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The graviton propagator in de Donder gauge on de Sitter background

S. P. Miao,[1,a)] N. C. Tsamis,[2,b)] and R. P. Woodard[3,c)]
[1]*Institute for Theoretical Physics and Spinoza Institute, Utrecht University, Leuvenlaan 4, Postbus 80.195, 3508 TD Utrecht, The Netherlands*
[2]*Department of Physics, Institute of Theoretical Physics and Computational Physics, University of Crete, GR-710 03 Heraklion, Greece*
[3]*Department of Physics, University of Florida, Gainesville, Florida 32611, USA*

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We construct the graviton propagator on de Sitter background in exact de Donder gauge. We prove that it must break de Sitter invariance, just like the propagator of the massless, minimally coupled scalar. Our explicit solutions for its two scalar structure functions preserve spatial homogeneity and isotropy so that the propagator can be used within the larger context of inflationary cosmology; however, it is simple to alter the residual symmetry. Because our gauge condition is de Sitter invariant (although no solution for the propagator can be) renormalization should be simpler using this propagator than one based on a noncovariant gauge. It remains to be seen how other computational steps compare. © *2011 American Institute of Physics*. [doi:10.1063/1.3664760]

I. INTRODUCTION

Mathematical physicists have long maintained that the graviton propagator is de Sitter invariant because explicit, de Sitter invariant solutions to the propagator equation arise from adding covariant gauge fixing terms to the action and analytically continuing from Euclidean de Sitter space.[1] It has recently been realized that this argument is wrong for three reasons:

- There is a topological obstacle to adding covariant gauge fixing terms on any manifold, and for any gauge theory, which possesses a linearization instability;[2]
- analytic continuation incorrectly subtracts off any power law infrared divergences;[3] and
- solutions exist to the propagator equation which do not correspond to propagators in the sense of being the expectation value of the time-ordered product of two fields in the presence of some state.[4] (It has been conjectured that this shows up in mathematical physics as a violation of reflection positivity[5]).

The first point occurs even for flat space electromagnetism on the manifold $T^3 \times R$: the invariant equations' linearization instability requires the total charge to vanish, whereas the Feynman gauge equations can be solved for any charge. The second point is familiar to everyone who has encountered the automatic subtraction of dimensional regularization. And a trivial example of the third point comes from multiplying the entirely real, retarded propagator by a factor of i.

These insights resolve a number of puzzles in the literature. For example, employing the Feynman gauge fixing term for scalar quantum electrodynamics on de Sitter[6] produces a one loop self-mass-squared which possesses on-shell singularities.[7] These singularities seem to be the quantum field theory representation of what one would expect classically from an $A_0 J^0$ interaction energy in view of the erroneous temporal growth of A_0 in Feynman gauge. The simplest noncovariant

a)Electronic mail: S.Miao@uu.nl.
b)Electronic mail: tsamis@physics.uoc.gr.
c)Electronic mail: woodard@phys.ufl.edu.

gauge[8] fails to show on-shell singularities.[7] Nor is there any problem using the de Sitter invariant, Lorentz gauge propagator.[9,10] The conclusion for de Sitter electromagnetism is, therefore, that one must avoid adding covariant gauge fixing terms, but no physical breaking of de Sitter invariance occurs.

The situation for gravitons is different owing to infrared divergences. It has long been noted that certain discrete choices of the two covariant gauge fixing parameters result in infrared divergences, if one insists on a de Sitter invariant solution.[11,12] These choices had been dismissed as unphysical, "singular gauges" which must simply be avoided.[13] However, we can now see that they are precisely the cases for which the order of the omnipresent infrared divergence in the formal, de Sitter invariant mode sum changes from power law to logarithmic.[3] The power law infrared divergences of other choices were automatically—but incorrectly—subtracted by analytic regularization techniques to produce solutions of the propagator equations that are not true propagators. The correct procedure in all cases is to allow free gravitons to resolve their infrared problem by breaking de Sitter invariance.

The purpose of this note is to construct the graviton propagator in an allowed covariant gauge, without employing analytic continuation techniques to subtract off infrared divergences. Our procedure is to express the propagator in terms of covariant projectors acting on scalar structure functions, without making any assumption about the eventual de Sitter invariance of the result. These structure functions obey completely de Sitter invariant equations, but they fail to possess de Sitter invariant solutions on account of infrared divergences. The procedure is so general that we implement it as well for a vector particle of general mass M_V and check that it agrees with the known de Sitter invariant solutions[3] for $M_V^2 > -2(D-1)H^2$ in the transverse sector and $M_V^2 > 0$ in the longitudinal sector. When de Sitter breaking must occur we have chosen to give explicit solutions which preserve the symmetries of homogeneity and isotropy that are relevant to cosmology. However, our equations for the structure functions are invariant, so one can easily derive solutions which respect any of the allowed subgroups.

Our notation is laid out in Sec. II. Section III presents a general treatment for minimally coupled scalars of any mass M_S. In Sec. IV, we solve for the propagator of a vector with general mass M_V, including longitudinal and transverse parts. Section V applies the same technique to solve for the graviton propagator in de Donder gauge. Our results are summarized and discussed in Sec. VI.

Because this work represents a long and mostly technical exercise we have thought it right to briefly discuss the physical motivation. The point is to facilitate the study of quantum effects from the epoch of primordial inflation for which the de Sitter geometry provides an excellent paradigm. Just how good can be quantified using the deceleration parameter $q(t) = -a\ddot{a}/\dot{a}^2$, which measures minus the fractional cosmic acceleration. Its value for de Sitter is $q = -1$, and the threshold between inflation and deceleration occurs at $q = 0$. If one assumes single scalar inflation, then the measured result for the scalar amplitude (Ref. 14), and the bound on the tensor-to-scalar ratio (Ref. 14), imply 95% confidence that $q(t) < -0.986$ when the largest observable perturbations experienced first horizon crossing (Ref. 15). Because this would have been near the end of inflation, when $q(t)$ was growing, most of the inflationary epoch was likely even closer to de Sitter.

The effects we seek to study arise from particle production. The small amount of particle production which has long been known to occur in an expanding universe[16] becomes explosive during inflation for any particle which is both massless and not conformally invariant on the classical level.[17] This includes massless, minimally coupled scalars, and gravitons.[18] Of course, this phenomenon is the origin of the tensor[19] and scalar[20] perturbations which are such an exciting tree order prediction of inflation. Our motivation is getting at the fascinating loop effects which should also be present.

There have been extensive studies of the quantum loop effects from inflation producing massless, minimally coupled scalars. Complete, dimensionally regulated and fully renormalized results have been derived at one and two loop orders for a real scalar with a quartic self-interaction,[21] for a massless fermion Yukawa-coupled to a real scalar,[22] and for scalar quantum electrodynamics.[7,10,23] These scalar effects are simpler than those from gravitons because there is no issue about general coordinate invariance. They are also generally stronger because they can avoid derivative interactions. However, scalar effects are less universal and less reliable because they depend upon the existence of light, minimally coupled scalars at inflationary scales. In four models with gravitons there are complete, dimensionally regulated, and fully renormalized results:

- For pure quantum gravity the graviton 1-point function has been computed at one loop order.[24] This result shows that the effect of inflationary gravitons at one loop order is a slight increase in the cosmological constant.
- For quantum gravity plus a massless fermion, the fermion self-energy has been computed at one loop order.[25] This result shows that spin-spin interactions with inflationary gravitons drive the fermion field strength up by an amount that increases without bound.[26]
- For quantum gravity plus a massless, minimally coupled scalar there are one loop computations of the scalar self-mass-squared[27] and the graviton self-energy.[28] The scalar effective field equations reveal that the scalar kinetic energy redshifts too rapidly for there to be a significant interaction with inflationary gravitons.[27] The effects of inflationary scalars on dynamical gravitons, and on the force of gravity, are still under study.[28]
- The nonlinear sigma model has been exploited to better understand the derivative interactions of quantum gravity,[29] and explicit two loop results have been obtained for the expectation value of the stress tensor.[30]

There are also a variety of other, sometimes less complete results, including:

- The graviton 1-point[31] and 2-point[32] functions in pure quantum gravity;
- the unregulated graviton 2-point function from scalars;[33]
- in scalar-driven inflation the one loop back-reaction,[34] loop corrections to the scalar power spectrum,[35] including corresponding work on how to correctly quantify effects,[36] a powerful theorem by Weinberg which limits the maximum secular growth,[37] and the problem of untangling infrared effects from ultraviolet divergences;[38] and
- in gravity plus generic matter much interest has been devoted to the recent proposal by Polyakov and co-worker[39] (following numerous antecedents[11,31,32,34,40]) that runaway particle production may destabilize de Sitter space.[41]

II. NOTATION

In Secs. III–V we shall study the de Sitter background propagators of three kinds of fields: minimally coupled scalars with arbitrary mass M_S, vectors with arbitrary mass M_V, and gravitons. The respective Lagrangians are

$$\mathcal{L}_S = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - \frac{1}{2}M_S^2\varphi^2\sqrt{-g} \; , \tag{1}$$

$$\mathcal{L}_V = -\frac{1}{2}\partial_\mu A_\rho\partial_\nu A_\sigma g^{\mu\nu}g^{\rho\sigma}\sqrt{-g} - \frac{1}{2}\Big[(D-1)H^2 + M_V^2\Big]A_\rho A_\sigma g^{\rho\sigma}\sqrt{-g} \; , \tag{2}$$

$$\mathcal{L}_G = \frac{1}{16\pi G}\Big[R - (D-2)(D-1)H^2\Big]\sqrt{-g} \; . \tag{3}$$

Here, D is the dimension of spacetime, H is the Hubble constant (which gives the cosmological constant $(D-1)H^2 = \Lambda$) and G is Newton's constant. We make no assumption that the vector is transverse, although the form of its mass term in (2) obviously derives from partially integrating and commuting covariant derivatives in the Maxwell Lagrangian, and then adding a spurious longitudinal kinetic term. The propagator of such a field appears in projection operators, even though the associated field cannot be dynamical.

We define the graviton field as the perturbation of the full metric $g_{\mu\nu}(x)$ about its background value $\overline{g}_{\mu\nu}$,

$$h_{\mu\nu}(x) \equiv g_{\mu\nu}(x) - \overline{g}_{\mu\nu}(x) \; . \tag{4}$$

Once this definition has been made, there is no more point to distinguishing the background from the full metric, so we drop the overbar and refer to the de Sitter background as simply $g_{\mu\nu}(x)$. Graviton indices are raised and lowered using this background field, for example, $h^{\mu}_{\nu} \equiv g^{\mu\rho} h_{\rho\nu}$. Covariant derivative operators D_μ and other geometrical quantities are similarly constructed with respect to the background. Of special importance is the Lichnerowicz operator which, when simplified using the de Sitter result for the curvature $R_{\rho\sigma\mu\nu} = H^2(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho})$, takes the form,

$$\mathbf{D}^{\mu\nu\rho\sigma} \equiv D^{(\rho} g^{\sigma)(\mu} D^{\nu)} - \frac{1}{2}\Big[g^{\mu\nu} D^\rho D^\sigma + g^{\rho\sigma} D^\mu D^\nu\Big] + \frac{1}{2}\Big[g^{\mu\nu} g^{\rho\sigma} - g^{\mu(\rho} g^{\sigma)\nu}\Big]\square + (D-1)\Big[\frac{1}{2} g^{\mu\nu} g^{\rho\sigma} - g^{\mu(\rho} g^{\sigma)\nu}\Big] H^2 \; . \tag{5}$$

Parenthesized indices are symmetrized, and $\square \equiv g^{\mu\nu} D_\mu D_\nu$ is the covariant d'Alembertian operator. With the help of (5) we can express the free part of the gravitational Lagrangian (3) in a convenient form,

$$\mathcal{L}_G = \frac{(D-1)H^2}{8\pi G}\sqrt{-g} + \Big(\text{Surface Term}\Big) - \frac{1}{2} h_{\mu\nu} \mathbf{D}^{\mu\nu\rho\sigma} h_{\rho\sigma}\sqrt{-g} + O(h^3) \; . \tag{6}$$

Much of our work will be valid in any coordinate realization of de Sitter space. However, when breaking de Sitter is necessary we shall always do so on the D-dimensional open conformal submanifold in which de Sitter can be imagined as a special case of the larger class of homogeneous, isotropic and spatially flat geometries relevant to cosmology. A spacetime point $x^\mu = (x^0, x^i)$ takes values in the ranges,

$$-\infty < x^0 < 0 \quad \text{and} \quad -\infty < x^i < +\infty \quad \text{for} \quad i = 1, \ldots, (D-1) \; . \tag{7}$$

In these coordinates the invariant element is

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = a_x^2\Big[-(dx^0)^2 + d\vec{x}\cdot d\vec{x}\Big] = a_x^2 \eta_{\mu\nu} dx^\mu dx^\nu \; , \tag{8}$$

where $\eta_{\mu\nu}$ is the Lorentz metric, and $a_x \equiv -1/Hx^0$ is the scale factor.

