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THE HYDROGEN-DEFICIENT CARBON STARS

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Summary

Abundance analyses of the five known non-variable carbon stars with weak hydrogen lines are carried out. With the possible exception of Na all observable elements heavier than O have normal abundances relative to Fe. H is deficient in four of the stars by factors of more than 10^5; in the remaining star a deficiency of a factor of 50 is found. C is over-abundant by factors of 3 to 10.

The similarity of the compositions, space and velocity distributions and absolute magnitudes of the above stars, the R Cr B variables and the helium stars is stressed. These stars are collectively called hydrogen-deficient carbon (Hd C) stars and their various properties are discussed. They appear to form a class of star entirely different from the normal R and N stars.

The continuous opacity in the cool Hd C stars is shown to arise from contributions from He^-, C^-, electron scattering, Rayleigh scattering by He and C, and photoionization of carbon. It is shown that the number of Fe atoms per gram of stellar material lies in the range 2 to 0·3 times that in the Sun. Most indications are that the Hd C stars are slightly metal deficient, in agreement with their large space velocities.

It is suggested that the peculiar abundances of H, C, N and O in these objects arise from loss of outer envelope at a stage shortly after He-burning has started in a star of 1 $M_\odot$. The predicted abundances are then in reasonable agreement with observations, and it emerges that most of the O will be in the form of O^{18}.

The lifetime of the Hd C stage appears to be $\sim 10^3$ years, in which case secular changes should be observable. The proposed scheme of evolution shows how these helium stars appear as red supergiants and evolve into the white dwarf region of the HR diagram.

1. *Introduction*

1.1 *General.* The carbon stars are the most common form of late-type peculiar star, comprising perhaps a few per cent of all G, K and M giants. Within the carbon star class we may distinguish temperature differences, and to some extent abundance differences, from one star to another, but the dominant population seems to be Population I. This contrasts with the heavy-metal stars, which have a Population I (Ba and S stars) and an extreme Population II (CH stars) component. However, there are a few moderately high velocity stars that have been classed as carbon stars (notably the R Cr B stars) and it is important to investigate their properties in order to ascertain whether they too can be considered as Population II analogues of normal R and N stars, or whether they are in fact a quite distinct type of star. This latter possibility is a strong contender because, for example, it is known that the R Cr B stars have very weak hydrogen lines, which is not a normal characteristic of Population II objects.

Arising from this we shall discuss in this paper the objects known as hydrogen-deficient carbon stars (Hd C stars). In so doing we select stars according to spectral peculiarity, but it happens that we include almost all carbon stars having large peculiar space velocities (it should be noted that some normal N stars have moderately large radial velocities arising from effects of differential galactic rotation). If we anticipate that the observed strength of carbon features in these objects is a consequence of excess carbon in their atmospheres, then we can widen the discussion to include helium stars, which are known to be carbon-rich and hydrogen-poor (Hill 1965). These latter stars also have high space velocities.

1.2 *The hydrogen-deficient carbon stars.* Bidelman (1953) first pointed out that there exists an apparently well-defined class of stars that have enhanced carbon features and weakened hydrogen lines in their spectra. Earlier, Ludendorff (1906), Berman (1935) and Herbig (1949) had remarked on these peculiarities in the irregular variable R Cr B, and Bidelman (1948) had shown similar features in the spectrum of XX Cam. The non-variable star HD 182040 had also been known to have similar spectral peculiarities (McKellar & Buscombe 1948). It is generally assumed that all variables of the R Cr B type will have spectra similar to the type star (about fifteen have actually been observed spectroscopically). Bidelman (1953) listed 4 non-variable stars in the class, and the writer (Warner 1963) has added a further example. The latter star (HD 148839) is of additional interest because, although showing C_2 and CI enhanced and CH absent, it does have medium strength Balmer lines, and thus represents an intermediate type between normal C stars and the more extreme hydrogen deficiency occuring in other members of the class.

According to Payne-Gaposchkin (1963) about 30 R Cr B variables are known in the Galaxy. These range in effective temperature from about 2 500 °K (RS Tel : R8) through 6 550 °K (R Cr B : F8p) to about 15 000 °K (MV Sgr : early B). The variable V348 Sgr (Herbig 1958a) has a light curve with some R Cr B characteristics, is C and He rich and with an estimated equivalent spectral type of O9 may be an even hotter member of the Hd C class. The variable members of the class thus extend over almost the whole range of the stellar temperature scale.

The non-variable members consist of the five stars mentioned above, a possible member discovered by Stephenson (1965), and the five known helium stars. The latter, with the exception of BD + 37° 442 (Rebeirot 1966), have been discussed by Klemola (1961), and have temperatures of up to 20 000 °K.

Abundances have been derived in R Cr B itself by Berman (1935), Fujita (1947) and Searle (1961) and in the R Cr B type variable RY Sgr by Danziger (1965). Their principal conclusions were that whereas most metals have the same abundance in the R Cr B stars as in the Sun, the abundance of carbon is enhanced by a factor of about 20 and that of hydrogen depleted by $\sim 10^4$.

The analysis by Hill (1965) of three helium stars led to the conclusion that carbon is overabundant by factors of 5 to 10 and hydrogen is deficient by a factor $\sim 10^5$; no reliable results could be obtained for the heavier metals (e.g. Ca and the iron group) because their lines appear to be formed in a circumstellar envelope with unknown electron density.

2. *Basic data for the hydrogen-deficient carbon stars.* In Tables I(a) and I(b) we list some of the basic properties of the nonvariable members of the Hd C class and for a selection of the variable objects. The R Cr B variables are noticeably concentrated towards the galactic plane and galactic centre. Whereas the cool non-variable Hd C stars all appear quite close to the galactic centre the helium stars are more widely

distributed. Some of the implications of the space distributions and motions of these stars will be discussed later.

TABLE I(a)

Non-variable Hd C stars

Star (HD)	α(1900) h	m	δ(1900) °	′	m_V	m_{pg}	l^{II} °	b^{II} °	RV km/s	μ_α ″/yr	μ_δ ″/yr	Ref.
137613	15	21·9	−24	49	7·40		342	26	+55	−0·012	−0·026	(1)
148839	16	25·7	−66	55	8·33	9·07	322	−13	−31	+0·006	−0·030	(2)
173409	18	40·0	−31	28	9·18	9·85	4	−13	−65	−0·024	−0·035	(1)
175893	18	52·4	−29	38	9·3	9·9	7	−14	+42	+0·021	+0·016	(1)
182040	19	17·7	−10	53	7·04		27	−12	−47	−0·014	+0·012	(1)
+37° 442	01	55·4	+38	20	9·3		137	−23	−156			(3)
+10° 2179	10	33·7	+10	34	10·0		235	+54	+155	−0·028	+0·001	(4)
124448	14	08·6	−45	49	9·98	9·91	317	+14	−66	−0·010	−0·010	(5)
160641	17	36·0	−17	51	9·8		9	+7	+100			(6)
168476	18	14·7	−56	41	9·37	9·36	339	−18	+165	−0·008	+0·011	(7)

The first 5 stars are the cool Hd C stars analysed in this paper, the remaining are helium stars.

(1) Bidelman (1953).
(2) Warner (1963).
(3) Rebeirot (1966).
(4) Klemola (1961).
(5) Popper (1942).
(6) Bidelman (1952).
(7) Thackeray & Wesselink (1952).

