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The identification of directed technical change revisited — Source link

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The Identification of Directed Technical Change Revisited

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Abstract

Technical change that augments capital and labor input in a non-neutral way plays an important role in explaining the relation between growth and other macroeconomic outcomes. Previous research has shown that restricting technical change to be neutral leads to overestimating the elasticity of substitution between capital and labor. I extend this line of analysis to misspecification of the functional form. Evidence from Monte Carlo simulations shows that the problem of biased estimates of the direction of technical change is relevant in the estimation of aggregate CES and translog production functions. In particular, I find examples where true technical change is neutral and estimated technical change is strongly directed towards one factor.

Keywords: directed technical change, estimation of production functions, CES functions, translog functions.

JEL Classification Numbers: C15, O30, O47.

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1 Introduction

Directed technical change means that not all inputs and sectors are affected by technical change in an equal way. There are many reasons for technical change to be directed. They have to do with properties of innovative technologies, supply of production factors, market imperfections, tastes and incentives. Directed technical change has important implications for the relation of economic growth to various other outcomes such as distribution, employment, and structural change (see, i.a., Acemoglu (2002), Acemoglu and Guerrieri (2008)). A major empirical challenge is how to identify the direction of technical change. In a standard two-factor setting this means to identify whether technical change is more capital-augmenting or labor-augmenting.

Ideally, we would identify technical change directly in terms of innovation output or technology input. While there is work on aggregate R&D and on intangible assets in a broader sense, the measures produced to date cannot claim to be proxies for total technical change at a macroeconomic level. Residual and indirect measures of technical change like multifactor productivity or estimated functions of time continue to play an important role. They are considered in this paper.

The CES (constant-elasticity-of-substitution) production function with two factors of production has been an important workhorse for analyzing directed technical change in both theoretical and empirical research. Klump et al. (2007) and León-Ledesma, McAdam and Willman (2010 and forthcoming) made major contributions laying out in a coherent way the nonlinear systems estimation of normalized CES functions, providing evidence on the elasticity of substitution and directed technical change in the U.S., and analyzing the numerical and econometric properties of nonlinear systems estimation in Monte Carlo simulations. In their simulation work, which produced a number of critical insights, the true data generating process is assumed to be CES.

Building on this work, I go one step further and assume that the true data generating process is not CES. At the macroeconomic level, which mixes many influences such as firm technologies, reallocation and institutions, it can be particularly difficult to impose a functional form on aggregate technology. While this paper does not answer the question how we should do it, I want to give some examples of the kind of deviations from CES or translog functions (another functional form frequently used in empirical research) we may have to account for when we want to measure the direction of technical change at the macroeconomic level.

In a first approach I employ a more general production function, the generalized linear homogenous Box-Cox function, which encompasses both the linear homogenous translog and the linear homogenous CES production function as special cases. The functions are normalized around sample averages. In a second approach I use a Cobb-Douglas style production function with time-varying exponents and factor-augmenting progress. While previous papers were interested in the elasticity of substitution between capital and labor, I focus on the drivers of growth to be explained by a production function: the contributions of capital deepening, neutral technical change and directed technical change to labor productivity growth.

In the Monte Carlo simulations I estimate both CES and translog functions using a normalized systems approach. The simulations with the data generated by Box-Cox functions exhibit little systematic deviation in the contributions to growth that would arise from misspecification. I conclude that normalization around averages makes the production functions too close to introduce severe misspecifications.

Monte Carlo simulations that generate data out of a Cobb-Douglas function with varying capital shares as exponents exhibit much stronger problems from misspecification. When true technical change is directed towards capital or labor, estimated contributions to growth from the non-neutral part of technical change look low. On the other hand, when true technical change is purely neutral, the estimated contribution of directed technical change turns out to be high.

Section 2 briefly summarizes the literature on macroeconomic production functions and directed technical change to which this paper contributes. Section 3 explains how CES and translog production functions are used to identify directed technical change and contributions to growth. Novel aspects in this are the identification of factor-augmenting technical change with the translog production function and the explicit comparison of contributions to growth from both functional forms in a growth-accounting style. Section 4 describes the parameters and data generating processes used in the Monte Carlo simulations and section 5 reports their results. Section 6 concludes.

2 Background

The functional form that is best suited to the theoretical modeling of factor-augmenting technical change is the production function with a constant elasticity of substitution and constant returns to scale. David and van de Klundert (1965) were the first to introduce directed technical change into an econometric estimation of this function. With better understanding of the formal properties of the CES function achieved, i.a., by the contributions by de La Grandville (1989), Klump and de La Grandville (2000) and Klump and Saam (2008) on normalization, it was possible to more systematically reveal the properties of the nonlinear CES estimation with directed technical change. Using Monte Carlo simulations, León-Ledesma, McAdam and Willman (2010 and forthcoming) show that the normalized systems approach is superior to other approaches in identifying the highly nonlinear CES function. Moreover, they find that restricting technical change to be neutral may severely bias the estimate of the elasticity of substitution between capital and labor towards unity. The systems estimation of CES production functions with directed technical change has been recently applied, i.a., by Herrendorf et al. (forthcoming) and Baccianti (2013).

The standard approach of identifying technical change in a macroeconomic production function consists in modeling it as a function of time. A different but conceptually related approach is growth accounting (Hulten (2001)). In both approaches technical change is not directly measured. Contrary to econometric estimation, standard growth accounting does not allow for the identification of non-neutral technical change. Attempts to directly measure technical change build on R&D data or more broadly on reducing residual multifactor productivity (MFP) by accounting for intangible assets, or on measuring embodied technical change through investment price decline (see, e.g., Oulton (2012)). None of these measures can, however, be considered to date as a proxy for all productivity growth not driven by factor accumulation at the aggregate level. On the other hand, MFP growth and measures based on time trends lack the connection to their drivers and thus remain "measures of ignorance" (Abramovitz (1956)). If time trends permit the identification of the direction of technical change, this allows at least to describe which inputs are augmented by technical change. Also the plausibility of the result can be checked against micro-level or anecdotal evidence. The purpose of the present paper is to gain further insight on the identification of the direction of technical change in estimations of aggregate production functions.

With regard to identification problems, Diamond et al. (1978) already noted that "measurements of the direction of technical change and the elasticity of substitution (...) have employed untested restrictions on the nature of production possibilities" (p.125). The authors analytically discuss under which conditions technical change is or is not identified by a given sequence of empirical observations. One way of identifying the direction of technical change is obviously to impose a particular functional form. Some empirical studies, like the early contribution by Appelbaum (1979), on which the Box-Cox function used in this paper is based, use econometric testing to discriminate between general and restricted functional forms. But in applied macroeconomic research a common practice which hypotheses to test and how to assess the power of the test under typical circumstances has not been established. Saltari and Federici (2013) analyze misspecification of the parameters of a CES function arising from the fact that one wants to estimate parameters that are valid in the long-run without taking this time horizon into account in the econometric specification. I take a different perspective looking at misspecification of the functional form.

3 Identification of Directed Technical Change

3.1 Normalized Neoclassical Production Functions

While it is by now well-known how to estimate directed technical change and the elasticity of substitution using a CES function, much less attention has been paid to the two-factor translog function in this context. The constant returns translog function can be interpreted as a second-order approximation of a CES function (Kmenta (1967)). The translog function has the advantage for econometrics of being linear in variables. On the other hand it does not exhibit globally positive and diminishing returns. For this reason it is much less used in theory (see, however, Ostbye (2010)). An early contribution by May and Denny (1979) estimates a translog production function with non-neutral rates of factor-augmenting progress, but this approach has not become standard in empirical research on economic growth (Somewhat more common is the identification of directed technical change from translog production or cost functions in energy economics.). León-Ledesma et al. (2010) note that the estimated coefficient on which the identification of directed technical change depends is close to zero in a normalized function. While this may represent a problem, it does not preclude econometric identification a priori.

As previous contributions on CES functions, I am interested in factor-augmenting progress of the multiplicative form. Macroeconomic identification of directed technical change will in many cases require some assumptions that are based on definitions or theory rather than on econometric testing. Defining technical progress to be of the multiplicative factor-augmenting form is one of them. The general form of a technology for producing output Y as a function of capital (K) and labor (N) with progress functions Γ_K and Γ_N depending on time t is then:

$$Y = F(\Gamma_K(t)K, \Gamma_N(t)N). \tag{1}$$

Moreover I assume constant returns to scale and I normalize the production function. The procedure of normalizing CES and translog functions builds on the work by Kmenta (1967), Klump and de La Grandville (2000) and León-Ledesma et al. (2010). I will summarize here only the most important properties and intuitions. Most work is related to CES functions, though a normalization is implicit in the very definition of the translog function, too.

Normalization allows to address three challenges related to the highly nonlinear CES functions. First and most importantly, it allows to isolate the effect of the substitution parameter. CES and translog functions can be constructed as functions that are tangent to a Cobb-Douglas function and also tangent to one another. The point of tangency is specified in a two-factor setting by the input and output values in the point of normalization (K_0, N_0, Y_0) and the output elasticity of capital, which equals the factor share of capital under competitive remuneration (π_0). Any point can be chosen as a point of tangency. In consequence, the question on which function we end up if, e.g., a constant elasticity of substitution declines from one to a lower value, does not have a well-determined answer without choosing a particular point of normalization. The point of normalization implicit in the standard CES production function introduced by Arrow et al. (1961) is $Y_0 = K_0 = N_0 = 1$. In most empirical applications, normalization is necessary to rescale this point to the data or vice versa. The choice of the point of normalization is not trivial (see Klump and Saam (2008)). Current practice in empirical work is to use either initial values or minima of data, or geometric averages for inputs and outputs and the arithmetic average of the capital share. Second, normalization allows to calibrate the distribution parameter of the CES and the translog function in an empirically meaningful way. Third, normalization can be used to stabilize the nonlinear estimation of a CES function, by calibrating instead of estimating the distribution parameter at the average of the capital share. This alleviates the joint identification of the distribution and the substitution parameter (León-Ledesma et al. (2010)). In macroeconomic applications, where the explanatory power of the estimated equations is usually high, not much is imposed in calibrating the normalized factor share instead of estimating it.

