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The homogeneity and isotropy of the Universe*

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Summary. We analyse those models of the Universe consistent with the observed isotropy, entropy, element abundances and with the existence of galaxies. The finiteness of the entropy per baryon in the Universe today, $S_b \sim 10^8$, limits the amount of dissipation that could have taken place in the past and hence the degree of irregularity allowed in the singularity structure. This observation essentially rules out the chaotic cosmology in its full generality and appears to constrain the singularity to be of simultaneous Robertson–Walker character containing only small curvature fluctuations.

1 Criteria

There are four important observational facts about the Universe, for which theoretical explanations are still being sought:

(a) The existence of galaxies.

(b) The present large heat content or entropy per baryon. This is usually presented by quoting the observed number ratio of photons to baryons, $S_b \sim n_\gamma/n_b \sim 10^8$. Although this is a number much greater than unity, it is far smaller than a typical 'cosmological' number $\sim 10^{40}$.

(c) The requirement of similar entropy $S_b \gtrsim 10^7$ at the time of element synthesis ($T \sim 10^9$ K) [1].

(d) The large-scale homogeneity and isotropy as indicated directly by the absence of structure in the intensity of the microwave background on large angular scales ($\Delta T/T \lesssim 3 \times 10^{-3}$), and by the apparently Planckian spectrum of the microwave background, and indirectly by the observed cosmic abundance of $He^4 \sim 0.3$ and $D \sim 2 \times 10^{-5}$ by mass which require small anisotropy at $T \sim 10^9$ K [2, 3]. In all models calculated except one (Bianchi Type VII_h) the nucleosynthesis limits on the current anisotropy are more stringent than

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limits from the observed blackbody radiation temperature anisotropy, even with the usual assumption of $Z \sim 10^3$ for the last scattering of the microwave radiation [3].★

The three facts (b)–(d) suggest that a considerable amount of dissipation occurred in the past. Moreover – and this is the point of this note – *the finiteness of S_b restricts the amount of dissipation that may have occurred in the past, and thus restricts the class of acceptable cosmological models to those with a finite degree of irregularity in the past.* We claim in fact that, except for a special model to be described below, the four facts (a)–(d) mean that the Universe must always have been close to homogeneity and isotropy and arbitrarily chaotic cosmologies are ruled out.

The crucial factor in our argument is that a universe which is irregular now must have been more irregular in the past. The exception to this idea, and the 'special model' referred to above, is familiar to any one who has read Lifshitz' [7] or any other treatment of perturbations of homogeneous isotropic cosmologies: there are growing modes, for which the density contrast $\delta\rho/\rho$ grows as some power of time t^s, $s \sim 1$. *Our special model is just one in which only growing modes are present*, thus having a simultaneous singularity, and in which the density contrast $\delta\rho/\rho$ becomes large (perhaps as large as $\delta\rho \sim \rho$) only late in the evolution of the model.†

In a recent paper [10], Liang has identified the growing modes as arising from fluctuations of the spatial curvature ${}^3R$ of the model. Another way of stating this observation is to note that growing modes have excess binding energy (hence more positive ${}^3R$) compared to neighbouring regions. Liang associates the decaying modes with primordial shear fluctuation. A chaotic universe near the singularity would of course have both growing and decaying modes, so the purely growing mode model is indeed special.

The argument upon which we base our criteria for deciding the acceptability of cosmological models goes as follows. A model which had large-amplitude inhomogeneities near the singularity must have been very anisotropic then. (We exclude very special 'horizonless' models, so the inhomogeneities should have had a scale larger than the horizon size, and their effect could be idealized as homogeneous shear.) The persistence of anisotropy is not tolerable because of point (d) above. Hence this anisotropy – if it existed – must have been dissipated prior to $Z \sim 10^{10}$. An energy density ρ_β may be associated with the anisotropy;