Although important de Sitter breaking occurs, it turns out that the vast majority of our propagator is de Sitter invariant. This suggests to express it in terms of the de Sitter invariant length function $y(x; z)$,

$$y(x;z) \equiv a_x a_z H^2\Bigg[\Big\|\vec{x}-\vec{z}\Big\|^2 - \Big(|x^0 - z^0| - i\epsilon\Big)^2\Bigg] \; . \tag{9}$$

Except for the factor of $i\epsilon$ (whose purpose is to enforce Feynman boundary conditions), the function $y(x; z)$ is closely related to the invariant length $\ell(x; z)$ from x^μ to z^μ,

$$y(x;z) = 4\sin^2\Big(\frac{1}{2} H \ell(x;z)\Big) \; . \tag{10}$$

With this de Sitter invariant quantity $y(x; z)$, we can form a convenient basis of de Sitter invariant bi-tensors. Note that because $y(x; z)$ is de Sitter invariant, so too are covariant derivatives of it. With the metrics $g_{\mu\nu}(x)$ and $g_{\mu\nu}(z)$, the first three derivatives of $y(x; z)$ furnish a convenient basis of de Sitter invariant bi-tensors,[7]

$$\frac{\partial y(x;z)}{\partial x^\mu} = H a_x \Big(y \delta^0_\mu + 2 a_z H \Delta x_\mu\Big) \; , \tag{11}$$

$$\frac{\partial y(x;z)}{\partial z^\nu} = H a_z \Big(y \delta^0_\nu - 2 a_x H \Delta x_\nu\Big) \; , \tag{12}$$

$$\frac{\partial^2 y(x;z)}{\partial x^\mu \partial z^\nu} = H^2 a_x a_z \Big(y \delta^0_\mu \delta^0_\nu + 2 a_z H \Delta x_\mu \delta^0_\nu - 2 a_x \delta^0_\mu H \Delta x_\nu - 2\eta_{\mu\nu}\Big) \; . \tag{13}$$

Here and subsequently we define $\Delta x_\mu \equiv \eta_{\mu\nu}(x - z)^\nu$. Acting covariant derivatives generates more basis tensors, for example,[7]

$$\frac{D^2 y(x;z)}{Dx^\mu Dx^\nu} = H^2(2-y)g_{\mu\nu}(x) \; , \tag{14}$$

$$\frac{D^2 y(x;z)}{Dz^\mu Dz^\nu} = H^2(2-y)g_{\mu\nu}(z) \; . \tag{15}$$

The contraction of any pair of the basis tensors also produces more basis tensors,[7]

$$g^{\mu\nu}(x)\frac{\partial y}{\partial x^\mu}\frac{\partial y}{\partial x^\nu} = H^2\Big(4y - y^2\Big) = g^{\mu\nu}(z)\frac{\partial y}{\partial z^\mu}\frac{\partial y}{\partial z^\nu} \; , \tag{16}$$

$$g^{\mu\nu}(x)\frac{\partial y}{\partial x^\nu}\frac{\partial^2 y}{\partial x^\mu \partial z^\sigma} = H^2(2-y)\frac{\partial y}{\partial z^\sigma} \; , \tag{17}$$

$$g^{\rho\sigma}(z)\frac{\partial y}{\partial z^\sigma}\frac{\partial^2 y}{\partial x^\mu \partial z^\rho} = H^2(2-y)\frac{\partial y}{\partial x^\mu} \; , \tag{18}$$

$$g^{\mu\nu}(x)\frac{\partial^2 y}{\partial x^\mu \partial z^\rho}\frac{\partial^2 y}{\partial x^\nu \partial z^\sigma} = 4H^4 g_{\rho\sigma}(z) - H^2\frac{\partial y}{\partial z^\rho}\frac{\partial y}{\partial z^\sigma} \; , \tag{19}$$

$$g^{\rho\sigma}(z)\frac{\partial^2 y}{\partial x^\mu \partial z^\rho}\frac{\partial^2 y}{\partial x^\nu \partial z^\sigma} = 4H^4 g_{\mu\nu}(x) - H^2\frac{\partial y}{\partial x^\mu}\frac{\partial y}{\partial x^\nu} \; . \tag{20}$$

III. SCALAR PROPAGATORS

Scalar propagator equations play an important role in our analysis because our strategy is to enforce the de Donder gauge condition, without making assumptions about de Sitter invariance, using covariant derivative projectors acting on scalar structure functions. The graviton propagator equation will then be used to infer invariant equations for these scalar structure functions. The point of this section is to review and systematize previous work[2,3] about how to solve such equations. We begin giving a general scalar propagator equation and explaining why infrared divergences for $M_S^2 \leq 0$ preclude a de Sitter invariant solution. We review the two fixes in the literature, and then give a simple approximate implementation for our favorite one. The section closes with some powerful results for integrating propagators.

One can see from (1) that the propagator of a minimally coupled scalar with mass M_S obeys the equation

$$\Big[\Box - M_S^2\Big] i\Delta(x;z) = \frac{i\delta^D(x-z)}{\sqrt{-g}} \; . \tag{21}$$

The plane wave mode function corresponding to Bunch-Davies vacuum is[42]

$$u_\nu(x^0,k) \equiv \sqrt{\frac{\pi}{4H}}\, a_x^{-\frac{D-1}{2}}\, H_\nu^{(1)}(-kx^0), \quad \text{where} \quad \nu = \sqrt{\Big(\frac{D-1}{2}\Big)^2 - \frac{M_S^2}{H^2}} \; . \tag{22}$$

The Fourier mode sum for the propagator on infinite space is

$$i\Delta_\nu^{\rm dS}(x;z) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}}\, e^{i\vec{k}\cdot(\vec{x}-\vec{z})}\Big\{\theta(x^0-z^0)u_\nu(x^0,k)u_\nu^*(z^0,k) + \theta(z^0-x^0)u_\nu(x^0,k)u_\nu(z^0,k)\Big\}. \tag{23}$$

The result is de Sitter invariant when the integral converges[43,44]

$$i\Delta_\nu^{\rm dS}(x;z)$$
$$= \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D-1}{2}+\nu)\Gamma(\frac{D-1}{2}-\nu)}{\Gamma(\frac{D}{2})} \, {}_2F_1\Big(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu; \frac{D}{2}; 1-\frac{y}{4}\Big) , \tag{24}$$

$$= \frac{H^{D-2}\Gamma(\frac{D}{2}-1)}{(4\pi)^{\frac{D}{2}}} \Big\{ \Big(\frac{4}{y}\Big)^{\frac{D}{2}-1} {}_2F_1\Big(\frac{1}{2}+\nu, \frac{1}{2}-\nu; 2-\frac{D}{2}; \frac{y}{4}\Big)$$
$$+ \frac{\Gamma(\frac{D-1}{2}+\nu)\Gamma(\frac{D-1}{2}-\nu)\Gamma(1-\frac{D}{2})}{\Gamma(\frac{1}{2}+\nu)\Gamma(\frac{1}{2}-\nu)\Gamma(\frac{D}{2}-1)} \, {}_2F_1\Big(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu; \frac{D}{2}; \frac{y}{4}\Big)\Big\}, \tag{25}$$

$$= \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Big\{ \Gamma\Big(\frac{D}{2}-1\Big)\Big(\frac{4}{y}\Big)^{\frac{D}{2}-1}$$
$$- \frac{\Gamma(\frac{D}{2})\Gamma(1-\frac{D}{2})}{\Gamma(\frac{1}{2}+\nu)\Gamma(\frac{1}{2}-\nu)} \sum_{n=0}^{\infty} \Big[\frac{\Gamma(\frac{3}{2}+\nu+n)\Gamma(\frac{3}{2}-\nu+n)}{\Gamma(3-\frac{D}{2}+n)\,(n+1)!} \Big(\frac{y}{4}\Big)^{n-\frac{D}{2}+2}$$
$$- \frac{\Gamma(\frac{D-1}{2}+\nu+n)\Gamma(\frac{D-1}{2}-\nu+n)}{\Gamma(\frac{D}{2}+n)\,n!} \Big(\frac{y}{4}\Big)^{n} \Big]\Big\}. \tag{26}$$

The gamma function $\Gamma(\frac{D-1}{2} - \nu + n)$ on the final line of (26) diverges for

$$\nu = \Big(\frac{D-1}{2}\Big) + N \qquad \Longleftrightarrow \qquad M_S^2 = -N(D-1+N)H^2 \, . \tag{27}$$

Its origin can be understood by performing the angular integration in the naive mode sum (23) and then changing to the dimensionless variable $\tau \equiv k/H\sqrt{a_x a_z}$,

$$i\Delta_\nu^{\rm dS}(x;z) = \frac{(a_x a_z)^{-(\frac{D-1}{2})}}{2^D \pi^{\frac{D-3}{2}} H} \int_0^\infty dk\, k^{D-2} \Big(\frac{1}{2}k\Delta x\Big)^{-(\frac{D-3}{2})} J_{\frac{D-3}{2}}(k\Delta x)$$
$$\times \Big\{ \theta(x^0-z^0) H_\nu^{(1)}(-kx^0) H_\nu^{(1)}(-kz^0)^* + \theta(z^0-x^0)\Big(\text{conjugate}\Big)\Big\}, \tag{28}$$

$$= \frac{H^{D-2}}{2^D \pi^{\frac{D-3}{2}}} \int_0^\infty d\tau\, \tau^{D-2} \Big(\frac{1}{2}\sqrt{a_x a_z}\, H\Delta x\tau\Big)^{-(\frac{D-3}{2})} J_{\frac{D-3}{2}}\Big(\sqrt{a_x a_z}\, H\Delta x \tau\Big)$$
$$\times \Big\{ \theta(x^0-z^0) H_\nu^{(1)}\Big(\sqrt{\frac{a_z}{a_x}}\,\tau\Big) H_\nu^{(1)}\Big(\sqrt{\frac{a_x}{a_z}}\,\tau\Big)^* + \theta(z^0-x^0)\Big(\text{conjugate}\Big)\Big\}. \tag{29}$$

In these and subsequent expressions we define $\Delta x \equiv \|\vec{x} - \vec{z}\|$. That the divergence at (27) is infrared can be seen from the small argument expansion of the Bessel function and from its relation to the Hankel function

$$J_\nu(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (\frac{1}{2}x)^{\nu+2n}}{n!\Gamma(\nu+n+1)} \, , \tag{30}$$

$$H_\nu^{(1)}(x) = \frac{i\Gamma(\nu)\Gamma(1-\nu)}{\pi} \Big\{ e^{-i\nu\pi} J_\nu(x) - J_{-\nu}(x)\Big\} \, . \tag{31}$$