I will use normalization for both data generation and estimation. Deterministic data are generated around the point of normalization with inputs and output at unity, which is implicit in the standard CES function. Since stochastic components may lead to deviations from this, the data are normalized again for estimation using arithmetic averages for the factor shares and time and geometric averages for input and outputs. The geometric average of output can be expected to correspond to the geometric average of inputs only for a Cobb-Douglas function. For this reason, an additional "normalization constant" (León-Ledesma et al. (2010), p.1341) is added to the CES function. It is expected to be close to one.

3.2 Estimation of Translog Functions

The translog function with factor-augmenting technical progress is obtained as a second-order approximation to the function (1), considering as arguments of the function efficiency units of capital and labor, $\Gamma_K K$ and $\Gamma_N N$. Approximation in logarithms of (1) around the point of normalization (Y_0, K_0, N_0, t_0) yields:

$$\begin{aligned}\ln Y &= \ln Y_0 + a_K \ln\left(\frac{\Gamma_K(t,t_0)K}{K_0}\right) + a_N \ln\left(\frac{\Gamma_N(t,t_0)N}{N_0}\right) + \frac{b_{KK}}{2}\ln\left(\frac{\Gamma_K(t,t_0)K}{K_0}\right)^2 \\ &+ \frac{b_{NN}}{2}\ln\left(\frac{\Gamma_N(t,t_0)N}{N_0}\right)^2 + b_{KN}\ln\left(\frac{\Gamma_K(t,t_0)K}{K_0}\right)\ln\left(\frac{\Gamma_N(t,t_0)N}{N_0}\right). \quad (2)\end{aligned}$$

Under constant returns to scale, the following restrictions are valid (see, e.g., May and Denny (1979)):

$$a_K + a_N = 1 \quad (3)$$

$$b_{KK} + b_{KN} = 0 \quad (4)$$

$$b_{NN} + b_{KN} = 0. \quad (5)$$

The coefficient of logarithmic capital can under competitive remuneration be identified as the capital share at the point of normalization, $a_K = \pi_0$. Moreover, I define $\beta = b_{KK}$. After multiplying out, rearranging and introducing per capita values of variables $y = \frac{Y}{L}$ and $k = \frac{K}{L}$, a formulation analogous to May and Denny (1979), p.763, is obtained:

$$\begin{aligned}\ln y &= \ln y_0 + \pi_0 \ln\left(\frac{k}{k_0}\right) + \pi_0 \ln\Gamma_K(t,t_0) + (1-\pi_0)\ln\Gamma_N(t,t_0) \\ &+ \frac{\beta}{2}\ln\left(\frac{k}{k_0}\right)^2 + \beta\left(\ln\Gamma_K(t,t_0) - \ln\Gamma_N(t,t_0)\right)\ln\left(\frac{k}{k_0}\right) \\ &+ \frac{\beta}{2}\left(\ln\Gamma_K(t,t_0) - \ln\Gamma_N(t,t_0)\right)^2. \quad (6)\end{aligned}$$

I consider how constant factor-augmenting technical change can be identified with this function. Defining

$$\Gamma_K(t,t_0) = e^{\gamma_K(t-t_0)} \quad (7)$$

$$\Gamma_N(t,t_0) = e^{\gamma_N(t-t_0)} \quad (8)$$

and $\gamma_k = \gamma_K - \gamma_N$, one obtains:

$$\begin{aligned}\ln y &= \ln y_0 + \pi_0 \ln\left(\frac{k}{k_0}\right) + (\gamma_N + \pi_0\gamma_k)(t-t_0) \\ &+ \frac{\beta}{2}\ln\left(\frac{k}{k_0}\right)^2 + \beta\gamma_k \ln\left(\frac{k}{k_0}\right)(t-t_0) + \frac{\beta}{2}\gamma_k^2(t-t_0)^2. \quad (9)\end{aligned}$$

In the Appendix I compare this with the more general function without restrictions on the coefficient of the quadratic trend. This function allows for a time-varying neutral component of technical change.

The translog function is estimated with the following system (one equation is omitted because of redundancy):

$$\ln\left(\frac{y}{y_0}\right) = \ln\zeta + \pi_0 \ln\left(\frac{k}{k_0}\right) + \frac{\beta}{2}\ln\left(\frac{k}{k_0}\right)^2 + \kappa(t-t_0)\ln\left(\frac{k}{k_0}\right) + \frac{\theta}{2}(t-t_0)^2 \tag{10}$$

$$\pi = \pi_0 + \beta \ln\left(\frac{k}{k_0}\right) + \kappa(t-t_0) \tag{11}$$

imposing the restriction

$$\theta = \frac{\kappa^2}{\beta}, \tag{12}$$

where ζ is a normalization constant expected to be close to one. Normalized values are set equal to arithmetic averages in the case of the capital share and time and to geometric averages in the case of inputs and output. Because of the restriction the system is estimated using nonlinear seemingly unrelated regression (SUR). The numerical problems of nonlinear estimation are, however, much less severe in this case than with the CES function, because the function is linear except for the parameter restriction. The parameters of the production function are then recovered writing them as in (9).

3.3 Estimation of CES Functions

The CES function is presented more briefly, since I follow the approach introduced by Klump et al. (2007) and León-Ledesma et al. (2010). The normalized CES production function with constant returns to scale and factor-augmenting technical change can be written as

$$Y = Y_0\left[\pi_0\left(\frac{K}{K_0}\Gamma_K(t,t_0)\right)^{\psi} + (1-\pi_0)\left(\frac{N}{N_0}\Gamma_N(t,t_0)\right)^{\psi}\right]^{\frac{1}{\psi}}, \tag{13}$$

with $\sigma = 1/(1-\psi)$ as elasticity of substitution between capital and labor. The factor-augmenting functions Γ_K and Γ_N are defined in the same way as in the previous section.

As discussed in León-Ledesma et al. (2010), the translog function represents a second-order approximation to an arbitrary production function, thus also to a CES function. Since I do not assume the true data generating process to be CES, it is, however, not in the focus of this paper how well the translog approximates the CES function.

Following Klump et al. (2007) and León-Ledesma et al. (2010), I estimate the CES function using nonlinear SUR with the following system:

$$\begin{aligned}
\ln\left(\frac{y}{y_0}\right) &= \ln\zeta + \gamma_N(t-t_0) \\
&+ \frac{\sigma}{\sigma-1}\ln\left(\pi_0\left(\frac{k}{k_0}\gamma_k(t-t_0)\right)^{\frac{\sigma-1}{\sigma}} + 1 - \pi_0\right) \quad (14)\\
\ln\pi &= \ln\pi_0 + \frac{1-\sigma}{\sigma}\ln\left(\frac{yk_0}{ky_0} - \ln\zeta - \gamma_K(t-t_0)\right) \quad (15)\\
\ln(1-\pi) &= \ln(1-\pi_0) + \frac{1-\sigma}{\sigma}\ln\left(\frac{y}{y_0} - \ln\zeta - \gamma_N(t-t_0)\right). \quad (16)
\end{aligned}$$

Normalized values are again set equal to arithmetic averages in the case of the capital share and time and to geometric averages in the case of inputs and output.

3.4 Factor Shares and Contributions to Growth

The elasticity of substitution between capital and labor is a parameter that plays an important role in models of growth and distribution. It is usually not of interest for itself in applied empirical research, but helps explaining empirical phenomena. Rather than reporting the estimated substitution elasticities and rates of technical progress, I put measured technical progress in relation to economic growth. This is done using the production function estimates for a growth accounting decomposition. Before turning to this method, I offer a brief discussion on the factor shares and substitution elasticities generated by translog and CES production functions. Theoretical and empirical contributions on the CES functions have highlighted the crucial role that the interdependence between the elasticity of substitution and the evolution of factor income shares plays for growth and distribution. The following summary gives an overview how both evolve under the CES and under the translog function. For the translog function, the conditions for a positive and diminishing marginal product are also considered in this context.

From the CES function one obtains the following capital share under competitive remuneration:

$$\pi = \frac{\pi_0\left(\Gamma_k(t)\frac{k}{k_0}\right)}{\pi_0\left(\Gamma_k(t)\frac{k}{k_0}\right) + 1 - \pi_0}. \quad (17)$$

As discussed in Klump and de La Grandville (2000) and León-Ledesma et al. (2010) the derivative of the capital share with respect to the (augmented) capital intensity depends on the elasticity of substitution $\sigma = \frac{1}{1-\psi}$:

$$\begin{aligned}
\frac{\partial \pi}{\partial(\Gamma_k(t)k)} &> 0 \qquad \text{if } \psi > 0 \\
\frac{\partial \pi}{\partial(\Gamma_k(t)k)} &= 0 \qquad \text{if } \psi = 0 \\
\frac{\partial \pi}{\partial(\Gamma_k(t)k)} &< 0 \qquad \text{if } \psi < 0.
\end{aligned} \tag{18}$$

Under competitive factor remuneration, the capital income share resulting from the translog function (10) and restriction (12) is

$$\pi = \pi_0 + \beta \ln\left(\frac{k}{k_0}\right) + \kappa(t - t_0) \tag{19}$$

$$= \pi_0 + \beta\left(\ln\left(\frac{k}{k_0}\right)\gamma_k(t - t_0)\right). \tag{20}$$

While the CES function with constant returns to scale has globally the neoclassical properties of positive and diminishing marginal products, the translog function exhibits them only locally. The translog function exhibits a positive marginal product of capital and labor when the capital share lies between 0 and 1:

$$0 < \pi = \pi_0 + \beta \ln\left(\frac{k}{k_0}\right) + \kappa(t - t_0) < 1. \tag{21}$$

