The History of the Undergraduate Program in Mathematics in the United States

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Abstract

This article describes the history of the mathematics major and, more generally, collegiate mathematics, in the United States. Interestingly, the Mathematical Association of America was organized about 100 years ago around the same time that academic majors came into existence.

The undergraduate program in mathematics in America has had a punctuated evolution. The Mathematical Association of American was organized 100 years ago at the end of a period of dramatic rethinking of American education at all levels, one product of which was the introduction of academic majors. The mathematics major was static in its first 40 years, followed by great changes from 1955 to 1975, and then a period of relative stability to the present.

The educational concerns of the Mathematical Association of America also changed in the 1950's. Initially its educational recommendations focused on preparing high school students for college mathematics, but starting in 1958, the MAA's Committee on the Undergraduate Program in Mathematics (CUPM) played the leading role in promoting a major reworking of the mathematics major.

1 Early History.

Mathematics actually had a greater role in the college curriculum in America in the 1700's and 1800's when it was studied as a classical training of the mind rather than as the language of science and engineering. The first colleges in the colonies were modeled on English colleges whose curriculum was largely prescribed and focused on Latin, Greek, Hebrew, and mathematics. The education was dry and involved mostly rote learning. Colleges' original purpose was to train ministers. In time, colleges came to be populated mostly with children of the well-to-do. The faculty had attended college, but nothing more. College mathematics initially was Euclidean geometry and some algebra. In the 1700's, there were few high schools and students entered college at age 15 or 16 with a couple of years of education, typically from tutors, beyond primary school. (For readers wanting more information about this early history, see Cajori [7], Smith and Ginsberg [33], the first chapter of Hofstadter [20], and the beginning of Duren [14]. These references were the primary sources of this section. Also see Cohen [8] and Hofstadter [19].)

In the nineteenth century, colleges welcomed middle-class students, and the growing number of private academies and public 'Latin' high schools in larger cities gave college-preparatory instruction that caused the age of freshman to rise to today's 18 years. The admission requirements in mathematics also rose over the century to Euclidean geometry and a year of algebra. Until the late 1800's, school teachers rarely had a college degree instead going to normal schools, which were high schools that specialized in teacher training.

As students arrived knowing more mathematics, the level of mathematics taught in colleges rose slowly through the 19th century. In the early 1800's, calculus had not yet entered the regular U.S. college curriculum, although a few institutions, e.g., Yale and West Point, offered classes about 'fluxions.' In the second half of the 1800's, the typical college curriculum in mathematics was: freshman year –algebra and geometry; sophomore year –more algebra and trigonometry. Technically oriented students continued with a junior year of analytic geometry; possibly calculus started then or else in the senior year. Well prepared students at better colleges took calculus in the sophomore year after a year of college algebra and trigonometry.

In the late 1800's, mathematics offerings at the University of Michigan (a university with strong academic standards then as now) were: algebra (several courses), determinants, Euclidean geometry, trigonometry, axiomatic geometry, analytic geometry, calculus, differential equations, calculus of variations, quaternions and elliptic functions. At this time, Harvard offered a similar curriculum supplemented by additional courses in theory of functions, higher plane curves, mechanics, and Fourier series.

Before 1850, all engineers were trained at the U. S. Military Academy or at Rensselaer Polytechnic Institute, but by the second half of the 19th century there was a growing demand for technically trained college graduates who required a more practical training than the classical curriculum offered. Scientific schools offering B.S. degrees were established within Harvard, Yale, and other leading colleges. The 1862 Morrill Act created 'land grant' public universities whose primary purpose was to "teach such branches of learning as are related to agriculture and the mechanic art" (U.S. Code [37]). However, the level of mathematics was not high in B.S. programs; engineers used mostly finite differences instead of calculus.

Even when Ph.D. programs were instituted —the first mathematics Ph.D. was awarded by Yale in 1862 —almost all faculty teaching in these programs had only a B.A. or possibly an M.A. degree. Mathematics Ph.D. production increased from 1 or 2 a year in the 1860's and 1870's to 10 a year by 1900 (see Richardson [30] for more early Ph.D. data).

A period of rethinking American education at all levels. The end of the nineteenth century and start of the twentieth century were a time of dramatic change in education in the U.S. The fraction of population going to college had started declining in 1840. In reaction to students' unhappiness with the classical curriculum, and seeking to have a more open 'democratic' college experience, Harvard's president Charles Eliot changed his college to an all-elective curriculum in the middle 1880's. The model for this change was the all-elective curriculum in German universities. By the early 1900's, most U.S. institutions had changed to an elective curriculum. The program of study tended to be very practical, in keeping with the spirit of the country. The reaction to these changes was dramatic: college enrollments soared from 150,000 in 1890 to 350,000 in 1910 (p. 31, Hofstadter [20]). This increase was also fueled by the growth of public high schools, even though they still were serving fewer than 10% of eligible students in 1910 (U.S. Census [36]).

The elective system led to a dramatic decline in the study of mathematics. What saved collegiate mathematics from being totally decimated was the growing need for engineers and technically oriented professionals in industry and agriculture. Thus, it came to be that while mathematics was previously associated with a classical education, henceforth it became associated with science and engineering. However, mathematicians also continued to hold on to the classical view that the study of mathematics is valuable as a general training of the mind.

The decline in mathematics instruction then came to high schools. While the college preparation curriculum standards had long been set by the colleges, early in the 20th century, K-12 education came under the control of education specialists, led by John Dewey, who argued that the primary purpose of school education was social development and personal fulfillment. An influential 1918 National Education Association report [26] had 14 subject area subcommittees, but mathematics was not one of them. Mathematics became an elective subject in many high schools. (It is worth noting that in Europe at this time, calculus was being made a mandatory subject in college-preparatory high schools.) The impact of this movement on college mathematics instruction was a substantial increase in precollege (remedial) mathematics courses in arithmetic and beginning algebra.

2 The First 40 Years of the MAA, 1915-1954.

The first MAA president, E.R. Hedrick, laid out a platform for the MAA and its Mathematics Monthly (p. 28, Hedrick [17]): "the great majority of the work fostered by the Association will be on questions directly affecting collegiate courses in mathematics." This included articles and talks on ways to improve instruction in introductory as well as advanced courses. Mathematics preparation for college was also a concern of his, given the situation just described. A broader concern was that the elective system in colleges was being abused with many students graduating with little more than a collection of freshmen-level courses.

