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ABSTRACT

This paper proposes that idiosyncratic firm-level fluctuations can explain an important part of aggregate shocks, and provide a microfoundation for aggregate productivity shocks. Existing research has focused on using aggregate shocks to explain business cycles, arguing that individual firm shocks average out in aggregate. I show that this argument breaks down if the distribution of firm sizes is fat-tailed, as documented empirically. The idiosyncratic movements of the largest 100 firms in the US appear to explain about one third of variations in output and the Solow residual. This "granular" hypothesis suggests new directions for macroeconomic research, in particular that macroeconomic questions can be clarified by looking at the behavior of large firms. This paper's ideas and analytical results may also be useful to think about the fluctuations of other economic aggregates, such as exports or the trade balance.

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1. Introduction

This paper proposes a simple origin of aggregate shocks. It develops the view that a large part of aggregate fluctuations arises from idiosyncratic shocks to individual firms. This approach sheds light on a number of issues that are difficult to address in models that postulate aggregate shocks. Although economy-wide shocks (inflation, wars, policy shocks) are no doubt important, they have difficulty in explaining most fluctuations (Cochrane 1994, Summers 1986). Often, the explanation for year-to-year jumps of aggregate quantities is elusive. On the other hand, there is a large amount of anecdotal evidence for the importance of idiosyncratic shocks. For instance, in December 2004, the $24 billion one-time Microsoft dividend boosted growth in personal income from 0.6% to 3.7%.[1] A macroeconomist would have difficulty in explaining this jump in personal income without examining individual firm behavior. The OECD (2004) analyzes that in 2000, Nokia contributed 1.6 percentage points of Findland's GDP growth. Likewise, shocks to GDP may stem from a variety of events, such as successful innovations by Wal-Mart, the difficulties of a Japanese bank, new exports by Boeing, a strike at General Motors.[2]

Since modern economies are dominated by large firms, idiosyncratic shocks to these firms can lead to non-trivial aggregate shocks. For instance, in Korea, the top two firms (Samsung and Hundyai) together account for 35% of exports, and the sales of those two firms account for 22% of Korean GDP (di Giovanni and Levchenko 2009). In Japan, the top 10 firms account for 35% of the exports (Canals *et al.* 2007). For the U.S., Figure 1 reports the total sales of the top 50 and 100 firms as a fraction of GDP. On average, the sales of the top 50 firms are 24% of GDP, while the sales of the top 100 firms are 29% of GDP. The top 100 firms hence represent a large part of the macroeconomic activity, and so understanding their actions gives a good insight into aggregate economy.

In this view, many economic fluctuations are not due, primitively, to small diffuse shocks that directly affect every firm. Instead, many economic fluctuations are attributable to the incompressible "grains" of economic activity, the large firms. I call this view the "granular" hypothesis. In the granular view, idiosyncratic shocks to large firms have the potential to generate small aggregate shocks that affect GDP, and via general equilibrium, all firms.

The granular hypothesis offers a microfoundation for the aggregate shocks of real business cycle models (Kydland and Prescott 1982). Hence, real business cycle shocks are not, at heart, mysterious "aggregate productivity shocks" or "a measure of our ignorance" (Abramovitz 1956). Instead, they are well-defined shocks to individual firms. The granular hypothesis sheds light

[1] Source: Bureau of Economic Analysis, January 31, 2005.

[2] The example of Nokia is extreme but may be useful. In 2003, worldwide sales of Nokia were $37 billion, representing 26% of Finland's GDP of $142 billion. This is not sufficient for a proper assessment of Nokia's importance, but gives some order of magnitude, as the Finnish base of Nokia is an important residual claimant of the fluctuations of Nokia International.

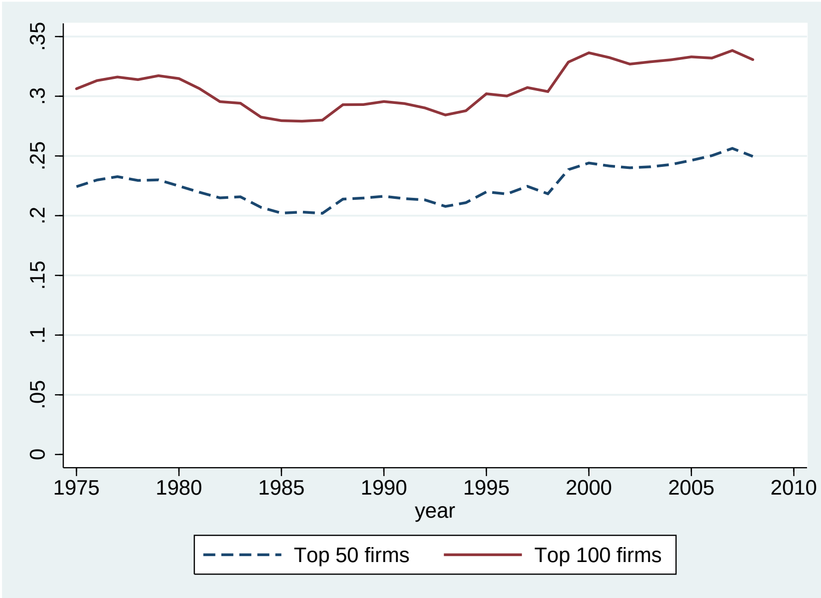

Figure 1: Sum of the sales of the top 50 and 100 non-oil firms in Compustat, as a fraction of GDP. Hulten's theorem (Appendix B) motivates the use of sales rather than value added.

on a number of other issues, such as the dependence of the amplitude of GDP fluctuations on GDP level, the microeconomic composition of GDP, the distribution of GDP and firm-level fluctuations.

In most of this paper, the standard deviation of the percentage growth rate of a firm is assumed to be independent of its size.[3] This explains why individual firms can matter in the aggregate. If Wal-Mart doubles its number of supermarkets and thus its size, its variance is not divided by two — as would be the case if Wal-Mart was the amalgamation of many independent supermarkets. Instead, the newly acquired supermarkets inherit the "Wal-Mart" shocks, and the total percentage variance of Wal-Mart does not change. This paper conceptualizes these shocks as productivity growth, but the analysis holds for other shocks.[4]

The main argument is summarized as follows. First, it is critical to show that $1/\sqrt{N}$ diversification does not occur in an economy with a fat-tailed distribution of firms. A simple diversification argument shows that, in an economy with N firms with independent shocks, aggregate fluctuations should have a size proportional to $1/\sqrt{N}$. Given that modern economies

[3] The benchmark that the variance of the percentage growth rate is approximately independent of size ("Gibrat's law" for variances) appears to hold to a good first degree, see section 2.5.

[4] The productivity shocks can come from a decision of the firm's research department, of the firm's chief executive officer, of how to process shipments, inventories, or which new line of products to try. They can also stem from changes in capacity utilization, and particularly strikes. Suppose a firm, which uses only capital and labor, is on strike for half the year. For many purposes, its effective productivity that year is halved. This paper does not require the productivity shocks to arise from any particular source.

can have millions of firms, this suggests that idiosyncratic fluctuations will have a negligible aggregate effect. This paper points out that, when firm size is power-law distributed, the conditions under which one derives the central limit theorem break down, and other mathematics apply (see Appendix A). In the central case of Zipf's law, aggregate volatility decays according to $1/\ln N$, rather than $1/\sqrt{N}$. The strong $1/\sqrt{N}$ diversification is replaced by a much milder one that decays according to $1/\ln N$. In an economy with a fat-tailed distribution of firms, diversification effects due to country size are quite small.

Section 4 then investigates accordingly the proportion of aggregate shocks that can be accounted for by idiosyncratic fluctuations. I construct the "granular residual" Γ_t, which is a parsimonious measure of the shocks to the top 100 firms:

$$\Gamma_t := \left(\sum_{i=1}^{K} \text{Sales}_{i,t-1}\right)^{-1} \left(\sum_{i=1}^{K} \text{Sales}_{i,t-1}\left(g_{it} - \overline{g_t}\right)\right),$$

where $g_{it} - \overline{g_t}$ is a simple measure of the *idiosyncratic* shock to firm i. Regressing the growth rate of GDP on the granular residual yields an R^2 of roughly one third. *Prima facie*, this means that idiosyncratic shocks to the top 100 firms in the U.S. can explain one third of the fluctuations of GDP and the Solow residual.

Having established that idiosyncratic shocks do not die out in the aggregate, I show in section 5 that they are of the correct order of magnitude to explain business cycles. A result based on Hulten (1978) shows that, if firm i has a productivity shock $d\pi_i$, these shocks are i.i.d., and there is no amplification mechanism, then the standard deviation of TFP growth is $\sigma_{TFP} = \sigma_\pi h_S$, where σ_π is the standard deviation of the i.i.d. productivity shocks, and h_S is the sales Herfindahl of the economy. Using the estimate of volatility of productivity of $\sigma_\pi = 12\%$/year, the sales Herfindahl of $h_S = 6.1\%$ for the US in 2002, one predicts a TFP volatility equal to $\sigma_{TFP} = 12\% \cdot 6.1\% = 0.9\%$. Standard amplification mechanisms generate the order of magnitude of business cycle fluctuations, $\sigma_{GDP} = 1.4\%$. Non-US data leads to even larger business cycle fluctuations. I conclude that idiosyncratic volatility seems quantitatively large enough to matter at the macroeconomic level.

Previous economists have proposed mechanisms that generate macroeconomic shocks from purely microeconomic causes. A pioneering paper is Jovanovic (1987), whose models generate non-vanishing aggregate fluctuations owing to a multiplier proportional to $\sqrt{N}$, the square root of the number of firms. However, Jovanovic's theoretical multiplier of $\sqrt{N} \simeq 1000$ is much larger than is empirically plausible.[5] Nonetheless, Jovanovic's model spawned a lively intellectual quest. Durlauf (1993) generates macroeconomic uncertainty with idiosyncratic shocks and local

[5] If the actual multiplier was so large, the impact of trade shocks, for instance, would be much higher than we observe.

interactions between firms. The drivers of his results are the non-linear interactions between firms, while in this paper it is the skewed distribution of firms. Bak, Chen, Scheinkman, and Woodford (1993) apply the physical theory of self-organizing criticality. While there is much to learn from their approach, it generates fluctuations more fat-tailed than in reality, with infinite means. Nirei (2006) proposes a model where aggregate fluctuations arise from (s,S) rules at the firm level, in the spirit of Bak *et al.* (1993). These models are conceptually innovative, but they they are hard to work with theoretically and empirically. The mechanism proposed in this paper is tractable, and relies on readily observable quantities.

Long and Plosser (1983) suggest that sectorial (rather than firm) shocks might account for GDP fluctuations. As their model has a small number of sectors, those shocks can be viewed as mini-aggregate shocks. Horvath (1998, 2000) and Conley and Dupor (2003) explore this hypothesis further. They find that sector-specific shocks are an important source of aggregate volatility. Finally, Horvath (1998) and Dupor (1999) debate whether N sectors can have a volatility that does not decay according to $1/\sqrt{N}$. I find an alternative solution to their debate, which is, formalized in Proposition 2. My approach relies on those earlier contributions, and clarifies that the fat-tailed nature of the sectoral shocks is important theoretically, as it determines whether the central limit theorem applies.

Studies disagree somewhat on the relative importance of sector specific shocks, aggregate shocks, and complementarities. Shea (2002) quantifies that complementarities play a major role in aggregate business cycle fluctuations. Caballero, Engel and Haltiwanger (1997) find that aggregate shocks are important, while Horvath (1998) concludes that sector-specific shocks go a long way toward explaining aggregate disturbances. Many of these effects in this paper could be expressed in terms of sectors. Carvahlo (2009) studies granular effects in the economy viewed as a network, by looking at the size of sectors and their interconnectedness.

Pareto (1896) was the first to discover that the income distribution follows a power law. A growing number of other economic variables appear to follow power laws: in particular Zipf's law, which is a power law with an exponent close to 1. This includes the size of cities (Zipf 1949, Gabaix 2009), firms (Axtell 2001, Fujiwara *et al.* 2004, Okuyama *et al.* 1999), mutual funds (Gabaix *et al.* 2006), Internet sites (Barabasi and Albert 1999). The origin of this Zipf distribution is becoming better understood (Simon 1955, Gabaix 1999, 2009, Luttmer 2007, Rossi-Hansberg and Wright 2007).

