# The haptic radial-tangential effect: Two tests of Wong's "moments-of-inertia" hypothesis 

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#### Abstract

When a given length is haptically traced, the direction of hand movements (relative to the body) influences the length perceived. A recent theory by Wong (1977) states that the source of the illusion is undetected differences in the moments of inertia associated with different directions of hand movement: Radial directions produce greater resistance to inertia (and, therefore, slower movements) than do tangential directions. Because subjects rely on time estimates to determine a fixed distance and because they are unable to perceive that they are moving more slowly in the radial than in the tangential direction, they overestimate "radial" lengths relative to "tangential" lengths. Experiment 1 examines the effect of altering inertia by changing the distance of the hand from the axis of rotation; Experiment 2 does so by changing the mass of the moving hand. Both manipulations fail to support the predictions derived from the momente-of-inertia hypothesis.


Imagine a series of concentric circles radiating along a plane from a common origin, for example, the subject's body. Any hand movement that extends along the radius of any of the circles is defined as a radial movement. Such movements entail motions toward and away from the subject's body. For any given circle, a tangent can be drawn that is, by definition, orthogonal or normal to the radial extension. Hand movements along any of the tangents possible are defined as tangential movements.

Reid (1954) used the corner component of a square frame (laid horizontally on a table in front of the subject) to investigate the haptic equivalent of the horizontal-vertical illusion found in vision (Finger \& Spelt, 1947; Künnapas, 1957). His subjects used a stylus to explore the length of part of one side of the square. Once the standard length was determined, the subjects' task was to reproduce the length just explored, but along an orthogonal axis of the square. Basically, the arm movements involved were radial and tangential. Reid found that with the square placed horizontally, subjects systematically overestimated the length produced with radial motions (away-toward movements) relative to those made with tangential movements (i.e., left-to-right movements, in Reid's case). The overestimation of the perceived movement was reflected in an objectively longer length generated for tangential motion than that generated for radial motion. No such illusion occurred, however, if the square was positioned vertically, that is, perpendicular to the table surface.

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The illusion was termed the "radial-tangential effect" by Davidon and Cheng (1964; see also Cheng, 1968). Davidon and Cheng investigated the illusion in more detail than Reid (1954), and they established that the direction of motion, radial or tangential, is central to the illusion. Further, they agreed with Reid that the illusion was due to the subjects' inability to detect that their hand speed in a radial direction is slower than that in a tangential direction. Because subjects assumed their hand was moving at a constant rate, regardless of the direction of motion, they relied on temporal cues to calculate the distance traveled. The inequality in speed, however, resulted in an error in reproduced length. Unfortunately, Davidon and Cheng did not provide a testable hypothesis regarding the source of the differential speed incurred during radial vs. tangential hand movements.

In 1972, Deregowski and Ellis made a major step toward understanding the limits of the radial-tangential effect. They established a mathematical function that predicted the size of the radial-tangential illusion by relating it to the length of the components used for exploration and response, and to the degree to which these components are explored in either a purely radial or a purely tangential manner. Using the degree of error between the two types of movements obtained at the point of subjective equality, they estimated their single free parameter. Their equation indicates that when a hand motion is purely radial and it is followed by a purely tangential motion, the illusion is at a maximum. It diminishes, however, as the two types of movements become mixed, that is, as the radial movement becomes less purely radial and the tangential movement becomes less purely tangential. Although Deregowski and Ellis were able to integrate descriptively across past empirical work, they did not propose a mechanism that could account for the haptic spatial illusion.

In 1977, Wong offered an explanation of the radial-
tangential effect. He argued that radial movements have inherently more resistance to inertia than do tangential movements; during radial motions, the hand is moving away from the axis of rotation (which he assumed was the shoulder) and, thus, "the moment of inertia is accordingly greater along radial directions" (Wong, 1977, p. 162) than along tangential ones. As a test of Wong's hypothesis, the moments of inertia were manipulated either by changing the distance of the hand from the fulcrum, defined as the shoulder joint, or by changing the limb mass to be moved, that is, by adding different weights to the hand.

## EXPERIMENT 1: USING CHANGES IN DISTANCE TO CHANGE INERTIA

Inertia increases as the distance from the axis of rotation increases (Hay, 1953). Thus, placing a stimulus at varying distances from the subject should increase the inertia that must be overcome and, therefore, the resistance to limb movement. According to Wong's (1977) hypothesis, increasing the resistance to the movement should cause an overestimation of perceived length. Because the overestimation is due to a general increase in limb resistance with increases in distance, an overestimation should occur for both radial and tangential motions at the different distances. Because the overestimation should occur for both types of motions, the size of the radial-tangential illusion should be constant across the increases in distance, but the lengths reproduced should change.

