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# THE HELICITY OF THE FREE ELECTROMAGNETIC FIELD AND ITS PHYSICAL MEANING

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# 1 Introduction

The difficulties associated with the probability density for photons were recognized by Landau and Peierls as early as 1930 [1]. However, density obtained by them was not positive definite and, thus, had no physical meaning. Later, Zeldovich [2] obtained the following bilinear representation for a number of photons:

$$N = \frac{1}{16\pi^3 \hbar c} \int \frac{\vec{E}(\vec{x})\vec{E}(\vec{y}) + \vec{H}(\vec{x})\vec{H}(\vec{y})}{|\vec{x} - \vec{y}|^2} d^3x d^3y \tag{1.1}$$

Here $\vec{E}$ and $\vec{H}$ are electromagnetic field strengths. The relation of photon nonlocalizability to other fundamental problems of modern physics has been discussed recently in ref. [3] which, in fact, initiated this investigation. For the static magnetic field there is a topological invariant called helicity which is defined as [4-6]

$$S = \int \vec{A} \cdot \vec{H} dV \tag{1.2}$$

Here $\vec{A}$ is the usual magnetic vector potential ($\vec{H} = curl\vec{A}$). The main advantage of $S$ is that it is invariant under the gauge transformation $\vec{A} \to \vec{A} + grad\, f$ under the condition that either $f\vec{H}$ decreases sufficiently fast at infinity or the normal component of $H$ vanishes at some boundary inside which $H$ and $A$ are confined. For the static magnetic field $S$ characterizes to what extent magnetic lines are coupled with each other. It has meaning even for the single magnetic line. In this case it estimates the screwness of this line. The relativistic generalization of helicity was introduced in ref. [7]. It is defined as an integral over the zeroth component of the vector

$$j^\mu = \tilde{F}^{\mu\nu} \cdot A_\nu, \quad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta},\ F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \tag{1.3}$$

where $\epsilon^{\alpha\beta\gamma\delta}$ is a completly antisymmetric fourth rank tensor with $\epsilon^{0123} = 1$. The components of the 4-current density

$$j^0 = \vec{A} \cdot \vec{H}, \quad \vec{j} = \vec{H} \cdot \Phi + \vec{A} \times \vec{E} \tag{1.4}$$

satisfy the equation

$$\partial_\mu j^\mu = -2\vec{E} \cdot \vec{H} \tag{1.5}$$

It follows from this that $j_\mu$ is conserved only if $\vec{E} \cdot \vec{H} = 0$. This means that in a relativistic case helicity (1.2) has physical meaning only for the very special electromagnetic fields. Another approach adopted in refs. [8,9] was grounded on the observation made by Stratton [10] that for the free electromagnetic field the standard representation of field strengths

$$\vec{E} = -grad\Phi - \dot{\vec{A}}/c, \quad \vec{H} = curl A \tag{1.6}$$

coexists with the following one

$$\vec{E} = -curl\vec{V}, \quad \vec{H} = -grad\Psi - \dot{\vec{V}}/c \tag{1.7}$$

Obviously, $\vec{E}$ and $\vec{H}$ may be united into the 2-nd rank tensor

$$\tilde{F}^{0i} = -H_i, \quad \tilde{F}^{ij} = \epsilon_{ijk} \cdot E_k, \quad \tilde{F}_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \quad V_\mu = (\Psi, -\vec{V})$$

The following 4-current can be constructed from $F^{\mu\nu}$ and $V^\mu$

$$\tilde{j}^\mu = F^{\mu\nu} \cdot V_\nu$$

Or, explicitly,

$$\tilde{j}^0 = \vec{E} \cdot \vec{V}, \quad \vec{\tilde{j}} = \vec{E} \cdot \Psi - \vec{H} \times \vec{V}$$