This depressing situation gave rise to a counter-movement in higher education. The change was again led by Harvard, whose president Lawrence Lowell instituted in 1910 a system of required academic majors. A few years later, Woodrow Wilson, the president of Princeton, expanded on Lowell's efforts by adding a core curriculum, what we now call general education requirements. The academic major and general education requirements were soon widely adopted, although the number of courses required in a major was initially modest. (Note that England and many European countries had by this time moved to a system where students applied to study an academic discipline upon admission to college and virtually all their studies were related to that discipline.)

The social sciences and humanities, which had the majority of college majors, led an effort to update the old classical curriculum with what became known as a liberal arts curriculum. This curriculum was driven by the mission of training a mind to think and explore knowledge as a foundation for future on-the-job learning. Mathematics was frequently a required subject at liberal arts colleges.

At this time, there were also calls to raise the standards for freshman instruction. A leading American mathematician, E. B. Wilson [41], called for requiring all freshmen to take a yearlong mathematics course that would be half calculus and half what he called 'choice and chance' –much of what later came to be called finite mathematics. Some selective institutions led by Harvard soon made calculus the second semester of freshman mathematics, with preparatory instruction available.

Nonetheless, for most college students, there were no longer any mathematics requirements. For general education purposes, mathematics was grouped with the natural sciences, so that science courses could be taken in place of any mathematics. When students needed to take college-level mathematics for their major, they often first needed extensive precollege work. Students who had a good high school mathematics preparation usually started college mathematics with freshmen study of college algebra and trigonometry followed by analytic geometry and calculus in the sophomore year.

The educational reforms led by Dewey also called for extending mandatory schooling through 12th grade (motivated as much by keeping students out of grueling factories as by greater learning). This recommendation was widely implemented after World War I. From 1910-19 to 1920-29 college graduates grew by over 150% (for such data see U.S. Census [36]) to meet the demand for high school teachers (high school graduates grew even faster) as well as for the growing number of engineers and other professionals required in the commerce, infrastructure and technology of the twentieth century world. Colleges, eager for these larger enrollments, had to lower standards to accommodate many of the applicants. Precollege mathematics enrollments grew even more.

The overall enrollment growth was accompanied by an increased production of mathematics Ph.D.'s —from about 25 a year in the 1910's to about 50 a year by 1925 (Richardson [30]). Still, typical tenured college/university mathematics faculty had at most a Master's degree, since most mathematics instruction was at a low level. However, the Mathematical Monthly had several articles in the first third of the 20th century discussing how to ensure that new mathematics Ph.D.'s would be good teachers; for example, see Slaught [32]. These articles highlight the fact that, despite the advent of doctoral programs in many university mathematics departments, teaching was still viewed as the primary duty of university mathematics faculty. Beginning mathematics faculty at leading doctoral mathematics departments were typically teaching 12 to 15 hours a week up through the 1940's.

While the early MAA educational activities focused on secondary school preparation in mathematics, there was a 1928 MAA report (MAA [22]) addressing complaints about the first two years of college mathematics; also see Schaaf [31]. It acknowledged calls to offer a survey course of mathematical ideas with historical aspects for non-technical students. The report made recommendations for enrichment readings to address these concerns, but in the end it defended the mental discipline developed by traditional drill and problem-solving courses.

Up until World War II, a typical student who chose to major in mathematics had in mind either becoming a high school teacher or an actuary. This was a pragmatic career-oriented major, although professors also lectured about general theory and the beauty of mathematics. It should be noted that actuary work was an offshoot of broader statistical concerns. Measuring and counting things had interested business-minded Americans from the republic's founding. For example, the American Statistical Association was founded in 1839, half a century before the American Mathematical Society.

Nonetheless, despite the practical values of American society, there were always a small number of college students who became fascinated with mathematics. Although the undergraduate mathematics offerings at most colleges and universities were limited in the first half of the 20th century, the higher level of mathematical knowledge among faculty with Ph.D. and M.S. degrees created opportunities for undergraduates to learn more mathematics. There were topic seminars, reading tutorials and better mathematical libraries. During the 1920's and 1930's, U.S. institutions educated a generation of mathematicians who went on became world leaders in their fields by the 1950's. More broadly, during this period American universities evolved from places that passed along knowledge to places that also created knowledge.

While the mathematics instruction did not change much in the 1930's and early 1940's, major world events during this period had a dramatic influence on higher education. Enrollments plunged during the Great Depression and some institutions closed. These hard times heightened the focus on career training in mathematics and other disciplines. During World War II, most colleges and universities either ran accelerated specialized programs for selected soldiers or educated women or else closed during the war.

Mathematics offerings at some institutions. To look more closely at the undergraduate program in mathematics in the first 40 years of the Association's existence, we present a sampling of mathematics offerings and requirements for the mathematics major at several institutions, gleaned from old college catalogs at these institutions. In 1920 (soon after the institution of academic majors) the University of Pennsylvania ([39]) required just six courses for an academic major, but most mathematics majors took more. All Pennsylvania students had to take two courses in the physical sciences, choosing among offerings in chemistry, mathematics and physics. Math majors were expected to complete at least one semester of calculus by the end of their sophomore year.

Low-level offerings: Math 1- Solid Geometry, Math 2- Plane Trigonometry,

Math 3- Intermediate Algebra, and Math 5- College Algebra.
Intermediate-level offerings: Math 6- Analytic Geometry, Math 7- Elements of Analytic Geometry and Calculus (not open to math majors), Math 8- Differential and Integral Calculus, Math 9- Advanced Plane Trigonometry and Spherical Trigonometry, Math 10- History of Mathematics, Math 11- Determinants and the Theory of Equations, Math 12- Determinants and Elimination, Math 13- Solid Analytic Geometry, Math 14- Infinite Series and Products.
Advanced offerings (not all offered annually): Math 15- Projective Geometry, Math 16- Modern Analytic Geometry, Math 18- Quaternions and Vector Methods, Math 19- Differential Equations, Math 20- Advanced Calculus, Math 21- Foundations of Geometry, Math 25- Mathematical Analysis (a deeper look at series, representations of functions and other classical topics), and Math 26- Functions of a Complex Variable.