TABLE I(b)

Variable Hd C stars

Star	α(1900) h	m	δ(1900) °	′	m_V (or m_{pg})	l^{II} °	b^{II} °	Notes
XX Cam	04	01·0	+53	06	7·1–8·7	149	0	RV = +16 km/s μ = 0″·003
SU Tau	05	43·2	+19	02	9·5–16·0	189	−5	
UW Cen	12	37·6	−53	59	(9·6–16)	301	9	
Y Mus	12	59·3	−64	59	(10·5–12·0)	304	−2	
DY Cen	13	19·3	−53	44	(12–16·4)	308	−8	
S Aps	14	59·4	−71	40	9·6–15·2	313	−12	
R Cr B	15	44·4	+28	28	5·8–>14	45	52	RV = +21 km/s μ = 0″·017
RT Nor	16	15·8	−59	07	(11·3–16·3)	327	−6	
WX Cr A	18	02·0	−37	20	(11·0–>16·5)	355	−8	
RS Tel	18	11·4	−46	35	(9·3–>13·0)	348	−14	μ = 0″·017
GU Sgr	18	18·1	−24	18	(11–15·0)	8	−5	
V Cr A	18	40·7	−38	16	(9·4–>14)	358	−15	
SV Sge	19	03·7	+17	28	(10·8–14·5)	50	+4	
RY Sgr	19	10·0	−33	42	6·5–14·0	4	−19	μ_α = +0″·056, μ_δ = −0″·080
V482 Cyg	19	55·9	+33	42	10·9–12·2	70	+2	
UV Cas	22	58·1	+59	04	(11·8–16·5)	109	−1	
VZ Sgr	18	16·8	−27	28	(11·8–>14)	5	−6	HD 317333 Sp B8
MV Sgr*	18	32·7	−21	03	(12·0–15·6)	13	−6	Sp B RV = −68 km/s
V348 Sgr	18	34·3	−23	00	(11·0–>16·5)	11	−8	Sp O9:
W Men	05	27·7	−71	16	(13·8–16·0)			In Large Magellanic Cloud

The first group of stars are cool Hd C stars.

* Strong CII and no HI according to Herbig (1964). Several of the unidentified emission lines in the spectrum may be attributed to CII and possibly HeII.

3. *Composition of the Hd C stars.* As mentioned above, two of the R Cr B stars have already been analysed for composition. In order to extend our knowledge of the Hd C stars the writer has undertaken a study of abundances in the non-variable cool Hd C stars. As can be seen from Table I(a), these are best observed from the southern hemisphere, and accordingly during the writer's stay at the Radcliffe Observatory in 1965 he secured spectra of the five members of this group (one plate had already been obtained in 1962).

3.1 *Observations and microphotometry.* The Z camera of the coude spectrograph on the 74-inch reflector of the Radcliffe Observatory was used in this study. A dispersion of 15·6 Å/mm was employed with IIaO emulsion. Fortunately the main mirror and the coude train were freshly aluminized, allowing the two faintest stars to be reached in reasonable exposure time. The material obtained is listed in Table II.

Table II

Coude spectra

Star (HD)	Plate	Date	Exposure (min.)
137613	DZ 719	14.6.62	180
	DZ 1492	15.8.65	130
148839	DZ 1467	16.7.65	275
173409	DZ 1493	15.8.65	280
175893	DZ 1487	14.8.65	350
182040	DZ 1468	16.7.65	90
	DZ 1481	13.8.65	120

The plates were traced with the recording microphotometer at the Yerkes Observatory, and photographic calibration curves were derived from intensitometer spots superimposed on the plates (or on plates from the same box and developed with the stellar plates).

The stars studied have equivalent spectral types of approximately G8, so we should expect to be able to estimate the position of the spectral continuum sufficiently well to measure equivalent widths. It is apparent, however, that the stars will have a large spectrum region complicated by the presence of the C_2 molecular bands. As a consequence, no lines were measured in the region 4570–4737 Å. On the other hand, whereas in normal stars of this temperature the CH band prevents reliable equivalent widths being measured at wavelengths shorter than about 4315 Å, no such restriction exists in these hydrogen-deficient stars, and lines may be measured almost to the head of the CN band at 4215 Å.

Equivalent widths of nearly 500 lines were thus measured in the ranges 4225–4570 Å and 4737–4960 Å. The procedure was that described previously (Warner 1965) and the reductions were carried out on the IBM 1620 computer at the Yerkes Observatory.*

Identification of spectrum lines was carried out with the aid of the list for normal K giants given by Gratton (1952) and the list of lines in R Cr B given by Keenan & Greenstein (1963).

* The writer is indebted to Dr C. R. Cowley for writing the FORTRAN program with which the reductions were made.

3.2 *Abundance determinations.* In general, especially when using material of a somewhat low dispersion, it is advantageous to perform a differential analysis relative to a standard star, the latter being observed with the same spectrographic equipment as that for the peculiar stars. Then systematic errors in spectrophotometry, and stratification effects in the stellar atmospheres, tend to be compensated. However, in the case of the Hd C stars this has not proved possible with the present plate material. In any standard star we would not have reliable line strengths for wavelengths shorter than 4315 Å, and in the Hd C stars the region 4570–4737 Å is unusable; consequently the overlap between stars would be considerably restricted and many important lines at the short wavelength end would be rejected. The writer has therefore carried out the analysis with the aid of laboratory oscillator strengths. These were selected from various sources, the most useful being those of Corliss & Bozman (1962), Corliss & Warner (1964) & Warner (1967). When the final derived abundances were compared with solar abundances (see later) care was taken to ensure that the source of the f-values was the same in both cases.

TABLE III

Atmospheric parameters

Star (HD)	137613	148839	173409	175893	182040	R Cr B
θ_{ex}	1·06	0·98	1·0	1·0	1·00	0·92
$\log v$	0·60	0·50	0·55	0·55	0·55	0·90
$\log a$	−3·0	−3·1	−3·1	−3·0	−3·0	−3·0
$\log P_e$	−0·4	0·4	0·0	−0·3	−0·2	0·0
f(C)	0·15	0·028	0·068	0·13	0·10	0·56
$\log \frac{N}{\kappa_\lambda} \cdot \frac{\kappa_\lambda^\odot}{N^\odot}$	2·6	2·1	2·5	2·8	2·6	1·8
$\log \kappa_{4550}(1)$	−3·6	−3·0	−3·3	−3·6	−3·3	−2·9
$\log \kappa_{4550}(0{\cdot}1)$	−4·2	−3·0	−4·0	−4·2	−4·0	−3·7
$\log \frac{N(Fe)}{N^\odot(Fe)}$ [$y = 1$]	0·0	0·1	0·2	0·2	0·3	−0·1
$\log \frac{N(Fe)}{N^\odot(Fe)}$ [$y = 0{\cdot}1$]	−0·6	0·1	−0·5	−0·4	−0·4	−0·9
$\Delta_{3463}(1)$	0·44	0·24	0·55	0·48	0·57	0·52
$\Delta_{3463}(0{\cdot}1)$	0·13	0·03	0·19	0·13	0·22	0·29
$\Delta_{3278}(1)$	0·31	0·18	0·34	0·30	0·35	0·34
$\Delta_{3278}(0{\cdot}1)$	0·10	0·04	0·14	0·10	0·16	0·21

Stellar excitation temperatures $\theta_{ex}(= 5\,040/T_{ex})$ were derived from Fe I lines in the usual manner and are listed in Table III. A sample curve of growth is given in Fig. 1, those for the other stars are very similar. It will be noticed that the curve of growth is very flat, indicating very low gas pressure in the stellar atmosphere, and it is also evident that we have only just reached down to the start of the weak-line portion of the curve of growth. For this reason the derived values of θ_{ex} are rather more uncertain (about $\pm 0{\cdot}04$) than is normally achieved in the coarse analysis; and as a result of this uncertainty, and also the difficulty of determining accurate shifts of curves of growth of one species relative to another, the abundances in this paper must only be regarded as preliminary. Much higher dispersion will be needed in order to reach the important weak lines in these stars.

Arising from these various difficulties it was found necessary to make some approximations in the analysis. The ionization temperature θ_{ion} was assumed to be given by $\theta_{\text{ion}} = \theta_{\text{ex}} - 0{\cdot}10$, a relationship generally valid for stars near solar temperature. The electron pressure P_e was then found from the relative positions of the Fe I and Fe II curves of growth. It may be thought disturbing that the resulting P_e depend on having the correct absolute scales in the Fe I and Fe II oscillator strengths, but the writer feels confident that at least the relative shifts between Fe I and Fe II are correct because these f-values give almost identical abundances of Fe from Fe I and Fe II lines in the Sun. The excitation temperature of C I and H I were taken equal to θ_{ion} because these lines arise from 7–10 volt levels and are formed deep in the stellar photosphere. A detailed calculation for the Sun supports this approximation. From multicolour photometry Mendoza & Johnson (1965) derive $\theta_{\text{eff}} = 1{\cdot}06$ and $0{\cdot}91$ for HD 137613 and HD 182040 respectively. These are in tolerable agreement with the θ_{ion} we have deduced.