The small τ behavior of the integrand (29) derives from three factors, the first being τ^{D-2}. The second factor takes the form

$$\Big(\frac{1}{2}\sqrt{a_x a_z}\, H\Delta x \tau\Big)^{-(\frac{D-3}{2})} J_{\frac{D-3}{2}}\Big(\sqrt{a_x a_z}\, H\Delta x \tau\Big) = \frac{1}{\Gamma(\frac{D-1}{2})} \sum_{n=0}^{\infty} C_1(n) \tau^{2n} \, . \tag{32}$$

And the final factor from the Hankel functions is

$$H_\nu^{(1)}\Big(\sqrt{\frac{a_z}{a_x}}\,\tau\Big)H_\nu^{(1)}\Big(\sqrt{\frac{a_x}{a_z}}\,\tau\Big)^* = \frac{2\Gamma(\nu)\Gamma(2\nu)}{\pi^{\frac{3}{2}}\Gamma(\nu+\frac{1}{2})\tau^{2\nu}}\sum_{n=0}^{\infty} C_2(n)\tau^{2n}\,. \tag{33}$$

Hence, the small τ expansion of the integrand has the form

$$\tau^{D-2}\times\frac{1}{\Gamma(\frac{D-1}{2})}\sum_{k=0}^{\infty}C_1(k)\tau^{2k}\times\frac{\Gamma^2(\nu)2^{2\nu}}{\pi^2\tau^{2\nu}}\sum_{\ell=0}^{\infty}C_2(\ell)\tau^{2\ell}$$
$$=\frac{2\Gamma(\nu)\Gamma(2\nu)}{\pi^{\frac{3}{2}}\Gamma(\frac{D-1}{2})\Gamma(\nu+\frac{1}{2})}\tau^{D-2-2\nu}\sum_{n=0}^{\infty}C_3(n)\tau^{2n}\,. \tag{34}$$

The naive mode sum (23) is infrared divergent for

$$D-2-2\nu\le -1 \qquad\Longleftrightarrow\qquad M_S^2\le 0\,. \tag{35}$$

However, there will only be a *logarithmic* infrared divergence, either from the leading term in (34) or from one of the series corrections at $n = N$, if one has

$$D-2-2\nu+2N=-1 \qquad\Longleftrightarrow\qquad M_S^2=-N(D-1+N)H^2\,. \tag{36}$$

This is precisely the condition (27) for the formal, de Sitter invariant mode sum (26) to diverge.

The infrared divergence we have just seen was first noted in 1977 for the special case of $M_S = 0$ by Ford and Parker.[45] The appearance of an infrared divergence signals that something is unphysical about the quantity being computed. The correct response to an infrared divergence is not to subtract it off, either explicitly or implicitly with the automatic subtraction of some analytic regularization technique. One must instead understand the physical problem which caused the divergence and then fix that problem. As we will see, the fix involves breaking de Sitter invariance, which was realized in 1982 for the special case of $M_S = 0$.[46] Allen and Folacci later gave a rigorous proof that de Sitter invariance must be broken.[47]

The divergence (35) occurs because of the way the Bunch-Davies mode functions (22) depend upon k for small k. The unphysical thing about having Bunch-Davies vacuum for arbitrarily small k is that no experimentalist can causally enforce it (or any other condition) for super-horizon modes. This has led to two fixes:

1. One can continue to work on the spatial manifold R^{D-1} but assume the initial state is released with its super-horizon modes in some less singular condition[48] or
2. One can work on the compact spatial manifold T^{D-1} with its coordinate radius chosen so the initial state has no super-horizon modes.[49]

We will adopt the latter fix. This makes the mode sum discrete, but the integral approximation should be excellent, and gives a simple expression for the propagator which differs from (23) only by an infrared cutoff at $k = H$.

From the preceding discussion, we see that the infrared corrected propagator $i\Delta(x; z)$ is just (29) with the lower limit cutoff at $\tau = 1/\sqrt{a_x a_z}$,

$$i\Delta(x;z)=\frac{H^{D-2}}{2^D\pi^{\frac{D-3}{2}}}\int_{\frac{1}{\sqrt{a_xa_z}}}^{\infty}d\tau\,\tau^{D-2}\frac{J_{\frac{D-3}{2}}(\sqrt{a_xa_z}\,H\Delta x\tau)}{(\frac{1}{2}\sqrt{a_xa_z}\,H\Delta x\tau)^{\frac{D-3}{2}}}$$
$$\times\Big\{\theta(x^0-z^0)H_\nu^{(1)}\Big(\sqrt{\frac{a_z}{a_x}}\,\tau\Big)H_\nu^{(1)}\Big(\sqrt{\frac{a_x}{a_z}}\,\tau\Big)^*+\theta(z^0-x^0)\Big(\text{conjugate}\Big)\Big\}. \tag{37}$$

Of course, we can express the truncated integral as the full one minus an integral over just the infrared

$$\int_{\frac{1}{\sqrt{a_xa_z}}}^{\infty}d\tau=\int_0^{\infty}d\tau-\int_0^{\frac{1}{\sqrt{a_xa_z}}}d\tau \qquad\Longleftrightarrow\qquad i\Delta(x;z)\equiv i\Delta_\nu^{\rm dS}(x;z)+\Delta_\nu^{\rm IR}(x;z)\,. \tag{38}$$

In this case, it does not matter if dimensional regularization is used to evaluate both $i\Delta^{\rm dS}_\nu(x;z)$ and $\Delta^{\rm IR}_\nu(x;z)$ because the errors we make at the lower limits will cancel.

A further simplification is that $\Delta^{\rm IR}_\nu(x;z)$ only needs to include the infrared singular terms which grow as $a_x a_z$ increases. These terms come entirely from the $J_{-\nu}$ parts of the Hankel function and they are entirely real

$$\Delta^{\rm IR}_\nu(x;z) = -\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{2\Gamma(\nu)\Gamma(2\nu)}{\Gamma(\nu+\frac12)} \int_0^{\frac{1}{\sqrt{a_x a_z}}} d\tau\, \tau^{D-2} \frac{J_{\frac{D-3}{2}}(\sqrt{a_x a_z}\, H\Delta x\tau)}{(\frac12\sqrt{a_x a_z}\, H\Delta x \tau)^{\frac{D-3}{2}}} \times \frac{\Gamma^2(1-\nu)}{2^{2\nu}}\, J_{-\nu}\Big(\sqrt{\frac{a_z}{a_x}}\,\tau\Big) J_{-\nu}\Big(\sqrt{\frac{a_x}{a_z}}\,\tau\Big)\,. \tag{39}$$

The final result is[3,50]

$$\Delta^{\rm IR}_\nu(x;z) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\nu)\Gamma(2\nu)}{\Gamma(\frac{D-1}{2})\Gamma(\nu+\frac12)} \times \sum_{N=0}^{\infty} \frac{(a_x a_z)^{\nu-(\frac{D-1}{2})-N}}{\nu-(\frac{D-1}{2})-N} \sum_{n=0}^{N} \Big(\frac{a_x}{a_z}+\frac{a_z}{a_x}\Big)^n \sum_{m=0}^{[\frac{N-n}{2}]} C_{Nnm} (y-2)^{N-n-2m}\,, \tag{40}$$

where the coefficients C_{Nnm} are

$$C_{Nnm} = \frac{(-\frac14)^N}{m!\,n!\,(N-n-2m)!} \times \frac{\Gamma(\frac{D-1}{2}+N+n-\nu)}{\Gamma(\frac{D-1}{2}+N-\nu)} \times \frac{\Gamma(\frac{D-1}{2})}{\Gamma(\frac{D-1}{2}+N-2m)} \times \frac{\Gamma(1-\nu)}{\Gamma(1-\nu+n+2m)} \times \frac{\Gamma(1-\nu)}{\Gamma(1-\nu+m)}\,. \tag{41}$$

Of course, there is no point in extending the sum over N to values $N > \nu - (\frac{D-1}{2})$ for which the exponent of $a_x a_z$ becomes negative. Those terms rapidly approach zero, and they can be dropped without affecting the propagator equation because they are separately annihilated by $\square - M_s^2$.

It might be worried that the approximations made in deriving the infrared correction do violence to delicate consistency relations in quantum field theory, but this is not the case. For the $M_S = 0$ scalar renormalization has been successfully implemented at one and two loop orders.[7,10,21–23] Because the physical graviton polarizations obey the same mode functions as massless, minimally coupled scalars,[18] one can also test the integral approximation with the graviton propagator. There is no disruption of powerful consistency checks such as the Ward identity at tree order[51] and one loop.[32] Nor is there any problem with the allowed one loop counterterms.[24,25,27,28]

It is worthwhile to summarize these results in the context of a consistent notation. Consider a general scalar whose mass obeys $M_s^2/H^2 = (\frac{D-1}{2})^2 - b^2$. Its propagator $i\Delta_b(x;z)$ obeys the equation

$$\Big[\square_x + b^2 H^2 - \Big(\frac{D-1}{2}\Big)^2 H^2\Big] i\Delta_b(x;z) = \frac{i\delta^D(x-z)}{\sqrt{-g}}\,. \tag{42}$$

We define the final result for $i\Delta_b(x;z)$ as the limit as ν approaches b of two functions which we wish to consider for general index ν. The first term in the sum is $i\Delta^{\rm dS}_\nu(x;z)$ as defined by expression (26). The second term is $\Delta^{\rm IR}_\nu(x;z)$, as defined by expression (40), except that the sum over N is cutoff at the largest non-negative integer for which $N \le b - (\frac{D-1}{2})$, with $\Delta^{\rm IR}_\nu(x;z)$ defined as zero for $b < (\frac{D-1}{2})$. Hence, our final result is

$$i\Delta_b(x;z) = \lim_{\nu\to b} \Big[i\Delta^{\rm dS}_\nu(x;z) + \Delta^{\rm IR}_\nu(x;z)\Big]\,. \tag{43}$$

We shall make significant use of four special cases for which a separate notation has been introduced

$$b_B = \left(\frac{D-3}{2}\right) \Longleftrightarrow i\Delta_B(x;z) = B(y) \; , \tag{44}$$

$$b_A = \left(\frac{D-1}{2}\right) \Longleftrightarrow i\Delta_A(x;z) = A(y) + \delta A(a_x, a_z, y) \; , \tag{45}$$

$$b_W = \left(\frac{D+1}{2}\right) \Longleftrightarrow i\Delta_W(x;z) = W(y) + \delta W(a_x, a_z, y) \; , \tag{46}$$

$$b_M = \frac{1}{2}\sqrt{(D-1)(D+7)} \Longleftrightarrow i\Delta_M(x;z) = M(y) + \delta M(a_x, a_z, y) \; . \tag{47}$$

Although the B-type propagator is de Sitter invariant, its A-type, W-type, and M-type cousins have de Sitter breaking parts

$$\delta A = k \ln(a_x a_z) \; , \tag{48}$$

$$\delta W = k\left\{(D-1)^2 a_x a_z - \left(\frac{D-1}{2}\right)\ln(a_x a_z)(y-2) - \left(\frac{a_x}{a_z} + \frac{a_z}{a_x}\right)\right\} , \tag{49}$$

$$\delta M = k_M\left\{\frac{(a_x a_z)^{b_M - b_A}}{b_M - b_A} - \frac{(a_x a_z)^{b_M - b_A - 1}}{b_M - b_A - 1} \times \frac{(y-2)}{4b_A} \right. \\ \left. - \frac{(a_x a_z)^{b_M - b_A - 1}}{4b_A(b_M - 1)} \times \left(\frac{a_x}{a_z} + \frac{a_z}{a_x}\right)\right\} . \tag{50}$$