Taking the second derivative $\partial y^2/\partial^2 k$ yields the condition for diminishing marginal productivity (see also Ostbye (2010)):

$$\pi(1 - \pi) > \beta. \tag{22}$$

For $\beta \leq 0$, a positive marginal product of both factors of production entails necessarily that it is also diminishing. For $\beta > 0$, this is not the case. The elasticity of substitution between capital and labor of a production function F is defined as

$$\sigma = \frac{d \ln(K/N)}{d \ln(F_N/F_K)}. \tag{23}$$

Along a CES function, it is constant. Along a translog function it corresponds to (see also Ostbye (2010) for the case without technical progress):

$$\sigma = \frac{\pi(1-\pi)e^{\gamma_k(t-t_0)}\frac{k}{k_0}}{\pi(1-\pi)e^{\gamma_k(t-t_0)}\frac{k}{k_0} - \beta}. \tag{24}$$

An important implication is that in the neoclassical region of a translog function, the elasticity of substitution always remains positive and above or below the threshold of one, depending on β. The point of normalization represents the point in which the function approximates a CES function. The elasticity of substitution in this point equals (see also Kmenta (1967)):

$$\sigma_0 = \frac{\pi_0(1-\pi_0)}{\pi_0(1-\pi_0)-\beta}. \tag{25}$$

The relation between capital share and elasticity of substitution is analogous to the relation under a CES function:

$$\begin{aligned} \frac{\partial \pi}{\partial(\Gamma_k(t)k)} &> 0 \qquad \text{if } \beta > 0 \\ \frac{\partial \pi}{\partial(\Gamma_k(t)k)} &= 0 \qquad \text{if } \beta = 0 \\ \frac{\partial \pi}{\partial(\Gamma_k(t)k)} &< 0 \qquad \text{if } \beta < 0. \end{aligned} \tag{26}$$

Now I turn to the application considered in this paper: the contribution of directed technical change to growth. Under constant returns to scale, competitive factor remuneration and in the absence of measurement error, the standard growth accounting decomposition describes the contributions of capital deepening and technical progress (or more generally multifactor productivity - MFP) to labor productivity growth. Assuming constant factor-augmenting progress and using the Törnqvist index (which is exact in the translog case and a good approximation in the CES case, see Diewert (1976)), MFP growth can be rewritten as a weighted average of the progress rates:

$$\Delta \ln y(t) = \bar{\pi}(t)\Delta \ln k(t) + \Delta \ln MFP(t) \tag{27}$$

$$= \bar{\pi}(t)\Delta \ln k(t) + \bar{\pi}(t)\gamma_K + [1-\bar{\pi}(t)]\gamma_N, \tag{28}$$

with $\bar{\pi}(t) = (\pi(t) + \pi(t-1))/2$. I use either the true production function or the estimated production function for this decomposition and then take the average of contributions over all periods. For determining the contribution of directed technical change, I define the lower of both progress rates as the rate of purely neutral technical change (which may even be negative). If capital-augmenting progress is faster, $\gamma_K > \gamma_N$, the contribution of neutral technical progress in each period is γ_N and the contribution of directed capital-augmenting technical change is $\bar{\pi}(t)(\gamma_K - \gamma_N) = \bar{\pi}(t)\gamma_k$. If technical change is directed towards labor, the neutral contribution is γ_K and the non-neutral contribution is $(\bar{\pi}(t) - 1)\gamma_k$. Stochastic disturbances are added to technical progress in the simulations. The main question addressed in the Monte Carlo experiment will be whether CES and translog functions estimate the direction and the order of magnitude of the non-neutral part of technical change correctly if the true data generating process is not CES or translog.

4 Methodology of Monte Carlo Simulations with More General Data Generating Processes

4.1 General Approach

In constructing a data generating process for Monte Carlo simulations that examine the macroeconomic identifiability of the direction of technical change, I am facing a trade-off: On the one hand, the aim is to extend the previous literature towards more general functional misspecification assuming that we might not know nor reliably test whether the true function is CES or not. On the other hand, going this avenue to an extreme might imply that any identification of technical change breaks down if misspecification is just made severe enough. The latter outcome seems a trivial result. An intermediate approach is to choose data generating processes that have some relation to CES or translog functions without being exactly of that form. I consider the data generating processes I introduce to be likely alternatives in a world where CES and translog functions are plausible descriptions of potential paths of growth and income distribution.

A function that encompasses both the CES and the translog function as special cases is the generalized Box-Cox function (Appelbaum (1979)). When constant returns and symmetry are imposed, it can be described intuitively in the following way: It consists of two additive components, one is the CES function, the other is the collection of quadratic terms that yield a translog function if added to a Cobb-Douglas function. The function becomes translog if the first component is really Cobb-Douglas and it becomes CES if the second component is zero.

A second data generating process is based on the observation that most factor income shares generated by my Box-Cox simulations evolve in an approximately linear way over time. Fitting the trend, I use the generated output elasticities as if they were time-varying exponents of a Cobb-Douglas function where directed technical change is again of the factor-augmenting form. Even though exponents vary over time, I do not consider this variation as technological change. Rather everything that is not accounted for by the given rates of factor-augmenting change is attributed to change in inputs.

My primary interest in the second specification is not to analyze the bias in particular parameter estimates that arises, nor even to know whether the data generating processes can be the result of a particular parametric production function with inputs and multiplicative levels of technology as arguments. I just want to know how the contribution of directed technical change is assessed by the estimation of translog and CES functions if the true data generating process is different, but not too different. To my knowledge a similar approach to assessing the performance of translog and CES estimation has not been realized before. The two data generating processes I construct are by no means to be viewed as exhaustive of what could be done along these lines.

The steps of both Monte-Carlo simulations done with the statistical software STATA are the following:

1. Take generated time-series of capital and labor input $(K(t), N(t))$ and technology shocks
2. Generate factor shares
   a) from the first-order conditions of the Box-Cox function
   b) from a linear function of time derived from the Box-Cox simulations
3. Add a stochastic component to factor shares
4. Generate observed output
5. Perform CES and translog regressions
6. Perform growth accounting decompositions using predicted growth, predicted factor shares and estimates of progress rates.

The procedure, especially for the Box-Cox simulations, closely follows León-Ledesma et al. (2010). I leave aside some features they introduce, such as autocorrelation, since I suspect they are not central for the main conclusions of the approach. Additional robustness checks, however, can be easily implemented.

For both simulation settings, I use the same time-series for inputs and technology shocks. The stochastic component in this part of the simulation is kept fix over consecutive Monte Carlo draws. The deterministic part of the function is normalized to $Y_0 = K_0 = N_0 = 1$, which in the absence of stochastic shocks would be reached at t_0. In this way, I control the common point of the production functions.

As in León-Ledesma et al. (2010), logarithmic capital and labor are assumed to follow an I(1) process:

$$\ln N(t) = n + \ln N(t-1) + \epsilon^N(t) \tag{29}$$

$$\ln K(t) = \kappa + \ln K(t-1) + \epsilon^K(t). \tag{30}$$

I assume that $\kappa = 0.02$, $n = 0$, $\ln K(0) = -t_0\kappa$ and $\ln N(0) = 0$. The difference between $\ln K(t)$ and $\ln N(t)$ is then trend-stationary. Factor-augmenting technology is assumed to deviate from its trend by shocks $\epsilon^{\Gamma K}$ and $\epsilon^{\Gamma N}$. The actual levels of technology are thus:

$$\Gamma_K(t, t_0) = e^{\gamma_K(t-t_0)+\epsilon_t^{\Gamma K}} \tag{31}$$

$$\Gamma_N(t, t_0) = e^{\gamma_N(t-t_0)+\epsilon_t^{\Gamma N}}. \tag{32}$$

4.2 The Linear Homogenous Generalized Box-Cox Function

Appelbaum (1979) specifies a function that encompasses both the CES and the translog function as special cases: the generalized Box-Cox transformation. In order to focus on capital-deepening versus directed technical change, I assume that linear homogeneity and the constancy of rates of factor-augmenting technical change are known. Appelbaum's function can be written for two factors of production (efficiency units of capital and labor, $E_K = \Gamma_K K$ and $E_N = \Gamma_N N$) in the following way:

$$\begin{aligned} Y_{det}(\delta) &= \alpha_K E_K(\lambda) + \alpha_N E_N(\lambda) + \frac{1}{2} b_{KK} E_K(\lambda)^2 + \frac{1}{2} b_{KN} E_K(\lambda) E_N(\lambda) \\ &+ \frac{1}{2} b_{NK} E_K(\lambda) E_N(\lambda) + \frac{1}{2} b_{NN} E_N(\lambda)^2, \end{aligned} \tag{33}$$

with the following conditions imposing linear homogeneity

$$\alpha_K + \alpha_N = 1 \tag{34}$$

$$\lambda\alpha_K = b_{KK} + b_{KN} \quad \text{and} \quad \lambda\alpha_N = b_{NN} + b_{NK} \tag{35}$$

$$\lambda = \delta. \tag{36}$$

Symmetry is also assumed, that is

$$b_{KN} = b_{NK}. \tag{37}$$

$E_i(\lambda)$ with $i \in \{K, N\}$ and $Y_{det}(\lambda)$ are the Box-Cox transformation functions defined by

$$E_i(\lambda) = (E_i^{\lambda} - 1)/\lambda \tag{38}$$

$$Y_{det}(\lambda) = (Y^{2\lambda} - 1)/2\lambda. \tag{39}$$

For $\lambda \to 0$ these functions become logarithmic functions. The subscript *det* signifies in this context that output is considered at given inputs and technology, without shocks that temporarily deviate factor payments from equilibrium values (see below).

In the following, I will always work with the symmetric linear homogenous case and write $\alpha = \alpha_K$ and $1 - \alpha = \alpha_N$. The b's are also interdependent now, only one of them remains a free parameter. The symmetric linear homogenous translog is obtained by assuming additionally $\lambda = 0$ and the linear homogenous CES by assuming $\lambda\alpha_K = b_{KK}$ and $\lambda\alpha_N = b_{NN}$.