It is easy to check that $\partial_\mu \tilde{j}^\mu = - - 2\vec{E} \cdot \vec{H}$. It follows that 4-current $J^\mu = j^\mu - \tilde{j}^\mu$ is conserved: $\partial_\mu J^\mu = 0$. The explicit components of $J^\mu$ are

$$J^0 = \vec{H} \cdot \vec{A} - \vec{E} \cdot \vec{V}, \quad \vec{J} = \vec{H} \cdot \Phi + \vec{E} \times \vec{A} - \vec{E} \cdot \Psi + \vec{H} \times \vec{V} \tag{1.8}$$

Some words should be added concerning the alternative representation of the electromagnetic strengths (1.7). In ref. [11] the nontrivial configurations of electric dipoles were found which are described adequately by the electric vector potential rather than electric scalar one. Further, a quite different functional form of the Fourier transforms of $\vec{A}$ and $\vec{V}$ (see Eq.(2.2)) suggest that they describe different degrees of freedom of the electromagnetic field.

# 2 Relativistic helicity and its physical meaning

The conservation of $J^\mu$ suggests that the integral

$$S = \int (\vec{H} \cdot \vec{A} - \vec{E} \cdot \vec{V}) \quad d^3x \tag{2.1}$$

does not depend on time. It is the relativistic generalization of helicity for an arbitrary free electromagnetic field. For this field only transversal components of $\vec{E}$ and $\vec{H}$ have physical meaning (however, sometimes (see, e.g., [12,13]) the physical sense is ascribed to the longitudinal component of electromagnetic field). The longitudinal component is most easily eliminated if the Coulomb gauge is used

$$\Psi = \Phi = 0, \quad div\vec{A} = div\vec{V} = 0$$

To clarify the physical meaning of S, we perform the Fourier expansion of field strengths and potentials according to the following rule

$$G(\vec{x}) = \int G(\vec{k}) \exp(i\vec{k} \cdot \vec{x}) d^3k$$

The requirement of $\vec{E}, \vec{H}, \vec{A}, \vec{V}$ to be real leads to the following representation of the Fourier components [14]

$$\vec{E}(\vec{k}) = \frac{\sqrt{\omega}}{2\pi}(\vec{f}(\vec{k}) + \vec{f}^*(-\vec{k})), \quad \vec{H}(\vec{k}) = \frac{c}{2\pi\sqrt{\omega}} \vec{k} \times (\vec{f}(\vec{k}) - \vec{f}^*(-\vec{k}))$$

$$\vec{A}(\vec{k}) = \frac{-ic}{2\pi\sqrt{\omega}}(\vec{f}(\vec{k}) - \vec{f}^*(-\vec{k})), \quad \vec{V}(\vec{k}) = \frac{-c^2 i}{2\pi\omega^{3/2}}\vec{k} \times (\vec{f}(\vec{k}) + \vec{f}^*(-\vec{k})) \qquad (2.2)$$

Here $\omega = c|\vec{k}|$. The function $\vec{f}(\vec{k})$ being the photon wave function in the momentum space satisfies the equations

$$i\partial_t \vec{f} = \omega \vec{f}, \quad \vec{k} \cdot \vec{f} = 0$$

(in fact, $\vec{f} = \exp(-i\omega t) \cdot \vec{f}_0$ where $\vec{f}_0$ does not depend on time). Using (2.2) we evaluate the energy of the electromagnetic field:

$$\mathcal{E} = \int \frac{\vec{E}^2 + \vec{H}^2}{8\pi} d^3x = \int \omega \vec{f}^*(k)\vec{f}(k) d^3k$$

It follows from this that $\rho(\vec{k}) = \vec{f}^*(\vec{k})\vec{f}(\vec{k})/\hbar$ is the photons density in the momentum space. The total number of photons is given by

$$N = \int \rho(\vec{k}) d^3k \qquad (2.3)$$

The same expression is obtained if we substitute the Fourier expansions of $\vec{E}$ and $\vec{H}$ into (1.1) and perform integration over the spatial variables. To clarify the physical meaning of $S$ we change $\vec{E}, \vec{H}, \vec{A}, \vec{V}$ in (2.1) by their Fourier expansions and get

$$S = -8\pi c^2 i \int \vec{k} \cdot (\vec{f}^*(\vec{k}) \times \vec{f}(\vec{k})) \frac{d^3k}{\omega} \qquad (2.4)$$

