

[a]N = number of valley cross sections represented.

all valleys are asymmetrical to the same degree. The following factors are associated with an increase in asymmetry: the main ice stream is linear where the tributary joins it, the tributary is linear for some distance upstream from the junction point, and the tributary joins the main stream at nearly right angles. The most asymmetrical first order channel meets all of these stipulations and has the following models:

Upstream side: $y = 0.0053 \, x^{2.0177}$ (9)

Downstream side: $y = 0.0044 \, x^{1.6996}$ (10)

Ten reference points were used in the calculation of each regression.

The b values for all the average models indicate that in the study area the valleys have cross sections

that are best approximated by parabolas, but that the best model is not always a normal parabola. Instead, the valleys varied between a normal parabola (b value of 2) and a semicubic parabola (b value of 1.5). The generality of this needs testing in other areas with different climates and different geology.

RESIDUALS

The mean standard errors of the estimate in log values are as follows: first order, 0.47035; second order, 0.29141; third order, 0.36202. Small residuals are produced by two primary factors: postglacial modification and geology. Third order segments are difficult to measure because in many cases the bedrock profile is obscured in whole or in part by postglacial modification through mass movement or fluvial processes. Filling of the bottom of the cross section with alluvium and talus reduced the form ratio and caused the model to have a and b values somewhat smaller than expected. This is the result of age, since these segments have not been glaciated since the Pinedale I maximum, while most of the other segments have been reglaciated by subsequent advances.

Although the geology is relatively similar throughout the area, geologic structures, particularly faults, have a very noticeable effect. One valley cross section has large residuals from the model because a fault along one valley side produces an almost box-like channel.

Valley side-slope orientation seemed to have little effect, probably because sufficient time has not elapsed since glacial evacuation for the factors that are controlled by orientation to operate. Most other variables were controlled by the study area selection procedure.

CONCLUSIONS

The results of this study show that the glacial valley cross section is that of a parabola. Further, as the intensity of the process of valley glaciation increases, the form of the valley changes to become relatively more narrow and deep. Erosional intensity appears to increase with valley order, with order difference (the change in energy gradient), with the activity of the glacier regimen (controlled by location and orientation of cirques), and on the outside of a cornering stream of ice.

These results also imply that the concept of the drainage basin as a geomorphic unit can be usefully applied to a glacial as well as a fluvial situation. The descriptive techniques available in fluvial analysis are valuable additions to the tools of the glacial geomorphologist. For example, the longitudinal profile of glacial valleys may also be susceptible to analytic techniques used to study fluvial profiles.

