The Heckscher-Ohlin Theorem in the Multi-Commodity Case

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Ronald Jones, in his seminal paper (1957) on Heckscher-Ohlin theory, has argued that, for the case of two countries, two factors, and several commodities, the Heckscher-Ohlin theorem would remain valid in the following weak sense: "Ordering the commodities with respect to the capital-labor ratios employed in production is to rank them in order of comparative advantage. Demand conditions merely determine the dividing line between exports and imports; it is not possible to break the chain of comparative advantage by exporting, say, the third and fifth commodities and importing the fourth when they are ranked by factor intensity" (p. 85).

It is easy to show, however, that this proposition, although correct for the case where factor prices are *not* equalized, is untenable as literally stated. When factor-price equalization is realized, a not unimportant case, a variety of crisscrossings are possible.[1]

Thus, let there be two countries, I and II, with endowments of two factors, K and L, as shown in figure 1 by the two rays from the origin, such that Country I is K-abundant and Country II is L-abundant, in the physical sense. Let us also assume that there are four commodities, w, x, y, and z, all of them characterized by linearly homogeneous production functions with the standard restrictions. We also assume that the strong Samuelson factor-intensity rankings obtain, such that the four commodities can be ordered according to their K/L ratios unambiguously. Let the K/L ratios decline successively for commodities w, x, y, and z.

Using the Lerner-Findlay-Grubert technique, we can now take the case

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[1] I have long taught the Jones proposition to my graduate students, of whom only Rick Diesen (in the fall of 1970) was certain that it was invalid and prompted me to write this note. Ronald Findlay, in his excellent Penguin text (1970, pp. 66–59) has lucidly explained the Jones proposition, without again noticing its invalidity as a general proposition.