The constants k and k_M are

$$k \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}, \qquad k_M \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(b_M)\Gamma(2b_M)}{\Gamma(b_A)\Gamma(b_M + \frac{1}{2})} \; . \tag{51}$$

The main, de Sitter invariant parts of each propagator consist of a few, potentially ultraviolet divergent terms (at $y = 0$), plus an infinite series. For the M-type propagator, there are no cancellations with the de Sitter breaking terms: just replace ν everywhere by b_M in expression (26) to find $M(y) = i\Delta^{\rm dS}_{b_M}(x;z)$. However, there are cancellations when this replacement is done for the A-type and W propagators

$$B(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\left\{\Gamma\left(\frac{D}{2} - 1\right)\left(\frac{4}{y}\right)^{\frac{D}{2}-1} \right. \\ \left. + \sum_{n=0}^{\infty}\left[\frac{\Gamma(n+\frac{D}{2})}{(n+1)!}\left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} - \frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})}\left(\frac{y}{4}\right)^{n}\right]\right\}, \tag{52}$$

$$A(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\left\{\Gamma\left(\frac{D}{2} - 1\right)\left(\frac{4}{y}\right)^{\frac{D}{2}-1} + \frac{\Gamma(\frac{D}{2}+1)}{\frac{D}{2}-2}\left(\frac{4}{y}\right)^{\frac{D}{2}-2} + A_1 \right. \\ \left. - \sum_{n=1}^{\infty}\left[\frac{\Gamma(n+\frac{D}{2}+1)}{(n-\frac{D}{2}+2)(n+1)!}\left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} - \frac{\Gamma(n+D-1)}{n\Gamma(n+\frac{D}{2})}\left(\frac{y}{4}\right)^{n}\right]\right\}, \tag{53}$$

$$W(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Biggl\{ \Gamma\Bigl(\frac{D}{2}-1\Bigr)\Bigl(\frac{4}{y}\Bigr)^{\frac{D}{2}-1} + \frac{\Gamma(\frac{D}{2}+2)}{(\frac{D}{2}-2)(\frac{D}{2}-1)}\Bigl(\frac{4}{y}\Bigr)^{\frac{D}{2}-2} + \frac{\Gamma(\frac{D}{2}+3)}{2(\frac{D}{2}-3)(\frac{D}{2}-2)}\Bigl(\frac{4}{y}\Bigr)^{\frac{D}{2}-3} + W_1 + W_2\Bigl(\frac{y-2}{4}\Bigr) + \sum_{n=2}^{\infty}\Biggl[\frac{\Gamma(n+\frac{D}{2}+2)(\frac{y}{4})^{n-\frac{D}{2}+2}}{(n-\frac{D}{2}+2)(n-\frac{D}{2}+1)(n+1)!} - \frac{\Gamma(n+D)(\frac{y}{4})^n}{n(n-1)\Gamma(n+\frac{D}{2})}\Biggr]\Biggr\}, \tag{54}$$

and the D-depdendent constants A_1, W_1, and W_2 are

$$A_1 = \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Biggl\{-\psi\Bigl(1-\frac{D}{2}\Bigr) + \psi\Bigl(\frac{D-1}{2}\Bigr) + \psi(D-1) + \psi(1)\Biggr\}, \tag{55}$$

$$W_1 = \frac{\Gamma(D+1)}{\Gamma(\frac{D}{2}+1)}\Biggl\{\frac{D+1}{2D}\Biggr\}, \tag{56}$$

$$W_2 = \frac{\Gamma(D+1)}{\Gamma(\frac{D}{2}+1)}\Biggl\{\psi\Bigl(-\frac{D}{2}\Bigr) - \psi\Bigl(\frac{D+1}{2}\Bigr) - \psi(D+1) - \psi(1)\Biggr\}. \tag{57}$$

A problem we shall often encounter consists of integrating one propagator against another. The result can be represented as the solution of a modified propagator equation

$$\Bigl[\square + b^2H^2 - \Bigl(\frac{D-1}{2}\Bigr)^2 H^2\Bigr] i\Delta_{bc}(x;z) = i\Delta_c(x;z)\ . \tag{58}$$

The solution is easily seen to be[2,3]

$$i\Delta_{bc}(x;z) = \frac{1}{(b^2-c^2)H^2}\Bigl[i\Delta_c(x;z) - i\Delta_b(x;z)\Bigr] = i\Delta_{cb}(x;z)\ . \tag{59}$$

For the special case that the indices b and c agree, one gets a derivative

$$i\Delta_{bb}(x;z) = -\frac{1}{2bH^2}\frac{\partial}{\partial b} i\Delta_b(x;z)\ . \tag{60}$$

We can obviously continue the process *ad infinitum*. For example, consider the case where the source is not a propagator, but rather a singly integrated propagator

$$\Bigl[\square + b^2H^2 - \Bigl(\frac{D-1}{2}\Bigr)^2 H^2\Bigr] i\Delta_{bcd}(x;z) = i\Delta_{cd}(x;z)\ . \tag{61}$$

The solution can be written in a form which is manifestly symmetric under any interchange of the three indices a, b, and c,

$$i\Delta_{bcd}(x;z) = \frac{i\Delta_{bd}(x;z) - i\Delta_{bc}(x;z)}{(c^2-d^2)H^2}\ , \tag{62}$$

$$= \frac{(d^2-c^2)i\Delta_b(x;z) + (b^2-d^2)i\Delta_c(x;z) + (c^2-b^2)i\Delta_d(x;z)}{(b^2-c^2)(c^2-d^2)(d^2-b^2)H^4}\ . \tag{63}$$

The case in which two of the indices are the same gives

$$i\Delta_{bcc}(x;z) = -\frac{1}{2cH^2}\frac{\partial}{\partial c} i\Delta_{bc}(x;z) = \frac{i\Delta_{cc}(x;z) - i\Delta_{bc}(x;z)}{(b^2-c^2)H^2}\ . \tag{64}$$

And equating all three indices produces

$$i\Delta_{bbb}(x;z) = -\frac{1}{2bH^2}\frac{\partial}{\partial b} i\Delta_{bc}(x;z)\Big|_{c=b} \; , \tag{65}$$

$$= -\frac{1}{8b^3H^4}\left[\frac{\partial}{\partial b} i\Delta_b(x;z) - b\Big(\frac{\partial}{\partial b}\Big)^2 i\Delta_b(x;z)\right] . \tag{66}$$

IV. VECTOR PROPAGATORS

One can see from (2) that the vector propagator obeys the equation

$$\Big[\square - (D-1)H^2 - M_V^2\Big] i\Big[{}_\mu\Delta_\rho\Big](x;z) = \frac{i g_{\mu\rho}\delta^D(x-z)}{\sqrt{-g}} \; . \tag{67}$$

Note that we do not assume transversality; indeed, the full vector propagator cannot be transverse because the right-hand side of Eq. (67) is not transverse. The first part of this section describes how to decompose the full propagator into its transverse and longitudinal parts, *without* making any assumptions about its eventual de Sitter invariance. Our technique is to express these parts using projectors formed from covariant derivative operators, acting on scalar structure functions. In the second part, we derive a scalar equation for the longitudinal structure function and solve it using the techniques of Sec. III. In the final part, we carry out the same analysis for the transverse structure function. The techniques employed here are a paradigm for the work of the subsequent section on the graviton propagator.

A. Transverse and logitudinal parts

The full vector propagator can be written as the sum of a transverse part and a longitudinal part

$$i\Big[{}_\mu\Delta_\rho\Big](x;z) = i\Big[{}_\mu\Delta^T_\rho\Big](x;z) + i\Big[{}_\mu\Delta^L_\rho\Big](x;z) \; . \tag{68}$$

In previous studies,[6,9] the vector propagator was expressed as a linear combination of de Sitter invariant basis tensors like those introduced at the end of Sec. II. Then the coefficient functions were chosen to enforce transversality (or longitudinality). This method is not open to us because we cannot assume de Sitter invariance for general mass M_V. What we require instead is a covariant decomposition which entails no assumption about de Sitter invariance.

The longitudinal part is easy

$$i\Big[{}_\mu\Delta^L_\rho\Big](x;z) \equiv \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial z^\rho}\Big[\mathcal{S}_L(x;z)\Big] \; . \tag{69}$$

This expression is longitudinal for any choice of the longitudinal structure function $\mathcal{F}_L(x;z)$. After much consideration, we decided to express the transverse part as

$$i\Big[{}_\mu\Delta^T_\rho\Big](x;z) = \mathcal{P}_\mu^{\alpha\beta}(x) \times \mathcal{P}_\rho^{\kappa\lambda}(z) \times \mathcal{Q}_{\alpha\kappa}(x;z) \times \Big[\mathcal{R}_{\beta\lambda}(x;z)\,\mathcal{S}_T(x;z)\Big] \; . \tag{70}$$

These symbols require explanation. The differential operator $\mathcal{P}_\mu^{\alpha\beta}$ is defined by writing the Maxwell field strength tensor as $F^{\alpha\beta} = \mathcal{P}_\mu^{\alpha\beta} A^\mu$,

$$\mathcal{P}_\mu^{\alpha\beta} \equiv \delta_\mu^\beta D^\alpha - \delta_\mu^\alpha D^\beta \; . \tag{71}$$

Note that acting $\mathcal{P}_\mu^{\alpha\beta}(x) \times \mathcal{P}_\rho^{\kappa\lambda}(z)$ on any 4-index, symmetric function of x and z produces something with the right properties to be a transverse propagator. Of course, some choices for the 4-index function give simpler final results than others! The best selection seems to be taking two of the four indices to be more covariant derivatives and the other two to belong to Sec. II basis tensor (13) which gives a Lorentz metric in the flat space limit. This corresponds to form (70) with the definitions

$$\mathcal{Q}_{\alpha\kappa}(x;z) \equiv -\frac{1}{2H^2}\frac{D}{Dx^\alpha}\frac{D}{Dz^\kappa} \; , \tag{72}$$

$$\mathcal{R}_{\beta\lambda}(x;z) \equiv -\frac{1}{2H^2}\frac{\partial^2 y(x;z)}{\partial x^\beta \partial z^\lambda} \; . \tag{73}$$

B. Solution for the longitudinal part

To derive an equation for the longitudinal structure function, we take the divergence of the full propagator equation (67), substitute relations (68)–(70), and then commute the derivative to the left

$$D_z^\rho\Big[\square_x - (D-1)H^2 - M_V^2\Big] i\Big[{}_\mu\Delta_\rho\Big](x;z)$$

$$= \Big[\square_x - (D-1)H^2 - M_V^2\Big]\frac{\partial}{\partial x^\mu}\square_z \mathcal{S}_L(x;z) \; , \tag{74}$$

$$= D_\mu^x\Big[\square_x - M_V^2\Big]\square_z \mathcal{S}_L(x;z) \; , \tag{75}$$

$$= -D_\mu^x\left(\frac{i\delta^D(x-z)}{\sqrt{-g}}\right) . \tag{76}$$

Hence, we conclude

$$\Big[\square_x - M_V^2\Big]\square_z \mathcal{S}_L(x;z) = -\frac{i\delta^D(x-z)}{\sqrt{-g}} \; . \tag{77}$$

From relation (42) of Sec. III this implies

$$\square_z \mathcal{S}_L(x;z) = -i\Delta_b(x;z) \qquad \text{for} \qquad b^2 = \left(\frac{D-1}{2}\right)^2 - \frac{M_V^2}{H^2} \; . \tag{78}$$

The final solution for $\mathcal{S}_L$ follows from relations (58) and (59):

$$\mathcal{S}_L(x;z) = \frac{1}{M_V^2}\Big[i\Delta_A(x;z) - i\Delta_b(x;z)\Big] = -i\Delta_{Ab}(x;z) \; . \tag{79}$$

We remind the reader of special case A with index $b_A = (\frac{D-1}{2})$ and the explicit expansion for $i\Delta_A(x;z)$ given by expressions (45), (48), and (53).