Now we represent $\vec{f}$ in the form [14]

$$\vec{f} = \vec{e}_R f_R + \vec{e}_L f_L \qquad (2.5)$$

Here $\vec{e}_R$ and $\vec{e}_L$ are the unit vectors for the right and left circularly polarized photons ($\vec{e}_R^* \cdot \vec{e}_R = \vec{e}_L^* \cdot \vec{e}_L = 1, \quad \vec{e}_R^* \times \vec{e}_R = i\vec{e}_K, \quad \vec{e}_L^* \times \vec{e}_L = -i\vec{e}_K, \quad \vec{e}_K = \vec{k}/k$).

Substitution of (2.5) into (2.4) gives

$$S = 8\pi c \int (|\vec{f}_R|^2 - |\vec{f}_L|^2) d^3k \qquad (2.6)$$

It is easy to check that the photon density in the momentum space is given by $\rho(\vec{k}) = (|\vec{f}_R|^2 + |\vec{f}_L|^2)/\hbar$. This means that $S/8\pi c\hbar$ coincides with the difference of the right and left circularly polarized photons. Hence, $(|\vec{f}_R|^2 - |\vec{f}_L|^2)/\hbar$ and $(\vec{H}\vec{A} - \vec{E}\vec{V})/8\pi c\hbar$ are the densities corresponding to this difference in the momentum and coordinate space, resp. Now we express $\vec{f}(\vec{k})$ in Eq.(2.3) through its Fourier transform ($\vec{f}(\vec{k}) = \frac{1}{(2\pi)^3}\int \vec{f}(\vec{x}) \exp(-i\vec{k} \cdot \vec{x}) d^3x$). Then, $N = \int \rho(\vec{x}) d^3x$, where $\rho(\vec{x}) = |\vec{f}(\vec{x})|^2/\hbar$. Since $\rho(\vec{x})$ is positive definite it seems at first that $\rho(\vec{x})$ may be viewed as a candidate for the photon density. However, the observables of the electromagnetic field are the

field strengths $\vec{E}$ and $\vec{H}$. The vector function $\vec{f}(\vec{x})$ is a highly nonlocal function of them. To see this, we express $\vec{f}(\vec{k})$ through the Fourier components of $\vec{E}$ and $\vec{H}$

$$\vec{f}(\vec{k}) = \frac{2\pi}{\sqrt{\omega}}(\vec{E}(\vec{k}) + \vec{e}_K \times \vec{H}(\vec{k}))$$

It follows from this that the Fourier transform of $\vec{f}(\vec{k})$, that is, $\vec{f}(\vec{x})$ depends on the values of $\vec{E}$ and $\vec{H}$ in the whole space, not at the point $x$ only. This is generally considered as a serious drawback [14].

For the free electromagnetic field Lipkin [15] has obtained the conserved 3-rd rank tensor (the so-called zilch) composed of field strengths and its derivatives. It was traceless and symmetric with respect to the first two indices. Later, Ragusa has discovered [16] the antisymmetric counterpart of the zilch tensor. He explicitly showed [17] that its components in the momentum space are reduced to the integral over the difference of right and left circularly polarized photons multiplied by the first or second power of $\omega$. Because of this the physical meaning of the zilch-type tensors is rather obscure. On the other hand, the helicity $S$ given by (2.1) reduces to the difference of right and left photons. It measures the screwness of the electromagnetic field and may be considered as a missing link in the list of the Lipkin-Ragusa invariants. Obviously, the helicity $S$ equals zero for the plane linearly polarized electromagnetic wave. The following important theorem concerning massless particles was formulated in ref. [18].

