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The growth of short fatigue cracks: a review

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Abstract
The behavior of short fatigue cracks differs from that of long cracks because of greater sensitivity to the microstructure, a greater size of the plastic zone relative to crack length, and a lesser extent of crack closure. Advances have been made in the understanding of the fatigue crack growth process of short cracks, and this understanding has been employed in the development of analytical treatments of short fatigue crack growth. The present paper reviews this progress and also discusses the relevance of short fatigue crack behavior to technologically significant areas.

1. Introduction

> "Sometimes a few hours in the library may be worth more than six months in the laboratory". Anon.

One of the more active areas of research in the field of fatigue in recent years has been the study of the fatigue crack growth behavior of short (small) cracks. Such cracks are of interest not only because their growth can occupy a significant portion of the fatigue lifetime but also because they can grow at rates much higher or lower than might be expected on the basis of long crack behavior.

Ritchie and Lankford (1) list four classes of small fatigue cracks:

1. Mechanically-small crack:
   crack length less than the plastic zone size
2. Microstructurally-small crack:
   crack length less than a critical microstructural dimension such as grain size
3. Physically-small crack:
   crack length less than that at which crack closure is fully developed, usually less than 1 mm in length
4. Chemically-small crack:
   crack length may be up to 10 mm, depending upon the crack tip environment

In the following recent developments concerning short fatgue cracks will be reviewed, with particularly emphasis placed upon the first three of these categories.

2. Fatigue Crack Initiation

Hunsche and Neumann (2) observed that in fatigue tests of copper single crystals persistent slip bands (PSBs) protruded about 20 μm from the surfaces of the specimens, and that a peak and valley topography existed within the PSBs. These valleys had a vertex angle of about 30° and a depth of 3 μm. Crack nuclei formed at the vertices of these valleys, with valleys near the interface between PSBs and the matrix being preferred sites for crack nucleation. These crack nuclei were truly closed cracks with unresolvably small crack tip angles. They concluded that crack nucleation was not a simple continuation of PSB formation process. Ma and Laird (3) observed that in single crystals of copper, Stage I fatigue crack growth rates at crack depths of more than 10 μm were constant at about one Angstrom per cycle for strain amplitudes in the plateau of the cyclic stress-stain curve. The crack growth rate was somewhat higher for crack depths less than 10 μm. Observations similar to those of Hunsche and Neumann were made by Boettner and McEvily (4) who examined extrusion formation in Fe-3% Si single crystals. In those crystals fatigue cracks originated at the interface

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between extrusion and matrix on that side of the extrusion which made the smaller angle with the matrix.

For polycrystalline copper, Bayerlein and Mughrabi (5) found that the persistent slip band/grain boundary crack was the most important crack nucleation site at low ($\Delta\varepsilon_p$ = 4.4 x 10^{-4}) and intermediate amplitudes ($\Delta\varepsilon_p$ = 1 x 10^{-3}). At $\Delta\varepsilon_p$= 1 x 10^{-3} crack nucleation at twin boundaries was relatively rare, however this type of crack propagated relatively rapidly. At $\Delta\varepsilon_p$ = 4.4 x 10^{-4} about 30% of the cracks investigated had originated at twin boundaries. At all plastic strain amplitudes slip-band cracks were generally the most frequent crack nuclei. However their damaging potential was small due to their low tendency to propagate. Their main contribution to the cracking process was to accelerate quickly growing crack types by being linked up into their crack paths. Grain boundary cracks appeared to play the most important role at all amplitudes, and because of their high frequency which increased with amplitude (in contrast to the observations on stainless steels mentioned above) and their strong tendency to propagate, their damaging potential was very high.

Heinz and Neumann (6) found for an austenitic stainless steel, that for lives greater than 3 x 10^5 cycles, cracks initiated at twin boundaries, whereas in a twin-free ferritic stainless steel, grain boundaries were the favored sites for crack initiation, with the relative frequency of cracks initiating at grain boundaries increasing with decreasing stress amplitude. Both of the alloys were planar-glide materials in which elastic anisotropy led to localized stresses at both twin and grain boundaries, and these stresses were thought to be responsible for the observed tendency for crack nucleation at twin and grain boundaries.