The final abundances, taking log N(Fe) = 0·0, are presented in Table IV. Solar abundances, and their sources, are also listed.

3.3 *Atmospheric parameters.* The atmospheric parameters found from the coarse analysis are given in Table III. From the low value of the damping constant, a, and of the electron pressure it is apparent that the nonvariable cool Hd C stars have extended atmospheres and must be of high luminosity. The moderately large velocity parameter, v, is in accordance with this conclusion.

3.4 *Discussion of abundances.* With the exception of C and H the abundances relative to Fe given in Table IV are seen to be similar in all the stars analysed. The variations are probably due to the difficulties of analysis, so a final weighted mean has been given. Comparison of this with solar abundances shows that, with the possible exception of Zn, the elements heavier than Na have normal (i.e. solar) abundances in the Hd C stars. The Zn abundance is based on only one line and is on the plateau of the curve of growth where an error of only about 80 per cent in equivalent width can lead to an error of an order of magnitude in abundance. The Na abundance is similarly unreliable (based on two lines), but it should be noted that Danziger (1965) obtained a similar, but more certain, over-abundance of Na in RY Sgr, and Fujita (1947) obtained a normal abundance of Na, Mg and Zn in R Cr B.

Carbon is overabundant by factors of 3 to 10, compared with the Sun. In the region where the high volt lines of CI are formed, formation of CO should be so small as to cause a negligible reduction in the free C. Only in the intermediate star HD 148839 (see Section 1.2) could the Balmer lines be detected with certainty. Consequently only upper limits on the hydrogen abundance can be given in the four other stars. In HD 148839 we have had to assume that HI will follow the same curve of growth as Fe I (with appropriate corrections for difference in turbulent velocity of H atoms) and although the H abundance is uncertain a deficiency of a factor of 50 is indicated. For the other stars the deficiency is a factor of 10^5 or more compared to the H/Fe ratio in the Sun.

Similar results were obtained by Searle (1961) and Danziger (1965) for the stars R Cr B and RY Sgr, and Hill's (1965) analysis of the Helium stars gave comparable C/H ratios.

4. *Absolute abundances.* In order to obtain absolute abundances (i.e. number of atoms per gram of stellar material) of the elements in the atmospheres of our stars we need to know the value of the continuous absorption coefficient. And in order to calculate this we must first know the composition of the atmosphere.

TABLE IV

Logarithmic relative abundances (log N(Fe) = 0·0)

Star (HD)	137613	148839	173409	175893	182040	Mean	Sun	Ref.
H I	<0·8	3·7	<0·7	<0·6	<0·1	—	5·4	(2)
C I	2·6	2·4	2·9	2·6	2·9	—	1·9	(3)
Ca I	−0·8	−0·6	−0·5	−0·6	−0·4	−0·6	−0·6	(1)
Na I	0·5	0·5	0·7	0·8	0·7	0·6	−0·4	(2)
Sc I	—	−2·6:	−2·6:	−2·9:	−2·7:	−3·3	−3·6	(4)
Sc II	−3·3	−3·1	−3·3	−3·2:	−3·2			
Ti I	−1·5	−1·5	−1·5	−1·5	−1·8	−1·8	−2·0	(1)
Ti II	−2·2	−1·8	−1·8	−1·9	−2·0			
V I	−2·0	−2·2	−2·2	−1·9	−2·5	−2·2	−2·5	(1)
V II	−2·8:	—	—	—	−2·9:			
Cr I	−0·8:	−1·3	−1·1	−0·7	−1·1	−1·1	−1·5	(1)
Cr II	−1·1	−1·0	−1·1	−1·1:	−1·5			
Mn I	−2·5	−2·1	−2·1	−2·3	−2·3	−2·3	−1·8	(1)
Co I	−2·0:	−2·0	−2·1	−1·8	−1·6	−1·9	−2·2	(1)
Ni I	−1·3	−1·2	−1·1	−1·0	−1·0	−1·1	−0·9	(5)
Zn I	−4·2:	−4·0:	−4·2:	−4·2:	−4·1:	−4·1:	−3·2	(4)
Y II	−3·9	−4·0	−4·0	−3·6	−4·0	−3·9	−4·5	(6)
Zr II	−3·8	−4·4	−4·0	−4·4:	−4·1:	−4·1	−4·4	(6)
Ba II	−4·8	−5·3:	−5·6:	−5·1:	−4·8	−5·1	−4·9	(6)
La II	−4·6	−5·1	−5·3	−4·8:	−4·7	−4·9	−4·9	(6)
Ce II	−5·7	−5·6	−5·7	−5·4	−5·3:	−5·5	−5·2	(6)
Nd II	−5·2	−5·4	−5·1	−5·0	−5·0	−5·1	−5·0	(6)
Sm II	−5·2	−5·6	−5·5	−5·2	−4·7	−5·2	−5·5	(6)

Solar abundance sources: (Fe abundance from Goldberg *et al.* 1964)

(1) Muller & Mutschlecner 1964.
(2) Goldberg *et al.* 1960.
(3) Lambert 1967.
(4) Aller 1965.
(5) Cowley 1966.
(6) Wallerstein 1966.

In normal stars this circularity is avoided because hydrogen is the dominant source of opacity and is also, neglecting He, by far the most abundant element. Consequently metal-to-hydrogen ratios can be determined without ambiguity. However, in the cool Hd C stars the low abundance of H shows that some other source of continuous opacity must be available. At the same time it is evident that the dominant constituent of the atmospheres of the cool Hd C stars is not spectroscopically prominent. It is probable that helium is this constituent. Both Searle and Danziger adopted this hypothesis in their analysis, partially justified by the unexpected presence of the He I triplet at 5876 Å in both R Cr B and RY Sgr, and also (by process of elimination) by the normal strength of OI and NI in the infrared spectra of these stars. For these same reasons, and with the example of the helium stars in which He quite clearly is the dominant constituent, we adopt He as the principal component. Then the problem is to determine the Fe/He ratio in the atmospheres of our stars. The procedure is one of self-consistency: assume an Fe/He ratio, which (with Table IV) fixes the abundance of those elements responsible for the continuous opacity; calculate the continuous absorption coefficient κ_λ; finally test whether the assumed Fe/He ratio is recovered by combining the calculated κ_λ with the observed line strengths.

Fitting a theoretical curve of growth to the observed curves indicates the position of the weak-line portion of these curves. Clearly this cannot be done with any great precision because of the flatness of the empirical curves (Fig. 1). Then

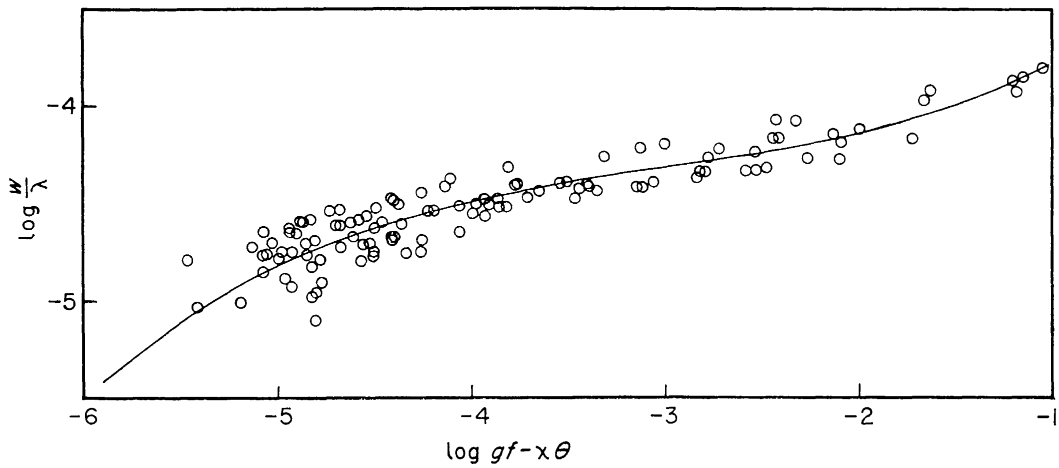

FIG. 1. *Curve of growth for Fe I in HD* 182040.