C. Solution for the transverse part

Substituting our explicit solution (79) for the longitudinal structure function into the full propagator equation (67) allows us to derive an equation for the transverse part that was previously guessed[9]

$$\Big[\square - (D-1)H^2 - M_V^2\Big] i\Big[{}_\mu\Delta_\rho^T\Big](x;z) = \frac{i g_{\mu\rho}\delta^D(x-z)}{\sqrt{-g}} + \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial z^\rho} i\Delta_A(x;z) \; . \tag{80}$$

The most effective technique for solving this equation is to reduce each side of the equation to the standard transverse form (70). We then read off a scalar equation for the transverse structure function $\mathcal{S}_T(x;z)$, which can be solved by the methods of Sec. III.

It is best to begin by establishing some important properties of the quadratic differential operator

$$\mathbf{P}_\mu^\beta \equiv \mathcal{P}_\mu^{\alpha\beta} \times D_\alpha = \delta_\mu^\beta \square - D^\beta D_\mu \; . \tag{81}$$

We shall always contract $\mathbf{P}_\mu^\beta$ into some vector T_β, so it is possible to commute the final covariant derivatives to reach the form

$$\mathbf{P}_\mu^\beta T_\beta = \left(\delta_\mu^\beta\Big[\Box - (D-1)H^2\Big] - D_\mu D^\beta\right)T_\beta \; . \tag{82}$$

It is tedious but straightforward to show that the covariant d'Alembertian commutes with $\mathbf{P}_\mu^\beta$,

$$\Box \mathbf{P}_\mu^\beta T_\beta = \mathbf{P}_\mu^\beta \Box T_\beta \; . \tag{83}$$

Note also that $\mathbf{P}_\mu^\beta$ is transverse on both left and right

$$D^\mu \mathbf{P}_\mu^\beta T_\beta = 0 = \mathbf{P}_\mu^\beta D_\beta T \; . \tag{84}$$

$\mathbf{P}_\mu^\beta$ must therefore be proportional to transverse projection operator. The proportionality factor can be found by squaring. Comparing relations (84) and (82) implies

$$\mathbf{P}_\mu^\alpha \times \mathbf{P}_\alpha^\beta T_\beta = \Big[\Box - (D-1)H^2\Big]\mathbf{P}_\mu^\beta T_\beta = \mathbf{P}_\mu^\beta\Big[\Box - (D-1)H^2\Big]T_\beta \; . \tag{85}$$

The relevance of $\mathbf{P}_\mu^\beta$ is that it gives the differential operators in front of the general transverse form (70),

$$\mathcal{P}_\mu^{\alpha\beta}(x) \times \mathcal{P}_\rho^{\kappa\lambda}(z) \times \mathcal{Q}_{\alpha\kappa}(x;z) = -\frac{1}{2H^2}\mathbf{P}_\mu^\beta(x) \times \mathbf{P}_\rho^\lambda(z) \; . \tag{86}$$

Substituting (70) into Eq. (80), and making use of relations (86) and (83), implies

$$\begin{aligned}&\Big[\Box - (D-1)H^2 - M_V^2\Big] i\Big[{}_\mu\Delta_\rho^T\Big](x;z) \\ &\quad = \mathcal{P}_\mu^{\alpha\beta}(x) \times \mathcal{P}_\rho^{\kappa\lambda}(z) \times \mathcal{Q}_{\alpha\kappa}(x;z)\Big[\Box - (D-1)H^2 - M_V^2\Big]\Big[\mathcal{R}_{\beta\lambda}\mathcal{S}_T\Big] \; .\end{aligned} \tag{87}$$

We need next to consider what the d'Alembertian gives when acting on the factors to the far right

$$\Box_x\Big[\mathcal{R}_{\beta\lambda}\mathcal{S}_T\Big] = \Big[\Box_x \mathcal{R}_{\beta\lambda}\Big]\mathcal{S}_T + 2g^{\alpha\gamma}\frac{D\mathcal{R}_{\beta\lambda}}{Dx^\alpha}\frac{\partial \mathcal{S}_T}{\partial x^\gamma} + \mathcal{R}_{\beta\lambda}\Box_x \mathcal{S}_T \; . \tag{88}$$

Recalling the definition (73) of $\mathcal{R}_{\beta\lambda}(x;z)$, and making use of relation (14) from Sec. II, gives two identities we shall use in this section and the next

$$\frac{D}{Dx^\alpha}\mathcal{R}_{\beta\lambda}(x;z) = \frac{1}{2}\, g_{\alpha\beta}(x)\frac{\partial y}{\partial z^\lambda} \; , \tag{89}$$

$$\Box \mathcal{R}_{\beta\lambda}(x;z) = -H^2 \mathcal{R}_{\beta\lambda}(x;z) \; . \tag{90}$$

Substitute these in (88) and pass the single derivative back outside to obtain

$$\Box_x\Big[\mathcal{R}_{\beta\lambda}\mathcal{S}_T\Big] = \frac{\partial y}{\partial z^\lambda}\frac{\partial \mathcal{S}_T}{\partial x^\beta} + \mathcal{R}_{\beta\lambda}\Big[\Box_x - H^2\Big]\mathcal{S}_T \; , \tag{91}$$

$$= \frac{\partial}{\partial x^\beta}\left[\frac{\partial y}{\partial z^\lambda}\mathcal{S}_T\right] - \frac{\partial^2 y}{\partial x^\beta \partial z^\lambda}\mathcal{S}_T + \mathcal{R}_{\beta\lambda}\Big[\Box_x - H^2\Big]\mathcal{S}_T \; , \tag{92}$$

$$= \frac{\partial}{\partial x^\beta}\left[\frac{\partial y}{\partial z^\lambda}\mathcal{S}_T\right] + \mathcal{R}_{\beta\lambda}\Big[\Box_x + H^2\Big]\mathcal{S}_T \; . \tag{93}$$

The first term on the right of Eq. (93) is longitudinal. In view of relation (84), we therefore conclude

$$\begin{aligned}&\Big[\Box - (D-1)H^2 - M_V^2\Big] i\Big[{}_\mu\Delta_\rho^T\Big](x;z) \\ &\quad = \mathcal{P}_\mu^{\alpha\beta}(x) \times \mathcal{P}_\rho^{\kappa\lambda}(z) \times \mathcal{Q}_{\alpha\kappa}(x;z)\Big[\mathcal{R}_{\beta\lambda}\Big[\Box - (D-2)H^2 - M_V^2\Big]\mathcal{S}_T\Big].\end{aligned} \tag{94}$$

It remains to reduce the right-hand side of (80) to the standard form (70), we have adopted for transverse bi-tensors,

$$i\Big[{}_\mu P_\rho\Big](x;z) \equiv \frac{i g_{\mu\rho}\delta^D(x-z)}{\sqrt{-g}} + \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial z^\rho} i\Delta_A(x;z) \; , \tag{95}$$

$$= \mathcal{P}^{\alpha\beta}_\mu(x) \times \mathcal{P}^{\kappa\lambda}_\rho(z) \times \mathcal{Q}_{\alpha\kappa}(x;z)\Big[\mathcal{R}_{\beta\lambda}(x;z)\mathcal{P}_1(x;z)\Big] \; . \tag{96}$$

This is easily accomplished by acting $\mathbf{P}^\mu_\nu(x) \times \mathbf{P}^\rho_\sigma(z)$ on both forms. Acting this operator on (95) and making use of relation (84) gives

$$\mathbf{P}^\mu_\nu(x) \times \mathbf{P}^\rho_\sigma(z)\, i\Big[{}_\mu P_\rho\Big](x;z)$$

$$= -2H^2\mathcal{P}^{\alpha\beta}_\nu(x) \times \mathcal{P}^{\kappa\lambda}_\sigma(z) \times \mathcal{Q}_{\alpha\kappa}(x;z)\Big[g_{\beta\lambda}\frac{i\delta^D(x-z)}{\sqrt{-g}}\Big] \; , \tag{97}$$

$$= -2H^2\mathcal{P}^{\alpha\beta}_\nu(x) \times \mathcal{P}^{\kappa\lambda}_\sigma(z) \times \mathcal{Q}_{\alpha\kappa}(x;z)\Big[\mathcal{R}_{\beta\lambda}(x;z)\frac{i\delta^D(x-z)}{\sqrt{-g}}\Big] \; . \tag{98}$$

Acting instead on (96) and making use of relations (85) and (93) gives

$$\mathbf{P}^\mu_\nu(x) \times \mathbf{P}^\rho_\sigma(z)\, i\Big[{}_\mu P_\rho\Big](x;z)$$

$$= \mathcal{P}^{\alpha\beta}_\nu(x) \times \mathcal{P}^{\kappa\lambda}_\sigma(z) \times \mathcal{Q}_{\alpha\kappa}(x;z)\Big[\square-(D-1)H^2\Big]^2\Big[\mathcal{R}_{\beta\lambda}\mathcal{P}_1\Big] \; , \tag{99}$$

$$= \mathcal{P}^{\alpha\beta}_\nu(x) \times \mathcal{P}^{\kappa\lambda}_\sigma(z) \times \mathcal{Q}_{\alpha\kappa}(x;z)\Big[\mathcal{R}_{\beta\lambda}\Big[\square-(D-2)H^2\Big]^2\mathcal{P}_1\Big] \; . \tag{100}$$

Comparing expressions (98) and (100) implies

$$\Big[\square-(D-2)H^2\Big]^2\mathcal{P}_1(x;z) = -2H^2\frac{i\delta^D(x-z)}{\sqrt{-g}} \; . \tag{101}$$

Relation (42) from Sec. III—with the special case of $b = (\frac{D-3}{2})$—to infer

$$\Big[\square-(D-2)H^2\Big]\mathcal{P}_1(x;z) = -2H^2 i\Delta_B(x;z) \; . \tag{102}$$

Now apply relations (58)–(60) to finally obtain the structure function for the transverse projection functional

$$\mathcal{P}_1(x;z) = -2H^2 i\Delta_{BB}(x;z) \; . \tag{103}$$

We have now reduced the transverse propagator equation to the form

$$\mathcal{P}^{\alpha\beta}_\mu(x) \times \mathcal{P}^{\kappa\lambda}_\rho(z) \times \mathcal{Q}_{\alpha\kappa}(x;z)\Big[\mathcal{R}_{\beta\lambda}\Big[\square-(D-2)H^2-M_V^2\Big]\mathcal{S}_T\Big]$$

$$= \mathcal{P}^{\alpha\beta}_\mu(x) \times \mathcal{P}^{\kappa\lambda}_\rho(z) \times \mathcal{Q}_{\alpha\kappa}(x;z)\Big[\mathcal{R}_{\beta\lambda}\Big[-2H^2 i\Delta_{BB}\Big]\Big] \; . \tag{104}$$

The transverse structure function therefore obeys

$$\Big[\square-(D-2)H^2-M_V^2\Big]\mathcal{S}_T = -2H^2 i\Delta_{BB}(x;z) \; . \tag{105}$$

Again making use of relations (58)–(60), our solution for it is

$$\mathcal{S}_T = +\frac{2H^2}{M_V^2} i\Delta_{BB} + \frac{2H^2}{M_V^4}\Big[i\Delta_B - i\Delta_c\Big], \quad \text{where} \quad c = \sqrt{\Big(\frac{D-3}{2}\Big)^2 - \frac{M_V^2}{H^2}} \; . \tag{106}$$

V. THE GRAVITON PROPAGATOR

Section IV provides a model for the analysis of this section, except that we immediately specialize to gravitons which obey de Donder gauge

$$D^{\mu} h_{\mu\nu} - \frac{1}{2} D_{\nu} h^{\mu}_{\mu} = 0 \; . \tag{107}$$

The first task is to express the propagator of such a graviton in terms of covariant projectors acting on scalar structure functions. With just a small extension of our previous results, we can then commute the differential operator to act directly on the structure functions. The final step is identifying the de Donder gauge projection functionals.