Goto and Nisitani (7) examined the statistics of fatigue crack initiation behavior for a 1045 steel. Fatigue cracks were found to initiate either at slip bands or defects with the percentage of initiation at defects decreasing from 75% to 50% with increase in stress amplitude. Murakami and Endo (8) have recently provided a comprehensive review of the effects of defects, inclusions, and inhomogeneities on fatigue strength. Included therein is a discussion of their $\sqrt{area}$ parameter which is used to assess the severity of a defect or inclusion in terms of its projected area.

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3. Short Fatigue Crack Growth

Forrest and Tate (9) showed for polycrystalline brass of large grain size, that if the stress amplitude was high enough to nucleate a Stage I crack, that stress would also be sufficient to cause the crack to propagate through the adjacent grains. On the other hand, in fine grained brass cracks could be initiated at stress levels that were insufficient for propagation into the adjacent grains. Therefore if fatigue failure is to occur, for those materials wherein the spacing of microstructural barriers is large, the critical event resulting in final failure is the nucleation of a crack. On the other hand, for those materials in which the spacing of microstructural barriers is small, the critical event resulting in final failure can be the ability to propagate a crack. It is also to be expected that on going from high cycle, near endurance level fatigue to low cycle fatigue, fatigue crack nucleation rather than fatigue crack propagation will become the critical event. The growth of short fatigue cracks can involve both Stage I and Stage II propagation, with the Stage I cracks growing in slipbands under combined Mode I and Mode II loading. In the absence of environmental effects, the Mode I contribution may be important, for Chen and McEvily (10) have observed that in a single crystal of a nickel-base superalloy pure Mode II propagation did not occur in vacuum (whereas it did occur in air).

Goto and Nisitani (7) carried out a Weibull analysis of fatigue crack initiation and propagation life behavior for 1045 steel at three stress amplitudes, 500, 600 and 700 MPa, using 16 specimens at each amplitude. In this analysis the number of cycles to crack initiation, N_i, was taken to be the number required to develop a 35 μm crack. As might be expected, there was more of a variation in the crack initiation life than in the crack propagation life. The results indicated that the later in life a crack was initiated the slower was its propagation. Therefore the local stress state which favored late initiation, whether it be intrinsic or extrinsic, also favored slow propagation.

Tokaji and Ogawa (11) studied microstructurally small fatigue crack growth behavior of seven metals: low carbon steel, medium carbon steel, high tensile steel, low alloy steel,

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dual-phase stainless steel, 7075-T6 aluminum, and pure titanium. The microstructurally affected crack length was usually less than 1-3 times that of the microstructural unit size. The overall growth behavior of microstructurally small cracks was similar in all materials, with crack growth rates markedly decreasing at grain boundaries, triple points and boundaries between phases. Crack deflection also led to a decrease in crack growth rate. The smaller the grain size the larger was the decreases in crack growth rate, and hence the longer the lifetime. The microstructural effect was more apparent at negative stress ratios, and Stage I facets were found at R = -1 and -2, but not at R = 0. It is not clear whether this reflects the role of compression in promoting Stage I growth or the role of K_{max} and the ambient environment in promoting Stage II growth. The transition from Stage I to Stage II at R = -1 occurred at a crack length of about 8d, where d is the size of the microstructural unit size, but at R = 0, since stage I was not observed, a similar relation could not be established.

Stage I and Stage II propagation of short and long cracks in Al-Zn-Mg alloys was investigated by Petit and Kosche at positive R values (12). At a given ΔK_{eff}, Stage I propagation was faster than Stage II, which in turn was faster than Stage I-like growth. Stage I-like growth was similar to Stage I at the scale of individual grains, but macroscopically the crack plane was normal to the stress axis. Stage II propagation was similar in single and polycrystals, with cracks growing alternatively along two (or more) {111} slip planes. Stage I growth, an extension of the initiation process in single- and polycrystals, was also a characteristic of near-threshold growth in single crystals.

In Ti-6Al-4V Petit et al. (13) found that the short crack effect in air on two-dimensional cracks was related to the development of crack closure over distances of 0.7-1.5 mm, depending upon the specimen thickness. However in vacuum the short crack effect was present only for cracks less 0.1 mm in length. The transition from Stage I to Stage II growth was aided by the environment. Stage I cracks developed to depths of about 35-65 μm in vacuum, but only to depths of 8-16 μm in air.