Using first-order conditions, one obtains the following expression for the capital share determined by inputs and technology:

$$\pi_{det} = \frac{\alpha E_K^{\lambda} + b_{KK} E_K^{\lambda} E_K(\lambda) + b_{KN} E_K^{\lambda} E_N(\lambda)}{\alpha E_K^{\lambda} + (1-\alpha) E_N^{\lambda} + b_{KK} E_K^{\lambda} E_K(\lambda) + b_{KN} E_K^{\lambda} E_N(\lambda) + b_{NN} E_N^{\lambda} E_N(\lambda) + b_{NK} E_N^{\lambda} E_K(\lambda)}. \tag{40}$$

With linear homogeneity and symmetry, the Box-Cox function (33) depends on three parameters, α, λ and b_{KK}, and additionally on the two rates of technical progress γ_K and γ_N.

As León-Ledesma et al. (2010) I assume the distribution parameter α to be 0.4. The values of λ are generated from commonly considered cases of the CES function. The elasticity of substitution of the CES function is $\sigma = 1/(1-\psi)$ with $\psi = 2\lambda$ if we write the CES as a special case of the Box-Cox function (33). For the CES function I use the values $\lambda \in \{-0.25, -0.1, -0.05, -0.02, 0.02, 0.05\}$, which correspond to the elasticities of substitution $\sigma \in \{0.44, 0.67, 0.8, 0.91, 1.11, 1.3\}$. León-Ledesma et al. (2010) use the values $\{0.2, 0.5, 0.9, 1.3\}$. I discard very low values since they lead to extreme declines in the capital share. Since $b_{KK} = \lambda\alpha$ for the CES function, the CES substitution parameters considered imply $b_{KK} \in \{-0.625, -0.25, -0.125, -0.05, 0.05, 0.125\}$. We know that the translog function, on the other hand, is obtained by $\lambda = 0$. To generate more variants of the Box-Cox, I take each pair (b_{KK}, λ) generated by the CES calibration and consider the variants $(b_{KK}, \omega\lambda)$ with $\omega \in \{-0.7, 0, 0.5, 1, 1.7\}$. This includes the CES function ($\omega = 1$), the translog function ($\omega = 0$), one case in between, and two more extreme cases in terms of λ for a given b_{KK}. Note, however, that this procedure does not match the CES with the translog function that represents its second-order approximation.

Writing the first-order conditions from (33) as:

$$w_{det} = \frac{\partial Y_{det}}{\partial N} = (1 - \pi_{det})\frac{Y_{det}}{N} \tag{41}$$

$$r_{det} = \frac{\partial Y_{det}}{\partial K} = \pi_{det}\frac{Y_{det}}{K} \tag{42}$$

I compute the deterministic (or equilibrium) part of real wages and real interest rates from equations (33) and (40). As León-Ledesma et al. (2010), I add a disturbance representing shocks to equilibrium factor payments:

$$w = \frac{\partial Y_{det}}{\partial N}(1 + \epsilon^w) \tag{43}$$

$$r = \frac{\partial Y_{det}}{\partial K}(1 + \epsilon^r) \tag{44}$$

where $\epsilon^w \sim N(0, 0.05\bar{w}_{det})$ and $\epsilon^r \sim N(0, 0.05\bar{r}_{det})$. "Observed" output corresponding to GDP in an aggregate economy is then obtained for each period as (time was omitted up to here)

$$Y(t) = r(t)K(t) + w(t)N(t). \tag{45}$$

As discussed in León-Ledesma et al. (2010), Y/Y_{det} is stationary. Table 4.1 summarizes the parameters used for calibrating the Box-Cox simulations.

Table 4.1: Parameters of Box-Cox Simulations

	Parameter	Values
π_0	Distribution parameter	0.4
γ_K	Deterministic growth rate of capital-augmenting progress*	0.005, 0.01, 0.015
γ_N	Deterministic growth rate of labor-augmenting progress*	0.015, 0.01, 0.005
σ	Substitution elasticity of CES cases	$0.44, 0.67, 0.8, 0.91, 1.11, 1.3$
λ_{CES}	CES substitution parameter in Box-Cox	$-0.25, -0.1, -0.05, -0.02, 0.02, 0.05$
b_{KK}	Box-Cox parameter dependent on π_0 and λ_{CES}	$-0.625, -0.25, -0.125, -0.05, 0.05, 0.125$
ω	Parameter to generate non-CES cases	$-0.7, 0, 0.5, 1, 1.7$
κ	Trend in capital growth	0.02
n	Trend in labor growth	0
$se(\epsilon_t^K)$	Standard deviation of shock to capital	0.02
$se(\epsilon_t^N)$	Standard deviation of shock to labor	0.02
$se(\epsilon_t^r)$	Standard deviation of interest rate shock	$0.05\bar{r}_{det}$
$se(\epsilon_t^w)$	Standard deviation of wage shock	$0.05\bar{w}_{det}$
$se(\epsilon_t^{\Gamma K})$	Standard deviation of technology shock to Γ_K	0.0005
$se(\epsilon_t^{\Gamma N})$	Standard deviation of technology shock to Γ_N	0.0005
T	Sample size (annual)	100
M	Monte Carlo draws	400

*Note: As León-Ledesma et al. (2010) I assume $\gamma_K + \gamma_N = 0.02$.

4.3 A Technology with Time-Linear Factor Shares

To motivate the specification introduced in the section I take a short preview on the results: The general Box-Cox function makes little difference to CES and translog functions as special cases in my simulations. One reason for this may be the normalization. The functions all have a common point around the variable averages, so there may be too little variation in total. Different normalizations are investigated in ongoing research. But even if this observation vanished then, it seems desirable to have more general data generating processes. We cannot expect that the aggregate technology of a country is usefully depicted by a particular functional form. We can, however, impose some a priori restrictions that enable us to make sense of the relation between aggregate output and inputs. Already the aggregation of these variables themselves is based on particular assumptions. With regard to technology, some assumptions may be tested, but I suspect that it will not be possible to test all assumptions at the aggregate level in a meaningful way.

At the aggregate level, I want to generate data that do not follow any standard functional form, but are not too far away from such a functional form. The data generating processes here are just examples of what is possible along these lines. I take the capital income shares generated by each Box-Cox parametrization. Visual inspection revealed that their behavior over time in the interval considered is almost always close to linear. I estimate a linear trend from

$$\pi(\pi_0, \lambda, b_{kk}, shocks) = a + b_\pi t + u_t. \tag{46}$$

Then I take the observed Box-Cox share from period 1 (which is, with minor exceptions resulting from error terms, either the minimum or the maximum) and generate the deterministic part of the capital share from the new "technology", adding a normally distributed disturbance with standard deviation 0.01:

$$\pi^{lin}(t) = \pi(0) + b_\pi t + \epsilon_t^\pi. \tag{47}$$

The input values and the rates of factor-augmenting technological progress are the same as assumed in the Box-Cox simulations.

I assume that output is a Cobb-Douglas aggregate of inputs in efficiency units:

$$Y(t) = E_K(t)^{\pi^{lin}(t)} E_N(t)^{1-\pi^{lin}(t)}. \tag{48}$$

The exponents of the Cobb-Douglas aggregate are the factor shares and they are time-varying. This variation, however, is not counted as technological progress, only the assumed rates of factor-augmenting progress are. The variation of capital shares could also be generated by the variation of inputs according to an assignment we do not know.

A difference between this specification and the previous Box-Cox specification is that I do not control to which extent factor prices deviate from the marginal product of inputs. There may be alternative ways to explicitly model such deviations through more complex shocks or market imperfections. Future research could analyze more in detail the nature of the misspecification of marginal productivity and the resulting estimation bias.

5 Results

5.1 Simulations with Box-Cox Technology

I report results of Monte Carlo simulations with three different constellations of true technical change: directed more towards labor (Tables 5.1 and 5.2), purely neutral (Table 5.3) and directed more towards capital (Table 5.4). Table 5.1 summarizes median contributions to labor productivity growth when the true function is Box-Cox with technical change directed more towards labor ($\gamma_K = 0.005$, $\gamma_N = 0.015$) and estimation is carried out with translog and CES functions. I take this specification of technical progress rates as a baseline case, since both theoretical and empirical research find labor-augmenting progress to be most relevant in the long run (Acemoglu (2007); Klump et al. (2007)), while evidence also points to the presence of a capital-augmenting component in different cases (Klump et al. (2008)). Median rather than mean contributions are reported, since the nonlinear CES estimation is prone to outliers. Contributions to growth are reported as shares of total labor productivity growth falling to capital accumulation, neutral technical change and directed technical change summing up to 100 percent. Contributions are averaged over 100 periods of observation.

The first result that springs to the eyes is that the contribution of capital ranging from around 41 to 44 percent seems to be tied down by the capital share (which is on average about 40 percent), in the same way it is in growth accounting. Note however that the growth accounting contribution of capital is in general only proportional to the capital share, not equal. The near equality here has to do with the fact that average capital growth (2 percent) and average output growth (1.9 percent for all specifications) are nearly equal. I calibrate the capital share rather than estimating it. But even with an estimated distribution parameter the estimated capital share can be expected to lie close to the actual capital share if the error term is small, which is the case in many time-series applications. So in the macroeconomic context, the information gained on the contribution of capital to growth in a translog or CES systems estimation is essentially the same as the information gained from growth accounting, unless one allows for market imperfection (e.g., by introducing a mark-up into the estimation as in Klump et al. (2007)). Residual MFP growth is thus also very similar. The additional insight that estimation offers is then about the direction of technical change.