## Method

Subjects. Nine undergraduates (who wrote with their right hand) from Queen's University were paid subjects.

Apparatus. Five steel rods ( 2 mm thick), ranging in length from 3.0 to 18.0 cm in $3.75-\mathrm{cm}$ steps, were used as stimulus rods. A sixth rod, 25.0 cm long, served as a response rod, along which subjects traced the length of the standard stimulus.

The $25.0-\mathrm{cm}$ response rod was always placed 10.0 cm horizontally in front of the subject so that movements along it were always tangential. The stimulus rods were arranged such that exploration was either in a radial or a tangential orientation; the resulting points of intersection in front of the subject were 11.5, $23.0,34.5$, or 46.0 cm away.

Procedure. The experimenter drew a line down the center of the subject's right index finger. The line served as a reference for measuring the subject's tangential responses. Radial responses were measured using the fingertip as the reference point.

All subjects were blindfolded. At the start of each trial, the subject's hand rested on a pad. One of the five stimulus rods was chosen and placed at one of the intersections and orientations, as randomly predetermined. The experimenter took the subject's index finger and placed it on the standard rod. Whether the subject explored initially from left to right or vice versa was altered daily and counterbalanced across subjects. When the subject was ready to trace a response, he verbally signaled the experimenter, who then placed the subject's finger on the edge of the response rod according to that day's direction of exploratory movement. Feedback was not provided. Seven sessions (each $40-50 \mathrm{~min}$ ) were run, and the last six sessions constituted data sessions.

## Results and Discussion

An ANOVA was run on the within-subjects factors of
rod size, orientation of the stimulus rod, and distance from the subject. Not surprisingly, rod size influenced response length. Estimated mean lengths were 3.25, $6.75,10.01,13.17$, and 16.29 cm (for stimuli of 3 , $6.75,10.5,14.25$, and 18 cm in length, respectively) $[F(4,32)=221.02, p<.0001]$.

Replicating the radial-tangential effect, the length of radially oriented stimuli was overestimated relative to that of tangentially oriented rods ${ }^{1}$ [ 10.25 vs. 9.55 cm , respectively; $\mathrm{F}(1,8)=14.96, \mathrm{p}<.005$ ]. With tangential movements, subjects produced lengths (collapsed across the stimulus-size factor) of 9.92, 9.59, 9.37, and 9.32 cm as the distance from the observer increased (from 11.25 to $22.5,33.75$, and 45.0 cm , respectively) However, while subjects increasingly underestimated tangential movements with increases in distance, they increasingly overestimated radial movements with the same increases in distance. The produced lengths with radial exploration were $10.18,9.94,10.20$, and 10.64 cm , respectively.

Because the length of radially oriented stimuli was increasingly overestimated with increases in distance from the subject and the length of tangentially oriented stimuli was increasingly underestimated, the size of the illusion actually increased with distance, resulting in an interaction $[\mathrm{F}(3,24)=5.04, \mathrm{p}<.01]$. The presence of the interaction undermines Wong's (1977) hypothesis, because as increases in distance cause increases in the amount of overall limb resistance, both radial and tangential movements should be overestimated; that is, increases in distance should affect the illusion in a proportionally constant manner. As a parenthetical note, similar findings were obtained when a magnitude estimation procedure was used instead of the present method, thereby increasing the generality of the findings (Marchetti, 1979, Experiment 1).

## EXPERIMENT 2: USING MASS LOADING TO CHANGE INERTIA

The predictions made from Wong's (1977) moments-of-inertia hypothesis were not supported when distance was manipulated. Another way to alter inertia is to change the mass of the limb. From Wong's (1977) hypothesis, with increases in the mass introduced prior to the response, the limb's resistance to acceleration increases. The increased resistance should cause the subject's hand to move more slowly than otherwise. Consequently, assuming in accord with Wong that (1) the subjects use temporal cues to calculate distance, and (2) they always perceive their hands to be moving at a constant speed, a length produced under conditions of increased mass should be shorter than one produced under normal conditions. (The shorter length is inferred as an overestimation of the movement.) In general, introducing increases in mass to the hand should cause a monotonic decrease in the length reproduced.

## Method

Subjects. Fifteen right-handed female undergraduates from Queen's University participated as unpaid volunteers.

Apparatus. An L figure with two $50-\mathrm{cm}$ aluminum rods, 5 mm thick, served as the stimulus base. A $2.5-\mathrm{cm}^{2}$ pad was positioned at what would otherwise have been the corner of the L figure. The rods formed orthogonal components positioned at the edges of the pad.