A. Enforcing de Donder gauge

In de Donder gauge (107), the propagator must obey the gauge condition on either coordinate and its associated index group

$$\Big[\delta^{\alpha}_{\mu} D^{\beta}_{x} - \frac{1}{2} D^{x}_{\mu} g^{\alpha\beta}(x)\Big] \times i\Big[{}_{\alpha\beta}\Delta_{\rho\sigma}\Big](x;z) = 0 \; , \tag{108}$$

$$\Big[\delta^{\alpha}_{\rho} D^{\beta}_{z} - \frac{1}{2} D^{z}_{\rho} g^{\alpha\beta}(z)\Big] \times i\Big[{}_{\mu\nu}\Delta_{\alpha\beta}\Big](x;z) = 0 \; . \tag{109}$$

Just as was the case for the vector propagator with the analogous conditions of transversality and longitudinality, we seek here to enforce (108) and (109) by acting covariant projectors on scalar structure functions. It turns out there are two ways to do it, corresponding to the spin zero and spin two parts of the graviton propagator

$$i\Big[{}_{\alpha\beta}\Delta_{\rho\sigma}\Big](x;z) = i\Big[{}_{\alpha\beta}\Delta^{0}_{\rho\sigma}\Big](x;z) + i\Big[{}_{\alpha\beta}\Delta^{2}_{\rho\sigma}\Big](x;z) \; . \tag{110}$$

The spin zero part of the graviton propagator is almost as simple as the longitudinal part of the vector propagator. It is a linear combination of longitudinal and trace terms from each index group

$$i\Big[{}_{\mu\nu}\Delta^{0}_{\rho\sigma}\Big](x;z) = \mathcal{P}_{\mu\nu}(x) \times \mathcal{P}_{\rho\sigma}(z)\Big[\mathcal{S}_0(x;z)\Big] \; . \tag{111}$$

The projector $\mathcal{P}_{\mu\nu}$ is

$$\mathcal{P}_{\mu\nu} \equiv D_{\mu} D_{\nu} + \frac{g_{\mu\nu}}{D-2}\Big[\square + 2(D-1)H^2\Big] \; . \tag{112}$$

Unlike the spin zero part of the graviton propagator, the spin two part is both transverse and also traceless within each index group. Recall that we obtained the key projector for the transverse part of the photon propagator by writing the Maxwell field strength tensor as $F^{\alpha\beta} = \mathcal{P}^{\alpha\beta}_{\mu} A^{\mu}$. We similarly define the projector $\mathcal{P}^{\alpha\beta\gamma\delta}_{\mu\nu}$ by expanding the Weyl tensor in powers of the graviton field $C^{\alpha\beta\gamma\delta} = \mathcal{P}^{\alpha\beta\gamma\delta}_{\mu\nu} h^{\mu\nu} + O(h^2)$. The final result takes the form[28]

$$\mathcal{P}^{\alpha\beta\gamma\delta}_{\mu\nu} \equiv \mathcal{D}^{\alpha\beta\gamma\delta}_{\mu\nu} + \frac{1}{D-2}\Big[g^{\alpha\delta}\mathcal{D}^{\beta\gamma}_{\mu\nu} - g^{\beta\delta}\mathcal{D}^{\alpha\gamma}_{\mu\nu} - g^{\alpha\gamma}\mathcal{D}^{\beta\delta}_{\mu\nu} + g^{\beta\gamma}\mathcal{D}^{\alpha\delta}_{\mu\nu}\Big]$$
$$+ \frac{1}{(D-1)(D-2)}\Big[g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}\Big]\mathcal{D}_{\mu\nu} \; . \tag{113}$$

The various pieces of this are

$$\mathcal{D}^{\alpha\beta\gamma\delta}_{\mu\nu} \equiv \frac{1}{2}\Big[\delta^{\alpha}_{(\mu}\delta^{\delta}_{\nu)} D^{\gamma} D^{\beta} - \delta^{\beta}_{(\mu}\delta^{\delta}_{\nu)} D^{\gamma} D^{\alpha} - \delta^{\alpha}_{(\mu}\delta^{\gamma}_{\nu)} D^{\delta} D^{\beta} + \delta^{\beta}_{(\mu}\delta^{\gamma}_{\nu)} D^{\delta} D^{\alpha}\Big] \; , \tag{114}$$

$$\mathcal{D}^{\beta\delta}_{\mu\nu} \equiv g_{\alpha\gamma}\mathcal{D}^{\alpha\beta\gamma\delta}_{\mu\nu} = \frac{1}{2}\Big[\delta^{\delta}_{(\mu} D_{\nu)} D^{\beta} - \delta^{\beta}_{(\mu}\delta^{\delta}_{\nu)}\square - g_{\mu\nu} D^{\delta} D^{\beta} + \delta^{\beta}_{(\mu} D^{\delta} D_{\nu)}\Big] \; , \tag{115}$$

$$\mathcal{D}_{\mu\nu} \equiv g_{\alpha\gamma} g_{\beta\delta}\mathcal{D}^{\alpha\beta\gamma\delta}_{\mu\nu} = D_{(\mu} D_{\nu)} - g_{\mu\nu}\square \; . \tag{116}$$

Acting $\mathcal{P}^{\alpha\beta\gamma\delta}_{\mu\nu}(x) \times \mathcal{P}^{\kappa\lambda\theta\phi}_{\rho\sigma}(z)$ on any eight index, symmetric function of x and z would produce a transverse and traceless tensor but, as with the vector, it pays to select a simple form. The best choice seems to be taking half the indices in the form of more covariant derivative operators, and the other half from two factors of the mixed second derivative (13) of the length function,

$$i\Big[{}_{\mu\nu}\Delta^2_{\rho\sigma}\Big](x;z) = \mathcal{P}^{\alpha\beta\gamma\delta}_{\mu\nu}(x) \times \mathcal{P}^{\kappa\lambda\theta\phi}_{\rho\sigma}(z) \times \mathcal{Q}_{\alpha\kappa} \times \mathcal{Q}_{\gamma\theta}\Big[\mathcal{R}_{\beta\lambda}\mathcal{R}_{\delta\phi}\,\mathcal{S}_2(x;z)\Big] . \tag{117}$$

We remind the reader of the definitions (72) and (73) of $\mathcal{Q}_{\alpha\kappa}(x;z)$ and $\mathcal{R}_{\beta\lambda}(x;z)$.

We close this subsection by giving the propagator equation. Acting the Lichnerowicz operator (5) on the graviton field and making use of the de Donder gauge condition (107) gives

$$-\mathbf{D}^{\mu\nu\rho\sigma}h_{\rho\sigma} = \frac{1}{2}\Big[\square - 2H^2\Big]h^{\mu\nu} - \frac{1}{4}g^{\mu\nu}\Big[\square + 2(D-3)H^2\Big]h^{\rho}_{\ \rho} . \tag{118}$$

This means the propagator obeys a relation of the form

$$\frac{1}{2}\Big[\square_x - 2H^2\Big]i\Big[{}_{\mu\nu}\Delta_{\rho\sigma}\Big](x;z) - \frac{1}{4}g_{\mu\nu}(x)\Big[\square_x + 2(D-3)H^2\Big]i\Big[{}^{\alpha}_{\alpha}\Delta_{\rho\sigma}\Big](x;z)$$
$$= g_{\mu(\rho}g_{\sigma)\nu} \times \frac{i\delta^D(x-z)}{\sqrt{-g}} + \Big(\text{Other Terms}\Big), \tag{119}$$

where the "Other Terms" make the right-hand side consistent with the gauge condition. However, the right-hand side of (119) cannot be symmetric under the interchange of x^μ and z^μ (and their associated indices) because the left-hand side of the equation obeys de Donder gauge on z^μ but not on x^μ. It is better to subtract off a term proportional to the trace

$$\frac{1}{2}\Big[\square_x - 2H^2\Big]i\Big[{}_{\mu\nu}\Delta_{\rho\sigma}\Big](x;z) - \frac{1}{4}g_{\mu\nu}(x)\Big[\square_x + 2(D-3)H^2\Big]i\Big[{}^{\alpha}_{\alpha}\Delta_{\rho\sigma}\Big](x;z)$$
$$- \frac{g_{\mu\nu}}{D-2} \times -\Big(\frac{D-2}{4}\Big)\Big[\square + 2(D-1)H^2\Big]\,i\Big[{}^{\alpha}_{\alpha}\Delta_{\rho\sigma}\Big](x;z) , \tag{120}$$

$$= \frac{1}{2}\Big[\square_x - 2H^2\Big]i\Big[{}_{\mu\nu}\Delta_{\rho\sigma}\Big](x;z) + H^2 g_{\mu\nu}(x) i\Big[{}^{\alpha}_{\alpha}\Delta_{\rho\sigma}\Big](x;z) . \tag{121}$$

It is easily checked that (121) obeys the de Donder gauge condition on both x^μ and z^μ. Hence, the right-hand side of the equation is symmetric under interchange of x^μ and z^μ and can in fact be guessed[3]

$$\frac{1}{2}\Big[\square_x - 2H^2\Big]i\Big[{}_{\mu\nu}\Delta_{\rho\sigma}\Big](x;z) + H^2 g_{\mu\nu}(x) i\Big[{}^{\alpha}_{\alpha}\Delta_{\rho\sigma}\Big](x;z)$$
$$= \Big[g_{\mu(\rho}g_{\sigma)\nu} - \frac{g_{\mu\nu}g_{\rho\sigma}}{D-2}\Big]\frac{i\delta^D(x-z)}{\sqrt{-g}} + \frac{1}{2}\left\{\begin{matrix} D^x_\mu D^z_\rho i[{}_\nu\Delta^W_\sigma] + D^x_\mu D^z_\sigma i[{}_\nu\Delta^W_\rho] \\ + D^x_\nu D^z_\rho i[{}_\mu\Delta^W_\sigma] + D^x_\nu D^z_\sigma i[{}_\mu\Delta^W_\rho] \end{matrix}\right\} . \tag{122}$$

Here, $i[{}_\mu\Delta^W_\rho]$ is the full vector propagator for the tachyonic mass $M^2_V = -2(D-1)H^2$, which obeys the equation

$$\Big[\square + (D-1)H^2\Big]i\Big[{}_\mu\Delta^W_\rho\Big](x;z) = \frac{i g_{\mu\rho}\delta^D(x-z)}{\sqrt{-g}} . \tag{123}$$

Recall from Sec. IV that it has the form given by Eqs. (68)–(70). From Eqs. (79) and (106), we see that the longitudinal and transverse structure functions are

$$\mathcal{S}_L(x;z) = -i\Delta_{AM}(x;z) , \tag{124}$$

$$\mathcal{S}_T(x;z) = \frac{1}{D-1}\Big[-i\Delta_{BB}(x;z) + i\Delta_{BW}(x;z)\Big] . \tag{125}$$

B. The spin zero part

To derive an equation for the spin zero structure function, we simply take the trace of the propagator equation (122). Tracing on the left-hand side and making use of relations (110)–(112) gives

$$\frac{1}{2}\Big[\square_x+2(D-1)H^2\Big]\, i\Big[{}^{\alpha}_{\alpha}\Delta_{\rho\sigma}\Big](x;z)$$
$$=\Big(\frac{D-1}{D-2}\Big)\Big[\square_x+2(D-1)H^2\Big]\Big[\square_x+DH^2\Big]\mathcal{P}_{\rho\sigma}(z)\Big[\mathcal{S}_0(x;z)\Big]\,. \tag{126}$$