In single crystals of the nickel-base superalloy Udimet 720 Reed and King (14) found that stage I growth occurred along slip planes of maximum resolved shear stress giving rise to faceted fatigue fracture surfaces. Short crack growth behavior was observed in that the crack growth rates were higher than for short cracks in polycrystals, an indication that grain boundaries in the polycrystals retarded fatigue crack growth. The threshold level for the single crystals at R = 0.5 was about 3 MPa $m^{1/2}$ as compared to about 6 MPa $m^{1/2}$ for polycrystalline material. In polycrystals an R-effect was observed which was attributed to surface roughness associated with small-scale faceted growth. Additional results implied that crack closure played a role in single crack growth crystals as well.

Bernede and Remy (15) reported for Astroloy, a P/M nickel-base superalloy, that no two-dimensional short-crack effect was found at 650° C, whereas a short crack effect had been found at room temperature.

4. Analysis of Short Fatigue Crack Growth

Newman et al. (16) modified the Dugdale strip yielding model to include the effects of a plastically deformed crack wake on crack closure behavior in a study of the growth of short cracks from notches. To reflect the state of stress at a crack tip, i.e., whether plane stress or plane strain, a constraint factor on the yield stress was used. This factor was set equal to 1.1 for plane stress conditions and to 1.73 for plane strain conditions. The crack-closure model predicted that the plasticity-induced closure effect on small cracks was not significant for positive stress ratios but became progressively more significant as the stress ratio became more negative. To make predictions of fatigue life, an initial surface defect void size of 3 by 12 by 0.4 μm was used which was consistent with experimental observations of initiation sites at inclusion particle clusters. With the use of model parameters evaluated from constant-amplitude fatigue data at R = -1, good agreement was found between experimental and predicted fatigue lives at other stress ratios. The development of crack closure in the short crack regime has also been analyzed by Nakai and Ohji (17), Tanaka (18), and by McEvily et al. (19). Weiss et al. (20) have also analyzed the effect of three-dimensional defects on the fatigue limit.

Tanaka et al. (21) used a continuous distribution of dislocations to model the interaction of the slip deformation ahead of the tip of a small crack with grain boundaries. They assumed that crack growth rate was controlled by the crack opening displacement, i.e., da/dN = C(ΔCTOD), where C is a material constant. A Monte Carlo simulation of the growth of a crack in a polycrystalline medium was used where the grain size and frictional stress were random variables of two-parameter Weibull distributions. The strength of the grain boundary blocking crack propagation was characterized by the critical value of the stress intensity factor. Rapid and irregular propagation of small cracks was derived from the simulation, and in the simulation interaction of the slip band with the first grain boundary encountered was critical. A calculation of the number of cycles required to propagate a crack from a size of 1 μm to 2 mm as a function of the applied stress was made. In a figure showing log stress vs. log lifetime a slope - 1/2 was observed. Of particular interest is that the method provides some insight into a matter of interest to designers, i.e., the causes of scatter in fatigue lifetimes.

Tanaka and Akinawa (22) have found that for cracks with large scale yielding, the J-integral range, ΔJ, is an appropriate correlating parameter. Chan (23) has pointed out that the driving force for the growth of short cracks can be influenced by microstructure, large scale yielding and crack closure. In the case of a small crack, a high applied stress to yield stress ratio will lead to larger than expected crack-tip ΔJ values. However, Edwards and Güngör (24) found that for short fatigue crack growth rates, the use of the elastic/plastic parameter ΔJ as a correlating parameter resulted in more scatter than did the use of the linear elastic parameter ΔK. They raised the interesting question as to why there was such a large difference in the S/N curves of aluminum alloys when there was so little difference in their rate of propagation of short fatigue cracks. They concluded that the difference in the S/N curves was attributable to differences in crack initation behavior.