from the weak-line portion we can derive log N(Fe I)/κ_λ, and an ionization correction gives log N(Fe)/κ_λ. Application of the same procedure to solar lines provides log N$^\odot$(Fe)/$\kappa_\lambda{}^\odot$. Values of

$$\log \frac{\mathrm{N(Fe)}}{\mathrm{N}^\odot(\mathrm{Fe})} \frac{\kappa_\lambda{}^\odot}{\kappa_\lambda}$$

are given in Table III. Using the calculated κ_λ ($\kappa_{4550}{}^\odot = 0{\cdot}10$) leads to evaluation of

$$\log \frac{\mathrm{N(Fe)}}{\mathrm{N}^\odot(\mathrm{Fe})}.$$

5. *Sources of continuous opacity.* The continuous absorption in the five stars we have analysed above will be formed in the region where the optical depth is of the order of unity. The temperature at this level will be close to θ_{ion}, so we adopt $\theta = 0{\cdot}90$ as characteristic of the continuum for the five non-variable cool Hd C stars. The possible contributors to the opacity are detailed below, H$^-$ is only of importance in HD 148839. Because of the absorption edges at 3463 Å and 3278 Å in photoionization of carbon we will need to calculate the total opacity on each side of these wavelengths, and also in the middle of the range of our equivalent widths, i.e. 4550 Å.

(i) *H*$^-$. Doughty & Fraser (1966) give, for $\theta = 0{\cdot}9$ and $\lambda = 4550$ Å,

$$\kappa(\mathrm{H}^-) = 4{\cdot}1 \times 10^{-26}\,\mathrm{cm}^4\,\mathrm{dyne}^{-1}\,\mathrm{atom}^{-1}$$

$$= 0{\cdot}6 \times 10^{-2} \frac{\mathrm{N(H)}}{\mathrm{N(He)}} P_e\,\mathrm{cm}^2\,\mathrm{g}^{-1}$$

where we have assumed that the atmosphere consists mostly of He. The corresponding coefficient for both 3463 and 3278 Å is $4{\cdot}2 \times 10^{-3}$.

(ii) *He⁻*. The calculations of McDowell *et al.* (1966) show that at $\theta = 0{\cdot}9$

$$\kappa(\mathrm{He}^-) = 3{\cdot}5 \times 10^{-28}\ \mathrm{cm}^4\ \mathrm{dyne}^{-1}\ \mathrm{atom}^{-1}$$

with little variation over the range 3250–4700 Å or sensitivity to θ. Hence

$$\kappa(\mathrm{He}^-) = 4{\cdot}8 \times 10^{-5}\ P_e\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$

for a He atmosphere.

(iii) *C⁻*. The results of McDowell & Myerscough (1966) give, for $\theta = 0{\cdot}9$

$$\kappa(\mathrm{C}^-) = 7 \times 10^{-26}\ \mathrm{cm}^4\ \mathrm{dyne}^{-1}\ \mathrm{atom}^{-1}$$

again nearly constant over our wavelength range. Hence

$$\kappa(\mathrm{C}^-) = 1{\cdot}0 \times 10^{-2}\ \frac{\mathrm{N(C)}}{\mathrm{N(He)}}\ \{1-f(\mathrm{C})\}P_e\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$

where $f(\mathrm{C})$ is the fraction of C in the once-ionized state.

(iv) *Electron scattering.* $\sigma(\mathrm{e}) = 6{\cdot}65 \times 10^{-25}\ \mathrm{cm}^2$ per electron

$$= 0{\cdot}10\ \frac{P_e}{P_g}\ \mathrm{cm}^2\ \mathrm{g}^{-1} \text{ for He atmosphere.}$$

(v) *Rayleigh scattering by He.* $\sigma(He) = 5{\cdot}60 \times 10^{-14}\ \lambda^{-4}\ \mathrm{cm}^2\ \mathrm{atom}^{-1}$ (λ in Å) which gives:

$$\left.\begin{array}{ll} 3278\ \text{Å} & 7{\cdot}3 \\ 3463\ \text{Å} & 5{\cdot}5 \\ 4550\ \text{Å} & 2{\cdot}2 \end{array}\right\} \times 10^{-5}\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$

for He atmosphere.

(vi) *Photoionization of carbon.* Reliable values for the photoionization cross-sections of neutral carbon can be obtained from the Quantum Defect method (Seaton 1958) and have been given by Griem (1964). In Fig. 2 we illustrate $\kappa(\mathrm{C})$ as a function of wavelength for $\theta = 0{\cdot}9$. $\kappa(\mathrm{C})$ is highly sensitive to temperature due to the fact that to contribute to the opacity in the visible region photoionization must take place from 7 or 8 volt excited levels. From Fig. 2 we obtain, for $\theta = 0{\cdot}9$ and $\lambda = 4550$ Å,

$$\begin{aligned}\kappa(\mathrm{C}) &= 1{\cdot}7 \times 10^{-25}\ \mathrm{cm}^2\ \mathrm{atom}^{-1} \\ &= 2{\cdot}6 \times 10^{-2}\ \frac{\mathrm{N(C)}}{\mathrm{N(He)}}\ \{1-f(\mathrm{C})\}\ \mathrm{cm}^2\ \mathrm{g}^{-1}.\end{aligned}$$

Either side of the absorption edge at 3463 Å the appropriate coefficients are $2{\cdot}0 \times 10^{-2}$ and $5{\cdot}0 \times 10^{-2}$, and either side of 3278 Å they are $5{\cdot}0 \times 10^{-2}$ and $8{\cdot}0 \times 10^{-2}$. Approximate values for other temperatures may be obtained by multiplying these $\kappa(\mathrm{C})$ by $10^{-7\cdot7(\theta-0\cdot90)}$.

(vii) *Rayleigh scattering by C.* Dalgarno (1962) gives a value for the polarizability of C which leads to

$$\sigma(\mathrm{C}) = 5{\cdot}6 \times 10^{-12}\ \lambda^{-4}\ \mathrm{cm}^2\ \mathrm{atom}^{-1}\ (\lambda \text{ in Å})$$

Hence we may write

$$\sigma(\mathrm{C}) = 100\ \frac{\mathrm{N(C)}}{\mathrm{N(He)}}\ \{1-f(\mathrm{C})\}\sigma(\mathrm{He})\ \mathrm{cm}^2\ \mathrm{g}^{-1}.$$