In interpreting the results on technical change in Table 5.1, first remember that $\lambda = 0$ represents the translog case and the CES case is found always two lines further down, with λ also printed in italics. The translog function estimates the true contribution of neutral and directed technical change quite well. The fit does not depend in any visible way on how close the Box-Cox variant is to the translog in terms of the parameter λ. The CES overestimates the contribution of neutral technical change and underestimates the contribution of directed technical change. Looking at the estimated rates of technical progress (not reported here) documents that their bias towards neutrality is higher with CES (up to 0.005) than with translog (around 0.001). The result seems driven more by numerical problems with the highly nonlinear CES estimation than by misspecification. It is also possible that stronger deviations would be visible for shorter periods. Here I only present average growth rates and contributions over 100 periods.

Table 5.2 yields further evidence on the dispersion of estimated contributions, reporting the 10th and the 90th percentile of contributions of directed technical change and the share of estimates that imply technical change to be directed more towards labor (so the correct direction). For the translog function, the dispersion of the contributions increases with the closeness to $b_{KK} = 0$, which is the value corresponding to a Cobb-Douglas function if λ is also 0. This is intuitive since with a Cobb-Douglas function the identification of directed technical change is not possible. Close to $b_{KK} = 0$, the share of correctly estimated directions also declines from nearly 100 percent to around 80 percent. With the CES function, estimated contributions of directed technical change exhibit higher dispersion. The share of correctly estimated directions is still high, markedly lower than 80 percent only for $b_{KK} = -0.02$. In this problematic case, there are also a number of implausible results with contributions exceeding 100 percent. Note that the numbers on the contribution of directed technical change do not indicate the sign of the direction. In those cases with high dispersion, the contributions at the 10th and the 90th percentile may be directed into opposite ways.

Tables 5.3 and 5.4 report results with data generating processes that assume different rates of technical progress. True technical progress is purely neutral ($\gamma_K = \gamma_N = 0.01$) in Table 5.3 and directed more towards capital ($\gamma_K = 0.015$, $\gamma_N = 0.005$) in Table 5.4. I report again the 10th and the 90th percentile of the contribution of directed technical change. In the case of true technical progress being neutral, I additionally report the percentage of results with a non-neutral part below a certain threshold ($|g_k| < 0.0025$), that is, close to neutral. In the case of true technical progress being directed more towards capital, I report the percentage of results with the correct direction ($\gamma_K > \gamma_N$). The other results corresponding to Table 5.1 are not reported for the alternative specifications of technical change, since the contribution of capital remains pinned down by the capital share and the contribution of neutral technical change is residual when the three contributions sum up to 100 percent.

If the true production function exhibits only neutral technical progress (Table 5.3), the translog and CES estimations also tend to produce low estimates of directed technical change, although the estimated contribution exceeds the true contribution (A true non-zero contribution is caused by technology shocks.). The precision of the estimates is again worse close to $b_{KK} = 0$.

If the true production function exhibits technical change that is directed more towards capital (Table 5.4), the direction of technical change is quite accurately estimated, except for the lowest value of b_{KK}. The direction is determined with less error by the CES than by the translog function. Contrary to the first case with true technical progress being more labor-augmenting, the CES function now tends to identify higher contributions of directed technical change than the translog function. Again no systematic influence of misspecification in parameters can be detected, except for the lowest value of b_{KK}. In sum, it seems that the simulated Box-Cox specifications are too close to CES and translog functions to make any systematic effect of misspecification visible.

Table 5.1: Contributions to growth: Box-Cox function, $\gamma_K = 0.005$, $\gamma_N = 0.015$

Parameters		**Contrib. k**			**Neutral Tech. Change**			**Directed Tech. Change**		
b_{KK}	λ	Box-Cox	Translog	CES	Box-Cox	Translog	CES	Box-Cox	Translog	CES
-0.25	0.4375	42.2	42.3	42.3	26.6	29.7	39.8	31.0	28.0	17.9
	0	42.9	42.8	42.9	26.5	30.3	39.1	30.6	26.9	18.1
	-0.3125	43.2	43.2	43.2	26.5	30.3	38.2	30.4	26.5	18.5
	-0.625	43.4	43.4	43.6	26.5	30.3	37.5	30.1	26.2	18.9
	-1.0625	43.7	43.8	43.9	26.4	30.2	36.9	29.9	26.0	19.2
-0.1	0.175	41.8	41.7	41.6	26.6	29.9	45.2	31.6	28.5	13.3
	0	41.9	41.8	41.8	26.6	30.7	43.7	31.5	27.5	14.7
	-0.125	42.0	42.0	41.9	26.6	30.7	42.8	31.4	27.3	15.3
	-0.25	42.1	42.0	42.0	26.6	30.8	42.8	31.3	27.2	15.2
	-0.425	42.2	42.1	42.1	26.6	30.8	41.8	31.2	27.0	16.1
-0.05	0.0875	41.6	41.5	41.3	26.6	30.8	46.1	31.8	27.6	12.8
	0	41.6	41.5	41.4	26.6	30.2	47.1	31.7	28.4	11.5
	-0.0625	41.7	41.6	41.4	26.6	31.0	47.9	31.7	27.5	10.6
	-0.125	41.7	41.6	41.4	26.6	31.0	47.2	31.7	27.5	11.4
	-0.2125	41.8	41.6	41.5	26.6	30.6	46.9	31.6	27.7	11.5
-0.02	0.035	41.4	41.4	41.3	26.7	29.7	25.8	31.9	28.9	33.0
	0	41.4	41.3	41.2	26.7	28.7	30.4	31.9	30.1	28.3
	-0.025	41.4	41.3	41.2	26.7	29.1	36.0	31.9	29.5	22.7
	-0.05	41.5	41.3	41.2	26.7	29.8	35.5	31.9	29.0	23.1
	-0.085	41.5	41.4	41.2	26.7	31.3	41.6	31.9	27.2	17.2
0.02	-0.035	41.2	41.1	41.3	26.7	29.0	28.0	32.1	29.9	30.7
	0	41.2	41.1	41.2	26.7	28.7	28.6	32.1	30.3	30.1
	0.025	41.2	41.1	41.2	26.7	30.5	32.0	32.1	28.5	26.8
	0.05	41.2	41.1	41.2	26.7	28.4	31.1	32.1	30.6	27.9
	0.085	41.1	41.0	41.2	26.7	30.5	31.2	32.2	28.5	27.7
-0.05	-0.0875	41.0	40.9	41.0	26.7	30.6	34.0	32.3	28.5	24.9
	0	41.0	40.9	41.0	26.7	30.5	33.9	32.3	28.5	25.1
	0.0625	41.0	40.8	40.9	26.7	30.6	35.3	32.3	28.7	23.8
	0.125	40.9	40.8	40.9	26.7	30.2	34.2	32.3	29.0	24.9
	0.2125	40.9	40.98	40.9	26.7	29.9	35.0	32.4	29.3	24.2

Median values of Monte Carlo simulations with 400 replications.
The values for the Box-Cox function are computed with true parameters.

Table 5.2: Contributions of directed technical change: Box-Cox function, $\gamma_K = 0.005$, $\gamma_N = 0.015$

Parameters		**True Technology** Percentiles		**Translog Estimation** Percentiles			**CES Estimation** Percentiles		
b_{KK}	λ	10th	90th	10th	90th	Share $b_N > b_K$	10th	90th	Share $b_N > b_K$
-0.25	0.4375	30.9	31.1	25.6	30.6	1.0	13.2	21.4	1.0
	0	30.5	30.7	24.7	28.8	1.0	14.2	21.9	1.0
	-0.3125	30.3	30.4	24.7	28.2	1.0	15.1	21.9	1.0
	-0.625	30.0	30.2	24.5	27.6	1.0	15.5	22.0	1.0
	-1.0625	29.8	30.0	26.4	27.4	1.0	16.0	21.6	1.0
-0.1	0.175	31.5	31.7	21.5	33.4	1.0	3.8	24.3	0.940
	0	31.4	31.5	22.1	32.1	1.0	5.3	22.2	0.970
	-0.125	31.3	31.5	23.0	31.4	1.0	6.8	22.6	0.973
	-0.25	31.2	31.4	23.2	31.4	1.0	7.8	22.3	0.998
	-0.425	31.1	31.3	22.9	30.3	1.0	8.7	22.2	0.998
-0.05	0.0875	31.7	31.9	12.8	36.9	0.965	7.3	65.7	0.793
	0	31.6	31.8	16.5	36.7	0.965	9.2	46.1	0.820
	-0.0625	31.6	31.8	17.1	35.2	0.988	10.6	46.1	0.837
	-0.125	31.6	31.8	19.0	34.7	0.995	8.3	42.9	0.845
	-0.2125	31.5	31.7	19.8	34.3	0.995	9.0	40.8	0.858
-0.02	0.035	31.8	32.0	7.3	65.7	0.793	4.5	155.7	0.535
	0	31.8	32.0	9.2	46.1	0.820	6.4	148.3	0.458
	-0.025	31.8	32.0	10.6	46.1	0.837	4.8	130.6	0.470
	-0.05	31.8	32.0	8.3	42.9	0.845	5.7	123.2	0.447
	-0.085	31.8	32.0	9.0	40.8	0.858	4.2	56.9	0.452
0.02	-0.035	32.0	32.2	8.6	52.5	0.783	6.9	50.5	0.790
	0	32.0	32.2	7.3	47.8	0.842	6.9	46.8	0.813
	0.025	32.0	32.2	6.4	44.6	0.822	8.0	43.8	0.830
	0.05	32.0	32.2	9.6	44.0	0.863	6.6	43.3	0.825
	0.085	32.1	32.3	9.4	44.1	0.863	6.5	41.2	0.860
-0.05	-0.0875	32.2	32.4	15.6	37.6	0.970	8.1	36.1	0.923
	0	32.2	32.4	16.7	36.1	0.993	9.4	34.7	0.952
	0.0625	32.2	32.4	17.9	36.5	0.980	9.0	34.2	0.965
	0.125	32.2	32.4	19.7	35.9	0.990	11.8	33.3	0.978
	0.2125	32.3	32.5	20.7	36.9	0.998	21.4	31.9	0.988

Median values of Monte Carlo simulations with 400 replications.
The values for the Box-Cox function are computed with true parameters.