Theorem 1. *A theory that allows the construction of a Lorentz-covariant conserved four-current $J_\mu$ cannot contain massless particles of spin $J > 1/2$ with non-vanishing values of the conserved charge $\int J^0 d^3x$.*

In our case, for the particles of the spin 1 (photons) there are the conserved 4-vector given by (1.8) and conserved charge given by (2.1). At first glance, this disagrees with Theorem 1. However, the proof of this theorem given in [18] contains the implicit assumption that the wave function of a massless particle with definite values of 4-momentum and *helicity* is an eigenfunction of rotation (around 3-momentum) in an arbitrary Lorentz reference frame (written in italics *helicity* means the projection of the spin onto the direction of motion in contradistinction to helicity defined by (2.1)). This in turn means that the wave function of a particle is uniquely (up to a nonessential phase factor) defined by its 4-momentum and *helicity* and, thus, is gauge invariant. In quantum electrodynamics there is no complete agreement as to what one means by the wave function of the photon. Some authors (see,e.g., [19]) mean by it the 4-potential $A_\mu$, while others (see,e.g., [20]) prefer to deal with stress tensor $F_{\mu\nu}$. Even in different editions of the same book ([14],[21]) various definitions are sometimes adopted. The gauge-invariant definition of photon wave function adopted in ref. [18] corresponds to the the second definition, i.e., to $F_{\mu\nu}$. In this basis (i.e., in $F_{\mu\nu}$) all the matrix elements of the 4-vector (1.8) are equal to zero. This is not the case for the basis associated with the 4-potential $A_\mu$.

We briefly summarize the content of this section: the quantity (2.1) is found which generalizes helicity notion for the arbitrary free electromagnetic field. It coincides with the difference of the right and left photons composing this field. The density corresponding to this generalized helicity taken at some space point $\vec{x}$ is expressed

through the values of electromagnetic strengths and potentials taken at the same point.

# 3 Gauge-invariant representation for the energy of weak gravitational field

In the mentioned above ref. [18] another theorem was proved as well.

Theorem 2. *A theory that allows the construction of a conserved Lorentz-covariant energy-momentum* $\Theta^{\mu\nu}$ *for which* $\int \Theta^{0\nu}\, d^3x$ *is the energy-momentum four-vector cannot contain massless particles of spin* $j > 1$.

This means, in particular, that a gauge-invariant density of the field energy does not exist for the Lorentz-covariant field of the spin 2. It was shown in ref. [22] that Einsteinian gravitational equations in the weak field limit and in the absence of masses coincide with the equations describing the massless spin 2 field. Then, the above Theorem reflects the well-known difficulty with the energy density problem in the General Relativity [23]. In this section, we find the gauge-invariant expression for the energy of the weak gravitational field consisting of gravitational waves. However, this expression reduces to a double integral similar to (1.1). Following ref. [22], we introduce the gauge-invariant (in the sense defined below, see Eq.(3.7)) quantities

$$E_{ij} = R_{4j4i} = \frac{1}{4}\epsilon_{jkl}\epsilon_{imn}R_{klmn}, \quad H_{ij} = \frac{i}{4}\epsilon_{imn}R_{4jmn} = \frac{i}{2}\epsilon_{imn}R_{mn4j} \tag{3.1}$$

Here $R_{\mu\nu\rho\sigma}$ is the Riemann tensor. According to [22], the symmetric traceless tensors $E_{ij}$ and $H_{ij}$ in the weak field limit satisfy equations strongly resembling the Maxwellian ones

$$\epsilon_{ikl}\partial_k E_{lj} + \frac{1}{c}\partial_t H_{ij} = 0, \quad \partial_i H_{ij} = 0$$

$$\epsilon_{ikl}\partial_k H_{lj} - \frac{1}{c}\partial_t E_{ij} = 0, \quad \partial_i E_{ij} = 0 \tag{3.2}$$

For the weak gravitational field ($g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$) the standard equations connecting the curvature tensor with Christoffel symbols have the form [24]

$$R_{\mu\nu\rho\sigma} = \partial_\rho \Gamma_{\mu\nu\sigma} - \partial_\sigma \Gamma_{\mu\nu\rho}$$