★ It may be appropriate at this point to interject a comment on the theoretical significance of the isotropy of the 3 K blackbody radiation. Doroshkevich, Lukash & Novikov [4] have discussed models of several Bianchi types of homogeneous cosmologies. In these models they find that anisotropy often decays very slowly, and they conclude that the observed $\Delta T/T \lesssim 10^{-3}$ puts limits on the anisotropy all the way back to the Planck epoch $t_p \sim 10^{-43}$ s. However, it must be noted that their limit on the epoch of isotropization depends exponentially on the epoch of last scattering and is unstable to small changes in estimates of this parameter. The physical reason is that in their models the only collisionless radiation is a test gas of photons which is the relict radiation giving the 3 K background. But as the work of Misner [5] has shown, the dynamical effects of collisionless radiation can be extremely important. The fact that the spatial curvature is non-zero at the instant that the radiation first becomes collisionless is a complication in the Bianchi types VII, VIII and IX considered by Doroshkevich *et al.* However, Matzner [6] has shown that observations imply that the spatial curvature terms have been unimportant in the observed Universe at least since the instant $Z \sim 10^3$ when hydrogen recombined. In this case the calculations done by Misner for negligible spatial curvature apply, and yield $\Delta T/T \lesssim 10^{-3}$ now.

† Compare with Peebles [8], who argues that because the Universe is *regular* today and irregularities grow in time it must have been even more regular in the past. This argument seems to beg the question since Peebles' Newtonian arguments correspond to studying only $R-W$ models with a growing scalar perturbation mode. We argue that the presence of (possibly interacting) shear fluctuations and tensor modes together with spatial curvature anisotropy must lead to significantly anisotropic behaviour at some time in the past. (See for example Collins & Hawking [9] who give the latest time at which anisotropy can have been significant, given $\Delta T/T \lesssim 2 \times 10^{-3}$.) The special model is the General Relativistic analogue of Peebles' Newtonian model run on computer.

this energy density is roughly proportional to $(1+Z)^6$. The presence of particles with long mean free paths is *not* necessary for the definition of ρ_β. In the simplest cases involving fluid-filled anisotropic cosmologies of Bianchi Type I, $\rho_\beta = \sigma^2$, where σ is the shear of the fluid. If particles with long mean free paths are present, ρ_β is increased to include the additional energy such particles gain from the work done on them by the anisotropic expansion.

By some dissipative mechanism, the anisotropy energy ρ_β may be converted to photons whose energy density subsequently evolves as $(1+Z)^4$. Thus the amount of anisotropy present at the dissipation epoch is limited because the entropy cannot exceed the 10^8 photons per baryon now observed, and the cosmological production of He and D requires a similar value $S_b \gtrsim 10^7$ at $Z \sim 10^9$–10^{10}. Prior to the dissipation epoch, the expansion θ (the logarithmic proper time derivative of the volume of the Universe, i.e. $(R^3)^\cdot/R^3$ [11]) obeys:

$$\tfrac{1}{3}(\theta)^2 \cong \rho_{\beta 10}\,[(1+Z)/(1+10^{10})]^6 + \rho_{b10}[(1+Z)/(1+10^{10})]^3 \tag{1.1}$$

with $\rho_{\beta 10}, \rho_{b10}$ constants. Here the first term is ρ_β, the anisotropy energy density and the second term is ρ_b, the matter density of baryons ($= m_b n_b$ with m_b the mass and n_b the number density of the baryons). In writing (1.1), it has been assumed that the initial entropy S_b is very low, so that the two terms on the right adequately describe the expansion. If, just prior to the helium production epoch at $Z \sim 10^{10}$ the anisotropy were dissipated and converted to photons, we would have subsequently

$$\tfrac{1}{3}\theta^2 = \rho_{\gamma 10}[(1+Z)/(1+10^{10})]^4 + \rho_{b10}[(1+Z)/(1+10^{10})]^3. \tag{1.2}$$

where $\rho_{\gamma 10} = \rho_{\beta 10}$ is a constant which gives the energy density in the photons. Using 10^8 photon/baryon now we find $\rho_{\gamma 10} = \rho_{\beta 10} \sim 10^5 \rho_{b10}$. This may seem a large number, but this anisotropy would have decayed away adiabatically in about 50 expansion times at $Z \simeq 2 \times 10^8$ anyway. Further, because of the Z^6 behaviour of the anisotropy energy density, dissipation earlier produces more photons. Hence, in general, the original anisotropic model is limited by

$$\rho_{\beta 10} \lesssim 10^5 \rho_{b10} \times (10^{10}/Z_{\rm diss})^2 \tag{1.3}$$

where $Z_{\rm diss}$ is the earlier epoch at which the dissipation takes place. For instance, the quantum-cosmology era ended at $t_P \sim 10^{-43}$ s, i.e. $Z_P \sim 10^{32}$. If all of the entropy now observed were created at that epoch, the original description of the model would have required

$$\rho_{\beta 10} \lesssim 10^{-39} \rho_{b10} \tag{1.4}$$

and for all dynamical purposes the Universe would have been considered isotropic even if the dissipation had not occurred. It is amusing that another cosmological coincidence appears here; this one involves $\hbar$, G from the quantum era, and the weak interaction constant (which determines the nucleosynthesis epoch $Z \sim 10^{10}$).