Tracing the right-hand side of (122) and making use of relations (124) and (58)–(60) implies

$$\frac{1}{2}\Big[\square_x+2(D-1)H^2\Big] i\Big[{}^{\alpha}_{\alpha}\Delta_{\rho\sigma}\Big](x;z)$$
$$=-\frac{2}{D-2}\frac{g_{\rho\sigma}\, i\delta^D(x-z)}{\sqrt{-g}}-2D^z_\rho D^z_\sigma i\Delta_M(x;z)\,, \tag{127}$$

$$=-2\mathcal{P}_{\rho\sigma}(z) i\Delta_M(x;z)\,. \tag{128}$$

The equation for $\mathcal{S}_0(x;z)$ derives from comparing expressions (126) and (128),

$$\Big[\square+2(D-1)H^2\Big]\Big[\square+DH^2\Big]\mathcal{S}_0(x;z)=-2\Big(\frac{D-2}{D-1}\Big)i\Delta_M(x;z)\,. \tag{129}$$

Its solution follows easily from relations (58)–(66),

$$\mathcal{S}_0(x;z)=\frac{2i\Delta_{MM}(x;z)-2i\Delta_{MW}(x;z)}{(D-1)H^2}=-2\Big(\frac{D-2}{D-1}\Big)i\Delta_{WMM}(x;z)\,. \tag{130}$$

C. The spin two part

This is the most complicated analysis we shall have to make and it is greatly facilitated by the analogy with what was done for the transverse part of the vector propagator in Sec. IV C. Here, as for that case, the first step is to derive an equation for the remaining (spin two) part of the propagator by subtracting off the part we already have. We then establish some identities for a differential projector which comprises the exterior operators of the spin two part (117) of the propagator. These properties allow us to pass the d'Alembertian in the propagator equation through to act on the spin two structure function $\mathcal{S}_2(x;z)$. Squaring this operator also allows us to express the right-hand side of the propagator equation in the same form (117) with a known structure function. Comparing the two sides of the equation leads to a scalar differential equation which can be solved by the techniques of Sec. III.

We derive an equation for the pure spin two part of the propagator from the full Eq. (122) by substituting the spin zero structure function (130), with definitions (111) and (112). Now move everything known to right-hand side to reach the form

$$\frac{1}{2}\Big[\square_x-2H^2\Big]\, i\Big[{}_{\mu\nu}\Delta^2_{\rho\sigma}\Big](x;z)\equiv i\Big[{}_{\mu\nu}P^2_{\rho\sigma}\Big](x;z)\,, \tag{131}$$

$$=\Big[g_{\mu(\rho}g_{\sigma)\nu}-\frac{g_{\mu\nu}g_{\rho\sigma}}{D-2}\Big]\frac{i\delta^D(x-z)}{\sqrt{-g}}+\Big(\frac{D-2}{D-1}\Big)\mathcal{P}_{\mu\nu}(x)\times\mathcal{P}_{\rho\sigma}(z)i\Delta_{WM}(x;z)$$
$$+\frac{1}{2}\left(\left(\begin{matrix} D^x_\mu D^z_\rho i[{}_\nu\Delta^W_\sigma](x;z)+D^x_\mu D^z_\sigma i[{}_\nu\Delta^W_\rho](x;z)\\ +D^x_\nu D^z_\rho i[{}_\mu\Delta^W_\sigma](x;z)+D^x_\nu D^z_\sigma i[{}_\mu\Delta^W_\rho](x;z)\end{matrix}\right)\right)\,. \tag{132}$$

It can easily be checked that the right-hand side of (132) is transverse and traceless on each index group. We will eventually reduce this transverse-traceless projector to standard form

$$i\Big[{}_{\mu\nu}P^2_{\rho\sigma}\Big](x;z) = \mathcal{P}^{\alpha\beta\gamma\delta}_{\mu\nu}(x) \times \mathcal{P}^{\kappa\lambda\theta\phi}_{\rho\sigma}(z) \times \mathcal{Q}_{\alpha\kappa} \times \mathcal{Q}_{\gamma\theta}\Big[\mathcal{R}_{\beta\lambda}\mathcal{R}_{\delta\phi}\mathcal{P}_2\Big] . \tag{133}$$

However, it is best to first concentrate on the left-hand side of the propagator equation (132).

In analogy with the transverse projector $\mathbf{P}^{\beta}_{\mu}$ defined in Eq. (81), we define the transverse-traceless projector

$$\mathbf{P}^{\beta\delta}_{\mu\nu} \equiv \mathcal{P}^{\alpha\beta\gamma\delta}_{\mu\nu} \times D_\alpha D_\gamma . \tag{134}$$

We shall always consider this acted on a second rank tensor $F_{\beta\delta}$. From the expressions (113)–(116) which define $\mathcal{P}^{\alpha\beta\gamma\delta}_{\mu\nu}$ it is straightforward but tedious to reach the form

$$\begin{aligned}\mathbf{P}^{\beta\delta}_{\mu\nu} \times F_{\beta\delta} = \frac{1}{2}\Big(\frac{D-3}{D-2}\Big)\Big\{ & D_\mu \square D^\alpha F_{\alpha\nu} + D_\mu \square D^\beta F_{\nu\beta} - \square^2 F_{\mu\nu} \\ & - D_\mu D_\nu D^\alpha D^\beta F_{\alpha\beta} + \frac{1}{D-1}\Big[D_\mu D_\nu - g_{\mu\nu}\square\Big]\Big[D^\alpha D^\beta F_{\alpha\beta} - \square F^\alpha_\alpha\Big] \\ & + H^2\Big[-2g_{\mu\nu} D^\alpha D^\beta F_{\alpha\beta} - g_{\mu\nu}\square F^\alpha_\alpha - 2D_\mu D_\nu F^\alpha_\alpha + D_\mu D^\alpha F_{\alpha\nu} \\ & + D_\mu D^\beta F_{\nu\beta} + (D+2)\square F_{\mu\nu}\Big] + H^4\Big[2g_{\mu\nu}F^\alpha_\alpha - 2DF_{\mu\nu}\Big]\Big\}. \end{aligned} \tag{135}$$

(Note the multiplicative factor of $D - 3$ which derives from the fact that the Weyl tensor vanishes for $D = 3$.) It is easy to see from (135) that $\mathbf{P}^{\beta\delta}_{\mu\nu}$ is traceless on both the left and the right

$$g^{\mu\nu}\mathbf{P}^{\beta\delta}_{\mu\nu} F_{\beta\delta} = 0 = \mathbf{P}^{\beta\delta}_{\mu\nu}(g_{\beta\delta}F) . \tag{136}$$

It is also transverse on any index, both on the left and the right

$$D^\mu\Big(\mathbf{P}^{\beta\delta}_{\mu\nu}F_{\beta\delta}\Big) = 0 = D^\nu\Big(\mathbf{P}^{\beta\delta}_{\mu\nu}F_{\beta\delta}\Big) , \tag{137}$$

$$\mathbf{P}^{\beta\delta}_{\mu\nu}(D_\beta F_\delta) = 0 = \mathbf{P}^{\beta\delta}_{\mu\nu}(D_\delta F_\beta) . \tag{138}$$

These two properties are very important because the only terms in expression (135) which do not involve either divergences or traces are

$$\begin{aligned}\frac{1}{2}\Big(\frac{D-3}{D-2}\Big)\Big\{-\square^2 F_{\mu\nu} + (D+2)H^2\square F_{\mu\nu} - 2DH^4 F_{\mu\nu}\Big\} \\ = -\frac{1}{2}\Big(\frac{D-3}{D-2}\Big)\Big[\square - 2H^2\Big]\Big[\square - DH^2\Big]F_{\mu\nu} .\end{aligned} \tag{139}$$

Hence, squaring $\mathbf{P}^{\beta\delta}_{\mu\nu}$ gives

$$\mathbf{P}^{\alpha\gamma}_{\mu\nu} \times \mathbf{P}^{\beta\delta}_{\alpha\gamma} F_{\beta\delta} = -\frac{1}{2}\Big(\frac{D-3}{D-2}\Big)\Big[\square - 2H^2\Big]\Big[\square - DH^2\Big]\mathbf{P}^{\beta\delta}_{\mu\nu} F_{\beta\delta} , \tag{140}$$

$$= -\frac{1}{2}\Big(\frac{D-3}{D-2}\Big)\mathbf{P}^{\beta\delta}_{\mu\nu}\Big[\square - 2H^2\Big]\Big[\square - DH^2\Big]F_{\beta\delta} . \tag{141}$$

We note in passing that the covariant d'Alembertian commutes with $\mathbf{P}^{\beta\delta}_{\mu\nu}$, just as it did for the transverse projector $\mathbf{P}^{\beta}_{\mu}$.

Of course, the relevance of the transverse-traceless projector $\mathbf{P}^{\beta\lambda}_{\mu\nu}$ is that two factors of it give the exterior operators of the spin two part of the propagator

$$i\Big[{}_{\mu\nu}\Delta^2_{\rho\sigma}\Big](x;z) = \frac{1}{4H^4}\mathbf{P}^{\beta\delta}_{\mu\nu}(x) \times \mathbf{P}^{\lambda\phi}_{\rho\sigma}(z)\Big[\mathcal{R}_{\beta\lambda}(x;z)\mathcal{R}_{\delta\phi}(x;z)\mathcal{S}_2(x;z)\Big] . \tag{142}$$

From the fact that the d'Alembertian commutes with $\mathbf{P}^{\beta\delta}_{\mu\nu}$ we see

$$\frac{1}{2}\Big[\square - 2H^2\Big] i\Big[{}_{\mu\nu}\Delta^2_{\rho\sigma}\Big](x;z) = \frac{1}{4H^4}\mathbf{P}^{\beta\delta}_{\mu\nu}(x) \times \mathbf{P}^{\lambda\phi}_{\rho\sigma}(z) \times \frac{1}{2}\Big[\square - 2H^2\Big]\Big[\mathcal{R}_{\beta\lambda}\mathcal{R}_{\delta\phi}\mathcal{S}_2\Big] . \tag{143}$$

The next step is to pass the differential operator through to the structure function, making use of identities (89) and (90) from Sec. IV,

$$\square\Big[\mathcal{R}_{\beta\lambda}\mathcal{R}_{\delta\phi}\mathcal{S}_2\Big] = \mathcal{R}_{\beta\lambda}\mathcal{R}_{\delta\phi}\square\mathcal{S}_2 + 2g^{\alpha\gamma}(x)\Big[\frac{D\mathcal{R}_{\beta\lambda}}{Dx^\alpha}\mathcal{R}_{\delta\phi} + \mathcal{R}_{\beta\lambda}\frac{D\mathcal{R}_{\delta\phi}}{Dx^\alpha}\Big]\frac{\partial\mathcal{S}_2}{\partial x^\gamma} + 2g^{\alpha\gamma}(x)\frac{D\mathcal{R}_{\beta\lambda}}{Dx^\alpha}\frac{D\mathcal{R}_{\delta\phi}}{Dx^\gamma}\mathcal{S}_2 + \Big[(\square\mathcal{R}_{\beta\lambda})\mathcal{R}_{\delta\phi} + \mathcal{R}_{\beta\lambda}(\square\mathcal{R}_{\delta\phi})\Big]\mathcal{S}_2 , \tag{144}$$

$$= \mathcal{R}_{\beta\lambda}\mathcal{R}_{\delta\phi}\Big[\square + 2H^2\Big]\mathcal{S}_2 + \frac{D}{Dx^\beta}\Big[\frac{\partial y}{\partial z^\lambda}\mathcal{R}_{\delta\phi}\mathcal{S}_2\Big] + \frac{D}{Dx^\delta}\Big[\mathcal{R}_{\beta\lambda}\frac{\partial y}{\partial z^\phi}\mathcal{S}_2\Big] - \frac{1}{2}g_{\beta\delta}(x)\frac{\partial y}{\partial z^\lambda}\frac{\partial y}{\partial z^\phi}\mathcal{S}_2 . \tag{145}$$