The work of Bataille et al (25) illustrates that simulation of microcrack growth processes can be quite complex, particularly when microcrack coalescence is taken into account. For the case of low cycle fatigue they used a numerical simulation procedure to model random microcrack nucleation and growth processes. These processes involved 90% of the fatigue lifetime. Results of the simulation process were compared with experimental results for 316L stainless steel for which the microcracking process was transgranular. Four classes of microcracks were introduced which were dependent upon their surface length.

A two-dimensional network of hexagonal grains of equal size was used in the modeling process. The orientation of each grain was random and differed from that of its neighbors. Each grain was sub-divided into 36 squares of equal size. Cracks were nucleated in slip bands only in those grains which were above a plastic-strain dependent nucleation threshold, and there is but one crack per such grain and it could be nucleated anywhere within the grain. A propagation threshold given as the nucleation threshold plus a function of the crack length, misorientation, plastic strain range and percentage of fatigue life was also used.

The modeling correlated with experimental data, including microcrack densities and fatigue lifetimes. In the simulation the density of isolated major cracks decreased upon the occurrence of coalescence, although the decrease was not as pronounced as found in the experimental results, perhaps due to the two-dimensional nature of the simulation. The approach was also extended to deal with the subject of cumulative damage in fatigue. They concluded that fatigue damage could be described in terms of a statistical study of surface microcrack populations, which can also be linked to the Manson-Coffin relationship

de los Rios et al. (26) used reversed torsional loading to study short fatigue crack growth in a 0.12C-0.38Mn steel of two different microstructures; a ferritic-pearlitic banded microstructure (tensile yield stress: 312 MPa; cyclic yield stress in shear: 230 MPa) and a ferritic-bainitic microstructure (tensile yield stress: 371 MPa; cyclic yield stress in shear: 275 MPa). The shear stress levels employed were 1.11, 1.16 and 1.30 times the cyclic yield stress in shear. In the banded microstructure slip-band cracks formed in the ferrite and propagated unhindered by the ferrite boundaries until they reached a pearlite band. There growth was hindered to a greater or lesser extent depending upon the local microstructure of the band. Decelerations in crack growth rate generally occurred when the short cracks were either one or two inter-band spacings long (50-100 μm), but the minimum growth rates were reached at crack lengths between 70 and 30 μm. It was considered that the slowing down was due to the thin ferrite phase component of the pearlite, on the assumption that crack propagation should be slow across a thin ferrite plate, whereas the cementite plate should fracture relatively easily. However it seems also possible that if the local stress were not high enough to fracture the cementite plate, the plate would act as a reinforcement and limit the extent of plastic deformation nearby and hence retard the rate of crack growth at the surface. With further torsional cycling the crack could grow in depth in the ferrite until the local stress concentration increased to the point where fracture of the cementite plate could occur.

In the ferrite bainite microstructure cracks were initiated in the ferrite and grew in the ferrite often across the entire prior austenitic grain. The crack then either changed direction if a ferrite plate was available in the next prior austenitic grain, or fractured and crossed the cementite elongated particles. The crack growth rates for the two microstructural conditions were similar at τ/τ_{yc} of 1.11 and 1.16, but cracks in the banded microstructure grew at a faster rate at τ/τ_{yc} of 1.30.

The behavior of dominant cracks was analyzed using empirical coefficients generated in the experimental phase using all data. The surface crack growth rate, da_s/dN, was assumed to be given by

$$\frac{da_s}{dN} = f\phi \qquad (1)$$

where ϕ is the crack tip displacement and f is an empirical coefficient representing the degree of plastic irreversibility. (For these experiments $f = 7.2 \times 10^{-42}(\Delta\tau)^{15.26}$.) The plastic displacement at the tip of crack is given by

$$\phi = \frac{1}{G}\frac{\sqrt{(1-n^2)}}{n}\,\Delta\tau\, a_s \qquad (2)$$

where G is the shear modulus and $n = \frac{a_s}{a_s + pzs}$, where pzs is the plastic zone size.