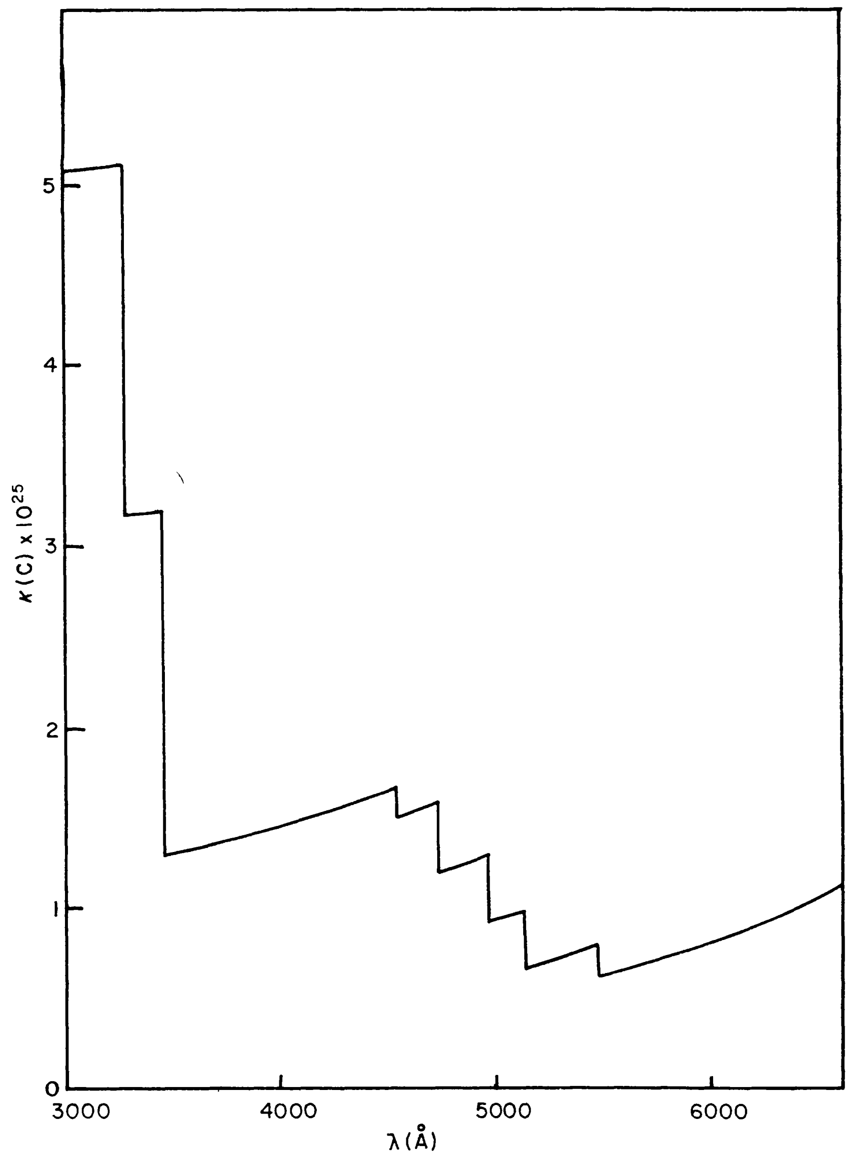

FIG. 2. *Bound–free absorption coefficient for neutral carbon at θ = 0·90.*

6. *Evaluation of the opacity.* We will follow through the details of the opacity calculation for HD 182040. From Tables III and IV we have

$$\theta = 0{\cdot}9, \log P_e = 0{\cdot}2, \log \frac{\mathrm{N(C)}}{\mathrm{N(Fe)}} = 2{\cdot}9 \text{ and } f(\mathrm{C}) = 0{\cdot}10.$$

Because C is so abundant it turns out to be the dominant source of electrons, so

$$\frac{P_e}{P_g} = \frac{\mathrm{N(C)}}{\mathrm{N(He)}} f(\mathrm{C}).$$

Let us first assume that the number of Fe atoms per gram of stellar material is the same as that in the Sun. We define y as the *assumed* ratio $\mathrm{N(Fe)/N^{\odot}(Fe)}$ and calculate $\kappa_\lambda(y)$. Then N(C)/N(He) for $y = 1$ is $1{\cdot}0 \times 10^{-2}$ in HD 182040. Hence we have (all κ will be given in $\mathrm{cm^2\,g^{-1}}$)

		3278 Å	3463 Å	4550 Å
$\kappa(\text{He}^-)$		$3\cdot0\times10^{-5}$	$3\cdot0\times10^{-5}$	$3\cdot0\times10^{-5}$
$\kappa(\text{C}^-)$		$5\cdot7\times10^{-5}$	$5\cdot7\times10^{-5}$	$5\cdot7\times10^{-5}$
$\sigma(\text{e})$		$1\cdot0\times10^{-4}$	$1\cdot0\times10^{-4}$	$1\cdot0\times10^{-4}$
$\sigma(\text{He})$		$7\cdot3\times10^{-5}$	$5\cdot5\times10^{-5}$	$2\cdot2\times10^{-5}$
	blue	$7\cdot2\times10^{-4}$	$4\cdot5\times10^{-4}$	
$\kappa(\text{C})$				$2\cdot3\times10^{-4}$
	red	$4\cdot5\times10^{-4}$	$1\cdot8\times10^{-4}$	
$\sigma(\text{C})$		$6\cdot6\times10^{-5}$	$5\cdot0\times10^{-5}$	$2\cdot0\times10^{-5}$

giving $\kappa_{4550}(1) = 4\cdot6\times10^{-4}$.

We further define

$$\Delta_\lambda(y) = \frac{\kappa_{\lambda_b}(y)-\kappa_{\lambda_r}(y)}{\kappa_{\lambda_r}(y)}$$

where λ_b and λ_r refer respectively to wavelengths just to the blue and to the red of the discontinuity at λ. Then for HD 182040 $\Delta_{3463}(1) = 0\cdot57$ and $\Delta_{3278}(1) = 0\cdot35$.

On the other hand, if we adopt $y = 0\cdot1$, we have

		3278 Å	3463 Å	4550 Å
$\kappa(\text{He}^-)$		$3\cdot0\times10^{-5}$	$3\cdot0\times10^{-5}$	$3\cdot0\times10^{-5}$
$\kappa(\text{C}^-)$		$5\cdot7\times10^{-6}$	$5\cdot7\times10^{-6}$	$5\cdot7\times10^{-6}$
$\sigma(\text{e})$		$1\cdot0\times10^{-5}$	$1\cdot0\times10^{-5}$	$1\cdot0\times10^{-5}$
$\sigma(\text{He})$		$7\cdot3\times10^{-5}$	$5\cdot5\times10^{-5}$	$2\cdot2\times10^{-5}$
	blue	$7\cdot2\times10^{-5}$	$4\cdot5\times10^{-5}$	
$\kappa(\text{C})$				$2\cdot3\times10^{-5}$
	red	$4\cdot5\times10^{-5}$	$1\cdot8\times10^{-5}$	
$\sigma(\text{C})$		$6\cdot6\times10^{-6}$	$5\cdot0\times10^{-6}$	$2\cdot0\times10^{-6}$

giving $\kappa_{4550}(0\cdot1) = 9\cdot3\times10^{-5}$, $\Delta_{3463}(0\cdot1) = 0\cdot22$ and $\Delta_{3278}(0\cdot1) = 0\cdot16$. Any further lowering of y will not greatly reduce $\kappa_{4500}(y)$ because $\kappa(\text{He}^-)+\sigma(\text{He}) = 5\cdot2\times10^{-5}$ is independent of y.

Similar results may be obtained for other stars and they are summarized in Table III. It can be seen that the assumption of normal Fe abundance results in the recovery of the same abundance (within the limits of error), whereas adoption of one-tenth normal abundance does not produce a consistent solution. However, for $y > 1$ the opacity is dominated by $\sigma(\text{e})$ and $\kappa(\text{C})$, and then $\kappa_{4550}(y) \propto y$, so any assumed y will result in a consistent solution. Allowing for observational error it turns out that any $y \gtrsim 0\cdot3$ leads to self consistency.

7. *Application to the R Cr B stars.* The writer has reanalysed the equivalent widths given by Searle (1961). The work of Danziger (1965) has shown that RY Sgr is almost identical to R Cr B, so we need discuss only R Cr B itself. Some changes in Searle's analysis have been found necessary. Searle used the same θ_{ex} as for the Sun ($\theta_{\text{ex}}^\odot \simeq 1\cdot00$), whereas the writer, using Fe I f-values, finds $\theta_{\text{ex}} = 0\cdot92$. In addition, Searle (and Danziger) adopted $\theta_{\text{ion}}^\odot = 0\cdot89$ and $\log P_e^\odot = 1\cdot51$, whereas coarse analyses using f-values (Warner, unpublished) and model atmospheres (Cayrel & Jugaku 1963) lead to $\theta_{\text{ion}}^\odot = 0\cdot95$ and $\log P_e^\odot = 0\cdot50$. A direct consequence of this change is that $\kappa^\odot$ is much smaller than adopted by Searle and thus the opacity in R Cr B (assuming $y = 1$) is reduced by the same factor (of five). The new analysis leaves unaltered the conclusions that elements heavier than Na have normal relative abundances in R Cr B, but C/Fe is slightly reduced to a value of about 10 times that in the Sun.

Thus R Cr B (and also RY Sgr) is somewhat hotter than the five stars analysed in the previous Section. This confirms the conclusion reached by Bidelman (1953) from visual intercomparison of low dispersion spectra of these stars.

The atmospheric parameters adopted for R Cr B, and the resulting opacities, are listed in Table III. It can be seen that any assumed y greater than about 0·3 will fit the observations. In order to fix the absolute abundances within more restricted limits we must appeal to the observations of the continuous spectra of these stars.