Here $\Gamma_{\mu\nu\sigma} = \frac{1}{2}(\partial_\nu h_{\mu\sigma} + \partial_\sigma h_{\mu\nu} - \partial_\mu h_{\nu\sigma})$. Having taken into account (3.1) these Eqs. can be presented in the form similar to the electrodynamic ones (1.6) and (1.7):

$$H_{ij} = \epsilon_{imn}\partial_m A_{nj}, \quad E_{ij} = -\frac{1}{c}\partial_t A_{ij} - \partial_i A_{0j} \tag{3.3}$$

$$E_{ij} = -\epsilon_{imn}\partial_m V_{nj}, \quad H_{ij} = -\frac{1}{c}\partial_t V_{ij} - \partial_i V_{0j} \tag{3.4}$$

Here

$$A_{ij} = i\Gamma_{4ji}, \quad A_{0j} = \Gamma_{4j4}, \quad V_{ij} = -\frac{1}{2}\epsilon_{jmn}\Gamma_{mni}, \quad V_{0j} = \frac{i}{2}\epsilon_{imn}\Gamma_{mn4} \tag{3.5}$$

Using a gauge $A_{0j} = 0, V_{0j} = 0$ we find the following expressions for the Fourier transforms of $E_{ij}(\vec{x}), H_{ij}(\vec{x}), A_{ij}(\vec{x}), V_{ij}(\vec{x})$ strongly resembling electromagnetic ones (2.2):

$$E_{ij}(\vec{k}) = \frac{\sqrt{G\omega^3}}{\pi^2 c^2\sqrt{2}}(f_{ij}(\vec{k}) + f_{ij}{}^*(-\vec{k})), \quad H_{ij}(\vec{k}) = \frac{\sqrt{G\omega}}{\pi^2 c\sqrt{2}}\epsilon_{imn}k_m(f_{nj}(\vec{k}) - f_{nj}{}^*(-\vec{k})) \tag{3.6}$$

$$A_{ij}(\vec{k}) = -i\frac{\sqrt{G\omega}}{\pi^2 c\sqrt{2}}(f_{ij}(\vec{k}) - f_{ij}{}^*(-\vec{k})), \quad V_{ij}(\vec{k}) = -i\frac{\sqrt{G}}{\pi^2\sqrt{2\omega}}\epsilon_{imn}k_m(f_{nj}(\vec{k}) + f_{nj}{}^*(-\vec{k}))$$

Here $G$ is a Newtonian gravitational constant. Symmetric traceless tensors $f_{ij}(k)$ satisfy the equations

$$i\partial_t f_{ij} = \omega f_{ij}, \quad k_i f_{ij} = 0$$

The Hilbert condition $\partial_\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h_{\rho\rho} = 0$ usually imposed on $h_{\mu\nu}$ [24] leads to the following equation on $h_{\mu\nu}$:

$$\Box h_{\mu\nu} = 0$$

The gauge transformation

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu v_\nu + \partial_\nu v_\mu, \quad \Box v_\mu = 0 \tag{3.7}$$

does not change the tensor $R_{\mu\nu\rho\sigma}$, conserves the Hilbert condition and does not change the total energy and momentum of the weak gravitational field (as it adds complete divergence to the energy-momentum pseudotensor). The gauge transformation (3.6) may be used to obtain $h_{\mu\nu}$ satisfying the following transversal gauge conditions [25]

$$h_{44} = 0, \quad h_{4i} = h_{i4} = 0, \quad h_{ii} = 0, \quad \partial_k h_{ik} = 0$$

By taking into account of (3.6) this gets

$$A_{ij} = -\frac{1}{2c}\partial_t h_{ij}, \quad A_{0j} = 0, \quad V_{ij} = \frac{1}{2}\epsilon_{imn}\partial_m h_{nj}, \quad V_{0j} = 0$$

Using the standard definition of the energy-momentum pseudotensor [24], one easily finds the following expression for the energy of the weak gravitational field

$$\mathcal{E} = \frac{c^4}{16\pi G}\int (A_{ij}(\vec{x})^2 + V_{ij}(\vec{x})^2)d^3x \tag{3.8}$$