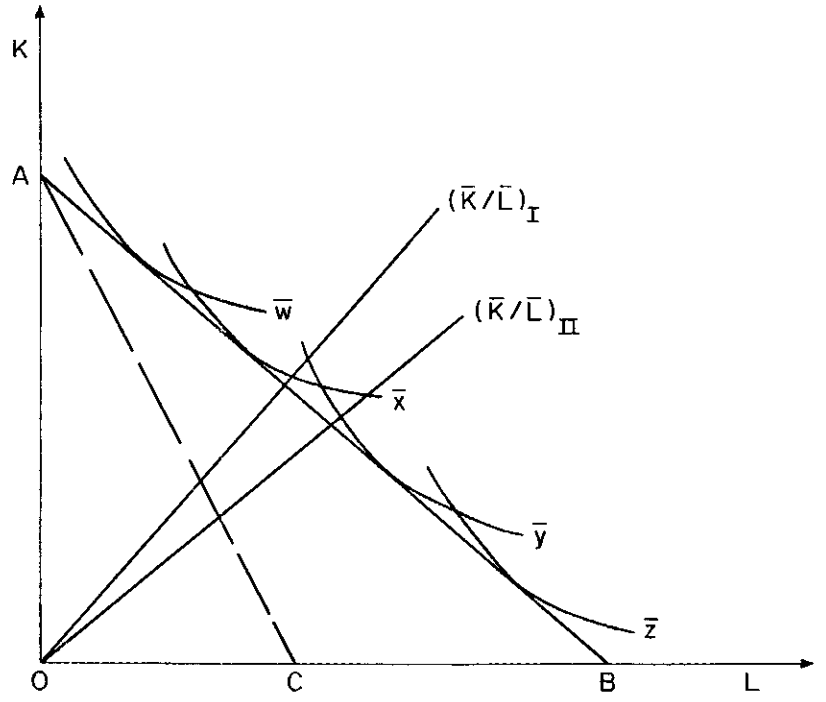

FIG. 1

where the commodity price ratio which obtains in free-trade equilibrium is one where $\bar{w}$, $\bar{x}$, $\bar{y}$, and $\bar{z}$ exchange for one another on the market. This commodity price ratio also leads, in figure 1, to factor price ratio AB. Note that, since *all* four commodities are viable at these prices in *each* country on the assumption of internationally identical production functions, we clearly have multiple production equilibria possible in each country. Readers of Tinbergen (1949) and Meade (1950) are quite familiar with this. Assume, then, that Country I produces w and y whereas Country II produces x and z; assuming further that each country consumes some of each commodity (an assumption which is consistent with identical, homothetic tastes across countries), we have Country I exporting w and y while Country II exports x and z. But commodities are ordered in a chain of K/L ratios such that $K_w/L_w > K_x/L_x > K_y/L_y > K_z/L_z$, and the Jones proposition would imply that, since $(\bar{K}/\bar{L})_I > (\bar{K}/\bar{L})_{II}$, with homothetic tastes, *all* of I's exports would be K-intensive in relation to all of I's imports. But this is *not* so. In short, the "comparative advantage" chain can be crisscrossed by the actual trade pattern in the Heckscher-Ohlin model. Q.E.D.

Why did Jones arrive at this erroneous proposition? A close look at his

paper reveals that it was an incorrect inference from his *correct* proposition that, in a Heckscher-Ohlin model with two countries (1 and 2) and two factors (capital and labor), many commodities (x, y, z) ordered by K/L intensities, and production functions identical across countries and characterized by factor-intensity nonreversals, "regardless of demand conditions in the two countries, if commodity-x is cheaper relative to y in country 1 than in country 2, so also must y be cheaper relative to z in country 1" (1957, p. 83).

This is, of course, a valid proposition. For, if x is K-intensive in relation to y and y in turn in relation to z, commodity x can be cheaper relative to y in country 1 than in country 2 only if country 1 has cheaper capital relative to labor than country 2. In figure 1, for example, country 1 would have factor price ratio AC and country 2 would have factor price ratio AB. But, in that event, y which is K-intensive in relation to z would also be relatively cheaper than z in country 1 than in country 2. Geometrically, this can be seen by noting that the isoquants which would be tangential to AC, which would then define the corresponding equilibrium commodity price ratios, have to be closer to the origin the less K-intensive the commodity.

But this proposition, as we have seen, is compatible with the invalidity of the Heckscher-Ohlin theorem. The error came through Jones's incorrectly inferring from this "chain" proposition that the chain would never be crisscrossed in free trade—an inference by analogy with the theory of Ricardian chains, where such an inference *is* correct.

Lest the reader infer that therefore the Heckscher-Ohlin pattern of trade is extremely unlikely to occur in the multi-commodity case, let me stress that the Heckscher-Ohlin pattern of trade *can* arise even under factor price equalization and *must* arise if factor prices are not equalized.

Furthermore, as Jones has pointed out to me, if we introduce transport costs, the theorem can be revalidated. With transport costs on *every* commodity, commodity prices would no longer be equal across countries in trade, and therefore factor prices also could not be equalized via commodity price equalization. Thus, while a commodity in the middle of a chain of exportables may be priced out of the export market into being a nontraded good by high transportation costs, it is impossible for it to be turned into an imported good. Hence, the trade pattern cannot register a crisscrossing of the chain; each exportable must thus have a higher K/L ratio than each importable, in the K-abundant country (with identical homothetic tastes across countries).

Note, finally, that, as Samuelson has commented to me, we always have the truism (in the case of internationally identical, homothetic tastes) that the country with the lower endowment of one of the factors cannot have *all* its exports more intensive in that factor than *all* its imports. This is because, with homothetic and identical tastes, each country's average

(identical) consumption K/L ratio must lie between the K/L endowment ratios of each country; hence the K-abundant country must be exporting a higher K/L *bundle* of exports than its bundle of imports,[2] which implies that *all* its exports cannot be L-intensive in relation to *all* of its imports.

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[2] This proposition was stated earlier by Melvin (1968, 1971) and is also discussed in Vanek (1968).