8. *The continuous spectra of the cool Hd C stars.* We can set upper limits on the C/He ratio by searching for the discontinuities in the continuous spectra resulting from the wavelength variation of κ(C) (see Fig. 2). The predicted changes in flux at these discontinuities, roughly proportional to the change in opacity, are given as a function of composition by $\Delta_\lambda(y)$ (Table III). The decrement in magnitudes at the absorption edge is $-2{\cdot}5 \log \{1+\Delta_\lambda(y)\}$.

Observations pertaining to the continuum of the Hd C stars are meagre. The spectral scan of R Cr B at maximum light given by Oke (Payne-Gaposchkin 1963) extends to 3390 Å and thus crosses the C absorption edge at 3463 Å. No decrement greater than about 0·2 mag is evident, and it is probable that an even smaller decrement is indicated by this scan. The six colour photometry of RY Sgr given by Danziger (1965), compared with the supergiant β Aqr, shows reasonable agreement in all six colours. The U magnitude is actually somewhat brighter in the Hd C star. The peak sensitivity of the U band almost coincides with the 3478 Å edge, and the additional discontinuity at 3278 Å would also just lie in this band, so any large decrement would result in U faint compared to a normal star. On the other hand, the Balmer continuum is absent in the Hd C star. It is probably reasonable to conclude that no large C decrements can exist in RY Sgr.

The writer's coude spectra of the cooler Hd C stars extend in some cases just shortward of 3463 Å, and no excessive weakening of the lines is discernable on the shortward side.

From Table III we find that for $y = 1$ in R Cr B a decrement of 0·52 mag would be observed, whereas for $y = 0{\cdot}1$ the decrement would be 0·29 mag. These values are not of high accuracy because of uncertainties in the C abundance and the atmospheric parameters. It is evident, however, that for all of the stars studied no value of y much greater than unity would be consistent with the observations. It is still possible that R Cr B and RY Sgr are metal-deficient by a factor of ten, but the possible range in the cooler non-variable stars is about 2 to 0·3 times that in the Sun.

9. *Discussion.* From Section 6 it is evident that all the opacity sources (other than H^-) considered contribute significantly to the total continuous absorption throughout the range 3000–4550 Å, and calculations show that this is so even out to 7000 Å. Any model atmospheres calculated for the cool Hd C stars must therefore incorporate all of these contributors in the opacity code.

The fact that photoionization of carbon is not the dominant opacity source in any of the Hd C stars is a conclusion different from that reached by Searle and Danziger. This difference arises primarily because the photoionization cross-sections used in this paper are considerably smaller than those calculated by Searle from an hydrogenic approximation. On the other hand this same conclusion does solve a problem posed by Searle, namely that completely different sources of opacity would be needed in the cooler members of the R Cr B class. He noted that in all the R Cr B stars the metallic line spectrum at low dispersion appears similar in strength to normal stars of equivalent spectral type—indicating that the continuous absorption also be similar in value. In fact this is a *non sequitur*; because of the supergiant nature of these stars, and the consequent small value of the damping constant, large changes in continuous opacity (or abundance) can take place without altering the appearance of the strong lines. In the coolest of the R Cr B stars the opacity may be very much less than in normal stars of equivalent type, but the resulting increases in line strength produce such small changes in equivalent width as to be unobservable at low dispersion.

The continuous opacity in the intermediate star HD 148839 is just dominated by H^- absorption. It is pleasing to find that the value of $\kappa_{4550}(1)$ calculated for a one-fiftieth reduction in hydrogen content leads to normal Fe abundance and thus supports the hydrogen abundance deduced from the Balmer lines.

An improvement in the accuracy of the carbon abundance could be achieved by measuring the strength of the [CI] line at 8727 Å. This arises from a 1·26 volt level and has an equivalent width of 6·5 mÅ in the Sun. In the presently investigated stars, with temperatures similar to that of the Sun but opacities reduced by a factor of 100 and C abundance increased by a factor of 10, the equivalent width of this line should lie between 50 and 100 mÅ.

Finally, while it has not been possible to decide on the absolute abundance of metals in these stars, other than in the range 2 to 0·3 times normal (and down to 0·1 for R Cr B), Wallerstein (1965) has pointed out that to account for the C, N and O abundances in the helium stars it is necessary to assume that initial abundances of metals heavier than He in these stars were *lower* by a factor of 3 than solar ones. This value lies within the range permitted by the present study of the cooler Hd C stars and may indicate that the R Cr B stars may be metal-deficient by a similar amount.

10. *The nature of the hydrogen-deficient carbon stars*. Before continuing with discussion of the abundances we examine other properties of the Hd C stars that are germane to their stage of evolution.

Klemola (1961) has summarized data concerning the luminosities of the helium stars. Various estimates have been made, based on strength of interstellar lines and on proper motions. All agree that the helium stars are intrinsically quite luminous, with M_v probably lying in the range -2 to -4. This would put $M_{\rm bol}$ in the range -4 to -6. Herbig (1964) finds $M_{\rm bol} \sim -5$ for MV Sgr. Herbig (1958b) summarizes data on the absolute magnitudes of the R Cr B variables. Values of M_v around -4 or -5 are indicated; certainly the R Cr B star W Mens in the Large Magellanic Cloud has $M_v \sim -5$. R Cr B itself will have a small bolometric correction, but the later spectral types will have substantial corrections, so the mean $M_{\rm bol}$ will lie in the range -5 to -6. The similarity to the helium stars is marked.

The nonvariable cool Hd C stars have proper motions large enough (Table I(a)) and reliable enough (the sources are Cape and Yale) to be accepted as of correct order of magnitude. If these stars are as luminous as their variable analogues, and the atmospheric parameters of Table III certainly indicate high luminosity, then with allowance for galactic absorption they have transverse velocities of about 400 km/s. As these stars are all close to the direction of the galactic centre their radial velocities will not necessarily be of the same order as their transverse velocities (though some large ones might be expected), but space velocities in excess of 400 km/s indicate membership of a halo population. Similarly, as the five stars in this class are all in the same part of the sky, the spread in radial velocities and proper motions indicates a large velocity dispersion, which is a characteristic of a high velocity group. On the other hand, the galactic distribution of R Cr B stars (Table I(b)) does not show the characteristics of a halo population. Because of the ease of discovery of R Cr B stars and the relative difficulty of finding the nonvariable members it is perhaps valid to conclude that we have found only the few brightest members of a much more populous non-variable class. They may have $M_v > -5$ and more consistent space velocities.

The attempt at determining absolute magnitudes of carbon stars from multi-colour photometry by Mendoza & Johnson (1965), in which they assign $M_v \sim -1$ for HD 137613 and HD 182040, cannot be taken seriously. As they assume that all the carbon stars are giants, circular argument results in their conclusions that the $M_{\rm bol}$–log T_e diagram for carbon stars is similar to that for giant stars of normal composition.

The helium stars seem to be spread more widely in the Galaxy than the cooler Hd C stars. Their somewhat larger radial velocities is probably a result of this. A selection effect is probably in action here: helium stars have been found by looking at faint high galactic latitude B stars. Any closer to the galactic plane would not only have to be distinguished from the myriads of faint B stars but would in any case be very faint because of interstellar absorption.

It is significant that the mean apparent magnitude difference between the five brightest cool Hd C stars and the five known helium stars is comparable to the difference in bolometric correction between the two groups, indicating that the two groups are at comparable mean distances. There are many reasons to believe, therefore, that all the Hd C stars are similarly distributed in the Galaxy, and that they belong to a population of at least moderately high space velocity, say Intermediate Population II.

The masses of the Hd C stars are uncertain. The data given by Klemola (1961) for BD$+10°$ 2179, and the effective surface gravity determinations by Hill (1965) in two other helium stars (with $M_{\rm bol} \sim -4$ and $T_{\rm eff}$ obtained from scaled up spectroscopic temperatures) give masses in the range 0·1 to 0·5 $M_\odot$. These calculations assume hydrostatic equilibrium, the validity of which is not known for the atmospheres of the helium stars. In the cool Hd C stars masses $\sim 0{\cdot}1\ M_\odot$ are derived from the spectroscopically determined P_g, but similarly small masses are obtained for normal supergiants which should have masses $\sim 5\ M_\odot$. In these cases the hydrostatic equation almost certainly is at fault because of turbulence in the supergiant atmospheres.