When the external operators are contracted into this, the terms on the final line of (145) all drop by virtue of either transversality or tracelessness. Hence, we have

$$\frac{1}{2}\Big[\square - 2H^2\Big] i\Big[{}_{\mu\nu}\Delta^2_{\rho\sigma}\Big](x;z) = \frac{1}{4H^4}\mathbf{P}^{\beta\delta}_{\mu\nu}(x) \times \mathbf{P}^{\lambda\phi}_{\rho\sigma}(z)\Big[\mathcal{R}_{\beta\lambda}\mathcal{R}_{\delta\phi} \times \frac{\square}{2}\mathcal{S}_2\Big] . \tag{146}$$

It is now time to reduce transverse-traceless projection functional (132) to standard form (133). Just as we did with the transverse projection functional of Sec. IV, this is accomplished by acting $\mathbf{P}^{\mu\nu}_{\alpha\gamma}(x) \times \mathbf{P}^{\rho\sigma}_{\kappa\theta}(z)$ on both forms. When acting on expression (132), tracelessness or transversality make all but the first term drop out

$$\mathbf{P}^{\mu\nu}_{\alpha\gamma}(x) \times \mathbf{P}^{\rho\sigma}_{\kappa\theta}(z)\, i\Big[{}_{\mu\nu}P^2_{\rho\sigma}\Big](x;z) = \mathbf{P}^{\mu\nu}_{\alpha\gamma}(x) \times \mathbf{P}^{\rho\sigma}_{\kappa\theta}(z)\Big[g_{\mu\rho}g_{\nu\sigma}\frac{i\delta^D(x-z)}{\sqrt{-g}}\Big] , \tag{147}$$

$$= \mathbf{P}^{\mu\nu}_{\alpha\gamma}(x) \times \mathbf{P}^{\rho\sigma}_{\kappa\theta}(z)\Big[\mathcal{R}_{\mu\rho}\mathcal{R}_{\nu\sigma}\frac{i\delta^D(x-z)}{\sqrt{-g}}\Big] . \tag{148}$$

On the other hand, acting the same operator on (133), and making use of relations (141) and (146), tells us

$$\mathbf{P}^{\mu\nu}_{\alpha\gamma}(x) \times \mathbf{P}^{\rho\sigma}_{\kappa\theta}(z)\, i\Big[{}_{\mu\nu}P^2_{\rho\sigma}\Big](x;z) = \frac{1}{4H^4}\mathbf{P}^{\mu\nu}_{\alpha\gamma}(x) \times \mathbf{P}^{\rho\sigma}_{\kappa\theta}(z) \times\Big[\mathcal{R}_{\mu\rho}\mathcal{R}_{\nu\sigma}\frac{1}{4}\Big(\frac{D-3}{D-2}\Big)^2\square^2\Big[\square-(D-2)H^2\Big]^2\mathcal{P}_2\Big] . \tag{149}$$

Comparing (148) with (149), we infer an equation for the structure function of the transverse-traceless projection functional

$$\square^2\Big[\square-(D-2)H^2\Big]^2\mathcal{P}_2(x;z) = 16H^4\Big(\frac{D-2}{D-3}\Big)^2 \times \frac{i\delta^D(x-z)}{\sqrt{-g}} . \tag{150}$$

The solution is easily constructed using relation (42) and successive applications of (58)–(60),

$$\mathcal{P}_2(x;z) = \Big(\frac{4}{D-3}\Big)^2\Big[i\Delta_{AA}(x;z) - 2i\Delta_{AB}(x;z) + i\Delta_{BB}(x;z)\Big] . \tag{151}$$

The long-sought equation for the spin two structure function derives from the substitution in Eq. (131) of relations (146) and (151),

$$\frac{1}{2}\square\mathcal{S}_2(x;z) = \Big(\frac{4}{D-3}\Big)^2\Big[i\Delta_{AA}(x;z) - 2i\Delta_{AB}(x;z) + i\Delta_{BB}(x;z)\Big] . \tag{152}$$

The solution can be found using relations (61)–(66) from the end of Sec. III,

$$\mathcal{S}_2(x;z) = \frac{32}{(D-3)^2}\Big[i\Delta_{AAA}(x;z) - 2i\Delta_{AAB} + i\Delta_{ABB}(x;z)\Big] . \tag{153}$$

VI. DISCUSSION

We have constructed the graviton propagator on de Sitter background in exact de Donder gauge (107). Our result takes the form (110) of a spin zero part and a spin two part. Both parts are represented in terms of covariant differential projectors which automatically enforce the gauge condition, acting on scalar structure functions. Our form for the spin zero part is given by relations (111) and (112). The spin two part (117) has a complicated definition involving relations (113)–(116) and (72) and (73). By taking appropriate traces and commuting differential operators, we eventually derive scalar equations (129) and (152) for the structure functions of the respective parts. These equations are then solved using the general scalar techniques explained and summarized in Sec. III.

We emphasize that our forms for the spin zero and spin two parts of the propagator involve no assumption about de Sitter invariance, nor specialization to any particular portion of the de Sitter manifold. Equations (129) and (152), we derive for the two structure functions are scalar equations, valid in any coordinate system and with no inherent assumption about de Sitter invariance. To emphasize this, we act extra derivatives so as to make the source on the right-hand side proportional to a delta function in each case

$$\Big[\square + DH^2\Big]\Big[\square + 2(D-1)H^2\Big]^2\mathcal{S}_0(x;z) = -2\Big(\frac{D-2}{D-1}\Big)\frac{i\delta^D(x-z)}{\sqrt{-g}} , \tag{154}$$

$$\square^3\Big[\square - (D-2)H^2\Big]^2\mathcal{S}_2(x;z) = 32\Big(\frac{D-2}{D-3}\Big)^2 H^4\frac{i\delta^D(x-z)}{\sqrt{-g}} . \tag{155}$$

It happens that neither the spin zero structure function (130) nor its spin two counterpart (153) is de Sitter invariant. For the spin zero case, this is obvious from the presence of tachyonic mass terms in both of the differential operators on the left-hand side of Eq. (154). The mass $M_S^2 = -DH^2$ includes a logarithmic singularity which shows up even in analytic regularization techniques. For the spin-two equation (155), the squared operator has positive mass-squared $M_S^2 = (D-2)H^2$ and would not lead to breaking of de Sitter invariance were it alone. However, the cubed operator is the same as that for a massless, minimally coupled scalar—as might have been expected from Grishchuk's old result.[18] Allen and Folacci long ago proved that this has no de Sitter invariant solution.[47]

Exact de Donder gauge is interesting because de Sitter invariant constructions based on analytic continuation methods had previously dismissed it as an infrared divergent special case.[13] In fact, all valid gauges show infrared divergences. The special thing about de Donder gauge is that some of its infrared divergences are logarithmic so that they are not automatically (and incorrectly) subtracted by analytic continuation. In all the cases, the right way to resolve the infrared divergence is by breaking de Sitter invariance.

We have gone to considerable lengths—in previous work[2,3] and again in Sec. III— to elucidate precisely what goes wrong with previous constructions[1] which seemed to produce de Sitter invariant results. However, it worth pointing out that the fact of de Sitter breaking was already obvious to cosmologists from the scale invariance of the tensor power spectrum, which becomes exact in the de Sitter limit.[8] It was also obvious from the explicit form of a propagator constructed by mode sums on the open submanifold (for which there is no linearization instability).[8,52] On the open submanifold the $\frac{1}{2}D(D+1)$ elements of the de Sitter group break down into four parts:

1. $(D - 1)$ spatial translations;
2. $\frac{1}{2}(D-2)(D-1)$ spatial rotations;
3. A single dilatation; and
4. $(D - 1)$ spatial special conformal transformations.

The gauge condition only breaks the last of these, but the solution for the propagator additionally breaks dilatation invariance.[8,52] The physical de Sitter breaking of this propagator was demonstrated by Kleppe, who augmented a naive de Sitter transformation by the compensating gauge transformation needed to restore the gauge condition.[53] Had the propagator been physically invariant this technique would have revealed it.

We should also comment on the apparent conflict of our result with the pro-invariance argument given by Marolf and Morrison,[54] based on work by Higuchi.[55] They dealt with free dynamical gravitons in a noncovariant gauge on the full de Sitter manifold and they were able to construct the complete panoply of mode solutions and inner products. This should imply a vacuum which is physically de Sitter invariant—that is, invariant once the compensating gauge transformation is included. We know of no problem with this work but it should be noted that the propagator one gets using only dynamical gravitons (that is, the spatial, transverse-traceless polarizations) is not complete. It is like the purely spatial and transverse photon propagator of flat space electrodynamics in Coulomb gauge. To fully describe electromagnetic interactions also requires the instantaneous Coulomb interaction. Both of these are part of the same propagator in a covariant gauge such as the one we employ here.

The constrained, spin zero part of our propagator—which is missing from the transverse-traceless part—provides the largest source of the de Sitter breaking we found. It is relatively simple to show that the de Sitter breaking terms in $\mathcal{S}_0(x;z)$ do not drop out when acted upon by the spin zero projector $\mathcal{P}_{\mu\nu}(x) \times \mathcal{P}_{\rho\sigma}(z)$. The spin two structure function contains less severe de Sitter breaking terms of the form

$$\Big[\mathcal{S}_2(x;z)\Big]_{\substack{\text{de Sitter}\\ \text{breaking}}} = \sum_{k=1}^{3} s_k \Big[\ln(a_x a_z)\Big]^k . \tag{156}$$

It is possible that these drop out from the spin two part of the propagator (117) after all eight of the derivatives have been taken. In that case our work would be fully consistent with that of Marolf and Morrison. However, what we expect is that one of the infrared logarithms survives, which seems to be indicated by the scale invariance of the tensor power spectrum.

The fact of de Sitter breaking in this system cannot be disputed, but there is wide freedom as to how one chooses to manifest that breaking. This freedom amounts to picking the initial state. We have chosen the explicit solutions of Sec. III so as to preserve the symmetries of homogeneity and isotropy, which allow one to view de Sitter as a special case of a spatially flat, Friedman-Robertson-Walker geometry. This choice is known in the literature as the "$E(3)$ vacuum." Readers who prefer to preserve another subgroup can do so by starting from our scalar equations (154) and (155).

We wrote this paper to help resolve the long-standing controversy about de Sitter breaking for free gravitons; however, it has other applications. One of these is to test for gauge dependence in quantum gravitational loop corrections from primordial inflation. Of course gauge-fixed Green's functions will show such dependence, mingled with valid physical information. In flat space, we would sift out the gauge dependence by forming the S-matrix. That observable is not available in cosmology,[56] and there is not yet any consensus for what replaces it. One technique is simply to carry out computations in different gauges. It may be that the leading infrared logarithm contributions (e.g., the one loop contribution to the fermion field strength from inflationary gravitons[25]) are independent of the choice of gauge. Now we can test this conjecture using a completely different gauge from the one[8,52] employed in all previous computations.

Our propagator should also make renormalization simpler because it precludes the appearance of noninvariant counterterms. These complicated the analysis for previous computations.[25,27] It may also be that the gauge condition (107) and the special properties of the differential projectors in our propagator make actual computations simpler. That turned out to be the case with the vector propagator in Lorentz gauge[9] for a variety of one and two loop computations.[10,23]

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