We note that Advanced Calculus in those days covered basics of multivariable calculus, some special functions, some functions of a complex variable (up to Cauchy's theorem), and possibly a little partial differential equations; e.g., see Woods [42]).

There were limited changes over the next thirty years at Pennsylvania. By 1930, there were new courses in statistics and projective geometry, but the complex variables course was dropped. By the late 1930's, Intermediate Algebra was dropped but was reinstated in the late 1940's for returning GI's. Around 1940, a topics course was added along with a new three-semester calculus sequence specifically for engineers. In the late 1940's, a course in abstract algebra was added. Throughout this period, calculus was a sophomore-year course for mathematics, natural science and engineering majors. In contrast to the fairly static undergraduate program at Penn, the graduate mathematics offerings expanded extensively.

The University of Nebraska's mathematics offerings ([38]) over this period were very similar to Penn's. Nebraska also required six courses for an academic major. Nebraska required all students to have two majors or else a major and two minors. In mathematics, one minor had to be the sciences and the other in French, German, Latin or philosophy. The Nebraska mathematics department had a number of courses in secondary school mathematics and in insurance mathematics (for actuarial training); Penn was unusual in its paucity of such courses. In 1920, Nebraska had an advanced course in functions of a real variable that was dropped within a decade. Nebraska did not yet offer abstract algebra in 1950.

For another example, consider the University of Rochester ([40]), then a technical institution (heavily influenced by the major employer in Rochester, Eastman Kodak). It initially had just five majors: arts, chemical engineering, mechanical engineering, chemistry and economics. The limited mathematics offerings were mostly courses for engineers, up through advanced calculus. In 1920 Rochester, like Harvard, had only one course below the level of calculus, indicating highly selective admission requirements. While there was still no mathematics major in 1940, there were by then increased elective offerings for engineers and scientists, including a rudimentary abstract algebra course and a course in complex variables, a course not then offered by Pennsylvania and Nebraska. A mathematics major was finally started shortly after World War II, but few new courses were added. Note that by 1960, Rochester had a highly regarded research faculty and doctoral program in mathematics.

We turn now to some liberal arts colleges. Colgate ([9]) required two mathematics courses of all students. A preamble to the mathematics offerings in the old Colgate catalogs reflected a liberal arts focus independent of career training: the goal of mathematics instruction was "to form habits of accurate and precise expression and to develop the power of independent and logical thinking." In 1920, there were seven courses below calculus, two sophomore calculus courses and only occasional mathematics electives beyond that. The pre-calculus courses decreased over time and were eliminated by 1940. Business mathematics, investment mathematics and statistics (for actuaries) were added in the late 1920's along with more electives (not taught every year) in number theory, axiomatic geometry, projective geometry, and mathematics for teachers. One sees here that Colgate had felt the pressure to offer a career-oriented curriculum side-by-side with liberal arts offerings. By 1940, decreased enrollments forced junior-senior courses to be reclassified as seminars that were offered when there was adequate demand. By 1950, the electives were almost all gone and the pre-calculus courses were also gone.

At smaller Macalester College ([21]), mathematical offerings in 1920 consisted of four pre-calculus courses and two calculus courses. By 1940, there were four semesters of calculus with differential equations; other additions were an investment mathematics course and a seminar in higher mathematics. By 1950, there were also courses in advanced calculus, mathematical statistics, and engineering mechanics, and, unlike Colgate, there were still four pre-calculus courses.

So what sequence of courses did a strong mathematics major take in this period? An example close to home comes from my mother's college transcript of her 1934-38 mathematics studies at Northwestern University. She went to a highly regarded high school, came from a mathematical family (her father was a Northwestern mathematics professor), and was accepted for graduate study in mathematics at Radcliffe upon graduation. She took courses in College Algebra and Analytic Geometry in her freshman year. She took courses in Differential and Integral Calculus in her sophomore year. In her junior year, she took courses in Differential Equations, Advanced Calculus and Theory of Equations. In her senior year, she took Higher Geometry, Functions of a Real Variable, and Honors Seminar. Note that these first two years of study were little changed from what a well-prepared student took in the late 1800's.

Impact of World War II. Two dramatic changes in the stature of mathematics in America in the 1930's and 1940's foreshadowed major changes in the mathematics major in the 1950's. First, the immigration of great European mathematicians fleeing Nazi Germany catalyzed America's emergence as the world's leading center of mathematics research. Second, mathematicians showed how mathematics could make major contributions in a wide range of allied efforts in World War II, including aerial combat, fluid mechanics, shock fronts, code breaking, logistics, and designing the atomic bomb (for more, see Rees [29]). Significantly, these mathematicians mostly had backgrounds in pure mathematics yet proved extremely adept at solving a range of important practical problems. John Von Neumann epitomized this versatility with his seminal work in theory of games, neural networks, design of programmable computers, and numerical analysis as well as applications of partial differential equations. After the war, mathematical and statistical models became an integral part of the study of engineering, economics, and quantifiable aspects of commerce, as well as the physical sciences.

At the end of World War II, mathematics majors had risen, in the eyes of industry, to be valued almost as much as engineers. Since pure mathematicians had been responsible for the success of mathematical models, the study of core mathematics was seen as an ideal combination of the classical values of an intellectual training of the mind along with a preparation to solve a range of problems of importance to industry. This led to an increase in the proportion of college students interested in mathematics as well as the sciences and engineering. After the war there was also a substantial increase in the overall college population. The GI Bill caused much of the increase by putting any returning soldier who wanted through college. All these factors led to a growth in mathematics enrollments at all levels.

On the other hand, as noted above, the mathematics offerings still had changed little in 1950 from what they were in 1920 at most institutions. However incoming students planning on scientific and engineering majors were better prepared. Many of these freshmen were ready to study calculus, as the level of high school mathematics for such students had come by 1950 to include two years of algebra, Euclidean geometry and a 'precalculus' course of analytic geometry and trigonometry.