A $25-\mathrm{cm}$ length, defined by the presence of a removable clip, was used as a standard (i.e., the length to be reproduced). (The clip's edges were taped in order to minimize contact noises and blunt its sharp edges.) The orientation of the movement along the standard rod, radial or tangential, was determined randomly on each trial.

Four identical plastic pill containers were used as weights. Each container was 6.5 cm long and had a locking cap. The cylindrical containers had a diameter of 3.5 cm at the closed end and 4.0 cm at the capped end. These containers were filled with varying amounts of lead to establish weights of 9.8 (empty container), 122,245 , and 367 g . (The range was determined empirically; Marchetti, 1979, Experiment 2.)

Procedure. After the subject was blindfolded and seated at a table, the apparatus was uncovered. The subject's fingertip was placed on the rest pad, and the table was moved to align the pad with her subjective midline.

The experimenter initiated a trial by moving the subject's index finger from the resting pad to the adjacent edge of the standard rod. The subject moved her finger 25.0 cm along the rod until she touched the clip. She was free to explore the distance as often as she desired (usually two complete runs). When ready, the subject returned her finger to the resting pad, thus signaling the experimenter to place her finger on the response rod. The subject's task was to move her finger along the rod a distance equal to the one just explored. When she stopped moving along the response rod, the experimenter read the response to the nearest millimeter by visually aligning the mark on the finger (or the fingertip) with a scale fixed alongside the rod. The subject's finger was then returned to the rest pad in preparation for the next trial. (If the same rod that served as the standard was to serve for the response, the clip was removed.) No feedback was given.

The mass loading was effected by placing one of the plastic pill containers in the subject's hand. No container was carried by the subject during the exploration of the standard. ${ }^{2}$ When the subject signaled she was ready, the experimenter placed one of the four containers (always with the capped end facing the subject's thumb) into the subject's hand and positioned her fingertip on the edge of the appropriate response rod. The intertrial interval was 5 sec .

Design. Four within-subjects factors were manipulated: orientation of the standard (radial or tangential), orientation of the response (radial or tangential), container weight during the response phase (four levels), and repetitions (four levels). All factors were factorially combined and administered in a random sequence determined uniquely for each repetition and for each subject. Although subjects received four repetitions, only the last three were used in the analysis. Thus, of the 64 observations from a session, only the last 48 were included in the analysis. Each session lasted about 60 min , including a 1 -min rest period halfway.

## Results

By definition, a response of 25 cm (the standard length) means no illusion. A response length of less than 25 cm is interpreted as a perceived overestimation of the response movement relative to the exploration movement. A response length greater than 25 cm defines a perceived underestimation of the response movement.

Application of an ANOVA on the four within-subjects factors revealed no effect of repetition $[F(2,28)=1.61$,
p>.2]; thus, the data were collapsed across that factor. The main effect of orientation of the standard was significant $[F(1,14)=7.03, p<.02]$, as was the main effect of orientation of the response $\operatorname{rod}[F(1,14)=4.49$, $\mathrm{p}=.05]$.

With a radially explored standard, subjects produced an average length of 25.02 cm or 25.3 cm , depending on whether the response was made in a radial or tangential orientation, respectively. The pattern suggests that a slight underestimation of the tangential movement is made to a radial standard. However, for a tangentially oriented standard, the average produced length was 22.65 cm or 25.07 cm for a radial or a tangential response orientation, respectively. Thus, in contrast to the results found with the radial standard, subjects greatly overestimated responses made radially, given a tangential standard. The contrast results in a significant interaction between the orientation of the response rod and the orientation of the standard $[\mathrm{F}(1,14)=28.11$, $\mathrm{p}<.001$ ]. The interaction indicates that little error occurred when subjects had to respond in the same orientation as they explored, but that subjects over- or underestimated their response if they had to make a movement orthogonal to their exploration.

The important triple interaction, Orientation of the Response by Orientation of the Standard by Mass of the Containers Carried, failed to reach significance $[F(3,42)=2.59, p<.07]$. However, it did contain a significant linear component $[F(1,14)=9.86, p<.01]$, demonstrating an asymmetrical influence of the mass factor as a function of the orientation of the standard and response rods. The finding, illustrated in Figure 1, replicated previous work (Marchetti, 1979, Experiment 2).


Figure 1. The figure displays the triple interaction between the orientation of the standard stimulus, the orientation of the response rod, and the weight carried during the response phase. Panel A shows the length of the response in centimeters made in both a radial and a tangential orientation, as a function of the weight carried when the standard was tangentially oriented. Panel B shows the same interaction between the weight carried and the orientation of the response rod when the standard was radially oriented.