Granular effects are likely to be even stronger outside the U.S, as the U.S. is more diversified than most other countries. One number reported in the literature is the value of the assets controlled by the richest 10 families, divided by GDP. Claessens, Djankov and Lang (2000) find a number equal to 38% in Asia, including 84% of GDP in Hong Kong, 76% in Malaysia, 39% in Thailand. Faccio and Lang (2002) also find that the top 10 families control 21% of listed assets in their sample of European firms. It would be interesting to transpose the present analysis to those countries, and to other entities than firms – for instance, business groups, or sectors.

This paper is organized as follows. Section 2 develops a simple model. Section 3 provides a richer model that gives a foundation for the measurement of idiosyncratic shocks, and spells out how production linkages can make all microeconomic and macroeconomic variables comove. Section 4 shows directly that the idiosyncratic movements of firms appear to explain, year by year, about one third of actual fluctuations in GDP and the Solow residual. Section 5 provides a calibration that indicates that the effects are of the right order of magnitude to account for macroeconomic fluctuations. Section 6 concludes.

2. The Core Idea

2.1. A Simple "Islands" Economy

This section uses a concise model to illustrate the idea. In this economy there are only idiosyncratic shocks to firms. Let S_{it} represent firm i's production in year t. It experiences a growth rate:

$$\frac{\Delta S_{i,t+1}}{S_{i,t}} = \frac{S_{i,t+1} - S_{it}}{S_{it}} = \sigma_i \varepsilon_{i,t+1} \tag{1}$$

where σ_i is firm i's volatility and $\varepsilon_{i,t+1}$ are uncorrelated random variables with mean 0 and variance 1. Total GDP is:

$$Y_t = \sum_{i=1}^{N} S_{it} \tag{2}$$

and GDP growth is

$$\frac{\Delta Y_{t+1}}{Y_t} = \frac{1}{Y_t} \sum_{i=1}^{N} \Delta S_{i,t+1} = \sum_{i=1}^{N} \sigma_i \frac{S_{it}}{Y_t} \varepsilon_{i,t+1}.$$

As the shocks $\varepsilon_{i,t+1}$ are uncorrelated, the standard deviation of GDP growth is $\sigma_{GDP} = \left(var \frac{\Delta Y_{t+1}}{Y_t} \right)^{1/2}$:

$$\sigma_{GDP} = \left(\sum_{i=1}^{N} \sigma_i^2 \cdot \left(\frac{S_{it}}{Y_t} \right)^2 \right)^{1/2}. \tag{3}$$

Hence the variance of GDP, σ_{GDP}^2, is the weighted sum of the variance σ_i^2 of idiosyncratic shocks with weights equal to $\left(\frac{S_{it}}{Y_t} \right)^2$, the squared share of output for which that firm i accounts. If the firms all have the same volatility $\sigma_i = \sigma$, we obtain:

$$\sigma_{GDP} = \sigma h \tag{4}$$

where h is the square root of the sales Herfindahl of the economy:

$$h = \left[\sum_{i=1}^{N} \left(\frac{S_{it}}{Y_t} \right)^2 \right]^{1/2}. \quad (5)$$

For simplicity, h will be referred to as the "Herfindahl" of the economy.

This paper works first with the basic model (1)-(2), which can be viewed as the linearization of a host of richer models, such as the one in section 3. The arguments apply if general equilibrium mechanisms are added.

2.2. The $1/\sqrt{N}$ Argument for the Irrelevance of Idiosyncratic Shocks

Macroeconomists often appeal to aggregate (or at least sector-wide) shocks, since idiosyncratic fluctuations disappear in the aggregate if there is a large number of firms N. Consider firms of initially identical size equal to $1/N$ of GDP, and identical standard deviation $\sigma_i = \sigma$. Then (4)-(5) gives:

$$\sigma_{GDP} = \frac{\sigma}{\sqrt{N}}.$$

To estimate the order of magnitude of the cumulative effect of idiosyncratic shocks, take an estimate of firm volatility $\sigma = 12\%$ from Section (5), and consider an economy with $N = 10^6$ firms[6]. Then

$$\sigma_{GDP} = \frac{\sigma}{\sqrt{N}} = \frac{12\%}{10^3} = 0.012\% \text{ per year.}$$

Such a GDP volatility of 0.012% is much too small to account for the empirically-measured size of macroeconomic fluctuations of around 1%. This is why economists typically appeal to aggregate shocks. More general modeling assumptions predict a $1/\sqrt{N}$ scaling, as shown by the next Proposition, whose proof is in Appendix B.

Proposition 1 *Consider an islands economy with N firms whose sizes are drawn from a distribution with finite variance. Suppose that they all have the same volatility σ. Then the economy's GDP volatility is:*

$$\sigma_{GDP} = \frac{E\left[S^2\right]^{1/2}}{E\left[S\right]} \frac{\sigma}{\sqrt{N}}. \quad (6)$$

Proposition 1 will be contrasted with Proposition 2 below, which shows that different models of the size distribution of firms lead to dramatically different results.

[6] Axtell (2001) reports that in 1997 there were 5.5 million firms in the United States.

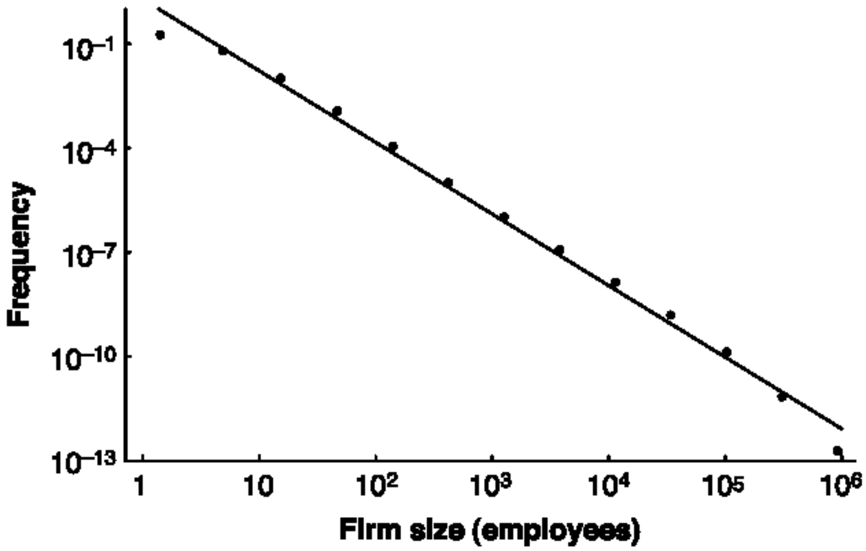

Figure 2: Log frequency $\ln f(S)$ vs log size $\ln S$ of U.S. firm sizes (by number of employees) for 1997. OLS fit gives a slope of 2.059 (s.e.= 0.054; R^2 =0.992). This corresponds to a frequency $f(S) \sim S^{-2.059}$, i.e. a power law distribution with exponent $\zeta = 1.059$. This is very close to Zipf's law, which says that $\zeta = 1$. Source: Axtell (2001).

2.3. Empirical Evidence that for the Fat-Tailed Distribution of Firms

Empirical evidence suggests that a good parameterization for the firm size distribution is a power law distribution:

$$P(S > x) = ax^{-\zeta}. \tag{7}$$

for $x > a^{1/\zeta}$. To estimate (7), it is useful to take the density $f(x) = \zeta a/x^{\zeta+1}$, and its logarithm

$$\ln f(x) = -(\zeta + 1)\ln x + C \tag{8}$$

where C is a constant. An extensive literature has estimated the size distribution of firms, but typically the sample includes only firms listed in the stock market. Axtell (2001) extends the literature by using the Census, which lists all the U.S. firms.

I reproduce his plot of (8) in Figure 2. The horizontal axis shows $\ln x$, where x is the size of a firm in number of employees. The vertical axis shows the log of the fraction of firms with size x, $\ln f(x)$. We expect to see a straight line in the region where (8) holds, and indeed the Figure shows a very clear fit.[7] An OLS fit of (8) yields an estimate of $R^2 = 0.992$, and $\zeta = 1.059 \pm 0.054$.

[7] Power law fits are typically less good at the two extremes of the distribution. In Axtell's data, which are binned in powers of 3, the fit is less good for firms between 1 and 3 employees. This does not affect our analysis, which deals with large firms. The fit may also be less good in the last bin, for the firms between 3^{11} =309,000 and

The size distribution of U.S. firms is well approximated by the power law with exponent $\zeta = 1$.[8]

The rest of the paper will pay special attention to the case $\zeta = 1$, the "Zipf" value. This value ($\zeta \simeq 1$) is often found in the social sciences, for instance in the size of cities (Zipf 1949), and in the amount of assets under management of mutual funds (Gabaix *et al.* 2003) and banks (Pushkin and Hassan 2004).[9] The origins of this distribution are becoming better understood[10].

The power law distribution (7) has fat tails, and thus produces some very large firms. The next section studies its implications for GDP fluctuations.

2.4. The Failure of the $1/\sqrt{N}$ Argument when the Firm Size Distribution is Power Law

The next Proposition examines behavior under a "fat-tailed" distribution of firms. The proof is in Appendix B.

Proposition 2 *Consider an islands economy with a large number N of firms with volatility of growth rate σ, and whose size distribution is a power law distribution $P(S > x) = ax^{-\zeta}$ with exponent $\zeta \geq 1$. Then its GDP volatility is:*

$$\sigma_{GDP} \sim \frac{v_\zeta}{\ln N}\sigma \text{ for } \zeta = 1 \quad (9)$$

$$\sigma_{GDP} \sim \frac{v_\zeta}{N^{1-1/\zeta}}\sigma \text{ for } 1 < \zeta < 2 \quad (10)$$

$$\sigma_{GDP} \sim \frac{v_\zeta}{N^{1/2}}\sigma \text{ for } \zeta \geq 2 \quad (11)$$

where v_ζ is a random variable that is independent of N and σ.

The firm size distribution has thin tails, i.e. finite variance, if and only if $\zeta > 2$. Proposition 1 states that if the firm size distribution has thin tails, then σ_{GDP} decays according to as $1/\sqrt{N}$. In contrast, Proposition 2 states that if the firm size distribution has fat tails ($\zeta < 2$), σ_{GDP} decays much more slowly than $1/\sqrt{N}$.

In the limit case of Zipf's law ($\zeta = 1$), larger countries are barely more diversified than small countries.[11] The reason is that, if Zipf's law holds, the top K firms of a country account for

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3^{12} =920,000 employees. But there are so few such firms, that it is unclear whether the deviation is statistically significant.

[8] Okuyama *et al.* (1999) and Fujiwara *et al.* (2004) also find that $\zeta \simeq 1$ for Japanese and European firms respectively.

[9] Axtell (2001) shows also $\zeta \simeq 1$ for sales.

[10] See Simon (1955), Gabaix (1999), Gabaix (2009) for a survey of various candidate explanations, and Luttmer (2007) and Rossi-Hansberg and Wright (2007) for recent developments.

[11] If there are N identical firms, $1/h_N^2 = N$. So $1/h_N^2$ reveals the "effective" number of firms in the economy, for diversification purposes. So, in a Zipfian world (where $\zeta = 1$), the effective number of firms is not N but $(\ln N)^2$. For $1 < \zeta < 2$, the effective number of firms scales as $N^{2-2/\zeta}$. This notion of the "effective" number of firms is important as long as diversification plays a role, as is the case in Caballero and Engel (2004) and the present paper.

a finite, as opposed to infinitesimal, fraction of the total output.[12] To see the result, take two countries, 1 and 2, and suppose that country 2 is twice as large as country 1, in the sense that it has twice as many firms as country 1. Firms in both countries are drawn from the same distribution. If Zipf's law holds, then the largest firm of country 2 will be, on average, twice as large as the largest firm in country 1. Indeed, the largest K firms will be, on average, twice as large in country 2 than in country 1. Hence, the relative share of the top K firms will be the same in country 2 and country 1. The Herfindahls are the same, and, as GDP volatility comes only from firm-level volatility in the scenario considered in Proposition 2, GDP volatilities are the same. If the distribution is a power law with exponent ζ between 1 and 2, the same reasoning holds, except that the largest firms in country 2 are $2^{1/\zeta}$ larger than in country 1, so their share of GDP is $2^{1/\zeta-1} < 1$ that of country 1. Thus the volatility of country 2 is that of country 1, times $2^{1/\zeta-1}$, as given by equation 10.