To accommodate the growing demand for high school teachers, regional teachers colleges expanded from training primary teachers to educating secondary school teachers, but their curriculum for prospective high school mathematics teachers was limited and typically did not extend beyond one year of calculus. At the other extreme, some leading universities were offering modern mathematics courses such as point-set topology, abstract algebra, and logic.

Most mathematics faculty still did not have Ph.D.'s, and in the early 1950's faculty at many leading research departments still saw teaching as their primary mission. Even senior administrators often taught two courses a semester. When my father A.W. Tucker was chair of the Princeton Mathematics department in the 1950's, not only did he have the same teaching load as other senior faculty but every other semester he also was the professor in charge of the freshman calculus course taken by almost all students. When I questioned him years later why he took on this huge extra obligation, he responded, "The most important thing that the Princeton Mathematics Department did was teach freshman calculus and so it was obvious that as chair I should lead that effort."

3 The Golden Age of the Mathematics Major, 1955-1974.

This was the only time of substantial change in the 100-year history of the mathematics major in America. The period was marked by a growing demand for mathematics instruction and mathematics graduates at all levels. This demand was driven by a quadrupling of college enrollments from 1950 to 1970 (U. S. Census [36]) and an increasingly quantitative world, highlighted by the growing use of computers. The launching of Sputnik in 1957, in the larger context of the Cold War competition with the Soviet Union, made mathematicians, scientists, and engineers the country's Cold War heroes. These 20 years saw the largest fraction of incoming students interested in a mathematics major —5% in the early 1960's —and largest fraction of Bachelor's degrees in mathematics —around 4% in the late 1960's (CBMS [10]). Mathematics' appeal among the brightest students was even greater. For example, in 1960 half my freshmen class at Harvard stated an intention to major in mathematics or physics. While many of these bright students ended up with successful careers in engineering, economics, and computer sciences, a number of them, e.g., several Economics Nobel Laureates, first earned a Bachelor's or even a Ph.D. in mathematics.

In 1953, the MAA formed the Committee on the Undergraduate Program (CUP), later renamed the Committee on the Undergraduate Program in Mathematics (CUPM). Its initial focus was a common first-year mathematics syllabus for all students, paralleling the common syllabus for introductory courses in the natural and social sciences. The course, called Universal Mathematics, consisted of one semester of functions and limits, the real number system, Cartesian coordinates, functions (with focus on exp(x) and log(x)), limits, and elements of derivatives and integrals, followed by one semester of mathematics of sets, logic, counting and probability. Note that this syllabus was very similar to the first-year mathematics syllabus proposed by E. B. Wilson in 1913. The second semester component was based on the expectation that newer areas of applied mathematics, such as statistics and operations research that were so useful in the war effort, would become a major part of the mathematics used in many disciplines. There was also a proposed 'technical laboratory' for engineers and physicists with more extensive work in calculus. The CUP-prepared 1954 text (MAA [23]) for the functions half of Universal Mathematics offered a highly theoretical approach on odd-numbered pages (e.g., replacing sequences by filters) along with a traditional approach on even-numbered pages. The theoretical approach was laying the foundation for a more theoretical mathematics major to prepare students for graduate study. See Duren [13] for more about the early history of CUP/CUPM.

CUP's proposal, with textbook, for an important new college course appears to have been without precedent. Professional academic organizations make reports about high school preparation but normally stay clear of telling their members what to teach, much less proposing a major organization of the introductory course in the discipline. However, the Association for Computing

Machinery followed the CUPM example in 1968 with recommendations for a curriculum to define the newly emerging computer science major.

Universal Mathematics was doomed, no matter what its reception might have been by mathematicians, by the decision in the mid-1950's of physicists to use calculus in their freshman physics course for engineers. These physics courses started with supplementary workshops on basic calculus formulas. Mathematics departments, wary of losing calculus instruction, quickly made a full-year course in calculus the standard freshman sequence for engineers and scientists. As calculus came to be appreciated as a foundation for the modern world and one of science's greatest intellectual achievement, a number of selective private institutions instituted a general education requirement that all students take three semesters of calculus or else, for the math-phobic, three semesters of a foreign language. Calculus, or preparation to take calculus, was thus firmly established as the primary focus of first-year college mathematics.

Moving in another direction, John Kemeny and colleagues at Dartmouth reworked the mathematics-of-sets part of the Universal Mathematics course into their 1956 textbook, Introduction to Finite Mathematics. Along with some logic, discrete probability (with some combinatorics) and matrix basics, they chose two modern topics, developed in the last 50 years, that were accessible and had interesting applications. These topics were Markov chains and linear programming. This course, with gentler textbooks, is still widely taught to business and social science majors. (Interestingly, the mathematics requirement for many business majors evolved to cover finite mathematics and a gentle introduction to calculus, essentially the curriculum of Universal Mathematics and that advocated by E.B. Wilson in 1913.) Over the next 20 years, other alternatives to precalculus and calculus were developed, including introductory statistics, mathematical modeling, and surveys of mathematical ideas. As an aside, we note that in the mid-1950's Kemeny also anticipated that personal computing that would soon be extensively used in mathematics instruction (it was 25 years before the IBM PC). His group created the easy-to-use programming language BASIC and developed the country's first (non-military) time-sharing system supporting terminals in every Dartmouth dormitory.

At the high school level, mathematicians led a Commission on Mathematics of the College Board that made recommendations for upgrading the school curriculum, including set theory and inequalities, topics that were introduced for their role in newer areas such as statistics and linear programming. The College Board forced schools to adopt this curriculum by including these new topics on the SAT tests.

Around the same time, a Ford Foundation report led to the establishment of Advanced Placement exams to give college credit for college-level courses taken in high school, although initially the program was active only at selective private schools. Calculus was one of the first subjects with an AP exam.

Good data about mathematics undergraduate degrees is not available before 1960, but it is estimated that the number of math Bachelor's degrees at least doubled from the early 1950's to 1960 when there were 13,000 math degrees. In 1966, there were 20,000 math degrees. In 1970, the number had grown to

27,000 a year, over 4% of all college graduates (NSF [27]).