As the crack grows with the plastic zone blocked by a barrier, the parameter n increases to a critical value n_c given by

$$n_c = \cos\left[\frac{\pi}{2}\frac{\tau - \tau_{Li}}{t_{comp}}\right] \qquad (3)$$

where τ_{comp} is a constant assumed to be of the order of the ultimate tensile strength, τ is the applied stress, and τ_{Li} is the minimum stress for slip propagation which is a function of the grain orientation factor and the slip band unlocking stress. This latter stress is considered to be a function of the fatigue limit, τ_{FL}:

$$\tau_{Li} = \frac{\tau_{FL}}{\sqrt{i}} \qquad (4)$$

with i = 1, 3, 5,... being the number of half grains affected by the fatigue process.

When n achieves the critical value n_c, the stress ahead of the plastic zone attains a level sufficiently high to initiate a new slip band and the plastic zone then extends across the next grain. Upon this occurrence, the stress concentration ahead of the new exended plastic zone decreases, affected by the new lower value of n_s, related to the larger plastic zone size

$$n_s = \frac{i}{i + 2} \qquad (5)$$

With this procedure upper and lower bounds to the rate of short fatigue crack growth were established, with the latter showing the effect of a microstructural barrier on the extent of retardation. In this treatment it is noted that crack closure is not treated explicitly.

McEvily and Shin (27) have used the following relationships in the analysis of short crack growth behavior:

$$\frac{da}{dN} = A(\Delta K_{eff} - \Delta K_{effth})^2 \qquad (6)$$

$$\text{or } \frac{da}{dN} = A(M)^2 \qquad (7).$$

where A is a material constant, and M is given by

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$$M = [\sqrt{2\pi r_e} + Y\sqrt{\frac{\pi}{2} a(\sec\frac{\pi}{2}\frac{\sigma_{max}}{\sigma_y} + 1)}](\sigma_{max} - \sigma_{min}) - (1 - e^{-k\lambda})(K_{opmax} - K_{min}) - \Delta K_{effth} \quad (8)$$

Here r_e is a material constant of the order of 1 μm which links the threshold and fatigue strength levels, Y is a geometrical factor, a is the crack length, σ_y is the yield strength, k is a material constant of dimensions mm^{-1} which reflects the rate of development of crack closure, λ is the length of a newly formed crack in mm, K_{opmax} is the level of crack closure for a macroscopic crack at the R level being analyzed, K_{min} is the minimum stress intensity factor, and ΔK_{effth} is the range of the stress intensity factor at threshold. Short crack growth data obtained by Nisitani and Goto (28) were analyzed using these equations, and the results were found to be in reasonably good agreement with experiment.

5. Significance of Small Cracks

Wanhill and Schra (29) evaluated the significance of short cracks in an experimental/analytical durabilty evaluation of the Fokker 100 wing/fuselage structure, and found that 0.127 mm corner flaws at simulated fastener holes grew to an estimated length of 7.7 mm after 90,000 flights at the current design mean-stress-in-flight level of 65 MPa. A length of 7.7 mm is close to the NDT minimal-normally- detectable-length of 6.35 mm, and they therefore concluded that widespread, detectable cracking at fastener holes would not be expected to occur within the design economic repair life of 90,000 flights even if 0.127 mm flaws were initially present. The behavior of microstructurally short cracks was therefore of even less significance in this case. However, if in the future a higher mean stress in flight were to be used then a more comprehensive assessment of the significance of short fatigue crack growth behavior would be needed. They also noted that short cracks initiated at the interfaces between inclusions and the matrix of 2024-T3 aluminum alloy used in the experiment phase of the study, a finding consistent with other results for low stress/long life fatigue. Individual short cracks grew very slowly, but at higher crack growth rates than expected on the basis of long crack data.

Examples of the use of small crack fatigue data in the prediction of the fatigue life of components have been given by Usami (30), and Lindley and Tomkins (31) have provided similar information in relation to integrity assessment in the power generation industry where corrosion pits, inclusions, fretting, and thermal loading are of concern. Howland (32) et al. have dealt with research on short crack behavior as related to the integrity of major rotating aero engine components. Blom (33) has discussed the topic with respect to aircraft. He points out that when damage tolerant design of engines is introduced, the behavior of short cracks will be of paramount importance.

6. Concluding Remarks

In a brief survey it is difficult to cover all of the pertinent work that has been done on short fatigue cracks. Nevertheless it is hoped that a perspective has been provided which reflects the current state knowledge concerning the experimental and analytical research in this field.

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