Proposition 2 offers a resolution to the debate between Horvath (1998, 2000) and Dupor (1999). Horvath submits evidence that sectorial shocks may be enough to generate aggregate fluctuations. Dupor (1999) debates this on theoretical grounds, and claims that Horvath is able to generate large aggregate fluctuations only because he uses a moderate number of sectors ($N = 36$). If he had much more disaggregated sectors (e.g. 100 times as many), then aggregate volatility would decrease in $1/\sqrt{N}$ (e.g. 10 times smaller). Proposition 2 illustrates that both viewpoints are correct, but apply in different settings. Dupor's reasoning holds only in a world of small firms, when the central limit theorem can apply. Horvath's empirical world is one where the firm size distribution of firms is sufficiently leptokurtic that the central limit theorem does not apply. Instead, Proposition 2 applies, and GDP volatility remains substantial even if the number N of subunits is large.

Though the benchmark case of Zipf's law is empirically relevant, and theoretically clean and appealing, most of the arguments in this paper do not depend on it. The results only require that the Herfindahl of actual economies is sufficiently large. For instance, if the distribution of firm sizes was lognormal with a sufficiently high variance, then quantitatively very little would change.

2.5. GDP Volatility When the Volatility of a Firm Depends on its Size

I now study the case where the volatility of a firm's percentage growth rate decreases with firm size. I examine the functional form $\sigma^{\text{Firm}}(S) = kS^{-\alpha}$, from equation 12. If $\alpha > 0$, then large firms have a smaller standard deviation that small firms. This was Hymer and Pashigan (1962)'s original finding. A series of papers (Stanley *et al.* 1996, Amaral *et al.* 1997, Canning *et al.* 1998) quantify the relation more precisely, and showed that (12) holds for firms in Compustat,

[12] This is true up to a slowly varying factor, $1/\ln N$.

with $\alpha \simeq 1/6$.

It is unclear whether the conclusions from Compustat can generalize to the whole economy. Compustat only comprises firms traded on the stock market and these are likely to be more volatile than non-traded firms, as small volatile firms are more likely to seek outside equity financing, while large firms are in any case very likely to be listed in the stock market. This selection bias implies that the value of α measured from Compustat firms alone is likely to be larger than in a sample composed of all firms. It is indeed possible α may be 0 when estimated on a sample that includes all firms, as random growth models have long postulated. Axtell and Teitelbaum (2005), using on two years of data from the U.S. census, conclude that $\alpha \simeq 0$. Further research is needed to verify this on a comprehensive set of large firms over a long time period.

In any case, any deviations from Gibrat's law for variances are likely to be small, i.e. $0 \leq \alpha \leq 1/6$. If there is no diversification as size increases, then $\alpha = 0$. If there is full diversification, and a firm of size S is composed of S units, then $\alpha = 1/2$. Empirically, firms are much closer to the "Gibrat" benchmark of no diversification, $\alpha = 0$.

The next Proposition extends Propositions 1 and 2 to the case where firm volatility decreases with firm size.

Proposition 3 *Consider an islands economy, with N firms that have power law distribution $P(S > x) = x^{-\zeta}$ for $\zeta \in [1, \infty)$. Assume that the volatility of a firm of size S is*

$$\sigma^{Firm}(S) = kS^{-\alpha} \tag{12}$$

for some $\alpha > 0$. Define

$$\alpha' = \min\left(\frac{1}{2}, 1 + \frac{\alpha - 1}{\zeta}\right) \tag{13}$$

GDP fluctuations have the form:

$$\frac{\Delta Y_t}{Y_t} = kN^{-\alpha'} g_t \text{ if } \zeta > 1 \tag{14}$$

$$\frac{\Delta Y_t}{Y_t} = k\frac{N^{-\alpha'}}{\ln N} g_t \text{ if } \zeta = 1 \tag{15}$$

such that when $N \to \infty$, g_t converges to a non-degenerate distribution. When $\zeta > 1$, g_t converges to a Lévy stable distribution with exponent $\min\{\zeta/(1-\alpha), 2\}$.

In particular, the volatility $\sigma(S)$ of GDP growth decreases as a power law function of GDP

S:[13]

$$\sigma^{\mathrm{GDP}}(S) \sim S^{-\alpha'}. \tag{16}$$

To see the intuition for Proposition 3, we apply the case of Zipf's law ($\zeta = 1$) to the two-country example of Proposition 2.[14] Country 2 has twice as many firms as country 1. Its largest K firms are twice as large as the largest firms of country 1. However, scaling according to (12) implies that their volatility is $2^{-\alpha}$ times the volatility of firms in country 1. Hence, the volatility of country 2's GDP is $2^{-\alpha}$ times the volatility of country 1's GDP, i.e. (16). Putting this another way, under the case presented by Proposition 3, and $\zeta = 1$, large firms are less volatile than small firms (equation 12). The top firms in big countries are larger (in an absolute sense) than top firms in small countries. As the top firms determine a country's volatility, big countries have less volatile GDP than small countries (equation 16).

Also, one can reinterpret Proposition 3 by interpreting a large "firm" as a "country" made up of smaller entities. If these entities follow a power-law distribution, then Proposition 3 applies and predicts that the fluctuations of the growth rate $\Delta \ln S_{it}$, once re-scaled by $S_{it}^{-\alpha}$, follow a Lévy distribution with exponent $min\left\{\zeta/\left(1-\alpha\right), 2\right\}$. Amaral *et al.* (1997) and Canning *et al.* (1998) plot this empirical distribution, which looks roughly like a Lévy stable distribution. It could be that the fat tails distribution of firm growth come from the fat tail distribution of the subcomponents of a firm.[15]

A corollary of Proposition 3 may be worth highlighting.

Corollary 1 *(Similar scaling of firms and countries.) When Zipf's law holds ($\zeta = 1$) and $\alpha \leq 1/2$, we have $\alpha' = \alpha$, i.e. firms and countries should see their volatility scale with a similar exponent:*

$$\sigma^{Firms}(S) \sim \sigma^{GDP}(S) \sim S^{-\alpha}. \tag{17}$$

Interestingly, Canning *et al.* (1998) and Lee *et al.* (1998) present evidence that supports (17), with an small exponent $\alpha \simeq \alpha' \simeq 1/6$ (see also Koren and Tenreyro 2007). A more systematic investigation of this issue would be interesting.

3. A Model with Comovement

The previous section has shown that how idiosyncratic firm shocks might explain a significant portion of aggregate fluctuations. This section considers whether they can also create plausibly

[13] In this paper, $f(S) \sim g(S)$ for some functions f, g, means that the ratio $f(S)/g(S)$ tends, for large S, to be a positive real number. So f and g have the same scaling "up to a constant real factor".

[14] When $\zeta = 1$, the limiting distribution of g is a more complicated distribution, a ratio of two non-independent Lévy distributions. Logan et al. (1973) provide an analysis of some such ratios.

[15] See Sutton (2002) for a related model; and Wyart and Bouchaud (2003) for a related analysis, which acknowledges the contribution of the present article (which was first circulated in the Fall 2001).

the strong comovements between the various firms or sectors of the economy, as observed, for instance, by Long and Plosser (1983), Shea (2002), Franco and Philippon (2008) and Foerster, Sarte and Watson (2008). Horvath (1998), Long and Plosser (1983) and Shea (2002) present models that generate comovement.[16] Horvath (2000) calibrates a dynamic general equilibrium model with many sectors. I present a simplified version of those models. Its main virtue is that it is solvable in closed form, so that the mechanisms are fairly transparent.

After a shock to firm i, all the other firms adjust instantaneously, rather than over time through the input-output matrix. There is an aggregate good. Each intermediate good firm i uses L_i, K_i, X_i of labor, capital and aggregate good, to produce:

$$Q_i = \frac{A_i \left(L_i^{\alpha} K_i^{1-\alpha}\right)^b X_i^{1-b}}{b^b (1-b)^{1-b}} \tag{18}$$

GDP is production net of the intermediate inputs, the X_i's:

$$Y = \left(\sum_i Q_i^{1/\psi}\right)^{\psi} - \sum_i X_i \tag{19}$$

with $\psi > 1$. b is the share of intermediate inputs, and will also be the ratio of value added to sales, both at the level of the firm, and of the economy.

The representative agent's utility function is $U = C - L^{1/\xi}$.[17] There is no investment, so $C = Y$. Thus the social planner's program is: $\max_{\{K_i, L_i, X_i\}} Y - L^{1/\xi}$ subject to $\sum K_i = K; \sum L_i = L$.

To abstract from a potential inefficiency arising from positive markups, I assume that the prices equal marginal cost. Several devices can generate this assumption. Firms could be competitive because there is free entry – markets are contestable in the sense of Baumol (1982). Another interpretation is that the "firms" are sectors made of competitive firms. A last possibility is that the government may have set an input subsidy equal to ψ for the intermediary firms. In any case, even if there was a strictly positive, constant markup, the formulas for the main results of this section (Proposition 4) would be unaffected.

[16] Long and Plosser (1983) impose a Cobb-Douglas structure, which imposes zero idiosyncratic movement in the sales per employee and dollar sales.

[17] Here the utility function $C - L^{1/\xi}$ simply captures the flexibility of labor supply along the business cycle, as formalized in equations (26)-(27). The linearity in C is for convenience only, as the model abstracts from interest rate movements and capital accumulation.

The model gives:

$$\text{GDP}: Y = \Lambda L^{\alpha} K^{1-\alpha} \quad (20)$$

$$\text{TFP}: \Lambda = \left(\sum_i A_i^{1/(\psi-1)}\right)^{(\psi-1)/b} \quad (21)$$

$$\text{Sum of sales}: H = \sum p_i Q_i = Y/b \quad (22)$$

$$\frac{\text{Sales}_i}{\text{GDP}}: \frac{p_i Q_i}{Y} = \left(\frac{A_i}{\Lambda^b}\right)^{1/(\psi-1)} /b \quad (23)$$

The result is standard, except for the b term in Eq. (21), which indicates that $1/b$ is a "productivity multiplier". If all firms increase their productivity A_i by 1%, TFP increases by $1/b$ %. This effect comes from the fact that a Hicks-neutral productivity shock increases gross output (sales), not just value added, and has been analyzed by Domar (1961), Hulten (1978) and Jones (2009).[18]

I use the "hat" notation to indicate a proportional change: $\widehat{Z} = dZ/Z$.[19] I assume that we start from a steady state equilibrium, and that, in the short run, labor but not capital is reallocated across firms.[20]

Models such as (18) always deliver a Sales / Employees ratio that is independent of the firm's productivity.[21] The reason for this almost surely counterfactual prediction, is that labor is assumed to be costlessly adjustable. To capture the realistic case of labor adjustment costs, I assume that a fraction $1-\lambda$ of labor is a quasi-fixed factor, in the sense of Oi (1962). Technically, I represent: $L_i = L_{V,i}^{\lambda} L_{F,i}^{1-\lambda}$, where $L_{V,i}$ and $L_{F,i}$ are respectively the variable part labor and the quasi-fixed part of labor. After a small shock, only $L_{V,i}$ adjusts. The disutility of labor remains $L^{1/\xi}$, where $L = L_V^{\lambda} L_F^{1-\lambda}$ is aggregate labor.[22] Likewise, I assume that capital is quasi-fixed in the short run.[23]

One can now study the effect of a productivity shock $\widehat{A}_i$ to each firm i. I call $S_i = p_i Q_i$ the

[18] To see this effect most clearly, consider an economy with a production function which, at the level of the representative firm, is $Q = A(L/b)^b (X/(1-b))^{1-b}$, where X is the intermediary inputs. GDP is $Y = \max_X A(L/b)^b (X/(1-b))^{1-b} - X$. Solving for X yields $Y = A^{1/b} L$. Though TFP is A at the firm level, it is $A^{1/b}$ at the aggregate level.

[19] The rules are well-known, and come from taking the logarithm and differentiating. For instance, $\widehat{X^{\alpha} Y^{\beta} Z^{\gamma}} = \alpha\widehat{X} + \beta\widehat{Y} + \gamma\widehat{Z}$.

[20] This formulation allows for other variants. For instance, if both capital and labor can be reallocated, then one replaces α, the current share of labor, by 1.

[21] If the sales are $S_i = f(A_i) L_i^{\theta}$, the frictionless optimum labor supply maximizes $f(A_i) L_i^{\theta} - wL_i$, and so at the optimum, the ratio of sales per employee, $S_i/L_i = w/\theta$, is independent of the productivity A_i.

[22] Alternatively, one could formulate the disutility of labor as any function $v(L_V, L_F)$. ξ is then defined as: $\xi = \lambda / \left(L_V \frac{\partial^2 v/\partial^2 L_V}{\partial v/\partial L_V} + 1\right)$.