Above we saw that the Hd C stars are distributed like Intermediate Population II stars which, to be in an advanced stage of evolution and to be of an age comparable with that of the Galaxy, would mean that they initially had masses slightly in excess of 1 $M_\odot$.

The presence of numerous sharp lines in the spectra of the helium stars (Hill 1964) indicates low rotational velocity for these objects, which is strong support for our conclusion that they are old stars.

11. *Interpretation of the abundances.* The extreme hydrogen deficiency of the Hd C stars meets with only one plausible interpretation—that of extensive mass loss resulting in disappearance of the hydrogen-rich envelope at least down to the point where only the helium-rich core remains. None of the Hd C stars is known to be a close binary so we conclude that these stars lost their outer layers during the natural course of evolution. Possible reasons for mass loss will be mentioned later, but if this hypothesis is correct it is clear that in the Hd C stars we are afforded a rare insight into the internal composition of stars in an advanced stage of evolution.

The predominance of He is the result of conversion from hydrogen. Hill (1962) has shown that in the helium stars the helium is mostly He^4. The excess of carbon leads to the supposition that He-burning by the triple-alpha process has taken place in the core (or a shell source surrounding a carbon-rich core), followed by mixing to the surface. It is well known from low-dispersion studies (Bidelman 1953, 1956) that the cool Hd C stars show no signs of molecular features due to C^{13}. Searle and Danziger failed to detect any such features at high dispersion in R Cr B and RY Sgr, and the present writer has found no indications of C^{13} in the cooler non-variable Hd C stars. This is a further indication that the excess carbon comes from $3\alpha \rightarrow C^{12}$.

It is possible that there are several ways of accounting for the observed abundances in the Hd C stars. The importance of attempting to interpret the composition of these stars by using current knowledge of stellar evolution and nuclear reactions lies in the expectation that by so doing we may find that many elements, some so far unobserved, will have predicted abundance or isotopic peculiarities thus providing a stimulus for extending observations in some specific direction. One explanation of the abundance anomalies (the only one here considered satisfactory) is given later in this section.

Wallerstein (1965) has used the calculations on the equilibrium of the CNO cycle (Caughlan & Fowler 1962) to interpret the helium star compositions. He proposed three different schemes, the first to account for the fact that in HD 168476, HD 124448 and BD + 10° 2179 oxygen is not observed (and is deficient by a factor of at least 3), and the other two to account for the nearly normal abundance of O in HD 160641. His first scheme results in a predicted O deficiency of a factor $\sim$200, which, while consistent with the absence of O lines in the above 3 stars, seems excessive when it is noted that O is *just* detectable in HD 160641 and BD + 37° 442.

The abundances of N and O in the non-variable Hd C stars cannot be determined from the present plate material. Danziger's (1965) analysis of RY Sgr suggested that N was normal (or somewhat enhanced), and O deficient by a factor of 3. Keenan & Greenstein (1963) from a careful comparison of R Cr B with α Per find from weak lines that slight enhancements of both N and O are indicated. In view of the importance of the C, N and O abundances in these objects it would be valuable to attempt more accurate determinations with high dispersion spectra, perhaps using the forbidden lines in the R Cr B stars (c.f. [CI] in Section 9).

From the space and velocity distributions of the Hd C stars it is clear that they are fairly old stars, formed near the end of the initial collapse of the Galaxy. To have reached an advanced stage of evolution in a time of about twice the age of the Sun they would have initial masses very close to 1 $M_\odot$; the times to reach the helium-burning phase for stars of 1, 1·25 and 1·5 $M_\odot$ are about 10·9, 4·5 and

$2 \cdot 3 \times 10^9$ years respectively (Iben 1967a). It seems likely that if the Hd C stars are metal-deficient then it is only by a small factor and will not affect conclusions based on evolution of Population I stars.

Apparently the loss of envelope occured at some stage in the evolution of a star of closely 1 $M_\odot$. In order to trace the possible history of the Hd C stars we need to follow through the evolutionary changes in a 1 $M_\odot$ stars as calculated by Iben (1965, 1967a, 1967b). The full development of evolution for such a star has not been given, but Iben (1967a) gives results for stages just prior to the helium-flash in a $2 \cdot 25$ $M_\odot$ star in order that similarity arguments may indicate analogous stages in stars of smaller masses.

We are interested in development and composition of the *core* of a 1 $M_\odot$ star. According to Iben (1967a) although the *p–p* chain provides the energy in a non-convective core for most of the hydrogen-burning phase, when the shell source is well established the CNO cycle eventually dominates. This processes all initial C, N and O and redistributes towards equilibrium proportions (Caughlan & Fowler 1962). The result is that a large proportion of the O^{16}, and practically all the C^{12}, are converted to N^{14}. Just before helium-burning the core contains about 20 per cent of the total mass of the star.

Subsequently the temperature in the degenerate He core increases to the point where helium-burning by $N^{14}(\alpha, \gamma)\, F^{18}(\beta^+\nu)\, O^{18}$ takes place (starting with an 'N^{14}-flash'). There results a convective core in which N^{14} is fed from the periphery and converted to O^{18}. At some later stage helium-burning by $3\alpha \rightarrow C^{12}$ starts. The central temperature is probably too low for the reaction $C^{12}(\alpha, \gamma)\, O^{16}$ to work.

It is at this stage that we postulate a major change in the structure of the star. By nova-like, or planetary nebula-like, processes the outer envelope is removed. The resulting lowered opacity of the outer regions causes a change in structure and complete mixing ensues. The star will rapidly diminish in size. Just prior to this collapse the star consists of an outer zone rich in He^4 and N^{14} (but depleted in C^{13} and O^{16}) in which some C^{12} may have already been mixed from the interior, and an inner core in which N^{14} has been converted to O^{18} and C^{12} has been produced from He^4.

Let us suppose that in the core of the original star a fraction z of the O^{16} was converted to N^{14}. If we define A_X as the initial abundance of element X (we will use the scale on which $\log A_{\mathrm{H}} = 12 \cdot 00$ for normal composition) then in the core the abundance A_{N}' of nitrogen just before helium-burning is given by

$$A_{\mathrm{N}}' = A_{\mathrm{N}} + A_{\mathrm{C}} + zA_{\mathrm{O}},$$

and effectively $A_{\mathrm{O}}' = A_{\mathrm{C}}' = 0$. After an inner core has been established, in which $N^{14} \rightarrow O^{18}$ takes place, we have for the abundances in this region,

$$A_{\mathrm{N}}'' = 0 \text{ and } A_{\mathrm{O}}'' = A_{\mathrm{N}}'$$

Suppose that the inner core contains a mass fraction x of the total core, then after loss of the H-rich envelope and total mixing the ratio R_X of predicted to normal (original) abundance is:

$$R_{\mathrm{N}} = (1-x)\frac{A_{\mathrm{N}}'}{A_{\mathrm{N}}} = (1-x)\left(1+\frac{A_{\mathrm{C}}}{A_{\mathrm{N}}}+z\frac{A_{\mathrm{O}}}{A_{\mathrm{N}}}\right)$$

$$R_{\mathrm{O}} = x\frac{A_{\mathrm{N}}'}{A_{\mathrm{O}}} = x\left(z+\frac{A_{\mathrm{N}}}{A_{\mathrm{O}}}+\frac{A_{\mathrm{C}}}{A_{\mathrm{O}}}\right).$$

Taking $A_C = 3 \cdot 5 \times 10^8$, $A_N = 8 \cdot 5 \times 10^7$ and $A_O = 5 \cdot 9 \times 10^8$ (Lambert 1967) we have

$$R_N = (1-x)(6 \cdot 9z + 5 \cdot 1)$$

$$R_O = x(z + 0 \cdot 74).$$

These represent the abundance ratios (observed : normal) predicted from the model. R_C will depend on how much C^{12} is produced before the mixing takes place.