Table 5.3: Direction and contributions of directed technical change:
Box-Cox function $\gamma_K = \gamma_N = 0.01$

Parameters		**True Technology** Percentiles		**Translog Estimation** Percentiles			**CES Estimation** Percentiles		
b_{KK}	λ	10th	90th	10th	90th	Share $\lvert\gamma_k\rvert < 0.0025$	10th	90th	Share $\lvert\gamma_k\rvert < 0.0025$
-0.25	0.4375	2.3	2.3	3.0	8.7	0.518	5.6	13.4	0.108
	0	2.3	2.3	0.2	3.2	0.998	6.1	12.3	0.095
	-0.3125	2.3	2.3	2.3	7.7	0.945	3.7	9.4	0.303
	-0.625	2.3	2.3	3.0	9.4	0.708	3.1	8.1	0.643
	-1.0625	2.2	2.3	2.9	9.8	0.610	2.8	7.1	0.778
-0.1	0.175	2.3	2.3	1.2	11.1	0.540	4.1	16.7	0.175
	0	2.3	2.3	0.8	7.7	0.847	4.9	15.2	0.132
	-0.125	2.3	2.3	0.6	9.2	0.877	4.7	14.6	0.178
	-0.25	2.3	2.3	0.9	8.8	0.892	4.3	13.1	0.180
	-0.425	2.3	2.3	0.8	8.8	0.762	4.2	12.7	0.195
-0.05	0.0875	2.3	2.3	1.4	21.5	0.425	3.6	26.1	0.168
	0	2.3	2.3	1.5	18.7	0.498	3.8	24.9	0.183
	-0.0625	2.3	2.3	0.9	16.4	0.555	3.5	23.0	0.160
	-0.125	2.3	2.3	1.0	15.1	0.643	3.6	20.7	0.185
	-0.2125	2.3	2.3	0.9	13.0	0.673	3.8	20.5	0.165
-0.02	0.035	2.3	2.3	4.2	40.0	0.207	5.6	85.7	0.123
	0	2.3	2.3	2.6	34.3	0.228	5.8	55.8	0.165
	-0.025	2.3	2.3	2.6	35.6	0.267	5.1	50.4	0.863
	-0.05	2.3	2.3	2.0	31.4	0.280	4.6	46.5	0.150
	-0.085	2.3	2.3	2.0	27.0	0.317	4.9	41.9	0.170
0.02	-0.035	2.3	2.4	3.2	39.2	0.175	4.1	45.2	0.192
	0	2.3	2.4	2.8	35.5	0.238	2.8	39.5	0.212
	0.025	2.3	2.4	3.8	31.7	0.235	2.8	34.1	0.250
	0.05	2.3	2.4	3.5	32.6	0.225	2.1	31.3	0.267
	0.085	2.3	2.4	2.8	28.4	0.31	2.6	28.9	0.232
0.05	-0.0875	2.3	2.4	2.0	19.4	0.498	1.5	18.7	0.423
	0	2.3	2.4	1.3	15.9	0.542	1.2	16.0	0.478
	0.0625	2.3	2.4	1.2	16.3	0.555	1.5	16.2	0.520
	0.125	2.3	2.4	1.2	15.5	0.525	1.3	13.6	0.50
	0.2125	2.3	2.4	1.0	13.4	0.607	1.1	12.7	0.555

Median values of Monte Carlo simulations with 400 replications.
The values for the Box-Cox function are computed with true parameters.

Table 5.4: Direction and contributions of directed technical change:
Box-Cox function $\gamma_K = 0.015, \gamma_N = 0.005$

Parameters		**True Technology** Percentiles		**Translog Estimation** Percentiles			**CES Estimation** Percentiles		
b_{KK}	λ	10th	90th	10th	90th	Share $\gamma_K > \gamma_N$	10th	90th	Share $\gamma_K > \gamma_N$
-0.25	0.4375	24.3	24.4	16.7	20.8	1.0	200.0	204.8	0
	0	24.6	24.7	18.4	21.6	1.0	16.2	22.5	1.0
	-0.3125	24.9	25.0	35.1	39.9	0.095	23.5	29.0	1.0
	-0.625	25.1	25.2	36.2	39.7	0.087	25.3	30.1	1.0
	-1.0625	25.2	25.3	36.9	39.4	0.055	25.8	30.1	1.0
-0.1	0.175	24.0	24.1	21.2	28.7	1.0	28.4	40.9	1.0
	0	24.2	24.3	16.0	22.5	0.990	26.9	36.0	1.0
	-0.125	24.3	24.3	13.3	21.0	0.970	25.3	34.0	1.0
	-0.25	24.3	24.4	12.4	19.9	0.937	24.8	32.4	1.0
	-0.425	24.4	24.5	12.2	34.3	0.883	24.5	31.7	1.0
-0.05	0.0875	24.0	24.1	15.6	28.4	0.982	22.5	41.7	1.0
	0	24.0	24.1	12.8	26.7	0.980	22.8	41.1	1.0
	-0.0875	24.1	24.1	12.2	23.5	0.993	22.5	38.3	1.0
	-0.125	24.1	24.2	11.4	22.8	0.998	23.0	37.6	1.0
	-0.2125	24.1	24.2	12.0	22.4	0.990	23.1	37.3	1.0
-0.02	0.035	23.9	24.0	5.0	45.4	0.825	14.0	68.9	0.975
	0	23.9	24.0	6.3	38.9	0.843	18.3	62.3	0.995
	-0.025	23.9	24.0	4.5	37.3	0.870	16.1	59.3	0.993
	-0.05	23.9	24.0	7.6	34.4	0.903	17.1	56.4	0.993
	-0.085	24.0	24.0	7.3	33.8	0.910	18.8	53.1	1.0
0.02	-0.035	23.8	23.9	5.2	43.7	0.797	4.8	56.3	0.870
	0	23.8	23.9	6.9	34.8	0.855	6.1	51.7	0.905
	0.025	23.8	23.9	5.8	35.1	0.873	6.6	48.6	0.917
	0.05	23.8	23.9	8.7	34.7	0.920	4.9	47.0	0.945
	0.085	23.8	23.9	8.0	32.7	0.942	7.4	43.7	0.973
0.05	-0.0875	23.7	23.8	8.2	22.3	0.990	15.0	35.1	1.0
	0	23.7	23.8	12.7	24.6	0.998	16.1	33.5	1.0
	0.0625	23.7	23.8	13.8	26.5	1.0	16.5	34.4	1.0
	0.125	23.7	23.8	15.4	26.3	1.0	16.5	32.4	1.0
	0.2125	23.7	23.8	17.0	26.5	1.0	19.0	32.4	1.0

Median values of Monte Carlo simulations with 400 replications.
The values for the Box-Cox function are computed with true parameters.

5.2 Simulations with Linear Factor Shares

With linear factor shares and output generated as a Cobb-Douglas aggregate using these shares, I again investigate three different constellations of true technical change. In the estimation the capital contribution is pinned down by the factor share in same way as in the Box-Cox simulations. Neutral technical change is largely overestimated and directed technical change underestimated in the case when true technical change is directed towards labor (Table 5.5). Whereas the true contribution of directed technical change lies between 30 and 33 percent, it is estimated with values between 3 and 10 percent by both the CES and the translog function. At these small values, individual estimations are likely to exhibit insignificant estimates. Thus with the present data generating process, technical change looks essentially neutral when estimated with CES and translog functions, although there is an important labor-directed component in the true specification. Looking at the dispersion of the estimates and the sign of the direction in Table 5.6, we see that most translog estimations yield the wrong sign, since estimates with more labor-augmenting technical change make up less than 50 percent, in some cases even 0 percent. The CES function captures the direction better, except for the lowest value of b_{KK}. The dispersion of estimated contributions becomes very large close to $b_{KK} = 0$, in the other cases it is low.

As in the previous section, I report results with alternative rates of technical progress using a more summary tabulation of results. In Table 5.7, true technical progress is assumed to be neutral ($\gamma_K = \gamma_N = 0.01$) and in Table 5.8 it is more capital-augmenting ($\gamma_K = 0.015$, $\gamma_N = 0.005$).

When true technical change is directed more towards labor (Table 5.8), the correct direction ($\gamma_K > \gamma_N$) is rather found by the translog than by the CES function. Though the estimated contributions by directed technical change remain low and in total, technical change would be seen as neutral from the estimations. The dispersion of estimates is lower than when true technical change is more labor-augmenting.

The case in which true technical change is purely neutral (Table 5.7) yields the strongest and most disturbing result of my analysis. Every estimation conducted with CES and translog functions finds non-neutral technical change (As a threshold I choose differences in technical progress of at least 0.0025.). The translog function comes up with a high share of degenerate results with implausibly high technology growth rates. The 10th percentile of the estimated contribution of directed technical change turns out to be stable at around 30 percent. The CES function yields a quite precisely estimated contribution of directed technical change of around 40 to 45 percent. Non-neutral technical change can be found although the true technology is neutral.

While it should be seen as open to discussion whether the data generating process chosen here is typical for the kind of misspecification we should be wary of, the simulations so far lead to somewhat puzzling conclusions: The presence of non-neutral technical change is detected by estimation if true technical change is purely neutral, and vice versa.