[23] Proposition 4 describes the short term behavior. In a next iteration of this paper, the long term analysis should be provided, including with the adjustment of quasi-fixed labor, and capital. One can anticipate that these extensions will not change materially the calibration for the short run.

dollar sales of firm i.

Proposition 4 *Suppose that each firm i receives a productivity shock $\widehat{A}_i$. Macroeconomic variables change according to:*

$$TFP: \ \widehat{\Lambda} = \sum \frac{S_i}{Y} \widehat{A}_i = \sum \frac{Sales_i}{GDP} \widehat{A}_i \tag{24}$$

$$GDP: \ \widehat{Y} = \frac{1}{(1-\alpha\xi)} \widehat{\Lambda} \tag{25}$$

$$Employment: \widehat{L} = \xi \widehat{Y} \tag{26}$$

$$Wage: \widehat{w} = (1-\xi) \widehat{Y} \tag{27}$$

and firm-level variables change according to:

$$Dollar\ sales: \ \widehat{S}_i = \widehat{X}_i = \beta \widehat{A}_i + (1 - \beta b (1-\alpha\xi)) \widehat{Y} \tag{28}$$

$$Production: \widehat{Q}_i = \psi\beta \widehat{A}_i + (1 - \psi\beta b (1-\alpha\xi)) \widehat{Y} \tag{29}$$

$$Price: \widehat{p}_i = -(\psi-1) \beta \widehat{A}_i + (\psi-1) \beta b (1-\alpha\xi) \widehat{Y} \tag{30}$$

$$Employment: \widehat{L}_i = \lambda\beta \widehat{A}_i + \lambda (\xi - \beta b (1-\alpha\xi)) \widehat{Y} \tag{31}$$

$$Dollar\ sales\ per\ Employee: \widehat{S_i/L_i} = (1-\lambda) \beta \widehat{A}_i + (1 - \lambda\xi - (1-\lambda) \beta b (1-\alpha\xi)) \widehat{Y} \tag{32}$$

where

$$\beta = 1/(\psi - b\alpha\lambda - 1 + b) \tag{33}$$

Equation (24) is Hulten's (1978) equation. TFP is entirely the sum of idiosyncratic firm-level shocks. Otherwise equations (25)-(27) are standard. GDP growth is TFP growth, multiplied by an amplification mechanism, labor supply.

The new results are the firm-level changes, in equations (28)-(32). The economy behaves like a one-factor model, with an "aggregate shock", the GDP shock $\widehat{Y}$. Again, this shock stems from a multitude of idiosyncratic shocks. The "aggregate shock" causes all firm-level quantities to comove. Aggregating, industry-level quantities would comove too. Economically, when firm i has a positive shock, it makes the aggregate economy more productive (equations 21 and 24), and affect the other firms in three different ways. First, other firms can use more intermediary inputs produced by firm i, hence increasing their production. Second, firm i demands more inputs from the other firms (equation 28), which leads their production to increase. Third, given firm i commands a large share of output, it will use more of the inputs of the economy, which tends to reduce the other firms' output.[24] The net effect depends on the magnitudes of the elasticities.

[24] In a more general framework (e.g., Acemoglu 2002), firm i could use less of some inputs.

I calibrate the model using conventional parameters to the extent possible.[25] The labor share is $\alpha = 2/3$. For the share of intermediate inputs Jorgensen, Gollop and Fraumeni (1987), updated in 1996, provide $b = 1/2$.[26] The elasticity ψ is a conventional $\psi = 1.2$. [27] To set $\xi \in (0, 1)$, I am guided by (26)-(27). In the business cycle, the volatility of hours is greater than the volatility of the compensation, which means $\xi \in (1/2, 1)$. Indeed, the ratio of these volatilities is about 2.5 empirically[28], which together with (26)-(27) implies $\xi = 0.7$. By contrast, there is little inherited guidance about the share of labor that is flexible in the short run, λ. Given λ is between 0 and 1, I rely on Laplace's principle and set $\lambda = 1/2$.

Of particular interest is the measure used in section 4, the change in sales per employee, $\widehat{S_i/L_i}$. It varies with the true productivity $\widehat{A}_i$, with a coefficient, $(1 - \lambda)\beta$. Hence, idiosyncratic movements in $\widehat{S_i/L_i}$ are a good measure of the idiosyncratic shocks in productivity. The above parameter values generate a coefficient of $(1 - \lambda)\beta = 0.94$ in equation (32), which is very close to one. Hence, labor productivity is a good measure of true productivity.

All the variables in Proposition 4 have a positive loading on the GDP factor $\widehat{Y}$, i.e. they all comove positively with GDP. I conclude that the above model is a useful benchmark to understand comovement the business cycle.

4. Tentative Empirical Evidence from the Granular Residual

4.1. The Granular Residual: Motivation and Definition

This section presents tentative evidence that the idiosyncratic movements of the top 100 firms explain an important fraction (one third) of the movement of total factor productivity (TFP). The key challenge is to identify idiosyncratic shocks. Large firms could be volatile because of aggregate shocks, rather than the other way round. I use a variety of ways to attempt to do this.

[25] Model (18) allows only for 1 firm-level shock, $\widehat{A}_i$, which generates a perfect correlation between firm-specific movements in sales and employment. However, the data shows the an imperfect correlation between sales and employement of about 1/2, and a correlation between employment and labor productivity (as measured by sales / employees) of about -1/2, which indicates two shocks – perhaps one Hicks neutral productivity or demand shock, and one labor-saving shock. This type of difficulty is familiar, and there is no consensus solution. Hence, the above calibration can only be indicative a definitive one would require a richer model with two types of firm-level shocks. See Altig, Christiano, Eichenbaum and Linde (2005) for a calibration that takes into account many relevant frictions

[26] I thank Susanto Basu for providing this updated number.

[27] A higher ψ may be required to better fit the short run behavior. As Golosov and Lucas (2007) note, this implies that a 10% reduction in price induces a very high quantity response of $\psi/(\psi - 1) \times 10\% = 60\%$, which is probably unrealistically large as a short-run response response. Hence, for the purposes of a calibration of short-run shocks, one may need a higher ψ. This problem, they point out, is "endemic" in constant elasticity of substitution models. Other frictions need to be included to solve this problem. I do not attempt to resolve the problem. For the limited purposes of this calibration, $\psi = 1.2$ is probably a good benchmark.

[28] Indeed, in Cooley and Prescott (1995, p.30), the volatility of hours is 1.64% while the volatility of the wage is 0.65%.

I start with a parsimonious proxy for the labor productivity of firm i, the log of its sales per worker:

$$z_{it} := \ln \frac{\text{Sales of firm } i \text{ in year } t}{\text{Number of employees of firm } i \text{ in year } t}. \tag{34}$$

This measure is selected as it requires only basic data that is more likely to be available for non-US countries, unlike more sophisticated measures such as a firm-level Solow residual. Most studies that construct productivity measures from Compustat data use (34). I define the productivity growth rate as $g_{it} = z_{it} - z_{it-1}$. I focus on the $K = 100$ firms that had the largest sales in year $t-1$. (Results are similar for other choices of K).

Many models, including the model of section 3 (eq. 32) predict that labor productivity growth rate behaves according to:

$$g_{it} = f_t + \gamma \widehat{A}_{it}. \tag{35}$$

where f_t is a factor (proportional to GDP growth in equation 32), and $\widehat{A}_{it}$ is the growth rate of total factor productivity of firm i. The same models also support the following decomposition:

$$g_{it} = a_t + a_{I_i(t)} + \varepsilon_{it} \tag{36}$$

where a_t is a shock common to all firms, $a_{I_i(t)}$ is a shock specific to the industry I_i of firm i, and ε_{it} is a shock that is purely idiosyncratic to firm i. Hence equations 35 and 36 link firm total productivity growth $\widehat{A}_{it}$ to market and industry factors, and an idiosyncratic component. Section 4.3 analyzes more complex models.

Hulten (1978)'s theorem, and the model of section 3, predict that TFP in the economy evolves according to:

$$\widehat{\Lambda}_t = \sum \frac{\text{Sales}_i}{\text{GDP}} \widehat{A}_{it} = \sum \frac{S_i}{Y} \widehat{A}_{it} \tag{37}$$

My goal is to investigate whether ε_{it}, the idiosyncratic component of the total factor productivity growth rate $\widehat{A}_{it}$ of large firms, can explain aggregate TFP.[29] Owing to the relationship between $\widehat{A}_{it}$ and g_{it}, this leads to the definitions:

Definition 1 *The granular residual Γ_t is defined as:*

$$\Gamma_t := \frac{\sum_{i=1}^{K} Sales_{i,t-1} \left(g_{it} - \overline{g_t}\right)}{\sum_{i=1}^{K} Sales_{i,t-1}}. \tag{38}$$

[29] The same analysis, with Γ_t, can be used to explore the weak form of the granular hypothesis, namely that idiosyncratic industry level shocks affect a large part of GDP.

where $\overline{g}_t$ is the equal–weighted average productivity growth rate of the top K firms:

$$\overline{g}_t = K^{-1}\sum_{i=1}^{K} g_{it}. \tag{39}$$

Definition 2 *The granular residual with industry de-meaning Γ_t^{ind} is defined as:*

$$\Gamma_t^{ind} = \frac{\sum_{i=1}^{K} Sales_{i,t-1}\left(g_{it} - \overline{g_{I(i)t}}\right)}{\sum_{i=1}^{K} Sales_{i,t-1}} \tag{40}$$

where $\overline{g}_{I(i)t}$ is the equal–weighted average productivity growth rate, amongst the top Q largest firms, for the for firms in i's industry:

$$\overline{g_{I_it}} = \text{Mean of } g_{jt}, \text{ for firm } j \text{ belonging to firm } i\text{'s industry,} \tag{41}$$
$$\text{and in the top } Q \text{ firms by sales at } t-1.$$

The justification for the definitions is as follows. In the residual Γ_t, the term $\overline{g}_t$ removes the common shock a_t if (36) holds. In the residual Γ_t^{ind}, the term $\overline{g_{I_it}}$ removes the industry shock $a_t + a_{I_i(t)}$ if (36) holds. Although Γ_t^{ind} offers a better control than Γ_t for industry shocks, it does not have uniformly better properties than Γ_t. Indeed, if the common component of volatility is much greater than industry-specific fluctuations (as often found, for instance in Stock and Watson 2005, and also predicted by the model in section 3), then Γ_t^{ind} is a noisier proxy for the true residual $\widehat{\Lambda}_t$ than Γ_t, as Lemma 1 in Appendix B indicates. In addition, the Γ_t^{ind} requires more data. In any case, Γ_t and Γ_t^{ind} are highly correlated, and I use both in the empirical analysis.

Lemma 1 in Appendix B indicates that the definition of Γ_t and Γ_t^{ind} is optimal, in the sense of maximizing the correlation with TFP while purging common shocks.[30] Also, the online appendix to this paper shows that identification is achieved if the number K of firms in the granular residual is very large.[31]

A simple example illustrates the granular residual.[32] Suppose that the economy is made of one big firm, which produces half of output, a hundred small ones. The standard deviation of all growth rates is 10%, and growth rates are given by $g_{it} = a_t + \varepsilon_{it}$, where a_t is a common shock.

[30] Finally, an alternative definition would be to place GDP in the denominator of (38). In practice, such a measure is well correlated with the granular residual, and yields very similar results.

[31] As the online appendix to this paper shows, with small K, the R^2 is underestimates the true R^{*2} of the idiosyncratic shocks, by a factor $1 - \frac{1}{KH_K}$, where $H_K = \sum_{i=1}^{K} S_i^2 / \left(\sum_{i=1}^{K} S_i\right)^2$ is about 0.6. Hence, if R^2 empirically found R^2 is 1/3, the R^2 of true idiosyncratic shocks is $R^{*2} = 1/2$. On the other hand, if the number of firms K becomes very large, $1 - \frac{1}{KH_K}$ tends to 1, and the R^2 becomes unbiased. I do not pursue that route, because for very large K the homogeneity postulate (36) is less likely to hold.

[32] I thank Olivier Blanchard for this example.

Suppose that in a given year, GDP increases by 3%, and the big firm has growth of, say, 6%, while the average of the small ones is close to 0%. What can we infer on the origins of shocks? If one thinks of all this being generated by an aggregate shock of 3%, then the distribution of implied idiosyncratic shocks is 3% for the big firm, and −3% on average for all small ones. The probability that the average of the i.i.d. small ones is −3%, given the law of large number for these firms, is very small. Hence, it is more likely that the average shock a_t is around 0%, and the economy-wide growth of 3% comes from an idiosyncratic shock to the large firm equal 6%. The estimate of the aggregate shock is captured by $\overline{g}_t$, which is close to 0%, and the estimate of the contribution of idiosyncratic shocks is captured by the granular residual, $\Gamma = 3\%$.