Changing the Mathematics Major. The success of pure mathematicians in war efforts supported efforts to move the mathematics major away from an explicit career focus (high school math teacher or actuary) to a major that included more theoretical courses, but was also believed to be a good preparation for industrial careers such as computer programming. The result was a dramatic change in the undergraduate program in mathematics that started in doctoral institutions and leading private colleges during the 1950s.

In 1960 at the University of Pennsylvania, ten courses were required for the math major and there were now only two pre-calculus courses, one a liberal arts survey course. There was a sophomore-level 'introduction to proofs' course. There were now upper-level courses in real variables, abstract and linear algebra, number theory, topology, differential geometry, Fourier analysis, numerical analysis and axiomatic set theory. At Nebraska, the major in 1960 now required five courses beyond Calculus III. There were still five courses before calculus in algebra, trigonometry and analytic geometry along with a new Elements of Statistics course, but there was also now a three-semester honors calculus sequence. There were many new upper-level courses similar to Pennsylvania. Colgate had a similar expansion although not as many new courses as the universities.

On the other hand, smaller colleges like Macalester were slower to change. In 1960, Macalester still had four pre-calculus courses. A two-semester abstract algebra course was added, but there were only two other post-calculus courses offered, advanced calculus and honors seminar. Another sign of the slow rate of change was experienced by this writer who took an AP Calculus course in 1960-61 at a well-regarded prep school using the text by Granville, Smith and Longley, first published in 1904.

The addition of more modern, theoretical courses and honors sequences at leading institutions was motivated in part by a desire to prepare more mathematics majors to continue on to doctoral study in mathematics. In 1962 the President's Science Advisory Committee optimistically estimated (Gilliland [16]) that there would be a need for seven times as many new Ph.D.'s annually by the 1970's, and indeed the number of Ph.D.'s grew from about 300 a year in the late 1950's to 1200 a year in 1970 (a level not matched again for 30 years); see AMS [2] for yearly Ph.D. data. This demand for mathematics Ph.D.'s was driven by: i) growing college enrollments in the 1960's as 'baby boomers' entered college and the percentage of students going to college was rising; ii) replacing retiring faculty hired during the last expansion in college enrollments in the 1920's; and iii) the Cold War-focus on science, mathematics, and engineering.

In the late 1950's, there still were many able students going to graduate school from mathematics programs that did not teach the Riemann integral. They dropped out of graduate study when faced with first-year courses in modern topics like Lebesgue integration. CUPM took the lead in addressing this problem with its first comprehensive curriculum report, published in 1963, Pregraduate Preparation of Research Mathematicians. The report's preface notes that the recommendations were 'idealized' and intended for an honors program.

The four-semester sequence for the first two years covered calculus and differential equations along with some real analysis (results about compactness, absolute continuity, Riemann integration, Ascoli's theorem), linear algebra and other topics such as differential p-forms. For the junior and senior years, yearlong courses were recommended in real analysis, complex analysis, abstract algebra, geometry-topology and either probability or mathematical physics. Real analysis went up to Banach spaces and spectral resolution of self-adjoint operators. Complex analysis went up to harmonic measures and Hardy spaces. Needless to say, this report was swinging the pendulum too far in the other direction with a curriculum that was over the head of most freshmen who considered themselves talented in mathematics.

At this same time, the MAA and AMS presidents, in coordination with the NCTM president, created the School Mathematics Study Group (SMSG), most prominent of the 'New Math' efforts, to produce textbooks for a more abstract school mathematics curriculum for stronger students. The CUPM Pregraduate Preparation report mentioned the SMSG curriculum as a valuable preparation for its recommended major. SMSG was led by mathematicians who got carried away with an axiomatic approach to school mathematics, e.g., the field axioms were discussed in its junior high school texts, and produced in time a massive pubic backlash.

CUPM's General Curriculum in Mathematics for Colleges. A follow-up 1965 CUPM report called the General Curriculum in Mathematics for Colleges (GCMC) presented recommendations for a mainstream mathematics major that were a watered-down version of the Pregraduate Preparation recommendations. This curriculum of 11 courses was aimed at four-year colleges. However, this program was still too ambitious for many institutions and as a result, CUPM produced a gentler version of GCMC in 1972. The 1972 GCMC recommended a beginning core of six courses: calculus I, calculus II (with some differential equations), linear algebra, multivariable calculus I, abstract algebra, and linear algebra. Suggested additional courses were multivariable calculus II, real variables, probability and statistics, numerical analysis, and applied mathematics. The GCMC was widely adopted and formed the basis of the mathematics major by the mid 1970's. Virtually all college mathematics departments also retained their separate course in differential equations. The one new course that GCMC missed was the Introduction to Proofs course that many colleges and universities felt was needed to help math majors make the transition from calculus to more theoretical upper-division courses such as abstract algebra.

Possibly the most significant curricular change arising from GCMC was the introduction of a sophomore-year linear algebra course; it was adopted widely at universities and colleges. Linear algebra had entered the undergraduate curriculum in the 1950's at universities as part of a yearlong upper-division abstract algebra course. It is somewhat ironic that this sophomore linear algebra course evolved from Kemeny et al.'s Introduction to Finite Mathematics text. In 1959, Kemeny's group produced a variation of this text for mathematics majors called Finite Mathematical Structures with a major part of the text devoted to linear algebra.

CUPM issued other reports about teacher preparation, statistics, and computational/applied mathematics in the late 1960's and early 1970's, but none had the impact of the 1972 CGCM report. While the primary change for the mathematics major was a more theoretical approach, there were other additions to the undergraduate program, starting in the 1960's, involving new areas of applied mathematics such as computer science, numerical analysis, mathematical modeling, and operations research.

As undergraduate enrollments quadrupled from 1950 to 1970, the fraction of mathematics enrollments in precollege (remedial) mathematics and pre-calculus courses actually dropped substantially while the percentage of enrollments in calculus grew. This trend would seem to reflect better mathematics instruction in the schools. The largest percentage growth, 150

Statistics and Computer Science Spin Off. Statistics flourished after World War II based on its wide use during the war effort. Specialization in statistics initially involved graduate- level education. About 50 statistics departments split off from mathematics departments in the 1950's and 1960's and most started their own undergraduate majors. To this day however, there are a modest number of majors in these statistics departments, probably because most mathematically oriented students are not ready to specialize in a particular mathematical science at the time they must pick a major. On the other hand, in time a good fraction of mathematics departments offered the popular option of a concentration in statistics.