Iben (1967a) gives $z \simeq 0 \cdot 67$, $0 \cdot 80$ and $0 \cdot 90$ respectively for 1, 1·25 and 1·5 $M_\odot$ stars. The fact that x and z may vary from one star to another and the uncertainties of the abundance analyses preclude a complete test of the above equations. We note though that statistically we would expect a moderate overabundance of N correlated with a moderate deficiency of O, and this is probably a fair description of the observational results. Two important points to note are that nearly all of the O in the Hd C stars is predicted to be in the form of O^{18}; and it may be possible to find hydrogen-deficient stars for which $R_C < 1$. The latter conclusion depends on whether it is possible for stars to lose their envelopes before much C^{12} has been produced. (BD + 13° 3224, with strong HeI and NII, but weak HI and OII but no CII (Berger & Greenstein 1963) may be such a star). A search for O^{18} should be made in the infrared bands of CO; freedom from C^{13} will aid this study.

Finally Danziger (1965) finds an overabundance of Li by at least a factor of 60 in RY Sgr, and Keenan & Greenstein (1963) show that the Li 6708A line is quite strong in R Cr B. The initial Li content of the Hd C stars would be destroyed during the H-burning phase. However, any Li currently present in the outer layers of the Hd C stars would have a long life time. To account for its presence we must either postulate the existence of magnetic fields (producing Li by spallation processes) or that Li can be made during He-burning. The first suggestion seems unlikely because the loss of envelope should carry any magnetic field away with it. However, Babcock (1958) finds that R Cr B may have a magnetic field, in which case it must be possible for part of the original field to be retained or a new field established. Li production during He-burning should in any case be investigated.

12. *The evolution of the Hd C stars.* Because of their intrinsic brightness and characteristic light curves a large proportion of the R Cr B variables in the Galaxy must already have been found. A figure of about 100 such stars is consistent with the fact that only one R Cr B variable is known in the Magellanic Clouds. However, the selection effects discussed in Section 10 indicate that the total population of Hd C stars in the Galaxy is much higher, possibly ~1 000. Even so, if all stars eventually pass through the Hd C stage, the lifetime of this phase must be very short. The central stars of planetary nebulae are distributed in the Galaxy very similarly to Hd C stars, and have a total population $\sim 5 \times 10^4$ (Minkowski 1965). As the lifetime of the planetary nebula stage is $\sim 5 \times 10^4$ years (Seaton 1966) we find that the Hd C phase may last for only $\sim 10^3$ years. If this is really the case then significant evolutionary changes should be observable over a time of only a century.

In the scheme of nuclear synthesis outlined in the previous section the Hd C star, initially of $\sim 1\ M_\odot$, loses a substantial fraction of its mass and collapses. This would carry it from the righthand side of the HR diagram to the high temperature side on a short time scale. We tentatively identify the 10^3 year ' observed ' lifetime as the time scale of this collapse. Following mixing and collapse the star will terminate on a low-mass helium main sequence only if its mass is greater than 0·31 $M_{\odot}$ (Cox & Salpeter 1964). As far as Iben (1967a) has followed the evolution of a 1 $M_{\odot}$ star the core had reached 0·20 $M_{\odot}$. Subsequent stages are unlikely to substantially increase the size of this, so it appears probable that none of the Hd C stars will be massive enough to initiate He-burning. In which case they will collapse directly into a white dwarf configuration.

There are some difficulties with the proposed evolutionary scheme. For instance, it will have to be demonstrated that complete mixing can take place on a time scale short compared with 10^3 years. This may be possible because a sound wave could travel from the interior to the surface of a supergiant Hd C star in less than 1 year. Again, the 10^3 year time scale itself is too long to interpret as a pure collapse phenomenon. Clearly a detailed analysis of evolution following loss of H-envelope is required. There are however some attractions to the scheme. Studies of the evolution from the main sequence of pure helium stars (Cox & Salpeter 1964, L'Ecuyer 1966) do not produce supergiant stars in the cool part of the HR diagram. Our suggested evolutionary history shows how helium stars will appear near the red end of the normal giant branch. If we assume that the helium stars and R Cr B stars are equally luminous (Section 10) then the former have only one-tenth of the size of the latter. (If we assume constant radius then the helium stars would be brighter bolometrically than the R Cr B stars by 5 magnitudes, this is certainly not the case). This does not of course determine the direction of evolution.

During evolution the Hd C stars may pass through the Cepheid instability zone. RY Sgr is described by Campbell & Jacchia (1946) as being, in addition to an R Cr B star, a Cepheid variable. The light variation is semi-regular with an amplitude of half a magnitude in a period of 39 days. The amplitude is considerably less than that for a normal Cepheid of the same period, which may be a result of the absence of the hydrogen convection zone. A further result of the absence of hydrogen would be a much smaller phase lag between the light and radial velocity curves. No radial velocity curve has been published for RY Sgr. In view of the suggested rapid evolution of the Hd C stars, it would be advantageous to look for secular changes in the period of RY Sgr. Danziger (1965) reports a light variation with a period of one month, which is significantly less than the 39 day period mentioned above.

13. *The R Cr B phenomenon.* We will briefly comment on some aspects of the variability of some Hd C stars.

O'Keefe (1939) suggested that condensed particles of carbon could cause sufficient obscuration to explain the light curves of the R Cr B variables. This has been interpreted in various ways by different authors. For instance, Herbig (1949) assumed that the carbon cloud condensed at some distance from the star, whereas Payne-Gaposchkin (1963) suggested that obscuration must occur in the photosphere itself in order that the emission line spectrum observed at minimum was not itself obscured. If the R Cr B stars have extensive chromospheres then the simplest explanation of the obscuration is of prominence-like ejection of matter, followed by condensation of carbon and consequent absorption. As the cloud expands it would eventually obscure the star leaving the chromospheric spectrum almost unchanged. Subsequently the emission spectrum itself would be obscured—as is observed (Payne-Gaposchkin 1963)—and final dissipation of the cloud would result in return to normality. If there is some prefered axis along which ejection takes place, due to rotation or magnetic field, then only a small proportion of the Hd C stars would be variable, as already suggested in Sections 10 and 12. On the other hand, variability may be a rare event in most Hd C stars; MV Sgr has undergone only one diminution of light in the past 50 years.

Schmidt (1965) has calculated that the R Cr B stars are losing mass at a very high rate. His work is based on the assumption of a spherical cloud of carbon particles surrounding the star. It is clear that if prominence-like action is the actual cause of variability then mass-loss will be many orders of magnitude less than that given by Schmidt.

Some support for carbon obscuration at some distance from the surface of the star can be obtained from Oke's spectral scans of R Cr B (Payne-Gaposchkin 1963). Hoyle & Wickramasinghe (1962) have suggested that interstellar graphite grains originate in the atmospheres of cool carbon stars, and that the variation of opacity with wavelength for grains in the atmospheres of these stars will follow the same law as that for interstellar reddening. Oke's scans show that the differential reddening (5870–3570 Å) between maximum and minimum light of R Cr B is about 1 magnitude. The interstellar reddening law suggests that graphite flakes would produce about 0·7 magnitude over the same wavelength limits. On the other hand, if the obscuration were occuring at some depth in the photosphere the reddening would be very much greater, because unit optical depth in the ultraviolet would correspond to higher, cooler regions of the photosphere than unit optical depth in the visible, and the ultraviolet flux is extremely sensitive to temperature.

The possible association of graphite flakes and magnetic fields in R Cr B makes it an attractive proposition to look for polarization of the light during a fall in brightness.

14. *Normal carbon stars.* Ordinary R stars have hydrogen lines of similar strength to those in normal G and K giants of the same temperature. The N stars are more difficult to compare with M stars because of molecular absorption, but no obvious anomalies in the hydrogen absorption lines have been noted. Both groups have much lower space velocities than the Hd C stars. Many carbon stars have high C^{13} abundances, and show signs of anomalous abundances among the heavy elements. Despite the large number of normal carbon stars, none are known to exhibit R Cr B light variations. R and N stars appear to be giants rather than supergiants.

We conclude that the Hd C stars and the normal R and N stars are two quite distinct types of star. In this case the answer to Vardya's (1966) question 'Are all carbon stars helium rich?' which was based on a brief discussion of the Hd C stars alone, is a negation. There is as yet no indication of a well-defined class of Population II carbon stars, which contrasts with the relationship between CH and S stars mentioned in Section 1.1. On the other hand, there is no evidence that the Hd C phase occurs in any other than low mass stars.

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