4.2. Empirical Implementation

I use annual U.S. Compustat data from 1951 to 2008. For the granular residual, I take for each year $t-1$ the $K = 100$ largest firms in Compustat that are not in the oil or energy sector.[33] Compustat contains some large outliers, which may result from extraordinary events, such as a merger. To handle potential outliers, I winsorize the extreme growth rates. Specifically, I construct:

$$\Gamma_t := \left(\sum_{i=1}^{K} \text{Sales}_{i,t-1}\right)^{-1} \left(\sum_{i=1}^{K} \text{Sales}_{i,t-1} T\left(g_{it} - \overline{g_t}\right)\right) \tag{42}$$

$$\Gamma_t^{ind} = \left(\sum_{i=1}^{K} \text{Sales}_{i,t-1}\right)^{-1} \left(\sum_{i=1}^{K} \text{Sales}_{i,t-1} T\left(g_{it} - \overline{g_{I_i t}}\right)\right) \tag{43}$$

using the trimming function $T(x) = x$ if $|x| \leq M$, $T(x) = sign(x) \cdot M$ if $|x| > M$. (I use $M = 20\%$, but the results are not materially sensitive to this threshold.) In other words, when a growth rate is larger than 20%, I replace the growth rate by 20%, and I do the same for negative growth rates.

Table 1 presents regressions of GDP growth and the Solow residual on the granular residual. These regressions are supportive of the granular hypothesis. The granular residual, and its lagged values, explain slightly over 1/3 of the fluctuations of GDP growth and the Solow residual.

If only aggregate shocks were important (f_t in Eq. 35), then the R^2 of the regressions in Table 1 would be zero. Hence the good explanatory power of the granular residual is inconsistent with a representative firm framework. It is also inconsistent with the hypothesis that most firm-level volatility might be due to a zero-sum redistribution of market shares.

[Insert Table 1 about here]

[33] For firms in the oil and energy sector, the wild swings in world-wide energy prices make (34) too poor a proxy of total factor productivity.

Table 1 indicates that the lagged granular residual predicts GDP growth. This may reflect several mechanisms: autocorrelation at the firm level; imitation dynamics, where a successful technology is imitated by other firms; time aggregation; and the propagation of shocks along supply and demand chains, as in Long and Plosser (1983).

[Insert Table 2 about here]

We next turn to the granular residual with industry-specific de-meaning, Γ_t^{ind}, using 2 digit SIC codes to define the industries. Table 2 presents the results, which are consistent with those in Table 1. The R^2's are slightly higher, with an average of 41% (and 38% for the adjusted R^2) across specifications. The similarity of the results is not surprising, as the correlation between Γ_t and Γ_t^{ind} is 0.84.

In conclusion, idiosyncratic movements of the top 100 firms seem to explain a large fraction of the Solow residual and GDP fluctuations.

4.3. Robustness Checks

The main objection to the granular residual is that the control for the common factors may be imperfect. This section shows that the explanatory power of the granular residual is not diminished by controlling for previously studied common shocks. Following the work of Hamilton (2003) and Romer and Romer (2004), I control for oil and monetary policy shocks. To arrive at an annual frequency, I sum the shocks over the years.

Table 3 characterizes the explanatory power of those variables. Monetary and oil shocks explain 21% of GDP growth.

[Insert Table 3 about here]
[Insert Table 4 about here]

Table 4 shows the explanatory power of the granular residual, controlling for oil and monetary shocks. The mean R^2 across specifications is 53%. Hence, if oil and monetary shocks explain 22% of GDP shocks, the granular residual explains a minimum of 31% of GDP fluctuations. This estimate is close to the one of 1/3 found in section 4.2.

I conclude that adding controls for oil and monetary shocks confirms the initial estimate of good explanatory power of the granular residual, of approximately 1/3 of GDP fluctuations. Further analysis shows that the results seem reasonably robust to changes in the number of large firms K, the number of firms for the industry controls Q, and the trimming level.

Another potential objection to the granular residual is that, while Γ_t^{ind} controls for industry shocks and common shocks in model (36), it may not control well for them in an alternative model such as:

$$g_{it} = \beta_i a_t + \beta_i' a_{I_i(t)} + \varepsilon_{it} \tag{44}$$

Given the short sample, if the β_i's are completely unconstrained, there does not seem to be any manageable way to estimate (44) and obtain clean enough estimates of the ε_{it} to form a granular residual of their weighted sum, and test the granular hypothesis.[34] Thus, some parametric restriction seems necessary. The most natural is that the sensitivity to shock might depend on firm size. Accordingly, I consider the following model:

$$g_{it} = \left[1 + b\left(\ln S_{i,t-1} - \overline{\ln S_{j,t-1;,j\in I_i}}\right)\right] g_{I_i t} + \varepsilon_{it}, \tag{45}$$

where ε_{it} is orthogonal to g_{It}, and $\overline{\ln S_{j,t-1;,j\in I_i}}$ is the mean log size of firms in i's industry. I estimate it by running the OLS regression:

$$g_{it} - \overline{g}_{I_i t} = b\left(\ln S_{i,t-1} - \overline{\ln S_{j,t-1;,j\in I_i}}\right) \overline{g}_{I_i t} + \text{noise}_{it} \tag{46}$$

which yields $b = -0.34$ (s.e. 0.04).[35] This result mean that large firms are actually *less* sensitive to aggregate shocks than small firms. Hence, in the definition of Γ_t (respectively Γ_t^{ind}), $\overline{g}_t$ (respectively $\overline{g}_{I_i t}$) adds a negative loading on the industry and GDP shocks. Hence the results from Γ_t and Γ_t^{ind} are biased *against* the granular hypothesis. I conclude that consideration of models such as (45) reinforces the previous results, rather than contradicts them.

The above results are considered provisional. The situation is the analogue, with smaller stakes, to that of the Solow residual.[36] Solow understood at the outset that there are very strong assumptions in the construction of his residual, in particular fully capacity utilization, no fixed cost etc. But a "purified" granular residual took decades to construct (e.g., Basu et al. 2006), and requires much better data, is harder to replicate in other countries, and relies on special assumptions as well. Because of that, the Solow residual still endures, at least as a first pass. In the present paper too, it is good to have a first step in the granular residual, together with caveats that may help future research do construct a better residual. The conclusion of this article contains some other measures of granular residuals that build on the present paper.

4.4. A Brief Narrative of GDP and the Granular Residual

Figure 3 plots a time series of the simple and industry-demeaned granular residual. Figure 4 presents a scatter plot with $\Gamma_t + \Gamma_{t-1}$, a choice motivated by the fact that Γ_t and Γ_{t-1} have similar coefficients in the regressions of Table 1.

While a full narrative is outside the scope of this paper, this section proposes an interpretation

[34] For instance, if one regresses g_{it} on GDP, and take the residuals, then tautologically the residuals will be orthogonal to GDP, which by construction contradicts the granular hypothesis. Also, oil and monetary shocks are not good instruments, as they can affect an industry directly.

[35] Estimating: $g_{it} - \overline{g}_t = b\left(\ln S_{i,t-1} - \overline{\ln S_{j,t-1}}\right)\overline{g}_t + \text{noise}_{it}$, gives a similar $b = -0.21$ (s.e. 0.03).

[36] I thank Steve Durlauf for this intepretation.

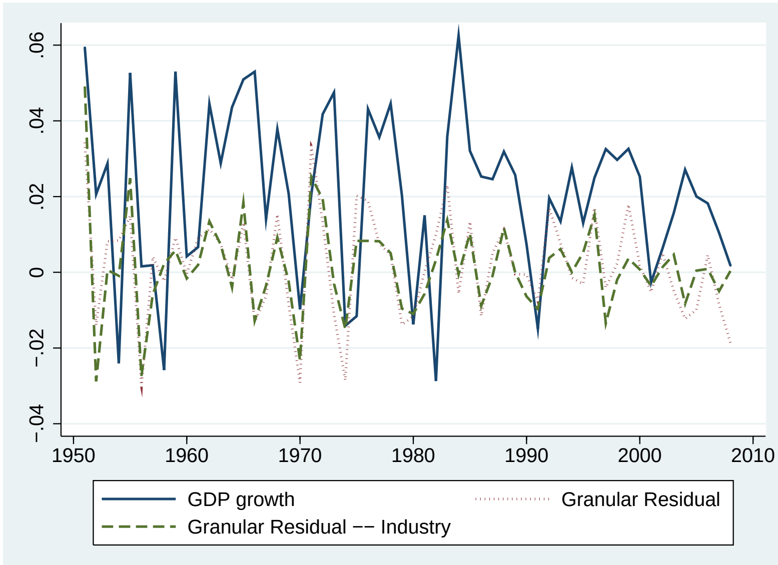

Figure 3: Time series of per capita GDP growth, the granular residual Γ_t, and the granular residual industry de-meaning Γ_t^{ind}, computed over the largest 100 firms by sales in the previous year.

of the most extreme points.[37] A general caveat is that the direction of the causality is generally hard to assess definitively, as the measures $\overline{g_t}$ for aggregate economy-wide and industry-wide movements are imperfect.

The bottom right quadrant of Figure 4 contains two outliers. 1954 can be attributed to the end of the Korean War. 1982 is commonly called the Volcker recession.

An interesting "granular year" may be 1955, which experiences a high GDP growth, and a reasonably high granular residual. The likely microfoundation is a boom in car production. Two main specific factors seem to explain the car boom: the introduction of new models of cars (Gordon 1980), and the fact that car companies engaged in a price war (Bresnahan 1987). In 1955, the granular residual is 1.5%, of which 81% is solely due to General Motors.[38] In 1956, the price war in cars ends, and sales drop back to their normal level (the sales of General Motors decline by 17%). The granular residual is -3.0%, of which 57% is due to General Motors. Hence, one may provisionally conclude the 1955-1956 boom-bust episode was in large part a granular event driven by new models and a price war in the car industry.[39]

An extreme negative granular residual occurs in 1970. This year features a major strike at General Motors, which lasts ten weeks (September 15 to November 20). Sales of GM fall by

[37] Gordon (1980), Temin (1998), and the reports of the Council of Economic Advisors provide useful narratives.

[38] This number is the fraction due to General Motors in the numerator of equation 38. By this definition, the sum of the shares is 1.

[39] To completely resolve the matter, one would like to control for the effect of the Korean war.

Figure 4: Growth of GDP per Capita against $\Gamma_t + \Gamma_{t-1}$, the granular residual and its lagged value. The display of $\Gamma_t + \Gamma_{t-1}$ is motivated by Table 1, which yields regression coefficients on Γ_t and Γ_{t-1} that are similar in magnitude.

31%. Hence, it is plausible to interpret 1970 as a granular year, whose salient event was the GM strike.[40] Additionally, one can interpret the positive granular shock in 1971 (which appears in 4 as label "72", for representing the sum of the granular residuals in 1971 and 1972) as a rebound from the negative granular 1970 shock. Hence the General Motors strike may explain the very negative "70" (1969+1970) point and the very positive "72" (1971+1972) point.

Another interesting granular event happens in 1971. The Council of Economic Advisors (1972, p.33) reports that "prospects of a possible steel strike after July 31st [1971], the expiration day of the labor contracts, caused steel consumers to build up stock in the first seven months of 71, after which these inventories were liquidated." Here, a granular shock – the possibility of a steel strike – creates a large swing in inventories. Without exploring inventories here, one notes that such a plausibly orthogonal inventory shock could be used in future macroeconomic studies.[41]

Figure 4 reveals that in the 1990s, granular shocks are smaller. Likewise, GDP volatility is

[40] Temin (1988) notes that the winding down of the Vietnam War (which ended in 1975) may also be responsible for the slump of 1970. This is in part the case, as during 1968 to 1972 the ratio of defense outlays to GDP was 9.5, 8.7, 8.1, 7.3, 6.7%. On the other hand, the ratio of total government outlays to GDP were respectively 20.6, 19.4, 19.3, 19.5, 19.6% (source: Council of Economic Advisors, 2005, Table B-79). Hence the aggregate government spending shock was very small in 1970.