The other major discipline to split off from mathematics was computer science. Academic study related to machine computation initially focused on numerical analysis and logic topics such as recursive function theory. So computer science started primarily as an area of mathematics, taught at the graduate level, although in some institutions it was associated with electrical engineering. In the 1960's there were few computer undergraduate degrees, but student demand and the growth of computing in industry soon created pressure for more undergraduate majors. Computer scientists did not have a good idea of what courses should be created for an undergraduate program in computer science. Copying CUPM, the Association for Computing Machinery (ACM) issued a report for an undergraduate curriculum in 1968 (ACM [3]), although it was almost as ambitious as CUPM's 1963 Pregraduate Preparation report, i.e., the initial textbooks for the proposed sophomore-year Discrete Structures course were actually aimed at a graduate audience. As with GCMC, two more reports were needed to settle on a workable undergraduate program.

There were 2000 computer science graduates in 1970, 5000 in 1975, 11,000 in 1980, and 50,000 in 1985 (NSF [27]). This growth is one of the reasons that math major enrollments declined after 1970. A limiting factor in the growth of computer science programs was the shortage of computer science faculty, since industry was hiring most new Ph.D.'s at high salaries. To help address this shortage, mathematics faculty with computing interests were encouraged, with financial incentives and summer training programs, to move into computer science. This was essentially the only option to staff computer science programs at liberal arts colleges. Even today, many colleges have combined mathematics and computer science departments reflecting this history of shared faculty.

University faculty turn focus to research. While in the early 1950's most faculty at doctoral institutions still saw undergraduate teaching as their primary mission, by 1970 that mission had changed with research becoming the primary focus of these faculty. One consequence was that freshman-level mathematics instruction at public universities now typically involved graduate student instructors or large lectures by faculty with supporting TA-led recitations.

During this period, there was a change in values that discouraged most faculty in doctoral departments from taking a serious interest in undergraduate education. To lure star research faculty to Berkeley from Ivy League institutions in order to raise Berkeley to their level, University of California Chancellor Clark Kerr had offers made to star faculty with no undergraduate teaching and a light graduate teaching load. I remember my father lamenting that there was no way Princeton could counter such offers, since Princeton believed that all faculty should teach at least one undergraduate course a semester and no one taught less than two courses a semester. Kerr succeeded in attracting Ivy League star faculty to Berkeley, but as a whole academia suffered. Other public universities building up their doctoral programs copied Berkeley, and eventually the Ivy League universities also reduced their teaching loads. The Berkeley model for recruiting star faculty got turned around and regular faculty starting pressing for reduced teaching loads and minimal undergraduate teaching as a sign that they too were elite researchers.

We note that the supposed incompatibility of research and undergraduate teaching was refuted by a 1957 National Research Council study (Albert [1]) that found little impact on research productivity for teaching loads of up to 15 hours a week (a common teaching load for junior faculty in top departments before World War II). This NRC study also found an important benefit of teaching-oriented liberal arts colleges. They produced a disproportionate number of future mathematics Ph.D.'s, despite the attraction that future mathematics Ph.D.'s should have found in undergraduate study at universities with their distinguished research faculty.

In the early 1970's, the attractiveness of the mathematics major started a dramatic decline. The percentage of freshman interested in a mathematics major dropped from 4.5% in 1966 to 3.2% in 1970 to 1.1% in 1975 (and never rose above 1.5% again) (data here come from CBMS [10] and NSF [27]). From 27,000 math degrees awarded in 1970, the number declined to 18,000 a year in 1975 (a 1970 CBMS estimate had projected 50,000 a year in 1975) and 11,000 a year in 1980. The 1975 CBMS enrollment survey revealed a shift of student interest away from pure mathematics courses, a number of which saw 70% enrollment declines from 1970 to 1975, while applied mathematics courses registered modest declines. Internally, CUPM's more theoretical GCMC curriculum may have overwhelmed some students at the same time that the new computer science major offered an attractive, career-oriented alternative. Externally, the Vietnam War was turning students off to working for the 'military-industrial' complex, a major employer of mathematics majors. Also, as baby boomer children started graduating from high school, the demand of high school mathematics teachers dropped. In short, the mathematics major in 1970's experienced a tidal wave of problems causing a sharp decline in its enrollments and general appeal.

4 The Last Forty Years, 1975-2015.

The decline in interest in the mathematics major stabilized and slowly the number of majors rebounded a little in the 1980's. Initially, there was a tension between students who wanted more career-oriented coursework, often in newer applied areas like statistics and operations research, and mathematics faculty who wanted the major to maintain its focus on the beauty and intellect depth of core mathematical theory. In 1981, CUPM published Recommendations for a General Mathematical Sciences Major to address this tension. The report provided a rationale for a mathematics major with a broader scope than the GCMC. The guiding philosophy of these recommendations was that a mathematical sciences major should: i) develop attitudes of mind and analytical skills required for efficient use and understanding of mathematics; and ii) should be designed around the abilities and needs of the average major (with supplementary work to attract and challenge talented students.)

This philosophy led to the recommendation that study in depth in one of the diverse areas of the mathematical sciences was a valid substitute for core courses in real variables and abstract algebra. While the Mathematical Sciences report was geared towards a single flexible major in a mathematics department, it actually helped catalyze the development of multiple tracks in the mathematics major, even at small colleges. Often, the applied tracks still required one or both of real variables and abstract algebra but then subsequent electives could be applied. This compromise worked for both students and faculty. The mathematics major today has the same multi-track structure.

One important area that the Math Science report neglected was the preparation of future high school mathematics teachers. Specifically, the report did not question the prevailing thinking that a standard mathematics major, with the new flexibility of the Math Science recommendations, developed a deep mastery of high school mathematics. Although this was a convenient assumption for mathematics faculty, a 1965 study by Begle [4] (and a later 1994 study by Monk [25]) challenged this assumption: Begle actually found a slightly negative correlation between the number of upper-level mathematics courses taken by high school mathematics teachers and the test performance of students of these teachers. In the late 1990's, an awareness developed among some mathematicians that there is substantial mathematics in the K-12 curriculum worthy of study in college courses, starting with place value in early grades. But it was not until 2012 that a CBMS teacher education report (CBMS [11]) recommended that a mathematics major for future high school teachers should include three courses in high school mathematics from an advanced perspective along with modifications to core courses such as emphasizing polynomial rings and fields in abstract algebra.