Table 5.5: Contributions to growth: linear capital share, $\gamma_K = 0.005$, $\gamma_N = 0.015$

Cap. Share		**Contrib. k**			**Neutral Tech. Change**			**Directed Tech. Change**		
Start	*End*	True	Translog	CES	True	Translog	CES	True	Translog	CES
0.537	0.297	42.6	41.9	42.0	26.6	54.8	54.5	30.9	3.3	3.5
0.561	0.282	42.8	42.1	42.2	26.5	54.6	52.1	30.6	3.3	5.6
0.576	0.277	42.9	42.2	42.3	26.5	54.5	50.7	30.5	3.4	7.0
0.595	0.264	43.3	42.5	42.7	26.5	54.1	49.3	30.2	3.4	8.1
0.617	0.249	43.6	42.8	43.0	26.5	54.4	48.1	30.0	3.4	9.2
0.456	0.358	41.9	41.5	41.5	26.6	54.8	56.5	31.5	3.2	2.0
0.463	0.357	41.8	41.4	41.4	26.6	54.8	56.7	31.5	3.3	1.9
0.470	0.349	41.9	41.4	41.3	26.6	54.7	56.9	31.5	3.4	1.6
0.474	0.345	41.7	41.3	41.3	26.7	55.2	57.4	31.6	3.3	1.3
0.490	0.338	42.4	41.9	41.9	26.6	55.1	57.2	31.0	3.3	0.9
0.426	0.381	41.4	41.1	41.1	26.7	54.8	57.8	31.9	4.2	1.1
0.434	0.378	41.8	41.4	41.4	26.6	54.8	57.4	31.6	3.8	1.2
0.438	0.373	41.9	41.5	41.5	26.6	54.7	57.0	31.5	3.8	1.5
0.437	0.374	41.5	41.2	41.2	26.7	55.2	57.3	31.8	3.7	1.5
0.445	0.370	41.9	41.5	41.5	26.6	55.1	56.8	31.5	3.4	1.7
0.408	0.397	41.2	41.1	41.1	26.7	50.4	46.9	32.1	8.7	12.1
0.412	0.391	41.4	41.4	41.4	26.7	51.5	50.6	32.0	7.4	8.2
0.416	0.392	41.6	41.5	41.5	26.6	51.9	53.0	31.7	6.8	5.6
0.415	0.388	41.4	41.2	41.2	26.7	52.5	54.3	31.9	6.4	4.6
0.417	0.389	41.4	41.5	41.5	26.7	53.0	55.6	31.9	5.9	3.3
0.387	0.409	41.0	40.9	40.9	26.7	50.8	53.3	32.3	8.4	5.8
0.386	0.408	41.1	41.0	41.0	26.7	52.4	55.5	32.2	6.6	3.5
0.387	0.410	41.2	41.1	41.1	26.7	52.4	52.4	32.1	6.6	3.6
0.383	0.411	41.0	40.8	40.8	26.7	53.3	53.3	32.3	5.9	3.0
0.385	0.413	41.3	41.1	41.2	26.7	53.5	53.5	32.0	5.4	2.4
0.372	0.422	41.0	40.9	40.9	26.7	55.3	57.9	32.3	3.8	1.3
0.369	0.423	41.1	41.0	41.0	26.7	55.3	57.5	32.2	3.7	1.5
0.363	0.424	40.8	40.7	40.7	26.7	55.7	57.6	32.4	3.6	1.7
0.359	0.429	40.7	40.6	40.6	26.7	56.0	57.7	32.5	3.4	1.7
0.359	0.432	41.0	40.9	41.0	26.7	55.6	56.9	32.3	3.5	2.2

Median values of Monte Carlo simulations with 400 replications.

Table 5.6: Contributions of directed technical change: linear capital share, $\gamma_K = 0.005$, $\gamma_N = 0.015$

Cap. Share		**True Technology**		**Translog Estimation**			**CES Estimation**		
		Percentiles		Percentiles			Percentiles		
Start	*End*	10th	90th	10th	90th	Share $\gamma_N > \gamma_K$	10th	90th	Share $\gamma_N > \gamma_K$
0.537	0.297	30.8	31.0	2.3	4.2	0	1.5	6.0	0.005
0.561	0.282	30.5	30.7	2.3	4.1	0	3.3	8.3	0
0.576	0.277	30.4	30.6	2.7	4.1	0	4.6	9.8	0
0.595	0.264	30.1	30.3	2.8	4.1	0	5.4	10.7	0
0.617	0.249	29.9	30.1	2.8	4.1	0	6.8	11.7	0
0.456	0.358	31.4	31.6	1.4	5.6	0.043	0.9	2.9	0.980
0.463	0.357	31.4	31.6	1.4	5.4	0.013	0.6	2.8	0.942
0.470	0.349	31.4	31.6	1.5	5.1	0.013	0.3	2.5	0.875
0.474	0.345	31.5	31.7	1.6	4.7	0.015	0.3	2.4	0.813
0.490	0.338	31.5	31.7	1.7	4.8	0.005	0.2	2.1	0.925
0.426	0.381	31.8	32.0	0.9	9.1	0.225	0.2	3.4	0.585
0.434	0.378	31.5	31.7	1.1	8.5	0.195	0.3	3.2	0.690
0.438	0.373	31.4	31.6	0.9	7.1	0.142	0.3	3.1	0.820
0.437	0.374	31.7	31.9	0.9	6.8	0.115	0.3	3.0	0.880
0.445	0.370	31.4	31.6	0.9	7.0	0.105	0.5	3.0	0.925
0.408	0.397	32.0	32.2	1.9	27.5	0.355	1.1	98	0.725
0.412	0.391	31.9	32.1	1.4	27.5	0.403	1.0	99.7	0.748
0.416	0.392	31.7	31.9	1.4	26.0	0.333	0.7	101.7	0.673
0.415	0.388	31.8	32.0	1.6	24.8	0.325	0.8	103.7	0.635
0.417	0.389	31.8	32.0	1.3	24.5	0.285	0.7	103.7	0.582
0.387	0.409	32.2	32.2	1.6	29.2	0.403	0.9	37.9	0.507
0.386	0.408	32.2	32.2	1.1	23.0	0.350	0.9	21.2	0.455
0.387	0.410	32.1	32.1	1.3	26.0	0.385	0.7	23.5	0.517
0.383	0.411	32.3	32.3	0.8	21.7	0.333	0.6	21.5	0.463
0.385	0.413	32.0	32.2	1.3	16.3	0.295	0.4	10.2	0.425
0.372	0.422	32.2	32.4	0.9	7.8	0.185	0.2	3.7	0.642
0.369	0.423	32.2	32.4	0.7	7.2	0.17	0.2	3.4	0.772
0.363	0.424	32.4	32.6	1.1	6.4	0.157	0.2	3.7	0.842
0.359	0.429	32.5	32.7	0.7	6.3	0.130	0.3	3.5	0.918
0.359	0.432	32.3	32.5	0.9	6.2	0.108	0.6	3.6	0.957

Median values of Monte Carlo simulations with 400 replications.

Table 5.7: Direction and contributions of directed technical change: linear capital share $\gamma_K = \gamma_N = 0.01$

Cap. share		**True Technology**		**Translog Estimation**			**CES Estimation**		
		Percentiles		Percentiles			Percentiles		
Start	*End*	10th	90th	10th	90th	Share $\lvert\gamma_k\rvert < 0.0025$	10th	90th	Share $\lvert\gamma_k\rvert < 0.0025$
0.165	0.646	2.3	2.3	33.2	22041	0	42.7	44.1	0
0.157	0.683	2.3	2.3	34	20968	0	43.0	44.3	0
0.152	0.706	2.3	2.3	34.3	21014	0	42.8	44.0	0
0.136	0.733	2.3	2.3	34.6	20560	0	42.5	43.7	0
0.115	0.765	2.2	2.3	34.9	21213	0	41.2	42.3	0
0.313	0.497	2.3	2.3	32.8	11520	0	43.5	44.8	0
0.307	0.512	2.3	2.3	32.9	12532	0	43.5	45.0	0
0.295	0.524	2.3	2.3	33.0	13054	0	43.6	45.1	0
0.287	0.533	2.3	2.3	33.0	13741	0	43.4	45.1	0
0.275	0.556	2.3	2.3	33.4	13862	0	43.9	45.7	0
0.359	0.446	2.3	2.4	32.8	9142	0	42.7	45.6	0
0.353	0.458	2.3	2.3	32.7	9760	0	43.2	45.4	0
0.346	0.465	2.3	2.3	32.7	10205	0	43.3	45.3	0
0.344	0.466	2.3	2.3	32.4	11019	0	43.2	44.8	0
0.337	0.478	2.3	2.3	32.6	11410	0	43.5	45.1	0
0.388	0.417	2.3	2.4	27.0	5257	0	35.3	51.1	0
0.381	0.421	2.3	2.3	27.6	5902	0	37.3	49.5	0
0.381	0.427	2.3	2.3	29.1	5946	0	39.1	49.0	0
0.376	0.427	2.3	2.3	29.4	6467	0	40.4	48.3	0
0.376	0.431	2.3	2.3	29.7	6658	0	41.1	47.7	0
0.417	0.378	2.3	2.4	26.0	5989	0	30.9	50.1	0
0.418	0.376	2.3	2.4	28.0	6480	0	38.5	48.7	0
0.421	0.376	2.3	2.3	27.7	6732	0	37.9	48.2	0
0.423	0.372	2.3	2.4	27.5	7036	0	38.6	47.7	0
0.426	0.372	2.3	2.3	29.3	7307	0	40.8	47.4	0
0.443	0.350	2.3	2.4	31.0	9724	0	42.3	45.1	0
0.449	0.345	2.3	2.4	31.4	10570	0	42.7	44.9	0
0.452	0.336	2.4	2.4	30.9	11491	0	42.3	44.5	0
0.459	0.330	2.4	2.4	31.4	11983	0	42.4	44.4	0
0.466	0.327	2.3	2.4	31.9	12630	0	42.7	44.5	0

Median values of Monte Carlo simulations with 400 replications.