For equations of state $p = \rho \propto R^{-6}$, sound perturbations carry an energy density $\epsilon \simeq \rho(\delta\rho/\rho)^2$ and there too the energy available to dissipate into photons is large at early epochs, and the observed baryon entropy sharply limits the parameters of the process.

We have not specified the mechanism of dissipation. Shear is dissipated in any process in which there is a relaxation time on the order of an expansion time of the Universe. The original suggestion of Misner [12] used neutrino collisions occurring near $T = 10^{10}$ K. Other dissipative processes can certainly occur at higher temperatures, e.g. hadron–hadron collisions at $T \gtrsim 10^{13}$ K (i.e. for $\infty > Z > 10^{13}$) [13]; particle collisions involving gravitons at

$T \sim 10^{20}$ K [14]; or if Hagedorn's [15] ideas concerning copious particle production near 10^{12} or 10^{13} K apply, very strongly dissipative effects which arise as anisotropic expansion causes blue shift of the random motions of at least some particles, which quickly dump their energy into numerous daughter particles. Near the singularity mini-black hole production may occur [16]. In this case the associated dissipation which occurs may produce entropy even before the production of black holes. If any black holes do survive whose mass is greater than the Planck mass, they can have an important effect in producing entropy past the quantum epoch $t \leqslant 10^{-43}$ s, by giving up their energy in elementary particles via the Hawking process [17]. In addition, near the singularity the cosmological quantum particle production processes recently discussed may be significant. These discussions are still unsatisfactory [18, 19] because of the difficulty of defining the concept of particle number in the very early Universe. None the less, most such calculations have an initial time t_0; the number of particles produced typically diverges at $t_0 \to 0$. The particle production process would be strongly dissipative of anisotropy, so the amount of particle production is itself limited by the $S_b \sim 10^8$ datum. There are thus dissipation mechanisms which would be effective arbitrarily early in the evolution of the Universe. And the argument above then shows that the chaos measured by $\rho_{\beta 10}/\rho_{b10}$ must have always been very small.

In addition to the mechanisms listed here which occur near the singularity, we may also consider those which produce dissipation of anisotropy late in the evolution, such as photon–photon dissipation which occurs during the decoupling epoch $Z \sim 10^3$. As we discuss in Section 2, this mechanism may have been relevant at the epoch of galaxy formation, but in any case the Universe must have been essentially isotopized before then in the sense that relatively little entropy would be generated. A further restriction is that the generation of significant amounts of entropy at $Z \sim 10^3$ would have seriously distorted the 3 K microwave spectrum [20]; see below.

Strong inhomogeneities must have generated a corresponding anisotropy. A qualitative argument which will be made gauge invariant shortly (see also [10]) is as follows:

$$\delta\rho \sim \delta M/R_0^3 \sim \dot{\sigma}$$

because $\delta M/R_0^3$ is like a Riemann tensor component and hence drives the shear. Further, $\theta^2 \sim t^{-2} \sim \rho$ in isotropic cosmologies; hence

$$\delta\rho/\rho \sim \delta\rho/\theta^2 \sim \dot{\sigma}/\theta^2 \sim (\sigma\theta)/\theta^2 \sim \sigma/\theta. \tag{1.5}$$

By this argument one expects large shear when the density perturbations are large. If the scale of the density perturbation is λ, then it will be expected to induce shear only on scales which are comparable with, or smaller than λ. We expect then that σ/θ according to (1.5) will be present in addition to any global homogeneous shear which might be primordially present.