While there have been no significant national changes in the mathematics major in the years since the CUPM Math Sciences report, there have been differing levels of interest in mathematics. Namely, liberal arts colleges have had a higher percentage of graduates majoring in mathematics than research universities, mirroring the 1950's Albert report ([1]) finding about the disproportionate success of liberal arts colleges in producing future Ph.D. mathematicians. Some liberal arts colleges have had a far larger percentage of mathematics graduates than the national average of 1%. A common theme at successful math major programs is the deep involvement of faculty who personally engage students in introductory courses. The mathematics chairman, Arnie Ostebee, at St. Olaf College, where up to 10% of graduates have been in mathematics, gave a sense of what drives such success when he told me, "calculus is the favorite course of our faculty because that is where we can change students' lives; teaching courses like real analysis is just preaching to the converted."

Enrollments Trends and Two-Year Colleges. Over the last 40 years, the annual number of the mathematics Bachelor's degrees has remained fairly constant in the range of 10,000–15,000 (about 50% of the high water mark in 27,000 in 1970), while overall college graduates increased by over 50

Total mathematics enrollments in (4-year) colleges and universities have also stayed fairly constant over this period at about 1,600,000 a semester while overall enrollments doubled. This discrepancy is likely explained by two factors: i) the growth of overall enrollments has been occurring in departments with light mathematics requirements; and ii) there has been no growth in precollege mathematics enrollments. (As noted below, a huge increase in precollege mathematics instruction has occurred at two-year colleges.) The overall division of mathematics enrollments has also remained fairly constant with precollege courses around 12%, courses below calculus around 45%, calculus courses around 40%, and post-calculus courses at 4-6%.

The growing role of mathematics in science and engineering resulted in an increase in mathematics courses being offered in other departments. While the large enrollments in statistics courses in the social sciences and business were well known, a study by Garfunkel and Young [15] found that in 1990 there were about 170,000 annual enrollments in post-calculus mathematical sciences courses in other departments as opposed to about 120,000 annual enrollments in post-calculus courses in mathematics departments.

A major feature of higher education in the last 40 years has been the growth of two-year college enrollments. They went from about 1,000,000 students in 1965 to around 6,000,000 students in 2000. Mathematics enrollments in two-year colleges have grown apace, but the majority were precollege mathematics enrollments, which grew from 200,000 in 1970 to 1,100,000 in 2010. At the same time, their enrollments in college-level mathematics only doubled. As a result of these surging precollege enrollments, current mathematics enrollments of 1,800,000 a semester in two-year colleges now exceed the mathematics enrollments in (4-year) colleges and universities even though colleges and universities have almost three times as many students as two-year colleges.

Initially two-year colleges focused on preparation for technician-type careers that did not require a 4-year degree, but they have increasingly become a starting place for Bachelor's degree study and have been offering the standard lower-level mathematics courses for mathematics majors: single and multivariable calculus, differential equations, and linear algebra. Many college graduates now complete all their mathematical coursework in two-year colleges. In particular, it is estimated that close to half of all elementary school teachers complete their mathematics requirements in two-year colleges.

Articulation with High Schools and Calculus Instruction. The greatest area of change and concern in the past 40 years has arguably been the articulation between high school and college mathematics. It used to be that freshmen were placed in mathematics courses based on which high school mathematics courses they took. However, by the early 1980's, mathematics faculty were dealing with large numbers of students in freshman courses who showed limited knowledge of needed algebra skills. States had been raising high school mathematics requirements, but courses were watered down to make these requirements workable. Further, cramming for tests became a widely accepted substitute for course-long learning. In response, mathematics departments instituted placement tests that all students, except those with AP calculus credit, were required to take to determine which mathematics courses they were eligible to take.

Another concern has been the unintended consequences of the AP calculus program. In time, large numbers of high school students were taking AP Calculus classes and then repeating first-semester calculus in college. A survey by Bressoud and colleagues (Bressoud [6]) found that 70% of calculus students at universities and colleges had taken calculus in high school, and 25% received a 3 or better on the AP Calculus AB exam (the numbers were lower at Master's and Bachelor's institutions). Half of all calculus students believed that one must take calculus in high school to succeed in college calculus. However, the effect was that many students coasted through most of the first semester of calculus and failed the second semester, i.e., second-semester calculus failure rates came to exceed first-semester failure rates at many institutions. Other students who rushed through the standard high school mathematics curriculum to take AP calculus (but not take the AP test), did so poorly on math placement tests that they had to start college with a college algebra course. Equally worrisome was the fact that by 2000 a number of the students who earned AP Calculus credit in high school never took a mathematics course in college.

There has also been a problem with superficial learning at the college level in calculus. As a broader audience of students from computer science, biology, business and economics started taking calculus– partly as a screening device– the contents got watered down. Many students reacted by learning calculus in a superficial way to pass tests, never expecting to see calculus again. A common 'horror' story was that when students were asked after taking a calculus course, what the derivative of a function is, they would answer, 'for x^2, it is $2x$.' A prominent computer scientist led a well publicized campaign in the early 1980's for computer science departments to stop requiring their majors to take any calculus because of these problems.

In reaction, dozens of efforts were undertaken to re-invigorate calculus instruction under the banner of 'lean and lively calculus' (Douglas [12]). Greater attention was given to understanding the derivative as a rate of change with graphs, numerical examples (using technology), and applications. Decreased attention was given to techniques of integration, epsilon-delta limit proofs, and series of constants. Another approach focused on more student-active modes of instruction, such as cooperative learning. This general initiative was provocatively labeled Calculus Reform. While many of the changes were not controversial, there were aspects, especially cooperative learning, that gave rise to heated criticism for making calculus too 'fuzzy.' While only one calculus reform textbook did become a commercial success, the less controversial aspects of calculus reform were incorporated into leading calculus texts.