Figure 1 shows two major patterns: (1) The radial movements were never influenced by changes in mass during the response phase, regardless of the orientation of the standard rod, and (2) the tangentially oriented responses were not affected by changes in mass if they were made to a tangentially oriented standard, but the responses followed the pattern predicted from the moments-of-inertia hypothesis when they were made to a radially oriented standard.

Panel A shows that with a tangentially oriented standard, responses made in the tangential orientation were both accurate and not affected by the mass of the containers carried. Also, radially oriented responses made to a tangential standard were flat as a function of changes in mass, but the average response length was 22.5 cm , revealing an overestimation of the radial movement relative to the tangential standard. Panel B shows that with a radially oriented standard, responses made in a radial orientation were both accurate and not influenced by the mass loading. However, the tangentially oriented responses made to the radial standard showed a monotonic (in fact, linear) decrease with increases in mass loading. This was the only instance (out of four) that supported the predictions derived from Wong's (1977) moments-of-inertia hypothesis.

## CONCLUSIONS

The inertia of a system is influenced by changes in the distance from the fulcrum that a mass is moved, in the amount of mass moved, or in both (Hay, 1953). Because radial movements involve changes in distance from the body, the moments of inertia for radially oriented movements change as the limb moves away or toward the body. In contrast, changes in inertial moments do not occur with tangentially oriented movements because the limb is a fixed distance from the body. The moments-of-inertia hypothesis asserts that as the inertia of a radial movement is increased, there is a corresponding increase in the resistance to the limb movement.

The increased resistance, associated with radial movements, causes limb movements made in radial directions to be executed more slowly than those made in tangential directions. Because the subjects are attempting to hold speed constant, while timing their movement to calculate the distance traveled, the undetected difference in speed between the two types of motions results in shorter lengths traced for radial movements than for tangential ones. The difference in physical lengths is interpreted as an overestimation of the radial motion relative to the tangential one.

In Experiment 1, we changed the resistance to limb movement by manipulating the distance from the fulcrum (the shoulder, as defined by Wong, 1977). We predicted that both the radial and the tangential orientations should have resulted in a constant and proportional reduction in perceived length. Consequently, no interaction was expected between the distance from the observer and the orientation of the stimuli. The presence of the interaction suggests either that the hypothesis is wrong or that the distance manipulation was ineffective in changing appreciably the degree of resistance offered to the limb.

In Experiment 2, we changed the inertia of the limb by changing its mass. We predicted that increases in resistance to the limb movement would result in a monotonic decrease in the length reproduced by the subjects. The decrease in reproduced length is interpreted as a perceived overestimation of the movement. The results suggest that length estimations based on radial
movements are in no way affected by the increases in limb mass. This particularly undermines Wong's (1977) hypothesis because it was formulated to account for the overestimation of reproduced (and produced) length during radial motion. The results for the tangential orientation, however, are equivocal. When exploration of the standard involved tangential motion, increasing the limb mass did not influence response lengths made in a tangential orientation. In contrast, when the standard was explored with a radial motion, the tangential responses followed the predicted monotonically decreasing path. As the mass loading manipulation was effective in only one instance, the failure of the data to support the hypothesis unequivocally suggests that the inadequacy lies in the hypothesis, not in the manipulation.

Accordingly, the results of the current study question the validity of the moments-of-inertia explanation and suggest that the radial-tangential effect does not have a simple biomechanical cause.

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## NOTES

1. Unfortunately, because of methodological differences, the length of a response does not have the same interpretation in the two experiments. For Experiment 1, because the responses were made only in a tangential orientation, underestimation of the explored length is interpreted from a response length that is shorter than the objective length of the exploration stimulus; overestimation of the explored length is inferred from a response length that is longer than the stimulus explored. The converse is true for Experiment 2.
2. Previous work (Marchetti, 1979, Experiment 2) suggests that carrying a weight during the exploration phase does not change overall performance, nor does it interact with any of the other factors. This curious outcome further undermines the moments-of-inertia hypothesis, which we claim would predict an interaction between the weight carried during the exploration phase and the weight carried during the response phase. The interaction did not obtain for either the group with a tangentially oriented standard or the group with a radially oriented standard $[F(4,56)=.54, p=.7$, and $F(4,56)=1.47, p<.2$, respectively], nor did the second-order interaction form a thirdorder interaction with group as the third factor $[F(4,112)=1.0$, $\mathrm{p}<.6$ ].