Perturbation analyses of this question are not too helpful. It is known from perturbation theory that in exactly isotropic backgrounds there is no coupling of the density and wave perturbations to first order. The wave quantities are indistinguishable from homogeneous shear, if their wavelength exceeds a horizon size [7].) However, calculations show that there is such coupling in strongly anisotropic backgrounds [21]). By integrating equation (5.5) of Perko, Matzner & Shepley [21] we find that in the absence of dissipation, the long-wavelength shear perturbation $\tilde{\sigma}$ (which has a wavelength $\tilde{\lambda}$ larger than the horizon size) satisfies

$$\tilde{\sigma}/\theta = (\delta\rho/\rho)(\sigma/\theta) + \text{constant}. \tag{1.6}$$

Here σ and θ are the background quantities and $\delta\rho/\rho$ is the density contrast with the same position dependence as $\tilde{\sigma}$ and equation (1.6) holds for those components of the shear transverse to the spatial gradients of $\delta\rho/\rho$. The quantity σ/θ is a constant of order unity in these strongly anisotropic models. For perturbations smaller than the horizon size σ is more complicated, with an oscillating time behaviour that depends on the equation of state. Consider now a second perturbation $\tilde{\tilde{\sigma}}$ whose length scale $\tilde{\tilde{\lambda}}$ satisfies $\tilde{\tilde{\lambda}} \gg \tilde{\lambda}$, and is at the same time much larger than the horizon size. So far as the perturbation $\tilde{\sigma}$ is concerned, $\tilde{\tilde{\sigma}}$ is essentially homogeneous. Hence a term $\tilde{\tilde{\sigma}}/\theta$ may be added into the factor σ/θ on the right-hand side of (1.6). In this case at least, the qualitative arguments and estimates of relevant length scales leading to the estimate (1.5) are exactly borne out.

Note that since $\delta\rho/\rho$ can grow as rapidly as $t^{8/3}$ in a strongly anisotropic model, the shear perturbations according to (1.6) can grow in time. (After the background shear becomes small, the shear perturbations always decay.) Our special model referred to above then is extremely special indeed. Any deviation from exact isotropy of the background on which the growing modes are superimposed would have made the model extremely anisotropic at earlier times. The special model avoids this fate by having only a small inhomogeneity until so late in its evolution that nothing can be done about it.

Before leaving the question of the special model, we pose a more specialized example in closed (Type IX) cosmologies. There the perturbation equations lead to v^i = constant, where the v^i is the velocity perturbation in a radiation-dominated closed universe. Although we might expect such a constant perturbation to have permanently small effect, an exact consequence of the Einstein equation in this case is (for homogeneous v^i, such as would be the case when the horizon size is small enough)

$$\sigma^2 > \text{const}/(R^6 \sinh^2 \sqrt{2}\, 3\beta_-) \qquad (1.7)$$

where the constant is proportional to the $(v^i)^2$ of the perturbation, R is the radius of the Universe, and where β_- is a parameter describing the anisotropy of the t = constant hypersurfaces [6]. *Hence at least one of the two anisotropy parameters σ^2, β_- must have been large near the singularity.* In either case we have a strongly anisotropic, essentially homogeneous cosmology, which shows that the perturbation solution could not have held all the way back to the singularity. The point of this exercise is that small perturbations may behave differently from finite ones. The discussion for the velocity just shows that this particular infinitesimal non-decaying mode stays arbitrarily close to a Robertson–Walker solution, for times $t > \epsilon$ for an arbitrarily small $\epsilon > 0$ if v^i is small enough. But there must have been a switch-over from anisotropic behaviour at some sufficiently early time $t < \epsilon$.

The v = constant perturbations induce a distortion in the shape of the model; compared to isotropic models the metric perturbation becomes large, $\delta g/g \sim 1$ at a redshift $\sim 10^4\,\Omega v^{-2}$ (where Ω is the ratio of matter density present to that required to close the Universe), [30]. So the isotropic Robertson–Walker model background becomes untenable, and for sufficiently early times an anisotropic model must be used. We suggest that similar behaviour may occur in the problem for non-infinitesimal density perturbations, so that even the growing modes may have required an anisotropic background very near the singularity.

2 Acceptable models

Based on the above discussion, an acceptable model of the Universe would be one which had undergone a moderate amount of dissipation $S_b \sim 10^8$ presumably very early in its history. Hence it must have been close to isotropy during essentially all of its early history, with only small irregularities subsequent to the dissipation. Such irregularities would have evolved like the perturbation modes of an isotropic model. They would eventually have been dominated by the growing modes with very small random velocities associated with the perturbations.