5 Conclusion.

This paper has tried to provide a chronicle of the evolution of the undergraduate program in mathematics in American colleges and universities. It also sketched out major trends in higher education that affected mathematics instruction. While the undergraduate program in mathematics has been relatively stable for the past 40 years, the same was true for the undergraduate program in the first 40 years of the Association's history. Who knows if this stability will soon be followed by dramatic changes comparable to those around the beginning and middle of the 20th century.

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References

[1] A.A. Albert, *A Survey of Research Potential and Teaching in the Mathematical Sciences*, University of Chicago Press, Chicago, 1957.

[2] American Mathematical Society, *Notices Amer. Math. Soc.*, annual reports on numbers of mathematics Ph.D.'s awarded.

[3] Association for Computing Machinery, Curriculum 68: Recommendations for an academic program in computer science, *Comm. A.C.M.* **11** (1968) 151-197.

[4] E. Begle, *Critical Variables in Mathematics Education: Findings from a Survey of Empirical Literature*, Mathematical Association of America, Washington, DC, 1979.

[5] G. Birkhoff, Mathematics at Harvard, 1836-1944, *A Century of Mathematics in America*, Vol. 2, American Mathematical Society, Providence, 1989.

[6] D. Bressoud et al., Description and selected results from the MAA national study of calculus, *Internat. J. of Math. Educ. in Sci. and Tech.*, 2013 (forthcoming).

[7] F. Cajori, *The Teaching and History of Mathematics in the United States*, Bureau of Education, United States Government Printing Press, Washington, DC, 1890.

[8] P. Cohen, *A Calculating People: The Spread of Numeracy in Early America*, Rutledge Press, New York, 1982.

[9] Colgate University, College Catalogs from 1920, 1930, 1940, 1950, 1960.

[10] Conference Board of Mathematical Sciences, *CBMS Survey: Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States*, undertaken every five years starting in 1965, Washington, DC.

[11] —, *The Mathematical Education of Teachers II*, Issues in Mathematics Education Vol. 17, American Mathematical Society, Providence, 2012.

[12] R. Douglas, ed., *Toward a Lean and Lively Calculus*, MAA Notes Vol. 6, Mathematical Association of America, Washington, DC, 1987.

[13] W. Duren, CUPM, The history of an idea, *Amer. Math. Monthly* **74** (1967) 23-37.

[14] — , Mathematics in American society, 1888-1988: An historical commentary, *A Century of Mathematics in America*, Vol. 2, American Mathematical Society, Providence, 1989.

[15] S. Garfunkel and G. Young, Mathematics outside of mathematics departments, *Notices Amer. Math. Soc.* **37** (1990) 321-325.

[16] E. Gilliland, ed. Meeting manpower needs in science and technology, *Graduate Training in Engineering, Mathematics, and Physical Sciences*, President's Science Advisory Committee, The White House, Washington, DC, 1962.

[17] E.R. Hedrick, A tentative platform of the Association, *Amer. Math. Monthly* **23** (1916) 31-33.

[18] R. Hofstadter, Education in a democracy, *Anti-Intellectualism in American Life*, Vintage Press, New York, 1962.

[19] R. Hofstadter and C.D. Hardy, *The Development and Scope of Higher Education in the United States*, Vintage Press, New York, 1952.

[20] R. Hofstadter and W. Smith, eds., *American Higher Education, a Documentary History*, University of Chicago Press, Chicago, 1961.

[21] Macalester College, College Catalogs from 1920, 1940, 1950, 1960.

[22] Mathematical Association of America, Report of committee on assigned collateral reading in mathematics, *Amer. Math. Monthly* **35** (1928) 221-228.

[23] —, Committee on the Undergraduate Program, *Universal Mathematics*, Washington, DC, 1954.

[24] —, Reports of Committee on the Undergraduate Program in Mathematics: *Pregraduate Preparation for Research Mathematicians*, 1963; *A General Curriculum in Mathematics for Colleges*, 1965 and 1972; *Recommendations for a General Mathematical Science Program*, 1981.

[25] D.A. Monk, Subject area preparation of secondary mathematics and science teachers and student achievement, *Economics of Education Review* **13** (1994) 125-145.

[26] National Education Association, *Cardinal Principles of Secondary Education*, United States Bureau of Education, Washington, DC, 1918.

[27] National Science Foundation, *S& E Degrees: 1966-2006*, Report 08-321, Washington, DC, 2008.

[28] K. Parshall and D. Rowe, American mathematics comes of age: 1875-1900, *A Century of Mathematics in America*, Vol. 3, American Mathematical Society, Providence, 1989.

[29] M. Rees, The mathematical sciences and World War II, *A Century of Mathematics in America*, Vol. 1, American Mathematical Society, Providence, 1989.

[30] R. Richardson, The Ph.D. degree and mathematical research, *Amer. Math. Monthly* **43** (1936) 199-211.

[31] W.L. Schaaf, Required mathematics in a liberal arts college, *Amer. Math. Monthly* **44** (1937) 445-448.

[32] H. Slaught, The teaching of mathematics in the colleges, *Amer. Math. Monthly* **16** (1909) 173-177.

[33] D. Smith and J. Ginzburg, *A History of Mathematics in America Before 1900*, Carus Monograph Vol. 2 , Mathematical Association of America, Washington, DC, 1934.

[34] State University of New York College at Geneseo, College Catalogs from 1920, 1940, and 1950.

[35] A.W. Tucker, personal communication.

[36] United States Bureau of the Census, *Education Summary: High School Graduates, and College Enrollment and Degrees: 1900 to 2001*, No. HS-21. Washington, DC, 2008.

[37] United States Code, Article 304.

[38] University of Nebraska, College Catalogs from 1920, 1930, 1940, 1950, 1960.

[39] University of Pennsylvania, College Catalogs from 1920, 1930, 1940, 1950, 1960.

[40] University of Rochester, College Catalogs from 1920, 1930, 1940, 1950.

[41] E.B. Wilson, Let us have calculus early, *Bull. Amer. Math. Soc.* **20** (1913) 30-36.

[42] F. Woods, *Advanced Calculus*, Ginn, Boston, 1926.

[43] R. Woods, How can interest in calculus be increased, *Amer. Math. Monthly* **36** (1929) 28-32.

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