Table 5.8: Direction and contributions of directed technical change:
linear capital share $\gamma_K = 0.015,\ \gamma_N = 0.005$

Cap. share		**True Technology** Percentiles		**Translog Estimation** Percentiles			**CES Estimation** Percentiles		
Start	*End*	10th	90th	10th	90th	Share $\gamma_K > \gamma_N$	10th	90th	Share $\gamma_K > \gamma_N$
0.008	0.771	33.0	33.3	2.8	3.4	1.0	5.5	8.5	0
0.033	0.805	31.1	31.3	3.0	3.6	1.0	2.8	4.2	0
0.043	0.824	30.3	30.5	3.1	3.7	1.0	1.3	3.0	0
0.035	0.849	29.6	29.7	3.2	3.7	1.0	1.1	2.9	0.005
0.013	0.883	29.2	29.4	3.2	3.8	1.0	3.5	5.3	0
0.265	0.539	31.8	32.0	2.5	4.0	1.0	3.0	7.9	1.0
0.257	0.561	31.7	31.9	2.6	4.0	1.0	4.4	9.6	1.0
0.243	0.578	31.5	31.6	2.6	3.9	1.0	5.2	10.4	1.0
0.232	0.592	31.5	31.7	2.7	3.8	1.0	5.7	10.7	1.0
0.215	0.620	30.9	31.1	2.7	3.9	1.0	6.9	11.2	1.0
0.337	0.467	32.0	32.2	1.5	5.0	0.990	0.3	2.4	0.178
0.328	0.483	31.6	31.8	1.7	4.8	0.993	0.2	2.0	0.382
0.319	0.492	31.5	31.7	2.0	4.5	1.0	0.2	2.2	0.640
0.315	0.496	31.8	32.0	2.1	4.4	1.0	0.2	3.1	0.820
0.305	0.511	31.5	31.7	2.3	4.3	1.0	0.4	4.4	0.930
0.379	0.425	32.0	32.2	0.9	8.4	0.843	0.2	3.2	0.325
0.371	0.431	31.9	32.1	0.8	6.8	0.865	0.3	3.4	0.158
0.370	0.437	31.7	31.9	1.1	6.9	0.873	0.3	3.2	0.090
0.364	0.439	31.8	32.0	0.8	6.4	0.892	0.5	3.3	0.058
0.363	0.444	31.8	32.0	1.0	6.0	0.932	0.8	3.3	0.020
0.426	0.370	32.1	32.3	0.8	7.1	0.832	0.2	3.3	0.313
0.428	0.367	32.0	32.2	0.9	6.3	0.905	0.2	3.5	0.137
0.432	0.365	32.0	32.2	0.6	6.2	0.890	0.4	3.7	0.077
0.435	0.360	32.1	32.3	0.9	5.8	0.885	0.6	3.6	0.027
0.440	0.359	31.8	32.1	1.1	5.9	0.837	0.8	3.3	0.043
0.465	0.328	32.2	32.3	1.7	4.7	0.990	0.2	2.6	0.238
0.474	0.320	32.0	32.2	1.8	4.5	0.987	0.2	2.4	0.495
0.480	0.310	32.2	32.4	1.7	4.3	1.0	0.2	2.7	0.650
0.489	0.302	32.2	32.4	2.0	4.2	1.0	0.3	3.6	0.805
0.500	0.296	31.9	32.1	2.1	4.3	1.0	0.4	4.6	0.915

Median values of Monte Carlo simulations with 400 replications.

6 Conclusion

Factor-augmenting technical change at the macroeconomic level has attracted much interest in theoretical and empirical research. Encompassing measures of technical change that can be directly observed seem out of reach. Identifying technical progress as a function of time in estimations of CES production functions has been a widely used method.

Building on previous methodological research by Klump et al. (2007) and León-Ledesma, McAdam and Willman (2010 and forthcoming), I have investigated the properties of systems CES estimations under the assumption of a production function that is in reality not CES. I have conducted Monte Carlo simulation analysis for both CES and translog systems estimation and looked at the results from the angle of a growth accounting decomposition. The first data generating process used is of the generalized Box-Cox form, which incorporates both the CES and the translog production function as special cases. The second data generating process builds on linear capital shares by using them as varying exponents of a Cobb-Douglas production function with factor-augmenting technical change. Sample averages are chosen as the point of normalization.

The contributions to growth obtained from estimating CES and translog functions on various versions of the Box-Cox data show little systematic effect of misspecification. By contrast, the data generated from time-linear capital shares lead to strong differences between the true and the estimated contribution of directed technical change. Interestingly, technical change that is neutral in the true specification always leads to high estimated contributions of directed technical change. Conversely, including a non-neutral component in technical progress in the true specification leads to estimated contributions that are close to neutral. This result may depend on the particular functional form chosen, but it remains a result that questions the current practice of estimating directed technical change. More research is clearly needed to better understand the conditions under which the identification of directed technical change at the aggregate level is plausible and reasonably robust.

Besides in the results, I take an interest in the question what would be useful data generating processes for analyzing misspecification in the estimation of directed technical change at the macroeconomic level. One conclusion from the results obtained in this paper is that a generalized Box-Cox function normalized at averages seems too close to CES and translog functions. Meanwhile the Cobb-Douglas function with factor shares as time-varying exponents may introduce two misspecifications at the same time: misspecification in the functional form and deviation of the expected factor prices from marginal products. The paper presents first insights about the kind of estimation biases that may arise from this.

The research undertaken in this paper should be extended in several directions. Misspecification of the system of equations arising from the deviation of factor prices from marginal products should be investigated more in detail. Also more analysis is needed to elicit which misspecification is relevant and which can be ruled out at a theoretical or empirical level.

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Appendix

7 Translog Function with Time-Varying Technical Progress

One way to extend this work might be to consider production functions with time-varying rates of technical progress as, e.g., in the work on CES functions by Klump et al. (2007). To my knowledge, the identification of time-varying technical progress with the standard translog function has never been discussed. This can be done approximating the following function:

$$Y = F(K, L, e^t). \tag{49}$$

In order to obtain the previous translog specification (6), technical progress both in the original function (1) and the second-order approximation was restricted to a the multiplicative factor-augmenting form. Function (49) depends on K, L and e^t in an unrestricted way. But when taking the translog approximation and imposing constant returns to scales, any parts of the function that are not represented by linear of quadratic functions of logarithmic capital intensity and time will vanish. The resulting translog function will differ only slightly from (6). The translog approximation to (49) is

$$\begin{aligned}\ln y &= \ln y_0 + a_k \ln\left(\frac{k}{k_0}\right) + a_t(t-t_0) \\ &+ \frac{a_{kk}}{2}\ln\left(\frac{k}{k_0}\right)^2 + a_{kt}\ln\left(\frac{k}{k_0}\right)(t-t_0) + \frac{a_{tt}}{2}(t-t_0)^2.\end{aligned} \tag{50}$$

Introducing the notation from (6) and defining additionally $a_t = \gamma$, $a_{kt} = \kappa$ and $a_{tt} = \theta$, this yields:

$$\ln y = \ln y_0 + \pi_0 \ln\left(\frac{k}{k_0}\right) + \gamma(t-t_0) + \frac{\beta}{2}\ln\left(\frac{k}{k_0}\right)^2 + \kappa \ln\left(\frac{k}{k_0}\right)(t-t_0) + \frac{\theta}{2}(t-t_0)^2. \tag{51}$$

I now consider which restrictions have to be placed on (51) in order to obtain constant factor-augmenting rates of technical change as in (9). In a next step, I ask which kind of more general multiplicative technical change can be derived if the restrictions are not imposed. The translog function with constant factor-augmenting technical change (9) corresponds to the general translog form with technical progress (51) under the following restrictions:

$$\gamma = \gamma_N + \pi_0\gamma_k \tag{52}$$

$$\kappa = \beta\gamma_k \tag{53}$$

$$\theta = \beta\gamma_k^2. \tag{54}$$

In order to identify the function with constant factor-augmenting progress empirically, one has to place a nonlinear restriction on the coefficients κ, θ and β:

$$\kappa^2 = \beta\theta. \tag{55}$$

If the restriction is not imposed, the translog function could allow the estimation of both a constant and a time-varying component of technical progress. One could either compute the constant rates γ_N and γ_k from γ and θ and the varying part from $\kappa - \beta\gamma_k$ or compute the constant rates from γ and κ and the varying part from $\theta - \beta\gamma_k^2$. From (55) we see that a real solution for θ is unique given κ, but the converse is not true. So we assume that for the general translog function (51), equations (52) and (53) identify constant rates of factor-augmenting changes γ_N and γ_k (or equivalently γ_K and γ_N). The last term of the following decomposition then represents an additional neutral, but time-varying technology component:

$$\begin{aligned}\ln y &= \ln y_0 + \pi_0 \ln\left(\frac{k}{k_0}\right) + \gamma(t-t_0) + \frac{\beta}{2}\ln\left(\frac{k}{k_0}\right)^2 \\ &+ \kappa \ln\left(\frac{k}{k_0}\right)(t-t_0) + \frac{\beta\gamma_k^2}{2}(t-t_0)^2 + \frac{\theta-\beta\gamma_k^2}{2}(t-t_0)^2 .\end{aligned} \tag{56}$$

One can note that the time-varying progress rate is non-monotonous. The factor-augmenting progress functions introduced in (49) now correspond to:

$$\begin{aligned}\Gamma_K(t) &= exp\left\{\frac{\theta-\beta\gamma_k^2}{2}(t-t_0)^2 + \gamma_K(t-t_0)\right\} \\ \Gamma_N(t) &= exp\left\{\frac{\theta-\beta\gamma_k^2}{2}(t-t_0)^2 + \gamma_N(t-t_0)\right\}\end{aligned} \tag{57}$$

with

$$\gamma_k = \frac{\kappa}{\beta} \tag{58}$$

$$\gamma_N = \gamma - \pi_0\frac{\kappa}{\beta} \tag{59}$$

$$\gamma_K = \gamma_k + \gamma_N. \tag{60}$$

The last three equations also apply to the case in which θ is restricted not to contain any time-varying component of technical progress.