[41] Although 1974 is not a granular year, the low value of the granular residual reflects the fact that the top three car companies, and particularly General Motors, were disproportionately affected by the shock. It is likely that, if large companies were producing more fuel efficient models, the granular residual would have been closer to 0, and the slump of 1974 could have been much more moderate.

smaller, a phenomenon explored in the literature[42], though of course that needs to be reassessed with the financial crisis that started in 2007.[43] Under a granular view, this might be because firm-level shocks were minor, particularly for large firms. Indeed, it seems that the average firm volatility has decreased since 1976 in the USA (Davis et al. 2006), even though firms in Compustat experienced a rise in volatility (Comin and Philippon 2005), perhaps because of a selection effect (it is the most volatile firms, with the largest growth options, that choose to raise money on the stock market).[44] Further research is needed to assess this hypothesis.

5. Empirical Evidence on Concentration and Firm-Level Volatility

This section illustrates that idiosyncratic fluctuations are indeed of the correct order of magnitude to explain aggregate shocks.

5.1. Large Firms are very Volatile

Most estimates of plant-level volatility find very large volatilities of sales and employment, with an order of magnitude $\sigma = 30\%$ to $\sigma = 50\%$ per year (Caballero and Engel 2004, Caballero, Engel and Haltiwanger 1997, Davis, Haltiwanger, Schuh 1996). Also, the volatility of firm size in Compustat is a very large 40% per year (Comin and Mulani 2006). Much of the work has been focused on the median firm, but the present paper requires an estimate of the volatility of large firms. This sub-section therefore studies the volatility of the top 100 non-oil industry firms each year.

Measuring firm volatility is difficult, because various frictions and identifying assumptions provide conflicting predictions about links between changes in total factor productivity and changes in observable quantities such as sales and employment (Proposition 4). I consider the volatility of three measures of growth rates: $\Delta \ln (\text{Sales}_{it}/\text{Employees}_{it})$, $\Delta \ln\text{Sales}_{it}$ and $\Delta \ln\text{Employees}_{it}$. For each measure and each year, I calculate the cross-sectional variance amongst the top 100 firms of the previous year, and take the average.[45] I find a standard

[42] See, for example, Blanchard and Johnson (2001), McConnell and Perez-Quiros (2000), and Stock and Watson (2003).

[43] It would be interesting to exploit the hypothesis that the financial crisis was largely caused by the (ex-post) mistakes of a few large firms, e.g. Lehman and AIG. Their large leverage and interconnectedness amplified into into a full-fledged crisis, what could have been a run-of-the-mill drop in asset values affecting on average the financial sector. Of course, those ideas are very tentative at this stage.

[44] As per Eq. 3, the weighing for the relevant "average" firm level volatility is the square of the sales, not the sales. This weighing gives an enormously higher weight to the top firms. A future paper will develop this point, which requires some statistical care, as standard analysis based on standard errors cannot be applied. Most moments here are infinite.

[45] In other term, for each year t, I calculate the cross-sectional variance of growth rates, $\sigma_t^2 = K^{-1} \sum_{i=1}^{K} g_{it}^2 -$

deviation of 12%, 12% and 14% for, respectively, for growth rates of the sales per employee, of sales, and of employees. Also, amongst the top 100 firms, the sample correlations are 0.023, 0.073 and 0.033 respectively, for each of the three measure.[46] Hence the correlation between growth rates is small. At the firm level, most variation is idiosyncratic.[47]

In conclusion, the top 100 firms have a volatility of 12% based on sales per employee. In what follows I use $\sigma = 12\%$ per year for firm-level volatility as baseline estimate.

5.2. Herfindahls and Induced Volatility

This sub-section discusses the theoretically appropriate measure of the firm size, for use in constructing the Herfindahl index. The key is given by Hulten's (1978) result, which shows that total sales, rather than value added, is the appropriate measure. Consider an economy with several competitive firms or sectors, and let firm i have a Hicks-neutral productivity growth $d\pi_i$. Hulten (1978) shows that the increase in TFP is:

$$\frac{d\text{TFP}}{\text{TFP}} = \sum_i \frac{\text{Sales of firm } i}{\text{GDP}} d\pi_i. \tag{47}$$

The weights add up to more than 1. This reflects the fact that productivity growth in a firm generates an increase in the economic value of all the inputs it uses. The firms' sales are the proper statistics for that social value. For completeness, Appendix B rederives and generalizes Hulten's theorem.

I now draw the implications for GDP volatility. Suppose productivity shocks $d\pi_i$ are uncorrelated with standard deviation σ_π. Then, the variance of productivity growth is:

$$var \frac{d\text{TFP}}{\text{TFP}} = \sum_i \left(\frac{\text{Sales of firm } i}{\text{GDP}} \right)^2 var\left(d\pi_i\right),$$

and so the volatility of the growth of TFP is:

$$\sigma_{TFP} = h_S \sigma_\pi, \tag{48}$$

$\left(K^{-1} \sum_{i=1}^{K} g_{it}\right)^2$, with $K = 100$. The corresponding average standard deviation is $[T^{-1} \sum_{t=1}^{T} \sigma_t^2]^{1/2}$.

[46] For each year, we measure the sample correlation $\rho_t = \left[\frac{1}{K(K-1)} \sum_{i \neq j} g_{it} g_{jt}\right] / \left[\frac{1}{K} \sum_i g_{it}^2\right]$, with $K = 100$. The correlations are positive. Note that a view that would attribute the major firm-level movements to shocks to the relative demand for a firm's product compared to its competitors, would counterfactually predict a negative correlation.

[47] Hence another indirect measure is the volatility of idiosyncratic stock market returns. If a firm produces a_{it} per year, of which a fraction f is paid in dividends, and the dividend grows at a rate μ, then the Gordon formula predicts a stock price $p_t = a_t f / (R - \mu)$, where R is the discount rate. In particular, the volatility of returns is equal to the volatility of productive capacity a. For the top 100 largest firms, I find an average annualized volatility of idiosyncratic returns of $\sigma = 27\%$.

where h_S is the sales Herfindahl

$$h_S = \left(\sum_{i=1}^{N} \left(\frac{\text{Sales}_{it}}{\text{GDP}_t} \right)^2 \right)^{1/2}. \tag{49}$$

Hulten's theorem allows us to simplify the analysis. For the total volatility, one does not need to know the details of the input-output matrix. The sales Herfindahl is a sufficient statistic. The international Herfindahls are from Acemoglu, Johnson and Mitton (2009). They analyze the Dun and Bradstreet dataset, which has a good coverage of the major firms in many countries.[48]

[Insert Table 5 about here]

Almost all models predict that GDP growth $\widehat{Y}$ is proportional to TFP growth $\widehat{\Lambda}$, when there are no other disturbances. For instance, in equation (25), $\widehat{Y} = \mu\widehat{\Lambda}$ with $\mu = 1/(1 - \alpha\xi)$. If $\widehat{\Lambda}$ is a geometrical random walk, in the neoclassical growth model where only capital can be accumulated, $\widehat{Y} = \mu\widehat{\Lambda}$ in the long run, with $\mu = 1/\alpha$, which gives $\mu = 1.5$.[49] So both in the short run and long run, one gets a relation of the type: $\sigma_Y = \mu\sigma_{TFP}$, i.e.

$$\sigma_{GDP} = \mu\sigma_\pi h_S. \tag{50}$$

The calibration of section 3 gives $\mu = 1/(1 - \alpha\xi) = 1.9$, which is the value I use for short term volatility. As seen above, a baseline estimate for the firm-level volatility is $\sigma_\pi = 12\%$. Table 5 displays the results. The sales Herfindahl h_S is quite large: $h_S = 22\%$ average over all countries, and $h_S = 6.1\%$ for the U.S. By Eq. 48 this implies a GDP volatility $\sigma_{GDP} = 1.9 \times 12\% \times 6.1\% = 1.4\%$ for the U.S., and $\sigma_{GDP} = 1.9 \times 12\% \times 22\% = 5.0\%$ for a typical country. This is very much in the order of magnitude of GDP fluctuations. As always, further amplification mechanisms can increase the estimate. I conclude that idiosyncratic volatility seems quantitatively large enough to matter at the macroeconomic level.

6. Conclusion

This paper shows that the forces of randomness at the micro level create a inexorable amount of volatility at the macro level. Because of random growth at the micro level, the distribution of

[48]There may be problems with multinationals. For instance, the sales of G.M. are probably counted as the worldwide sales of G.M.

[49]If $Y_t = \Lambda_t K_t^{1-\alpha} L^\alpha$, $\Lambda_t \sim e^{\gamma t}$, and capital is accumulated, then in balanced growth path, $Y_t \sim K_t \sim \Lambda_t^{1/\alpha}$. In the long run, $\Delta \ln Y_t = \mu \Delta \ln \Lambda_t$, with $\mu = 1/\alpha$. This holds also with stochastic growth. If $\ln \Lambda_t$ is a Brownian motion with drift, $\lim var\left(\ln Y_t/Y_0\right)/t = \alpha^{-2} var\left(\ln \Lambda_t/\Lambda_0\right)/t$.

firm sizes is very fat tailed (Simon 1955, Gabaix 1999, Luttmer 2007). That fat-tailness makes the central limit theory break down, and idiosyncratic shocks to large firms (or, more generally, to large subunits in the economy), affect aggregate outcomes.

This paper illustrates this effect by taking the example of GDP fluctuation. This paper argues that idiosyncratic shocks to the top 100 firms explain a large fraction (one third) of aggregate volatility. While aggregate fluctuations such as changes to monetary, fiscal and exchange rate policy, and aggregate productivity shocks, are clearly important drivers of macroeconomic activity, they are not the only contributors to GDP fluctuations. Using theory, calibration and direct empirical evidence, this paper makes the case that idiosyncratic shocks are an important, and possibly the major, part of the origin of business-cycle fluctuations.

The importance of idiosyncratic shocks in aggregate volatility leads to a number of implications and directions for future research. First, and most evidently, to understand the origins of fluctuations better one should not focus exclusively on aggregate shocks, but concrete shocks to large players, such as Wal-Mart, Intel, and Nokia.

Second, shocks to large firms (such as a strike, a new innovation or a CEO change), initially independent of the rest of the economy, offer a rich source of shocks for VARs and impulse response studies – the real-side equivalent of the "Romer and Romer" shocks for monetary economics.

Third, this paper gives a new theoretical angle for the propagation of fluctuations. If Wal-Mart innovates, its competitors may suffer in the short term and thus race to catch up. This creates rich industry-level dynamics (that are already actively studied in the industrial organization literature) that should be useful for studying macroeconomic fluctuations, since they allow one to trace the dynamics of productivity shocks.

Fourth, this argument could explain the reason why people, in practice, do not know "the state of the economy". This is because "the state of the economy" depends on the behavior (productivity and investment behavior, among others) of many large and interdependent firms. Thus the integration is not easy, and no readily-accessible single number can summarize this state. This contrasts with aggregate measures, such as GDP, which are easily observable. Conversely, agents that focus on aggregate measures may make potentially problematic inferences (see Veldkamp and Wolfers (2007) for research along those lines). For example, the aggregate dividend-price ratio is often used as a key business cycle variable. However, changes in aggregate dividends may stem only from the policies of a small number of firms, as found by DeAngelo et al. (2004). This paper therefore could offer a new mechanism for the dynamics of "animal spirits".

Finally this mechanism might explain a large part of the volatility of many aggregate quantities other than output, for instance, inventories, inflation, short- or long-run movements in productivity, and the current account. Fluctuations of exports due to "granular" effects are explored in Canals *et al.* (2007) and di Giovanni and Levchenko (2009). The latter paper in particular finds that lowering trade barriers increases the granularity of the economy (as the most productive firms as selected), and imply an increase in the volatility of exports. Blank, Buch and Neugebauer (2009) construct a "banking granular residual" and find that negative shocks to large banks impact negatively small banks. Malevergne, Santa-Clara and Sornette (2008) find that the granular residual of stock returns (the return of large firm, minus a return of the average firm) is an important priced factor in the stock market, and explains the performance Fama-French factor models.

In sum, this paper suggests that the study of very large firms can offer a useful angle of attack on some open issues of macroeconomics.

Appendix A: Lévy's Theorem

The basic theorem can be found in most probability textbooks, e.g. Durrett (1996, p.153).

Theorem 5 *Suppose that $X_1, X_2, \ldots$ are i.i.d. with a distribution that satisfies:*

(i) $\lim_{x\to\infty} P(X_1 > x)/P(|X_1| > x) = \theta \in [0,1]$

(ii) $P(|X_1| > x) = x^{-\zeta} L(x)$

with $\zeta \in (0,2)$ and $L(x)$ slowly varying[50]*. Let $s_n = \sum_{i=1}^{n} X_i$, and*

$$a_n = \inf\{x : P(|X_1| > x) \leq 1/n\} \text{ and } b_n = nE\left[X_1 1_{|X_1|\leq a_n}\right]$$

As $n \to \infty$, $(s_n - b_n)/a_n \to^d Y$ where Y is a Lévy distribution with exponent ζ.