All length scales, at least all those larger than the horizon size at the epoch of dissipation, should be present, and there is no strong reason to prefer one or another scale length. These perturbations may have grown to substantial perturbations very late in the evolution and may have exceeded the $\delta\rho/\rho \sim 10^{-4}$ amplitudes which would have led to galaxy formation. The restriction on the size of perturbations at recombination arises from the requirement that energy release associated with the fluctuations does not distort the shape of the microwave spectrum [22, 23]. For instance, Sunyaev & Zel'dovich estimate, for adiabatic perturbations in the mass range [22]

$$5 \times 10^5 \Omega^{4.9} < M/M_\odot < 10^9 \Omega^{7/9},$$

that adiabatic fluctuations must have satisfied

$$\delta\rho/\rho < 0.1\, \Omega^{7/8}. \tag{2.1}$$

Equation (2.1) provides the upper limit on the amplitude which the growing modes achieve prior to recombination, and restricts our class of acceptable models.*

To illustrate the concept of acceptable models, we comment on model universes which have appeared in the literature. The presence of the 3 K microwave background indicates that prior to a redshift $Z \sim 10^3$, radiation drag and radiation pressure would have strongly interfered with the growth of initial perturbations whose typical length scale was smaller than the horizon size at that time. The redshift $Z \sim 10^3$, when the recombination of hydrogen occurred, may be taken as a convenient benchmark in the evolution of the Universe, as the epoch when galaxies began to collapse. The simplest models currently considered for galaxy formation are based on small perturbations of isotropic background cosmologies [21, 24]. The slow power-law in the growth of perturbations in these models requires an *a priori* density perturbation of $\delta\rho/\rho \sim 10^{-4}$ at the epoch of decoupling, a perturbation much larger than anticipated from any random process. Such models have always been considered somewhat unsatisfying because of the requirement of a primordial perturbation of this order. On the other hand, from the viewpoint here, these modes are complete acceptable although not explicable since they are essentially a small $\delta\rho/\rho$ version of the special model, discussed above.

An especially interesting mode is due to Zel'dovich [25]. He assumes an extremely hard equation of state, $p = \rho$, near the singularity and an initial spectrum $\delta\rho/\rho = bt^2/\lambda^2 \propto \lambda^{-2}$ (where λ is the *proper* wavelength of the perturbation) which gives $\delta\rho/\rho \sim b$ as the perturbation comes within the horizon. Zel'dovich picks $b \sim 10^{-4}$. The minimum wavelength for these perturbations (which are considered to be sound waves) is given by the interparticle separation at the Planck time $t_P \sim 10^{-43}$ s. At t_P the horizon size is much less than the interparticle separation; at a later time ($t \sim 10^{-32}$ s) the horizon has grown to include this wavelength. Zel'dovich requires that the energy in the waves be dissipated as they come within the horizon. The entropy generated in this dissipation is $S_b \sim 10^{14} b^{3/2} \sim 10^8$ if $b \sim 10^{-4}$ and furthermore the assumption of the λ^{-2} spectrum also gives the correct behaviour for seed galaxies much later at the epoch $Z \sim 10^3$. This model apparently fulfils all our criteria and has the advantage that only one parameter, b, allows the model to fit both the entropy and the galaxy production problems simultaneously. The principal objection to this model is that it may be unstable to processes of a type described by Liang [13]. By considering a model with Zel'dovich's $p = \rho$ equation of state but with large anisotropy, Liang showed

*Large peturbations $\delta\rho/\rho \sim 1$ on the scale of the horizon at the recombination epoch would have produced large black holes of mass

$$M_H \sim 10^{15} \Omega^{-2} M_\odot$$

in flagrant contradiction with observation [16].

that the required entropy $S_b = 10^8$ can be produced by dissipation from the global shear. But the amount of shear removed in this process may be an arbitrarily small fraction of the total shear. The $S_b = 10^8$ observation is still unexplained because no reason has been given why dissipation should stop at that value of S_b. The density fluctuations postulated by Zel'dovich require a completely isotropic background. However, his models requires density perturbations with $\delta\rho/\rho \sim 10^{-4}$ on all scales as they enter the horizon. These induce a corresponding velocity field $v \sim (\delta\rho/\rho)\, c \sim 10^{-4} c$. Conservation of angular momentum gives $v \propto Z^{-2}$ subsequent to the epoch t_P when the model begins. Long prior to the dissipation epoch $t = 10^{-32}$ s assumed by Zel'dovich, a very strong local anisotropy would have arisen, associated with the relativistic velocities. Any sort of dissipation would allow the Liang mechanism to produce excessive amounts of entropy.