On the other hand, this energy may be presented as a double integral over the bilocal gauge-invariant density

$$\mathcal{E} = \frac{c^4}{64\pi^2 G}\int \frac{E_{ij}(\vec{x})E_{ij}(\vec{y}) + H_{ij}(\vec{x})H_{ij}(\vec{y})}{|\vec{x} - \vec{y}|}d^3x d^3y \tag{3.9}$$

Expressing in (3.8) and (3.9) $A_{ij}, V_{ij}, E_{ij}$ and $H_{ij}$ through their Fourier transforms and performing the integration over the spatial variables we arrive at

$$\mathcal{E} = \int |f_{ij}(\vec{k})|^2\omega d^3k \tag{3.10}$$

A posteriori the distinction of $|\vec{x}-\vec{y}|$ degrees in (1.1) and (3.9) may be realized by the dimensional considerations. In (3.9) the fields $E_i$ and $H_i$ have dimensions $[L]^{-2}$, the integral has dimension $[L]$ and with account of the factor standing in front of the integral one obtains the dimension of energy. In (1.1) the electromagnetic strengths $\vec{E}$ and $\vec{H}$ have dimensions $[e]/[L]^2$($e$ is a charge), the integral has dimension $[e]^2$ and the whole expression (1.1) is dimensionless, as it should be. The following fact remains unclear to us. In the case of a weak gravitational field there are two equivalent expressions for energy corresponding to the local (3.8) and bilocal (3.9) densities. In the electromagnetic case, there is only bilocal density for the sum (1.1) of right and left photons and local density for their difference (2.1).

# 4 Conclusion

We have proved that for the free electromagnetic field the conserved gauge-noninvariant 4-pseudovector can be constructed. The integral over its zeroth component is a gauge-invariant, independent of time quantity that coincides with the difference of the right and left photons composing the field. This conserved integral is a relativistic generalization of the helicity, well-known topological invariant widely used for the description of the static magnetic field. The existence of such a quantity does not contradict the well-known theorem prohibiting the existence of the conserved gauge-invariant 4-current composed only of the electromagnetic field strengths (likewise there is no gauge invariant density of the gravitational field density). For the weak gravitational field reducing to the gravitational waves it is possible to introduce the quantities strongly resembling the electromagnetic potentials. The energy density of weak gravitational field may be expressed through quadratic combinations of these potentials. On the other hand, the energy of the gravitational field may be represented as a double integral over the bilinear gauge-invariant density. The existence of these two representations does not contradict the theorem mentioned above. To the end, we note the similarity of the electrodynamic Eqs. (1.6),(1.7),(2.2) to the gravitational ones (3.2),(3.4),(3.6). Probably, this will be a balm for those who believes into the vector gravitational theory ( its nice exposition may be found in the book [26]). Yet, this similarity is limited by the weak gravitational fields.

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Спиральность свободного электромагнитного поля и ее физический смысл

Проанализировано понятие спиральности для свободного электромагнитного поля. Введена обобщенная спиральность, являющаяся сохраняющейся величиной,равной разности чисел правых и левых фотонов, составляющих электромагнитное поле. Найденная спиральность является естественным дополнением к набору «цильх»-инвариантов, построенных Липкиным и Рагузой. Для энергии слабого гравитационного поля найдено калибровочно-инвариантное выражение, напоминающее известное билинейное выражение для полного числа фотонов электромагнитного поля.

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The Helicity of the Free Electromagnetic Field and Its Physical Meaning

The notion of helicity for the free electromagnetic field is analyzed. The generalized helicity is introduced which is a conserved quantity coinciding with the difference of the right and left photons composing the electromagnetic field. It seems that it completes the list of the zilch-type invariants found by Lipkin and Ragusa. The gauge invariant expression for the energy of the free gravitational field is obtained which strongly resembles the well-known bilinear expression for the total number of photons composing the electromagnetic field.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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