The most typical use of Lévy's theorem is the case of a distribution with zero mean and power-law distributed tails, $P(|X_1| > x) \sim (x/x_0)^{-\zeta}$. Then $a_n \sim x_0 n^{1/\zeta}$, $b_n = 0$, and $\left(x_0 N^{1/\zeta}\right)^{-1} \sum_{i=1}^{N} X_i \to^d Y$, where Y follows a Levy distribution. The sum $\sum_{i=1}^{N} X_i$ does not scale as $N^{1/2}$, as it does in the central limit theorem, but it scales as $N^{1/\zeta}$. This is because the size of the largest units X_i scales as $N^{1/\zeta}$.

A symmetrical Lévy distribution with exponent $\zeta \in (0,2]$ has the distribution $\lambda(x,\zeta) = \frac{1}{\pi}\int_0^\infty e^{-k^\zeta}\cos(kx)\,dk$ and the cumulative $\Lambda(x,\zeta) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty e^{-k^\zeta}\frac{\sin(kx)}{k}dk.$

For $\zeta = 2$, a Lévy distribution is a Gaussian. For $\zeta < 2$, the distribution has power law tail with exponent ζ.

Appendix B: Longer Derivations

6.1. Hulten's Theorem with and without Instantaneous Reallocation of Factors

For clarity, I re-derive and extend Hulten (1978)'s result, which says that a Hicks-neutral productivity shock $d\pi_i$ to firm i causes an increase in TFP equal to:

$$\text{TFP growth} = \sum_i \frac{\text{Sales of firm } i}{\text{GDP}} d\pi_i.$$

There are N firms. Firm i produces good i, and uses a quantity X_{ij} of intermediary inputs from firm j. It also uses L_i units of labor, K_i units of capital. It has productivity π_i. If production is: $Q_i = e^{\pi_i} F^i(X_{i1}, \ldots, X_{iN}, L_i, K_i)$. The representative agent consumes C_i of good

[50] *$L(x)$ is said to be slowly varying (e.g. Embrechts et al. 1997, p.564) if $\forall t > 0, \lim_{x\to\infty} L(tx)/L(x) = 1$. Prototypical examples are $L(x) = a$ and $L(x) = a \ln x$ for a non-zero constant a.*

i, and has a utility function is $U(C_1, ..., C_N)$. Production of firm i serves as consumption, and intermediary inputs, so: $Q_i = C_i + \sum_k X_{ki}$. The optimum in this economy reads:

$$\max_{C_i, X_{ik}, L_i, K_i} U(C_1, ..., C_N) \text{ subject to}$$
$$C_i + \sum_k X_{ki} = e^{\pi_i} F^i(X_{i1}, ..., X_{iN}, L_i, K_i); \sum_i L_i = L; \sum_i K_i = K$$

The Lagrangian is:

$$W = U(C_1, ..., C_N) + \sum_i p_i \left[e^{\pi_i} F^i(X_{i1}, ..., X_{iN}, L_i, K_i) - C_i - \sum_k X_{ki} \right]$$
$$+ w \left[L - \sum_i L_i \right] + r \left[K - \sum_i K_i \right].$$

Assume marginal cost pricing.[51] GDP is this economy is $Y = wL + rK = \sum_i p_i C_i$. The value added of firm i is $wL_i + rK_i$, and its sales are $p_i Q_i$.

If each firm i has a shock $d\pi_i$ to productivity, I differentiate the expression of W to find TFP growth:

$$\frac{dW}{W} = \frac{1}{W} \sum_i p_i \left[e^{\pi_i} G^i(X_{i1}, ..., X_{iN}, L_i, K_i) \, d\pi_i \right] = \sum_i \frac{\text{Sales of firm } i}{\text{GDP}} d\pi_i,$$

which is Eq. 47.

The above analysis shows that Hulten's theorem holds even if, after the shock, the capital, labor, and material inputs are not reallocated. This is a simple consequence of the envelope theorem. Hence Hulten's result also holds if there are frictions to adjust labor, capital, or intermediate inputs.

6.2. Proof of Proposition 1

Since $\sigma_{GDP} = \sigma h$, I examine h.

$$N^{1/2} h = \frac{\left(N^{-1} \sum_{i=1}^N S_i^2 \right)^{1/2}}{N^{-1} \sum_{i=1}^N S_i}$$

The law of large numbers ensures that $N^{-1} \sum_{i=1}^N S_i^2 \rightarrow^{a.s.} E\left[S^2\right]$, and $N^{-1} \sum_{i=1}^N S_i \rightarrow^{a.s.} E[S]$. This yields: $N^{1/2} h \rightarrow^{a.s.} E\left[S^2\right]^{1/2} / E[S]$.

[51] Basu and Fernald (2001) provide an analysis with imperfect competition.

Proof of Proposition 2

Since $\sigma_{GDP} = \sigma h$, I examine h.

$$h = \frac{\left(\sum_{i=1}^N S_i^2\right)^{1/2}}{\sum_{i=1}^N S_i} \tag{51}$$

I treat the cases where $\zeta > 1$ and $\zeta = 1$ separately. When $\zeta > 2$, the variance of firm sizes is finite, and we use Proposition 1.

When $1 < \zeta \leq 2$. By the law of large numbers,

$$N^{-1} \sum_{i=1}^N S_i \rightarrow^d E[S].$$

In addition, S_i^2 has power law exponent $\zeta/2 \leq 1$, as shown by:

$$P\left(S^2 > x\right) = P\left(S > x^{1/2}\right) = a\left(x^{1/2}\right)^{-\zeta} = ax^{-\zeta/2}.$$

So to handle the numerator of (51), I use Lévy's Theorem from Appendix A. This implies:

$$N^{-2/\zeta} \sum_{i=1}^N S_i^2 \rightarrow u,$$

where u is a Lévy distributed random variable with exponent $\zeta/2$. So

$$N^{1-1/\zeta} h = \frac{\left(N^{-2/\zeta} \sum_{i=1}^N S_i^2\right)^{1/2}}{N^{-1} \sum_{i=1}^N S_i} \rightarrow^d \frac{u^{1/2}}{E[S]}.$$

When $\zeta = 1$. Additional care is required, because $E[S] = \infty$. Lévy's Theorem 5 in Appendix A gives $b_n = n \ln n$, hence:

$$N^{-1} \left(\sum_{i=1}^N S_i - N \ln N\right) \rightarrow^d g,$$

where g is a Lévy with exponent 1. I conclude $\ln N \cdot h \rightarrow^d u^{1/2}/g$.

Proof of Proposition 3

As $\Delta S_i / S_i = S_i^{-\alpha} u_i$:

$$\frac{\Delta Y_{t+1}}{Y_t} = \frac{\sum_{i=1}^N \Delta S_{it}}{Y_t} = \frac{\sum_{i=1}^N S_i^{1-\alpha} u_{it}}{\sum_{i=1}^N S_i}. \tag{52}$$

When $\zeta > 1$, by the law of large numbers:

$$N^{-1}Y_t = N^{-1}\sum_{i=1}^{N} S_i \to \overline{S}.$$

To tackle the numerator, I observe that $S_i^{1-\alpha}$ has power law tails with exponent $\zeta' = \zeta/(1-\alpha)$. I consider two cases.

If $\zeta' < 2$, $x_i = S_i^{1-\alpha}u_i$, which has power law tails with exponent ζ', and by Lévy's theorem:

$$N^{-1/\zeta'}\Delta Y_t = N^{-1/\zeta'}\sum_{i=1}^{N} S_i^{1-\alpha}u_{it} \to^d g,$$

where g is a Lévy with exponent ζ'.

If $\zeta' \geq 2$, $S_i^{1-\alpha}u_i$ has finite variance, and $N^{-1/2}\Delta Y_t \to^d g$, where g is a Gaussian.

In both cases:

$$N^{-\max(1/2,1/\zeta')}\Delta Y_t \to^d g$$

for a distribution g. So, when $\zeta > 1$:

$$N^{1-\max(1/2,1/\zeta')}\frac{\Delta Y_{t+1}}{Y_t} \to^d \frac{g}{\overline{S}}g.$$

Hence the proposition holds, with

$$\begin{aligned}\alpha' &= 1 - \max\left(1/2, 1/\zeta'\right) = 1 + \min\left(-1/2, -1/\zeta'\right) \\ &= \min\left(1/2, 1 - 1/\zeta'\right) = \min\left(1/2, 1 - \frac{1-\alpha}{\zeta}\right).\end{aligned}$$

When $\zeta = 1$, a $\ln N$ correction appears, as in the proof of Proposition 2.

Proof of Proposition 4

Step 1. Frictionless equilibrium. I define:

$$H = \left(\sum_i Q_i^{1/\psi}\right)^{\psi} \tag{53}$$

The price of firm i is: $p_i = \frac{\partial H}{\partial Q_i}$, so: $S_i/H = p_i Q_i/H = Q_i \frac{\partial H}{\partial Q_i}/H$, and

$$\frac{S_i}{H} = \left(\frac{Q_i}{H}\right)^{1/\psi} \tag{54}$$

Because H is homogenous of degree 1, $H = \sum \frac{\partial H}{\partial Q_i} Q_i = \sum S_i$. H is the sum of sales in the economy.

Firm i solves: $\max_{K_i,L_i,X_i} p_i Q_i - X_i - wL_i$, which gives: $(K_i, L_i, X_i) = ((1-\alpha)\, b/r, \alpha b/w, 1-b)\, S_i \propto S_i$. I use $\propto$ to mean that the variables are proportional, up to a factor that does not depend on i. So, $S_i^{\psi} \propto Q_i \propto A_i S_i$ by (18), so $S_i \propto A_i^{1/(\psi-1)}$. Calling $B = \sum A_i^{1/(\psi-1)}$, and using the adding up constraint $\sum (K_i, L_i, X_i) = (K, L, X)$, we find the constant of proportionality: $(K_i, L_i, X_i) = (K, L, X)\, A_i^{1/(\psi-1)}/B$. Plugging this in (53), we get:

$$H = B^{\psi-1} \left(\frac{L^{\alpha} K^{1-\alpha}}{b} \right)^{b} \left(\frac{X}{1-b} \right)^{1-b}$$

Now, we solve for X, i.e. solve $\max_X H - X$. The solution is: $Y = H - X = B^{(\psi-1)/b} L^{\alpha} K^{1-\alpha}$, i.e. the announced relation.

Also, as $Y = H - \sum X_i$, and $X_i = (1-b)\, S_i$, $Y = H - \sum (1-b)\, S_i = bH$.

Step 2. Changes, assuming $\lambda = 1$. To keep the proof streamlined, I first consider the case $\lambda = 1$, i.e. the case no frictions in the adjustment of labor.

We now look at the changes. TFP growth comes from (21), and is also Hulten's formula. $Y = bH$ gives $\widehat{Y} = \widehat{H}$. As the total production is: $Y = \Lambda D L^{\alpha}$, for a constant D, the optimal labor supply L maximizes $\Lambda D L^{\alpha} - L^{1/\xi}$, so $L = \Lambda^{\frac{1}{1/\xi - \alpha}}$ times a constant, and $\widehat{L} = \frac{\xi}{1-\alpha\xi}\widehat{\Lambda}$. This implies that $\widehat{Y} = \widehat{\Lambda} + \alpha \widehat{L} = \left(1 + \frac{\alpha\xi}{1-\alpha\xi}\right)\widehat{\Lambda} = \frac{1}{1-\alpha\xi}\widehat{\Lambda}$, the announced relation. The wage is $w = \frac{1}{\xi} L^{1/\xi - 1}$, so: $\widehat{w} = \left(\frac{1}{\xi} - 1\right)\widehat{L}$, which gives the announced value. It is convenient that one can solve for changes in the macroeconomic variables without revisiting the firms' decision problems.