The energy density associated with an acoustic perturbation in Zel'dovich's model is $\sim c_s^2(\delta\rho)^2/\rho$ [26], i.e. $\sim \rho(\delta\rho/\rho)^2$ since the sound speed $c_s = 1$ in a $p = \rho$ fluid. For a given $\delta\rho/\rho$ this behaviour follows the $\rho \propto (1+Z)^6$ behaviour which is the same as that of the shear in an anisotropic model. Hence the analysis leading to equation (1.3) applies, and dissipation which occurred arbitrarily early could have produced arbitrarily large amounts of entropy. At the earliest epoch considered, $t_P \sim 10^{-43}$ s, the amount of entropy S_b produced will be a function of b as in Zel'dovich's argument, but the value of b to give $S_b = 10^8$ would be much smaller than his, in fact of order 10^{-40}, by an analysis like that leading to (1.4). The coincidence between the b necessary to give the correct entropy and that for galaxy formation is broken. It appears inescapable that the Universe had to be miraculously isotropic near the singularity if shear damping were not to predict too much S_b.

The alternate galaxy formation viewpoint [27–29], [34], supposes the existence of large density perturbations (which, to agree with observation must obey (2.1)). Such models have been postulated to explain for instance the spin of galaxies, in which case the $\omega^2 \sim \rho$ is also often assumed (ω is the rotation). From the viewpoint of galaxy formation these models have been fairly successful, and they substitute the (perhaps) more satisfying assumption of a chaotic initial cosmology* for the arbitrary choice of the initial $\delta\rho/\rho \sim 10^{-4}$. The models with large rotation, or with chaotic turbulence at the time of galaxy formation, are definitely ruled out by the considerations here because they would necessarily have dissipated to isotropy long before that epoch by observation (d) of Section 1, and doubly ruled out by the entropy limits (b) and (c) of Section 1. The special solution would, however, allow a model in which density perturbations reach $\delta\rho/\rho \sim 0.1$, the maximum allowed by (2.1) as the perturbation came within the horizon. However, from our previous comments, at the end of Section 1 it is clear that random velocities or rotation are excluded from such a model. Additionally, Barrow [30] has concluded that turbulent models which are appropriate to form galaxies cannot give the correct light element synthesis.

The special model is in fact a version of a type due to Rees [31]. Rees supposes a model which is essentially cold, but strongly irregular, on all scales up to some maximum wavelength. The dissipation associated with this irregularity keeps the baryon matter density ρ_b approximately equal to the photon density ρ_γ as long as the dissipation continues. The largest contribution to the radiation field occurs for inhomogeneities which just come into the horizon at $Z \sim 10^4$; The inhomogeneity spectrum is arbitrarily cut off at this point (no longer wavelengths are allowed) because (a) this gives the correct $S_b \sim 10^8$ as observed now, and (b) this is the appropriate scale from which clusters of galaxies form.

However, Rees' model may be excluded on the basis that most of the entropy is produced

* Note contrary to popular opinion the vortex models are not really chaotic but require a very special initial state with only a flat spectrum of velocity perturbations [34], if they are to give galaxies as advertised.

too late, i.e. at $Z \sim 10^4$ rather than at $Z \sim 10^{10}$ as necessary for element synthesis [32]. (In Rees' model, $S_b \sim 10^3$–10^4 at element synthesis.) Only by appealing to clumping in the matter density can Rees produce the thermalization needed to give the clear 3 K background from such a late production of the entropy. An acceptable model of a hybrid-Rees type could be constructed [32], however, by having enough dissipation early on to produce the correct nucleosynthesis and effectively smooth the Universe, but still leaving growing modes which become large, consistent with (2.1) and with no rotation or random velocity at the horizon at $Z \sim 10^4$. A small amount of dissipation would be tolerated as the galaxies formed, if the inhomogeneities were sufficient to give thermalization via Rees' mechanism.