ACKNOWLEDGMENTS

REFERENCES

Bevan, A.
  1923 : Summary of the geology of the Beartooth Mountains. *J. Geol.*, 31: 441-465.
  1946 : Three stages of Pleistocene glaciation, Beartooth Mountains, Montana. *Geol. Soc. Amer. Bull.*, 57: 1178.
Charlesworth, J. K.
  1924 : The glacial geology of northwest Donegal. *Proc. Roy. Irish Acad.*, 36: 173-314.
Coleman, A. P.
  1913 : Cirques and U-shaped mountain valleys. *Can. Alpine J.*, 5: 49-52.
Corbato, C. E.
  1965 : Theoretic gravity anomalies of glaciers having parabolic cross sections. *J. Glaciol.*, 5: 255-258.
Crosby, W. O.
  1928 : Certain aspects of glacial erosion. *Geol. Soc. Amer. Bull.*, 39: 1171-1181.
Cotton, C. A.
  1958 : *Geomorphology.* Whitcomb and Tombs, Christchurch. 505 pp.
Davies, J. L.
  1969 : *Landforms of Cold Climates.* M.I.T. Press, Cambridge. 323 pp.
Davis, W. M.
  1900 : Glacial erosion in France, Switzerland, and Norway. *Proc. Boston Soc. Nat. Hist.*, 29: 273-322.
  1906 : The sculpture of mountains by glaciers. *Scot. Geog. Mag.*, 22: 76-89.
  1916 : The Mission Range, Montana. *Geog. Rev.*, 2: 267-288.
Dury, G. H.
  1964 : Aspects of the geomorphology of Slieve League Peninsula, Donegal. *Proc. Geol. Assoc.*, 75: 445-460.
  1969 : Perspectives on geomorphic processes. Assoc. Amer. Geog. Res. Paper No. 3.
Embleton, C. and King, C. A. M.
  1968 : *Glacial and Periglacial Geomorphology.* St. Martin's Press, New York. 608 pp.
Flint, R. F.
  1957 : *Glacial and Pleistocene Geology.* John Wiley, New York. 553 pp.
Foose, R. M., Wise, D., and Garbarini, G.
  1961 : Structural geology of the Beartooth Mountains, Montana and Wyoming. *Geol. Soc. Amer. Bull.*, 72: 1143-1172.
Giles, R. V.
  1962 : *Theory and Problems of Fluid Mechanics and Hydraulics.* Schaum's Outline Series, McGraw-Hill, New York. 274 pp.
Harker, A.
  1899 : Glaciated valleys in Cuillins, Skye. *Geol. Mag.*, 6: 196-199.
Hershey, O. H.
  1900 : Ancient alpine glaciers of the Sierra Costa Mountains, California. *J. Geol.*, 13: 42-57.
Horton, R. E.
  1945 : Erosional development of streams and their drainage basins: hydro-physical approach to quantitative morphology. *Geol. Soc. Amer. Bull.*, 56: 275-370.
Hughes, R. V.
  1933 : The geology of the Beartooth Mountain front in Park Co., Wyoming. *Nat. Acad. Sci. Proc.*, 19: 239-253.
Kanasewich, E. R.
  1963 : Gravity measurements on the Athabaska Glacier, Alberta, Canada. *J. Glaciol.*, 4: 617-631.
Koechlin, R.
  1924 : *Mechanisme de l'eau.* Librarie Polytechnique, Beranger, Paris.
Lane, E. W.
  1955 : Design of stable channels. *Amer. Soc. Civil Engrs. Trans.*, 120: 1234-1279.
Leopold, L. B. and Maddock, T.
  1953 : The hydraulic geometry of stream channels and some physiographic implications. U.S. Geol. Surv. Prof. Paper, 252.
Leopold, L. B. and Miller, J. P.
  1956 : Ephemeral streams—hydraulic factors and their relation to the drainage net. U.S. Geol. Surv. Prof. Paper, 282-A.
Leopold, L. B., Wolman, M. G., and Miller, J. P.
  1964 : *Fluvial Processes in Geomorphology.* Freeman, London. 522 pp.
Lewis, W. V.
  1947 : Cross section of glaciated valleys. *J. Glaciol.*, 1: 37-39.
Meier, M. F.
  1960 : Mode of flow of Saskatchewan Glacier, Alberta, Canada. U.S. Geol. Surv. Prof. Paper 351.
McGee, W. J.
  1883 : Glacial canons. *Proc. Amer. Assoc.*, p. 238.
Morisawa, M. E.
  1968 : *Streams: Their Dynamics and Morphology.* McGraw-Hill, New York. 175 pp.

Nordbeck, S.
    1965 : The law of allometric growth. Michigan Inter-Univ. Comm. Math. Geog., Disc. Paper No. 8.
Ostenso, N. A. and Holmes, G. W.
    1962 : Gravimetric determination of ice thickness of Jarvis Glacier, Alaska. U.S. Geol. Surv. Prof. Paper. 450-C: C93-C98.
Penck, A.
    1905 : Glacial features in the surface of the Alps. *J. Geol.*, 13: 1-17.
Playfair, J.
    1802 : *Illustration of the Huttonian Theory of the Earth.* Facsimile Reprint, 1956, Dover Publ., New York. 328 pp.
Richmond, G. M.
    1965 : Glaciation of the Rocky Mountains. *In* Wright, H. E. and Frey, D. G. (eds.), *The Quaternary of the United States,* Princeton Univ., Princeton. 922 pp.
Salisbury, N. E., Knox, J. C., and Stephenson, R. A.
    1968 : The valleys of Iowa—1: Valley width and stream discharge relations in the major streams. Univ. of Iowa, Iowa Studies in Geog., No. 5, Iowa City. 107 pp.

Scheidegger, A. E.
    1965 : The algebra of stream order numbers. U.S. Geol. Surv. Prof. Paper. 525-B: B18-B189.
Shreve, R. L.
    1966 : Statistical law of stream number. *J. Geol.,* 74: 17-37.
    1967 : Infinite topologically random channel networks. *J. Geol.,* 75: 178-186.
Strahler, A. N.
    1957 : Quantitative analysis of water-shed geomorphology. *Trans. Amer. Geophys. Union,* 38: 913-920.
Svensson, H.
    1959 : Is the cross section of a glacial valley a parabola? *J. Glaciol.,* 3: 362-363.
Thornbury, W. D.
    1954 : *Principles of Geomorphology.* John Wiley, New York. 618 pp.
von Engeln, O. D.
    1942 : *Geomorphology.* Macmillan, New York. 655 pp.
Werner, C.
    1969 : Topological randomness in line patterns. *Proc. Assoc. Amer. Geog.,* 1: 157-163.