We now turn to the firm-level changes. Optimization of the demand for labor gives $wL_i = (1-b)\, \alpha S_i$, so $\widehat{L}_i = \widehat{S}_i - \widehat{w}$. We have, from (18), $\widehat{Q}_i = \widehat{A}_i + (1-b)\, \alpha \widehat{L}_i + b\widehat{S}_i$. Eq. (54) gives $\widehat{Q}_i = \psi \widehat{S}_i + (1-\psi)\, \widehat{H}$, and using $\widehat{Y} = \widehat{H}$,

$$\widehat{Q}_i = \psi \widehat{S}_i + (1-\psi)\, \widehat{Y} = \widehat{A}_i + (1-b)\, \alpha \left(\widehat{S}_i - (1-\xi)\, \widehat{Y} \right) + b\widehat{S}_i,$$

which gives the announced expressions for $\widehat{S}_i$ and $\widehat{Q}_i$.[52] $\widehat{L}_i$ comes from $\widehat{L}_i = \widehat{S}_i - \widehat{w}$. S_i was defined as $S_i = p_i Q_i$, which gives $\widehat{p}_i = \widehat{S}_i - \widehat{Q}_i$.

Step 3. With a general $\lambda \in [0,1]$.

After the changes $\widehat{A}_i$, only $L_{V,i}$ can adjust. First, we remark that the utility function is $L^{\xi} = \left(L_V^{\lambda} L_F^{1-\lambda} \right)^{1/\xi}$, so the elasticity associated with L_V is $\xi' = \xi/\lambda$, while the production share of L_V is $\alpha' = \alpha\lambda$. Hence, the expression (25) for GDP holds, replacing (α, ξ) by $(\alpha', \xi') = (\alpha\lambda, \xi/\lambda)$. As $\alpha'\xi' = \alpha\xi$, (25) is invariant in λ.

Furthermore, the planner solves: $\max_{L_V} \Lambda L_V^{\alpha\lambda} L_F^{\alpha} K^{1-\alpha} - \left(L_V^{\lambda} L_F^{1-\lambda} \right)^{1/\xi}$. There are now

[52] In (33), $b\alpha\lambda + 1 - b$ is the share of flexible factors, i.e. of the factors that adjust in the short run.

two wages, w_V and w_F, which are the marginal products of respectively L_V and L_F. One finds: $L_V \sim \Lambda^{1/(\lambda/\xi-\alpha\lambda)}$, i.e. $\lambda\widehat{L}_V = \xi\widehat{\Lambda}/(1-\alpha\xi)$. As total employment varies as: $\widehat{L} = \lambda\widehat{L}_V + (1-\lambda)\widehat{L}_F = \lambda\widehat{L}_V = \xi\widehat{\Lambda}/(1-\alpha\xi)$, we get (26). Likewise, $\widehat{w}_V = \widehat{Y} - \widehat{L}_V, \widehat{w}_F = \widehat{Y}$, and the weighted wage changes as: $\widehat{w} = \lambda\widehat{w}_V + (1-\lambda)\widehat{w}_F = \widehat{Y} - \xi\widehat{\Lambda}/(1-\alpha\xi) = (1-\xi)\widehat{\Lambda}$. Hence expressions (24)-(27) do not change with a general λ.

For the firm-level variables, one replaces (α, ξ) by $(\alpha', \xi') = (\alpha\lambda, \xi/\lambda)$, which delivers the expressions (28)-(30) and (33). The expression for employment becomes:

$$\widehat{L}_{V,i} = \beta\widehat{A}_i + (\xi - \beta b(1-\alpha\xi))\widehat{Y}$$

as so total employment in firm i varies as $\widehat{L}_i = \lambda\widehat{L}_{V,i}$, which gives expression (31). The expression for (32) follows immediately from $\widehat{S_i/L_i} = \widehat{S}_i - \widehat{L}_i$.

Lemma 1

The following Lemma is used in section 4.

Lemma 1 *Suppose a world where* $g_{it} = f_t + \gamma\widehat{A}_{it}$, $\widehat{\Lambda}_t = \sum_i \frac{S_i}{Y}\widehat{A}_{it}$, *and the productivity shocks* $\widehat{A}_{it}$ *are uncorrelated and have equal variance. Consider the class of residuals of the form* $\Gamma'_t = \sum_{i=1}^{K} w_i g_{it}$, *with weights* $(w_i)_{i=1...K}$ *satisfying* $\sum_{i=1}^{K} w_i = 0$, *so that* Γ'_t *is not affected by* f_t. *One seeks residuals* Γ'_t *that have the greatest correlation with* $\widehat{\Lambda}_t$, *i.e. which solve* $\max_{w_i} corr\left(\sum_{i=1}^{K} w_i g_{it}, \sum_{i=1}^{N}\frac{S_i}{Y}\widehat{A}_{it}\right)$ *subject to* $\sum w_i = 0$. *The highest correlation is achieved by any multiple of the granular residual* Γ_t *given in (38).*

Proof of Lemma 1. Given for any $k > 0$, $corr\left(\widehat{\Lambda}_t, k\Gamma'_t\right)$ is independent of k, one first looks at the residuals Γ'_t with a given variance, say 1. The problem is then: $\max_{w_i} cov\left(\widehat{\Lambda}_t, \Gamma'_t\right)$ s.t. $var\left(\Gamma'_t\right) = 1$ and $\sum_{i=1}^{K} w_i = 0$. Calling $\sigma^2 = var\widehat{A}_{it}$, one forms the Lagrangian

$$\begin{aligned}
\mathcal{L} &= cov\left(\widehat{\Lambda}_t, \Gamma'_t\right) - \lambda var\left(\Gamma'_t\right) - \mu\sum w_i \\
&= cov\left(\sum_i \frac{S_i}{Y}\widehat{A}_{it}, \sum_i w_i\gamma\widehat{A}_{it}\right) - \lambda var\left(\sum w_i\gamma\widehat{A}_{it}\right) - \mu\sum w_i \\
&= Y^{-1}\gamma\sigma^2\sum_i S_i w_i - \lambda\gamma^2\sigma^2\sum w_i^2 - \mu\sum w_i
\end{aligned}$$

Forming $0 = \partial\mathcal{L}/\partial w_i = Y^{-1}\gamma\sigma^2 S_i - 2\lambda\gamma^2\sigma^2 w_i - \mu$, so the optimal weights are of the form: $w_i = aS_i + b$, with a and b independent of i. Condition $\sum w_i = 0$ gives: $w_i = a\left(S_i - K^{-1}\sum_{j=1}^{K} S_j\right)$, and a residual $\Gamma'_t = a\sum S_i\left(g_{it} - \overline{g}_t\right)$, which proves the Proposition.

Appendix C: Data Appendix

Firm level data. The firm-level data comes from Compustat, North America, Industrial Annual. The frequency is annual, the years 1950 to 2008. I download the following variables: company name (CONAME), industry name (INAME), industry classification code (DNUM), DATA 3 – Investments – Total ($MM), DATA 6 – Assets – Total ($MM), DATA 12 – Sales (Net) ($MM), DATA 29 – Employees (M), DATA 78: Inventories - Finished goods ($MM).

I filter out oil and oil-related companies (DNUM=2911, 5172, 1311, 4922, 4923, 4924 and 1389), and energy companies (DNUM between 4900 and 4940), as fluctuations of their sales come mostly from worldwide commodity prices, rather than real productivity shocks, and financial firms (DNUM between 6,000 and 7,000), because their sales do not mesh well with the meaning used in the present paper.[53] To exclude foreign firms based in the US, I filter out companies whose name ends with -ADR, -ADS, -PRE FASB.

An important caveat is in order for U.S. firms. With Compustat, the sales of Ford, say, represent the worldwide sales of Ford, not directly the output produced by Ford in the U.S. There is no simple solution to this problem, especially if one wants a long time series. An important task of future research is to provide a version of Compustat that corrects for multinationals.

Macroeconomic data. GDP per capita comes from the Bureau of Economic Analysis. The Solow residual is the multifactor productivity of the private business sector from the Bureau of Labor Studies. The data for the Romer and Romer (2004) monetary policy shocks come from David Romer's web page. Their original series (RESID) is monthly, from 1966 to 1996. Here the yearly Romer-Romer shock is the sum of the 12 monthly shocks in that year. For the years not covered by Romer and Romer, the value of the shock is assigned to be 0, the mean of the original data. This assignment does not bias the regression coefficient under simple conditions, for instance if the data is i.i.d. It does lower the R^2 by the fraction of years in which the assignment is done, which is 0.4.

The data for the Hamilton (2003) oil shocks come from James Hamilton's web page. A quarterly Hamilton shock is the amount by which oil price exceeds the maximum price over the previous year, zero otherwise. This paper's yearly shock is the sum of the quarterly Hamilton shocks.

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[53] The results with financial firms are very similar, and are available upon request.

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	GDP growth$_t$	GDP growth$_t$	Solow Residual$_t$	Solow Residual$_t$
Granular Residual Γ_t	0.574	0.747	0.756	0.871
	(0.193)	(0.186)	(0.155)	(0.163)
Granular Residual Γ_{t-1}	0.675	0.843	0.410	0.506
	(0.187)	(0.183)	(0.147)	(0.156)
Granular Residual Γ_{t-2}		0.628		0.320
		(0.180)		(0.154)
R^2	0.26	0.41	0.37	0.42
Adjusted R^2	0.24	0.37	0.34	0.39
Observations	57	56	50	49

Table 1: Explanatory Power of the Granular Residual. For the year $t = 1952$ to 2008, per capita GDP growth and the Solow residual are regressed on the granular residual Γ_t of the top 100 non-oil industry firms (equation 42). The firms are the largest by sales of the previous year. Standard errors in parentheses.

	GDP growth$_t$	GDP growth$_t$	Solow Residual$_t$	Solow Residual$_t$
Granular Residual Γ_t^{ind}	1.003	1.279	0.931	1.074
	(0.228)	(0.215)	(0.185)	(0.199)
Granular Residual Γ_{t-1}^{ind}	0.804	1.183	0.407	0.499
	(0.196)	(0.203)	(0.159)	(0.187)
Granular Residual Γ_{t-2}^{ind}		0.801		0.313
		(0.180)		(0.166)
R^2	0.34	0.53	0.36	0.41
Adjusted R^2	0.32	0.50	0.33	0.37
Observations	57	56	50	49

Table 2: Explanatory Power of the Granular Residual, with industry de-meaning. For the year $t = 1952$ to 2008, per capita GDP growth and the Solow residual are regressed on the granular residual Γ_t^{ind} of the top 100 non-oil industry firms (equation 42), removing the industry mean within this top 100. The firms are the largest by sales of the previous year. Standard errors in parentheses.

	GDP growth$_t$	GDP growth$_t$	GDP growth$_t$
Oil$_t$	-5.1e-06		.000037
	(.00015)		(.00016)
Oil$_{t-1}$	-.000643		-.000616
	(.00028)		(.00028)
Monetary Shock$_t$		.003464	-.00268
		(.05784)	(.05602)
Monetary Shock$_{t-1}$		-.099848	-.077234
		(.04805)	(.04757)
R^2	0.18	0.06	0.22
Adjusted R^2	0.15	0.02	0.15
Observations	49	53	49

Table 3: Explanatory Power of oil and monetary shocks. The shocks are the yearly aggregations of the measures of Hamilton (2003) and Romer-Romer (2004). Standard errors in parentheses.

	With Simple Granular Residual		With Industry Granular Residual	
Granular Residual$_t$	0.565	0.778	0.902	1.188
	(0.224)	(0.218)	(0.261)	(0.258)
Granular Residual$_{t-1}$	0.677	0.936	0.699	1.118
	(0.214)	(0.212)	(0.218)	(0.234)
Granular Residual$_{t-2}$		0.654		0.728
		(0.206)		(0.203)
R^2	0.40	0.53	0.43	0.58
Adjusted R^2	0.32	0.45	0.35	0.50
Observations	49	48	49	48

Table 4: Explanatory Power of the Granular Residual, with controls for oil and monetary shocks. For the year $t = 1952$ to 2001, per capita GDP growth is regressed on the granular residual Γ_t of the top 100 non-oil industry firms (equation 42-43), and the contemporaneous and lagged values of the Romer-Romer and Hamilton shocks. The firms are the largest by sales of the previous year. Standard errors in parentheses.

		All Countries	USA
Sales Herfindahl	h_S	22.0	6.1
GDP volatility induced by idiosyncratic firm-level shocks	$\sigma_{GDP} = \mu\sigma_\pi h_S$	5.0	1.4

Table 5: Sales herfindahl h_S (eq. 49) in 2002, and induced GDP volatility. Units are in %. For the induced GDP volatility, I take $\sigma_{GDP} = \mu\sigma h_S$, with a firm-level volatility $\sigma = 12\%$ (eq. 50), and an amplification factor $\mu = 1.9$, as discussed in the text. Source: Acemoglu, Johnson and Mitton (2009) for the international data, and Compustat for the U.S. data.