3 Conclusion

The conclusion of this work is that the chaotic cosmology is essentially ruled out. Since the anisotropy energy $\rho_\beta \propto Z^6$ decreases faster with expansion that does the photon density ρ_γ, this anisotropy, which must have been small at the element synthesis epoch, could not have been large at earlier times when dissipation is possible or entropy would have been overproduced. Thus only a moderate amount of chaos is permitted near the singularity. On the other hand, since the density of baryons $\rho_b \propto Z^3$ decreases more slowly than the photon density, dissipation of dust motions must not occur too late in the evolution or again entropy will be overproduced.

Rather than Misner's [33] hope of beginning with chaos and working toward a regular universe, we are constrained by the requirements that the anisotropy be small and the entropy moderate. This means that we live in a world which was approximately Robertson–Walker near a simultaneous singularity, and which is near Robertson–Walker now.

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References

[1] Wagoner, R., 1973. *Astrophys. J.*, **179**, 343.
[2] Thorne, K., 1967. *Astrophys. J.*, **148**, 51.
Hawking, S. & Tayler, R., 1966. *Nature*, **209**, 1278.
[3] Barrow, J. D., 1976. *Mon. Not. R. astr. Soc.*, **175**, 359.
[4] Doroshkevich, A. G., Lukash, V. N. & Novikov, I. D., 1973. *Zh. Eksper. Teor. Fiz.*, **64**, 1457. (Translated in 1973. *Sov. Phys. JETP*, **37**, 739.
[5] Misner, C. W., 1968. *Astrophys. J.*, **151**, 431.
[6] Matzner, 1971. *Ann. Phys.*, **65**, 438.
[7] Lifshiftz, E., 1946. *Zh. Eksper, Teor. Fiz.*, **16**, 587
[8] Peebles, P. J. E., 1972. *Comm. Astrophys. Sp. Sci.*, **4**, no. 2.
[9] Collins, C. B. & Hawking, S. W., 1973. *Mon. Not. R. astr. Soc.*, **162**, 307.
[10] Liang, E., 1975. *Phys. Lett. A*, **51**, 141.
[11] Matzner, R. & Misner, C., 1972. *Astrophys. J.*, **171**, 415.
[12] Misner, C., 1967. *Phys. Rev. Lett.*, **19**, 533.
[13] Liang, E., 1975. *Mon. Not. R. astr. Soc.*, **171**, 551.

[14] Matzner, R., 1968. *Astrophys. J.*, **154**, 1123.
[15] Hagedorn, R., 1970. *Astr. Astrophys.*, **5**, 184.
[16] Carr, B., 1975. *Astrophys. J.*, **201**, 1; **206**, 8.
[17] Hawking, S., 1974. *Nature*, **248**, 30.
[18] Berger, B. K., 1975. *Phys. Rev. D*, **11**, 2770.
[19] Parker, L., 1969. *Phys. Rev. (2nd Series)*, **183**, 1057.
[20] Zel'dovich, Ya. B. & Sunyaev, R. A., 1969. *Astrophys. Space Sci.*, **4**, 301.
[21] Perko, T., Matzner, R. & Shepley, L., 1972. *Phys. Rev.*, **6**, 969.
[22] Sunyaev, R. A. & Zel'dovich, Ya. B., 1970. *Astrophys. Space Sci.*, 7, 20; **9**, 368.
[23] Chan, K. L. & Jones, B. J. T., 1975. *Astrophys. J.*, **195**, 1.
[24] Gott, J. & Rees, M., 1975. *Astr. Astrophys.*, **45**, 365.
[25] Zel'dovich, Ya. B., 1972. *Mon. Not. R. astr. Soc.*, **160**, 1P.
[26] Landau, L. D. & Lifshitz, E. M., 1959. *Fluid mechanics*, Pergamon Press, London.
[27] Layzer, D., 1968. *Astrophys. Lett.*, **1**, 99.
[28] Saslaw, W., 1970. *Mon. Not. R. astr. Soc.*, **147**, 253.
[29] Harrison, E. R., 1971. *Mon. Not. R. astr. Soc.*, **154**, 167.
[30] Barrow, J. D., 1977. *Mon. Not. R. astr. Soc.*, **178**, 624.
[31] Rees, M., 1972. *Phys. Rev. Lett.*, **28**, 1669.
[32] Barrow, J. D., 1976. *Nature*, **267**, 117.
[33] Misner, C., 1969. *Phys. Rev.*, **186**, 1328.
[34] Ozernoi, L. M., 1976. *Confrontation of cosmological theories with observational data*, ed. Longair, M. S., Reidel, Dordrecht, Holland.